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Algorithms, microtonality, performance: eleven musical compositions

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Algorithms, Microtonality, Performance: Eleven Musical Compositions

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by

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Thesis Certification

CERTIFICATION

I, Warren Burt, declare that this thesis, submitted in partial fulfillment of the requirements for the award of Doctor of Philosophy, in the Faculty of Creative Arts, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. The document has not been submitted for qualifications at any other academic institution.

Warren Burt

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Table of Contents

List of Tables

List of Figures

Abstract

Algorithms, Microtonality, Performance: Eleven Musical Compositions,

by Warren Burt

Following a lifetime of creative work and investigation into algorithmic composition and microtonality, I became interested in the speculative mathematical music theory of Ervin Wilson. Encountering his work spurred me on to further investigations in sound and tuning, in a series of compositions using electronic, acoustic, and robotic acoustic instruments. Tuning ideas developed by Wilson and others were extended and expanded into several families of new microtonal musical scales, which were used as the basis for composing a series of algorithmic real-time musical works of extended duration. Some of these works involved collaborative relationships with other musicians, hardware and software instrument designers, and scientists. Wilson's ideas, such as Moments of Symmetry (MOS) scales, Euler-Fokker Genera, limit-ratios, the Scale Tree, and additive sequences and their derivation from number triangles, as well as other tuning ideas, such as permutations of the materials of the ancient Greek modal system were all extended and developed into families of interrelated microtonal scales. The desire to compose works of extended duration was aided by the large size of some of these scale families, which consist of between 60 and 276 new scales each. Acoustic instruments were built or adapted to perform some of these works, including microtonal plucked-string and percussion instruments, and the computer-controlled microtonal instruments of Godfried Willem Raes at the Logos Foundation, in Gent, Belgium. Other works used electronic timbres designed to explore placing of sound in space produced by the interaction of timbre, tuning and room-

acoustics. Software instruments designed to perform the algorithms used were developed in collaboration with John Dunn of Algorithmic Arts, in Fort Worth, Texas. Investigation into the role of timbre and tuning in sonification was carried out with the help of the Wollongong Room Calorimeter project, led by Professor Arthur Jenkins. This thesis discusses how these tunings, algorithms, real-time processes, instruments, and collaborative relationships were used in creating these compositions. Recordings of all compositions discussed are contained in the electronic Appendix, on the attached DVD-Rom. Catalogs of all scales used in the thesis, as well as *Scala* files for all scales, and all data used in composing the pieces is also contained in the Appendix. Additionally, a copy of the commercial CD release of *The Animation of Lists And the Archytan Transpositions*, one of the works discussed in this thesis, is also included.

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Introduction

I love playing with things. Language is a toy, as well as a tool. The eye is a collector of delights. And the ear: timbres, tunings, colours, pitches, rhythms, phrases, found objects, implied patterns, the lot. My musical work, though serious, is all a grand exploration for me. As a composer, I am a musical omnivore. I enjoy composing and making music in an extremely wide variety of forms and genres. And even within the confines of a specific area, I enjoy a diverse approach to my materials. . Unwilling to be hemmed in by any one musical or theoretical approach, I have ranged across a wide span of contemporary ideas, often with a sense of wit and humour. This humour, and this sense of play with materials, frequently expressed in the form of irreverence, is a vital and central part of my work. This might be shown by a quote from my 1996 article "Some parentheses around algorithmic composition" (Burt 1996a).

> In our collaborative 1979 super-8 film *Der Yiddisher Cowboy*, composer and cultural historian Ronald Al Robboy, discussing our collaborative and film-making methods, and alluding to the early film-making techniques of such cinema pioneers as Alan Dwan, says that I am the ideal collaborator on such a project because of my ability to make a piece out 'any thin thread of trashy material'. This scene, in which Robboy is talking directly to the camera in extreme close-up, is immediately followed by a short flickering burst of hand-painted film. On closer examination, or showing in slow motion, the hand-painted film is seen to be a portion of the Fibonacci series, written in my handwriting, one number per frame. (Since the film dealt with many aspects of Eastern European culture and its influence on our lives, this was my loving and irreverent homage to my maternal countryman and teen idol Bartok.)"

This film was a comedy, and Robboy's remark about "any thin thread of trashy material" was meant humorously – he was going for a laugh. However, underneath the laugh was a serious statement – my use of structural bases for my work has encompassed both serious sources, such as chaos equations and recursive sequences, and silly sources, such as gambling charts and the output of damaged consumer electronics. My approach to both these sources has been the same. I treat both in a playful manner, delighting in the arbitrary; and in a very serious manner, seeing where a rigorous following of these sources leads. For me, working with mathematical systems, different tunings, and algorithmic processes, among many other things – that's fun, and that's play. That spirit of playful exploration should be seen as pervading this thesis.

This thesis is a survey of a series of musical works composed between 2002 and 2006. All of the works were made by applying a) an algorithm (or a process, or an information source) to b) a microtonal tuning system with c) some element of real-time generation or real-time performance involved. The pieces were realised with acoustic instruments, acoustic instrument samples, or electronic timbres. Choice of timbre was often a critical part of the composing process, as a number of the works also explore the relationship between tuning and timbre. Many other works were written during this period, but the works under discussion were chosen because they most clearly represent the intersection of the three main interests listed above.

This thesis is in three parts. **Part One** is a series of short essays dealing with some of the main aesthetic, technical and tuning issues used in these works.

Part Two is a description of the particular algorithms, tuning systems, and performance processes used in each work.

Part Three is a digital appendix, found on the accompanying data DVD, which includes MP3 files of recordings of all works discussed in this thesis, articles by the author referred to in the text; lists of scales (including several extensive catalogs of tunings used in the works), and other data. Each Folder in the Appendix is numbered in agreement with its corresponding chapter. For example, the composition *For JSB and JT* is discussed in Chapter 2.1. The recording of this piece, its score, and all materials related to it are found in Appendix 3.1. The reader is referred to the Table of Contents for further navigational aid through the Appendix.

This thesis assumes familiarity with the concept of musical ratios and cents for expressing tuning. The freeware software *Scala* (Op de Coul 2007) was used extensively, and *Scala* charts and terminology will be extensively used throughout the thesis. For those unfamiliar with *Scala*, it can be downloaded from the site quoted in the References.

A general word on my approach: I am primarily a composer, and only secondarily a theorist. Therefore, my main interest is in using new tunings and interesting algorithms as worked out in the time, space and texture of sonic composition. I am not interested in complete theoretical explications of particular harmonic or algorithmic ideas. This thesis will not attempt complete explorations of any of the concepts discussed herein; rather it will concentrate on the ways that I develop and compose with particular tunings and algorithms.

In the tuning charts in this thesis, cents will be rounded off to a maximum of 3 decimal places, unless the example requires greater precision.

PART ONE – THEORETICAL CONCEPTS

This part of the thesis is a series of short essays dealing with some of the main aesthetic, technical and tuning issues used in the compositions discussed in Part Two. Definitions are given, and theoretical concepts are discussed with reference to the ways they were realized in the compositions.

1.1 Algorithms

The works discussed in this thesis were all generated using algorithms, or processes of some kind. I have been working in this way since 1968, when I was first exposed to both the Moog synthesizer and structuralist ways of composing, such as serialism and Cageian indeterminacy. Since then, I've done so much work with process-oriented composition that it has become my native musical language. My approach to algorithmic composing is a very broad one. Although I acknowledge the distinctions between mathematical, structural, rule-based, mechanical, and body-based processes, I do not prefer one over another in my work. Indeed, all these processes are useful, and choice of process uniquely affects the outcome of the piece.

A similarly enthusiastic diversity is present in my choice of performing modes. I enjoy a wide range of types of performance, choosing each for its own intrinsic qualities. In this thesis are works conceived for

- Live performance on acoustic instruments (with and without improvisation)
- Computer-controlled acoustic instruments
- Real time interactive performance with algorithmic electronic instruments

- Performance of recorded media
- Presentation in sound installations.

The issues involved in each of these performance types will be discussed in the Sections of Part Two which deal with the individual works.

1.1.1 History

In my earliest process-oriented pieces I combined control voltages from analog synthesizers to make complex real-time streams of control information, in which the moment-to-moment details could not be predicted, though the more general motion and overall ranges could be. This use of constrained random material continued in a series of works and machines right up to the present day. Some of the highlights of this work were the self-built composing machines *Aardvarks IV* (1975), *Aardvarks VII* (1978-79) and *Aardvarks IX* (1981-84), in which digital and analog electronics were combined in various ways to make a series of live electronic music performances. In *The Wilson Installations* (Burt 2007 forthcoming), I discuss these early days and my excitement at that time:

> Algorithmic composition was first shown to me by Joel Chadabe in the late 60s. The combining of control voltages on the early synthesizers to make musical patterns that were otherwise unavailable was an impetus to further investigation, as was contact with composers who were using processes to generate their music, such as John Cage, John White, Salvatore Martirano, and Chadabe himself. Later, my interest in numerical systems, such as chaos of various kinds, was provoked by my inability to "do the math." This was a challenge, so I learned the mathematics necessary for each new project/interest in turn. The possibilities of these kinds of composing thrilled me. I could get a larger variety of musics than if I just "felt" my way through traditionally

expressive kinds of composing. For me, variety of output is something I value highly. As well, the idea of interacting with a "would be intelligent" or "semi-intelligent" machine system is something I like. I enjoy creating systems which give me information that I can respond to at the moment of performance. In the early 1920s, Ernst Krenek had a similar experience when he first encountered counterpoint: "I was fascinated by the notion that music was not just a vague symbolization of *Gefuehl* instinctively conjured up into pleasant sounding matter, but a precisely planned reflection of an autonomous system of streams of energy materialized in carefully controlled tonal patterns (Krenek 1971).

In the mid-1980s, an interest in chaos theory and non-linear dynamics led to a number of works, such as *Voices, Tuning Forks, and Accordion* (1986) for choir, Just-Intonation tuning forks, and Just-Intonation accordion, and the electronic *Chaotic Research Music* (1989-90). The free-ranging use of different kinds of processes is best shown in *Music for Microtonal Piano Sounds* (1992-98), a five and one half hour piece in which a great number of processes and tuning systems were freely combined and used. In the late 1980s, I began working with software developers to realise some of my process-oriented ideas in software. I used the software to realise my ideas, and was also involved with the development of the software itself. Most recently, I have written two packages of mathematical composing resources for John Dunn's *ArtWonk* software, which have been incorporated into the published program (Dunn 2007). Elements from these packages were used in these works and will be discussed below.

1.1.2 Motivations

As stated above, I've been using process-oriented composing methods for so long that they have become my native compositional language. I have written extensively about my compositional processes and motivations. The essays *Some Parentheses around Algorithmic Composition* (Burt 1996a) and *Some Thoughts on Structure and Necessity* (Burt 2002a), both of which are in Appendix 3.12, give succinct descriptions of my use of processes. In both of these essays, I emphasize that perception is primary – that by close attention to a work of art, we learn how to perceive it, and that my use of processes is often aimed at giving us new materials that we might use to exercise and expand our perception. In *Some Parentheses around Algorithmic Composition* (Burt 1996a), I express this desire to use processes to make new things:

> One *proper* use of algorithmic composition techniques, chance, randomness, mappings, morphings, etc. (all of which, I acknowledge, are different things), is not duplicatory (assembling lookalikes and soundalikes of previously existing works), but exploratory (finding (making, uncovering, stumbling across, perceiving, learning to comprehend)) that which does not yet exist (or in Herbert Brun's too-often-quoted-to-even-be-bothered-with-footnoting phrase, 'We're interested in the music we *don*'*t* like, yet.').

In common with many of my colleagues, I have talked about my work as being a model of what music might become, or what music might be. However, I have been working in this way for so long, that it frequently seems to me that what I am doing *is* what music has become. I feel that what I am doing is, or has become, normal. I am concerned, however, that normal or exceptional, my use of these processes at least creates the potential of making works which reflect aspects of our environment; works which may at least serve as metaphors of new attitudes, new roles, rather than simply acting as structural hooks to hang our sound materials on. My colleague, composer David Dunn, expresses this thought succinctly:

> Within the western cultural context, musical practice has often been more or less coupled to the state of mathematical and scientific knowledge. The current

state of much musical practice continues this co-evolutionary scenario such that we now witness the almost complete collapse of prescriptive formal theoretical models or so-called "common practice." Instead we see a shift toward an integration of form and function that is organically determined through the compositional specification of systems and networks that give rise to behaviors reflecting current ideas of emergence, self-organization and biological autonomy. In other words, rather than musical composition as the specification of fixed details of structure over time, it now becomes the design of: a generative system of sufficiently high-dimensional complexity from which rich sonic behaviors can emerge.

This perspective is coincident with the most profound reasons for exploring these mathematical ideas through sound as a means to acquaint a larger audience with them. As the mathematician Ralph Abraham has stated:

'We are now at a hinge of history, not an apocalypse. Our own participation in the creation of the future may be improved in quality and power by mathematically illuminated images of space-time patterns in our history that are in harmony with similar patterns of the universe and Earth.' (Dunn 2007, Abraham 1994)

My own propensity to use a wide variety of processes, and to explore their uses freely, may be summed up by this excerpt from *Some Thoughts on Structure and Necessity* (Burt 2002a):

I no longer feel some sort of psychic or structural or historical mandate to use numerical structures in my pieces. If I want to explore a serious structural idea, I do, but if I want to explore something else, I do that also. Apart from exploring the unique attributes of any given material or structure, there seems to be no particularly urgent or meaningful reason to use one kind of structure over any other kind now. Since I can see/hear/feel structure in anything, and since, by choosing to give exact attention to something, I can make it interesting and beautiful for myself, all creative choices become simple

> options for me, with none having any particular monopoly on their ability to create "good works."

However, I do not mean to convey that I do not care about which process or system I am currently engaged with, or that my choices are entirely arbitrary. In the spirit of playful enquiry mentioned earlier, I am constantly testing, weighing, measuring, experimenting: making choices about which kinds of procedures "fit" with what kind of material, using these ideas as tools in a process of self-expansion and self-exploration. These works, and this thesis, are part of a process of dissemination in which I am trying to provide ways of making and thinking about music which, while they may have become normal for me, are still alternatives for many others. My attempt is to show that these alternatives are highly desirable.

1.1.3 Kinds of processes

In some of these works, I have used simple equal-weighted randomness applied to parameter selection; while in others, I have developed quite elaborate multi-level processes to construct the work, relying on the intersection of differing processes to create an interesting sonic surface. Some of the processes are used "straight", meaning that particular information is directly mapped to a specific set of music parameters. Other processes are designed for real-time interaction, in which compositional decisions are made at the moment of performance. Other processes were designed to provide materials for real-time improvisational shaping. In compositional processes designed for improvisation, intuition plays a greater or lesser role, depending on the kind of process I set up. Non-repeating sequences are used in a number of the works, and, in contrast, number sequences consisting of "found objects" produced by limitations in the functioning of an electronic calculator, or by sensors responding to the movement of participants in a scientific experiment are used in others.

Thomas DeLio (DeLio 2002) makes the distinction between what he calls an organic musical structure, in which every element of the piece proceeds from, or refers to, something inherent in the basic generating musical material, and inorganic structures, where various structural elements are juxtaposed, exploring what kinds of musical materials result from these juxtapositions, and how they can be shaped into larger compositions. The works in this thesis largely are based in the idea of inorganic structure, although in the works in which considerable real time choice among harmonic and rhythmic materials is made, the organic is never entirely absent.

The intensive use of arithmetic in my working method has led some to criticise this work as a new Pythagoreanism, in that it deals heavily with number and the concept of music being number made audible. There may be an element of truth to this, especially in those pieces where I am actually using Pythagorean methods, but I feel that because I'm doing all this to obtain and explore certain qualities of sound, I'm not involved with the mystical nature of number as the historical Pythagoreans were. (Although I admit I have a high regard for the moral philosophy of some of the later Pythagoreans, such as Iamblichus and Porphyry. (Guthrie 1987)) For me, in the pieces discussed in this thesis, the idea of serious listening to very subtly different shades of sound is paramount, and it shaped my choice of structures and how and where I choose to present them.

Each process used will be described in detail in the individual sections of Part Two, such as Sections 2.1.3; 2.2.3; 2.3.3, etc.

1.2 Microtonality

I have been exploring microtonal tuning systems since the very early 1970s. In *The Wilson Installations* (Burt 2007 forthcoming), I discuss my early motivations for this:

> I got into tuning because I couldn't do it very well, and there were inspiring people around who were doing it very well indeed (Harry Partch, for example), so I decided to see if I could do it too. Also, the mathematics involved looked absolutely intimidating. Mathematics was something I wanted to learn more about, but I'd never given it sufficient attention. I found that I could do both the mathematics and the tuning (and the retraining of my listening habits), and it became fascinating for me to think that pitch and interval quality could be such a deep and endlessly expanding field. An epiphanic moment for me was hearing the Indian composer and singer Pandit Pran Nath sing a descending semitone glide. For about 2 minutes. He told us what he was going to do, and then gave us a demonstration. I felt the floor fall away from under me. For those 2 minutes, I was literally floating in one of the biggest harmonic spaces in the universe, as he showed us, with absolute vocal control, that there was a lot of territory to explore there, if only one knew how to concentrate on it. Thirty years later, I'm still fascinated by the many different shades of sound that can exist within what we think of as a single interval.

As might be inferred from the above, my primary interest in tuning is perceptual, that is, it is an exploration of what we can hear. My main harmonic interest is in exploring the many different qualities of sound produced by different tunings, and although I have a fascination with aspects of tuning theory, I am not primarily concerned with, for example, conflicts between kinds of tuning, such as Equal-Temperament and Just-Intonation.

I have not limited myself to exploring any one kind of tuning, but rather, have ranged freely across the whole field of tuning, composing works in Just-Intonation; Equal-Temperaments; Non-octave, Non-just tunings; and Found Object tunings. Any sort of pitch relationships (including none) have been grist for my mill. Some of the pitch systems have been determined by instrumental design, such as the harp tuning in *For JSB and JT*, discussed in Section 2.1.2. In other cases, I designed tuning systems for acoustic instruments, such as the19-note Just-Intonation scale used by my tuning forks (Burt 1987a, 1987b). Other tuning systems have been exploratory, such as systems inspired by my interest in mathematics. Still others have been motivated by historical researches, such as my extensive use of the Greek modal system, both on its own and as a structural model for other tunings, while others have come from an interest in the theoretical works of Ervin Wilson. I have not participated to any great degree in debates over the meaning, or usefulness, of consonance versus dissonance. I am fascinated by all kinds of intervals, from extremely smooth to very rough. To quote Milton Babbitt (Babbitt 2001), I am intrigued by "that marvelous relation that we call an interval that is unique almost entirely to the perception of sound and nothing else."

It might be added parenthetically that the perception of interval is unique to hearing, and does not exist in other senses, such as sight, smell, taste, or touch. The idea of interval is one of the most powerful aspects of sound, and theories of sound-art that fail to consider the usefulness of interval are, in my view, seriously flawed.

Some of the more interesting uses of tuning in my work over the past four decades have included the following live electronic works:

- *Aardvarks VII: Le Grand Ni* (1978) in which a Just-Intonation system based on subharmonics 2-10 and their multiples was explored

- *Explorations of 31* (1981-82) in which this subharmonic based system was extended down to the 31st subharmonic
- *21 Studies in the Modes of Archytas* (1990-91), in which the Greek modal system as defined by Archytas was used
- *Justice, Equality and Beatings* (seven pieces, 1984-1990), in which beating tones produced by the juxtaposition of Just-Intonation and Equal-Temperaments were revealed; and
- *Bunjil's Soft Journey Through Two Hebdomekontanies* (*Music for Microtonal Piano Sounds, Part 8,* 1998), in which two related 70-note harmonic systems devised by Ervin Wilson were used.

Another series of works in which interesting harmonic systems have been explored are the works involving my Just-Intonation tuning forks, including

- *Voices, Tuning Forks and Accordion* (1986)
- *Pond* (1988)
- *Two Enharmonic Cycles* (1993), all for choir and tuning forks.

My recent work for multi-tracked and transposed tuning forks *The Animation of Lists And the Archytan Transpositions* (2005) for multi-tracked and transposed tuning forks will be discussed in Section 2.3.

I continue to make instruments in new tunings, both in physical form and in software. Most recently, I adapted a baritone ukulele to 24-tone Equal-Temperament and used it in *The Malleable Urn* (2004), as well as in a number of other recent works. *The Malleable Urn* is discussed in Chapter 2.2.

My deepest and most profound tuning investigation in recent years has been into the Just-Intonation scales derived from additive sequences and number triangles. *The*

MOSsy Slopes of Mt Meru and *Proliferating Infinities* are but two of the works made with these scales. The basic idea behind additive sequences and number triangles is discussed in Chapter 1.9. The compositions themselves are discussed in Chapters 2.6 and 2.11.

Finally, I've also used microtonal scales as a sound environment for text, based on their emotional qualities. In my settings of *Poems of Rewi Alley* (2001-2003) and of Damien Broderick's science fiction story *Schrodinger's Catch* (2003-2004), each section of the text is accompanied by music in a different Equal-Temperament, each one chosen because it contained harmonies that I felt reinforced the emotional content of the text. Further, in the case of *Poems of Rewi Alley* the exact choice of harmonies were derived from analysis of the spoken voice's resonant frequencies.

It might be mentioned here that all of my works referred to in this thesis are available from our website, www.tropicapricorn.com. Those wishing to hear the above works are encouraged to visit the site.

I freely use many kinds of structural or algorithmic processes, and I have a similar approach to tuning. I feel no loyalty to any one particular type of tuning, but find all tuning systems interesting. I enjoy the "smoothness" of much Just-Intonation, but I also enjoy the "roughness" of the more dissonant Equal-Temperaments. In fact, one of the reasons for my composing has been to make musical contexts in which I can enjoy the tuning I'm currently interested in. At least as far as my own perception is concerned, I find all hierarchical arguments as to the "rightness" of any one tuning over another moot.

I also embrace a wide variety of approaches to tuning systems, from the "extremely careful" to the "extremely careless," to use terms from John Cage's "The Future of Music" (Cage 1974):

> We can be extremely careful about harmony, as Lou Harrison, La Monte Young, and Ben Johnston are, or we can be, as I often am, extremely careless about harmony. Or we can make do as our orchestras do with grey compromise about which sounds sounded together are harmonious.

In most of the works under discussion here, the use of harmony tends more toward the "extremely careful", however, the application of some of the found object processes to tuning systems can be viewed as "extremely careless." Making works which traverse these positions is quite satisfying for me; my combination of extreme control with a delight in surprise juxtapositions creates a rich field of possibility.

It should be mentioned here that my concept of harmony is not one based on the idea of voice leading. Rather, it is one in which each harmonic unit – a chord, a scale, a mode – is treated like a particular colour. These harmonic colours may occur in any order, or any combination. This is harmony heard as timbre. In this way, my music relates to much other contemporary music in which timbre is the most important element.

Details of the tuning systems used in the works under consideration will be found in specific sections of Part Two, such as Sections 2.1.2; 2.2.2; 2.3.2, etc.

1.3. Why combine algorithmic composition with microtonality?

It might be asked why I choose to combine algorithmic composition with microtonality. The most honest and immediate answer is that it seems like the most natural thing in the world. In fact, given my interests, and the profound history of developments in both fields over the past century, it might be more appropriate to ask, "How could it be otherwise?"

However, it could be argued that certain aspects of some algorithmic systems might be more easily heard when applied to familiar pitch materials, and conversely, that certain aspects of a particular tuning system might best be revealed by traditional composing methods, rather than any sort of process. My reply to these arguments is that there are many others who have explored both of these options.

David Cope, for example (Cope 2006) with his *Experiments in Musical Intelligence* is developing a number of algorithmic methods, applying them to music for traditional instruments in 12-note Equal-Temperament. It could also be argued that the familiar sound of 12-note Equal-Temperament is absolutely crucial to the DNA music project of Mary Anne Clark and John Dunn (Dunn and Clark 1997, 1998, Clark 2001). For example, in Clark's DNA music, the same protein from several different species will be played simultaneously. Unison lines happen when the protein sequences are identical, and chords occur where there are differences in the protein between species. Quickly identifying the kinds of differences that occur between the different melodic lines may rely on having a familiar palette of intervals. Conversely, Brian McLaren (McLaren 2006) is continually composing, with intuitive methods, works which are designed to reveal selected aspects of the tuning system being used at the time. Another composer who, in some pieces, composes with the specific harmonic qualities of his given scale uppermost in mind is Ben Johnston. However, Johnston has also used structuralist (serialist, in some cases) ideas with microtonal scales in some works, as well as intuitive methods (Johnston 2006).

One composer who consistently combined algorithmic methods with microtonality was John Cage. Many of his late "number pieces" are good examples of this. One of the best discussions of Cage's late works is in Brooks 2002. In fact, one of my first exposures to microtonality was as a performer in the 1971 Albany realisation of John Cage & Lejaren Hiller's collaboration *HPSCHD*. This was the second performance of the work (Austin, Cage & Hiller 1969, Husarik 1983). *HPSCHD* has tape parts in every Equal-Tempered scale from 5 to 56 notes per octave, each one produced with an algorithmic composition program. In performance, these tapes are juxtaposed in numerous and unpredictable ways. Being immersed in this glittering indeterminate microtonal soundscape in 1971 was inspiring. Also crucial was attending a series of lectures given by Cage and Hiller in conjunction with that performance in which Cage talked about algorithmic composition in general, and Hiller talked in great depth about the technical details of the software and processes used to compose *HPSCHD*. The seeds of much of the work discussed in this thesis must have been sown then.

My own particular interest is in hearing how tunings and algorithmic systems intersect, producing works which may (or may not) reveal unexpected aspects of both the process and the tuning system. Some argue that these combinations may produce results that are too complex to be easily perceived. My reply is that although I have an interest in extremely simple and elegant musical structures, I also have an interest in complexity as well. If these combinations of algorithms and microtonality produce a complex surface, then this will provide us with opportunities to exercise our perceptual abilities. I feel that the giving of precise attention to artworks (even ambient ones!) is a good thing, and I wish to make works of art which provide us with opportunities to do so.

Further, I have a great interest in the emergent properties of process-oriented systems. This interest comes from my fascination with chaos and non-linear dynamics, in which unpredictable structures result from the combining of very simple actions. Using different tuning systems may reveal unsuspected aspects of the algorithms, and vice-versa.

Finally, I must state that my primary interest is in making works of art, and not research stimuli for experiments in perception, or works whose primary purpose is to demonstrate music theories. That is, my interest in is making works with an overall gestalt, an overall identity, and not works whose primary purpose is to prove or disprove aspects of musical perception theory. I am interested in ideas of musical perception, but they must inspire, and not determine, my music. How the various combinations of algorithms and tuning systems were used will be dealt with in detail in the relevant individual sections of Part Two.

1.4 Extreme duration

Of all the works discussed here, all but three are at least an hour in duration. Over the past few years, I have begun making works of longer and longer duration. I have made long single works and very long collections of works in the past, such as *Le Grand Ni* (1978), 70 minutes in duration, or *Some Kind of Seasoning* (1990-91), a suite of 16 works lasting between 20 minutes and 1 hour each, but in the past few years my motivation for making long works has become clearer. At first I just wanted to make works that filled the duration of a solo concert – an extension of the length and attention span of 19th century symphonism, as it were.

I also noticed that other composers were working with extended duration spans. Morton Feldman's long works of the late 1970s and early 1980s, La Monte Young's *The Well Tuned Piano*; John White's *Tuba and Cello Machine*, and installation works such as Charlemagne Palestine's 3 day long 1973 Theatre Vanguard Performance in Los Angeles, which I attended, were all inspirational. I also was involved in minimalist performance in the 1970s, and this developed my ability to concentrate on performance activities for considerable lengths of time. In 2002, I also participated in a performance of Erik Satie's *Vexations* at the Brisbane Powerhouse, which I enjoyed greatly. Long duration has not been a problem for me.

More recently, though, I have begun thinking about the political dimension of long duration, as expressed in the following quote from Benjamin Boretz (Boretz 2003):

> …Do I have to tell you about the spiritual cannibalism of the culture, our culture, which has been bombarding us with ultrasensory overstimulation aiming to reprocess us into fulltime consumption machines, stealing above all from us our time (not an inch of time without an imprint of message), and even our very *sense* of time (to be measured in lengths of no more than one

> message unit each) under the guise of entertainment, and even of 'art', commoditizing the eternal, hyping the primal? Our time is the *sine qua non* of our identity. We need to take extreme measures to reclaim it for ourselves and each other.

Agreeing with this quote, I, too, felt that extreme measures were necessary to reclaim a sense of time and identity. I felt that the "sound-byte, bullet-point" consumerist treatment of time in contemporary society and academia needed to be opposed, and I wanted to make works which demanded more serious attention for greater lengths of time than that which commercial media or academic culture participated in.

My concern with these issues has a positive aspect as well. In a time dominated by parsimonious bean-counting attitudes, I would like my compositions, with their extravagant length, their many algorithms, and their huge number of scales, to stand as examples of abundance, and generosity. I'm a great fan of abundance. I feel that we can experience many different kinds of composition, many different kinds of tuning, and many different performance situations. Rather than a situation of poverty, my work celebrates a luxuriousness of artistic experience that we have simply to allow ourselves to inhabit.

My desire for longer duration had another impetus as well. In recent years, I had become an enthusiastic participant in the improvised music scene. I liked the continual ongoing sense of "practice" that happened in the improv scene - the fact that you were always developing ideas and almost immediately trying them out in performance. But I believe that the nature of the music should dictate the nature of the performance space and manner of presentation, and not vice-versa, and I found that my ideas of making pieces in which very large sets of harmonic resources were explored for a long time just didn't fit into the variety show concept of 10-20 minute "sets" that had developed in the world of free improvisation. I also needed a different set of venues for this music – one that would promote close listening to the music, and the smoke-clogged, acoustically bad, primarily-designed-for-socializing venues that existed in Melbourne, the main city in which I performed and composed for 30 years, were just not appropriate for this. So my desire for longer works was not just making a distinction between my work and commercial culture, it was also making a distinction between my work and that of my highly respected colleagues in the free improvisation scene.

There was also another, more practical reason for the length of some of these pieces. In some of these, I've developed harmonic systems of great complexity, which generate families of hundreds of different scales. Even working at the rate of harmonic change of traditional commercial music, it would still take hours to explore all of those scales in even a superficial way. If I wanted to explore the complete set of harmonic resources from a given harmonic system, then, I would have to make works which lasted a long time.

1.5 Ervin Wilson

Many of the works in this series use developments of harmonic systems which were first proposed by Ervin Wilson. Wilson (born 1928) has been described as "the most productive speculative music theorist since Ptolemy" (Wolf 2003). A non-academic researcher, Wilson publishes his work as numerical diagrams only, with only minimal verbal explanation. He has been engaged in this research for more than half a century, and much of his work is now publicly available on the Anaphoria web site (Grady 2006). A remarkably open person, his work has been realised in a variety of musical genres, each of which he enthusiastically observes. I first met him in 1996, after he wrote to me, correcting an error in an article of mine about one of his structures (Burt 1995). Since then, we have been in constant correspondence. I have a large number of unpublished papers from him. It should be enough to keep me busy for several lifetimes. Every time I go through Los Angeles, I try to spend some time with him. He said to me once that in his youth, he wanted to study with either Arnold Schoenberg or Harry Partch. By the time he got around to making a decision, Schoenberg had died, so he hung out with Partch instead, absorbing his tuning ideas, and then extending them greatly.

Again, it must be stated that Wilson is not an academic, and his work does not indulge in either the clear explications or aesthetic justifications of academic writing. He has told me that he feels his method of presentation (pages of charts and diagrams, with little verbal explanation) is a way of making sure that only those who are really serious explore, use and develop his ideas.

It might also be parenthetically mentioned here that the bulk of microtonal research in the 20th century was, indeed, carried out by non-academic researchers. Only in recent years has an interest in microtonality become academically respectable, and it has been interesting comparing the recent work of academic theorists in microtonality with that of their non-academic colleagues. For me, in general, the academic theorists may have rigor and clarity on their side, but the non-academic theorists still seem far more adventurous, creative, and useful; providing a lot more room for exploration and discovery. Since Wilson feels that his work is best taught as part of an oral tradition, a personal story is relevant here. During the 2001 Microfest Symposium at Claremont College in Los Angeles, Wilson presented his paper "Euler Genus (3-5-7-9-11-13) Lattticed within the Triakontahedron" (Wilson 2001). Wilson had sent me an advance copy of the paper, and I had puzzled over it for months, finally understanding it, and writing the electronic composition *Memories of Cecil Street on a Hot Summer Day* (2000-2001) using theoretical ideas contained in it. As a result of all this work, I understood his presentation, which mostly consisted of presenting physical 3D models of his tuning diagrams. By chance, I was sitting next to Lou Harrison. He too, was enjoying Erv's presentation immensely, and both of us noted the bewilderment at Erv's presentation by the assembled roomful of academic colleagues. At one point Lou turned to me, and in an exaggerated stage whisper said, "Why there's more possibilities for modulation here than a *young boy* could ever possibly use!" A more full exploration of the modulatory resources of this Wilson paper was accomplished in my composition "Saturday in the Triakontahedron with Leonhard" (2004) which will be extensively dealt with in Section 2.8.

During the course of his researches, Wilson has elaborated on, invented, or developed a number of concepts. Some of these are

- Moments of Symmetry (MOS)
- Combination Product Sets

- Permutations of the Intervals of Tetrachords generating families of scales
- The Farey Series applied to musical intonation, and it's extension as
- The Scale Tree
- Euler Fokker Genus mappings
- Extensions of the ancient Greek Lambdoma
- Developments of Viggo Brun's algorithms
- Mt. Meru Scales - scales made from additive sequences derived from the sums of diagonals of number triangles
- Keyboard designs for any and all of the above systems, and most recently
- Scale systems based on the Co-Prime Grid.

It is not the purpose of this thesis to explain all of these. The complete user's guide to the work of Ervin Wilson has yet to be written. For the moment, Wolf 2003, and Kraig Grady's introductions to the Wilson website (Grady 2005a, 2005b), are probably the two best overview introductions to Wilson's work. In the course of this writing, however, those aspects of Wilson's work that were used and extended will be explained in detail. Chapters 1.6, 1.7 and 1.8, respectively, cover Wilson's developments of Moments of Symmetry, Euler-Fokker Genera, and the Scale Tree. An extensive listing of Wilson's writings is found in the References.

1.6 Moments of Symmetry (MOS)

At this point, we should look in detail at the first of Wilson's concepts that has informed many of the works in this thesis: the concept of Moments of Symmetry, or MOS.

1.6.1 Basic Definition

MOS is a term coined by Ervin Wilson around 1975. It describes a concept that is, to quote him, "almost embarrassingly simple" (Wilson 1975). According to John Chalmers (Chalmers 2006),

> in plain English, a MOS is generated by a cycle or stack of an interval relatively prime to the interval of equivalence, has only two sizes of step interval, and is the first such scale composed of such intervals as the stack increases in depth.

From this definition, it will be seen that there are three main requirements for a scale to be considered as a MOS scale. MOS scales

- Are created by stacking a generating interval and reducing the scale within some limit
- Contain two and only two interval steps
- Are the first such sub-set generated by stacking.

These properties are now described in detail.

1) Each degree of the scale is generated by stacking a selected interval on top of an arbitrary starting note N times, and then reducing the resulting intervals so that the pitches of the scale fall within the range of an octave (or other interval of equivalence – the interval at which the patterns of a given scale repeat). The process works in the same manner as the Pythagorean stacking of fifths to generate diatonic or pentatonic scales.

In MOS scales, the stack of intervals can also be reduced using an interval other than the octave. This is referred to as the *interval of equivalence* (Wilson 1975). Furthermore, the generating interval is *relatively prime* to the interval of equivalence. That is, it shares no common factors with the interval of equivalence. Initially it may appear that this relationship applies only in cases where the interval of equivalence and the generating interval are integers. However, this relationship can also be expressed using numerical units other than simple integers such as cents or just intonation ratios. In this thesis, all the work done with MOS scales is done with the octave as the interval of equivalence.

To understand how this stacking of intervals works, consider the familiar diatonic and pentatonic scales in 12-tone Equal-Temperament.

A diatonic scale is made by stacking the generating interval of an Equal-Tempered 5th – or 7 semitones. Starting on the note F, the following 7 pitches are formed: F C G' D'' A'' E''' and B'''; by reducing these to a compass of one octave and sorting into ascending order, seven pitches of the diatonic scale are formed: C D E F G A and B.

A pentatonic scale is made by stacking the generating interval of an Equal-Tempered 4th – or 5 semitones. Starting on the note Bb, the following 5 pitches are formed: Bb, Eb, Ab, Db' and Gb'; by reducing these to a compass of one octave and sorting into ascending order, five pitches of the pentatonic scale are formed: Db Eb Gb Ab and Bb.

2) MOS scales are made of two and only two kinds of intervals. Both these scales are MOS. In mathematics, this phenomenon of only two step sizes in a sequence is known as Myhill's property (Van Ravenstein 1988, Breed 2002). In the diatonic scale described above, the MOS intervals are tones and semitones; MOS intervals in the pentatonic scale are minor thirds and Major seconds. In 12-tone Equal-Temperament, one octave of this scale can therefore be thought of as two interleaved diatonic and pentatonic MOS scales. This concept of scales made from 2 other interleaved scales is greatly expanded on in Joseph Yasser's "A Theory of Evolving Tonality" (Yasser 1932).

The table below shows both the octave based diatonic scale with a generating interval of 700 cents, and the equivalent scale in Pythagorean 3-limit Just-Intonation, where the generating interval is 3/2. On the left, cents are used to show interval sizes. On the right, intervals are shown by ratios. Note that in both instances, only two sizes of step intervals exist.

Table 1: Comparison of interval sizes and steps in diatonic MOS scales in 12-tone Equal-Temperament and Pythagorean Just-Intonation

Scale degree	Pitch – equal temp. cents	Step size in cents		Scale degree	Pitch – Pythagorean just intonation - ratio and cents	Step size in ratios and cents
0	0 (C)	-		0	1/1 (0)	-
1	200 (D)	200		1	9/8 (204)	9:8 (204)
2	400 (E)	200		2	81/64 (408)	9:8 (204)
3	500 (F)	100		3	4/3 (498)	256:243 (90)
4	700 (G)	200		4	3/2 (702)	9:8 (204)
5	900 (A)	200		5	27/16 (906)	9:8 (204)
6	1100 (B)	200		6	243/128 (1110)	9:8 (204)
7	1200 (C)	100		7	2/1 (1200)	256:243 (90)

3) The scale is the first scale made up of two kinds of intervals as the generating interval is stacked up. For example, in 12-tone tuning, a scale made up of 8, 9, 10, or 11 perfect 5ths ALSO has only two sizes of interval – Major and minor 2nd. However, the lowest number of perfect 5ths that can be stacked to get a scale of Major and minor 2nds is 7. Therefore, by definition, the 7-note scale is the MOS scale, while the 8- through 11-note scales aren't.

1.6.2 Some Initial Examples

The following examples should make things clear. Note that although scales of 2, 3, and 4 degrees can exhibit MOS characteristics, in line with Wilson (Wilson 1975), we consider them as trivial cases.

Table 2: Pitches and step sizes of MOS pentatonic scale in 12-tone Equal-Temperament with step sizes of 300 and 200 cents, and a generating interval of 700 cents. (This is the pentatonic scale discussed above – (C D E G A))

Scale Degree	Pitch (cents)	Step Size (cents)
0	0	-
1	200	200
2	400	200
3	700	300
4	900	200
5	1200	300

Note that there are only 2 kinds of step intervals in the scale, 300 cents and 200 cents. This is also the scale with the smallest number of degrees that has only two step sizes of this type. Therefore, this scale is a Moment of Symmetry.

MOS scales can exist with any generator size. For example, Table 3 shows a 9-note scale made from stacking a 528.0 cent generator interval.

Table 3: Pitches and step sizes of MOS scale of 9 steps in 25-tone Equal-Temperament with step sizes of 144 and 96 cents, and a generating interval of 528 cents

Scale Degree	Pitch (cents)	Step Size (cents)
0	0	-
1	96	96
2	240	144
3	384	144
4	528	144
5	624	96
6	768	144
7	912	144
8	1056	144
9	1200	144

Note that once again, there are only two types of step intervals in the scale, and, in this case, it's the only scale with these two sizes of intervals. Therefore, this scale is a Moment of Symmetry.

To show that this number of degrees is indeed unique, here are interval listings for scales made by stacking eight and ten of the same 528 cent intervals. It will immediately be seen that in both cases, there are three kinds of step sizes in each of these scales.

Table 4: Pitches and step sizes of non-MOS scale of 8 steps in 25-tone Equal-Temperament with step sizes of 96, 144, and 240 cents, and a generating interval of 528 cents

Scale Degree	Pitch (cents)	Step Size (cents)
0	0	-
1	96	96
2	240	144
3	384	144
4	528	144
5	768	240
6	912	144
7	1056	144
8	1200	144

Table 5: Pitches and step sizes of non-MOS scale of 10 steps in 25-tone Equal-Temperament with step sizes of 48, 96 and 144 cents, and a generating interval of 528 cents

Scale Degree	Pitch (cents)	Step Size (cents)
0	0	-
1	96	96
2	240	144
3	384	144
4	528	144
5	624	96
6	768	144
7	912	144
8	1056	144
9	1152	144
10	1200	48

Note that in the both of these examples there are three kinds of step interval. Therefore, these scales are **not** Moments of Symmetry.

Finally, here is an example using an irrational number as a stacking interval. Phi, 1.618033988… when used as a ratio over 1, yields an interval of 833.09 cents.

Table 6: Pitches and step sizes of MOS scale of 10 steps with step sizes of 99.27 and 168.37 cents, and a generating interval of Phi, 833.09 cents

Scale Degree	Pitch (cents)	Step Size (cents)
0	0	-
1	99.27	99.27
2	198.54	99.27
3	297.81	99.27
4	466.18	168.37
5	565.45	99.27
6	664.72.	99.27
7	883.09	168.37
8	932.36	99.27
9	1031.63	99.27
10	1200	168.37

Once again, note that there are only two kinds of intervals in this scale. It is also the only scale that has these two kinds of intervals. Therefore this scale is a Moment of Symmetry.

1.6.3 Other Characteristics of MOS Scales

There are several other characteristics of MOS scales, which flow from these defining points:

A. MOS scales are made of repetitive patterns of a block of intervals

B. MOS scales can be made symmetrical in a conventional sense by rotation

C. The MOS principle shows complementarity. That is, if two different generating intervals are inversions within an octave, they will generate inverse scales

D. MOS scales are well-formed scales. Each occurrence of the generating interval in the scale covers the same number of scale steps.

E. The number of scale degrees in MOS scales, in many cases, end up in an additive sequence relationship.

A) MOS scales are made up of repetitive patterns of blocks of intervals.

Consider the scale in Table 2 – a simple pentatonic in 12-tone Equal-Temperament (C D E G A). The intervals in this scale are 200, 200, 300, 200, 300 cents. With s = small interval, and L = large interval, the pattern here is

ssLsL

Note that there is a larger block of intervals

ssL

followed by a similar, but smaller block.

sL

This kind of patterning will exist in every MOS scale. MOS scales do not divide the octave, or interval of equivalence, into equal blocks (Chalmers 2006). For example, the Pythagorean 3-limit Just- diatonic scale from Table 1 has the following pattern of step intervals:

Table 7: Listing of Step Intervals of the Pythagorean 3-limit Just- diatonic scale

Scale Degree	Ratio	Cents
1	9:8	204
2	9:8	204
3	9:8	204
4	256:243	90
5	9:8	204
6	9:8	204
7	256:243	90

With 9:8 (204 cents) being L, and 256:243 (90 cents) being s, the pattern is

LLLsLLs

Again, notice that there is a larger block

LLLs

followed by a shorter version of that same block.

LLs

Finally, consider the scale in Table 6, the ten-tone scale with Phi as a generator. It has a more complex repeating pattern.

Table 8: Listing of Step intervals from Table 6, the 10-tone scale with Phi as a generator

Scale Degree	Step Size (cents)
1	99.27
2	99.27
3	99.27
4	168.37
5	99.27
6	99.27
7	168.37
8	99.27
9	99.27
10	168.37

.With 99.270 cents being s, and 168.370 = L, the following pattern results.

sssLssLssL

It can clearly be seen that the pattern consists of two sizes of blocks of repeating intervals. Here, however, there are two repeats of a smaller block

ssL

preceded by a larger version of the same block.

sssL

Any MOS scale can be parsed out into repeating blocks on intervals such as these. Sometimes, however, the scale may need to be rotated for the repeating pattern to be clearly seen.

B) MOS scales can be made symmetrical in a conventional sense by rotation.

We have been using the term Moment of Symmetry, but it will clearly be observed that none of the scales we have been examining so far are symmetrical in the conventional sense. However, any MOS scale can be rotated to form modes. Consider our conventional diatonic Equal-Tempered scale, made up of Major 2nds (L) and minor 2nds (s).

C D E F G A B C
L L s L L L s

If this scale is rotated to the following interval pattern, it will be seen that the resulting scale, (shown both starting on C, and also on D for a "white key" solution), will indeed be symmetrical in the conventional sense.

L s L L L s L
C D Eb F G A Bb C
D E F G A B C D

Therefore, at least one mode (usually only one) of any Moment of Symmetry scale will be conventionally symmetrical. A number of 20th century composers, such as Bela Bartok, Kenneth Gaburo, and Harry Partch, placed great emphasis on the use of symmetrical scales (Lendvai 1979; Gaburo 1962, Partch 1974). The use of MOS scales can easily be incorporated into their interests.

C) The MOS principle shows complementarity.

If one uses a particular generating interval, such as 528 cents, to make a scale of N degrees, one can also use the interval that is complementary with that interval within an octave to make a MOS scale of N degrees which is an inversion of that scale.

For example, consider Table 3, the nine-tone MOS scale with step sizes of 144 and 96 cents, and a generating interval of 528 cents, which is a subset of 25-tone Equal-Temperament. For reading convenience, it is also given below as Table 9.

The interval which is complementary with 528 cents within an octave is 672 cents. (1200 cents – 528 cents = 672 cents.) Making a scale by stacking nine 672 cent intervals will make a MOS scale that is the exact inversion of the scale with a generator of 528 cents. I leave it to the reader to observe the two scales, and see how the interval of 96 cents which is at the bottom of the scale in Table 9 is at the top of the scale in Table 10, etc.

Table 9: Pitches and step sizes of a nine-tone MOS scale with step sizes of 144 and 96 cents, and a generating interval of 528 cents, which is a subset of 25-tone Equal-Temperament

Scale Degree	Pitch (cents)	Step Size (cents)
0	0	-
1	96	96
2	240	144
3	384	144
4	528	144
5	624	96
6	768	144
7	912	144
8	1056	144
9	1200	144

Table 10: Pitches and step sizes of the other MOS scale of 9 steps in 25-tone Equal-Temperament, with step sizes of 96 and 144 cents, and a generating interval of 672 cents

Scale Degree	Pitch (cents)	Step Size (cents)
0	0	-
1	144	144
2	288	144
3	432	144
4	576	144
5	672	96
6	816	144
7	960	144
8	1104	144
9	1200	96

D) MOS scales are well-formed scales.

In a masterly dissertation, Norman Carey writes in great detail and thoroughness the concept of well-formed scales. "A scale is said to be well-formed if a) it is generated, and b) the number of steps in the generator is constant in each of its N-1 appearances" (Carey 1998). Carey defines a scale as generated if "all of its pitch classes are formed by successive multiples (reduced modulo the interval of periodicity) of a single interval (called the *generator*)." This is the same method that we are calling "interval-stacking." The prototype of the generated scale is obviously the Pythagorean scale.

As an example, consider this 18-note MOS scale made by stacking intervals of 328.32 cents, one of over160 scales I made for *Pythagoras' Babylonian Bathtub.*

Table 11: 18-note MOS scale made by stacking a generator interval of 328.32 cents

Scale Degree	Pitch (cents)	Interval (cents)
0	0	-
1	11.52	11.52
2	113.28	101.76
3	124.8	11.52
4	226.56	101.76
5	328.32	101.76
6	339.84	11.52
7	441.6	101.76
8	453.12	11.52
9	554.88	101.76
10	656.64	101.76
11	668.16	11.52
12	769.92	101.76
13	781.44	11.52
14	883.2	101.76
15	984.96	101.76
16	996.48	11.52
17	1098.24	101.76
18	1200	101.76

It can be shown that each occurrence of the interval of 328.32 cents in this scale encompasses five steps. Table 12 shows this for the beginning of the scale.

Table 12: Number of scale steps contained in each occurrence of the generator in the scale shown in Table 11

Cents	0	328.32	656.64	984.9	113.280	441.600 etc
Scale degree	0	5	10	15	2	7
Steps between	-	5	5	5	5	5

Taking scales made from the lower limits of the Fibonacci Series will show how the concept of a well-formed scale and MOS are related. The intervals of the Fibonacci series tend to Phi, 1.618033. A full explanation of ratios "tending to" a limit in a convergent series, also known as limit-ratios, is given in Chapter 9, Section 1.9.2, below, especially in Tables 27-29, and in Figure 6. The MOS scales made by taking Phi as a generator are 7, 10, 13, 23, 36, 49, 85, etc. By making scales with 8 and 10 members of the Fibonacci series, we can clearly see which scale is well-formed, and which is not. Note that the generator is not a single scale degree, but a number sequence, the ratios of which approach a single generator as they get further and further out in the sequence.

Table 13: Number of steps contained in each generator interval, in a scale made with 8 elements of the Fibonacci Series from 3 to 89

Fib. number & ratio	3 3/2	5 5/4	8 2/1	13 13/8	21 21/16	34 17/16	55 55/32	89 89/64
Cents(rounded)	702	386	0	841	471	105	938	571
Scale degree	5	2	0	6	3	1	7	4
Steps between	-	5	6	6	5	6	6	5

Reading the last row of the above table ("steps between"), it is clear that each occurrence of a generating interval from the number sequence does not cover the same number of scale degrees. Some occurrences cover 5 scale degrees, while others cover 6. Therefore, a scale made with 8 elements of the Fibonacci Series is not well-formed. However, if we extend the scale to encompass 10 members of the Fibonacci Series, we will see that this scale is indeed well-formed. Not coincidentally, one of the MOS scales made with Phi, the limit of the Fibonacci series, is a 10-note scale.

Table 14: Number of steps contained in each generator interval, in a scale made with 10 elements of the Fibonacci Series from 3 to 233

Fib. no. Ratio	3 3/2	5 5/4	8 2/1	13 13/8	21 21/16	34 17/16	55 55/32	89 89/64	144 9/8	233 233/128
Cents	702	386	0	841	471	105	938	571	204	1037
Scl. deg.	6	3	0	7	4	1	8	5	2	9
Steps btw.	-	7	7	7	7	7	7	7	7	7

Reading the last row in this table ("Steps btw."), it can clearly be seen that every occurrence of a successive generating interval from the Fibonacci Series covers exactly the same number of scale degrees. Every occurrence of the generating interval covers 7 scale steps. Therefore, this scale is well-formed. Neither of these two scales is a Moment of Symmetry, strictly speaking, because they do not have only two step interval sizes. However, the scale that has the same number of scale degrees as a MOS scale made with the limit-ratio of the Fibonacci Series *is* well formed.

Scales made with limit-ratios will be explored in much more detail in Chapters 1.8 and 1.9. Pieces that use limit-ratio scales are discussed in Chapters 2.6; 2.7 and 2.11. Just-intonation scales which come close to, but do not precisely fit the definition of MOS will be discussed below, in Section 1.6.7, "Just-Intonation and Quasi-MOS."

E) The number of scale degrees in MOS scales, in many cases, contain additive sequence relationships.

These additive sequence relationships are not universal, but are common enough to merit observation. They occur when the numbers of notes in two successive MOS scales add up to the number of notes in a third adjacent MOS scale. Table 14 lists a number of generators, and the MOS scales that result from them. It will be noticed that in many cases, those scale sizes – the number of notes in a scale - which yield MOS scales will form a sequence where the next MOS scale is shown by the sum of the sizes of the two previous MOS scales. Sometimes these relationships continue, sometimes they do not.

Table 15: MOS scale sizes from various generators. Additive sequences are in **bold**

GENERATOR	MOS	Scale Sizes					
3/2 (701.955c)	**5**	7	**12 - - 12**	**17**	**29**	41	53
Phi (833.090)	7	**10**	**13**	**23**	**36 ---36**	**49**	**85**
656.604c	5	7	**9**	**11**	**20**	**31**	42
181.132c	4	5	**6**	7	**13**	**20**	**33**
439.024c	2	3	5	**8**	**11**	**19**	**30**

Some examples from the above chart may help to illustrate this.

- In the first row, the additive pattern 5 + 7 = 12. A 5-note scale is MOS. The next MOS scale has 7 notes, and the MOS scale after that has 12 notes. The sizes of the first two scales add up to the size of the third scale. Another additive pattern happens with the three scales beginning with 12 notes. 12 + 17 = 29. After that, the pattern breaks down.
- In the second row, scale sizes 10, 13, 23, 36 are additive, but after that, 49 breaks the pattern, but 36 + 49 = 85, so another additive relation begins there.
- In the 3rd row, 9, 11, 20, 31 are in an additive relationship, but the pattern breaks down with 42.
- In the 4th row, 7, 13, 20, 33 is an additive pattern, as is 8, 11, 19, 30 in the final row.

There are many generators where this kind of pattern does not occur. But where the pattern does occur, it can be useful. For example, one may choose to treat the sizes of the two smaller scales that add up to a larger scale as interlocking subsets. For example, consider the interlocking 7- and 5-note scales in our white key – black key 12-note scale. Wilson has proposed interlocking 10- and 13-note scales based on Phi as subsets of a larger 23-note scale. My compositions *The MOSsy Slopes of Mt. Meru*, and *Pythagoras' Babylonian Bathtub*, discussed in Sections 2.6 and 2.7, use dozens of such relationships. In fact, these pieces are made entirely of scales containing this kind of interlocking relationship.

1.6.4 Not all scales with only two step sizes are MOS

However, just as not every scale generated by stacking intervals is a Moment of Symmetry, many scales which have only two step sizes are not Moments of Symmetry either. Consider, for example, the octotonic scale, consisting of alternating tones (T) and semitones (S).

T S T S T S T S
C D Eb F F# G# A B C

It is true that this scale has only 2 step sizes, but it divides the octave into equal blocks – something that MOS scales do not do, AND it is impossible to generate this scale by stacking any of its component intervals 8 times. That is, none of the component intervals of the scale, if stacked 8 times, will generate this scale – they will generate other scales. Therefore, this scale is not a Moment of Symmetry.

Another example of a 2 step-interval scale that is not a Moment of Symmetry is Vaziri's Persian tuning, a 17-tone tuning involving quarter-tones.

Table 16: Vaziri's Persian tuning – a 17-tone, 2 step scale using quarter-tones

Scale Degree	Pitch (cents)	Step size (cents)
0	0	-
1	100	100
2	150	50
3	200	50
4	300	100
5	350	50
6	400	50
7	500	100
8	550	50
9	650	100
10	700	50
11	800	100
12	850	50
13	900	50
14	1000	100
15	1050	50
16	1100	50
17	1200	100

Although this scale has MOS-like characteristics, such as having only two sizes of step interval, and being conventionally symmetrical, which might suggest that in some rotation, it could be a Moment of Symmetry, none of the intervals within it, when stacked 17 deep, will generate the scale. And further, its intervals do not form repeating blocks with one shortened (or lengthened) version. Here are the intervals in the scale – "Q" = quartertone, and "S" = semitone.

S Q Q S Q Q S Q S Q S Q Q S Q Q S

There are a couple of ways to read this pattern. One is with a repeating block of SQQ, followed by two SQs and finally with SQQS at the end of the pattern. Another way would be to rotate the final S to the beginning of the pattern, creating

SSQQ
SQQ
SQ
SQ
SQQ

In either reading, one ends up with three different kinds of repeating or non-repeating blocks. Therefore, although this scale looks like it might be a Moment of Symmetry, on closer examination, it fails the test.

1.6.5 MOS and world music scales

Looking at the above scale was prompted by a conversation with Greg Schiemer, where he was looking for a Moment of Symmetry scale that was also a genuine scale from a non-Western culture. This search was prompted because there are many non-Western scales which are based on stacks of five or seven 3/2 intervals – the classic Pythagorean pentatonic and diatonic scales, which are indeed Moments of Symmetry. It was tempting to look for other scales from non-Western cultures to see if they were MOS. So far, the search has not been successful. However, Kraig Grady points out that Wilson calls scales based on the additive sequence rule

$$H_n = H_{n-3} + H_{n-2}$$

"Meta-Slendro" for their "uncanny sound compared to this family of Indonesian scales." Grady has created a whole family of instruments and performances based on these Meta-Slendro scales (Grady, 2005a). A fuller explanation of scales based on additive sequences will be found in Sections 1.9, 2.6 and 2.11.

1.6.6 Other mathematical properties related to MOS

Although relatively few MOS scales can be found in world music (except for ones resembling conventional diatonic and pentatonic scales), there are places in the world of mathematics where MOS scales can be found in great abundance. The Stern-Brocot Tree, a mathematical formation called the Scale Tree by Wilson, is one such, and it will be discussed in detail in Chapter 1.8. The Scale Tree forms the basis for my composition *Pythagoras' Babylonian Bathtub,* discussed in Chapter 2.7. For the moment, it is sufficient to note that every ratio on a node in the Scale Tree generates numerous scales that have MOS properties.

A mathematical proof which relates the concept of MOS to Myhill's property, and the concepts of Maximal Evenness and Propriety was given by Graham Breed (Breed 2002). As I have not used the concepts of Maximal Evenness or Propriety in any of the compositions referred to in this thesis, an explanation of them is beyond the scope of this writing. Those interested can refer to Breed's work for a fuller explanation and links to other work in this area. However, at the time of writing this Chapter, Breed's website seems to have disappeared from the internet. A pdf copy of Breed 2002 can be found in Appendix 3.13.

It should be noted that when MOS scales are formed using a transcendental number as a generator, such as Phi, or Pi, the scale formed is neither Just-, nor Equal-Tempered. Many of the MOS scales formed from the Scale Tree, discussed in Chapter 1.8, have this characteristic. This is but one way of generating Non-just, Non-equal-Tempered Scales. Brian McLaren has explored this and many other ways of generating these scales in "The Uses and Characteristics of Non-Just, Non-Equal-Tempered Scales" (McLaren 1993). This work parallels and in some cases anticipates Wilson's work from the same period.

1.6.7 Just-Intonation and Quasi-MOS

Some Just-Intonation scales also exhibit MOS-like characteristics. That is, they come close to being MOS, but don't quite meet the exact criteria. "Quasi-MOS" is the name I have coined for these scales, which may be Well-Formed, but do not have two step sizes. For example, we know that a scale of 10 notes, made from the generator Phi (1.618033 or 833.090 cents) will be a Moment of Symmetry. The Fibonacci Series consists of a series of numbers, the relationship between each two of which gradually gets closer and closer to Phi as was alluded to in Section 1.6.3D, and is fully explained in Section 1.9.2. If we use Phi as a generator, and use a fragment of the upper members of the Fibonacci Series, from 8 – 610, as harmonics, we should get fairly similar scales. And we do:

Table 17: Step intervals in a 10-note scale made with Phi as a generator

Scale Step Number	Step Size (cents)
1	99.267
2	99.267
3	99.267
4	168.377
5	99.267
6	99.267
7	168.377
8	99.267
9	99.267
10	168.377

Notice that in Table 17, the step sizes are 99.267 and 168.377 cents.

Table 18: Step intervals in a 10-note Just- scale made by treating the Fibonacci Series from 8 to 610 as harmonics

Scale Step	Step Size (ratio)	Step size (cents)
1	17:16	104.955
2	18:17	98.955
3	305:288	99.289
4	336:305	167.582
5	89:84	100.099
6	377:356	99.255
7	32:29	170.423
8	55:52	97.104
9	233:220	99.392
10	256:233	162.977

In Table 18, the three largest intervals are within 6 cents of 168.377 cents, while the seven smallest intervals are within 4 cents of 99.267 cents. Therefore the scale in Table 18 can be called "Quasi-MOS". It's "on-the-way" to becoming a MOS scale, but it isn't quite there. As shown in Table 14, and as defined in Section 1.6.3D, the scale *is* Well-Formed. However, the intervals between successive degrees of the Fibonacci scale, shown in Table 18, are not identical to those of the Phi Scale, shown in Table 17, but they are very close.

On the other hand, if a scale is made with the lower elements of the Fibonacci Series, this variance is more pronounced. For example, elements 8 through 610 of the Fibonacci Series were used to generate the Fibonacci scale shown in Table 18, whereas elements 2 through 233 of the Fibonacci Series were used to generate the Fibonacci scale shown in Table 19, below.

Table 19: Step intervals in a 10-note Just- scale made by treating the Fibonacci Series from 3 to 233 as harmonics

Scale Step	Step Size (ratio)	Step Size (cents)
1	17:16	104.955
2	18:17	98.955
3	10:9	182.404
4	21:20	84.467
5	89:84	100.099
6	96:89	131.075
7	13:12	138.573
8	55:52	97.104
9	233:220	99.392
10	256:233	162.977

Here, the divergences from the interval sizes shown in Table 17 are much greater. The three largest intervals diverge from the size of the large interval in Table 17, 168.377 cents, by as much as 30 cents. The nine smallest intervals diverge from the size of the small interval in the same table, 99.267 cents, by as much as 32 cents. Why is this? This is because the three lowest members of the series, 3/2 (formed by harmonic 3 and harmonic 8, octave-reduced), 5/3, and 8/5, at 701.955, 884.358, and 813.686 cents respectively, diverge most greatly from the mean of 833.090 cents towards which the ratios of the series tend. This creates a scale which is very “far from” a Moment of Symmetry. That is, it is still Well-Formed, but it’s really stretching the definition of “two interval sizes” to consider intervals that diverge from the true MOS interval sizes as widely as this to be even remotely in the same “class.” However, higher segments of the Fibonacci series do form scales which are both Well-Formed and which do have interval sizes that diverge only slightly from the two sizes of intervals found in the Phi scale, Table 17. It can be said, therefore that the scales made from the Fibonacci series “tend to” being MOS.

It becomes a matter of personal taste if one wants to use such a scale or not. The mere tendency towards MOS in additive sequences, like those discussed here, was a good enough reason for me to use them in a number of compositions, such as *The MOSsy Slopes of Mt. Meru*, discussed in Chapter 2.6, and *Proliferating Infinities,* discussed in Chapter 2.11. Theorists sometimes prefer more mathematically strict definitions. Composers, however, frequently prefer the uneven, the lumpy. Kraig Grady points out that "often the most musically interesting parts of these series lie before the area [of very similar and converging ratios], when the sequence is in the ball park so to speak and has not completely narrowed to small fluctuations" (Grady, 2005a). One of the things that make the Just- scales assembled from lower elements of additive sequences so attractive to me is their uneven, varied collection of intervals.

This is a property which can easily be heard, if, for example, one tunes both the Phi scale, shown in Table 17, and the scale made with the lower elements of the Fibonacci series, shown in Table 19, on to a keyboard. In the Phi scale, an interval of 7 scale steps is the same size, no matter what note it is based on – 833.09 cents. In the Fibonacci scale, the same interval of 7 scale steps has a different size for every single note in the scale. The sizes of the intervals vary from 813.69 to 884.36 cents, depending on the placement of the interval in the scale. This variability of interval size may annoy some, and may appeal to others. I simply regard each as a unique characteristic, which can be used in a variety of ways.

In any case, when I generated many scales from additive sequences, as will be discussed in Chapters 1.9, 2.6 and 2.11, I found that MOS scale sizes obtained by using the sequence limit as a scale generator were convenient stopping points for selecting Just-Intonation scale sizes and subset sizes. Further, although scales made from the lower numbers of an additive sequence were not strictly MOS, when these scales had the same number of scale degrees as limit-ratio generated scales that were MOS, they ***were*** well-formed. This satisfies my desire for structures which have some aspect of contradiction in them. They are almost MOS, but not quite, but are nevertheless well-formed. On a conceptual level, I'm quite pleased with this kind of contradiction.

1.6.8 MOS and equal-beating intervals

Kraig Grady (Grady, 2005a) points out another property of MOS scales, and scales made with additive number sequences – their propensity to form a series of equal-beating or proportional triads. To paraphrase him, each of these scales forms a series of equal beating or what are called proportional triads. These will generate difference tones that, if not simple consonances, are tones that occur in the scale or its seed. Thus these scales are constantly reinforcing themselves, making self-contained perceived acoustical units. The proportional triad is found by taking the "sum triplet" of the additive sequence in question and placing the top tone in between the two numbers used to generate it. For instance, consider the sequence made with this rule:

$$C_n = C_{n-3} + C_{n-2}$$

Using this rule, we get the sequence fragment 4 5 7 9 12 16 21 28 etc. Within this, we have the number sequence 7+9 =16. If these numbers, 7 9 and 16 are taken as harmonics, and if we raise the two lower tones an octave, we get three harmonics related to each other in the proportions 14: 16:18. This is a three note chord with the ratio 8/7 on the bottom and 9/8 on the top. The beats in each of these intervals are the same. One can do this for any sum triplet in any additive sequence.

I noted that this was the case, mathematically, but remained unconvinced that this would produce any distinctive results aurally. To investigate the equal beating interval phenomenon, I set up a stable triangle wave in both a 12-note scale made with the above rule, and also with a scale made with the limit-ratio of that rule, 1.324717, as a generator. (Again, limit-ratios as generators will be dealt with in Chapters 1.9; 2.6; 2.7

and 2.11.) I played chords in the proper voicing, and played all the possible “sum triplet” chords in both scales, holding them for about 20 seconds each. In all cases the resulting chords were **very** smooth and neutral but curiously satisfying. As was the case with the “similar interval” example discussed in Section 1.6.7, the Just-Intonation scale had more variety of harmonic type than the limit-ratio scale, but even the more dissonant chords produced by both scales sounded smooth and coherent. This suggests that one might use this quality compositionally and in many of his compositions, Grady has done just that. Although, in many cases, I remain unconvinced that a given mathematical property of sound can be clearly and easily perceived, in this case, through empirical testing, I was convinced.

1.6.9 Perception and MOS

Bringing up perception raises the question: can MOS properties be heard? In his early work on MOS, Wilson seemed to think so – he felt that MOS scales had a kind of aural coherence that other scales didn’t have. He speculated “It seems to me that the organizing principal must override the absolute acoustic value of the intervals, in this case [a chain of 4/3s], insofar as the perceptual apparatus is concerned” (Wilson 1975). And in 2003, in a personal note to me about David Finnemore’s CD *Golden Horagram Music Example* (Finnamore 2001b), he writes “The music examples are quite credible displays of MOS ‘scaleness’ (if I may). How about that?” (Wilson 2003a). So clearly, for Wilson, there is some kind of perceptual characteristic that MOS scales have. I and other listeners may or may not hear this characteristic. At times, as in the example discussed in Section 1.6.7, I do hear the difference; at other times, aural MOS-ness remains completely opaque to me. Hearing it may depend on musical context and on the subtlety and the quality of attention that is brought to one’s listening. Perhaps a series of sound experiments could be set up where pieces made in MOS scales and non-MOS scales differing by as little as one scale degree could be compared. Such work is beyond the scope of this thesis, but might be an interesting avenue for future exploration.

1.6.10 Ways I've used MOS

Regardless of its perceptibility or not, I have used the MOS principle in many of the works discussed in this thesis.

1) In *The MOSsy Slopes of Mt. Meru*, MOS numbers are used to determine the size of scales, and how to divide scales into subsets. See Chapter 2.6.
2) Similarly, when generating scales from additive number sequences found in Pascal's and other number triangles, MOS numbers are used to determine subsets within the 12-note scales, as in *Proliferating Infinities.* See Chapter 2.11.
3) In *Pythagoras' Babylonian Bathtub*, scales made from the nodes of the scale tree all exhibit MOS characteristics, although the way they are juxtaposed in this piece intentionally obscures that. See Chapter 2.7.
4) In *Homage to Wyschnegradsky*, the 7-note modes made with a generator of 550 cents are all MOS scales, as are the larger 13-note scale made by combining some of the 7-note scales. See Chapter 2.4.
5) In *Someone Moved in a Room*, the 18 movements are each in one of the eighteen possible 7-note Moment of Symmetry scales that have either 5L+2s or 2L+5s structures, made in 21- through 31-tone Equal-Temperament. In this case, I did indeed use the MOS structure to get scales that sounded perceivably diatonic, and was pleasantly surprised at the variety of "modal" sounds that resulted. See Chapter 2.10.

The concept of MOS is one I have returned to repeatedly, examining its musical and mathematical implications from a number of perspectives. In each of the compositions listed above, I used it in a different way. Chapters 1.8 and 1.9 also deal with other of Wilson's concepts, in which the MOS concept figures heavily.

1.7 Euler-Fokker Genera

Euler-Fokker Genera are Just-Intonation scales generated using a lattice of numbers. A lattice is an n-dimensional structure in which different harmonic factors are assigned to different axes. Though it can have many axes it is usually represented using a 2-dimensional diagram. Wilson discovered a new way to represent these scales visually vastly extending their musical possibilities (Wilson 2001). In their original form, Euler-Fokker Genera scales were generated from a limited set of harmonics multiplied in various combinations. For example, using two harmonics 3 and 5 as generators, one can form a very simple two-dimensional lattice consisting of two Perfect 5ths on one axis and a Major 3rd on the other. It would look like this:

```
A – E – B
|   |   |
F – C – G
```

Leonhard Euler proposed making scales by multiplying factors 3 (the perfect 5th) and 5 (the major 3rd) a certain number of times (Euler 1739). In the 1940s, Adriaan Fokker proposed the addition of the 7th harmonic to this (Fokker 1955, 1987), and developed a series of Genera of between 4 and 8 notes. He built a series of microtonal organs in the 1940s to play these, culminating with his 31 keyed organ of 1950. Fokker's Genera are explained in Op de Coul 2000. Adriaan Fokker writes about the genesis of his microtonal organ, and its unique keyboard in Fokker 1955. A detailed translation and consideration of Euler's original theories are found in Bailhache 1997.

Although Euler proposed the idea of a two-dimensional matrix of tones made by multiplying harmonics 3 and 5 in the early 1700s, he was not the first to do this. Large and extensive 3-5 matrices are described in ancient Babylonian and Indian writings. These are discussed in detail in McClain 1976. However, the extension of these matrices to make scales of 3, 4, 5, 6 and higher dimensions is, as far as we know, a strictly contemporary phenomenon.

Briefly, a Euler-Fokker Genus is any scale made by multiplying two or more intervals along axes to make a matrix. Some of the simplest Euler Fokker genera are simply Pythagorean scales. For example a genus of 3 3 3 would simply be a chain of three Perfect fifths, the ratio 3/2.

Table 20: Euler Genus 333 – a chain of three perfect 3/2 fifths

Note	Interval	Note	Interval	Note	Interval	Note
C	(3/2)	G	(3/2)	D	(3/2)	A
0c	(702c)	702c	(702c)	1404c	(702c)	2106c
				204c		906c

When two factors become involved, two-dimensional matrices are generated. For example a genus of 3 3 3 3 3 3 5 would be two chains of six 3/2 perfect fifths separated by a 5/4 major third, making a 14-note scale. Figure 1 shows the structure of this scale. This figure was made with the "2D Play" option in the freeware tuning software *Scala*.

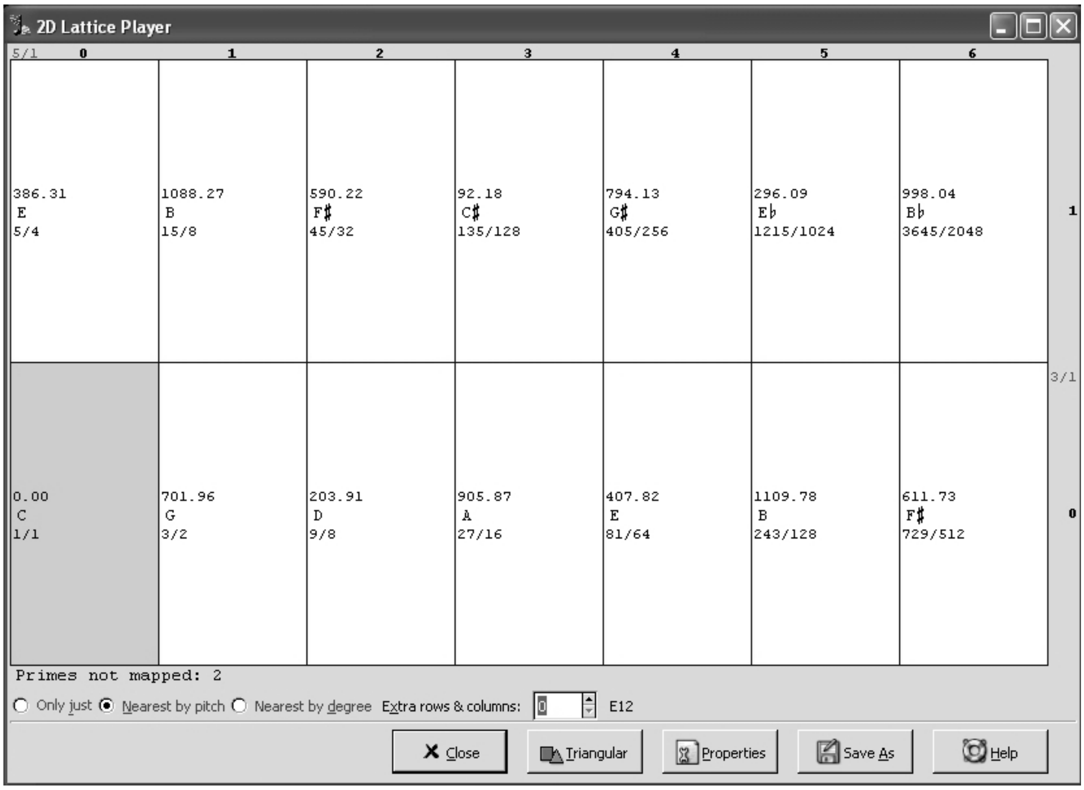


Figure 1: Euler-Fokker Genera 3 3 3 3 3 3 5 as displayed as a 2 dimensional matrix performance keyboard in *Scala* tuning software

This diagram is displayed on the computer screen and is a mouse-performable keyboard. It is a matrix of 7 x 2 notes, a scale of 14 notes. In terms of conventional tuning, there are 8 pitches represented once, with the E, B and F# each having two different versions. Notice there is no F natural.

If three chains of intervals were used, a three-dimensional lattice would be generated. Many people have used harmonic lattices like this, among them, Ben Johnston and James Tenney. Tenney's *Changes: 64 Studies for 6 Harps* is based on the mapping of a 3 5 7 matrix into 72-tone Equal-Temperament (Tenney 1987). Heidi von Gunden's *The Music of Ben Johnston* (Von Gunden 1986)**,** discusses some of the lattices Ben Johnston uses, and Johnston's "Rational Structure in Music" (1976, in Johnston 2006) discusses several examples of three-dimensional lattices used in his compositions. Finally, the harmony of Larry Polansky's *Movement for Lou Harrison*, with a harmonic series built on each of harmonics 3, 5, 7, and 11 of a very low fundamental, can be considered four-axial, if not strictly four-dimensional. A similar structure is found in his *B'rey'sheet*, for voice and interactive computer. In this piece a melody and its accompaniment are changed over time from 17-limit, through 13-, 11-, 7- and 5-limit, until the piece finishes in 3-limit Pythagorean intonation. This can be seen as a slow moving through a six-dimensional harmonic space (Polansky 1994a, 1994b).

The use of three factors, each taken only once, would be notated as 3 5 7. This matrix would involve all the possible products of all the factors, including the implied fundamental, 1. These factors are shown in Table 21. Each number represents a harmonic which will be octave-reduced to form a tone in the scale.

Table 21: All possible factors in Euler-Fokker Genus 3 5 7

Factors	1	3	5	7	3*5	3*7	5*7	3*5*7

Since there are three factors, this scale could be graphically represented as a cube, as Wilson does (Wilson 2001). This cube is given in Figure 2.

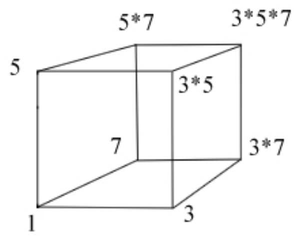


Figure 2: Euler-Fokker Genus 3 5 7 graphically represented as a cube

With each harmonic reduced so it fits within an octave, this forms the 8-note scale given in Table 22. John Chalmers, in *Divisions of the Tetrachord* (Chalmers 1993) gives several interesting ways of using this scale harmonically. These kinds of cubes form the basis for the harmonies in my composition *Saturday in the Triakontakedron with Leonhard*. In this piece, I use both Chalmers' ideas and other harmonic formulations, some of them generated algorithmically. This will be discussed in detail in Chapter 2.8.

Table 22: Listing of Euler Genus 3 5 7, factors, ratios, and cents values

Scale degree	Factor	Ratio	Cents
0	1	1/1	0
1	5*7	35/32	155
2	5	5/4	386
3	3*7	21/16	471
4	3	3/2	702
5	3*5*7	105/64	857
6	7	7/4	969
7	3*5	15/8	1088
8	1	2/1	1200

With more than three factors, lattices of 4, 5, 6, and n dimensions are obtained. In fact, it quickly becomes futile to describe these formations as "lattices." Things become very complex, with unmanageably large sets of pitches being generated. For example, the Euler Genus (3 5 7 9 11 13) is a scale made with 6 factors, each taken only once. This scale would be made by taking all the possible multiplications of all the factors, to make a scale of 64 notes. A lattice would be a totally inappropriate way to represent this scale visually. More multidimensional ways of drawing the scale are necessary, and these have been developed by Wilson, and are shown in Figures 37 and 39 in Section 2.8.2 (Wilson 2001). Elated by his ability to graphically represent complex multidimensional scales on paper, Wilson described this as "breaking through the lattice barrier." More detailed consideration of Wilson's work with these scales, and ways of dividing them into useful subsets will be found in Chapter 2.8.

In two of the works discussed later in this thesis, I have used Euler-Fokker Genera in different ways. In *18 New Fuguing Tunes for Henry Cowell,* I used all the 18 possible Euler-Fokker Genera using factors of 3 5 and 7 that would make scales of 12 notes. In *Saturday in the Triakontahedron with Leonhard,* I used a partitioning of the Euler Genus (3 5 7 9 11 13) into ten different 8 by 8 matrices. Both of these harmonic systems will be discussed in detail in Chapters 2.8 and 2.9.

1.8 The Scale Tree

The Scale Tree is another of Wilson's formations I have used. Also known as the Stern-Brocot Tree, and related to the Farey Series, it is comprised of a number of interlocking ratios. Wilson has shown that every one of these ratios, depending on how they are used, produces not only one MOS scale, but several. The Scale Tree is based on the 19th century work of George Farey, Charles Sanders Peirce, Achille Brocot, and M. Stern (Peirce 1933, Bogomolny 2006a, 2006b). Wilson has gone into great depth in his exploration of the musical implications of the Scale Tree, the Farey Series and related phenomena in a series of papers (Wilson 1994, 1996a, 1996b, 1997). David Finnemore has provided a further elegant explication of these papers in his work (Finnemore, 2002). I explored the musical implications of some of the MOS scales generated by the Scale Tree in my composition *Pythagoras' Babylonian Bathtub,* which will be discussed in Chapter 2.7.

The Scale Tree is a development of the Farey series, which is the set of all fractions in lowest terms between 0 and 1 whose denominators do not exceed N, arranged in order of magnitude (Bogomolny 2006b). Table 23 shows the Farey series of order 6.

Table 23: Farey Series of order 6

$\frac{0}{1}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$	$\frac{1}{1}$

In 1816, Farey noticed that in this series, the middle term of any three successive terms is the mediant of the other two. This can be also be expressed by saying that the each middle fraction is the sum of the numerators and denominators of the fractions on either side of it, reduced to lowest terms. For example, taking the 2nd, 3rd, and 4th elements of Table 23, we find that the sum of the numerators of 1/6 and 1/4 is 2 (1+1), while the sum of the denominators is 10 (4+6). 2/10, the resulting fraction, reduced to lowest terms is 1/5, which is the fraction between them in the series.

Wilson first considered that the Farey Series of order N is identical to the Lambdoma of the same order (Wilson 1996a). The Farey series order 4 is given in Table 24.

Table 24: Farey Series of order 4

$\frac{0}{1}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{1}$

For musical purposes, Wilson proposed that the Farey series be carried out fully from 0/1 to 1/0. This would create a symmetrical series of inverted fractions around 1/1, and would extend the series as shown in Table 25.

Table 25: Farey Series of order 4, carried out fully from 0/1 to 1/0

$\frac{0}{1}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{1}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{2}{1}$	$\frac{3}{1}$	$\frac{4}{1}$	$\frac{1}{0}$

In "A Brief History of the Lambdoma" (Hero 1995), Barbara Hero describes the history of the Lambdoma, a pre-Pythagorean multiplication and division table used as an anagram of ratios and musical harmonics. In the second century Nichomachus, among others, described it as an ordering of ratios of integers both above and below 1. Figure 3, below, gives the basic Lambdoma of order 4.

$$
\begin{array}{ccccccc}
 & & & \frac{1}{1} & & & \\
 & & \frac{1}{2} & & \frac{2}{1} & & \\
 & \frac{1}{3} & & & & \frac{3}{1} & \\
\frac{1}{4} & & & & & & \frac{4}{1}
\end{array}
$$

Figure 3: Basic Lambdoma of order 4

Figure 4 shows the lamboma filled in and octave-reduced:

$$
\begin{array}{ccccccc}
 & & & \frac{1}{1} & & & \\
 & & \frac{1}{2} & & \frac{2}{1} & & \\
 & \frac{1}{3} & & \frac{1}{1} & & \frac{3}{1} & \\
\frac{1}{4} & & \frac{2}{3} & & \frac{3}{2} & & \frac{4}{1} \\
 & \frac{1}{2} & & \frac{1}{1} & & \frac{2}{1} & \\
 & & \frac{3}{4} & & \frac{4}{3} & & \\
 & & & \frac{1}{1} & & &
\end{array}
$$

Figure 4: Lambdoma of order 4 filled in with ratios octave-reduced

The Lambdoma and the Farey series consist of the same ratios. A Lambdoma of order 11, when octave-reduced, is almost identical to the tonality diamond of Harry Partch and Augusto Novaro (Partch 1974, Novaro 1927). My 1996 composition *Portrait of Erv Wilson* from *Harmonic Colour Fields* was based on the order 11 Lambdoma / Farey Series. An explanation of its musical usage can be found in Burt 1997a.

Wilson further extended the use of the Farey series. He pointed out that when arrayed in a tree-like pattern, known in mathematics as the Stern-Brocot Tree, the ratios of successive Farey series made a series of paths based on the addition of their numerators and denominators. (Bogomolny 2006a has an elegant explanation of the many properties of the Stern-Brocot Tree, along with mathematical proofs.) In these paths each new ratio in an additive series of fractions converges onto the "Golden Section" mean (that is, Phi) of the original two ratios.

For two ratios a/c and b/d, the formula for finding the mean is (a+b)/(c+d). For example, taking two starting ratios 1/3 and 2/7, this gives a series 1/3, 2/7, 3/10, 5/17, 8/27, 13/44, 21/71 etc. As the fractions get farther out along the series, they converge on the mean between the original two fractions. Both Wilson and David Finnemore have made elegant diagrams showing these paths. Further, there are only 32 possible additive paths through this diagram. Finnemore's diagram is especially beautiful and revelatory (Finnamore 2001a, 2001c).

Please see print copy for Figure 5

Figure 5: David Finnemore's diagram of the Scale Tree (used by permission)

There are a number of ways to make scales from the ratios of the scale tree. The first is to take the ratio itself, treating the denominator D as the number of steps in an Equal-Temperament. Treat the numerator N as an interval of N steps out of the D Equal-Temperament. Use that interval of N steps, in cents, as a scale generator. As an example, consider the ratio 8/27 near the top of the middle of Figure 5, at the top of the path labelled with a blue 7. A generator of 8 steps out of a 27-tone Equal-Temperament scale yields an interval size of 355.56 cents. Stacking 7, 10 or 17 intervals of that size will produce MOS scales.

The second way of generating scales, which Wilson and Finnemore have dealt with in great detail, is to use the convergence interval of each branch of the tree as a scale generator. (These scales are also known as limit-ratio scales.) For example, the series 1/3, 2/7, etc. eventually converges on 0.295685999. Multiplying this by 1200, an interval of 354.82 cents is obtained. This interval can be used as a generator to make MOS scales. Note that this is very close to the interval of 355.56 cents we derived from 8/27.

In addition to the two ways of generating scales discussed here, I devised a third way to generate scales from these ratios. That was to take the Scale Tree fractions as Just-Intonation ratios, finding what interval they converged on. I took the convergent sequence 1/3 - 2/7 and treated each fraction as an octave-reduced Just-Intonation ratio. Table 26 shows the results.

Table 26: Convergence of Scale Tree intervals when treated as Just-Intonation ratios

Original Ratio	Octave Reduced	Cents
1/3	4/3	498
2/7	8/7	231
3/10	6/5	316
5/17	20/17	281
8/27	32/27	294
13/44	52/44	289
21/71	84/71	291
etc. down to		
377/1275	1508/1275	290.567

When 290.567 cents is used as a generator, MOS scales at 9, 13, 21 and 33 degrees result. Each of the 32 series of convergent fractions could be used to generate

scales in this way. Some examples of these scales can be seen in Tables 64, 65 and 67 in Section 2.7.3.2. A complete catalog of 11- to 19-note MOS scales made from the ratios of the Scale Tree, and used in *Pythagoras' Babylonian Bathtub*, can be found in Appendix 3.7.2.

Wilson also developed a method of visualising the MOS and non-MOS scales of a given generator (Wilson 1997a). This is called a horagram, and has been implemented in the free tuning software Scala (Op de Coul, 2007). The horagram is discussed in Section 2.7.2, and an example is given in Figure 34 in that Section.

Each of the 32 branches of the scale tree can generate a family of unique scales. Some, based on generators of N steps out of D sized Equal-Temperaments, and the convergence intervals (the Wilson-Finnemore methods) might be very similar. Scales based on my method would usually be extremely different from both. This use of convergent series, and their limits as scale generators was used extensively in *Pythagoras' Babylonian Bathtub, The MOSsy Slopes of Mt. Meru,* and *Proliferating Infinities.* A more detailed explanation of this method and its uses will be found in Chapters 2.6, 2.7 and 2.11. Further discussion of limits will be found in Section 1.9.2.

1.9 Additive Sequences, Limits and Number Triangles

1.9.1 Additive Sequences

Additive sequences have been extensively as a technique for generating new scales. An example of how scales can be generated using additive sequences was briefly introduced earlier in Section 1.6.3D, where the Fibonacci Series was used to generate various well-formed MOS scales. However, the Fibonacci Series is only one of an infinite number of additive sequences. Wilson has written extensively on these, and their special properties.

An additive sequence is any sequence of integers that follows the rule that the next element in the series is the sum of two (or more) preceding elements of the series. The best known of these sequences is the Fibonacci series:

1 1 2 3 5 8 13 21 34 55 89 etc

This series follows the rule that the next element in the series is the sum of the preceding two. This rule may be expressed as an equation.

$$A_n = A_{n-2} + A_{n-1}$$

The subscript "n" refers to the current number in the series, while the subscripts "n-2" and "n-1" refers to the 2nd and immediately previous elements in the series.

There are an infinite number of additive sequence rules, and any numbers may be used as the initial seed values for a sequence.

Number sequences may be realised as scales in a variety of ways. Three ways have been used here.

1. Treat the numbers as harmonics, and octave reduce them to form a scale.
2. Use the limit of a particular series as a ratio over 1 as a generator for a scale.

3. Multiply the limit by 1200. Read the result as cents. Octave-reduce to get a generator for a scale.

These methods are similar to the methods used with the ratios of the Scale Tree, described above in Section 1.8. We now consider limits further, and how to derive them.

1.9.2 Limits

One of the interesting aspects of additive sequences is that the ratios of all the successive pairs of elements in the series become more and more similar, converging toward a common point, the farther you go out in the sequence. For example, Table 27 lists the ratio of each successive two elements of the Fibonacci series.

Table 27: Ratio of each successive two elements of the Fibonacci series, as it tends to Phi

Series position	Ratio	Decimal Value
1	1:1	1.000000
2	2:1	2.000000
3	3:2	1.500000
4	5:3	1.666667
5	8:5	1.600000
6	13:8	1.625000
7	21:13	1.615384
8	34:21	1.619047
9	55:34	1.617647
10	89:55	1.618181

Eventually, at an indefinite point, the ratios of this series converge on Phi, the number known as the Golden Ratio, 1.618033. Phi has quite an illustrious history of its own. One of the best summaries of Phi and its uses in art and science is Livio 2002, although Livio is quite skeptical about the uses of Phi in art, architecture and music. It is important to stress here that *any* numbers plugged into the rule that governs the Fibonacci series:

$$A_n = A_{n-2} + A_{n-1}$$

will make a series that will tend to the same limit, Phi. Tables 28 and 29 give two examples of this. Table 28 uses the series 1 3 4 7 11 18 29 47 76 123 199…and Table 29 uses the series 6 5 11 16 27 43 70 113 183 296 479…

Table 28: Ratios of Additive Sequence (Fibonacci rule) with seeds of 1, 3, approaching Phi

Series position	Ratio	Decimal Value
1	3:1	3.000000
2	4:3	1.333333
3	7:4	1.750000
4	11:7	1.571428
5	18:11	1.636363
6	29:18	1.611111
7	47:29	1.620896
8	76:47	1.617021
9	123:76	1.618421
10	199:123	1.617886

Table 29: Ratios of Additive Sequence (Fibonacci rule) with seeds of 6, 5, approaching Phi

Series position	Ratio	Decimal Value
1	5:6	0.833333
2	11:5	2.200000
3	16:11	1.454545
4	27:16	1.687500
5	43:27	1.592592
6	70:43	1.627906
7	113:70	1.614285
8	181:113	1.619469
9	296:183	1.617486
10	479:296	1.618243

In all cases, the ratios approach Phi, 1.618033, as they go farther out in the series. The series of ratios is called a convergent series because its values weave around the limit value, with higher and lower values converging on the target as the series progresses. Figure 6 shows this graphically for the ratios of the Fibonacci series.

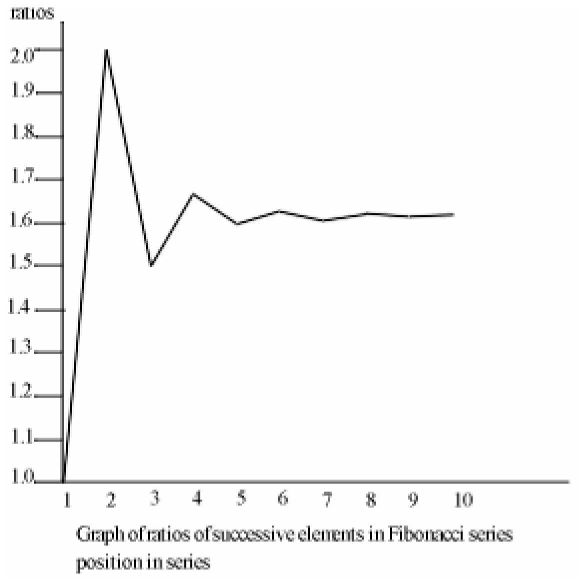


Figure 6: Graph of ratios of successive elements in the Fibonacci series

After graphing a number of these convergent ratio series, I noticed a similarity between them and the waveform of a quickly damped pulse or spike. The envelope of a quickly decaying percussive strike seemed very similar in shape to these quickly converging waveform series. This may someday be an interesting avenue of investigation.

1.9.3 Number Triangles

One way of deriving new additive sequences, and new rules for them is through the use of number triangles. Wilson has done considerable work in this area (Wilson 1993, 1997b, 2002, 2003b). A number triangle is a geometric arrangement of numbers on a page. The numbers are arrayed in horizontal rows, with the numbers of each lower row being the sum of the two numbers above them. The generating seed number, or numbers, of the triangle flow down its sides. The first of these triangles to be noticed was Pingala's Meru Prastara (The steps of Mt. Meru). Pingala, Indian mathematician, lived sometime between 200-440 BCE, and the earliest description of this triangle is in the Chandahsastra (Singh1936; Kak 2004). This triangle is made by putting a 1 at the top, and 1s down the sides of the triangle, with each other element being the sum of the two elements above it. Figure 7 shows the beginning of this triangle.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
etc.

Figure 7: Pascal's, or Pingala's, or 1-1 Triangle (beginning)

Historically, this triangle was known by many other names in different locations. In China, ca. 1250 AD, it appears as "Yang-hui's Triangle". In Italy, ca 1525 AD, it was called "Tartaglia's Triangle. And in France, around 1655, it became known as "Pascal's Triangle," the name under which it is most commonly known in the West today (Darling, 2006). For convenience, we will here adopt the convention of naming the triangles after their generating seed numbers – the numbers which flow down their sides. This triangle, therefore, can be called the 1-1 triangle.

Number triangles, however, need not have the same number flowing down both sides. For example, Figure 8 shows the beginning of the 3-5 triangle, which is made of 3s on the left side, and 5s on the right side.

3/5
3 5
3 8 5
3 11 13 5
3 14 24 18 5
3 17 38 42 23 5
etc.

Figure 8: 3-5 Triangle (beginning)

Pingala pointed out that if one added up numbers on the diagonals of the Meru Prastara, a new number series was obtained, which we now call the Fibonacci series. Figure 9 shows how this works. By adding up the numbers along the diagonals, the numbers that result (on the right edge of the triangle) form the Fibonacci series.

Please see print copy for Figure 9

Figure 9: Adding the numbers along the diagonals of the 1-1 triangle produces the Fibonacci series (diagram from Kak 2004)

Furthermore, as Wilson pointed out (Wilson 1993), within any one number triangle are an infinite number of additive sequences which are found by taking the sums of different diagonals of the triangle. Each additive sequence has its own rule and its own limit. Following Wilson's lead, I developed many different rules, triangles, scales, and limits in this manner, and used the scales in *The MOSsy Slopes of Mt. Meru* (2002-2003) and *Proliferating Infinities* (2006). In these works, I used 11 different sets of diagonals, which generated their own rules and limits. These are described in Chapters 2.6 and 2.11. Table 30 shows these rules and their limits.

Table 30: Rules and limits of additive sequences used in this thesis

Number	Sequence Rule	Limit
1	$A_n = A_{n-2} + A_{n-1}$	1.618033989
2	$B_n = B_{n-3} + B_{n-1}$	1.465571232
3	$C_n = C_{n-3} + C_{n-2}$	1.324717957
4	$D_n = D_{n-4} + D_{n-1}$	1.380277569
5	$E_n = E_{n-4} + E_{n-3}$	1.220744085
6	$F_n = F_{n-5} + F_{n-1}$	1.324717957
7	$G_n = G_{n-5} + G_{n-2}$	1.236505703
8	$H_n = H_{n-5} + H_{n-3}$	1.193859111
9	$I_n = I_{n-5} + I_{n-4}$	1. 167303978
10	$J_n = J_{n-6} + J_{n-1}$	1.285199033
11	$K_n = K_{n-6} + K_{n-5}$	1.134724138

1.9.4 Scale Realisation

As stated above, three ways of realising scales from number sequences are used in the works described in this thesis. The first is to simply take the integers of the sequence as harmonics, and octave reduce them to form a scale. One could, of course, just as easily use the integers as subharmonics, and thus form a scale that was an inversion of the scale made with harmonics. Harry Partch (Partch 1974) maintained that a “good” scale, for him, was one in which each harmonically related pitch had as its inverse companion the corresponding pitch made with a subharmonic relationship. For a series of 12 integers, this would imply combining a 12-note harmonically generated scale with its 12-note subharmonic inversion to get a composite scale of 23 notes. Subharmonic scales are not used in the present work, so they will not be dealt with here. Their potential presence, however, is acknowledged.

Table 31 shows the realisation of the first 15 elements of the Fibonacci series into a scale of 12 notes. Note that the integers 1 2 and 8 all octave reduce to 1/1, the fundamental.

Table 31: 12-note scale made by treating elements of the Fibonacci series as harmonics

Scale degree	Integer from series	Ratio octave reduced	Cents
0	1, 2, 8 (=1)	1/1	0.00
1	34 (/2 = 17)	17/16	104.96
2	144 (/16 = 9)	9/8	203.91
3	610 (/2 = 305)	305/256	303.20
4	5	5/4	386.31
5	21	21/16	470.78
6	89	89/64	570.88
7	377	377/256	670.11
8	3	3/2	701.96
9	13	13/8	840.53
10	55	55/32	937.63
11	233	233/128	1037.02
12	2	2/1	1200.0

The other way of generating scales from additive sequences used in this work is to use the limit of the scale as a generator, stack it N times, and octave-reduce the result. Two different ways, given above as methods 2 and 3, in Section 1.9.1, of deriving a generator from a limit were used. The first was taking the limit as a ratio over 1, realizing that as cents (using the pitch calculator in *Scala*, for example) obtaining the size of the generator to be used. The limit of the Fibonacci series is 1.618033989. Taken as a ratio over 1, and realised as cents, this is an interval of 833.09 cents. Table 32 shows the result of stacking this interval 11 times and octave reducing it.

Table 32: 12-note scale resulting from taking the limit of the Fibonacci series as a generator (first method)

Scale degree	Cents
0	0.000
1	99.271
2	198.542
3	297.813
4	466.181
5	565.451
6	664.722
7	763.993
8	833.090
9	932.361
10	1031.632
11	1130.903
12	1200.000

The other way of deriving a generator from a limit is simply to multiply the limit by 1200 (cents), reducing the result to make an interval within an octave. Multiplying the limit 1.618033989 by 1200 and octave reducing it gives an interval of 741.99 cents. Table 33 shows the result of stacking this interval 11 times and octave reducing it.

Table 33: 12-note scale resulting from taking the limit of the Fibonacci series as a generator (second method)

Scale degree	Cents
0	0.000
1	109.998
2	219.996
3	283.999
4	393.997
5	567.998
6	677.996
7	742.000
8	851.998
9	961.996
10	1025.999
11	1135.997
12	1200.000

Detailed explanations of these rules, their derivations, scales made from them, and how they were used in the compositions *The MOSsy Slopes of Mt. Meru* and *Proliferating Infinities* will be found in Chapters 2.6 and 2.11.

1.10 Tetrachords and the Greek modal system

One of the reasons I began studying microtonality seriously was that I was curious to see if Harry Partch's description of the ancient Greek modal system (Partch 1974) was accurate. This led me to extensive and ongoing readings in the field, including John Wallis's 1682 Latin translation of Claudius Ptolemy's harmonics, using high-school Latin and a dictionary (Wallis 1682), and later Andrew Barker's English translations of many of the source texts, including Ptolemy, Plato, Aristoxenus, Aristides Quintilianus, and others (Barker 1984, 1989). Other readings included John Landels' *Music in Ancient Greece and Rome* (Landels 1999), and Ernest McClain's more speculative *The Pythagorean Plato* (McClain 1978). From these readings, I found that not only was Partch's research accurate, but that the field of ancient music theory was one which provided a wealth of compositional resources. Since the late 1970s, I have been using the Greek modal system in my composition to such an extent that it has become one of the bases for my musical thinking. Even works in this thesis in which Greek modal ideas are not consciously used, such as *Proliferating Infinities*, or *Someone Moved in a Room,* show their influence.

The basis of the Greek modal system was the tetrachord, the 4/3 interval divided into 4 tones and 3 intervals. The primary source text for information on all things tetrachordal is John Chalmers' *Divisions of the Tetrachord* (Chalmers 1993), which not only gives ancient Greek usages, but dozens of others as well. Tetrachordal and modal ideas form the basis for the harmony of three of the works discussed in this thesis: *For JSB and JT*, *The Malleable Urn*, and *Homage to Wyschnegradsky,* as well as the two works mentioned above. Therefore, a very brief introduction to the Greek tetrachordal and modal system is given here.

Tetrachords are made by placing two pitches between the outer notes of a 4/3 perfect 4^{th}. The largest of the three resulting intervals, called the characteristic interval, was placed at the top of the tetrachord, and in ancient Greek usage, there were three types, or *genera,* of tetrachords: Enharmonic, Chromatic, and Diatonic.

- In an Enharmonic tetrachord the characteristic interval was a major 3^{rd} (of some type)
- In the Chromatic tetrachord the characteristic interval was a minor 3^{rd} (of some type)
- In the Diatonic tetrachord the characteristic interval was a major 2^{nd} (of some type)

In each case, the remaining interval was divided into two smaller intervals.

On top of this basic model, however, dozens of variations existed. The most inclusive summary of the varieties of tetrachordal tuning is found in Ptolemy's *Harmonics* (Barker 1989). Table 34 lists some of the principal tetrachords given by Archytas, Aristoxenus, Didymos, Eratosthenes, and Ptolemy. There are many other varieties of tetrachords given by ancient theorists, but these should suffice to show not only the various genera, but also the range of tolerance within which a tetrachord was considered as being of one type or another.

Table 34: Principal tetrachords listed in ancient Greek tuning theory

Archytas								
Ratios				Cents values of pitches				Name
28:27	36:35	5:4		0	63	112	498	Enharmonic
28:27	243:224	32:27		0	63	204	498	Chromatic
28:27	8:7	9:8		0	63	294	498	Diatonic
Aristoxenos								
Parts (30 parts to perfect 4th)				Cents values of pitches				Name
3	3	24		0	50	100	500	Enharmonic
4	4	22		0	67	133	500	Soft Chromatic
4.5	4.5	21		0	75	150	500	Hemiolic Chromatic
6	6	18		0	100	200	500	Intense Chromatic
6	9	15		0	100	250	500	Soft Diatonic
6	12	12		0	100	300	500	Intense Diatonic
Didymos								
Ratios				Cents values of pitches				Name
32:31	31:30	5:4		0	55	112	498	Enharmonic
16:15	25:24	6:5		0	112	183	498	Chromatic
16:15	10:9	9:8		0	112	294	498	Diatonic
Eratosthenes								
Ratios				Cents values of pitches				Name
40:39	39:38	19:15		0	44	89	498	Enharmonic
20:19	19:18	6:5		0	89	183	498	Chromatic
256:243	9:8	9:8		0	90	204	498	Diatonic
Ptolemy								
Ratios				Cents values of pitches				Name
46:45	24:23	5:4		0	38	113	498	Enharmonic
28:27	15:14	6:5		0	63	182	498	Soft Chromatic
22:21	12:11	7:6		0	81	232	498	Intense Chromatic
21:20	10:9	8:7		0	85	267	498	Soft Diatonic
16:15	9:8	10:9		0	112	316	498	Intense Diatonic
12:11	11:10	10:9		0	151	316	498	Equable Diatonic

These tetrachords were combined into various systems, such as the Lesser Perfect System, and the Perfect Immutable System. An explanation of these systems can be found in Chalmers 1993. The essential element of these systems was that two tetrachords were linked to form a scale of 7 or 8 notes. If two tetrachords were separated by a 9/8, or major 2nd, eight-note scales, the ancestors of our "white-key" modes, were formed.

In ancient Greek practice, the mode formed by combining two copies of any of the above listed tetrachords, separated by a 9/8, or major 2nd, was called Dorian. So, for example, the scale formed by Aristoxenus' intense diatonic tetrachord of 0, 100, 200, 500 cents, beginning on E:

E F G A B C D E

would be called "Aristoxenus' Intense Diatonic Dorian mode."

Any of the 8-note scales formed by combining tetrachords could be rotated, or eight-note scales could be started on any degree, forming modes. These form the basis for the modern "white-key" modes, although due to historical accident, the contemporary Church names for the modes are different than the ancient Greek names.

Table 35 shows the original Greek names and the contemporary Church names for each of the "white-key" modes. Note that if Aristoxenus' Intense Diatonic is used as the basic tetrachord, with its fundamental on E, the mapping to the modern piano keyboard is exact.

Table 35: Ancient Greek mode names compared with contemporary Church mode names

"White-key" mode	Greek name	Church name
C-c	Lydian	Ionian
D-d	Phrygian	Dorian
E-e	Dorian	Phrygian
F-f	Hypolydian	Lydian
G-g	Hypophyrgian, Ionian	Mixolydian
A-a	Hypodorian, Aeolian	Aeolian
B-b	Mixolydian, Hyperdorian	Locrian

Given that there are many varieties of tetrachords, and that these tetrachords can be combined in several ways to make scales, and that each of these scales can then be rotated into a series of modes, it can be seen that the resources offered by this system are huge. Further, if contemporary structural practices are used, such as permuting the order of the intervals in a tetrachord, or in a resulting scale, one encounters a further exponential increase in compositional resources. This ever expanding world of modal resources has provided me with much fertile ground for exploration in the past 30 years. The ways in which ancient Greek modal ideas have been used in the current work will be covered in detail in Chapters 2.1, 2.2, 2.3, 2.4, and 2.10.

1.11. *ArtWonk* Probability Distributions and Additive Sequence Generators

Ever since I started composing music, I have investigated many different kinds of "random" information generators, using them to control aspects of pieces. Some of these generators have been truly random, such as sample-and-hold white noise; others have been complex and deterministic, such as chaos equations; while others have been happily quirky and idiosyncratic, such as the use of Dream Books (gambling charts mostly used in African-American communities) as information generators (Rabo 1959, 1980). Since the late 1980s, one of my most useful tools has been the probability distribution module, where one can draw any probability distribution desired, using its scaled output in any way. This function has been implemented in many of the composing tools I use, most recently by John Dunn in his algorithmic composing programs *SoftStep* and *ArtWonk* (Dunn 2004a, 2007).

Over the past 10 years, I've worked as a volunteer beta-tester for John Dunn on these programs, and in 2004-7, using his user-functions module, designed a number of functions and macros for *ArtWonk* which have been included in the current commercial release of the program. These distributions were added to the already rich set of fractal, chaotic and random resources of the programs to provide a larger set of random resources than just the few standard distributions (Dodge and Jerse 1995) available in a number of programs. The distributions developed for *ArtWonk* include:

- Beta
- Borel
- Bradford (2 versions)
- Burr

- Cauchy
- Exponential
- Extreme LB
- Gaussian
- Generalized Logistic
- Gumbel
- Laplace (Bilex)
- Linear (6 different slopes)
- Pareto
- Reciprocal
- Triangle
- Weibull

Additionally, several chaotic attractors were programmed. These were:

- tENT attractor
- Sine attractor
- Logistic attractor
- Henon (2D) attractor.

Three other functions were included. These were:

- the Lehmer Function
- a Shift Register Feedback function
- a 4-variable Iterated Function System (IFS).

Those wishing a detailed description of each function are referred to the *ArtWonk* manuals (Burt 2005e), found in pdf form in Appendix 3.12.20.

In addition to Dodge and Jerse, the other source for the probability distributions was A *Compendium of Common Probability Distributions* (McLaughlin 1999). The source for the attractors and the IFS equations was Peak and Frame 1994. Ames 1992 and Battey 2004 were also useful, and provided the Lehmer function. Additionally, I developed a whole family of additive sequence generators which implement all of the additive sequence rules found in Table 30, Section 1.9.3.

Each of these software generators has its specific uses. For example, many of the modules have control inputs where the shape of the output can be changed in real-time in a continuous manner. This is a unique feature within these programs. For example, the Lehmer Function, in common with many of the probability distributions and attractors, has two inputs, A and B. For the Lehmer function:

- If A+B = <1, a single repeating value is produced.
- If A+B = slightly more than 1, repeating patterns with some degree of randomness are produced.
- As A + B get larger, the result gets closer and closer to a uniformly distributed pseudo-random sequence, until
- Uniform randomness is achieved when A+B > 10.

This kind of changing function can produce radical changes in musical gestures, This kind of change is at the heart of *Lehmer's Kookaburra.* A more disjunctive kind of changing musical gesture is found in *Homage to Wyschnegradsky*, where a variety of low and high weighted linear random distributions, or triangular random distributions of varying slopes were selected for each new section. This switching produced quite radical changes in musical texture. These uses will be discussed further in Chapters 2.4 and 2.5.

The attractors also produce changing results with changing inputs. Although none of the works described in this thesis use them, a quick description of the Sine Attractor will show this. The module has 3 inputs. The strobe input generates a new value with each clock pulse. The other two inputs, "S strt" and "X strt" change the nature of the attractor's output. Changing values of between 2.5 and 3.14 (pi) in the "S strt" input will produce radically different kinds of gestures from the output. Changing values of between 0 and 1 in the "X strt" input, while leaving the value in "S strt" unchanged, will produce different details in the output gestures, while leaving the overall sense of gesture intact.

Harry Partch, Kenneth Gaburo, and Herbert Brun all talked about the necessity for the artist to create their own tools, and they all did so; Partch with his theoretical work and his instruments (Partch 1974, Blackburn 1997), Gaburo with his work with ensembles and electronics, and especially, his use of the "scatter" technique, which treated the human body as the ultimate random information generator (Gaburo 1987); and Brun with his development of computer synthesis programs and his idea of "socially-beneficial information processors," which extend this principle well beyond the bounds of music, or even the arts in general (Brun 2004). My work in developing tuning systems, algorithmic resources, and new acoustic and electronic sound sources can be seen as an extension of this pioneering, cross-disciplinary work. I've found developing these tools exciting. And beta-testing work with a software designer enabled me to develop software tools in a collaborative, interactive way that allowed all members of the team to contribute their individual strengths. The following quote from a note to myself, made while programming these functions, reveals some of this excitement.

> At first the mathematics for the probability distributions were opaque, but after a series of long train rides where I could sink into the maths in an uninterrupted manner, I began to realise that all of them were just shaping of equally-weighted randomness in some way. That is, they were all just very slow versions of filtering white noise!
>
> As a way of proving this, I set up a patch with the Vaz Modular software synthesizer where white noise was passed through a variable state filter, which provided the input for a sample and hold. This controlled the pitch of a sine wave oscillator. By changing the resonance, the frequency, and the shape of the filter, different spreads of 'random' pitches were indeed obtained. This suggest a further avenue for exploration, as the kinds of random spreads of values produced by this filtered noise were often as expressive of the characteristics of the filter as of the noise source being used. More material to play with, to explore, to hear. The universe is unbelievably rich.

The additive sequence generators proved to be unexpectedly powerful. When the output of an additive sequence generator is divided by N, and the remainder is output (an operation known as modulo-N division), the result is unpredictable, but deterministic sequences of various lengths, from very short to extremely long. Especially at higher modulo values, such as 61, the sequences are not only several thousand elements long, but they have all sorts of interesting self-similar gestures contained within them, in a way that is quite unlike the output of simple random generators. This is probably due to the recursive nature of the additive sequence rules, but much more research needs to be done on this. A good description of lower modulo limits applied to additive sequences is in Armstrong 2005. For now, it is sufficient to note that the output of these additive sequence generators, when applied to pitch over a 5 octave range, produced unusually pleasing musical results. These modules were used to generate the main melodic material for *Proliferating Infinities.* This will be discussed in more detail in Chapter 2.11.

It can be seen that applying probability distributions, and other functions of various sorts to the scalar resources described earlier, especially if sieves (filters) of various kinds are used to select subsets of the total material available, can result in systems with a great deal of power and compositional subtlety. Xenakis 1992 covers sieves in extensive and exquisite detail. I find enormous potential in the combination of these kinds of systems and resources, and hope that the compositions described in this thesis provide some examples of that.

The complete catalogs of these functions, taken directly from the *ArtWonk* help files, are found in Appendix 3.12.20. More detailed explanations of how the specific distributions were used in specific pieces will be found in Chapters 2.4, 2.5, 2.6, 2.8, and 2.11.

1.12. Some notes on the technologies used in these pieces.

All of these works discussed in this thesis were composed with the aid of a computer, but not all use electronically generated sound. *After JSB and JT*, and *The Malleable Urn* are for harp and baritone ukulele, respectively. Several other works were for unique, home-made acoustic instruments. *The Animation of Lists And the Archytan Transpositions* is for my set of self-made Just-Intonation tuning forks built at the CSIRO National Measurement Lab, Melbourne, in 1985 (Burt 1987a, 1987b). *Homage to Wyschnegradsky* and *Lehmer's Kookaburra* are for the home-made computer-controlled acoustic instruments "tubi" and "puff" at the Logos Foundation, Gent, Belgium. The instruments were designed and built by Godfried-Willem Raes, and programmed by Kristof Lauwers (Raes 2006). The internet-based process by which these works were composed will be discussed in detail in Chapters 2.4 and 2.5. The rest of the works are either for electronic sound, or acoustic instrument samples. My principle software synthesizer for this work was Martin Fay's *Vaz Modular*. Matthias Ziegs' *VSampler3* was used for *18 New Fuguing Tunes for Henry Cowell*, and *Proliferating Infinities*. In the case of the Cowell work, the Emu systems *Proteus 1, 2, and 3* soundfont sets were used. This will be discussed in Chapter 2.9.

All of the work presented in this thesis was made with a Pentium IV Asus laptop, running a variety of synthesis, algorithmic, and sound editing software. The main software used included *Scala* (Op de Coul 2007), *ArtWonk*, *SoftStep*, and *DataBin* (Dunn 2007, 2004a, 2004c), *IC* (Culver-Cage 1984-93), *PD* (Puckette 2006), *Vaz Modular* (Fay 2004), *VSampler3* (Ziegs 2006), and *Adobe Audition* (Adobe 2006). Data for *Someone Moved in a Room* was provided by Arthur Jenkins and team at the Wollongong Room Calorimeter Project, University of Wollongong.

Several of these works were also designed as real-time interactive performances. Performance interfaces, mainly using the computer mouse as the performing device, were developed using *SoftStep*, *ArtWonk*, and *Scala*. Curiously, none of the works made here involved the use of keyboards, either of the traditional or experimental variety. The design of keyboards for microtonal performance is an area that I wish to pursue in the future, as is the design and construction of other microtonal acoustic instruments. Projects for the immediate future include construction of a 72-tone acoustic guitar, and a set of metallophones with removable keys tuned to Partch's 43-tone scale.

Most of the works composed for this project were performed and recorded by myself. Catherine Schieve was the co-performer for *The Animation of Lists And the Archytan Transpositions*. This work was subsequently commercially released as a double CD on the New York based XI label (Burt 2006). The recording was produced by Al Margolis for Phill Niblock. Kristof Lauwers, at the Logos Foundation, operated and recorded tubi and puff, the Logos instruments for *Homage to Wyschnegradsky* and *Lehmer's Kookaburra* (Raes 2006). Finally, the one work for a traditional classical acoustic instrument, *For JSB and JT*, for harp, remains unperformed. However, a MIDI realisation of the work was made, and is included, along with recordings of all the other works, in Appendix 3.1.1.

PART TWO- DISCUSSION OF WORKS

This part of the thesis is a discussion of a folio of eleven works based on the ideas in the previous Chapters. The context of each work will be introduced, followed by Sections in which the tuning system, the algorithm and the real-time process used in the piece are presented.

2.1 *For JSB and JT: Non-Directional Journey Out from the Enharmonic*

2.1.1 Introduction

In this piece, the tuning system is influenced by the construction of the instrument. Written in April 2004, it began with my wondering about the possibility of microtonal retuning on a modern chromatic pedal harp, and what the harmonic implications of those pedals on a microtonal tuning would be. I was intrigued by the ability of the pedals to transpose strings up and down an Equal-Tempered semitone from their initial tuning.

I had already written a series of works for microtonal guitar, including *My Monodies I & II* (1996), written for Larry Polansky, and *Fast Random Walk Around a Microtonal Fingerboard* (1999), written for Claudio Calmens, (Burt 1997b, 1999, Polansky 1997) in which retuned strings and 12-tone Equal-Temperament frets combined to make the required microtonal scales. I wondered if this notion could be extended to the harp. For simplicity's sake, I decided to start with 24-tone Equal-Temperament.

There are a number of good reasons for not writing in the 24-tone Equal-Tempered scale, and many people, including Harry Partch and Ben Johnston (Partch 1974, Johnston 2006) have enumerated them. The chief objection is that the 24-tone scale does not address the differences between 12-tone Equal-Temperament and Just-Intonation. The only three small-ratio Just-Intonation intervals found in 24-tone Equal-Temperament are the 3/2 (702c), 11/8 (551c), and 19/16 (297c). All other small number ratios are just as far from 24-tone pitches as they are from 12-tone pitches. Unless one wanted to work with a 3-11-19 Euler-Fokker matrix, a matrix of 3/2 Perfect 5ths along one axis, 11/8 flat tritones the second axis, and 19/16 minor 3rds on the third axis, 24-tone tuning is a very inadequate way to try to approximate Just-Intonation. Another objection to 24-tone tuning is its unyielding symmetry. The scale is simply two interlaced 12-tone scales. In certain harmonic systems, this will tend to limit possibilities for modulation.

However, as Kenneth Gaburo says, in *The Beauty of Irrelevant Music,* (Gaburo 1970)

> As a composer, the statement: "a given system is the only tenable one," constitutes the only challenge necessary to disprove that statement.

Restrictions and objections are often the only motivation a composer needs for a project or an idea. Therefore, I began investigating the intrinsic resources of the quarter-tone scale.

Three years on, having worked with the scale in a variety of both acoustic and electronic contexts, I find that I really like the sound of quarter-tone intervals. I also enjoy the contrast in moods between works composed in Just- scales, and works composed in 24-tone Equal-Temperament. The structure of a recent composition of mine, *3 Watermoods* (2006), is based on precisely such a contrast, the final quarter-tone movement having a much duskier, unfocussed sound, avoiding precisely the "Maximum Clarity" of the earlier Just-Intonation movements. (The reference here is to the title of Johnston 2006.) Most recently, I find that I'm once again being attracted to the 3-11-19

Euler-Fokker matrix. I'm intrigued to see what kind of music I can make with its somewhat limited resources.

In investigating the harp, I found that if some of the strings were detuned by a quarter-tone, interesting subsets of the 24-tone scale were obtained by using the pedals. This could apply equally well to any Equal-Tempered scale which had 12 as a factor. In the future, if I have access to a chromatic harp and / or an interested harpist, other studies could be done in, for example, 18-, 30-, 36-, 42-, and 48-tone Equal-Temperaments, and Just- tunings in which transposition by 18/17 (99 cents) was important. Detuning every string on the harp would also make non-octave based scales possible. A further possibility exists with multiple harps, detuned appropriately, such as James Tenney's *Changes: 64 Studies for 6 harps*, in 72-tone Equal-Temperament (Tenney 1987).

The title of the piece should be self explanatory. JSB and JT are, of course, Johann Sebastian Bach and James Tenney, the two models for the piece. The Enharmonic is the ancient Greek genus, and its Dorian form is the starting point for the piece. And due to the nature of the algorithm used, described below in Section 2.1.3, progress through the series of modes is non-directional, without an implied harmonic goal.

Hopefully, this piece has a kind of elegant simplicity in both its conception and notation, and yet produces an interesting result when sounded. I sometimes enjoy writing music which is much more complex sounding and complexly structured than this. However, when all the elements of composition, sound and thought come together in a very simple result that has a ring of "rightness" to it, I find one very particular kind of satisfaction.

2.1.2 Tuning System

Aristoxenus described the ancient Greek Enharmonic Genus as being in quarter-tone tuning (Barker 1989, Chalmers 1993). Even if our reading of Aristoxenus as an advocate for Equal-Temperaments of various kinds is mistaken, the placement of ancient Greek modes into those tunings produces very pleasant and evocative results. Investigations were begun to see if there was a harp tuning where the Dorian form of the Enharmonic was available.

Using the notation C+ for a quarter-tone (50 cents) higher than C and C- for a quarter-tone (50 cents) lower than C, this scale can be notated in the following manner:

C C+ C# F G G+ G# C

After much work, I evolved a tuning where this was possible, and where no string needed to be tuned down more than a whole tone (200 cents). The tuning was this:

1) C, F, G strings as normal. Table 36 shows the pitches available on these strings.

Table 36: Pitches available on C, F and G strings of harp in special tuning of *For JSB and JT*

String name:	C	F	G
Pedal up (tune flat):	B	E	Gb
Pedal mid (tune natural):	C	F	G
Pedal down (tune sharp):	C#	F#	G#

2) D and A strings are tuned down 1/4 tone. Table 37 shows the pitches available on these strings.

Table 37: Pitches available on D and A strings of harp in special tuning of *For JSB and JT*

String name:	D	A
Pedal up:	C+	G+
Pedal mid:	D-	A-
Pedal down:	D+	A+

3) E and B strings are tuned down a whole tone. Table 38 shows the pitches available on these strings.

Table 38: Pitches available on E and B strings of harp in special tuning of *For JSB and JT*

String name:	E	B
Pedal up:	C#	Ab
Pedal mid:	D	A
Pedal down:	D#	A#

Table 39 shows the pitches available in the entire tuning.

Table 39: Complete set of pitches available on all strings of the harp in special tuning of *For JSB and JT*

String name:	C	D	E	F	G	A	B
Pedal up:	B	C+	C#	E	Gb	G+	Ab
Pedal mid:	C	D-	D	F	G	A-	A
Pedal down:	C#	D+	D#	F#	G#	A+	A#

This tuning gives 18 out of the possible 24 tones. Some tones occur on two strings, and some tunings produce notes whose pitches are lower than their lower neighbour string, producing scales that are non-monotonic, that is, not all ascending or descending. Table 40 lists the complete 24-tone scale on one line, with the available pitches placed below that. A “2" below a pitch means that it occurs on two different strings.

Table 40: Available pitches in the special harp tuning of *For JSB and JT* compared with pitches of 24-tone Equal-Temperament

Complete:	C	C+	C#	D-	D	D+	D#	E-	E	E+	F	F+	F#	G-	G	G+	G#	A-	A	A+	A#	B-	B	B+
Available:	C	C+	C#	D-	D	D+	D#		E		F		F#		G	G+	G#	A-	A	A+	A#		B	
			2										2				2							

With this tuning, Aristoxenus’ Enharmonic Dorian mode was available with the harp pedals set in the following manner:

D-up C-mid B-up E-up F-mid G-mid A-up

Figure 10, below, shows this in harp pedal notation.

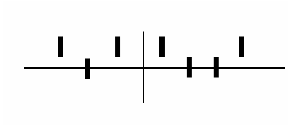


Figure 10: Harp pedal notation for Aristoxenus' Enharmonic Dorian tuning

Table 41 shows how this tuning produces the mode on the indicated strings.

Table 41: Pedal settings for Aristoxenus' Enharmonic Dorian on harp strings in special tuning of *For JSB and JT*

Strings:	C	D	E	F	G	A	B
Pitches:	C	C+	C#	F	G	G+	G#

The harp pedal notation, and the pitch resources available from changing pedals suggested an idea - if one pedal were changed, a different mode would result. A piece could be structured around a series of changing modes, each of which was the result of changing one pedal. This would create a constantly changing sense of harmony without excessive pedal changes, yet pedal technique would be an essential part of the piece. The piece would be uniquely for the chromatic harp, its tuning structure arising out of the instrument's design.

2.1.2 Algorithm

While developing this tuning, I heard James Tenney's *August Harp* (1971), a harp piece that consists of a single slow four note figure repeated over and over, with pedalling changes producing harmonic change (Tenney 2004). *August Harp* reminded me of the first prelude of JS Bach's *Well-Tempered Clavier*, which also consists of a single repeated gesture with only the harmonies changing from repeat to repeat.

The existence of two pieces of this type provided me with a precedent. I do not feel that precedents are necessary for composition, but their existence is at least comforting. I chose a figure that was extremely simple. This would be repeated over and over, with one pedal changed at the end of each figure, producing a different scale for each repeat. As with the Bach and Tenney pieces, I wanted to focus exclusively on harmonic change and colour, so I chose to eschew any rhythmic or gestural complication in my melodic figure.

The figure was the ascending scale available on the strings.

- First in seconds, to make scalar motion
- Next in thirds, to make traditional chordal arpeggiation,
- Then in fourths,
- Finally in fifths.

Each expansion of the figure highlighted different harmonic aspects of the available scale, and introduced a new register.   Further, as pedal changes were introduced which produced non-monotonic scales, the ascending nature of the notated figure would be broken up. For example, a scale with pitch C# produced on the C string, and pitch C+ produced on the D string will produce a descending motif within a mostly ascending scale. Figure 11 shows the chosen figure.

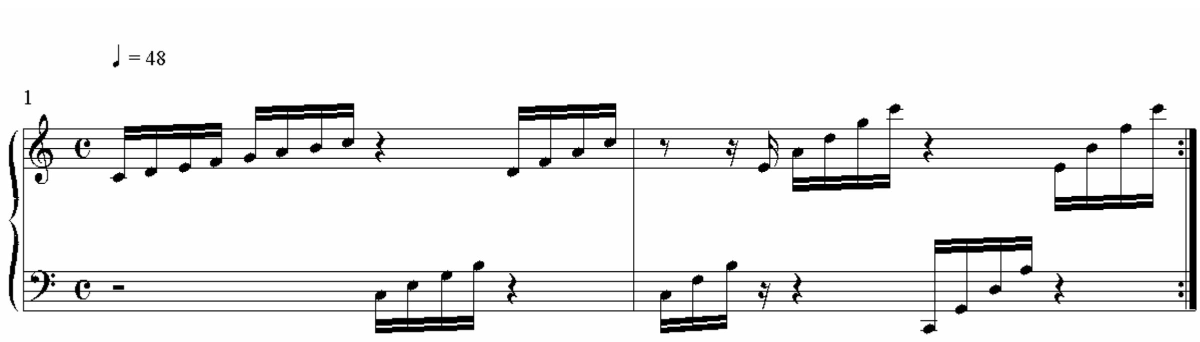


Figure 11: Basic motive of *For JSB and JT*, for retuned chromatic pedal harp

The use of this motive, repeated endlessly with only harmonic change from repeat to repeat implied that I was writing a minimalist piece. The next matter to decide was how to structure the progression of modes, each of which would be made by changing one pedal from the previous. I decided to eschew any sense of consciously chosen harmonic direction in favour of a non-directional randomly chosen harmonic sequence.

Aesthetically, I was much more comfortable with the idea of a journey which seemed to “go nowhere”, but which wandered freely, rather than with progressions which applied ideas of directional harmony to the resources of a new scale. In terms of harmonic motion, I was much more attuned to the ideas, say, of Jack Kerouac, than those of Johann Jacob Fux. A fuller discussion of non-directionality and Kerouac’s influence on my structural thinking is found in Burt 1988. A fuller discussion of Fux’ structural ideas is contained in Wellesz 1965, which also has some wonderful descriptions of Fux’s outdoor pageants, which sound like early baroque analogies to contemporary sound and light spectacles.

A very simple algorithm was developed to determine which string was to be changed for each new mode, and in which way the string was to be changed. Following Cageian procedure (Cage 1993), this algorithm can be expressed as a series of questions. For each new mode, two questions were asked:

1. Which string is to be retuned in the new section?

2. Which new position is the pedal to be moved to?

For the first question, Andrew Culver and John Cage's *IC* program (Culver and Cage 1993) was used to generate non-repeating sequences of random digits from 1-7, with no digit repeated until all were chosen. This chose the next pedal to change.

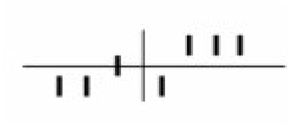
For the second question, either a 1 or a 2 was generated, with an equal probability of either occurring. This determined how the pedal was to be changed. Depending on the existing position of the pedal, there were three possibilities:

1. If original pedal position is flat (up), 1= natural (mid) and 2=sharp (down)
2. If original pedal position is natural (mid), 1= flat (up) and 2=sharp (down).

3. If original pedal position is sharp (down), 1= flat (up) and 2=natural (mid).

For each new measure, the appropriate numbers were generated. Harp pedal diagrams were made with the resulting scales listed afterwards. Figure 12 shows the result.

28.



(C# D+ D# Fb Gb G+ A C#)

(Ab)

Figure 12 Notation for *For JSB and JT*, for retuned chromatic pedal harp

- The number 28 indicates the section.
- The scale in parentheses gives the scale resulting from the pedal positions.
- The pedal diagram gives the pedal positions after changing the indicated pedal.
- The Ab in parentheses under the pedal diagram gives the name and direction of which pedal to change.

Except for the basic motive, it was decided to notate the piece as a series of pedal changes and resulting scales as in Figure 12, rather than notating the piece in traditional staff notation. It was decided that traditional staff notation would only be too confusing, and contradictory to the way in which the piece was actually played, with a constant repeating pattern on the strings altered by pedal changes. I feel that this notation is the most compact, easy, and efficient way of presenting the piece. The score to the piece is included in Appendix 3.1.2.

It can be seen that the process by which new modes are chosen is very simple. This is in keeping with the simplicity of the piece as a whole. The overall length of the piece is 57 repetitions of the opening motive. This length was also chosen by the *IC* random-number generating program.

Additionally, when I made a MIDI realisation of the piece, in order to avoid a metronomic feel, I used *IC* to pick tempi for groups of measures. Tempi varied between 44 and 52 beats per minute; a new tempo was picked every 2-6 measures. Both these values were chosen by *IC* within the given boundaries. The recording of the MIDI realisation is in Appendix 3.1.1.

2.1.3 Process – Real Time Usage

This piece is the most traditionally oriented of all the pieces discussed in this thesis. Although it has an unusual notation, it is a fully notated piece for a performer to play in the Western interpretive tradition. However, due to the nature of the instrument, and the process used, certain adjustments will have to be made in live performance.

For example, for those changes which involve adjusting the C pedal, and for those in which moving the pedal immediately after playing its string would introduce an unseemly glissando, a slight *luftpause* might be inserted between each of the subsequent repetitions of the motive. This will become a characteristic of the live performance, and will vary from performer to performer.

The use of changing tempi gives the MIDI realisation its own identity, just as the live version is given its identity by the insertion of pauses into the piece to deal with pedalling changes, and the performer's own way of playing. I am pleased that each instance of the piece has its own intrinsic characteristics.

I'm also quite pleased with the sound of the piece, based on its MIDI realisation. The harmonic changes are what I was aiming at, and the piece does indeed wander non-directionally in a quite pleasant manner. The metaphor for harmonic motion here might be that of the mobile, moving unpredictably within its constraints, rather than the arrow of traditional harmony, moving unerringly towards its goal.

The piece has yet to receive a live performance. I sent the piece to James Tenney, the one (at the time) living dedicatee of the piece. He enjoyed it, and gave it to a harpist colleague at California Institute of the Arts. However, I am still waiting to hear a live performance, to have the added joy of irregularity that a live performer can bring to a work.

2.2 *The Malleable Urn*

2.2.1 Introduction

The Malleable Urn is an hour-long piece for solo baritone ukulele, restrung and tuned in quarter-tones. It is an extremely quiet and very slow piece. It came about for several reasons. First, I met artist Carol Ruff at a party in Clovelly, a beachside suburb of Sydney, in July 2004, just after we moved from Melbourne to Wollongong. She told me about a show she was organizing called *Hula Dreams* which featured artist-painted ukuleles. I told her I would be happy to make a piece for baritone ukulele, and would perform it in her gallery on a Sunday afternoon during the show.

At the time of composing this piece, I was experiencing a sense of utter revulsion at the levels of high volume that seemed to be dominating both the electronic music and the improvisation scenes, and also a sense of total alienation from the higher-faster-louder fast-and-concise sound byte culture that seemed to be dominating commercial media. My difficulty with extreme amplitude is discussed in Burt 2005a. My difficulties with contemporary sound byte culture have already been referred to in Chapter 1.4. My response to these issues was to make a piece that was slow, soft, and demanded intense concentration on each individual sound or group of sounds as they occurred. This was an attempt to find what I regarded as one of the last beauties that remained available to us, the beauty that exists in quiet, intimate situations.

On a more practical level, I realised that writing a piece for my personal performance on baritone ukulele would necessitate it being fairly easy to play - my uke/guitar technique is hardly of the highest order. An interest in making the piece performable, even by an earnest amateur was a motivating factor.

Since my work with *Fatty Acid*, the trio of Ron Robboy, David Dunn, and myself, throughout the 1970s, in which we performed serious classical music on violin, mandolin and accordion with less than adequate technique, I have been exploring what I have called an aesthetic of radical amateurism. After the *Fatty Acid* experience, which largely had humour as its core, I turned to radically amateur performing of a more serious intent, developing a minimalist performance technique of great austerity and simplicity. In 1998, I discussed this aesthetic in an interview with Elizabeth Dempster, one of my choreographer collaborators from the early 1980s, with whom I had made several works in this vein (Burt-Dempster 1999).

Working with dancers was a primary motivation for developing both the more serious side of the radical amateur aesthetic and in making works for baritone ukulele. I bought the baritone ukulele, basically a sweeter sounding, half-size, four-string guitar, in 1978, for duo performance work with choreographer Eva Karczag, because a portable, easy-to-play string instrument was needed. I could play ukulele, four strings seemed easier to control than six, and the lower range of the baritone uke (from D below middle C) gave a more mellow timbre than the smaller, shriller ukulele. Among the works I composed for the instrument, and for my work with her, was the hour-long minimalist *Pocket Calculator Music I - 39 Penguins by Moonlight* (1978), which we performed at the Little Lonsdale Street Studio in Melbourne and the Watters Gallery in Sydney in 1978. This was a slow, repetitive, monophonic work, in normal 12-tone tuning, which was generated by a particular algorithm I developed for deriving random numbers from an ordinary pocket calculator. The use of a pocket calculator, rather than a computer, was also motivated by my low tech, radical amateur aesthetic. I returned to this algorithm twice more, once in the early 1980s, and again in 2005. The work produced in 2005, *18 New Fuguing Tunes for Henry Cowell (Pocket Calculator Music III)* will be discussed in Chapter 2.9.

2.2.2 Tuning System

As discussed in Chapter 2.1, I had previously made a number of microtonal guitar pieces which involved retuning the strings of the guitar while leaving the frets unchanged (Polansky 1997, Burt, 1997b, 1999). This strategy worked so well, I decided to pursue it again. After several months of experimentation, I realised that I would also have to restring the ukulele if I wanted a quarter-tone tuning that had a consistent timbre. The normal baritone ukulele tuning is D G B E, exactly like the upper four strings of the guitar. The tuning I decided on was D D+ A A+. The + sign indicates a string raised a quarter-tone. In order to efficiently achieve this, the uke was restrung with two wound steel D strings and two nylon B strings: D D B B. This stringing and tuning gives just over two complete octaves of pitches in 24-tone Equal-Temperament, and is an easy tuning to learn and to play. This restringing was quite successful. After making this work, the quarter-tone baritone ukulele has remained in my office, constantly available. Almost every day, I improvise a little on it, savouring its tuning and dusky, mellow timbre.

In this piece, pitch range is limited to the lowest octave. There were at three reasons for this:

- First was the sense of constrained resources that I wished to have in the piece
- Second was my limited technique - I found that I could easily get around in one octave without too much finger stretching
- The third reason was harmonic - with the two outer D's forming a kind of harmonic drone just by reason of their always being there and never changing,

the harmonic focus of the piece went inward, to the structures of the scales I was using.

As with *For JSB and JT*, discussed in Chapter 2.1, in this piece I decided to use Aristoxenus' Enharmonic Dorian mode as a starting point. My approach here would be different, though. I used the Enharmonic tetrachord itself as my primary motive.

The enharmonic tetrachord, as defined by Aristoxenus (Barker 1989, Chalmers 1993) consists of two quarter-tones followed by a Major 3^{rd}. For convenience, notate this as

Q Q 3.

The Enharmonic Dorian mode was made by having two of these tetrachords separated by a major 2^{nd}. This can be notated as

Q Q 3 2 Q Q 3.

A set of seven modes could be made by rotating the order of these intervals, just as the "white key" modes on the piano can also be conceived of as rotations of a basic intervallic order. Table 42 shows the seven possible modes of this scale.

Table 42: Possible modes made by rotating order of intervals of Aristoxenus' Enharmonic Dorian mode

1:	Q Q 3 2 Q Q 3
2:	Q 3 2 Q Q 3 2
3:	3 2 Q Q 3 Q Q
4:	2 Q Q 3 Q Q 3
5:	Q Q 3 Q Q 3 2
6:	Q 3 Q Q 3 2 Q
7:	3 Q Q 3 2 Q Q

The intervals of the enharmonic tetrachord can also exist in 2 other orders. Table 43 shows the three possible orders of two quarter-tones and a Major 3^{rd}.

Table 43: The three possible orders of two quarter-tones and a major 3rd

1:	Q Q 3
2:	3 Q Q
3:	Q 3 Q

The second and third of these tetrachords can also be made into 7-note scales by having two identical tetrachords separated by a major 2nd. This is shown in Table 44.

Table 44: Seven-note scales made from tetrachords 2 and 3 in Table 43

1:	3 Q Q 2 3 Q Q
2:	Q 3 Q 2 Q 3 Q

Each of the seven-note scales in Table 44 can also exist in seven different rotations. A total of 21 different scales can be formed from rotation of the scales made by combining the three tetrachords in the traditional ancient Greek manner. That is, two copies of the identical tetrachord are separated by a major 2nd. Making scales by combining different tetrachords, although discussed in Ptolemy's *Harmonics* (Barker 1989), is not used here. Table 45 gives a complete listing of the 21 scales made with this permutation and rotation procedure. The numbers given to the scales in Table 45 corresponds with their numbering as used in this piece.

Table 45: Complete listing of quarter-tone modal scales used in *The Malleable Urn*

1:	Q Q 3 2 Q Q 3	**2**:	3 Q Q 2 3 Q Q	**3**:	Q 3 Q 2 Q 3 Q
4:	Q 3 2 Q Q 3 Q	**5**:	Q 3 Q Q 2 3 Q	**6**:	3 Q 2 Q 3 Q Q
7:	3 2 Q Q 3 Q Q	**8**:	Q Q 3 Q Q 2 3	**9**:	Q 2 Q 3 Q Q 3
10:	2 Q Q 3 Q Q 3	**11**.	3 Q Q 3 Q Q 2	**12**:	2 Q 3 Q Q 3 Q
13:	Q Q 3 Q Q 3 2	**14**:	2 3 Q Q 3 Q Q	**15**:	Q 3 Q Q 3 Q 2
16:	Q 3 Q Q 3 2 Q	**17**:	Q 2 3 Q Q 3 Q	**18**:	3 Q Q 3 Q 2 Q
19:	3 Q Q 3 2 Q Q	**20**:	Q Q 2 3 Q Q 3	**21**:	Q Q 3 Q 2 Q 3

These scales, when taken together, play 20 of the possible tones in 24-tone Equal Temperament. Only pitches 3, 7, 17 and 21 are unused. This is a result of the structure of the basic tetrachord in its three permutations. None of the combinations of intervals in any of the permutations allows either pitch 3 or 7 to occur.

2.2.3 Algorithm

The structure of this piece, and the means used to generate score materials, are both extremely simple. Each of the 21 scales in Table 45 is used in one section. In line with repetitive, minimalist aesthetics, each section has the same form.

- First, the scale is played twice over one octave, ascending and descending.
- Second, there is a period of improvisational play with the resources of the scale.
- Third, the scale is played one last time, ascending and descending.

The 21 sections can be played in any non-repeating order. All 21 scales must be played once before any repeats occur. In each performance, a different order of sections is to be used. The performance and recording in November 2004 were 21 sections long; future performances can be longer than 21 sections.

The second part of each section has its own smaller score. This was made with the “urn” module in *PD* (Puckette 2006), which will generate a random series of numbers between 1 and n, where each number appears only once. This function is found in a number of other programs, such as the “-nr” option in *IC* (Culver and Cage 1993), and the “count / scatter” functions in the sequencer modules of *ArtWonk*, *SoftStep* and *BankStep*, or the “ScatterArray” function in *ArtWonk* (Dunn 2004a, 2004b, 2007). I found non-repeating sequences central to the pieces described in Chapters 2.1, 2.2, and 2.3. My use of them is derived from serial technique, but applied in ways unlike traditional serial composition.

PD’s urn module is useful because one can specify its upper limit, and this limit can change in real time. This changing nature of the urn module gave the piece its title. The title is a pun on the name of Ern Malley, one of the most revered non-existent writers in Australian literary history. Malley and his milieu are discussed in Heyward

1993. Further discussion of the non-existent artist phenomenon and its applicability to contemporary composition is found in Futrelle and Cooper 2006.

For each section, three orders of the numbers 1-8 were generated. These indicated the seven pitches of the scale, and the top octave. The three random orders of 1-8 were listed in the manner shown in Table 46.

Table 46: Method of displaying random number sequences in the score of *The Malleable Urn*

6 4 2 3 7 8 1 5
1 4 7 2 5 8 6 3
5 4 7 1 8 2 3 6

These sets of random numbers were used as a source for improvisation. The rules for this improvisation will be discussed in the next Section.

2.2.4 Process – Real Time Usage

The random number lists were used as scores for the improvisatory middle part of each section. They were used in a number of ways:

1. One was to play dyads, repeating them a number of times. For example, the first two numbers 6 and 4 could be played as a dyad of scale degrees 6 and 4, and this dyad could be repeated 10 times (6 + 4 times), or it could be repeated 4 times, or 6 times, or even just played once (6 + 4 = 10, 1 + 0 = 1). If the scale in question made playing pitches 6 and 4 simultaneously impossible (if they both occurred on the same string, for example), they could be played as an evenly spaced two note riff, or with one as a grace note to the other.
2. Another technique would treat the stream of digits as pitches for a slow melody. Or a few of the pitches could make a slow loop.

3. Vertical columns could be used for harmonies, trichords, or three note repeating riffs, depending on how the pitches of the scale mapped onto the strings and frets of the ukulele.

These were the three principal improvisational strategies employed, but others were used, and still others will probably be developed during future performances.

The piece used a tablature notation. I felt it was easier for me to learn the piece if I worked from tablature, rather than learning the fingering in addition to a form of traditional notation. Even though the scales could be notated traditionally, I felt that tablature notation was more true to the spirit of the piece as I conceived of it - as an exploration of sound resources made accessible to an amateur player, where each interval is treated as if it were a separate timbre. The tablature consisted of a fretboard diagram for the first 5 frets of the ukulele, with a dot indicating finger position, or an open circle indicating an open string. The interval between each scale degree was listed at the top, between the fretboard diagrams. Each fretboard diagram also had a number 1 - 8 below it, indicating which pitch the fingering referred to. The 3 x 8 number grid for improvising was placed, centred, below the fretboard diagrams. Each section of the piece was printed on to a separate A5 page, which could be re-ordered before each performance of the piece. Figure 13 shows the tablature notation. The complete score is included in Appendix 3.2.2.

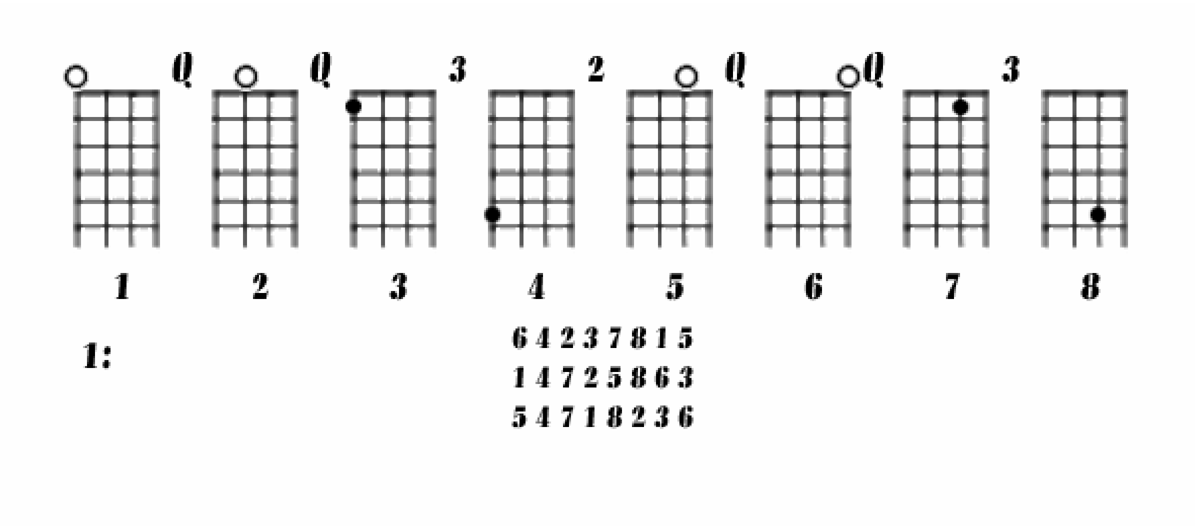


Figure 13: Tablature notation for Section 1 of *The Malleable Urn* -One of 21 pages

The piece was rehearsed for a period of 6 weeks before the performance on November 14, 2004, at Gallery East, Clovelly. A Sunday afternoon crowd passed through the Gallery, with a permanent audience for the piece of four: Stephen Jones, Roz Chaney, Joan Brassil and Carol Ruff. All seemed quite moved by the piece, which quietly filled the gallery, and its allotted time. In fact, it was, I believe, the last live performance that Joan Brassil, a dear friend and a much-loved inspiration for the Australian experimental art world, attended before her death early the next year. I was very happy to have given her such a quiet sonic offering in thanks for her lifetime of work, interest, and, indeed, personal support. Performance for a very small audience is something that I enthusiastically embrace. A fuller discussion of this is in Burt 2005b. Figure 14 is a photo from the live performance, taken by Carol Ruff.

Figure 14: Warren Burt performing *The Malleable Urn* - Gallery East, Clovelly, November 14, 2004

Later, at home, I recorded and edited the piece. In editing, the meditative nature of the piece forcefully struck me again, and an intense listening resulted, where I became aware of the tiniest nuances of each playing of every interval or note. A piece for a mono-timbral instrument became a highly focussed, meditative *klangfarbenmelodie*. My desire to blur the distinction between tuning, timbre and harmony was realised well here. I played the recording at several friends' homes, quietly in the next room. This always seemed to calm the atmosphere, and contribute to a sense of slowing down the environment. I'm quite happy with this extremely slow, extremely simple, yet highly effective piece, as it seems to have the potential to change the way I, and hopefully others, listen, and when performed in intimate circumstances, seems to allow for a very direct form of communication. I would certainly perform it again if the opportunity arises.

2.3 *The Animation of Lists And the Archytan Transpositions*

2.3.1 Introduction

The Animation of Lists And the Archytan Transpositions is a recorded composition for my self-built Just-Intonation tuning forks, multi-tracked and computer transposed. I was trying to create a beautiful sound world that went beyond what I had previously done with my tuning forks. Seeing as how people have said that my most beautiful work is that which involves the forks, I set myself a fairly high goal. In making this beauty, I explored ideas of more complex Just-Intonations, long scale permutational systems, and work that exists in multiple versions created by pitch and rhythm transpositions of pre-recorded materials.

The piece had its origins in a request in 2002 from composer Phill Niblock for a recording he could release on his XI label. Phill is one of the people who loves the sound of the forks, so I began mulling over what I could do with them that would work as a CD. After about 2 years of thought, I came up with the idea of a piece that would be multitracked, where each track would be transposed by the computer, thus making a richer, more complex Just-Intonation world than the forks themselves had.

The piece was composed and recorded between October 2004 and January 2005. It was produced by Al Margolis, and released on the XI label in mid-2006. Since its release it has been enthusiastically received, with about 40 reviews, all positive, in the international press (Burt 2006). MP3 files of the work are included in Appendix 3.3.1.1 and 3.3.1.2. Copies of the reviews will be found in Appendix 3.3.5. A copy of the commercial CD is also included in the back pocket of the thesis as Appendix 3.3.7. Figures 15 and 16 show the cover and back of the CD.

Please see print copy for Figure 15

Figure 15: Cover to XI 130: *The Animation of Lists and the Archytan Transpostions*

Please see print copy for Figure 16

Figure 16: Back cover, XI 130: *The Animation of Lists and the Archytan Transpositions*

The tuning forks are made of aluminium, and were made by me in 1985, when I was artist in residence with the CSIRO, the Australian government science body. This work was funded by an Australia Council Artists and New Technologies grant. The forks are of two kinds, treble and bass. The treble forks are made of 25 x 40 mm aluminium construction bar, with a 10 mm wide slit down the middle. They are struck with a variety of percussion beaters and may be mounted in a frame or hand held. An essential part of their design was that they would be individual forks, not mounted together, and could be hand held. This was to encourage their use by community music groups and ensembles of untrained performers. They have received their greatest use in this context in the series of works for the Astra Choir and tuning forks described in Chapter 1.2. The treble forks produce a very pure tone (practically a sine wave) after the initial attack. The timbre of the initial attack is determined by the kind of beater used. The forks have a decay which lasts between 30 seconds to about a minute. Figure 17 shows the construction and proportions of the treble forks.

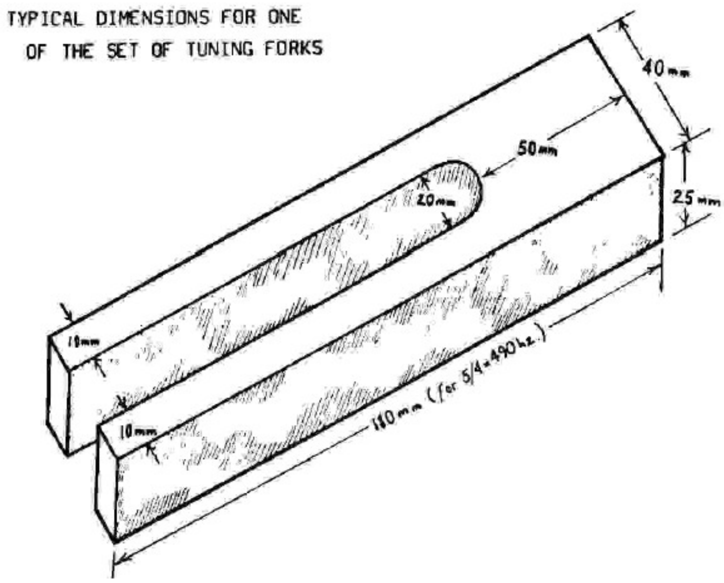


Figure 17: Typical dimensions for one of the set of tuning forks

The bass forks are made of 40 x 40 mm aluminium construction bar, and must be mounted in a frame in order to be played. Very soft beaters are used for them, and resonators must be used for them to be heard. The resonators consist of 100 mm plastic sewer pipe cut to the appropriate lengths for each pitch. In 1998, a new frame for the forks was designed by composer and sculptor Anne Norman. It holds 4 bass forks and their resonators and 11 treble forks. Figure 18 shows the frame with bass and treble forks mounted in it, with resonators. Figure 19 shows another use – the treble forks hand held and played in a recording situation. A fuller description of the original tuning fork project can be found in Burt 1987a and 1987b.

Figure 18: Tuning forks frame with treble forks, and bass forks with resonators

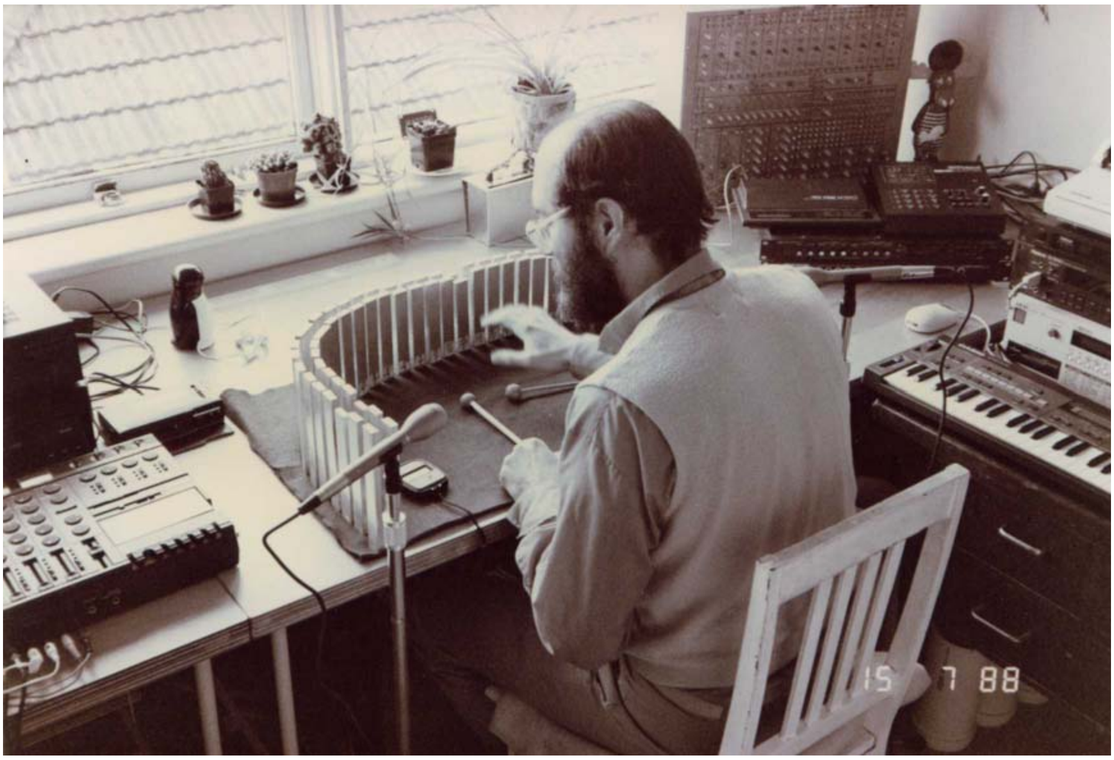


Figure 19: Warren Burt with hand held tuning forks in studio, 1988

2.3.2 Tuning System

The forks are tuned to a 19-note Just-Intonation scale, derived from modes described in Claudius Ptolemy's 2nd century CE *Harmonics* (Barker 1989). This scale enables one to have a large variety of Just-Intonation resources available, including Enharmonic, Chromatic and Diatonic versions of various Dorian modes, and their inversions. Harry Partch's idea of symmetry in scale design was followed here, as it allowed me to experiment with inverted genera. These were not used in ancient Greece, but provide some lovely harmonic resources (Partch 1974). Table 47 lists the scale, with traditional interval names included for reference. In this scale, the fundamental 1/1 = G 392 Hz, the same fundamental used by Partch for his instruments. These forks cover a 4-octave range from G on the bottom of the bass clef (96 HZ) to the G one octave above the treble clef (1568 Hz.). There are 39 bass forks (2 octaves of 19 tones plus a top octave), covering from G 96 Hz up to G 392 Hz; and 39 treble forks, covering from G 392 Hz to G 1568 Hz. I have other forks, covering several other pitches, but the forks tuned to this scale are the "core" forks, and for this piece, I limited myself to them.

Table 47: Tuning forks scale – Just-Intonation ratios, cents, interval names

Scale Degree	Ratio	Cents	Interval names
0:	1/1	0.000	unison
1:	28/27	62.961	Archytas' 1/3-tone
2:	16/15	111.731	minor diatonic semitone
3:	10/9	182.404	minor whole tone
4:	9/8	203.910	major whole tone
5:	6/5	315.641	minor third
6:	5/4	386.314	major third
7:	9/7	435.084	septimal major third
8:	4/3	498.045	perfect fourth
9:	7/5	582.512	septimal or Huygens' tritone
10:	10/7	617.488	Euler's tritone
11:	3/2	701.955	perfect fifth
12:	14/9	764.916	septimal minor sixth
13:	8/5	813.686	minor sixth
14:	5/3	884.359	major sixth
15:	16/9	996.090	Pythagorean minor seventh
16:	9/5	1017.596	just minor seventh
17:	15/8	1088.269	classic major seventh
18:	27/14	1137.039	septimal major seventh
19:	2/1	1200.000	octave

For this composition, I was interested in expanding this scale, but I didn't want to do metalwork again, and make more forks. Fortunately, I realised that if I used the computer to transpose recordings of the forks slightly, I could get other pitches than the ones the forks themselves produce. Since this was a piece designed for the recorded medium, I could do this easily. A variety of intervals were tried out, but the interval of transposition that seemed to work best for me was the 28/27, Archytas' large 63 cent version of the quarter-tone, referred to above as "Archytas' 1/3-tone." Table 48 shows the scale resulting from transposition up by 28/27. Table 49 shows the scale resulting from transposition down by the same interval.

Table 48: Tuning fork scale transposed up 28/27

Scale degree	Ratio	Cents
0:	1/1	0.000
1:	28/27	62.961
2:	784/729	125.922
3:	448/405	174.692
4:	280/243	245.365
5:	7/6	266.871
6:	56/45	378.602
7:	35/27	449.275
8:	4/3	498.045
9:	112/81	561.006
10:	196/135	645.473
11:	40/27	680.449
12:	14/9	764.916
13:	392/243	827.877
14:	224/135	876.647
15:	140/81	947.320
16:	448/243	1059.051
17:	28/15	1080.557
18:	35/18	1151.230
19:	2/1	1200.000

Table 49: Tuning fork scale transposed down 28/27

Scale degree	Ratio	Cents
0:	1/1	0.000
1:	36/35	48.770
2:	15/14	119.443
3:	243/224	140.949
4:	81/70	252.680
5:	135/112	323.353
6:	243/196	372.123
7:	9/7	435.084
8:	27/20	519.551
9:	135/98	554.527
10:	81/56	638.994
11:	3/2	701.955
12:	54/35	750.725
13:	45/28	821.398
14:	12/7	933.129
15:	243/140	954.635
16:	405/224	1025.308
17:	729/392	1074.078
18:	27/14	1137.039
19:	2/1	1200.000

Table 50: 49-note scale made by combining original scale with its transpositions

Scale degree	Ratio	Cents
0:	1/1	0.000
1:	36/35	48.770
2:	28/27	62.961
3:	16/15	111.731
4:	15/14	119.443
5:	784/729	125.922
6:	243/224	140.949
7:	448/405	174.692
8:	10/9	182.404
9:	9/8	203.910
10:	280/243	245.365
11:	81/70	252.680
12:	7/6	266.871
13:	6/5	315.641
14:	135/112	323.353
15:	243/196	372.123
16:	56/45	378.602
17:	5/4	386.314
18:	9/7	435.084
19:	35/27	449.275
20:	4/3	498.045
21:	27/20	519.551
22:	135/98	554.527
23:	112/81	561.006
24:	7/5	582.512
25:	10/7	617.488
26:	81/56	638.994
27:	196/135	645.473
28:	40/27	680.449
29:	3/2	701.955
30:	54/35	750.725
31:	14/9	764.916
32:	8/5	813.686
33:	45/28	821.398
34:	392/243	827.877
35:	224/135	876.647
36:	5/3	884.359
37:	12/7	933.129
38:	140/81	947.320
39:	243/140	954.635
40:	16/9	996.090
41:	9/5	1017.596
42:	405/224	1025.308
43:	448/243	1059.051
44:	729/392	1074.078
45:	28/15	1080.557
46:	15/8	1088.269
47:	27/14	1137.039
48:	35/18	1151.230
49:	2/1	1200.000

Because the 28/27 figures prominently in the original 19-note scale, transposing by that interval results in several pitches in common between one or more of the scales, such as 1/1, 4/3, 3/2, 28/27, 27/14, 9/7 and 14/9. Also, a number of interesting small number ratio intervals are found in the transposed scale which didn't exist in the original scales, such as 12/7, 15/14, 28/15, and 7/6. In total, the three scales together make a composite 49-note scale, which has many more harmonic possibilities than the original 19-note scale. (In the liner notes to the CD, I describe the composite scale as having 53 tones. This was an error. (Now how *did* I do that?)) Table 50 (above) lists the composite 49-note scale made by combining the original scale with its two transpositions. *Scala* tuning files for all four scales can be found in Appendix 3.3.4.

Various methods of computer transposition were tried out. FFT based methods of transposing pitch but not rhythm were tried and rejected. The forks' sustain is so pure that no matter what frequency I chose to sample the original tones with, one or two pitches always ended up sounding garbled after transposition. In the end, only traditional "tape-style" transposition, in which both pitch and rhythm are changed by the same ratio, produced no timbral modification to the original. Using this would mean that when I mixed transposed tracks together, they would not only have different harmonies, they would also have different rhythms than when I mixed together untransposed tracks. This would mean that with a very simple technique, I could generate different versions of the piece. Further, there were potentially many different versions, each one using a different ratio of transposition.

I decided that I would make a piece that would be made in three passes, and that there would be two versions of the piece. The first would be one in which the three passes would be mixed together untransposed. The second would be one in which pass

1 was untransposed, pass 2 was transposed up a 28/27, pass 3 was transposed down a 28/27, all three passes then being mixed together.

In each of the three passes there would be bass forks playing, frame-mounted treble forks playing, and hand-held treble forks being played and moved around in space. This moving of the hard-held treble forks created phase shifting which added a delicious degree of animation and liveliness to the sound. At first I tried recording the frame-mounted forks as one track and the hand-held forks as another, but this didn't seem to work - the mixing of the two tracks produced combinations of events which seemed really clumsy. However, I found that having two players, one (myself) playing the frame-mounted forks, and the other (Catherine Schieve) playing the hand-held forks produced a much more musical result. Even though the individual parts are related heterophonically, without precise co-ordination, somehow the energy of two people recording at the same time resulted in a much more satisfying performance. I also found that three of these duo performances mixed together sounded much more coherent than a mix of six solo performances did.

Awareness of this use of transposition to make different versions of a piece from the same starting material, and of the performance technique outlined above influenced the design of the compositional algorithms, and it is to these we now turn our attention.

2.3.3 Algorithm

In contrast to the algorithms used in *For JSB and JT* and *The Malleable Urn*, the process used to generate this piece was complex and multilayered. It consisted of the following steps:

1) Decide that each "pass" is divided into sections

2) Generate the number of measures that will occur in each section

3) Generate sections until the total duration of a "pass" exceeds 1 hour

4) Choose bass forks for each section

5) Choose treble forks for each section

6) Determine how many forks will be played in each measure

7) Determine which forks are to play in each measure.

All these decisions were then incorporated into a score. The score allowed the performer considerable freedom, yet specified other elements exactly. The score is included as Appendix 3.3.2. Each of the above steps will now be discussed in detail.

1) Decide that each "pass" is divided into sections

I decided that each pass would consist of a number of sections, which would be subdivided into "measures" of 15 seconds each.

2) Generate the number of measures that will occur in each section

Each section would have between 16 and 32 of these "measures", resulting in each section being between 4 and 8 minutes long. The number of measures in each section was generated by a random number generator in John Dunn's *ArtWonk*.

3) Generate sections until the total duration of a "pass" exceeds 1 hour

Each section was generated, and its length added to the total duration of the previous sections until a duration longer than 1 hour resulted. Pass 1 ended up with 10 sections, Pass 2 also had 10 sections, and Pass 3 had 12 sections.

Very occasionally two sections on different passes might begin together, but for the most part, sections began independently of each other. Due to the length of decay of the forks, however, each section actually lasted from 30 seconds to 1 minute longer than specified. These decays were kept, resulting in a final mix using six stereo tracks for the three passes, so that the overlapping final decays were a part of the mix. In a similar manner, all throughout the composing process, the acoustical realities of the forks continually influenced the way the algorithms were designed and used.

4) Choose bass forks for each section

Pitch selection was a made by a series of random selection processes. Each pass was conceived of having two independently progressing harmonic streams. Bass forks and treble forks were treated as independent entities. I was interested in the composite harmonies that resulted from the juxtaposition of these two streams.

The selection process for the bass forks was as follows: For each section of each pass, choose 4 forks. Select all forks for a single pass at the same time. Using the Urn module in *PD*, generate a random ordering of numbers 0 - 38. (The Urn module is described in more detail in Section 2.2.3) Divide this ordering of 0 - 38 into sections of four numbers each. Assign the first four numbers to pass 1, section 1; the second four numbers to pass 1, section 2, etc. When you run out of numbers, use the Urn again to generate another set of numbers from 0-38. Use as many of these as you need to complete the pass. Generate a new set of numbers when you start the next pass. The bass forks to be used in any section are now chosen. This results in three independent travellings through the complete set of bass forks, one per pass. Since the three passes are mixed, all kinds of simultaneous combinations will occur, but in the long term, all possible bass pitches will occur at least once during the course of the piece.

This making a row out of a gamut of several octaves of a scale is a practice that can be traced to the very early works of John Cage, where all pitches of a 25-tone gamut had to be used before one could be repeated (Pritchett 1993). The use of a number of related complete, and independent rows has, in my mind anyway, at least a tangential relationship to certain compositional techniques of Milton Babbitt's, and in fact, the title of the first version of the piece, the mix of the three untransposed passes, is taken from

Joseph Dubiel's description of Babbitt's work: *The Animation of Lists* (Dubiel 1992). This description seems to fit this work as well. The use of Just-Intonation and home-made instruments is clearly influenced by the work of Harry Partch. Thus, in this piece, in my opinion, three completely incompatible historical influences mingle, making a different work than any of those three fine gentlemen would have made.

5) Choose treble forks for each section

The treble forks to be used in each section were chosen as follows. The Anne Norman designed frame shown in Figure 18 has four slots for mounting bass forks and their resonators, three slots between them for treble forks, and eight slots at the right end of the frame for mounting treble forks. If used in this configuration, the following setup results: (T = treble fork slot; B = bass fork and resonator slot, and 1-8 = treble fork slots at the right end of the frame.)

B T B T B T B 1 2 3 4 5 6 7 8

Choosing the four bass forks was dealt with above. For each section I found that I needed three treble forks to mount between the bass forks, eight treble forks to mount at the right end of the frame, and an additional four treble forks to be hand-held and played by the second player. This is a total of 15 forks.

For each section, I generated a set of 39 numbers with the Urn module, but only used the first 15 of them. The non-repeating random sequence of the Urn was used so that the same fork couldn't be picked to be played at the same time, say, in the frame and hand-held. The use of partial rows, continually regenerated, allowed for the possibility of repeated pitches from section to section, perhaps creating a sense of harmonic overlap between each section. However, since the overall pace of the work is one of almost glacial slowness, I wonder if that sense of harmonic change will actually be heard. In any case, it's there if people want to listen for it.

Of the 15 pitches chosen, the first three, put in ascending order, became the three treble forks put between the bass forks, the next eight, put in ascending order, became the eight forks on the right side of the frame, and the final four, again put in ascending order, became the hand-held forks. Therefore, for each section there was a completely different set of forks placed in the frame, and a completely different set of hand-held forks used.

Once the forks are mounted in the frame, as shown above, a different ordering process is used to choose the actual pitches to play. Once the treble and bass forks (with resonators) are mounted in the frame, they are each assigned a number from 1-15 from left to right. The hand-held forks are each assigned a number from 1-4 based on ascending pitch order. Figure 20 shows this numbering of the forks, using small bits of paper taped to the frame.

Figure 20: Numbering of forks in tuning fork frame

6) Determine how many forks will be played in each measure

The next stage involved selecting how many forks were to be played in each 15 second measure. For the frame-mounted forks, *ArtWonk's* random generator was used to pick a number between 1 and 5: between 1 and 5 frame-mounted forks will be played in each measure. For the hand-held forks, the same generator picked a number from 0-2. This means that in each measure, either no, one or two hand-held forks will be played.

7) Determine which forks are to play in each measure.

The actual pitches played in each measure are selected by the *PD* Urn module. For the frame-mounted forks, a random ordering of numbers 1-15 is generated by the Urn, and these are assigned to each measure, determined by the number of forks per measure generated above. For example, if the first four measures had 1, 5, 3, and 2 tuning forks assigned to each one, and the series of numbers generated was 3, 10, 2, 8. 15, 4, 1, 5, 6, 7, 14, etc. then the assignments shown in Table 51 would result. As soon as all 15 pitches are used, another set of 15 is generated, until all the pitches needed for a given section are in place. If a repeat, or near-repeat of some pitch takes place on the boundary between one sequence of 15 pitches and another, some kind of gesture is improvised in performance that emphasizes, however slightly, this connection.

Table 51: Assignment of pitches to measures for frame mounted forks

Meas.	#Forks	Pitches Played
1	1	3
2	5	10, 2, 8, 15, 4
3	3	1, 5, 6
4	2	7, 14

A similar process is used to pick pitches for the hand-held forks. Here, the Urn is used to generate different orders of the numbers 1-4. These determine which forks are played in which measures. For example, for six measures with 0, 1, 0, 2, 0, 1 forks assigned to them, a fork ordering of 1, 4, 3, 2, was generated. The result is shown in Table 52.

Table 52: Assignment of pitches to measures for hand held forks

Meas.	#Forks	Pitches Played
1	0	
2	1	1
3	0	
4	2	4, 3
5	0	
6	1	2
etc.		

It is to be noted that two unrelated processes are used to determine the pitch content of the piece. Steps 4 and 5 are used to determine which forks are used in each section. That is, which specific pitches will occur in that section. The choice of which pitches are to be played in each measure, however, is done in Steps 6 and 7, and is independent of the choices made in Steps 4 and 5. That is: in Steps 6 and 7, pitch 4, say, is specified. But whether that "4" applies to, say 3/2, or 8/5, is determined in Steps 4 and 5. The process of Steps 6 and 7 doesn't "know" anything about specific pitch content. It only deals with the ordering of a pre-given set of pitches.

2.3.4 Process – Real Time Usage

The score generated by this multi-layered process allowed for considerable performer choice. This was in keeping with my radical amateur aesthetics, which seek to make structures that untrained performers can use (Burt-Dempster 1999). In any case, I had no desire to write a piece that called for precise rhythmic synchronization between parts, as this would contradict my essential idea for the piece, which was to create a series of shifting clouds of harmonies that would effortlessly float between one harmonic region and another. The method of assembling the piece, three independent recordings mixed together to get complex indeterminate rhythms, also did not allow for precise rhythmic co-ordination.

The rules for the frame mounted fork player were as follows: In each 15 second measure, a player must play all the forks in the given order during the measure. However, the placement of the sequence within each 15 second measure is free. Thus, harmonic rhythm and pitch order is determined, but small-scale rhythm is improvised. The performance technique here is similar to that of some of John Cage's late "number" pieces.

The rules for the hand held forks player specify that for a measure with one fork in it, the hand held player is free to strike the fork once any time during the 15 seconds of the measure, and then move the fork around in the air until they have to put it down to pick up the fork or forks for the next section. If no forks are needed for the next measure, the player may keep moving the fork struck in the previous measure. If two forks are to be played, they are held in the same hand, and struck fairly closely together (in time), and then moved about as a unit. This is occasionally quite difficult, if low treble forks of some weight are called for.

Each section of each pass is recorded separately. For each section, both frame-mounted and hand held fork players use beaters of different hardness. This gives a considerable timbral variety to the tracks. At any moment in the piece, the aim is to have hard, medium and soft beaters employed simultaneously.

The spatial nature of the recording, especially as regards placement of pitches in space, was also carefully planned. Table 53, below, shows the setup used for recording.

Table 53: Spatial setup of tuning forks for recording session

Left	Middle	Right
Frame (at 45 deg. angle)	Mic (angled 45 deg down)	Hand held forks.

The recording space (our dining room) was about 3.5 by 3 meters, with a fairly live but carpeted floor acoustic. An Audio-Technica 802 stereo cardioid mic was used, and was recorded to a Sony MiniDisc recorder. The automatic gain control and compression on the MiniDisc were left on, as it was found that these controlled the loudness peaks of the forks without adversely affecting the sound. In fact, in over three hours of source recordings, fewer than ten attacks had to be edited, where the compression and automatic gain control had produced adverse “pumping” effects. Once recorded (over a period of about two weeks in late December 2004 and early January 2005), the tracks were then transferred into a computer for further processing.

Adobe Audition (the program formerly known as *Cool Edit*) was used for processing and mixing the recordings. Each track was first edited. This involved finding the beginning and ending points, and occasionally editing out a click, or a scrape, or rarely, a complete attack of a note. For these, simple cut and splice editing usually proved adequate, but occasionally, *Audition's* spectral editing tools (which allow one to cut out only a portion of the spectrum of the sound) were needed to cut out the rumble of distant traffic, or even, on two or three occasions, the odd bird call or insect chirp.

The three passes then received different spatial treatments:

Pass 1 was edited and noise-reduced, then put through a “slow rotate” process, normalized to -.5 db, and then saved for final mixing. The “slow rotate” process uses the “Rotate Stereo Field” module in *Audition*, which moves left and right channels of a stereo track through a certain number of degrees of stereo shifting. I made a sine shaped graph that very slowly moved the tracks in a circle once over the course of a section. This produced, in this track, the effect of the same pitch coming from different places in space on subsequent attacks. In other words, the Left-Centre-Right setup of the recording session was here very slowly rotated over the course of a section.

Pass 2 was simply edited, noise-reduced, normalized, and saved. However, a second version was then made, which was the track transposed up 28/27 for use in the transposed version.

Pass 3 was edited, noise-reduced, the stereo channels were swapped, and then it was normalized and saved. Then a second version was made, which was the track transposed down 28/27 for use in the transposed version. The swapping of the stereo channels reverses the Left-Centre-Right orientation of the recording session.

Since all three Passes are mixed together, this means that there is a great variety of spatial assignments for each pitch and movement of forks within the stereo field, and that these assignments will change over the course of the piece. Spatial positioning will be continually surprising, and fresh.

All three passes are then mixed together. As explained earlier, six stereo tracks were necessary for all the sections of the three passes to be mixed. All the tracks were attenuated by -4 db, and then the few overload peaks that occurred were modified, and the final mix was made. Two versions were made. *The Animation of Lists* lasts 64:30, and is the mix of the 3 passes untransposed. This piece, therefore, is in the 19-note Just-Intonation tuning forks scale. *And the Archytan Transpositions*, with a duration of 64:47, mixes the untransposed version of Pass 1 with the transposed versions of Passes 2 and 3. This piece is in the 49-note Just-Intonation scale described earlier. Because the transposition by 28/27 affects both pitch and rhythm, this piece has a completely different rhythmic and harmonic feel. Whereas the *The Animation of Lists* has a very calm and placid feeling, in *And the Archytan Transpostions* the resultant rhythms are more jagged, and there is an almost constant sense of acoustical beating because of the many 28/27 third-tone relations that exist between the transposed passes. This is especially prominent with the bass forks. Low throbs abound in *And the Archytan Transpositions*, where in *The Animation of Lists*, such throbs only occur when, for example a low 28/27 and a low 1/1 are played in the same or adjacent measures.

Thus, a complex sonic object is assembled, which is now available for contemplation, enjoyment or study. I will be most interested to find out if with repeated hearings, any of the long-term structuring I built into the piece is audible. My first listening reveals a constantly changing world of harmonic surprise and timbral delight. Whether these moment to moment details yield a sense of larger scale motion remains to be heard - most reviewers so far have noted a sense of timelessness and seamless evolution in the course of the piece. Hearing longer-term structuring may require repeated hearings.

2.4 *Homage to Wyschnegradsky*

2.4.1 Introduction

The compositions described in this Chapter and in Chapter 2.5 were written for unique instruments. Tubi and Puff are two of the many computer-controlled acoustic instruments built by Godfried-Willem Raes and installed at the Logos Foundation in Gent, Belgium. A full description of the Robot Orchestra project, and the M&M Ensemble, of which these two instruments form a small part, can be found at the Logos website (Raes 2006). At Godfried's invitation, I composed a series of six works for the some of the instruments of the ensemble in 2004 and 2005. The instruments were in Belgium, but I was in Wollongong, Australia. I composed for these works long distance, using the Internet. The compositions were made with real time interaction using algorithmic programs. The programs controlled samples of the instruments, which were downloaded from the Logos website. The outputs of these programs were MIDI files that I sent over the internet to Logos, where Kristof Lauwers, Logos' programmer, played them on the instruments, and recorded the result. He then sent me an MP3 file of the recording, and I heard the works in this way. The works were also performed live on the monthly Logos M&M Ensemble concerts in 2005. Held at the Logos Concert Hall in Gent, *Homage to Wyschnegradsky* was performed on 9 February 2005, and *Lehmer's Kookaburra* was performed on 15 March 2005. The complete set of six pieces, and the Internet-based composing process, is described in detail in "Long Distance Composing for Computer Controlled Microtonal Acoustic Instruments" (Burt 2005d).

2.4.1.1 The Logos Foundation

The Logos Foundation is a research institution and concert-giving organization in Gent, Belgium, founded in 1969 by Godfried-Willem Raes and Moniek Darge. It is funded by the Flemish state government and the City of Gent. Over 65 concerts of new music are given there each year. Since the mid-90s, one of Godfried-Willem Raes' projects has been designing and building computer-controlled acoustic instruments. This began with a player piano mechanism, designed in collaboration with the German-American composer and instrument inventor Trimpin, and has progressed from there to include percussion, organ, wind, brass and original instruments. Some of the unique instruments include Flex, a computer-controlled musical saw; Belly, an automated carillon; and So, a computer-controlled tuba. All the instruments are controlled by MIDI, and ensembles of the instruments can be set up by assigning different instruments to different channels. Godfried has been inviting composers to work with the instruments for several years. During a visit to the Logos Studio in December 2003, Godfried gave us a delightful demonstration of the instruments. Hearing those wonderful mechanical creations was inspiring. I immediately began thinking about composing for the ensemble. I was most intrigued with two of the microtonal instruments, Puff, a unique quarter-tone organ which has a separate bellows for each pipe; and Tubi, a quarter-tone tubulon. I was also interested in Belly, a computer-controlled carillon which uses a collection of found-object signal bells instead of tuned musical bells, and I was also attracted to the computer-controlled piano.

2.4.1.2 The Working Method

The working method on the pieces was as follows: I would make a MIDI-file of test sequences I would like to hear on an instrument. I would then email the MIDI-file to Kristof Lauwers at Logos. He would play the sequence, scaling parameters in the MIDI-file, if need be, record it, and put an MP3 file of the recording on his website where I could download it. My test sequences included single notes at a variety of pitches and loudness levels so that I could load the timbres into a sampler and work directly with them. On finishing a sketch, I would email the MIDI-file to Kristof, who would then again place the recording on his website for me to download. If there were things I wanted revised, I would make corrections and we would repeat the cycle until the results satisfied me. Following this long-distance collaborative procedure, we made six pieces, two of which are discussed in this thesis.

Of course, if I had been in Gent, I could have worked interactively with the instruments, but part of Godfried's invitation involved seeing what would happen when these instruments were worked with at a distance. I have been working interactively in electronic music for over 30 years, where hearing the exact sound being produced in real time was an intrinsic part of the process, and in some ways, this seems like returning to the days of writing instrumental music, and then waiting to hear it performed. The difference is that in this case, I'm highly reliant on a human designer (Godfried), and a human performer-programmer (Kristof) to operate the robots; the score and recordings are sent over the Internet (which I'm also highly reliant upon); and I'm not constrained by the limitations of human performers (just the limitations and idiosyncrasies of the machines). I'm not a devotee of "cyberculture", so I don't think the addition of Internet usage to any process makes all that much difference. I can see that the technological tools of this collaboration may be different, but structurally, I'm still a composer sending out a score for performance. And here I should convey a huge thank you to Kristof Lauwers, who has been as cheerful, helpful and responsive a collaborator as one could ask for.

The question might arise, are the recorded performances on the instruments any different than performances with samples of the instruments? The answer is a resounding yes. Not only are the sounds of the acoustic instruments more subtle than performances made with samples of them, but there is a space - a kind of room reverberation - around each individual sound, which just doesn't happen with sampled sounds. Further, the sampler is just too perfect - each of these instruments is experimental, and sometimes the response of the instruments is not 100% predictable. In my case, all of the instruments I chose to work with had idiosyncrasies of their own. For example, the aluminium tubes of Tubi are mounted vertically and have a bit of "give" and swing to them, so sometimes, in rapid passages, an attack might be missed. Puff has an individual bellows for each note, providing a rapid puff of air quite unlike that supplied to a normal organ - very delicate control of dynamics is called for to avoid (or get) overblowing. And the controllers for Puff click rather loudly, meaning that the mechanical noise of the instrument is an inherent part of writing for it. Although the use of samples of the instruments gave me a good idea of what the piece would sound like, there were always surprises when I heard the MP3 file of the performance on the intended instruments.

Figure 21 shows Tubi, the computer controlled quarter-tone tubulon. As can be seen, it consists of vertically mounted aluminium tubes, which are struck by solenoids. The instrument covers three octaves of pitches from C = 525 Hz (MIDI 72) in quarter-tones. MIDI notes 72-108 control the normal 12-tone pitches, while MIDI notes 36 to 71 control the quarter-tone shifted pitches. A quite wide range volume and repetition rate can be obtained from this instrument.   Like all the Logos instruments, it is mounted on a large mobile platform, enabling the instrument to be moved, and also to be used in outdoor contexts, such as community music celebrations, and Medieval-style typical Belgian street parades. In this respect, Logos' instruments are very much indigenous to their local area.

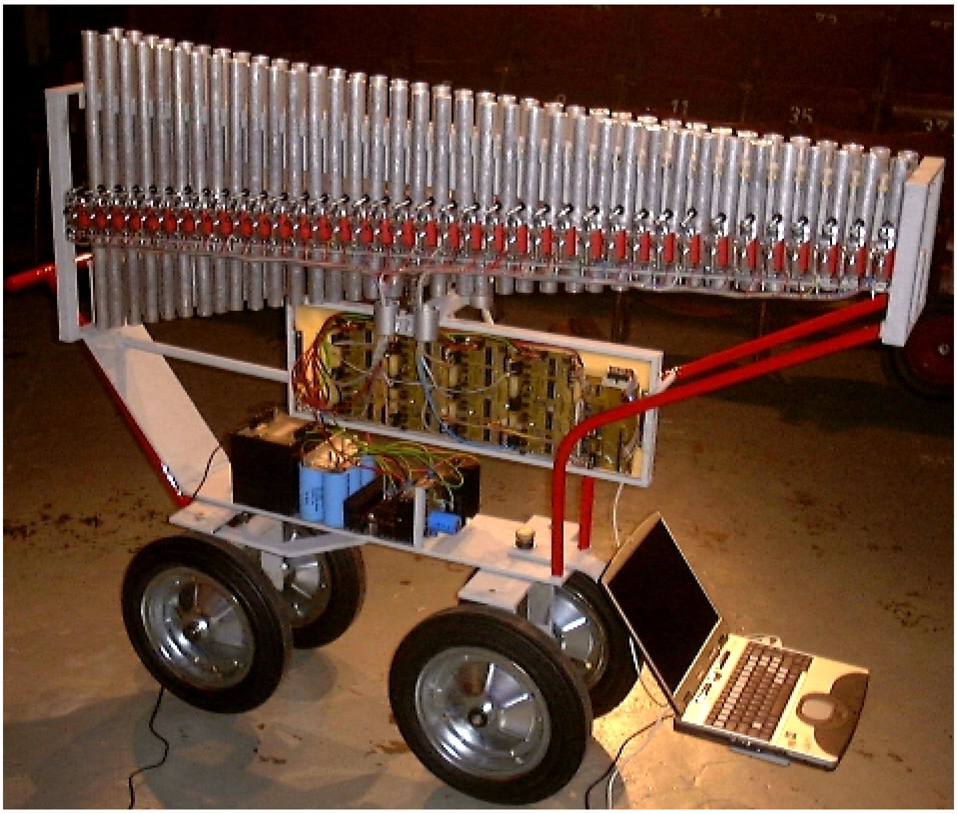


Figure 21: Tubi, the computer controlled quarter-tone tubulon at Logos Foundation

2.4.2 Tuning System

The Logos instruments use 4 different kinds of tuning: 12-tone Equal-Temperament, 24-tone Equal-Temperament, found object tunings, and instruments for which the concept of precise tuning is irrelevant, such as unpitched percussion instruments. Tubi and Puff are two of the instruments tuned in quarter-tones. Other quarter-tone instruments in the collection include QT, a quarter-tone organ, and Xy, a quarter-tone xylophone. These last two instruments were not completed at the time I wrote the works discussed in this thesis, but you can bet that I'll be writing for them soon. With enthusiasm.

The investigations of 24-tone Equal-Temperament modes that I began in *For JSB and JT* and *The Malleable Urn,* described in Chapters 2.1 and 2.2, were continued here. *Homage to Wyschnegradsky* uses 7-note subsets of the quarter-tone scale as its basic harmonic material. The starting scale was a 7-note Moment of Symmetry (MOS) scale made with a generating interval of 550 cents, 11 quarter-tone steps. This interval is only 1 cent flat of the 11/8 eleventh harmonic interval, so the scale can also be thought of as an Euler genus of stacked 11th harmonics. It is also one of the scales described in Wilson's Scale Tree, being found on Page 9 of Wilson 1994. It would also be the next scale described on branch 18, the farthest right branch, of Finnemore's diagram in Figure 5, if that diagram were extended to one more layer of depth (Finnemore 2001a). Table 54 shows this scale. Scale step size is on the top line, while scale degrees are on the bottom line:

Table 54: Basic 7-note quarter-tone mode used in *Homage to Wyschnegradsky*

Step size (in scale steps)		7		2		2		7		2		2		2	
Scale degrees	0		7		9		11		18		20		22		24

This 7-note mode, which has a quasi-gamelan sound due to it's 350 cent neutral thirds can have 7 rotations. This is similar to the organization of the "white key" modes on the piano. Table 55 gives a listing of all the modes, transposed to begin on scale degree 0, using the same notation as in Table 54.

Table 55: All 7 modes formed by rotating the basic quarter-tone mode used in *Homage to Wyschnegradsky*

Mode 1								
Step size (in scale steps)	7	2	2	7	2	2	2	
Scale degrees	0	7	9	11	18	20	22	24
Mode 2								
Step size (in scale steps)	2	7	2	2	7	2	2	
Scale degrees	0	2	9	11	13	20	22	24
Mode 3								
Step size (in scale steps)	2	2	7	2	2	7	2	
Scale degrees	0	2	4	11	13	15	22	24
Mode 4								
Step size (in scale steps)	2	2	2	7	2	2	7	
Scale degrees	0	2	4	6	13	15	17	24
Mode 5								
Step size (in scale steps)	2	2	7	2	2	2	7	
Scale degrees	0	2	4	11	13	15	17	24
Mode 6								
Step size (in scale steps)	2	7	2	2	2	7	2	
Scale degrees	0	2	9	11	13	15	22	24
Mode 7								
Step size (in scale steps)	7	2	2	2	7	2	2	
Scale degrees	0	7	9	11	13	20	22	24

When these seven modes are added together, they make a 13-note mode in 24-tone Equal Temperament. Surprisingly (or maybe not so surprisingly), this mode itself has MOS properties, that is, it has two and only two sizes of scale degrees. Not all 7-note modes in 24-tone Equal-Temperament exhibit this property. For example, the 7-note MOS mode of 4 3 4 3 4 3 3 steps can also be rotated to produce 7 modes. The sum of these modes is indeed a 13-tone mode, but the mode does not have MOS characteristics. Table 56 gives the 13-tone mode made by summing the 7-note modes in Table 55.

Table 56: 13-note scale made by summing the seven 7-note modes listed in Table 55

Step size	2	2	2	1	2	2	2	2	2	1	2	2	2	
Scale deg.	0	2	4	6	7	9	11	13	15	17	18	20	22	24

This mode is one of the modes that Ivan Wyschnegradsky called “diatonisée” modes of the quarter-tone scale in his composition *Vingt-quatre préludes dans tous les tons de l'échelle chromatique diatonisée à 13 sons, opus 22* (1934) (24 Preludes in all the tones of the chromatic scale diatonicized in 13 notes, opus 22) for two pianos in quarter-tones.

In this composition, he extracted a 13-tone mode from the 24-tone scale, and made a series of etudes, each one using a rotation of that basic 13-tone mode as its pitch set. Wyschnegradsky’s work predated the term Moment of Symmetry by four decades, but his diatonicization technique, which refers to having a “diatonic-like” structure (having only two step sizes in a scale) in the 13-tone modes he uses is as clear and elegant an example of MOS thinking as exists in almost any microtonal music. The fact that my 7-note modes added up to his 13-note mode, and that both were MOS scales, seemed a serendipitous enough coincidence to base a piece upon (Wyschnegradsky 1977, Jedrzejewski 2001).

Table 57 is a chart of the 7 modes used in the piece, with the modes covering the 3 octave range of Tubi, and changes from one mode to another indicated by underlining. Scale degree 0 is the lowest note of Tubi; C = MIDI 72 = 525Hz.

Table 57: The seven quarter-tone modes used in *Homage to Wyschnegradsky*, covering three octaves, and showing changes from one mode to another with underlining

Mode 1:	0 7 9 11 18 20 22 24 31 33 35 42 44 46 48 55 57 59 66 68 70 72
Mode 2:	0 2 9 11 13 20 22 24 26 33 35 37 44 46 48 50 57 59 61 68 70 72
Mode 3:	0 2 4 11 13 15 22 24 26 28 35 37 39 46 48 50 52 59 61 63 70 72
Mode 4:	0 2 4 6 13 15 17 24 26 28 30 37 39 41 48 50 52 54 61 63 65 72
Mode 5:	0 2 4 11 13 15 17 24 26 28 35 37 39 41 48 50 52 59 61 63 65 72
Mode 6:	0 2 9 11 13 15 22 24 26 33 35 37 39 46 48 50 57 59 61 63 70 72
Mode 7:	0 7 9 11 13 15 20 24 31 33 35 37 44 46 48 55 57 59 61 68 70 72

Note that in this arrangement of the modes, no more than two pitches per octave change as one advances from mode to mode. If we used this ordering of the modes, we could get a sense of "modulation" which had minimal change from mode to mode, and a maximum amount of "common tones" for each change.

2.4.3 Algorithm

2.4.3.1 Distributions

This piece was made with an interactive algorithmic process. The generators used were the linear probability distributions and the triangle probability distribution functions I wrote for *ArtWonk* described in Chapter 1.11. In this case, however, I imported these functions into John Dunn's other algorithmic program *SoftStep*, because I felt more comfortable with certain aspects of its interface, such as the way it displays sequencer information. I also expanded the number of linear and triangular distributions available from the nine in *ArtWonk* to 21 here, allowing for a greater variety of slopes in the distributions.

The piece consists of three independent lines, two of which are monophonic, while a third plays dyads. This third line plays two voices, the second of which is always 5 modal degrees higher than the first. These modal degrees will, of course, be of different sizes, depending on which mode is being used, but conceptually at least, I think of this line as being doubled in "fifths," where the size of the "fifth" varies with different interval structure of each mode. Each line is made by controlling its pitch, duration and loudness with numbers chosen from the 21 possible linear or triangle random number distributions. These are probability distributions made in the simplest possible manner.

A left-linear distribution (my term) is one that has more low values than high values. It's made by taking the lowest of N different equally-weighted random numbers chosen. There are seven different distributions like this, ranging from the lowest of two random numbers, to the lowest of eight random numbers.

There are also seven different **right-linear distributions** (again, my term), which, conversely, are simply the highest of N different random numbers, and have many more high values than low values. Again, there are seven different distributions, ranging from the highest of two random numbers, to the highest of eight.

Finally, there are seven **triangle distributions**, which have more values in the middle than at either end. These are made by taking the average of N different random numbers. Again, this varies from a low of two to a high of eight. The triangle distribution is sometimes referred to as the "poor man's bell curve."

2.4.3.2 Choice of distributions

Each of the three voices uses three different probability distributions, one each for pitch, duration and loudness. The outputs of the functions are scaled to desired levels and then applied to the required parameter. To control all three voices, a total of nine distributions are required (three each for three different voices). For this patch, I wanted there to always be nine different probability distributions controlling the nine parameters. I accomplished this through the use of the Multi-Rand module in *SoftStep.*

The Multi-Rand module, like the Urn in *PD*, generates a unique random series of all the non-duplicated integers within a given range at each clock input. The Multi-Rand can also have as many unique satellite Output modules as there are elements in the series. Each of these modules outputs a different member of the random series generated by the Multi-Rand. What this means is: If, for example, the range was set to 21, and there were nine MROutput modules, each clock input would generate a unique random ordering of the 21 numbers, and nine of these, all different, would be available, one each from each of the MROutputs. At each new clock input, then, one would get nine different numbers, all chosen from the range of 1-21. This is similar to the use, in *The Animation of Lists,* of the first 15 elements of a 39 number long non-repeating random series described in Section 2.3.3

Having already described my use of Urn, or Urn-like modules in Chapters 2.1, 2.2 and 2.3, it must be said that I continue to find these generators of random series very useful. An ordering of a series with all the values within a given range occurring once each derives, of course, from serialism. But the uses these series are put to depart significantly from the mainstream serial music composition practice.

2.4.3.3 Effects of distribution choice on musical texture

In the patch for this piece, when the "New Fcn" Strobe button is pushed, a Multi-Rand module selects nine different probability distributions. Each of these controls one of the nine required parameters. Pushing the "New Fcn" Strobe button, and selecting a new set of probability distributions can produce radically differing musical textures, because not only is there a possibility for changing the steepness of the slope of a particular distribution, but also the very ranges in which things are happening. For example, if a left-linear 2x distribution were controlling the pitch of a voice, there would be more slightly more low pitches than high pitches chosen. But if the distribution was changed to a right-linear 8x distribution, the result would be mostly very high pitches only, with only a very few occasional low pitches happening unpredictably. If one also considers rhythmic character, and loudness choices, and how a change of distribution would change those, one will realise that selecting a new set of probability distributions with the “New Fcn” Strobe button can radically change the musical texture.

Rhythm, or tempo, is subject to a similar change. The basic set of durations consists of 2 durations of 7 pulses and 5 durations of 2 pulses, a set derived from the interval content of the basic mode. The duration for a voice is chosen from the duration set by a probability distribution, and is then multiplied by a number between 3 and 11. This transposes the rhythm of that voice into a tempo 1/n times as slow as the original tempo. Each time a second Strobe button, called “Tempi,” is pushed, three different random numbers between 3 and 11 are chosen. One of each of these numbers is used as the multiplier for each of the three voices, resulting in the three lines having tempi related by whole number ratios. Polytemporal relations such as 8:10:7, or 3:7:11, or 5:3:4 are all possible, as are all the other combinations of 3 out of 8 elements. Given that a range of almost two temporal octaves (11:3) is covered by this control, it will be realised that the “Tempi” Strobe button can produce quite a radical change in musical texture as well.

2.4.4 Process – Real Time Usage

The *SoftStep* patch I made allowed me to perform interactively, hearing the music with samples of Tubi, in preparation for performance with the real instrument in Gent. The patch allowed me three levels of control:

- First, I could change modes freely. The *SoftStep* patch I made has seven "snapshots", each of which contains the pitch numbers for a particular mode. By changing snapshots, I could change the mode in which the music is playing.
- Second, by pressing either or both of the "New Fcn" Strobe, or the "Tempi" Strobe, I can change the structure of the music, and the polyrhythms that are occurring.
- Third, I can select how many voices, from a maximum of four, are actually playing at any one time. Each of the four MIDI Out modules used (one each for voices 1 and 2, and two for voice 3) has a disable/enable button on it, which can be clicked with the mouse, turning that voice off or on. In performance, I can select if I'm having anywhere from 0 to 4 voices occurring.

To make the piece, I practiced using the *SoftStep* patch controlling the sampler with tubi samples in it. I tried to get an intuitive sense of when it was appropriate to change modes, and when I should change probability distributions and polyrhythms, as well as selecting which lines, and which voices were sounding. This use of intuition to select aspects of a structure which combines precise specification with random distributions is one that I find especially satisfying. My inner-improviser, my inner-structuralist, and my inner indeterminist are all nicely engaged.

When I thought I had developed an interesting way of interacting with the patch, I recorded my five minute improvisation into a MIDI file, and sent that file to Kristof , who used it to play the piece on Tubi. He recorded the result, and sent the recording to me as an MP3 file. On hearing the recording, I felt that no changes needed to be made. By following Kristof's suggestions as to velocity levels (MIDI 20-50) and available repetition rates with Tubi I was able to obtain a performance in which all my notes were heard, with no machine malfunction.

The piece had that combination of modal, “gamelan-esque” harmony and complex polyrhythmic texture that I had been looking for, combined with a mercurial sense of constant changing of tempo and texture. The changing of modes was very clearly heard, and the changing rhythms (both subtle changes, and more abrupt ones), were very attractive. I really like the timbre of this instrument. It’s very glittery – the visual shine of the aluminium tubes is reflected by the jolly high glittering sound, especially in the upper half of the range, where all sorts of difference tones can be heard from combinations of notes. The changing modes and tempi, and the rhythms that are sometimes regular and pulse-like, sometimes polyrhythmically tumbling, seem to suggest to me a quick cutting and fading between fragments of gamelan music, but gamelan music coming from no specific culture or era.

I was very happy that my long-distance collaboration with Godfried, Kristof and Logos was cordial and produced interesting musical results. On hearing this piece, I felt ready to make a piece for Puff, Raes’ unique quarter-tone organ rank. This piece will be discussed in the next Chapter.

2.5 *Lehmer's Kookaburra*

2.5.1 Introduction

Puff is another of the quarter-tone computer controlled acoustic instruments at the Logos Foundation, and it's one that has a unique and arresting timbre. To paraphrase Godfried-Willem Raes' description of it on the Logos website (Raes 2006):

> This robot realises a percussive organ in which every single note is driven by an individual small bellows. All of these bellows are in their turn driven by strong and fast operating solenoids and hence, produce a single but precisely controllable puff of wind on each stroke of the solenoid. The instrument has no equivalent in existing musical instruments. It has 84 different notes. The compass is 3.5 octaves and the instrument is tuned in Equal-Temperament quarter-tones. The scale starts at midi note 55 (equivalent to the lowest note on the violin) running upward to 96. Because of its extended range in the high treble and its quarter-tone tuning, this robot lends itself particularly well to music using spectral harmony techniques. If enough energy is sent to the solenoids, the pipes can also be made to overblow. In this case, the interval of a twelfth will sound, since we used closed pipes.

The instrument does indeed possess a unique timbre. Each note consists of a tuned puff of wind, followed by a mechanical clack as either the solenoid, or some part of the organ pipe mechanism resets. The windy quality of the first part of the sound has the effect of obscuring the definition of each pitch, while the clacking aftersound is unpitched and, in contexts where pitch discrimination is crucial, can be quite distracting. When pipes are heard individually, it is hard to identify precisely which pitch is being played. However, heard in context, either with other pitches of its own, or with other instruments, pitch identification becomes much easier. Figure 22 shows Puff. The copper pipes stand on top of solenoids, which open sharply to provide the puff of air which sounds the pipe. Like all the Logos instruments, it stands on a wheeled frame, to facilitate movement and non-concert hall performance.

I find the sound of Puff incredibly jolly, a kind of good humored asthmatic, clacking wheezing. It very much evokes in me memories of a Northern European mechanical clock-making tradition. Sustain is impossible on this instrument: each tone lasts precisely the length of one short puff of air. This, and the windy tone quality, makes an instrument that sounds like some kind of strange, sputtering bird. Puff needn't sound humorous, though. In combination with other instruments, if used sensitively, it is also capable of a more serious and subtle tone. So far, I have used it to invoke a sense of fantasy and filigree, qualities I've wanted to have in my music for years.

Figure 22: Puff, the computer-controlled quarter-tone organ rank at Logos Foundation

2.5.2 Tuning System

Puff is tuned in quarter-tones, but unlike all the other pieces discussed in this thesis, here I did not use scale subsets or modes of any kind. That is, I did not consciously divide up the scale in any way. All 24 tones of the scale and all 3 and ½ octaves of its range are treated as raw material for the Lehmer equation to play. If any interesting subsets of the scale occur in this piece, they are due to the output of the Lehmer equation and the shaping of its output, and not to any conscious choice on my part. When I made this piece, I felt that Puff's timbre was its dominant characteristic, and this timbre did not lend itself to harmonic explorations, but rather to rhythmic play and timbral effects. However, when I subsequently used Puff in an ensemble situation, in *Beneath the Slopes of Mt Corrimal*, for Belly, Tubi, Puff and Player Piano, I found that Puff was indeed capable of harmonic subtlety, and could contribute very nicely to a composition's sense of harmonic progression and weight.

2.5.3 Algorithm

More than any other piece in this thesis, my compositional interest here was on an algorithm, and the shaping of its output. The algorithm used here was the Lehmer equation, described briefly earlier in Chapter 1.11. The Lehmer equation is a simple recursive function often used as the basis for pseudo-random number generation in computer hardware and software (Battey 2004, Ames 1992). I developed a version of it for use in *ArtWonk*, and later in *SoftStep,* which I used in this piece. As stated earlier in Chapter 1.11, the output of the equation depends on its two inputs, A and B.

- If A+B = <1, a single repeating value is produced.
- If A+B = slightly more than 1, repeating patterns with some degree of randomness are produced.
- As A + B get larger, the result gets closer and closer to a uniformly distributed pseudo-random sequence, until
- Uniform equal-weighted randomness is achieved when A+B > 10.

In my implementation, I have found that true uniform pseudo-randomness is not really achieved until A+B is well above 20. This gives me a wider range of "almost random" outputs to play with.

Using The Lehmer equation to select pitches on a pulse over a 5 octave range, from MIDI 36 to MIDI 96 (C just below the bass staff to C an octave and a Perfect 5th above the treble staff), the characteristics of these different settings can easily be seen and heard. Figure 23 shows the output where A + B is equal to 1. Notice that the output quickly ascends from a low note to a high note, and once it gets to its upper limit, just stays there.

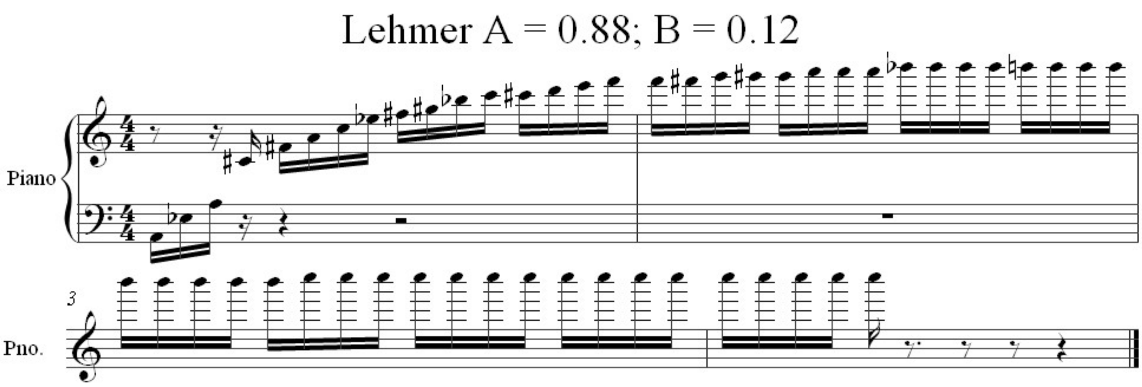


Figure 23: Output of the Lehmer equation mapped to pitch where A+B = 1.0

Figure 24 shows what happens when A+B = 2.0. Repeating downward figures result, but on each repeat, the length of the downward figure, and its pitch content changes. In measure 4 the downward figures get to their shortest length, before expanding and contracting again.

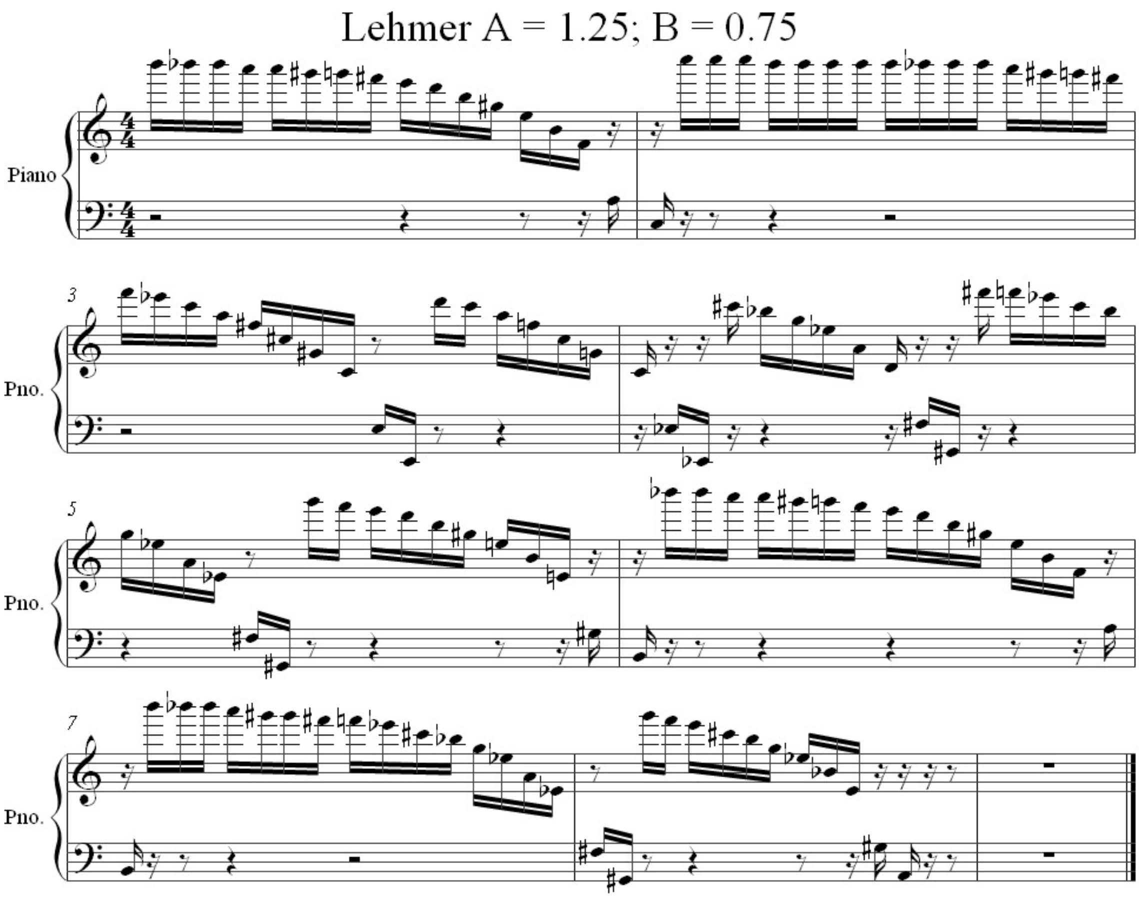



Figure 24: Output of the Lehmer equation mapped to pitch where A + B = 2.0

In Figures 25 and 26, A+B is even higher, and we can see the repeating figures of Figure 24 breaking up and tending more towards the kind of output that might be expected from a uniformly-distributed random sequence. In Figure 25, A+B = 2.87. As in Figure 24, it consists of descending figures, but the length of the figures gets shorter and shorter, until in measure 4, three note repeating figures occur, and by measure 5, the output is very similar to a "true random" distribution, with descending figures again becoming dominant in measure 6. It should be noted here that descending figures are not always characteristic of the output of the Lehmer equation. The numbers I chose for these two examples just happened to create descending patterns. By the time we get to Figure 26, in which A+B = 3.62, the output of the equation is getting much closer to what would be expected from an equally-weighted pseudo-random distribution. But even here, the sense of "clumping" and directional gesture is still somewhat removed from that produced by an equally weighted random distribution. Clearly, there are a lot of resources to explore in just this one equation.

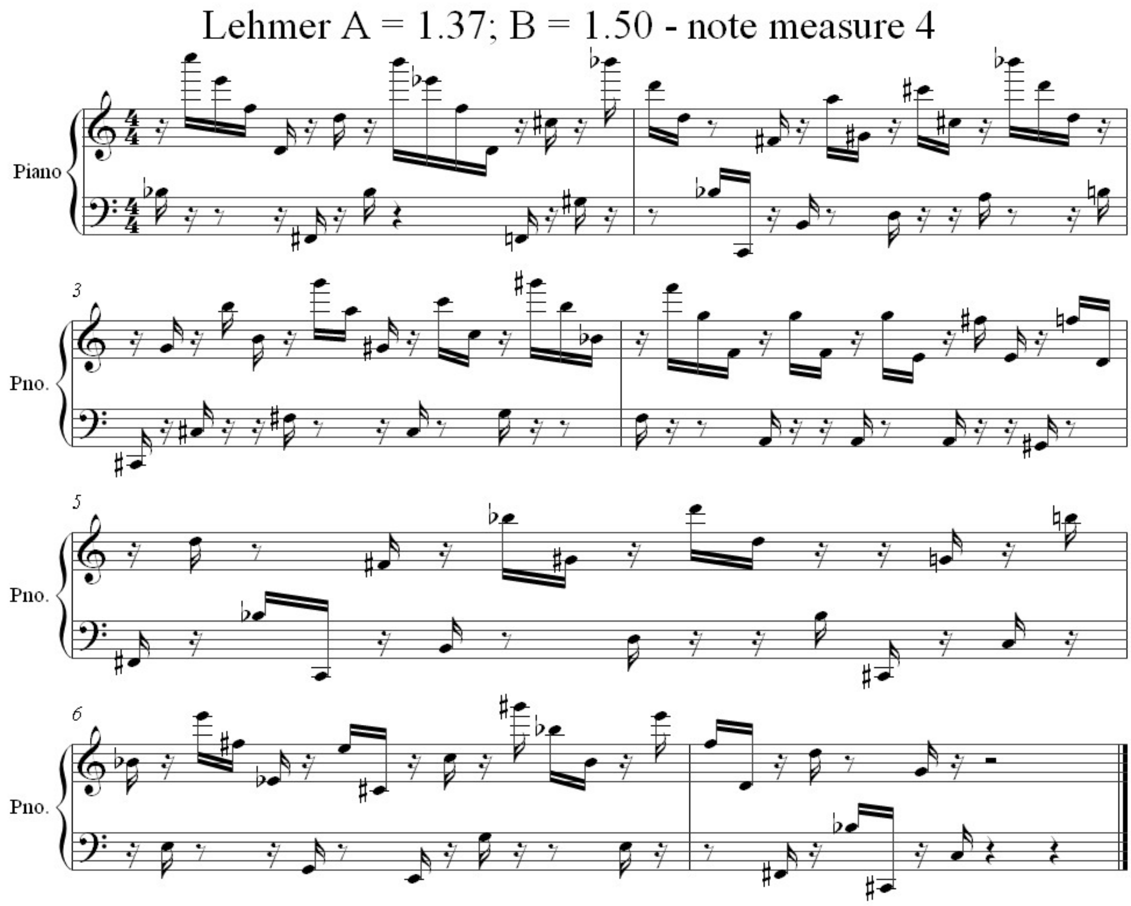


Figure 25: Output of the Lehmer equation mapped to pitch where A+B = 2.87

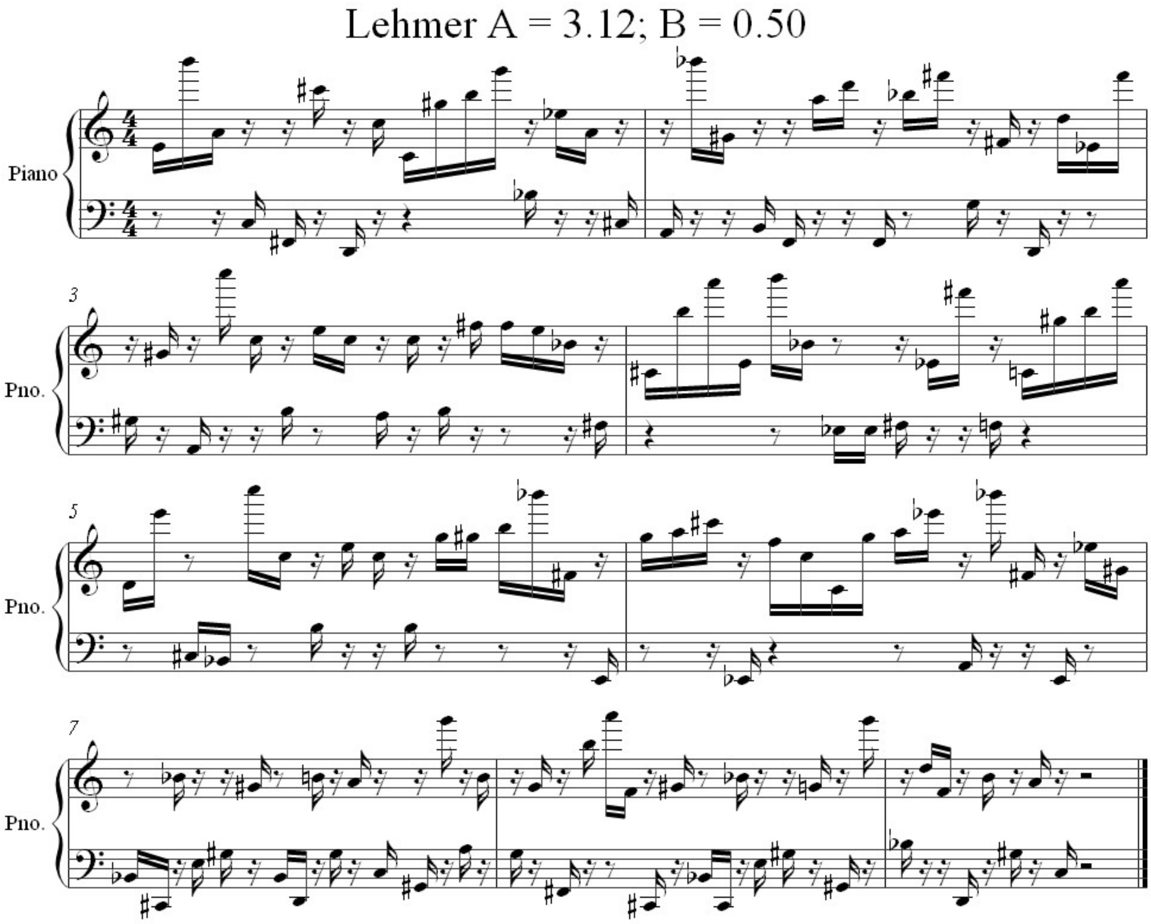



Figure 26: Output of the Lehmer equation mapped to pitch where A+B = 3.62

For *Lehmer's Kookaburra*, I had constantly changing values going into inputs A and B, and the output of the equation was constantly being scaled over different ranges.

I programmed a series of six very slow ascending ramps to act as overall formal controls in the *SoftStep* patch made for the piece. These six ramps were clocked by a common pulse but each ramp was a different length. The rhythmic proportion of the ramps, expressed in their shared clock pulses, was 13:17:19:23:29:31. That is, the first ramp (at speed 13) took approximately 35 seconds to reach the top of its range, while the last (at speed 31) took about 84 seconds to reach its maximum, with the durations of the other ramps arrayed proportionally between them. Clearly, once started, it will be a very long time before these six ramps will come into sync.

Two of these ramps (13 and 17) slowly raised the values of A and B in the Lehmer equation, until they independently snapped back to zero to begin climbing again. Considering the radically different kinds of musical gestures produced with different inputs into the equation, as shown in Figures 23-26, above, it can easily be imagined that constantly changing the input values of the equation will create a constantly evolving set of output gestures.

However, these gestures were further processed by the other ramps, which gradually expanded the ranges of pitches and durations chosen. In the case of pitch, this gradually increased the pitch range of the output of the Lehmer equation from a single low note to the full 3 and ½ octave range of Puff In the case of durations, the range of control was from 6 to 17 clock pulses, with 6 pulses producing sixteenth notes at 120 beats per minute, while 17 clock pulses produced durations of just under a dotted eighth note. At the low limit of the ramps, the Lehmer equation has no effect – all durations are 6 pulses long. At the high limit, the Lehmer equation is applied to the full range of 6 to 17 pulses to select varying durations.

Therefore, in this piece, pitch and duration output is controlled by a three level process:

- The basic Lehmer equation
- With its inputs A and B constantly being changed by ascending ramps, thus making continuously changing kinds of gestures
- And the output of the equation constantly being compressed and expanded by other non-synchronized ramps before being applied to pitch and duration choices, thus continuously changing the ranges of the continuously changing gestures.

The piece has two lines, one of which covers the entire range of the instrument, the other of which controls just the top half of the range. This two voice texture contributes a further level of complexity to the overall sound of the piece.

2.5.4 Process – Real Time Usage

This combination of a changing equation, and changing parameter ranges, all controlled by a series of non-synchronized ramps, provided a very rich set of disjointed, tumbling musical gestures to enjoy, especially when applied to the huffing, sputtering, clacking Puff. Although I composed the piece interactively, listening to samples of Puff while I adjusted the parameters of the *SoftStep* patch, when I finished the patch, I just sat back and enjoyed the show.

The piece is probably the least pitch oriented of any in this thesis. Its sputtering gestures, leaping from one texture to the next, present a very ragged surface to the listener. Because of the non-synchronized control ramps, nothing in the piece is ever in sync, but is in a process of constant change and evolution, although characteristic gestures, such as snapping back from a wide pitch range to a single repeating pitch are heard frequently. The sound of Puff is, as stated above, quite jolly, and when rapidly repeated pitches are played in some registers, the result has a great resemblance to the laugh of a kookaburra. Since our house is surrounded by kookaburras, we hear their laughter all through the day, and far into the night. That a lot of kookaburra got into this piece says much to me about the subconscious effect one's environment has on one's art: all of the structural fine-tuning, such as that discussed above, in addition to its exploratory value, might also be a means by which the composer's subconscious asserts itself.

2.6 *The MOSsy Slopes of Mt Meru – The Meru Expansion*

2.6.1 Introduction

The pieces discussed in this and the next two Chapters form a larger unit known as *The Wilson Installations.* This is a set of three very large-scale live solo electronic music performance installations that deal with extended tuning systems, the relation of tuning to timbre and spatiality of sound, concentration of attention for extended time periods, site specific installation and performance, live interaction with algorithmic generation of melody and harmonic choices, and the notion of the artist re-gaining control over the presentation and production of their work. The initial impetus for each work came from certain music theory articles by Ervin M. Wilson, and in each work, implications in his initial articles have been expanded upon, creating large families of interrelated scales and harmonies, with between 60 and 167 scales found in each.

All the works have been performed in a number of contexts. Performances of the later members of the series will be discussed in Chapters 2.7 and 2.8. All three pieces also exist in recorded form, with the entire cycle being available as a four CD set from our www.tropicapricorn.com website. MP3 files of all these pieces are included in Appendices 3.6.1.1, 3.6.1.2, 3.7.1.1, 3.7.1.2, and 3.8.1. I'm still waiting for the right circumstances to emerge to present all three pieces in the ideal way I envisage, as a day-long, or multiple-days long, series of installation/performances, where over the course of the time period, all three works would be performed.

The first, *The MOSsy Slopes of Mt. Meru*, was first composed in 2002 in a version for a single laptop computer. It was expanded the next year into a second version, *The MOSsy Slopes of Mt. Meru: The Meru Expansion,* for 2 laptop computers controlling a software synthesizer and a hardware synthesizer. The original version was premiered on 23 March 2002, at the ArtScience Laboratory, Santa Fe, NM. Subsequent performances were on 25 April 2002 in the Krannert Center for the Arts, University of Illinois; 26 May 2002 at the Deep Listening Space in Kingston, NY; and on 6 July 2002 at the Australasian Computer Music Association annual conference in Melbourne. The paper presented at that conference, discussing my first attempts at deriving these scales, will be found in Appendix 3.6.4, with its sound examples in Appendix 3.6.5. The expanded version, *The Meru Expansion,* was premiered on 14 June 2003 at the Rechabite Hall, Northcote, Melbourne, and subsequently performed on 1 August 2003 as part of the Sonic Connections Festival at the University of Wollongong. Performances lasted from 15 minutes up to 70 minutes. The recorded performance of the first version, *The MOSsy Slopes of Mt. Meru*, was made in March 2002 and lasts around an hour. The recording of the second version, *The Meru Expansion*, was made in May 2004 and lasts 78 minutes.

This piece represents some of my purest investigations into tuning. The timbre used in the work is pure sine wave only. Composite timbres are made by a combination of the tuning and the behaviour of the sine waves in the acoustic of the performance space. Spatial movement, as well, is created by the interaction of the sine waves, the tuning, and the acoustics of the space.

2.6.2 Tuning System

The scales used in this piece are all based on Wilson's work with additive number sequences which are derived from the sums of the diagonals of Pascal's triangle. This work has been introduced in Chapter 1.9, and we here expand on it further.

It should be mentioned that other music has also been based on Pascal's triangle. One such piece, representing a completely different approach, applying the relationships of the triangle to the piano in 12-tone tuning, was composed by Tom Johnson in *Music for 88* (Johnson 1988). In Wilson's 1993 article, "The Scales of Mt. Meru" (Wilson 1993), following up on work by Thomas M. Green (Green 1968) and A. N. Singh (Singh 1936), he shows how an infinite series of recurrent sequences can be derived by taking the sums of different diagonals of this diagram.

As discussed earlier in Section 1.9.3, in this triangle, each number in each lower row consists of the sum of the two numbers diagonally above it. If parallel diagonal lines are drawn across the diagram (in this case, parallel with the line from the third number down on the left outer edge to the second number down on the right outer edge), and the numbers that those parallel lines intersect are added up, the result is the additive sequence shown on the right side of the diagram. In this case, the series happens to be the well known Fibonacci series, in which each subsequent number is the sum of the two numbers preceding it. Figure 27 shows Wilson's diagram of this.

Please see print copy for Figure 27

Figure 27: Diagram from Wilson 1993 (used by permission) showing derivation of Fibonacci series from diagonals of Pascal's triangle

A choice of a different angle for the diagonal lines yields a different additive sequence with each additive sequence having its own rule describing which numbers will be added to produce the next element in the series. For example, in Figure 28, the diagonals parallel the line from the fourth number down on the left outer edge to the second number down on the right outer edge. The sums of these diagonals form a different additive series than the Fibonacci series. In this series, each new number is the sum of the number preceding it and the number three elements before it in the series. A listing of 11 different additive sequence rules is found in Table 58.

Please see print copy for Figure 28

Figure 28: Diagram from Wilson 1993 (used by permission) showing derivation of number series with rule $B_n = B_{n-3} + B_{n-1}$ from a different series of diagonals of Pascal's triangle

Yet a different number sequence, with a different rule, and a different limit, is made by summing the diagonals that parallel the line from the 4th number down on the left outer edge to the 3rd number down on the right outer edge. The rule for this series is to sum the third and second preceding elements in the series to form the next. This is shown in Figure 29.

Please see print copy for Figure 29

Figure 29: Diagram from Wilson 1993 (used by permission) showing derivation of number series with rule $C_n = C_{n-3} + C_{n-2}$ from yet another series of diagonals of Pascal's triangle

In Wilson 1993, he gives diagrams for 9 different additive sequences formed by the sums of the diagonals of Pascal's triangle. In Wilson 1997b, however, he lists rules for 192 different additive sequences. Figure 30, below, shows the first 31 of these. Note that as you get further and further out in the series, the limits (labeled in this diagram as "decimal") get closer and closer together:

Please see print copy for Figure 30

Figure 30: List from Wilson 1997b of the first 31, of 192 additive sequence rules, with their limits, formed by summing different diagonals of Pascal's triangle

In *The MOSsy Slopes of Mt. Meru*, I based the scales for the original version on the first 5 of these additive sequences. This is described in great detail in "Developing and Composing with Scales Based on Recurrent Sequences" (Burt 2002b). For *The Meru Expansion*, I used the first 11 of these rules to form scales. In *Proliferating Infinities*, to be discussed in Chapter 2.11, I continued to use the first 11 rules, but derived results from a whole family of number triangles, of which Pascal's was only the first.

As stated in Section 1.9.1, in the works discussed in this thesis, scales have been derived from additive sequences either by treating each number as a harmonic, or by using the limit of the sequence (in two different ways) as a generator for a scale made by stacking intervals. In both these cases, a scale can have a potentially infinite number of tones, so the question arises – where to stop? As discussed in Subsections D and E of Section 1.6.3, one way to get useable scales that have some sort of property of recognisability or coherence was suggested by Erv Wilson. He suggested that those scales that had MOS characteristics (for the stacked interval scales) or which were well formed (for the harmonics scales) would be useful subsets.

Further, as shown in Table 15, I noticed that in some cases, the size of consecutive MOS scales was such that the sizes of two consecutive scales added up to the size of the next adjacent scale. The first two scales would interlock to make the third, just as the 7 white and 5 black keys on the piano interlock to form a 12-note chromatic scale. I decided I would use this property, and I would only use the lowest three MOS scales available from my sequences which followed this rule.

For example, for the Fibonacci series, which follows the rule $A_n = A_{n-2} + A_{n-1}$, and which converges on 1.618033989, scales of 10, 13 and 23 elements are MOS. Using these scale sizes, I made 6 related scales.

- First, I made a 23-note Just-Intonation scale using the numbers of the series as harmonics and reducing them to an octave
- I then divided that scale up into interlocked 13-note and 10-note subsets. I determined the division by the order of the numbers of the series. That is, the first 13 numbers of the series generated the 13-note subset, and the

next 10 elements of the series generated the 10-note subset. When these were placed within an octave, they made interlocking patterns

- The 13-note subset was used as a second scale, and the 10-note subset used as a third scale
- I then made a 23-note stacked scale, using the limit as the generator, as described in Section 1.6.1
- I then took the first 13 elements of this stacked scale, and used them as one subset, and the next 10 elements of the scale were used as another subset. This gave me a total of six scales:
    1. The first 23 elements of the number series as (octave-reduced) harmonics. (Just- scale)
    2. The first 13 elements of that series as a subset. (Just- scale)
    3. The next 10 elements of that series as a subset. (Just- scale)
    4. An octave-reduced chain of 23 generator intervals. (stacked scale)
    5. The first 13 elements of that series as a subset. (stacked scale)
    6. The next 10 elements of that series as a subset. (stacked scale).

Table 58 shows the scale sizes I used for each additive sequence, as well as the rule which generates each sequence. With 11 additive sequences, and six scales each, this should generate 66 scales. However, the limit of series F is exactly the same as the limit of series C, so the three stacked scales for series C are exact duplicates of the three stacked scales for series F. The Just-Intonation scales made by the two series, however are different. Therefore, there are 63 unique scales used in this piece, which as an aggregate make up the composition's harmonic vocabulary. This raises the issues, alluded to before in this thesis, of "massive" harmonic vocabularies, thresholds of perception, and ideas of extreme duration as a means of dealing with these. I use the algorithmic process outlined in the next section to articulate these scales. Table 59 gives the 23-note MOS Just- scale made from the Fibonacci series. Table 60 shows how it is divided up into interlocking 13- and 10-note MOS subsets. A complete catalogue of all 63 scales can be found in the Appendix 3.6.2. *Scala* *.scl and *.tun files for all the scales are found in Appendix 3.6.3.

Table 58: Additive sequence rules and MOS divisions used in *The MOSsy Slopes of Mt Meru: The Meru Expansion*

Number	Rule	MOS formation
1	$A_n = A_{n-2} + A_{n-1}$	13 + 10 = 23
2	$B_n = B_{n-3} + B_{n-1}$	11 + 9 = 20
3	$C_n = C_{n-3} + C_{n-2}$	7 + 5 = 12
4	$D_n = D_{n-4} + D_{n-1}$	15 + 13 + 28
5	$E_n = E_{n-4} + E_{n-3}$	10 + 7 = 17
6	$F_n = F_{n-5} + F_{n-1}$	7 + 5 = 12
7	$G_n = G_{n-5} + G_{n-2}$	13 + 10 = 23
8	$H_n = H_{n-5} + H_{n-3}$	7 + 4 = 11
9	$I_n = I_{n-5} + I_{n-4}$	13 + 9 = 22
10	$J_n = J_{n-6} + J_{n-1}$	14 + 11 = 25
11	$K_n = K_{n-6} + K_{n-5}$	6 + 5 = 11

Table 59: 23-note MOS Just-Intonation scale made by treating the elements of the Fibonacci series as harmonics

Scale degree	Ratio	Cents
0:	1/1	0.000
1:	4181/4096	35.559
2:	17/16	104.955
3:	17711/16384	134.830
4:	9/8	203.910
5:	75025/65536	234.101
6:	305/256	303.199
7:	5/4	386.314
8:	323/256	402.468
9:	21/16	470.781
10:	5473/4096	501.739
11:	89/64	570.880
12:	1449/1024	601.010
13:	377/256	670.105
14:	3/2	701.955
15:	1597/1024	769.378
16:	13/8	840.528
17:	6765/4096	868.649
18:	55/32	937.632
19:	28657/16384	967.920
20:	233/128	1037.023
21:	121393/65536	1067.191
22:	987/512	1136.288
23:	2/1	1200.000

Table 60: Division of Just Fibonacci scale from Table 59 divided into 13 + 10-note interlocking groups. Numbers in boxes refer to scale degrees

10		1		3		5			8		10		12			15		17		19		21		
13	0		2		4		6	7		9		11		13	14		16		18		20		22	0(23)

2.6.3 Algorithm

I devised an algorithmic process which mirrors the structure of additive sequences to choose pitches and rhythms in this piece. Since I was using additive sequences, formed by the addition of previous elements, I thought that the "Accumulator" module in *Softstep* would be an appropriate melody generator. The Accumulator is an idiosyncratic up-down counter. That is, at every new clock pulse, it adds the number at its value input to the sum of the previous values. When it reaches a (user-settable) maximum value, it reverses direction, subtracting value inputs from the sum until it reaches the (user-settable) minimum value, at which point it begins adding the value input again. This produces a series of numbers which ascends and descends between specified limits, but the amount of ascent or descent can change constantly.

For the value input, I constructed an array of lower members of the number series which generated the scale being used at the time. In the case of the Fibonacci series, this array was 1 1 2 3 5 8 13 21. Selection of elements from this array was under control of a shift-register feedback random number generation algorithm derived from John Roy and Joel Chadabe's work on their early 1970s random information generator called *Daisy* (Chadabe 1997). Other people who used this algorithm at the time were myself (Burt 1975), Salvatore Martirano (Chadabe 1997), Greg Schiemer (Schiemer 1990) and Carl Vine (Vine 1977).

The shift-register feedback module selected one of the eight members of the Fibonacci series. This was then input into the Accumulator. The output of the Accumulator was input into a Modulo-n divider. This rescaled the output of the divider in an interesting way, and the resulting number controlled pitch selection from the current scale. This meant that the melodies used would consist only of intervals (number of scale degree steps) determined by the number series that generated the current scale. For the Fibonacci scales, that meant that only intervals of 1 2 3 5 8 13 or 21 steps would occur.

Given the up-down nature of the Accumulator, and the non-linear scaling that the Modulo-n divider produced, this meant that ascending and descending melodic lines formed the gestural vocabulary of the piece. These lines would not always alternate ascending and descending motion evenly, though, due to the size of the Modulo-n divider and the limits of the Accumulator sometimes being "out-of-sync." For example, if the Accumulator produced a series which goes up and down from 0 to 127 and back, but the Modulo-n divider was set to 30, the result would be a series of ascending lines, then a series of descending lines, as the accumulator values were scaled by the divider. Table 61 shows the effects of this.

Table 61: Input to Accumulator module in *SoftStep* and output of Modulo module with range of Accumulator from 0 to 127 and range of Modulo set to 30

Accum.	0	21	42	43	48	61	66	68	76	97	118	123	127	105	104	99	96	88	85	72	69	56
Mod.	0	21	12	13	18	1	6	8	16	7	28	3	7	15	14	9	6	28	25	12	9	26

The bottom line (Mod) can be read as the melody line, given in scale degrees. In this case, it is controlling a scale of ten steps per octave over three octaves. Notice how the series does not evenly alternate going up and down, but that the up/down gestural shape predominates. Notice also that when the accumulator hits its maximum (or minimum) limit, it simply stops there, producing the occasional interval not given by our array of possible interval sizes. For example, the Accum. line going from 123 to 127 (max limit) is an interval of 4 scale degrees, not a Fibonacci numbered interval. This occasional "error" is fine by me. It makes things a bit less consistent, and a bit more interesting.

Rhythmic durations, in the form of time points, are chosen with a similar algorithm. Each new time-interval is selected by an independent occurrence of the same shift-register feedback algorithm, which randomly selects from an array of possible durations. The values in this array are also chosen from the generating number series. In the case of the Fibonacci series scales, these values were 5, 8, 13, 21, 34, 55 and 89. Furthermore, a new time-interval value is chosen every 5 notes, which means that while there may be (and frequently are) sequences of 10 or 15 notes having the same time-interval; at least every 5 notes, there is the potential for a rhythmic change.

Interestingly, these are heard more as tempo changes than as rhythmic changes. There is a further level of rhythmic complication added, though. The rhythm pulse is fed into a module called a "Pattern." This sends trigger signals out only on certain defined steps of a sequence. In the case of this piece, as can be seen in Figure 31, below, there are six Patterns used. The top five of these modules turn on and off pitches of separate oscillators. The bottom one selects a new rhythmic duration. So the "notes" referred to above do not always have the same duration. In the settings shown in Figure 31, notes would start on pulses 1, 2, 3, 4 and 6 out of the 8 pulses of the length of the Pattern. This has the effect of breaking up what might otherwise become an unrelenting pulse-oriented rhythm. A different set of numbers and Pattern settings are used for each scale in the piece. In the course of the piece, all possible combinations of 5 pulses out of 8 are used in the Pattern modules.

Figure 31: Rhythm modules in *SoftStep* patch for *The MOSsy Slopes of Mt. Meru*

2.6.4 Process – Real Time Usage

The combinations of these melodic and rhythmic algorithms produce a monophonic line which has a pleasing combination of predictability and unpredictability. Since most of the notes chosen will have long sustains (longer than the time until the next note begins), they will hang over, creating chords. In effect, the *SoftStep* patch is a very big melody generating machine. I have an on/off control for this machine, and I use it to initiate very long phrases, or extremely short one, thus producing a larger scale sense of phrasing and breathing in the piece. Additionally, in the second composition, *The Meru Expansion,* the second computer is sampling individual pitches from this sequence, and choosing to play these with extremely long durations, thus assembling a series of quasi-drones, which play contrapuntally against the faster melodies produced by the first computer.

The Meru Expansion is played on two laptop computers, linked by MIDI, running John Dunn's *Softstep* algorithmic control program. The larger laptop is also running Martin Fay's *Vaz Modular* software synthesizer, while the smaller computer controls a Korg *XD5R* hardware synthesizer. Both synthesizers are programmed to produce sine waves, the only difference being in envelope lengths and note durations.

In performing these scales, a curious phenomenon occurs. Since the only timbre I use in this piece is a pure sine wave, and since many of the notes are quite long in duration, sustained chords are set up. As a listener, to move one's head even slightly causes the balance of the tones of the chords to change, sometimes quite dramatically. This is because the pure sine tones are setting up standing waves within the room, and each position in the room then has a different set of amplitudes of each waveform. This means that at each point in the room, the balance between the tones of any given chord will be different. And this difference will happen in whatever room the sound is played in, and will vary according to the size and shape of the room interacting with the frequencies of the piece. So in this piece, the spatiality of the sound is a direct bi-product of an interaction between the particular physical architecture the sound is projected into, and the frequencies of the piece itself. Here, timbre, tuning, and spatialisation are intimately linked. There are, of course, much more elaborate ways of relating tuning and timbre. The classic study of this is *Tuning Timbre Spectrum Scale* by William Sethares (Sethares 2005).

It might be asked why I used such plain, stable timbres in this piece and the other two pieces in *The Wilson Installations.* Since I was dealing mainly with harmonies, and sustained harmonies at that, I wanted to use timbres that focussed listening on harmonic quality. Complex and varying timbres, such as the honking of black swans or the screeching of cockatoos, or socially loaded timbres, such as electric guitar distortion or chickens clucking, didn't seem appropriate here.

In live performance then, the machine, under my programming, makes certain decisions as to pitch, duration, sustain, and interval choice. I control the overall phrasing, the thickness of texture, and choose which of the 63 available scales I will explore next. In some performances, I have gone through the scales in linear order (first the Just- scales, then the stacked ones); while in others, I have alternated Just- and stacked examples of the same scale-type; while in still others, I have wandered freely among all the scales. For shorter performances, I tended to wander freely around the available scales, while in longer performances, where time was allowed for subtle listening, I tended to use more linear orders, so that a sense of long term harmonic motion, or at least, long term change of harmonic colour, was also present. These performance decisions were also influenced by subtle aspects of the performing space architecture, outside sounds, time-frame, time of day, audience mood, etc. Keeping one's intuition alert allows one to shape the performance according to the needs and contingencies of the situation.

I and those who have witnessed either of the versions of *The MOSsy Slopes of Mt. Meru* find the piece very relaxing. The purity of the harmonies and the timbres, combined with the slow pacing, and the psychoacoustic effects produced by the interaction of the sine waves and the acoustics of the space, produce a sense of profound meditation and stillness in me and others who have witnessed the piece. Far from sounding boring, or plain, the simple harmonies and structures of this piece sound both meditative and compelling.

2.7 *Pythagoras' Babylonian Bathtub*

2.7.1 Introduction

Because of the way he publishes his work - as sets of diagrams, without much verbal explanation, Erv Wilson's work can frequently seem enigmatic. Several people, including Erv himself, have told me this is his way of making sure that people who work with his materials are really dedicated and interested - they have to "do the math" or the hard work involved in realising the import of the diagrams before any use can be made of them. In my case, this has sometimes meant that papers he sent me have often been first read and dealt with in a fairly superficial manner. They then sit around in the back of my head for several years, until I finally have an insight into them, whereupon I go back and study the paper again and finally understand, to some degree of depth, aspects of the ideas I hadn't understood before. This was definitely the case with his paper "The Golden Horagrams of the Scale Tree" (Wilson 1997a), which I first received from him in 1997. Horagrams are tuning diagrams showing the possible MOS scales made when a particular interval is stacked. They will be explained in Section 2.7.2 below. I made one small piece with the information found in his paper, *64 Golden Chords* (1997), treating each of the 32 horagrams as a template for a chord and its associated additive synthesis timbres. However, it wasn't until I received a paper from David J Finnamore, "5- to 9-tone, octave-repeating scales from Wilson's Golden Horagrams of the Scale Tree" (Finnamore 2001a), that I truly understood the import of Wilson's Scale Tree work.

Pythagoras' Babylonian Bathtub is the piece that came out of my study of the Scale Tree. This is a work for three laptops and three synthesizers, either hardware or software, depending on the capabilities of the computers in question. It is a long, harmonically thick work, designed for live performance in a gallery or installation situation. Although it uses very simple timbres, because of the way different tunings are combined in it, the acoustical effect is a quite complex and surprising timbral field. It has been performed live twice; on 14 June 2003, at Cecil St. Studios, Fitzroy, Melbourne, and on 1 August 2003 at the Sonic Connections Festival at the University of Wollongong. It is a piece I would very much like to perform again, because in it, I set myself a quite engrossing challenge in terms of real-time shaping of a harmonic world. This nature of this challenge will be discussed in Section 2.7.3.

The title is not the total joke it first appears. *Pythagoras* refers, of course, to the philosopher. Scales made of stacked Perfect 5ths are referred to as "Pythagorean." Extending the use of the term, I decided to refer to ***all*** scales made by stacking intervals of any kind as "Pythagorean." All scales derived from the scale tree are made by stacking intervals of one kind or another. Therefore, this piece (in which I have the possibility of drawing on 167 different scales, all made by stacking intervals of one sort or another) can be considered extravagantly Pythagorean. Legend tells us that Pythagoras studied in Egypt and Babylon, and brought those cultures' mathematical ideas, such as making scales by stacking intervals, back to Greece. That's the *Babylonian.* But *Bathtub?* This is for two reasons. First, the piece is an immersive experience – a real sound bath of shimmering beating harmonies. Secondly, it refers to a bit of myth-making on my part. I'm assuming that like many people who studied abroad, Pythagoras probably found aspects of the culture he visited to be very attractive. So why not a bathtub? In any case, the image of trying to import a bathtub from Babylon to southern Italy in 500 BCE is one that appeals to my sense of the ridiculous. So the title is more than a joke – it has historical references – but it is, of course, still partially a joke.

Figures 32 and 33 are photos taken by Catherine Schieve during the July 2003 performance of the piece. Figure 32 shows the setup with three laptops and small hardware synthesizers. Figure 33 is a view of the performing setup. The graphics on the computer screens show the performer the keyboard interface used by the Scala program. This interface was the principal tool I used to perform the piece. The standard black and white key keyboard visible in Figure 32 was only very rarely used to play a single tone or two, during the course of the performance. The computers are placed in the middle of the space. Four loudspeakers, not visible in the photos, are placed in the corners of the room, up high. A stereo signal is sent to these speakers in an X-Y configuration. This fills the space with sound quite nicely, and allows the architecture of the space and the timbres to interact to form subtle spatial effects. Rugs were placed on the floor on all sides of the equipment, allowing people to freely move around, or lie on the floor, in any part of the space they desired. People could also watch over my shoulder, viewing my performance technique and the computer screens. Every effort was made to make the space as inviting and comfortable as possible, a context in which quiet, meditative listening could occur. The performance lasted about 3 hours. A small audience of about 40 came and went throughout the afternoon.

Figure 32: Warren Burt performing *Pythagoras' Babylonian Bathtub* at Cecil St Studios, Melbourne, 14 June 2003

Figure 33: Warren Burt performing *Pythagoras' Babylonian Bathtub* at Cecil St Studios, Melbourne, 14 June 2003

2.7.2 Tuning System

As stated above, the tuning system of the piece is a set of 167 scales made from the ratios of the Scale Tree. The Scale Tree, and methods of deriving scales from its ratios, was discussed in detail in Chapter 1.8. We now describe in more detail how the particular harmonic world of this piece was derived.

Briefly summing up, the ratios on the Scale Tree can be used in three ways to make generators that are stacked to make scales:

1. The ratio can be used to describe a generator size of N steps out of an Equal-Temperament of M steps. I call this "Gen. N/M ET."
2. The limit of a particular chain of Scale Tree ratios, multiplied by 1200c, can be used as a generator. I call this the "Wilson/Finnamore Limit."
3. Each ratio on the scale tree can be treated as a Just-Intonation ratio, and the convergence ratio, in cents, of the ratios on a particular branch, can be used as a generator. I call this the "Burt Limit."

As with all scales made by stacking a generating interval, the problem of scale size arises. In common with all such scales, Moments of Symmetry occur, where a scale will have two and only two interval sizes. For example, stacking a generator interval of 8 steps out of 27-tone Equal-Temperament, or 355.556 cents, we find that Moments of Symmetry occur with scales of 2, 3, 4, 7, 10 and 17 steps.

Wilson evolved an ingenious diagram, the *horagram*, to show which scale sizes are MOS. This is a diagram, made of concentric circles, where each circle, progressing from the centre, shows a new MOS size. A circle of 1200 cents is used, and the generator interval line extends out from the centre. Each subsequent scale interval is then placed one ring of the circle outwards. If the scale generated is non-MOS, lines are added on that ring until the scale generated has MOS characteristics. In Figure 34, made with *Scala*, the generator is expressed as a number g, which shows the generator as a percentage of 1200 cents. 1200 x 0.29629667 = 355.556 cents. The width of the angle in degrees (formed between the generator and the origin), is given below g. MOS scale sizes are listed in pale red numbers, ascending the vertical generator line. The smaller numbers below each MOS number give the interval sizes in the scale, rounded to the nearest cent.

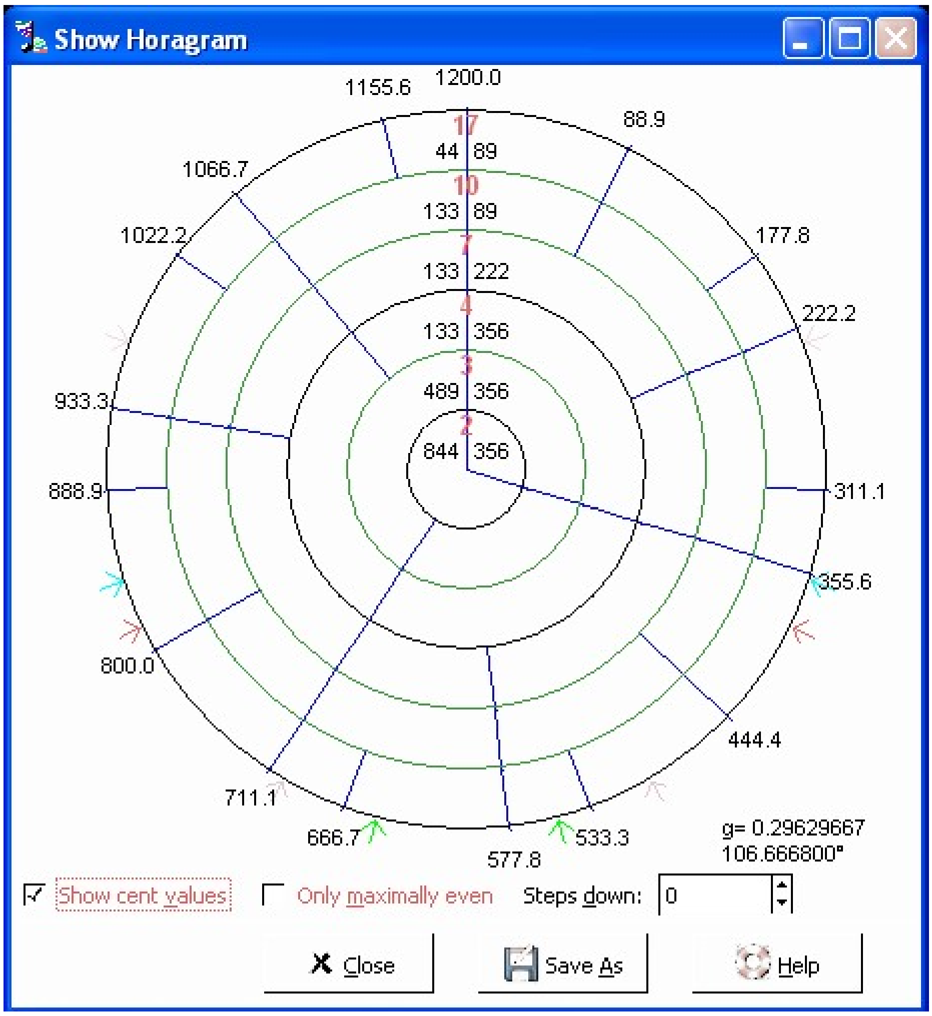


Figure 34: Horagram made by *Scala* showing MOS scales sizes for scale generator of 355.556 cents

The Scale Tree has 32 branches. I worked out the generating intervals for all 32 branches given in Finnamore's diagram, given in Chapter 1.8, Figure 5. For *Pythagoras' Babylonian Bathtub*, I decided to use MOS scale sizes of between 11- and

19-tones. I did this because David Finnamore had already composed a number of pieces in the 5- to 9-tone MOS scales, and I was interested in creating scales slightly bigger than that, but not so big that they would become unwieldy. Given the generators made by three different methods from 32 branches of the scale tree, and limiting ourselves to MOS sizes of 11- to 19-notes, a family of 167 possible scales are generated. It seemed to me that 167 scales was a suitably large set of harmonic resources to keep me involved for a while. Forgive the understatement: when I realised how many scales would be in this family, actually, I was shocked. Feelings of panic gripped me – how could I ever deal with that number of scales in a single piece, or even a set of pieces? The algorithm and performing structure evolved for this piece was my answer. It will be discussed in Sections 2.7.3 and 2.7.4.

Table 62 is a comprehensive chart showing the results of this work. All the generators, as well as all the 167 MOS scales, are listed. Complete listings of all the scales will be found in Appendix 3.7.2. *Scala* *.scl and *.tun files for each scale are found in Appendix 3.7.3. Proceeding from left to right, here is an explanation of the columns of the chart:

- The first two columns simply show the branch number of the Scale Tree in Wilson's and Finnamore's notations
- The next column gives the generator of N steps out of an Equal-Temperament of M steps, and the cents equivalent, as given in method 1
- The next column give the MOS scales of between 11 and 19 steps made with that generator
- The next two columns give the generator size for the Wilson / Finnamore method of using the limit-ratio of a particular branch of the Scale Tree as

a generator, as given in method 2, above, and the MOS scale sizes between 11- and 19-yones made with that generator

- The final two columns give the generator size for the Burt method of interpreting the Scale Tree ratios as Just-Intonation ratios, as given in method 3, above, and the MOS scale sizes between 11- and 19-tones made with that generator.

Some things should be noted about this chart. First, it might be noted that in most cases, the MOS scale sizes for the N/M ET generator, and the Wilson-Finnamore limit-ratios are the same. This is to be expected, as the generator sizes are so similar. Sometimes, however, this is not the case, as in Wilson branch 17, where the N/M ET generator has MOS sizes of 11 and 14, and the Wilson/Finnamore limit has MOS sizes of 11, 14 and 17. This occurs because the N/M ET generator here is 6 steps out of 17-tone Equal-Temperament, 423.53 cents. A 17-tone scale is indeed generated by that limit – the 17-tone scale itself, which of course is not MOS. The Wilson/Finnamore limit, however, is just sufficiently different – 425.23 cents - to generate a true MOS scale of 17 degrees. This occurs several times in this chart.

Second, it must be noted that the true size of the family of scales generated here is 165 scales, not 167. The 2 lowest N/M ET generators, 2/13 and 1/8, do not generate any MOS scales between 11 and 19 steps. To compensate for this, I substituted 3/20 (180.00 cents) for 2/13, and 3/23 (156.52 cents) for 1/8, to get MOS scales of 13 and 15 steps each. These two limits are close enough to the actual limits to give suitable substitutes.

Table 62: Generator sizes and 11- to 19-note MOS scales available on Scale Tree

Wilson Branch #	Finnamore Branch #	Gen. N/M ET - cents values in parentheses	MOS scale sizes 11-19	Wilson/ Finnamore Limit x 1200c Gen.	MOS scale sizes 11-19	Burt Limit Ratio as Cents Generator	MOS scale sizes 11-19
32	18	6/13 (553.85c)	11	550.96c	11,13	1052.4c	17
31	10	9/20 (540.00c)	11	541.38c	11	1022.02c	13
30	6	11/25 (528.00c)	16	527.15c	16	975.90c	16
29	27	10/23 (521.74c)	16	522.72c	16	961.29c	16
28	28	11/26 (507.69c)	12,19	506.94c	12,19	908.21c	13,17
27	4	13/31 (503.23c)	12,19	503.79c	12,19	897.416c	11,15,19
26	15	12/29 (496.55c)	12,17	495.90c	12,17	870.12c	11,18
25	23	9/22 (490.91c)	12,17	491.95c	12,17	856.245c	17
24	24	9/23 (469.57c)	13,18	468.62c	13,18	772.148c	11,14,17
23	16	12/31 (464.52c)	13,18	465.08c	13,18	759.03c	11,19
22	1	13/34 (458.82c)	13	458.36c	13	733.82c	13,18
21	32	11/29 (455.17c)	13	455.78c	13	724.056c	13,18
20	31	10/27 (444.44c)	11,19	443.74c	11,19	677.68c	16
19	8	11/30 (440.00c)	11,19	440.59c	11,19	665.38c	11
18	12	9/25 (432.00c)	11,14	431.12c	11,14	627.746c	11,13,15,17,19
17	20	6/17 (423.53c)	11,14	425.23c	11,14,17	603.937c	11,13,15,17,19
16	19	5/16 (375.00c)	13	373.07c	13,16	377.38c	13,16,19
15	11	7/23 (365.22c)	13	366.26c	13	345.47c	17
14	7	8/27 (355.56c)	17	354.82c	17	290.56c	13,17
13	29	7/24 (350.00c)	17	350.90c	17	271.295c	13
12	30	7/25 (336.00c)	11,18	335.18c	11,18	191.96c	13,19
11	2	8/29 (331.03c)	11,18	331.67c	11,18	173.75c	13
10	14	7/26 (323.08c)	11,15	322.27c	11,15	123.96c	19
9	22	5/19 (315.79c)	11,15	317.17c	11,15,19	96.35c	11,12,13
8	21	4/17 (282.35c)	13	280.61c	13,17	1084.33c	11
7	13	5/22 (272.73c)	13	273.85c	13	1042.11c	15
6	3	5/23 (260.87c)	14	259.85c	14	951.27c	14,19
5	26	4/19 (252.63c)	14	254.04c	14,19	912.13c	13,17
4	25	3/16 (225.00c)	11	222.97c	11,16	686.24c	12,19
3	5	3/17 (211.77c)	11	213.60c	11,17	611.92c	11,13,15,17,19
2	9	2/13 (184.62c)*	13 (0)*	181.32c	13	328.32c	11,18
1	17	1/8 (150.00)*	15 (0)*	157.52c	15	84.70c	11,13,15,17,19

It will also be noticed that in the N/M ET and the Wilson/Finnamore columns, the generator sizes decrease as one goes down the chart. In the Burt column, generator sizes decrease until Wilson branch 9, at which point they leap up to the top of the octave again, and decrease for the remainder of the chart. There is probably some underlying structure that gives rise to this kind of descending sawtooth-like structure here. This might be an area for further investigation. In performance, all 167 scales are available to me in real time. I freely select between them as part of the performance, as explained below. Since writing this piece in 2003, I have occasionally dipped into this collection of 167 scales on other occasions. Most recently, I used the 14-note MOS scales in *Scrambled Etudes*, a short piece in which I made deconstructions and microtonal re-mappings of MIDI files of Arnold Schoenberg's op. 19 piano pieces.

2.7.3 Algorithm

This and *The Malleable Urn* (see Chapter 2.2) are the only two pieces discussed in this thesis where moment-to-moment details of the piece are not determined by a machine-performed algorithmic process. In this piece, the selection of musical material is done by the performer, improvisationally, in real time. However, there are 3 different ways in which algorithmic thinking is involved in this piece.

1. A vocabulary of chords is selected with algorithmic methods.
2. The *Scala* keyboards (explained below) are used with these chords to produce harmonies that have a degree of unpredictability to them.
3. Chords in one tuning are combined with the same chord in another tuning to create composite beats, and very complex timbres.

2.7.3.1 Chord selection

The scales of *Pythagoras' Babylonian Bathtub* may have been inspired by the work of Ervin Wilson and David Finnamore, but the compositional and performance aspects of it were a direct response to the new performance devices Manuel Op de Coul built into *Scala*. Beginning in 2002, Op de Coul introduced a series of on-screen keyboards, matrices, and other performance devices. These were intended to act as analytical tools for work with microtonal scales, but were also eminently useable in live performance. For any given microtonal scale, *Scala* provides an on-screen “split-key” keyboard. The design of this keyboard is based on the normal pattern of 7 white and 5 black keys, but adapted to the needs of the scale. Figure 35 is an illustration of a 19-tone Equal-Tempered keyboard. Note the split keys for the accidentals, and the extra “E” and “B” keys for 19-tone.

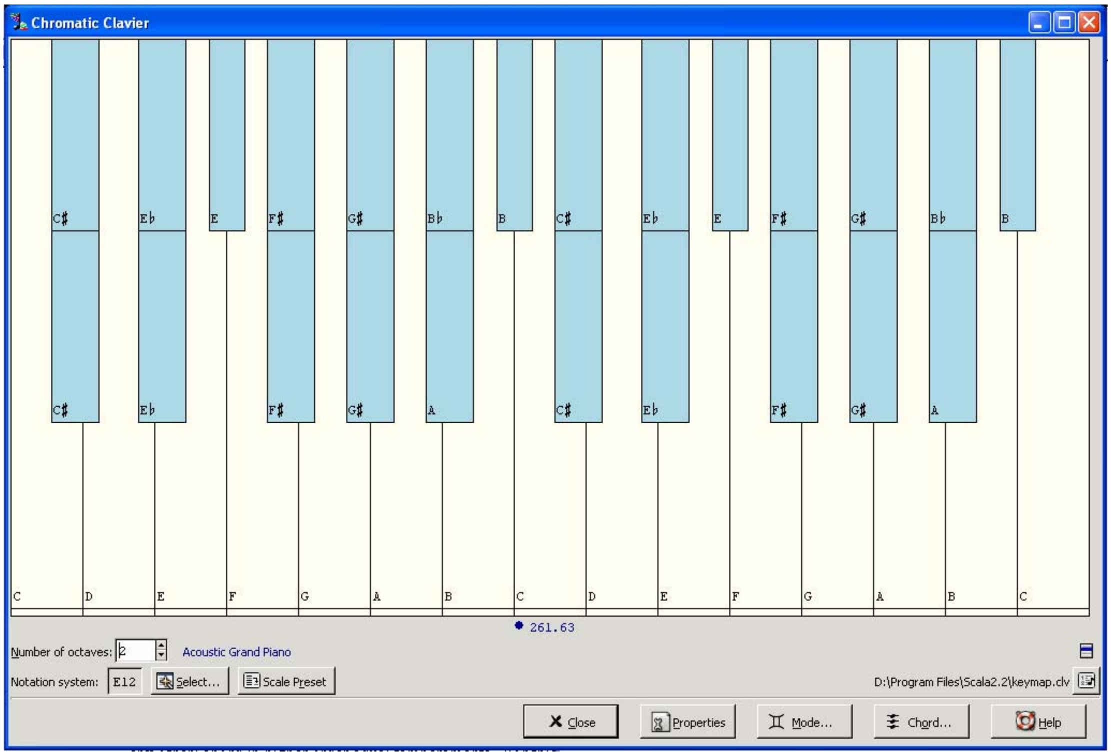


Figure 35: On-screen19-tone Equal-Temperament keyboard made by *Scala*

This keyboard can have up to 10 octaves, and can also sustain chords. These are built up by holding down “Shift” while clicking on the desired notes with a mouse. Using combinations of the arrow, shift, control, page up and page down keys, these chords can then be transposed by a wide selection of intervals.

The keyboard can also be set so that certain notes are on and others are off. This is a tool for exploring modes. By pressing the “Mode” button in the keyboard dialogue, the user can access a large selection of modes (subsets) for a scale of the current size. The user can also add other modes to this list. Figure 36 shows the 19-note scale in a particular mode, “Yasser’s Hexad”, a basic harmonic unit described by Joseph Yasser in *A Theory of Evolving Tonality* (Yasser 1932), and made of intervals of 3 3 3 3 3 4 scale steps. The darker notes shown in Figure 36, below, are non-modal notes, and don’t play. The notes with dots on them are notes I selected to play the chord - a particular voicing of this hexad. I have read various criticisms of Yasser’s choice of this hexad as the harmonic basis for his vision of 19-tone harmony. On setting this mode up, and hearing this chord played in various transpositions, it sounded fine to me. In fact, it sounded like it could be used as the basis for a neo-impressionist kind of harmony. Since Yasser had been trying to extend some of Scriabin’s ideas, this was not surprising.

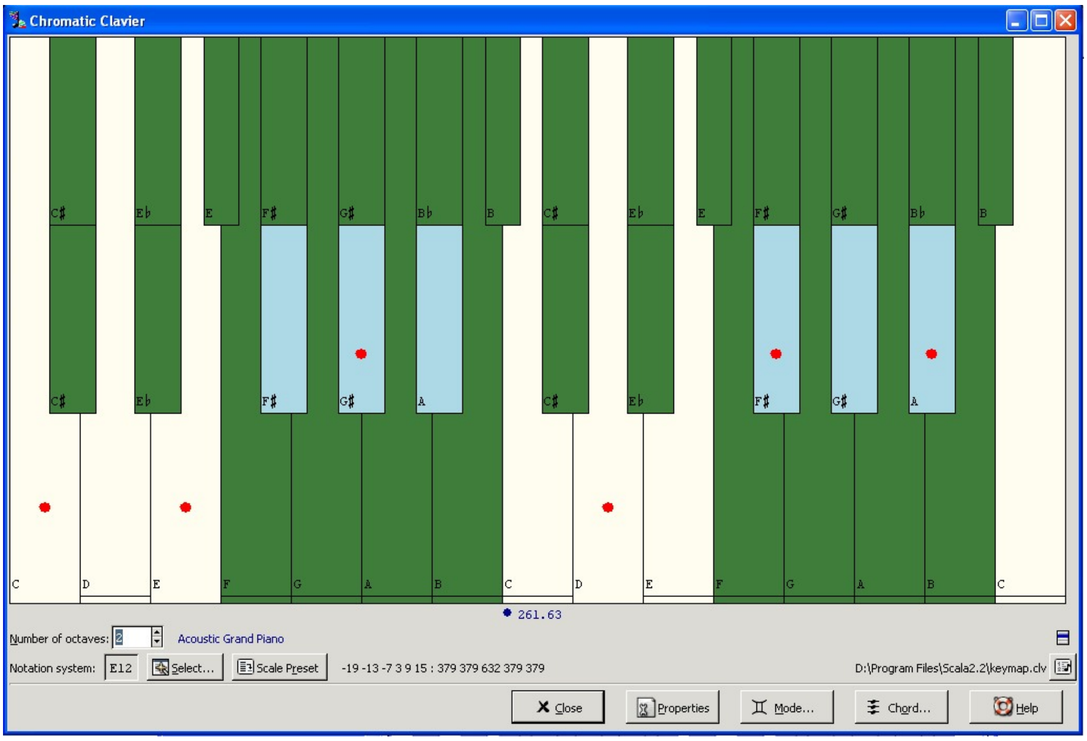


Figure 36: *Scala* 19-note keyboard with Yasser’s Hexad selected as mode. Green keys are non-modal tones and do not play. Red dots indicate sustained notes

Scala also allows you to play chords, and select them from a preset list. This list is selected by pressing the "Chord" button in the keyboard dialogue. It provides a selection of several hundred chords, and the user can add new chords to the list. On opening the chord dialogue, and selecting a chord with a left-click of the mouse, that chord can then be heard by selecting the keyboard window, and right-clicking on the desired fundamental. Right-clicking and then selecting Shift will sustain the chord. Left-clicking anywhere on the keyboard or pressing the space bar will then turn the sustained chord off. A bit of practice with the keys, mouse, chord and mode dialogues will reveal that this is a performance tool of considerable power. If it does not have the virtuosic potential of fingers on traditional keys, it more than compensates for that in immediate access to formidable harmonic resources.

For any given scale, *Scala* will try to find the closest approximation for the chord in question. This could be used, for example, to hear the closest equivalent to a major (or any other) chord in higher order Equal-Temperaments. For this piece, I made a vocabulary of chords based on selections of lower prime and Fibonacci numbers, a set of 13 chords made with the Culver-Cage *IC* program, and a set of chords which were stacks of lower Just- intervals. I placed these chords at the top of the list so that they were the first chords available when the chord window was opened. Table 63 lists these chords. Chords are listed here in three ways:

1. As a list of absolute frequency ratios separated by colons, as in 4:5:6
2. As a list of relative ratios, as in 11/10 13/12 21/20
3. As a list of relative intervals in cents, as in 387.1 197.99 990.2.

The first five of the I-Ching chords were made by selecting seven numbers from 1 to 64 to be absolute frequency ratios. The next seven were made by having *IC* select from a range of superparticular ratios. (A superparticular ratio is one in which the numerator is one larger than the denominator. Some theorists claim that they are inherently more consonant than other ratios.) The final I-Ching chord was made by having *IC* select interval sizes in tenths of a cent. The "Prime Hex" chords were made with sets of 6 consecutive prime numbers used as absolute intervals. The derivation of the rest of the chords shown in Table 63 should be obvious.

Table 63: Chord vocabulary made for *Pythagoras' Babylonian Bathtub*

Number	Chord intervals	Chord Name
1	1:2:3:4:5:6:7:8	Harmonics 1-8
2	(1:2:3:4:5:6:7:8)	Subharmonics 1-8
3	11:13:17:19:23:29:31	Prime numbers chord
4	2:3:5:7:11:13	Prime Hex 1
5	17:19:23:29:31:37	Prime Hex 2
6	41:43:47:53:59:61	Prime Hex 3
7	67:71:73:79:83:89	Prime Hex 4
8	1:2:3:5:8:13	Fibonacci Chord Hex 1
9	2:3:5:8:13:21	Fibonacci Chord Hex 2
10	3:5:8:13:21:34	Fibonacci Chord Hex 3
11	5:8:13:21:34:55	Fibonacci Chord Hex 4
12	8:13:21:34:55:89	Fibonacci Chord Hex 5
13	13:21:34:55:89:144	Fibonacci Chord Hex 6
14	1:3:4:7:11:18	Lucas Chord
15	7:9:11:15:32:39:52	I Ching Chord 1
16	9:21:37:43:47:53:61	I Ching Chord 2
17	13:20:25:27:40:49:56	I Ching Chord 3
18	5:18:35:36:43:56:61	I Ching chord 4
19	8:17:37:45:48:51:63	I Ching chord 5
20	11/10 13/12 21/20 11/10 8/7 22/21	I Ching Chord 6
21	3/2 3/2 3/2 7/6 4/3 4/3 8/7	I Ching chord 7
22	5/4 5/4 5/4 5/4 9/8 4/3 4/3	I Ching chord 8
23	4/3 9/8 7/6 6/5 6/5 7/6 5/4	I Ching chord 9
24	5/4 4/3 9/8 6/5 5/4 5/4 3/2	I Ching chord 10
25	6/5 7/6 6/5 5/4 5/4 8/7 4/3	I Ching chord 11
26	4/3 4/3 4/3 8/7 8/7 5/4 6/5	I Ching chord 12
27	387.1 197.9 990.2 585.8 403.0 614.4 1003.1	I Ching chord 13
28	3/2 3/2 3/2 3/2 3/2 3/2 3/2	Chain o fifths
29	4/3 4/3 4/3 4/3 4/3 4/3 4/3	Chain o fourths
30	5/4 5/4 5/4 5/4 5/4 5/4 5/4	Chain o major thirds
31	6/5 6/5 6/5 6/5 6/5 6/5 6/5	Chain o minor thirds
32	7/4 7/4 7/4 7/4 7/4 7/4 7/4	Chain o blues sevenths
33	8/7 8/7 8/7 8/7 8/7 8/7 8/7	Chain o 8/7s
34	6/5 5/4 6/5 5/4 6/5 5/4 6/5	minor 15th chord
35	5/4 6/5 5/4 6/5 5/4 6/5 5/4	major 15th chord
36	4/3 5/4 6/5 7/6 8/7 9/8 10/9 11/10	Contracting Intervals Chord

2.7.3.2 Chord variability

As mentioned above, *Scala* will try to find the closest approximation for any given chord. Applying the chord vocabulary given above to different scales will often produce chords that differ significantly from each other. The following example will show this. "I-Ching chord 2" is applied to two different scales, an 11-tone MOS scale with a generator of 3 steps out of 16-tone Equal-Temperament (called "04ETMOS-03-16-11ST"); and a 14-tone MOS scale with a generator of 6 steps out of 17-tone Equal-Temperament (called "17ETMOS-06-17-14ST"). Table 64 lists the pitches of the 11-tone scale; Table 65 lists the pitches of the 14-tone scale. Table 66 shows how the chord is realised in the two scales. This table is set out symmetrically. The outer columns give the names of the notes on the *Scala* keyboard. The next inner columns give the interval sizes on the keyboard. The middle three columns are the most significant. The central column, "Ideal cents" shows the cents values of the pitches of the chord's ratios. The columns on either side of it, labelled "Cents", show the actual cent values of the pitches as played in each scale. Comparison of these columns will show the difference between these two. Some of the differences between the pitches are 71, 57, 44, and 40 cents. These are fairly significant pitch differences. If heard sequentially, these two chords sound very different.

Table 64: Pitches of 11-tone MOS scale with generator of 3 steps out of 16 ET

"04ETMOS-03-16-11ST" Generator: 3 steps out of 16 ET Generator: 225.0c MOS scale of 11 notes	
Scale degree	Cents
0	0.00
1	150.00
2	225.00
3	375.00
4	450.00
5	600.00
6	675.00
7	825.00
8	900.00
9	1050.00
10	1125.00
11	1200.00

Table 65: Pitches of 14-tone MOS scale with generator of 6 steps out of 17 ET

"17ETMOS-06-17-14ST" Generator: 6 steps out of 17 ET Generator: 494.12c MOS scale of 14 notes	
Scale degree	Cents
0	0
1	70.59
2	141.18
3	211.77
4	282.35
5	423.53
6	494.12
7	564.71
8	635.29
9	705.98
10	847.06
11	917.65
12	988.24
13	1058.82
14	1200

Table 66: “I Ching chord 2” mapped to scales of Tables 64 & 65

Scale 1 “04ETMOS-03-16-11ST”				Scale 2 “17ETMOS-06-17-14ST”		
Note name	Scale deg. + octave	Cents	Ideal cents	Cents	Scale deg. + octave	Note name
A hi	8 + 2 oct.	3300	3313	3317	11 + 2 oct.	A hi
G hi	6 + 2 oct.	3075	3070	3035	8 + 2 oct.	F#2 hi
E2 hi	4 + 2 oct.	2850	2862	2894	6 + 2 oct.	F hi
E1 hi	3 + 2 oct.	2775	2708	2682	4 + 2 oct.	Eb hi
C hi	0 + 2 oct.	2400	2447	2471	1 + 2 oct.	C# hi
D mid	2 + 1 oct.	1425	1467	1482	4 + 1 oct.	Eb mid
C low	0	0	0	0	0	C lo

Approximating chords in various tunings is a feature of *Scala*, but I don’t think the software designer anticipated it being used as a performing and compositional resource. This unexpected use of features of equipment and software as composing material is one I find very attractive. I see it as related to other interests of mine such as circuit-bending (rewiring consumer electronics to make idiosyncratic synthesizers), radical amateurism (Burt-Dempster 1999), and observation and use of the emergent properties of a complex system. The first two of these come from what some might consider underground or non-establishment contexts. The third comes from the world of science and mathematics. Using software features in unexpected manners is a way in which I demonstrate the continuity between these fields, which are often thought of as separate.

2.7.3.3 Composite timbres and beats from chord combinations

Scala can be used to realise the same chord type into different tunings, resulting in very different sounding chords. If two scales are similar, mapping a chord into them might produce chords that are so similar that when they are played together they will produce very rich timbres made of closely tuned, beating tones. My lifetime of playing the accordion has shown me that combining very closely tuned tones to make beating timbres is a very powerful tool. The timbres of the accordion could not exist without this, as different sets of closely tuned reeds are combined to make its various registers.

Some of the scales made for this piece are quite similar to each other. A scale made with an N/M ET generator (method 1), and a scale made with the limit of the branch of the Scale Tree the N/M ratio comes from (method 2) will usually be very similar. Table 65 showed the pitches of the scale made with a generator of 6 steps out of 17-tone Equal-Temperament. Table 67, on the other hand, shows the pitches of a scale made with a limit of the branch on which 6/17 appears (called "17F20MOS14"). Table 68 shows how closely the two chords realised with "I-Ching chord 2" in these two scales resemble each other. As with Table 66, this Table is set out symmetrically. The important information is in the centre three columns. In these, the actual frequencies of the two chords, with a fundamental of C 261.6, are listed in Hz. The central column shows the differences between each set of two frequencies in Hz. If the two chords are played together, this will be the amount each note will beat, in cycles per second. In this chord, all the notes but two beat at rates of 4 cycles per second or less. This would create a chord of considerable richness, and create a composite timbre that would be quite unexpected, given the very simple electronic timbres used in this piece.

A large part of the history of intonation is concerned with the elimination of beats. More recently, a number of composers, such as Alvin Lucier (Lucier 1990, 2003), Phill Niblock (Niblock 1995) and myself, have begun exploring the acoustical properties of beating tones, using them as a powerful compositional resource. And, as mentioned above, in my history as an accordionist, I've revelled in the richness and timbral possibilities of beating tones. This non-conventional way of looking at beating is, of course, another aspect of the ideas discussed in the final paragraph of Section 2.7.3.2, above.

Table 67: Pitches of 14-tone MOS scale with generator of 425.23 cents

"17F20MOS14" Limit of Wilson branch 17 (Finnemore branch 20) Generator: 425.23c MOS scale of 14 notes	
Scale degree	Cents
0	0
1	75.69
2	151.38
3	227.07
4	302.76
5	425.23
6	500.92
7	576.61
8	652.30
9	727.99
10	850.46
11	926.15
12	1001.84
13	1077.53
14	1200

Table 68: "I Ching chord 2" mapped to scales of Tables 67 and 65, beat frequencies between different versions shown in Hz

Scale 1 "17F20MOS14"					Scale 2 "17ETMOS-06-17-14ST"			
Note name	Scale deg. + octave	Cents	Hz	Beats in Hz	Hz	Cents	Scale deg. + octave	Note name
A2 hi	11+2oct	3326	1493.3	2.1	1491.1	3317	11+2oct	A hi
G1 hi	8+2oct	3052	1427.9	3.7	1424.2	3035	8+2oct	F#2 hi
E hi	5+2oct	2825	1381.1	13.5	1394.6	2894	6+2oct	F hi
Eb hi	4+2oct	2702	1358.2	3.7	1354.5	2682	4+2oct	Eb hi
C# hi	1+2oct	2476	1319.9	0.8	1319.1	2471	1+2oct	C# hi
Eb mid	4+1oct	1502	835.1	4.0	831.1	1482	4+1oct	Eb mid
C lo	0	0	261.6	0	261.6	0	0	C lo

2.7.4 Process – Real Time Usage

Given the number of scales in *Pythagoras' Babylonian Bathtub*, and the number of preset chords I had available, the number of resources was so vast that most often, I would only have the most general idea of what the sonic result would be when placing any one chord into any particular scale. This meant that even though I was performing directly, by selecting chords and playing them, the sheer number of resources often meant that this was still a process-oriented piece with a high degree of uncertainty to it.

Here is the performance set-up: The piece is played on three laptop computers. *Scala* is running on each computer. On one of the computers, *Scala* controls a software synthesizer (*Vaz Modular*). On the other two, it produces MIDI output which controls hardware synthesizers (Emu *Proteus I* and Korg *XD5R*). A family of similar timbres is programmed into each of these synthesizers; steady state tones (no vibrato) with a few upper harmonics, short attacks and decays of between 1 and 2 seconds. These timbres, while richer than sine waves, are still fairly plain. The upper harmonics allow the qualities of the harmonies to be heard more easily than with sine waves.

Often in performance, I have used one laptop playing an Equal-Temperament based scale, another playing the related Wilson-Finnamore scale, and the third playing my version of that scale. I set up the exact same chord on the same fundamental on all three computers, allowing the differences between the scales to create beats, richly dissonant harmonies, and a sense of the spatial shaping of the sound through an interaction of harmony and room acoustics. Especially remarkable is the way that the different chords combine to make a wide variety of complex, sustained, shifting timbres. Every time I introduce a new chord to the mix, the composite timbre changes dramatically. In fact, I could easily conceive of this piece being analysed primarily in terms of timbre, with all the scales and chords being considered as components of a unique kind of additive synthesis. My performance technique, then, involves me selecting scales, modes, chords, and timbres in real time, playing *Scala* keyboard chords and melodies with the mouse, moving from laptop to laptop in order to do so, and gradually moving from family to family of these scales in an open-ended, improvisatory manner. Given the number of scales, modes, chords, and timbres I had available, it should be obvious that it will take a very long time before the harmonic implications of this world are even revealed, much less exhausted. The two performances of this piece so far have lasted in excess of two hours. Future performances will, hopefully, be of a similar or greater duration.

2.8 *Saturday in the Triakontahedron with Leonhard*

2.8.1 Introduction

The third piece of the Wilson installations is a real-time interactive performance-piece for a single laptop computer called *Saturday in the Triakontahedron with Leonhard*. The title is a paraphrase of Stephen Sondheim's *Sunday in the Park with George.* Leonhard is Leonhard Euler, 18th century mathematician, and originator of the Euler Genera, discussed in Chapter 1.7. The Triakontahedron is the geometric figure Erv Wilson used to map the 64 pitches of the Euler Genus (3 5 7 9 11 13). In his paper "Euler Genus (3 5 7 9 11 13) Latticed within the Triakontahedron", he shows the scalar resources of the Euler-Fokker Genus made with factors 3 5 7 9 11 and 13, and some ways of dividing this scale into musically useful partitions (Wilson 2001 – all illustrations in this section are either from, or based on diagrams in that paper). In my piece, I take one of these methods ("the cube of cubes"), and by exploring all possibilities of it, develop a series of 160 eight-note scales, which I use with an algorithmic performance program made with John Dunn's *ArtWonk* control program (Dunn, 2007). This controls a series of frequency-modulation (FM) timbres made with Martin Fay's *Vaz Modular* software synthesizer (Fay 2004).

This piece is much more "note-oriented" than either of the other two *Wilson Installations*. In both *The MOSsy Slopes of Mt. Meru* and *Pythagoras' Babylonian Bathtub*, I use very simple timbres, simple melodic shapes, and a series of more or less complex chords to make shifting clouds of sound that float or hang weightlessly in space. In these pieces, the sense of musical time, as measured out in beats, melodies, and changing texture, seems if not suspended, then somehow stretched into new territory.

With *Saturday in the Triakontahedron with Leonhard*, edgy, even awkward acoustic-instrument-like FM timbres; more complex melodic shapes; a slow and stately walking-pace tempo; and the frequent use of such traditional textures as melody with accompaniment make a piece that is much closer to the world of instrumental music. This more conventional musical texture is used as the basis for quite radical harmonic exploration. Starting from the most consonant of chords, the piece progresses through a series of 8-note scales, each of which have both extremely consonant and extremely dissonant harmonies within them. The consonant chords are familiar and comforting. The dissonant chords, while not unpleasant, by their very unfamiliarity evoke a sense of the strange. The piece is slow, and moody, constantly moving back and forth between areas of familiarity, and sometimes almost aching beauty, to unsettling areas of harmonic darkness and acoustic murkiness. Indeed, the shifting quality of light during a day of unsettled weather is a good metaphor for the way I explore this harmonic world. Like the weather on a stormy Melbourne day, with clouds constantly breaking up and shifting, we continually alternate between moments of brilliant golden sun, and dark, turbulent unsettled grey.

While performing, I improvisationally change from one kind of musical texture to another. Sometimes we hear only a single-line melody; at other times, chords seem to float by; while at others, fragments of counterpoint appear and disappear, almost as if in a dream. The stately, slow pace, and the constantly changing harmonies and musical textures call to mind another metaphor: a first slow amble through an exotic botanical garden – interestedly examining one plant, then another, each for only a little while before moving on to the next.

This piece has a rather complex harmonic system, and that system is then controlled by a set of complex algorithms. The piece has five separate strands: (1) a progression of 4-voice chords; (2) a progression of dyads; and (3) a single line melody, which all follow the same selection of pitch subsets of the 8-note scale currently in use. Two other independent single line melodies (4 & 5) are also available. These melodies draw freely on the complete pitch resources of the current 8-note scale played over 2 octaves.

2.8.2 Tuning System

Figure 37, below, reproduces page one of Wilson's "Euler Genus" paper: a diagram which is the most efficient way he has evolved to illustrate the structure of this scale. As can be seen from this diagram, it takes a very complex drawing to show all the "common factor" relationships in a scale like this. This scale also functions as a larger "master scale" incorporating many of the "combination product set" scales - the hexanies, dekanies, eikosanies, and so on that Wilson proposed in his 1969 paper "D'alessandro - like a hurricane" (Wilson 1969). This scale of 64 notes is made up of all the possible products of all the factors. The most complex member of the scale is 3x5x7x9x11x13 (the multiplication product of all the source numbers) treated as a very high harmonic and then reduced to an octave, while the simplest members of the scale would be the individual factors themselves, plus 1, the implied fundamental.

Please see print copy for Figure 37

Figure 37: Wilson's triakontahedron mapping of Euler Genus (3 5 7 9 11 13)

As was discussed in Chapter 1.7, and illustrated in Figure 2, any three factors of this scale, when taken once each, produce an 8-note scale. For convenience we here reproduce Figure 2 as Figure 38, and refer the reader to Table 22, Chapter 1.7, which gives a listing of the pitches of this scale.

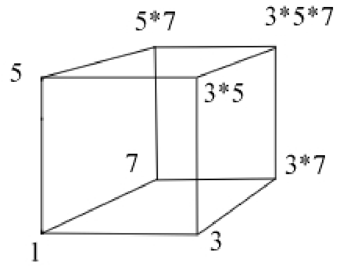


Figure 38: Euler-Fokker Genus 3 5 7 graphically represented as a cube

Any three factors can be combined to make a cubic 8 element scale like this. Wilson shows that because there are 6 factors, the scale can be broken down into 8 cubes like this, made of three factors, and these cubes can be placed at the corners of a larger cube, made of the other three factors. This is shown in Figure 39.

Please see print copy for Figure 39

Figure 39: Wilson's diagram of the Euler Genus as a "cube of cubes"

In this diagram, the 8 element scale shown in Figure 38 exists at 8 different transpositions levels within the resulting 64-note scale. That is, 3, 5, and 7 axes exist within the small cubes. These are then transposed on the 9, 11 and 13 axes of the larger square. The 64-note scale is completely comprised of 8 transpositions (the big cube) of an 8-note scale (the small cubes).

In Figure 39, the small cubes have axes of 3, 5, and 7, while the larger cube has axes of 9, 11, and 13. I realised that any combination of 3 factors could be used to make the small cubes; those would then be transposable to the 8 levels given by the other 3 factors. For example, if the small cubes were made of factors 7, 11 and 13, the resulting 8-note scale could be transposed 8 ways, along the 3, 5, and 9 axes. In fact, there are 10 ways of combining 3 out of 6 factors. This use of all the combinations of X out of Y factors is a common theme in Wilson's work. He has described many scale types formed like this, such as the 6-note hexany (every 2 out of 4 factors), and the10-note dekany (every 2 out of 5 factors). These combinational patterns are used to make particular scales. Here, however, they are used to show all the possible transpositional relations that can exist within a particular scale. Table 69 shows the 10 ways of combining sets of 3 out of 6 factors.

Table 69: The 10 ways of combining sets of 3 out of 6 factors

Number	First 3 elements	Second 3 elements
1	3 5 7	9 11 13
2	3 5 9	7 11 13
3	3 5 11	7 9 13
4	3 5 13	7 9 11
5	3 7 9	5 11 13
6	3 7 11	5 9 13
7	3 7 13	5 9 11
8	3 9 11	5 7 13
9	3 9 13	5 7 11
10	3 11 13	5 7 9

Each of these combinations can also be reversed. That is, an 8-note scale made of factors 3, 5, and 7 can be transposed onto the 8 combinations of factors 9, 11, and 13, while an 8-note scale made of factors 9, 11, and 13 can be transposed onto the 8 combinations of 3, 5, and 7, thus creating a total of 20 8-note scales that can each exist at 8 different transposition levels. This seems like enough different scales and transposition possibilities to keep even the most modulation-obsessed tonal composer happy. A similar observation was made by Lou Harrison in the anecdote recounted in Chapter 1.5.

I made charts for all these possible scales and their reciprocals. They each make an 8 x 8 matrix with vertical scales providing transposition levels for the horizontal scales, and vice-versa. The first two of these matrices are shown here in Tables 70 and 71. Each of these charts shows how the 8-note scale made from 3 factors exists in 8 different transpositions in the 64-note scale, and how the sum total of all these transpositions is the 64-note original scale.

In Table 70, rows read from left to right are scales made of factors of 3, 5 and 7, columns read vertically are scales made of factors 9, 11 and 13. The top number of each set of three is the ratio of the scale degree. The next number is the number of cents the tone is above the fundamental. The bottom number is the scale degree of the tone. The numbers in parentheses are scale degrees in a second (upper) octave. The tuning files for *Vaz Modular* allow any 128 tones to be assigned a microtonal retuning. Two octaves of this 64-note scale will therefore fit into the *Vaz* tuning. This lessens the necessity for changing octaves in the oscillators as often as otherwise might be the case, and also allows transpositions and modes to be played more freely. In Table 71, scales made of factors 3, 5 and 9 are read horizontally, while scales made of factors 7, 11 and 13 are read vertically.

The complete set of ten charts, diagramming the 160 eight-note scales, is found in Appendix 3.8.2. *Scala* *.scl and *.tun files for the master 64-note scale are found in Appendix 3.8.3. All of these scales together formed the harmonic vocabulary of this piece, accessed with the performance interface discussed below in Section 2.8.4. An

MP3 file of the studio performance of the piece recorded on 15 May, 2004 is Appendix 3.8.1. The CD cover for that performance is Appendix 3.8.4.

There are, of course, other ways of partitioning this scale. As mentioned earlier, in Chapter 1.5, in *Memories of Cecil Street on a Hot Summer Day,* from *Playing in Traffic* (2001), I also used this scale, partitioning it into four mutually exclusive 16-note subsets. There are 15 unique ways to do this, resulting in a family of 60 related scales. And there are many other ways of working with the Euler-Fokker Genera. For example, scale size might be a determining factor. If one wanted, say, to work with scales of only 12 notes, using 7 generating factors, this would give rise to a family of 196 related 12-note scales. Using 6 generating factors would yield 126 related 12-note scales, etc.

Table 70: Euler Genus 3 5 7 9 11 13 as 8 parallel 8-note scales with scale degrees 0-63 for *Vaz Modular* and *Scala* *.tun format. Combination 1: 3-5-7 x 9-11-13

1/1	35/32	5/4	21/16	3/2	105/64	7/4	15/8
0	155.1	386.3	470.8	702.0	857.1	968.8	1088.3
0	8	21	25	38	45	52	58
143/128	5005/4096	715/512	3003/2048	429/256	15015/8192	1001/512	2145/2048
191.8	347.0	578.2	662.6	893.9	1048.9	1160.7	80.114
10	19	31	35	47	56	61	5 (69)
9/8	315/256	45/32	189/128	27/16	945/512	63/62	135/128
203.9	359.0	590.2	674.7	905.9	1061.0	1172.7	92.2
11	20	32	36	48	57	62	6 (70)
1287/1024	45095/32768	6435/4096	27027/16384	3861/2048	135135/131072	9009/8192	19305/16384
395.8	550.9	782.1	866.5	1097.7	52.9	164.6	284.0
22	29	42	46	59	3 (67)	9 (73)	15 (79)
11/8	385/256	55/32	231/128	33/32	1155/1024	77/64	165/128
551.3	706.5	937.6	1022.1	53.3	208.4	320.1	439.6
30	39	51	54	4 (68)	12 (76)	17 (81)	24 (88)
99/64	3465/2048	495/256	2079/2048	297/256	10395/8192	693/572	1485/1024
755.2	910.4	1141.5	26.0	257.2	412.3	524.1	643.5
41	49	60	1 (65)	14 (78)	23 (87)	27 (91)	34 (98)
13/8	455/256	65/64	273/256	39/32	1365/1024	91/64	195/128
840.5	995.7	26.8	111.3	342.5	497.6	609.4	728.8
44	53	2 (66)	7 (71)	18 (82)	26 (90)	33 (97)	40 (104)
117/64	4095/2048	585/512	2457/2048	351/256	12285/8192	819/512	1755/1024
1044.4	1199.6	230.7	315.2	546.4	701.6	813.3	932.7
55	63	13 (77)	16 (80)	28 (92)	37 (101)	43 (107)	50 (114)

Table 71: Euler Genus 3 5 7 9 11 13 as 8 parallel 8-note scales with scale degrees 0-63 for *Vaz Modular* and *Scala* *.tun format. Combination 2: 3-5-9 x 7-11-13

1/1	135/138	9/8	5/4	45/32	3/2	27/16	15/8
0	92.2	203.9	386.3	590.2	702.0	905.9	1088.3
0	6	11	21	32	38	48	58
143/128	19305/16384	1287/1024	715/512	6435/4096	429/256	3861/2048	2145/2048
191.8	284.0	395.8	578.2	782.1	893.8	1097.7	80.1
10	15	22	31	42	47	59	5 (69)
77/64	10395/8192	693/512	385/256	3465/2048	231/128	2079/2048	1155/1024
320.1	412.3	524.0	706.4	910.4	1022.1	26.0	208.4
17	23	27	39	49	54	1 (65)	12 (76)
11/8	1485/1024	99/64	55/32	495/256	33/32	297/256	165/128
551.3	643.5	755.2	937.6	1141.5	53.3	257.2	439.6
30	34	41	51	60	4 (68)	13 (78)	24 (88)
91/64	12285/8192	819/512	455/256	4095/2048	273/256	2457/2048	1365/1024
609.4	701.5	813.3	995.7	1199.6	111.3	315.2	497.6
33	37	43	53	63	7 (71)	16 (80)	26 (90)
13/8	1755/1024	117/64	65/64	585/512	39/32	351/256	195/128
840.5	932.7	1044.4	26.8	230.7	342.5	546.4	728.8
44	50	55	2 (66)	13 (77)	18 (82)	28 (92)	40 (104)
7/4	945/512	63/32	35/32	315/256	21/16	189/128	105/64
968.8	1061.0	1172.7	155.1	359.1	470.8	674.7	857.1
52	57	62	8 (72)	20 (84)	25 (89)	36 (100)	45 (109)
1001/512	135135/131072	9009/8192	5005/4096	45045/32768	3003/2048	27027/16384	15015/8192
1160.7	52.9	164.6	347.0	550.9	662.6	866.5	1048.9
61	3 (67)	9 (73)	19 (83)	29 (91)	35 (99)	46 (110)	56 (120)

2.8.3 Algorithm

This piece is meant for live interactive performance. In performance, using an interface described in Section 2.8.4, I choose which scales, transpositions, and harmonies will be used by the five possible strands of music. The five strands are:

- Four voice chords, always playing in rhythmic unison
- Dyads, also playing in rhythmic unison
- Single line melody 3 which plays the same pitches as the chords and dyads
- Single line melody 1, which can freely choose among all notes of the given scale
- Single line melody 2, which duplicates melody 1, with slight variation.

I also choose which strands are playing, the register, note length, volume and panning of each voice, and the overall tempo. Beneath this, an algorithm is choosing the pitches, rhythms and durations of the different strands of music, and this is what we now describe.

The first choice the performer makes is which scale is happening at which transposition. Having made this choice, all the pitches will be chosen from a 2 octave gamut of that scale. Since the scales have 8 notes, the pitches are labeled 0-15. The next thing the performer must choose is a probability distribution to select from these 16 pitches. These probability distributions sometimes simply choose a chord – a subset – from the available pitches and select any pitch from that chord with equal probability. In other cases, more complex subsets are chosen with more complex probability distributions. Table 72 is a summary of the probability distributions used in this piece. These are selected with a large green knob in the bottom centre of the software performance interface shown in Figure 40, below. The selection of the probability distribution affects the pitches played by the chords, the dyads and melody 3, the "probability distributions melody." Melodies 1 and 2, as stated above, freely choose from all the pitches of the available scale. The model here was the idea of an accompaniment playing, say, only the three pitches of a C major chord, while the melody ranges freely over all the seven pitches of the C major scale.

Table 72: Probability Distributions for *Saturday in the Triakontahedron with Leonhard*

Number	Name	Pitches 0-15	Probabilities %
0	One7	0 2 4 6 8 10 12 14	Equal
1	Two7	1 3 5 7 9 11 13 15	Equal
2	3	0 3 6 9 12 15	Equal
3	4	0 4 8 12	Equal
4	5	0 5 10 15	Equal
5	Three7 Root favored.	2 4 6 9	35 22 22 22
6	Four7 Root favored.	3 5 7 10	35 22 22 22
7	Five7 Root favored.	4 6 9 11	35 22 22 22
8	Six7 Root favored.	5 7 9 11	35 22 22 22
9	Seven7 Root favored.	6 9 11 13	35 22 22 22
10	Eight7 Root favored.	7 10 12 14	35 22 22 22
11	Fifteenth Chord Inversion 1	0 2 4 6 9 11 13 15	Equal
12	Fifteenth Chord Inversion 2	1 3 5 7 8 10 12 14	Equal
13	IC Pentachord 1	1 3 8 14 15	10 10 30 28 21
14	IC Pentachord 2	1 7 10 13 15	6 8 35 15 36
15	IC Pentachord 3	5 8 12 13 15	6 11 26 15 42
16	IC Pentachord 4	8 10 12 13 14	10 5 38 19 28
17	IC Pentachord 5	0 4 5 8 12	26 21 3 27 24
18	Random all pitches 1	All 0 – 15	Eq. Random 2 - 10
19	1/f all pitches 1	All 0 – 15	1/f Random 3 - 8
20	Random all pitches 2	0 – 6, 8 – 14	Eq. Random 2 - 12
21	1/f all pitches 2	All 0 – 15	1/f Random 1 - 15
22	Random most pitches 3	0 – 6, 8 – 14	Eq. Random 1 – 13

In this chart, pitch numbers refer to two octaves of an 8-note scale. In each scale, the chords will sound quite different. For example, in Euler Genus (3 5 7) they will sound quite tonal: for the "One7", "Three7" etc chords, very much like traditional 7th chords on each degree of the scale. In this scale, distributions 13-22 will not sound like traditional chords. This enables me to switch between traditional harmonic chords, and non-traditional sounding harmonies.

Of course, if the 8-note scale being used does not have traditional tonal chords within it, such as Euler Genus (7 11 13), then even the "One7" or "Four7" chords will not sound like traditional 7^{th} chords. In that case, we will have an interesting harmonic vocabulary in which we alternate between chords made from alternating scale degrees (the "tonal" distributions 0 – 10) and chords that do not have that structure (the "atonal" distributions 11-22). These will still usually have distinct sounds, and a non-traditional harmonic world can then be explored.

In the "Probabilities %" column of Table 72, above, "Equal" means that there is an equal chance of choosing any of the given pitches, and them.

Numerical percentages given in the "Probabilities %" column means there is a one to one mapping of pitch numbers and probabilities. For example, pitches 2 4 6 9 with probabilities 35 22 22 22 means that pitch 2 will occur 35% of the time, and the other three notes each 22% of the time. (Percentages are rounded off to the nearest percent –this explains why the total of all probabilities can sometimes be more than 100%.)

The IC Pentachords were selected with *IC* (Culver-Cage 1993). 5 values from 0-15 were picked to choose pitches, then 5 values from 0 – 127 were chosen for the probabilities. The 0 – 127 values were put into the probability matrix which normalized the numbers to give the percentages listed here. So, for example Pitches 1 3 8 14 15, with probabilities 10 10 30 28 21 means that Pitches 1 and 3 each occur 10% of the time, Pitch 8 occurs 30% of the time, Pitch 14 occurs 28% of the time, and Pitch 15 occurs 21% of the time.

Distributions 18, 20 and 22 are "random." This means that *ArtWonk's* "Random Fill" function was used to generate an equal-weighted random number from 0 – 127 for each pitch from 0 -15. These values are normalized to make a distribution with the percentage values within the range given in their probabilities column. The pitch choice of 0-6 and 8-14 for distributions 20 and 22 resulted from the random generator picking a probability of 0 for those pitches.

Distributions 19 and 20 are like the previous three, except that the "random fill" function generated 1/f weighted random numbers from 0 – 127 for the probabilities of each pitch.

These distributions form a vocabulary of harmonic and melodic choices for me to select from in performance. They are applied to the three strands, 4-voice chords, dyads, and a single line melody, in the following way.

- For the 4-voice chords, the probability distributions control the pitch choice. Rhythm choice is made from another probability table – pulse values of 3, 5, 7, 9, 11 and 13 pulses are chosen with respective probabilities of 4, 6, 10, 15, 25, and 40%. These rhythm values are then multiplied by 5, making the chords have an overall slow tempo. Velocity is chosen by a random walk generator choosing MIDI velocities between 30 and 90. The maximum increment of change is 20. The probability of an upward change in this random walk is 3/2 greater than the probability of a downward change. This means there will be more crescendi than decrescendi in this voice.
- For the dyads, the values are the same as for the 4-voice chords, except that the rhythmic durations are multiplied by 3. This means that the dyads will generally play 3/5 as fast as the chords.

- For the probability distribution melody, values for pitch are the same as for the chords and dyads. Rhythm probabilities are the reverse of what they are for the chords and dyads. This means that where the chords had a much greater probability of long values than short values, in the case of the melody there is a much greater chance that short durations will occur. Rhythm values are not multiplied. This means that the overall melody will generally play 5 times faster than the chords and 3 times faster than the dyads. Velocity is chosen with a random walk generator choosing MIDI velocities between 65 and 125. This means the melody will generally be louder than the chords. The maximum increment of change is 20, again, but the probability of an upward change is 5/3 greater than the probability of a downward change. This means that the melody will be even more biased towards crescendi than either the chords or the dyads.

The two other melodies have pitches chosen with a different algorithm. The pitch algorithm follows these rules:

- Every 5 to 13 notes, alternate between one of two different pitch choosing routines, an equal-probability random generator and a more elaborate generator.
- The more elaborate generator is an up-down counter that steps in only one kind of interval at a time (chains of minor 2nds, minor 3rds, etc). The choice of the interval is also controlled by a probability generator. A 1 step interval will be chosen 39% of the time. Two step intervals will occur 19% of the time. 3, 4, 5, and 6

step intervals will occur 11, 19, 6, and 6% of the time respectively. A new interval is chosen every 1 to 7 notes.

- The up-down counter is also divided modulo 16, which will, like the up-down counter divided modulo N in *The MOSsy Slopes of Mt Meru* (described in Section 2.6.3) produce other intervals than the single chosen one if its limits are reached.

The overall effect of this is to make a melodic line that is biased towards one size of interval, but which does not use that interval exclusively. The interval being emphasized also constantly changes. And this interval-biased melody alternates with one in which any pitch can occur with equal probability. This produces melodies which alternate between a feeling of coherence and non-coherence.

Rhythm generation for these two melodies is the same as for the probability distribution melody. Short durations will occur more than long durations, and the overall tempo will be 5 times that of the chords. Similarly, velocities are chosen by a random walk generator with the same limits as for the probability distribution melody. The exception is that downward choices are favored over upward choices by 3/2. This means that these melodies will be more biased towards decrescendi, as distinct from the crescendi favoring probability distribution melody.

These algorithms provide the moment to moment details, and the structural underpinning for the live performance of the piece, using the performance interface. This aspect of the piece will now be discussed.

2.8.4 Process – Real Time Usage

Using *ArtWonk*, I made a performance interface for the piece, which is shown in Figure 40.

Figure 40: *ArtWonk* performance interface for *Saturday in the Triakontahedron with Leonhard*

This interface allowed me to pick any of the twenty 3-factor 8-note scales, and any transposition of them, in real time, and to hear them applied to any combination of chords and melodies, using frequency modulated timbres in 5 different registers. As discussed above, in Section 2.8.3, three of these lines, one consisting of 4-voice chords, another consisting of dyads, and the third a single voice melody, also had their pitches selected by a set of probability distributions (sieves), which allowed only certain chords, intervals, or subsets of the scale to be played. The other two lines were single-line melodies, which chose their pitches using similar probability distributions, but selecting from all the available tones of the scale. Pan, volume, and note duration were also controlled from this interface. Additionally, another control allowed selection of overall tempo.

With this interface, I could freely explore any of the scales, and a large number of harmonic subsets in each of them. Further, I could make this selection in real time, being able to perform changes and hear their effects immediately. What was most delightful was that even though the melodies and chords were randomly selected (through sieves), thus negating any intentionally composed directionality within them, when one changed from one scale to another, it sounded like a modulation, often a quite dramatic one. On listening back to a recorded performance of the piece, I was quite amazed that for me, both local and long term harmonic motion seemed to be happening.

And furthermore, this sense of harmonic motion was also occurring while moving between harmonic worlds that were relatively consonant (the 3 5 7 factor scale) or relatively dissonant (the 9 11 13 scale). That is, harmonic motion was heard as both a function of changing probability distributions within a scale, and also as a consequence of moving between scales. The overall "feel" of the harmony - whether more consonant or more dissonant - was clearly changing as well. This is unlike, say, in tonal harmony in the 12-note scale, where the overall harmonic "feel" doesn't change, even though scale type and fundamental may. That is, one might change from major to minor scales, but intervals used remain the same. Here, entire harmonic worlds are changing, rather than modulating within a single harmonic world.

In short, with this 64-note scale, partitioned in this "cube of cubes" way, I'd arrived at what I felt was a true sense of expanded tonality, one that also allowed an expanded kind of melodic vocabulary to exist within it. Non-directional melodies, as well as melodies with functional harmonic structuring, could both be used. The FM timbres chosen were more harsh and spiky than the timbres used in the first two pieces of *The Wilson Installations*, but were not modulated to such an extent that pitch perception was affected. One can clearly hear the fundamentals of these pitches, and how they combine harmonically. These timbres are still (mostly) harmonic-series timbres.

I have performed this piece live only once, so far, at the September 2004 Sonic Connections Festival at the University of Wollongong. That performance, which took place in David Worrall's wonderful multi-channel "Dome" environment, was a shortened version which was also marred by technical problems with the software (Worrall 1988). These problems have since been addressed. The earlier studio performance of May 2004, which was recorded, was much more successful. But I need to perform this piece more. Many more hours of exploration are needed to hear the wide range of the harmonic and melodic possibilities of this complex scale. Future developments of this piece will include making the *ArtWonk* patch more elegant and efficient, and refining of the FM timbres made with *Vaz Modular*. I eagerly await further performance opportunities for this piece.

Considering *The Wilson Installations* as a whole, and working with the three large mathematical-musical worlds contained within them, I keep getting hints that all three relate in some way. To borrow a conceit from physics, I think there may be some sort of "unified-harmonic-field" theory at work here, which would unite all three methods of scale formation (additive sequences, the Scale Tree, the Euler-Fokker Genera) under one sort of mathematical procedure. Unfortunately, these have so far been only been hints, illusive intuitions that there might be some kind of deeper structure at work here. In "The Triangle/Lambda Equivalence" (Wilson 2003b), Wilson proves that the Pingala / Pascal triangle, and the Farey Series / Lambdoma diagrams are equivalent. And Brian McLaren, in "General Methods for Generating Musical Scales" (McLaren 1991) briefly discusses the underlying relationship of scales made with additive series or processes of all kinds. And most recently, considering Wilson's 2006 work on the Co-Prime grid (Wilson 2006), I noticed two interesting phenomena:

1. All pairs of subscripts that give unique additive sequences and number triangles are co-prime

2. All ratios listed on the Scale Tree are co-prime.

But much more work needs to be done here. Further investigation may reveal if larger underlying principles underlay the structures of all these scales.

2.9 *18 New Fuguing Tunes for Henry Cowell*

2.9.1 Introduction

Eighteen New Fuguing Tunes for Henry Cowell (Pocket Calculator Music III) is an hour-long computer-music composition in eighteen movements composed in early 2005 in which, in common with many of the other pieces discussed in this thesis, three interests of mine come together. First, the generation of musical material and patterns using algorithmic processes; second, the realisation of those patterns into music which explores how those patterns articulate various microtonal tunings; and third, the shaping of that music into forms derived from various historical practices. This piece, and *Someone Moved in a Room*, discussed in the next Chapter, also form a pair, both being multi-movement recorded suites of canons based on found-object data which explore families of 18 scales each.

In the case of this piece, the algorithmic process is a “low-tech” one using a pocket calculator to generate deterministic, but eventually non-linear number sequences. This “low-tech” interest is then continued with the choice of “antique” low-fi samples in the realisation of the piece.

The scales used are Euler-Fokker Genera of 12-notes, using only factors 3, 5, and 7. There are exactly 18 of these, and they are all used in this composition, one per movement.

The historical form used here is the canon, treated freely in the manner of Henry Cowell’s *Fuguing Tunes*, a series of contrapuntal compositions for extremely diverse resources he composed between the years 1944 and 1964 (Shirley 1997). In fact, in my composition, a patch embodying the exact same canonic process generated each movement - only the parameters feeding into the process and the timbres used were changed with each movement.

As opposed to an organic musical structure, in which every element of the piece proceeds from, or refers to, something inherent in the basic generating musical material, these pieces (and indeed, all the works discussed in this thesis) stand as examples of what Thomas DeLio (DeLio 2002) refers to as inorganic structures, where various structural elements are juxtaposed, exploring what kinds of musical materials result from these juxtapositions, and how they can be shaped into larger compositions. MP3 files of the movements of the piece will be found in Appendix 3.9.1. Listings of the data, tunings, and parameter setting for each movement will be found in Appendices 3.9.5, 3.9.2, and 3.9.6 respectively. *Scala* *.scl and *.tun files for all scales used are in Appendix 3.9.3, and the cover for the CD of the piece will be found in Appendix 3.9.4.

2.9.2 Tuning System

This piece uses all the possible Euler-Fokker Genera of 12 notes that can be made with factors 3, 5, and 7. Euler-Fokker Genera can be made with any kind of factors, of course, but in this piece, I thought I would stick to the most traditional factors. This was further reinforced when I decided to use Cowell's *Fuguing Tunes* as a model. The archaic nature of the fuguing tune (an 18th century North American form) seemed to suggest keeping the harmony close to "the tradition." Further, Cowell wrote 18 compositions using the Fuguing Tune form, so the match of 18 scales with a series of 18 movements in homage to a series of 18 compositions seemed irresistible.

My interest in 12-note scales, however, was triggered by the malfunctioning ability of my software sampler of choice, Maz Sound's *Vsampler3*, to import *Scala* tuning files. There was, and, as of February 3, 2007, still is, a bug, so that Scala files with scales of more or less than 12 notes do not import properly into *Vsampler3*. I have figured out what the bug is, and have posted a rather lengthy workaround to the *Vsampler* on-line forum, so that, with effort, scales of any size can be used with *Vsampler3*, but for the moment, fixing the bug seems to be a rather low priority. This unedited document is included in Appendix 3.9.7. However, finding a work-around for this bug led me to think about what sort of interesting 12-note scales could be used. I began thinking of the Euler-Fokker Genera, which were discussed in Chapter 1.7. I found that there were 18 possible 12-note lattice-based Just scales using factors 3, 5 and 7. They are listed here in Table 73 in the order in which they are used in the piece. The scales are arranged in an order that produces the effect of similarity from scale to scale, because of the common factors within each adjacent scale. Notice the symmetrical arrangement of matrices within the sequence, as a transition from Scale 1, a scale based exclusively on Perfect fifths (3/2s) is made to Scale 6, a scale based exclusively on Major thirds (5/4s), etc. That is, Scale 2 is 3(5) 5(1), five 3/2 intervals on one axis with one 5/4 on the other; while Scale 5 is 3(1) 5(5), one 3/2 interval on one axis with five 5/4 intervals on the other. Even the three factor scales at the end try to follow this “least change from the previous scale” idea.

The complete listing of all the scales is found in Appendix 3.9.2. In Table 73, we simply describe some salient points about the nature of each scale. A number in parentheses after a factor number shows the number of times that factor is used. For example, 3(2) 5(1) 7(1) is a scale made of two 3/2s on one axis, one 5/4 on another axis, and one 7/4 on a third - a three dimensional matrix of fifths, thirds and sevenths.

Table 73: Euler-Fokker Genera used in *18 New Fuguing Tunes for Henry Cowell*

No.	Factors	Description
1	3(11)	a straight 12 note Pythagorean scale.
2	3(5) 5(1)	two 6-note chains of fifths, separated by a just major third
3	3(3) 5(2)	matrix of three 4-note chains of fifths, each separated by a just major third
4	3(2) 5(3)	matrix of three 4-note chains of just major thirds, separated by just fifths
5	3(1) 5(5)	two 6-note chains of just major thirds, separated by a just fifth
6	5(11)	a 12-note chain of just major thirds
7	5(5) 7(1)	two 6-note chains of just major thirds, separated by a just seventh harmonic
8	5(3) 7(2)	matrix of three 4-note chains of just major thirds, separated by just sevenths
9	5(2) 7(3)	matrix of three 4-note chains of just sevenths, separated by just major thirds
10	5(1) 7(5)	two 6-note chains of just sevenths, separated by a just major third
11	7(11)	a 12-note chain of just seventh harmonics
12	3(1) 7(5)	two 6-note chains of just sevenths, separated by a just fifth
13	3(2) 7(3)	matrix of three 4-note chains of just sevenths, separated by just fifths
14	3(3) 7(2)	matrix of three 4-note chains of just fifths, separated by just sevenths
15	3(5) 7(1)	two 6-note chains of just fifths, separated by a just seventh
16	3(2) 5(1) 7(1)	a three dimensional matrix of four 3-note chains of just fifths, separated by a just major third (on one axis) and a just seventh (on another axis)
17	3(1) 5(2) 7(1)	a three dimensional matrix of four 3-note chains of just major thirds, separated by a just fifth (on one axis) and a just seventh (on another axis)
18	3(1) 5(1) 7(2)	a three dimensional matrix of four 3-note chains of just sevenths, separated by a just major third (on one axis) and a just fifth (on another axis)

Illustrating some of these scales in matrix form may help clarify their structures. Figures 41 and 42 are *Scala* performance matrices of Scale 2 and Scale 9 respectively from Table 73. Scale 2 consists of two chains of 3/2 Perfect fifths, separated by a 5/4 Major third. Its traditional harmonic structure can be fairly easily seen, with Just- Major 7th chords formed by every square block of four adjacent tones. Scale 9, on the other hand, with its chains of 7/4 blues 7ths separated by Major 3rds, produces harmonies that are more strident and unfamiliar, with, for example “whole tone” scale fragments of linked 8/7s formed in the vertical axes, and Just- augmented chords formed in the horizontal.

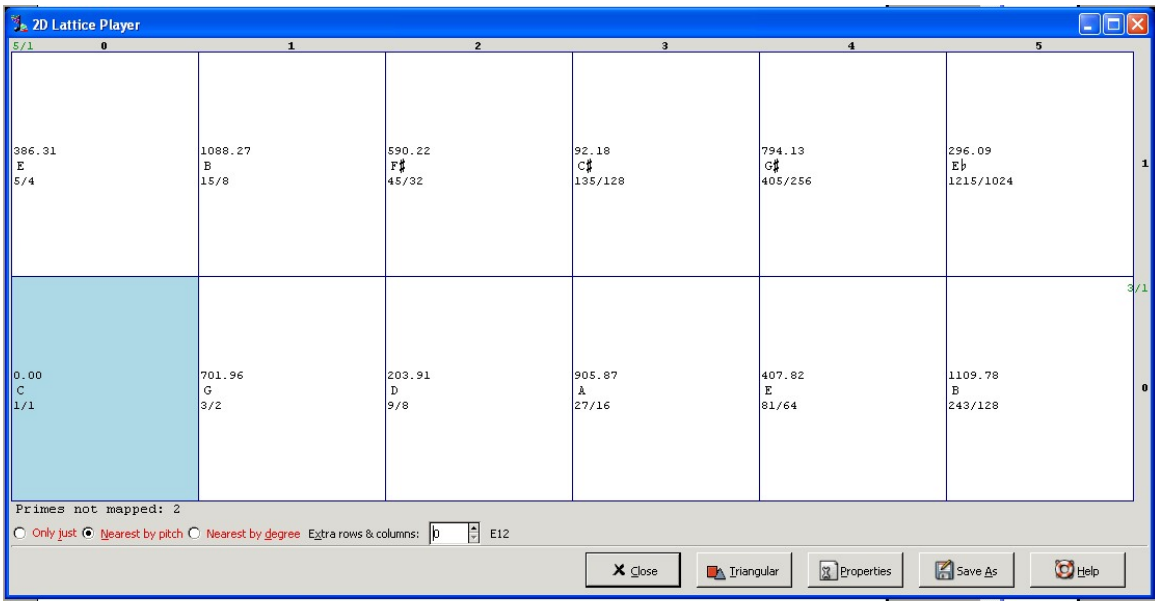


Figure 41: Scale 2, Euler Fokker Genera 3(5) 5(1) made in *Scala* 2D Play matrix

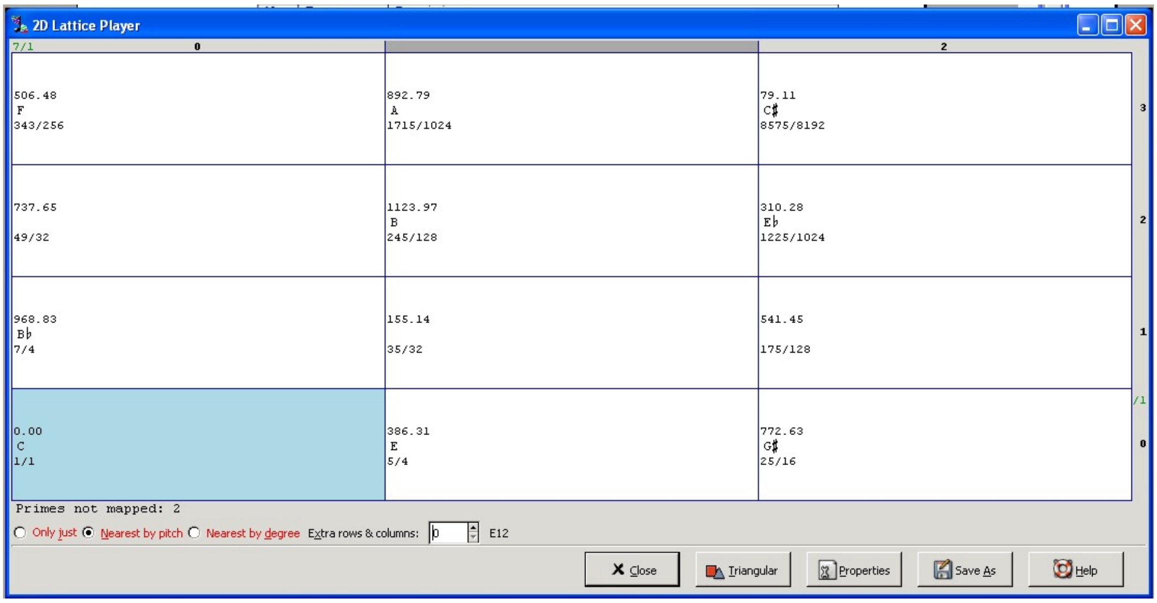


Figure 42: Scale 9, Euler Fokker Genera 5(2) 7(3) made in *Scala* 2D Play matrix

Some of these scales, such as Scale 1, are very close to our traditional 12-note Equal-Tempered scale. Others, such as Scale 9, made of Just- sevenths and pure thirds, are quite “modal”, and others, such as the “chain of thirds” scale, Scale 6, have a number of very small intervals within them. Playing with these scales, one is tempted down the “organic” path, constructing motivic material which plays with the inherent natures of each of these scales, and then building tightly constructed pieces based on that material in order to express one kind of “essence” of the scale. I have followed this path before, such as in *Three Cat Laxative Sonatas (Music for Microtonal Piano Sounds, Part 5a)* (Burt 1998) although I have not done so in any of the pieces discussed in this thesis. Likewise in this case, I did not want to make an organic structure. Here, sounding identity would be determined by the intersection of the pocket calculator patterns, the nature of these scales, how I chose to shape the patterns into melodies and contrapuntal complexes, and the timbres I chose to use.

Nonetheless, the characters of the scales do come through. The modal, anxious sound of Scale 6, for example, is heard quite clearly in the sixth movement, with its frantic electric organ polyrhythms; and the choice of melodic range in movement fifteen, for example, extracts a quite “Javanese” sounding electric piano melody out of the small intervals of that scale. Listening to the complete cycle, I believe I do hear some kind of logic of harmonic progression from movement to movement, despite the wild differences in timbre, rhythm, texture, and gestural type that exist between the movements.

2.9.3 Algorithm

In this section, we discuss the number sequences produced by the “low-tech” pocket calculator process mentioned earlier in Section 2.9.1. The number patterns produced in this way, properly scaled, control the choice of pitches, durations, and dynamics in the piece.

In 1978, I was playing with my pocket calculator (a Sanyo CX-8145). This particular calculator had keys for taking the square root of x, and squaring x, next to each other. This meant that you could take any number, take the square root of x, and then square it, returning to where you started. But the calculator had only a limited number of digits, and so round-off errors occurred, and one could go back and forth between the two functions indefinitely, generating a sequence of descending numbers based on the round-off errors generated by the calculator's limited capability. If one took only the last 2 or 3 digits of each number, one could get a deterministic sequence of numbers, but one that had some levels of surprise in it. Further, if one performed each function several times in a row, one could get sequences with a richer structure than the simple linear sequences that resulted from simply taking the square root of a number, and then squaring it. I made two pieces using this method. In an overly enthusiastic burst of misapplied proletarianism, I called these pieces *Pocket Calculator Music* - the reference being to what I satirically regarded as "upper-class" computer music. In line with my interests in making music with low technology (most exemplified at this time by my many compositions using cassette recorders and cheap homemade electronics), I thought that instead of a large expensive computer, I would use a lowly pocket calculator, and find what materials could be extracted from its inherent characteristics (others might call these characteristics "flaws").

The first piece, already mentioned in Chapter 2.2, was *Pocket Calculator Music I - 39 Penguins by Moonlight* (1978), for solo baritone ukulele, a 60-minute composition performed as accompaniment for a dance by Eva Karczag in Melbourne and Sydney during that year. The second piece was *The Wanderer - Pocket Calculator Music II* (1983) for english horn, bassoon, french horn, bass trombone, viola, cello and conductor, or six keyboard synthesizers with pitch-bend wheels and conductor. This 11-minute long microtonal composition has never been performed. These were two of a number of works that I have based on integer sequences of various kinds. Indeed, the works of N. J. A. Sloane, including the print *Handbook of Integer Sequences* (1973), and *Encyclopedia of Integer Sequences* (1995), and the current on-line version (Sloane 1973, 1995, 2006), as well as the S. I. Gelfand et al *Sequences, Combinations, Limits* (Gelfand et al 1969) have been continual sources of interest and inspiration for me. I am always willing to examine a new and interesting number sequence for its structural and musical possibilities.

In 2005, I began idly playing with a pocket calculator again, this time exploring the nature of the resulting number sequences in more detail. I started doing the simple "square root, then square" process on my wife Catherine's calculator (a Sharp Elsi-Mate EL-376C), and then began wondering if I would get the same results on other calculators. I quickly found that calculator technology had advanced, and that some calculators used many more significant digits that others, so that in some cases, the "square root, then square" process would simply bring you back to the number with which you started. These calculators were programmed to regard round-off errors as insignificant. I found that Catherine's calculator, and my old Casio H-1 calculator produced identical results - long trains of sequences based on, I presume, their similar chip architecture. However, my Casio fx-82B scientific calculator was too precise for this use, as was the Windows calculator on my computer. It was only when I expanded the process to many layers deep that I began to get significant round-off error sequences on either the scientific calculator or the Windows calculator. That is, in order to get round-off error sequences on these two machines, I had to, for example, take the square root of a number, say, 13 times, and then square the result 13 times before I got a number that was not identical with my starting number, and which allowed one to proceed into sequences based on round-off errors.

For this piece, I generated 9 sequences using the Casio H-1. Each sequence was generated with a certain number of iterations of the “square root - square” process, until 128 results were obtained. That is, the first sequence was generated by taking the square root of the initial number, writing down the last two digits on the calculator display, then squaring the number, and writing down the last two digits of that result on the display, and repeating that process until 128 results were obtained. This mostly results in interpolated descending sequences of numbers, with some numbers being much higher. These are numbers with zero on the end, which the calculator does not display. That is, if the last three digits of the numbers of a descending sequence were 643, 642, 641, 640, 639, 637, etc. because the calculator does not display terminal zeros, the sequence of the last two digits would be 43, 42, 41, 64, 39, 37, etc. Figures 43-51, given below, graph these sequences, and this “contrapuntal” introduction of “new” material can be seen clearly.

The second sequence was generated by taking the square root of the initial number, then taking the square root of that, then squaring that result, and squaring that result again, returning to a number very near, but not identical with the starting number. At each stage, the last two digits on the display were written down. Each subsequent sequence is then generated by taking the square root of the starting number N times, then squaring the result N times. In all cases, the starting number was 154, a number chosen off the top of my head, but having chosen it, I decided to stick with it.

Figures 43 – 51 display the results. In all cases these graphs display values from 1 to 99, with the 128 results displayed from left to right. Grid lines are not used here, because the aim of these graphs is to give a quick look at overall shape and patterning. Those wishing to see the precise values of the data should refer to Appendix 3.9.5, where all the data charts will be found.

Looking at these graphs reveals some curious features, which I don't have the mathematical abilities to investigate more deeply, but would be grateful if someone else did. For example, Figure 43 (Graph 1) shows the first level sequence: it's linear, consisting of 2 different descending sequences interleaved. This is as expected. Similarly, Figure 44 (Graph 2) shows the second level sequence, and it, too, is fairly linear, with a cycle length of 4 elements being fairly obvious, especially after the first 12 elements of the sequence. Figure 45 (Graph 3), showing the third level sequence, also is very linear, with its cycle length of 6 elements being obvious right from the beginning. What is most interesting is that all three sequences, however, have a leap to higher values at about 2/3 of the way through the 128 element length sequence. Since the sequences shown in Figure 44 (Graph 2) and Figure 45 (Graph 3) are expansions of the sequence of Figure 43 (Graph 1), I would have expected the "leap to higher value" of Figure 43 (Graph 1) to re-occur later, if at all. Some sort of "fractal expansion", where the same structure is revealed at different levels of magnification, seems to be operative here, but at present, I don't know what it is.

With Figure 46 (Graph 4), things begin to get more interesting. The cycle length of 8 elements produced by the 4^{th} level sequence (root, root, root, root, square, square, square, square) is less visible, and unlike Figures 43-45 (Graphs 1-3), does not exhibit the simple linearity of the previous three Graphs, even though it is produced by a simple, deterministic, linear process. Similarly, Figures 47-51 (Graphs 5-9) reveal more and more unpredictable patterns. Although in no sense random, by the time we get to Figure 49 (Graph 7) or Figure 50 (Graph 8), the patterns are certainly beginning to have the "look and feel" of sequences generated randomly. Interestingly, this seems to parallel the progression of certain chaotic sequences. For example, the logistic equation produces linear, or no, results when its variable S is between 0 and 3. Interesting, chaotic results occur when S is between 3 and 4, especially between 3.5 and 4. When S is greater than 4, the results either zoom off to infinity, or head to zero. An analogous thing is observed with these sequences. When S (for sequence length in this case) is between 1 and 3, the results are linear and easily predictable. When S = 4, the results have a combination of linearity and seeming non-predictability. As S goes to 5 and beyond, the results seem more and more random. Even at S = 9, however, there are still some interesting coherencies and symmetries in the sequence, more so than if the sequence were randomly generated. Therefore, it seemed to me that these sequences, and their progression, could make interesting material on which to base a musical composition.

As stated above, Figures 43-51 are presented without numerical coordinates here. These diagrams are meant to be seen simply as shapes. The actual number output for each sequence is given in Appendix 3.9.5.

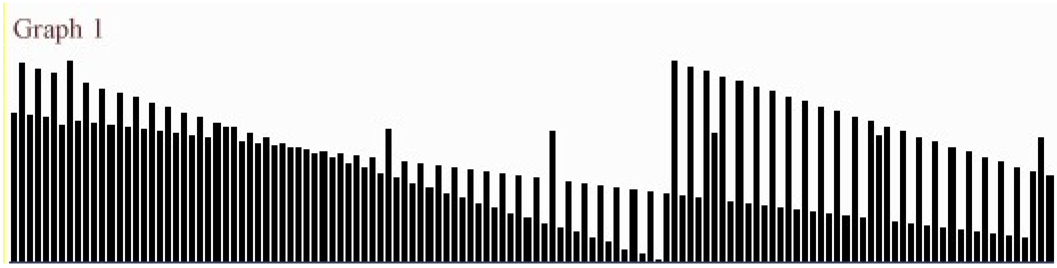


Figure 43: Graph 1 – output of pocket calculator process 1 level deep

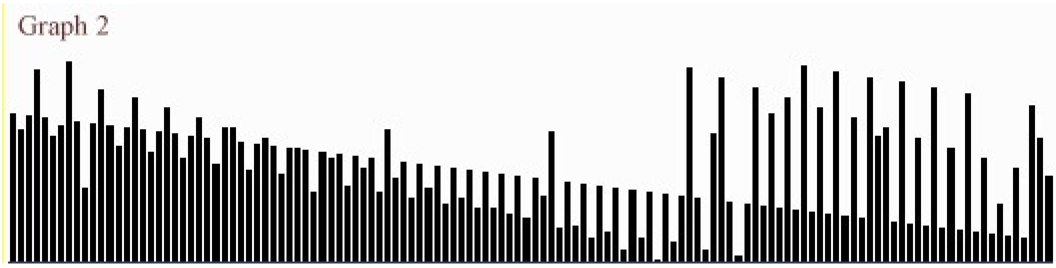


Figure 44: Graph 2 – output of pocket calculator process 2 levels deep

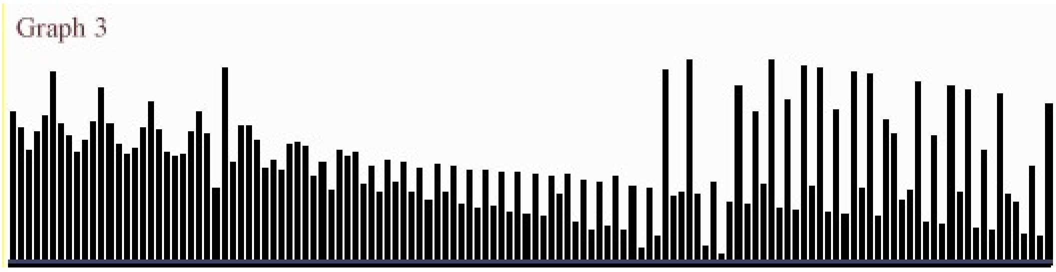


Figure 45: Graph 3 – output of pocket calculator process 3 levels deep

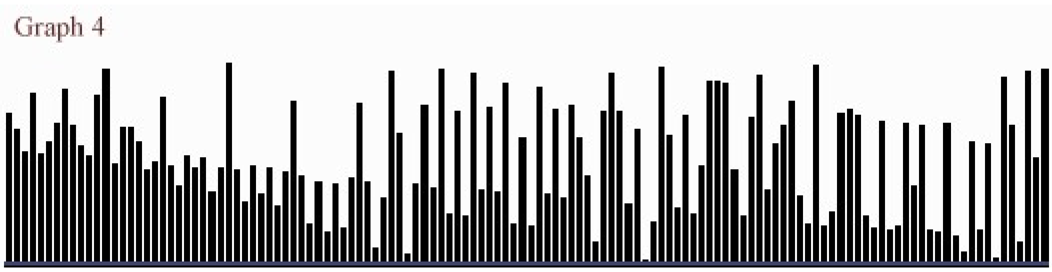


Figure 46: Graph 4 – output of pocket calculator process 4 levels deep

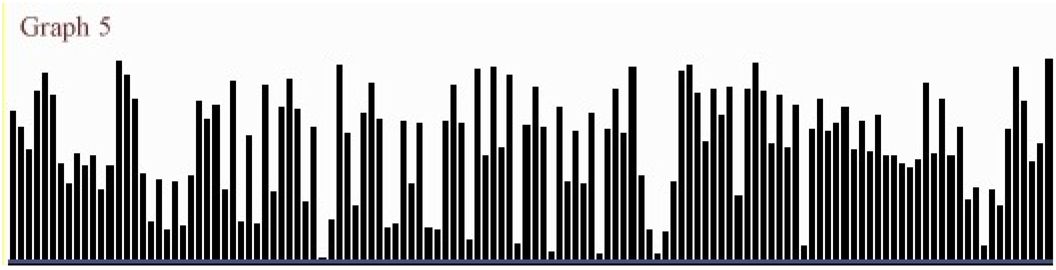


Figure 47: Graph 5 – output of pocket calculator process 5 levels deep

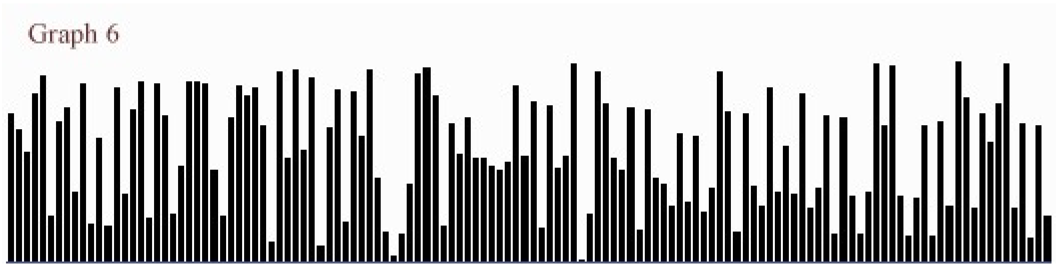


Figure 48: Graph 6 – output of pocket calculator process 6 levels deep

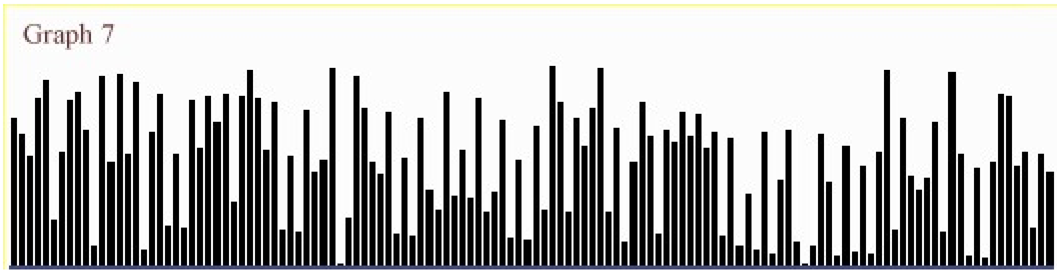


Figure 49: Graph 7 – output of pocket calculator process 7 levels deep

Figure 50: Graph 8 – output of pocket calculator process 8 levels deep

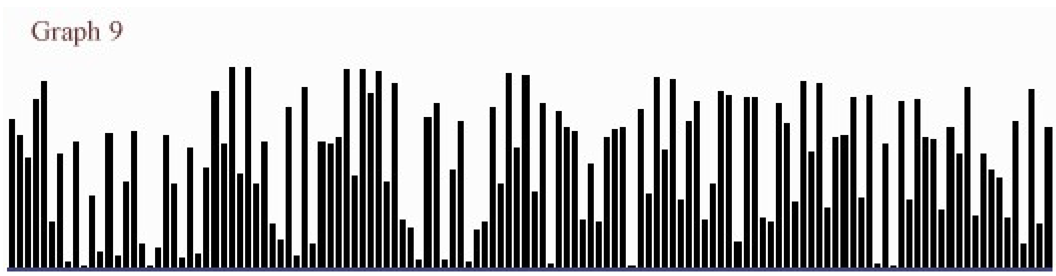


Figure 51: Graph 9 – output of pocket calculator process 9 levels deep

2.9.4 Process – Real Time Usage

2.9.4.1 Questions

As mentioned above, these pieces are based on Henry Cowell's series of 18 *Fuguing Tunes*. The fuguing tune is an 18th century North American contrapuntal form, which is canonic, but more freely constructed than its European relatives. William Billings was one of the composers most closely associated with the form, and Cowell had studied his work closely. Interestingly, Cowell's student, John Cage, also used the Fuguing Tunes and Hymns of Billings as material for a series of pieces in the 1970s (Brooks 1994). William Duckworth, a composer closely associated with Cage, also based a series of pieces on this material as well (Duckworth 1994). There seems to be an ongoing interest with composers from that tradition in this older contrapuntal material.

I thought it would be fun to emulate Cowell's *Fuguing Tune* form on my computer. Cowell himself had been extremely free in his application of the form, with his canons getting stricter and stricter the later he got in his series of compositions. For me, Henry Cowell's music is fascinating in its combination of radical ideas and very conservative phraseology. This gives his music a very unique sound, one that never, to my mind at least, seems to "fit in" with most of the other radical art music of the early 20th century. I was therefore fascinated to observe that my piece, constructed with some of Cowell's principles in mind, also seemed to exhibit the same curious combination of conservative and radical ideas, as well as seeming to not "fit in" with the vast bulk of contemporary musical investigation.

To further set up compositional constraints, I decided to set up a canon generation patch in John Dunn's *SoftStep*, and to limit myself to the same patch for all 18 canons. Any differences between the pieces would be produced not only by the different number sequences and scales used, but also by choices of elements such as melodic range, selection of durations, dynamic range, length of delay between voices (producing different lengths of canonic entry), transpositions of each voice, overall tempo, and choices of timbre. I wanted to see if I could generate sufficient variety between the movements while still maintaining a sense of coherent progression thorough the overall composition. Listening to the work, I believe I have done this.

Figure 52 shows the *SoftStep* patch used to produce the piece. The piece was interactively composed along with the construction of this patch – different values for treating parameters were tried out with different timbres, different modules were added and subtracted until the final fixed collection of modules were decided upon. Once the values for each movement were set, recording of the movement occurred as an automated process. Even if one does not understand the workings of the program, the graphic layout of the patch should reveal its simplicity.

- A clock drives counters
- The counters drive Page–sequencers, which contain all the data described in Section 2.9.3
- The data is processed and scaled (in Scale modules) and applied to pitch, duration and velocity choice for Voice 1
- The data is then delayed (three different ways) and pitch values scaled (in AddDiv modules), and the delayed data is applied to pitch and velocity choices for three other voices
- Each of the four voices have the same rhythms – but the rhythms are delayed a different amount between each voice.

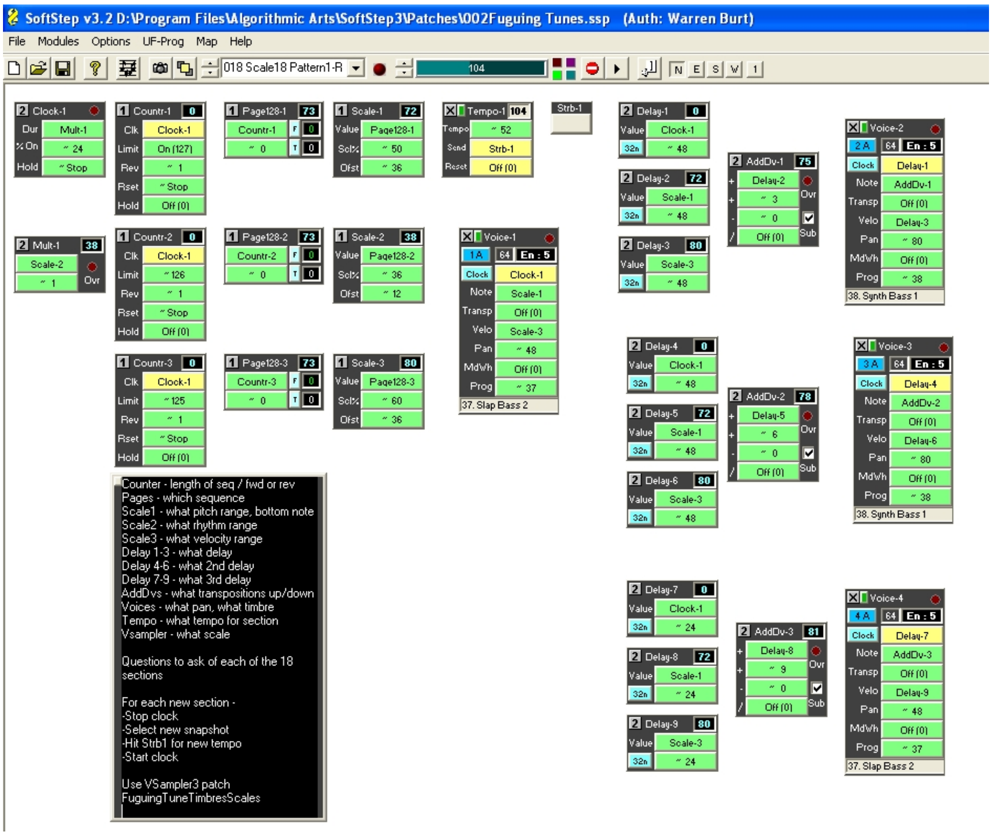


Figure 52: *SoftStep* patch used for *18 New Fuguing Tunes for Henry Cowell* with parameter values for Movement 18 shown

In this patch, the same number sequence is used to determine pitches, durations, and velocities for each note. Each parameter uses a different sequencer with the same sequence in it. The pitch sequencer counts all 128 elements of the sequence, while the duration sequencer uses only the first 127 elements, and the velocity sequencer uses the first 126 elements. The permuting of these three sequences against each other (each driven by the same clock) produces a ridiculously long sequence - in excess of 512,000 notes. Of course, long before that, certain recurring pitch motives, even with different rhythm and dynamic articulations would occur so many times that a sense of redundancy and saturation would occur. However, there is at least the possibility of a near-infinity of variation within each of the melodies in the composition.

The number sequence is scaled very differently for each parameter, of course. For example, in the first movement, the pitch sequence is scaled to 33% and offset by 60. This produces melodies with a width of 33 scale degrees (2 octaves and a "major sixth" - bearing in mind that a "keyboard major sixth" might not sound like a major sixth in the tuning being used) starting on middle C, midi note 60. The duration sequence is scaled to 10% with an offset of 3. This scales the sequence down to produce values of between 3 and 13 clock ticks - 24 clock ticks equal a quarter note - at the chosen tempo of quarter note = 132. The dynamics sequencer is scaled to 60% with an offset of 36. This scales the sequence to produce velocities between midi 36 and midi 96. In traditional dynamic terms, with the timbres I used, this means dynamics between *piano* and just a bit louder than *mezzo-forte*. By scaling these values differently, of course, radically different results can be obtained. For example, if one were to scale durations so that only values of, say, 1, 2, 3 and 4 were produced, and the output of the duration scaler was multiplied by 6, a melody would be produced with durations of a sixteenth, an eighth, a dotted eighth, and a quarter note. This would sound very different - much more regular, rhythmically - than melodies made in the way they were for the first movement. A full listing of the details of the scaling of each parameter in each movement is found in Appendix 3.9.6. Study of these details will reveal the compositional and structural essence of each movement.

Composite rhythms can also vary considerably based on the length of canonic delay. For example, a melody of even quarter notes at metronome 120, delayed 24 ticks, when combined with the original, would produce a rhythmic unison, a composite rhythm of a series of even quarter notes. However, the same melody delayed 29 ticks, when combined with its original, will produce a composite rhythm with a pronounced loping, uneven character. If both generating melodies and delays are complex, the resulting composite rhythms can be both surprising and swinging (in the full jazz sense of the word). In this piece, I tried to explore a wide variety of these.

Each movement consisted of four contrapuntal lines, with the three following lines being simply canonically delayed and transposed versions of the leading line. For each movement, I asked myself the following questions. My “freedom of form” (since the machine canon is much stricter than Billings’ or Cowell’s handling of the form) would result from the answers I gave to the questions given in the list below:

1. Sequencers (Counter and Page Modules): Which sequence are you using? Is it read forward or backward?

2. Melody pitches (Scale 1 Module): What pitch range is the melody using? What is the bottom note of the melody?

3. Melody durations (Scale 2 Module): What range of durations are you using? Are they multiplied at all?

4. Melody dynamics (Scale 3 Module): What range of velocities are you using?

5. First voice delay for canonic entry (Delay Modules 1-3): What length delay are you using?

6. Second voice delay for canonic entry (Delay Modules 4-6): What length delay are you using?

7. Third voice delay for canonic entry (Delay Modules 7-9): What length delay are you using?

8. Transpositions for the 3 following voices (AddDiv Modules): What transpositions up or down are you using? How do they relate to the structure of the scale being used?

9. Panning for each voice (Pan inputs on Voice Modules): Where is that voice in the stereo space?

10. Timbre for each voice: (Timbre selection on *VSampler3*): What timbre is being used for each voice? How does it combine with the other voices? Is the timbre used unaltered, or are there changes made in its envelopes, fine tuning, etc?

11. Tempo: In what tempo will this movement be played?

12. *Vsampler3*: What scale is being used for this movement? (In this piece, tuning is specified in the sampler, not in *SoftStep*.)

An interesting aspect of the harmonic contrapuntal structure of the piece is how doubling works in the Just-Intonation scales used here. If two voices were doubled at the interval, say, of three scale steps, depending on the structure of the scale, a series of different sounding intervals could result. For example, in Scale 18, depending on the fundamental chosen, an interval of three scale degrees can be any one of five different intervals: 5/4, 6/5, 7/6, 8/7 and 60/49.

Given the canonic permutation scheme outlined above which generates each melody, each movement could be, if not potentially infinite, then at least potentially very very long. For practicality, however, the versions recorded here had a time limit placed on them - the end of the first complete cycle of the pitch sequencer that occurred after 2 minutes would constitute the end of the movement. This still produced a wide variety of durations - everything from just over 2 minutes to over 10 minutes, yet allowed the cycle of 18 movements to last just over an hour, which seemed like a practical length for a cycle intended for CD.

2.9.4.2 The Timbres

In this piece, I decided to use only "legacy" (I very much dislike that word and the technological arrogance it implies) samples that were used on the Emu *Proteus 1, 2,* and *3* sample players of the late 80s / early 90s. These were recently made available by Emu as sample sets in the Soundfont format, for a very low price ($20 US per sample set). I had worked a lot with the *Proteus 1* in the early 90s, and still occasionally use it, so that timbre set was very familiar to me. Mostly, for this piece, I concentrated on the *Proteus 3* timbres - a set of samples called "World", which are mostly based around timbres from various non-Western cultures. I assembled colourful ensembles from these timbres, and tried to make the orchestration of each movement as different as possible from the others. This was both in line with Cowell's diverse orchestrations for his *Fuguing Tunes*, and also reflecting his interest in non-Western and folk musics of all types. As well, trying to get the most out of a very limited set of timbres has always appealed to me as a compositional challenge. In a time when multi-gigabyte sample sets promise absolute fidelity of imitation (a contradiction in terms?) to acoustic performance, I still find it charming to use a tiny, limited, obsolete set of samples, and try to make something interesting out of them. In doing so, I remain under no illusion that what I'm doing is in any sense a "good imitation" of acoustic performers. In fact, I celebrate the difference between my limited sound world – in constrained-timbre pieces such as this, and the world of acoustic performance. I also recognize that I am not making a simulacra - I don't have the financial or technological means to attempt that - and I celebrate that fact. Using these timbres is as low-tech a solution as is my use of a particular pocket calculator and its round off errors. It might be also be observed, however, that in these pieces, I am revealing a "Zen garden" sense of composition –

raking (with numerical found objects) white pebbles (found object instrumental samples) into patterns of contemplation.

2.9.4.3 The Overall Form

Overall, then, the piece consists of 18 movements. Each movement uses a different Euler-Fokker Genera scale as described in Section 2.9.2, in the order 1-18. Further, each of the first nine movements uses one of the nine pocket calculator number sequences, described in Section 2.9.3 in linear order. That is, movement one uses Sequence 1, movement two uses Sequence 2, etc. Each of these sequences is played in the forward direction. The second nine movements each use one of the nine sequences in reverse order, and the sequences themselves are played backwards, in retrograde. That is, movement ten uses Sequence 9 in retrograde, movement eleven uses Sequence 8 in retrograde, etc. Each movement has a unique orchestration, and a unique way of handling the canonic relationships of the patch. For example, one movement might have a standard four-voice canon with each voice delayed and transposed differently, while another might be treated as a two-part texture, with one part for a solo instrument, and three of the voices having a common delay time, and harmonically related transpositions. Again, the reader is referred to Appendix 3.9.6 for a complete listing of all parameter settings, and a description of each movement. Although the scaling of the individual sequences is different in each movement, there is considerable redundancy in the opening of each sequence, and this gives the work a kind of sonic coherency which comes through in spite of the differing timbres and textures of the movements. Similarly, the chosen progression of the scales provides a measure of harmonic similarity from movement to movement, further creating a sense of linear progression in what is timbrally as non-directional a progression as I could make. This combination of linear implications in melodic type and harmonic progression, juxtaposed with the diversity of orchestration and texture makes a form that sits uneasily between worlds - in fact, I was delighted to find that the piece made me feel a sense of "dislocation" similar to that which a number of Cowell's works give me - perhaps, in my following my instincts in constructing this piece, I penetrated more deeply into the essence of Cowell's composing than I knew.

Having finished this piece in my studio, the question arises, what to do with it? The collection obviously in intended to be released as a CD, and it's highly likely that it will reach its widest circulation in that way. But there are further possibilities: Each melody is the product of a potentially very long 128 x 127 x 126 element permutation scheme, and thus, each canon could be set up as an installation in a gallery or preferably, a dedicated listening space, and be allowed to play for hours at a time. Also, the short durations of most of these pieces would suit them for downloading, suggesting publication on the internet. The one form of musical presentation I think would be most difficult would be live performance in the context of academic computer music. The pitch oriented nature of these pieces is pretty far removed from mainstream computer music interests, which seem, at the moment, to be concentrating on timbral modification, digital signal processing, and juxtaposition and transformation of reference-laden sonic objects. My use of outdated timbres, in the competitive research-oriented context I perceive in academic computer music, would also probably be viewed as quaintly anachronistic. Greater than either of these difficulties, however, is the duration of the piece. If I want the whole cycle to be played (which lasts just over an hour), the only viable context is a self-organized concert, as most conference-style computer music concerts these days feature a selection of shorter works by a variety of composers. This is similar to the difficulties I expressed in regard to the world of free improvisation, as discussed in Chapter 1.4. Perhaps, like a number of the other pieces discussed in this thesis, I really have composed a piece which will find its best home, not in live concert performance, but in an installation, or intimate home listening.

2.10 *Someone Moved in a Room*

2.10.1 Introduction

Someone Moved in a Room is a 37 movement CD length composition which began as a sonification project in collaboration with the Wollongong Room Calorimeter Project, a research project in the Department of Biomedical Sciences led, at that time, by Associate Professor Arthur Jenkins. He expressed interest in collaborating with an artist; this piece is what resulted. My musical work with sonification questioned the role timbre, tuning, and mode play in the perception of sonic data. Sonification is an emergent field with many issues to consider, a few of which will be briefly mentioned in this Chapter. For purposes of this thesis, however, we can consider the data from the Calorimeter Project to be yet another form of numerical found objects such as the pocket calculator patterns used in *18 New Fuguing Tunes for Henry Cowell*, discussed in Chapter 2.9. Accordingly, in this Chapter, we will concentrate on the tuning system of this piece, and the manner in which the data was realised musically. Those wishing to examine some of the issues involved in sonification which led to this piece are referred to Appendix 3.10.6, in which the Power Point presentation and paper, with sound examples, "From Sonification to Sound Art and Music: The interaction of usefulness and aesthetics," will be found. This unpublished paper was delivered at the University of Wollongong Post-Graduate Research Conference in September 2005 (Burt 2005c).

2.10.2 Tuning System

When I began this project, I thought about the many ways it is possible to sonify data. Even if one limits oneself to realizing data as streams of pitch, the possibilities are still great. I began to reflect that three of the most important aspects of musical sound are timbre, tuning and mode. Perceptible by people even without knowledge of what these parameters are, they are often what makes a piece immediately memorable. As anecdotal proof of this, consider any of those contests where people are called on to identify an old rock song. The real experts successfully identify the tune in less than 2 or 3 seconds. I maintain that an essential part of what they are remembering is the timbre of the tune, and the characteristics of the attack, even before the melody is recalled. Those wishing to see a more scientific approach to this question should refer to Scheirer, Watson and Vercoe 2001. The most recent research in this area is documented in Levitin 2006. The choice of timbre, tuning, and mode are critical in the musical affect / effect of sonification, as they are with all music.

In my first attempts at sonification, I mapped the data I was given by the Wollongong Room Calorimeter Project to 6 octaves of 53 notes per octave. The data had so many values, I wanted to hear it mapped as finely as possible. These results were quite acceptable, but then, remembering Charles Dodge's *The Earth's Magnetic Field*, an early classic of sonification, I decided to try mapping the data to a diatonic pitch set. In Dodge's work, first released on LP on Nonesuch (Dodge 1970), one side of the LP featured the magnetic field data mapped to the chromatic scale, while the other side had the same data mapped to a diatonic scale. The diatonic music was much more attractive, in traditional musical terms, and when Dodge re-released the piece on CD, he chose to release only the diatonic version. Despite my distrust of the sonic familiarity of the diatonic scale, I decided to hear what the data would sound like when mapped to it. I immediately liked the sound of what I heard, and had two questions:

1. How would the use of a diatonic pitch set affect the perception of the data?

2. What kinds of diatonic scales are there, that can stretch our idea of what "diatonic" is?

I decided I wanted to keep the familiarity of "diatonic," but wanted to explore diverse tunings within this large field.

To explore these questions, I wrote this piece. The 37 movements of the piece explore 37 possible answers to the first question. To explore the second question, I began considering the structure of the traditional diatonic scale. It has 7 intervals, containing two different sizes of step intervals, arranged A A B A A A B. This, of course, is a Moment of Symmetry scale. MOS scales were discussed extensively in Chapter 1.6. Previously, I had used MOS scales because they seemed like a valuable source of scale sizes. Here, for the first time, I was using them to get a particular kind of scale sound.

I investigated in which Equal-Temperaments I could find diatonic MOS scales with either 5 large and 2 small intervals, or conversely, 5 small and 2 large intervals. For convenience, these will be abbreviated 5L + 2s, or 2L + 5s. *Scala* has a MOS dialogue in which one can specify tuning and size of scale, and get a read out of all the possible MOS scales that fit those criteria. Using this tool, I found there were 18 different scales of this type in Equal-Temperaments of between 21- and 31-tones. More interestingly, these scales had distinct characteristics, which approximated the sounds of the three types of ancient Greek genera: Enharmonic, Chromatic, and Diatonic. These scales were also made by stacking a generator interval of N steps out of an Equal-Temperament of M degrees. This kind of scale generation has already been discussed in Chapters 1.8 and 2.7. The Greek modal system has been discussed in Chapter 1.10.

With these "Greek-sounding" tunings, I could combine many of my tuning interests in one piece. Another, more light-hearted interest, could also be indulged – my love of coincidences. In *18 New Fuguing Tunes for Henry Cowell*, discussed in Chapter 2.9, I was delighted that there were 18 scales that fit my criteria, which matched the 18 available data sets and the 18 historical models of Cowell's *Fuguing Tunes.* Here, I had 18 scales that matched my criteria, and the Wollongong Room Calorimeter group had provided me with 18 different kinds of data. It seemed that 2005 was my year for pieces based on the number 18. Oh well, it's a living.

Table 74 shows all the eighteen 7-note MOS scales that have either 5L+2S or 2L+5S structures found in 21- through 31-tone Equal-Temperament. Only the form with the smaller generator has been shown (each scale has a partner made with its inversional interval counterpart, but the result is simply a mode of the shown scale). The scale is made by stacking intervals of the generator's size, and then collapsing everything within an octave, just as the diatonic 12-tone tuning scale can be made by piling up Perfect 4ths or Perfect 5ths.

All these generators hover around (mostly above) a Perfect 4th, from about 496 to 580 cents. This suggests that many diatonic MOS scales exist which are not related to Equal-Temperaments, but which can be found with generators about this size.

The scales have four qualities – which I call Enharmonic, Chromatic and Major and Minor Diatonic after the ancient Greek genera which they sound similar to. The Enharmonic scales have a pattern of two very small intervals followed by a much larger one; the Greek Chromatic scales has 2 intervals, hovering around a minor 2nd, followed by a minor 3rd type interval; and the Diatonic scales alternate between quasi-Major 2nds and minor 2nds. Each of these families of scales has a distinctive sound, but the Enharmonic scales sound the most distinctive; more like each other than they sound anything at all like the Diatonic or Chromatic scales, and vice versa. Note that Scale 5 in this chart is identical to the 7-note scale used as the basis for *Homage to Wyschnegradsky*, discussed in Chapter 2.4.

A complete listing of all scales discussed here can be found in Appendix 3.10.2. *Scala* *.scl and *.tun files for each scale can be found in Appendix 3.10.3.

Table 74a: 18 Diatonic type 7-note MOS scales found between 21- and 31-tone ET. Part 1

Number	Scale Data
1	21 tone equal temperament Generator - 10 steps: 571.429c Pitch levels: 0 8 9 10 18 19 20 21 = 2L+5S intervals Interval structure in scale steps: 8 1 1 8 1 1 1 - Enharmonic quality.
2	22 tone equal temperament Generator - 9 steps: 490.909c Pitch levels: 0 1 5 9 10 14 18 22 = 5L+2S intervals Interval structure in scale steps: 1 4 4 1 4 4 4 - Major Diatonic quality
3	23 tone equal temperament A Generator - 10 steps: 521.739c Pitch levels: 0 4 7 10 14 17 20 23 = 2L+5S intervals Interval structure in scale steps: 4 3 3 4 3 3 3 - Enharmonic quality
4	23 tone equal temperament B Generator - 11 steps: 573.913c Pitch levels: 0 9 10 11 20 21 22 23 = 2L+5S intervals Interval structure in scale steps: 9 1 1 9 1 1 1 - Minor Diatonic quality
5	24 tone equal temperament Generator - 11 steps: 550c Pitch levels: 0 7 9 11 18 20 22 24 = 2L+5S intervals Interval structure in scale steps: 7 2 2 7 2 2 2 - Chromatic quality
6	25 tone equal temperament A Generator - 11steps: 528.000c Pitch levels: 0 5 8 11 16 19 22 25 = 2L+5S intervals Interval structure in scale steps: 5 3 3 5 3 3 3 - Minor Diatonic quality
7	25 tone equal temperament B Generator - 12 steps: 576.000c Pitch levels: 0 10 11 12 22 23 24 25 = 2L+5S intervals Interval structure in scale steps: 10 1 1 10 1 1 1 - Enharmonic quality
8	26 tone equal temperament Generator - 11 steps: 507.692c Pitch levels: 0 3 7 11 14 18 22 26 = 5L+2S intervals Interval structure in scale steps: 3 4 4 3 4 4 4 - Major Diatonic quality
9	27 tone equal temperament A Generator 11 steps: 488.889c Pitch levels: 0 1 6 11 12 17 22 27 = 5L+2S intervals Interval structure in scale steps: 1 5 5 1 5 5 5 - Major Diatonic quality

Table 74b: 18 Diatonic type 7-note MOS scales found between 21- and 31-tone ET. Part 2

10	27 tone equal temperament B Generator - 13 steps: 577.778c Pitch levels: 0 11 12 13 24 25 26 27 = 2L+5S intervals Interval structure in scale steps: 11 1 1 11 1 1 1 - Enharmonic quality
11	28 tone equal temperament Generator - 13 steps: 557.143c Pitch levels: 0 9 11 13 22 24 26 28 = 2L+5S intervals Interval structure in scale steps: 9 2 2 9 2 2 2 - Chromatic quality
12	29 tone equal temperament A Generator - 12 steps: 496.552c Pitch levels: 0 2 7 12 14 19 24 29 = 5L+2S intervals Interval structure in scale steps: 2 5 5 2 5 5 5 - Major Diatonic quality
13	29 tone equal temperament B Generator - 13 steps: 537.931c Pitch levels: 0 7 10 13 20 23 26 29 = 2L+5S intervals Interval structure in scale steps: 7 3 3 7 3 3 3 - Chromatic quality
14	29 tone equal temperament C Generator - 14 steps: 579.310c Pitch levels: 0 12 13 14 26 27 28 29 = 2L+5S intervals Interval structure in scale steps: 12 1 1 12 1 1 1 - Enharmonic quality
15	30 tone equal temperament Generator - 13 steps: 520.000c Pitch levels: 0 5 9 13 18 22 26 30 = 2L+5S intervals Interval structure in scale steps: 5 4 4 5 4 4 4 - Minor Diatonic quality
16	31 tone equal temperament A Generator - 13 steps: 503.226c Pitch levels: 0 3 8 13 16 21 26 31 = 5L+2S intervals Interval structure in scale steps: 3 5 5 3 5 5 5 - Major Diatonic quality
17	31 tone equal temperament B Generator - 14 steps: 541.935c Pitch levels: 0 8 11 14 22 25 28 31 = 2L+5S intervals Interval structure in scale steps: 8 3 3 8 3 3 3 - Chromatic quality
18	31 tone equal temperament C Generator - 15 steps: 580.645c Pitch levels: 0 13 14 15 28 29 30 31 = 2L+5S intervals Interval structure in scale steps: 13 1 1 13 1 1 1 - Enharmonic quality

2.10.3 Algorithm

2.10.3.1 Sonification

This piece began as a sonification project. Although it is not the focus of this thesis, we need to briefly consider sonification and look at the origin of this data in the Wollongong Room Calorimeter Project. Sonification is defined by the International Community on Auditory Display (ICAD) as:

> the use of nonspeech audio to convey information. More specifically, sonification is the transformation of data relations into perceived relations in an

> acoustic signal for the purposes of facilitating communication or interpretation. By its very nature, sonification is interdisciplinary, integrating concepts from human perception, acoustics, design, the arts, and engineering. Thus, development of effective auditory representations of data will require interdisciplinary collaborations using the combined knowledge and efforts of psychologists, computer scientists, engineers, physicists, composers, and musicians, along with the expertise of specialists in the application areas being addressed. (Kramer et al 1997)

Roberto Morales-Manzanares, in the User's Manual to his *STEREOSpectro* sonification software gives a very lucid and succinct introduction to the field. His definition of three types of sonification is very useful.

> Sonification is the act of representing data as sound. An example of sonification is using a wind chime to convert wind speed to music. The higher the wind speed is, the more notes are played in one second. We often use the mathematical verb "to map" to describe converting data to sound: "We mapped the wind speed to the number of notes played in one second by the wind chime."
>
> Different types of sonification can be categorized in three ways*:
>
> **Iconic sonification:** This type of sonification is when someone maps data to sounds that are associated with certain phenomena. For example, if we gathered weather data, such as cloud cover, temperature, and humidity, to calculate the probability of rain tomorrow, then using the sound of rain to indicate when there is a high probability of rain would be an iconic sonification.
>
> **Direct Conversion Sonification:** This type of sonification is when someone maps data to sound to listen for patterns that are represented in the data. For example, space scientists map data of waves made up of magnetic and electric fields called electromagnetic waves to sound waves. This direct conversion sonification can be as simple as taking the frequencies of the waves and making sound waves with the same frequencies, which is most useful as long as the

frequencies are at pitches that our ears can hear. Earth's whistler wave is such an electromagnetic wave that scientists have been sonifying for over 30 years.

Musical Sonification: This type of sonification is when someone maps data to sound in a musical way. For example, we have created a computer software program that will convert data of very fast particles that have come from the Sun and are captured by an instrument on one of 2 satellites in space, called Helios 1 and Helios 2, to bell-like sounds. Several musicians have used musical sonfication of space data to create quartet or orchestra music pieces.

*J. Keller's definitions, 2003, private communication" (Morales-Manzanares 2006)

I have another definition of sonification, one that comes from the composer's, not the scientist's sensibility. It is this:

Sonification as Poetics:

Sonification becomes more than sonic data with a utilitarian purpose when the composer / sonifier feels the poetry in the numbers and attempts to make that audible.

The idea of *feeling* or *sensing* the inherent poetry in a set of numbers is central to my work. It is discussed in "Some Parentheses Around Algorithmic Composition" (Burt 1996) and several other articles. It is an idea which I would like to again emphasize here. It is an area of aesthetics potentially shared by mathematicians, musicians, and other artists of the structural. It is where "deep sensing" and structure come together.

We should also not forget that many years before sonification became an area of interest, composers were using non-musical data as a source for composition. A few early examples of this rich history would include Brazilian composer Heitor Villa-Lobos' compositions *New York Skyline* (1939), for piano, and his *Symphony No. 6: On the Profiles of the Mountains of Brazil* (1944) for orchestra (Villa-Lobos 1939, 1944), both of which used found object contours to determine thematic material; American composer John Cage's *Atlas Eclipticalis* (1961-62) for orchestra; *Etudes Australes* (1974-75) for piano; and *Etudes Boreales* (1978) for cello and/or piano, all made by tracing star maps onto score paper (Cage 1962, 1975, 1978); and American composer Charles Dodge's *The Earth's Magnetic Field* (1970) for computer, which mapped magnetic field measurements onto the notes of a diatonic musical scale (Dodge 1970). Also of interest from this early period is the essay "Any Bunch of Notes" (1953) by German emigre composer Stefan Wolpe, which deals with his Bauhaus and Dada influenced ideas about the shaping possibilities of any arbitrarily selected musical material (Wolpe 1953). More recent examples would include Czech composer Petr Kotik's *There is Singularly Nothing* (1971-73) for voices and instruments, which used, among other sources, readings of the brain waves of fruit flies (Kotik 1973); Australian composer Tristram Cary's *Contours and Densities at First Hill* (1976) for orchestra, which used topographical data and photographs from the Flinders' Ranges as source material (Cary 1976); and American Tom Hamilton's *London Fix* (2003) for computer, which used the fluctuating price of gold in London for the year 2002 as its source set (Hamilton 2003).

Two remarkable projects that cross the line between scientific sonification and musical composition are the DNA music project of Professor Mary Anne Clark and John Dunn, and the 2004 "Listening to the Mind Listening" project of ICAD. Clark and Dunn's work, especially their paper "Life Music: The Sonification of Proteins" and their several collaborative CDs of DNA music are paradigmatic for the field (Dunn and Clark 1997, 1998; Clark, 2001). Dunn has also developed a number of computer programs which can be used for sonification such as *SoftStep*, *MicroTone*, *BioEditor*, *DataBin*, and *ArtWonk*, most of which I have been involved in beta-testing, and which were used to realise most of the pieces discussed in this thesis (Dunn 2004a-e, 2007, Van Raaij 2004). The ICAD project involved ten composers, or groups of composers, from around the world using EEG data obtained from a person listening to music to generate further sound and music compositions. These pieces have an enormous range of style and sound, showing just how wide the possibilities of sonification are (Barrass et al 2004).

In each of these compositions or projects, non-musical source material is worked with and the resulting music often has quite unfamiliar aspects to it. Learning to hear this music on its own terms often results in expansion of one's musical tastes and expectations. For example, in Mary Anne Clark's music, discussed in Chapter 1.3, the same protein from several different species will be sonified simultaneously. Unison lines happen when the protein sequences are identical, and chords occur where there are differences in the protein between species. Here a musical patterning - the timing of chords and unison lines - is produced by biological information. The result is an unexpected musical timing and a revealing of aspects of a biological structure simultaneously. Similarly, any of the Cage star-map pieces are full of musical surprises - unexpected melodic leaps, sudden textural changes, etc, of the kind not normally found in traditionally produced musics. The joy of all these pieces is both in the finding of familiar musical gestures where none were suspected, and also in the opportunity they give us to discover musical material that we now can learn to hear, and use as a means of expanding our musical tastes and knowledge.

2.10.3.2 The Data

This data originated from the Wollongong Room Calorimeter Project, directed by Associate Professor Arthur Jenkins. The Wollongong Room Calorimeter project consists of a small room in the Department of Biomedical Sciences which can be sealed off so that all activity in the room, specifically intake and output of gasses, can be measured precisely. Motion detectors, built by engineer Harry Battam, are also in the room, and the amount of motion in the room can be monitored and recorded, and correlated with gas usage. The data from the movement detectors was the data they thought artists might be interested in sonifying. The movement in the room is monitored 20 times a second. Output from the detectors can range from 0 to 10 volts, but, in the data I used, ranged between 3.59 volts and 7.05 volts. Participants in the experiment are asked to perform a number of activities, listed in Table 75 below. Each of these activities produces different data, with different patterning. The data I was given consisted of a spreadsheet with 18 columns of data, each of which had 1200 elements. This spreadsheet, in original and normalized forms, can be found in Appendix 3.10.5. The data consisted of readings of the motion of participants doing 18 different activities for one minute each. It was this data set that formed the basis of this work.

Table 75: Data Types from Wollongong Room Calorimeter Project

Number	Data Type	Number	Data Type	Number	Data Type
1	Bike – 60 rpm	7	Thumb	13	Write
2	Knee bench	8	Type	14	Step
3	Bike – 40 rpm	9	Fidget	15	Song
4	Turn head	10	Sit	16	Bike – 80 rpm
5	Sit cycle	11	Lay down	17	Sit computer
6	Stand	12	Jump	18	Drink

After trying several different types of sonification, described in the paper in Appendix 3.10.6, I decided to use the data to control selection of pitch, duration, and dynamics of a set of electronic timbres. The first question was, of course, how to use the data? Each stream of data consisted of 1200 motion readings made at 20 readings per second, with values that varied between 3.59 and 7.05 volts overall. However, only two data streams used that full range. The rest seemed to hover somewhere in between, often covering only a very narrow range. If I used the raw data, some data streams would cover a very wide range, others a very narrow range. On the other hand, if I normalized each data stream independently, so that it covered the complete range the software could handle, finer details in the structure of each data stream might be made audible. Figures 53-56 show these relationships. Notice in Figures 53 and 54, how "Sit" is very narrow in range, while "Drink" is much wider. Once the data is normalized, as shown in illustrations Figures 55 and 56, each data stream covers the full range, and differences in the internal structure of each data stream are much more easily seen and heard.

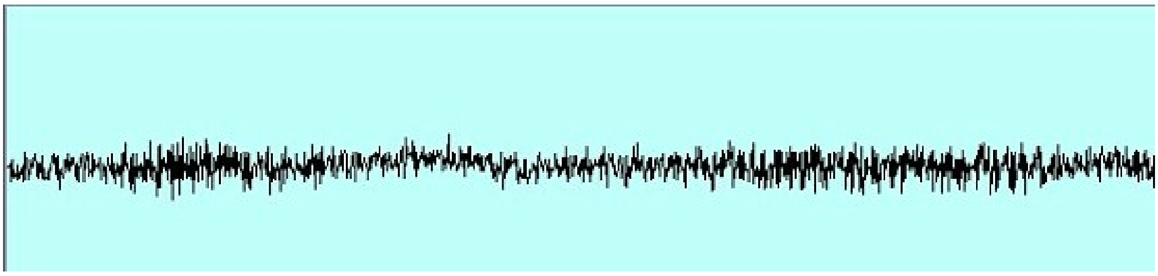


Figure 53: Sit – raw data from Calorimeter Project

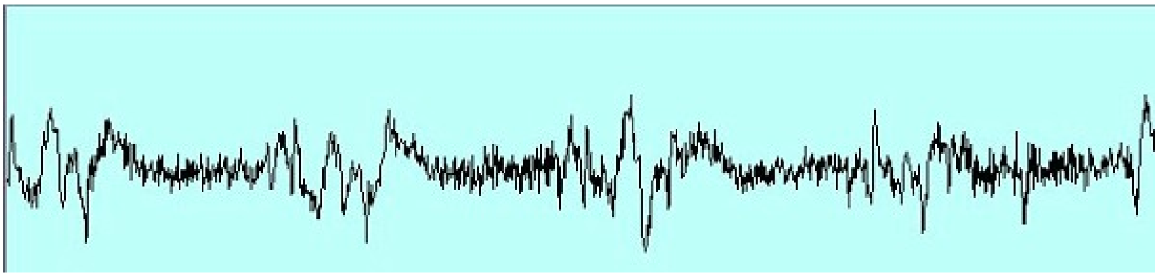


Figure 54: Drink – raw data from Calorimeter Project

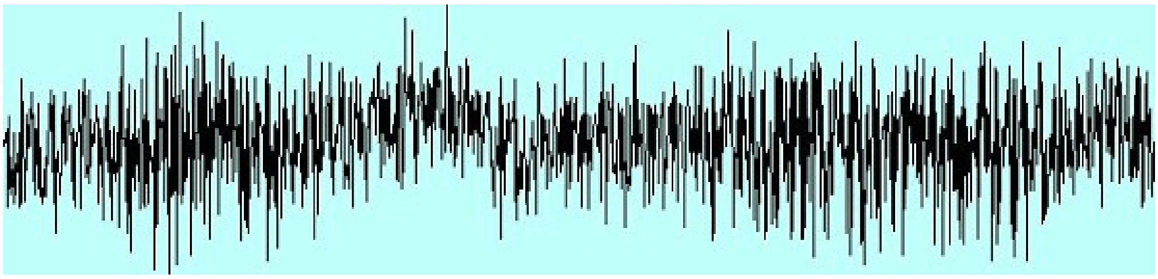


Figure 55: Sit – normalized data from Calorimeter Project

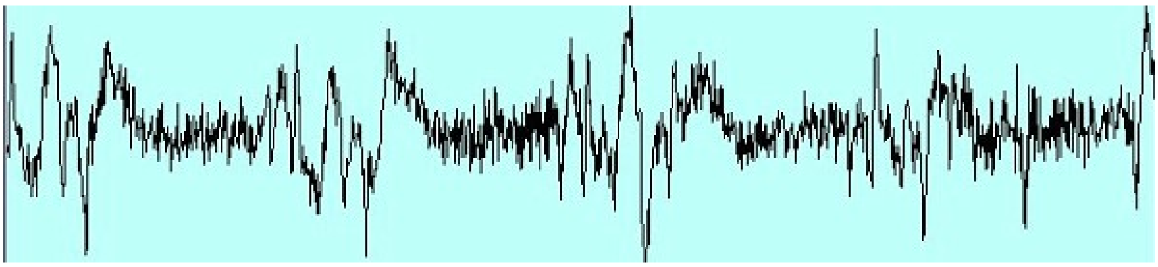


Figure 56: Drink – normalized data from Calorimeter Project

In the end, I decided to sonify both raw and normalized data, since the possibilities for establishing interrelationships between the different kinds of data seemed so rich. Two suites of 18 movements were made: one in which the raw data was used, the other which used the normalized data. How I used the data to control pitch, duration and dynamics of each musical line will be discussed in Section 2.10.4.

2.10.4 Process – Real Time Usage

The data was used to control pitch, duration and dynamics of a musical line. A single stream of data was put through a delay line and applied to the three parameters. The data was further delayed, and a second musical line played in canon with the first. I have frequently used the technique of canonically delaying a single stream of data to provide control for different musical parameters. As discussed in Chapter 2.9, this technique was a central aspect of the structure of *18 New Fuguing Tunes for Henry Cowell.* This technique is also used when mapping attractors in chaos science, where a single stream of data is often mapped onto an x-y axis by using two slightly differently delayed versions of the data. That is, every Nth element of a sequence is mapped onto the x axis, while every N+1th element is mapped onto the y axis. The resulting "delay maps" often reveal interesting aspects of the structure of the data (Peak and Frame 1994). Similarly, in music, delayed versions of the same data stream can control different aspects of sound, such as pitch, duration and dynamics. Such a *three-dimensional sonic fractal* has a name - in music, it's called a "melody."

Since we are using delayed data streams - in musical terms, a canonic relationship, to generate the melodies, we can also delay the resulting melodies against themselves, making an actual musical canon. This is another aspect this piece has in common with *18 New Fuguing Tunes for Henry Cowell.*

One characteristic of this data is that it doesn't have repeating patterns in it. Therefore, it's hard for the ear and mind to process: new information is always happening. A canon allows one to hear material again, often at a different pitch level. This kind of repetition and redundancy allows one to hear striking aspects of the material again, while allowing the data to progress unhindered. Sometimes the use of contrapuntal techniques like the canon can obscure things. Here, I hope I'm using it as an aid to comprehension.

To explore this set of 18 data sets and 18 possible tunings, I made an interface with *ArtWonk*. It allows me a number of controls for exploring the musical potential of this material. Data Sets and Scales are chosen with the grid controls at the upper left. Clicking on the name of an activity or a scale selects it. Horizontal control sliders allow selection of the following:

1. Pitch mode - what note does the chosen scale begin on?

2. Rhythm mode - the pitch levels are used to determine durations - which level of pitch are we using to start our set of durations with?

3. Bass octave - what is the lowest note used?

4. Duration multiplier - to get slower tempi, if desired.

5. Tempo in BPM - set the overall tempo. A tempo of 300 with a duration of 6 = 1200 notes per minute - the speed of the original data.

6. Canon Delay in Ticks - length of delay between the first and second voice (the second voice is always an octave higher than the first).

7. Timbre Select Voice 1 - choose between 18 possible timbres, produced by *Vaz Modular*, made especially for this piece.

8. Timbre Select Voice 2 - choose between 18 possible timbres, as above.

9. Counting by... sets how the data is stepped through. Counting by 1s gives every element in order. Counting by 3s gives every third element, in order, etc. Larger values of counting give shorter pieces and cruder scans through the data set.

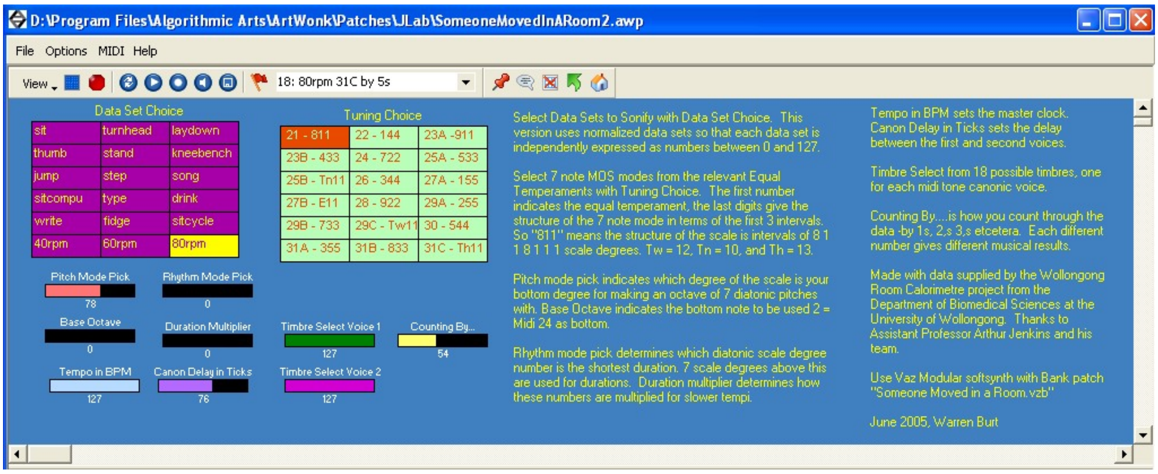


Figure 57: Performance interface for *Someone Moved in a Room* made with *ArtWonk*

This interface allows one to explore the different tunings and the different data sets. Matchings of tuning, data set and many other parameters can be worked with interactively. As with *18 New Fuguing Tunes for Henry Cowell,* once a good set of parameters are chosen, the results are automated and recorded.

Tuning and mode are important elements in musical perception, but the other important element mentioned earlier was timbre, and it has not been neglected here. A set of 18 timbres was designed for the piece using *Vaz Modular*. These timbres were designed "by ear," using the full set of resources of *Vaz Modular*, (specifically the many different kinds of oscillators available) in order to make a family of stable, non-vibrato timbres of clear pitch which sounded beautiful. I find that much sonification is done with very plain, unattractive timbres. I did not want that to be the case here. I wanted to make very attractive sounds, and I wanted them to be distinct from each other. A number of the sounds have a pluck-like character. Others are more bell-like. Still others resemble sustained pad sounds. A variety of techniques, from additive synthesis (with careful detuning of harmonic and inharmonic partials) to pulse width modulation and frequency modulation to waveshaping were all used and combined to make the 18 patches which constitute the timbral family of this piece. Although pitch, duration and dynamics were controlled by found object data, and the harmonies were a set of scales chosen for their theoretical qualities, interval structures which were similar to diatonic scales (and which just happened to sound good), as far as timbre is concerned, I relied on my ear to make timbres I would like to listen to for a long time. Hopefully, the listeners to this piece will want to do that too.

I was very happy with making these timbres by ear. Making timbres in this way was not dealing with the problems of sonification, nor was it dealing with the problems of tuning, or with problems of structure. It was dealing in a very immediate way with the problems of musical sound, in sound, with all the tools at my disposal. It was, in its own way, the most intensely musical part of the whole process. And while research may be engrossing, making "music" is really fun. In fact, it was so much fun that I felt almost slightly guilty about it!

My criteria for timbre were threefold. First, the timbres should have a "depth", that is, an inner life created by the movement of their partials. Second, each timbre should have enough high partials to cause them to "sparkle" without seeming edgy or causing ear-fatigue. Third, there should be a predominance of harmonic over inharmonic partials, so that pitch-perception was not obscured. Sharper, faster attacks also aid in pitch-perception, and the "note-oriented" quality of the timbres.

With this performing setup, which gives me 18 data sets, 18 scales, 18 timbres (of which you can have two at once), 10 ways of scanning through the data, and a variety of tempi, modes and octave choices, it's obvious that a very large number of different pieces can be made. To work within this infinity, I decided that sets of pieces should be generated, which would allow me to hear the relationships between the different data sets, using the following rules:

1. It's important to use the same algorithm - the same set of rules for all the pieces in a set - in order to really hear differences between the data sets - these then have the potential to become musically meaningful differences.
2. A set of pieces should use the same rate of stepping through the data set.

In the end, I made two suites, one with each of the 18 normalized data sets stepped through by 3s, and one with each of the 18 raw data sets stepped through by 8s. This gave a normalized data suite of moderate length (47 minutes) and a raw data suite of shorter length (22 minutes). As well, a version of "Drink" normalized was made stepping through the data by 1s, which took just over 9 minutes. This was done because I felt that the "Drink" data set had the most interesting musical material contained within it.

When I started out this project, I was fairly committed to maintaining the "integrity of the data." Even with all this work with different tunings, timbres, and rates of scanning through the data set, I can still hear that each data set produces music with its own unique qualities. However, this work has made me question the meaning of the concept of "integrity of the data". That is, although there may be certain number patterns in a set of data which imply, or suggest certain musical contours or structures, the sheer number of ways of realizing those contours creatively are so large that perhaps the "identity" or "integrity" of a data set, when used for artistic purposes, can only be metaphorical at best. Still, if a data set had, to give an extremely simple example, mostly high values at the start and mostly low values at the end, some musical way of reflecting that progression should be devised in order to reflect the overall structure implied by the data.

The completed computer music composition, *Someone Moved In A Room* consists of the two suites mentioned above, plus the 9 minute version of *Drink* made by stepping through every value. The completed work is a CD with 37 tracks with a total duration of 80 minutes. The MP3 files are contained in Appendix 3.10.1, and the cover for the CD in Appendix 3.10.4.. In this piece, the listener can hear and decide for themselves which data sets produce traditionally meaningful musical gestures, and which produce gestures that we might come to know as musical, given repeated listenings. Although the piece may not have had immediate applicability in analyzing the data, I feel it successfully posed a number of questions about the nature of sonification and musical perception.

2.11 *Proliferating Infinities*

2.11.1 Introduction

Proliferating Infinities is a very big work for recorded harp samples, which thoroughly explores a family of 264 scales made from additive sequences based on number triangles other than Pascal's triangle. Pitch is also controlled using the same additive sequences that generated the scales. The term very big is not used lightly. Of all the works discussed in this thesis, it is the longest. It is more than 13 hours long, and fills 12 CDs. In this piece, my desire for extreme duration, combined with my desire to thoroughly explore a large harmonic system, came together and reached what is, for now, probably their most extreme expression. The motivations for such an extreme piece will now be discussed.

There were six main motivations for making this piece.

1. Following up on suggestions in Erv Wilson's "Mt. Meru" papers, I had developed a family of 276 12-note Just-Intonation scales, using number triangles other than Pascal's triangle. This was the largest family of scales I have so far developed. Enumerating it took the better part of a year. I wanted to make a piece in which to hear this family of scales. That meant doing a very long piece, longer than even my extended attention span. One of the central tenets of my work as a composer is that one should not be afraid to follow where one's materials lead, especially if that is into unfamiliar territory.
2. My wife, Catherine, likes ambient harp music. We call it "plinkies." I was using a harp timbre to explore some of the implications of the additive sequence generators I had made for *ArtWonk*. She, from the next room, said that she liked what was happening with the composite melodies that were formed by the wide

ranging melody. Knowing that she likes harp music made me decide to use that timbre for this piece. It is a timbre I enjoy, and its clarity seems to lend itself to exploring a wide palette of harmonies.

3. My continuing desire for works of extreme duration was, of course, a factor. I decided to write one etude in each of my scales. Even at only three minutes per etude, this resulted in a piece longer than any one person's attention span could encompass at one sitting
4. I began the series of works documented in this thesis with a harp piece, *For JSB and JT*, discussed in Chapter 2.1. I thought that ending it with music for harp timbres gave a nice sense of conceptual completeness.
5. While working on this piece, I had been reading Eli Maor's *To Infinity and Beyond: A Cultural History of the Infinite* (Maor 1987) and was inspired by his historical discussions of mathematical, geometric, philosophic, aesthetic and cosmological infinity.
6. During the time I was reading this, I gave Canadian poet Christian Bők a tour of Wollongong's two architectural treasures, the Nan Tien Buddhist Temple and the Sri Venkataswara Hindu Temple. Bők, no stranger to infinite mathematical processes applied to art (Bők 2003) was quite overwhelmed by both buildings, especially with the many repeated images of religious figures they contained. The Nan Tien Temple contains large walls with thousands of repeated pressed metal relief images of the Buddha. The many varied images Ganesha and other gods in the Hindu temple may be many fewer in number than their Buddhist counterparts, but they make up in elaborate iconography what they lack in strength of numbers. In either case, the overall effect is one of almost dazzling complexity and a sense is created that both spaces extend, if not to infinity, then

very much farther than the confines of their physical dimensions. I wanted to encapsulate this sense of bountiful proliferation in a musical composition / object / environment.

Musicians, at least contemporary musicians, are used to the idea of infinite or near infinite sets. Permutation processes, such as the 479 million possible arrangements of 12 pitch classes, while finite, are far beyond the immediate comprehension abilities of people, and thus can seem infinite. Similarly, additive sequences, even when limited modulo N, can have such long cycles that they are, for all practical purposes, seemingly infinite and non-repeating. I reasoned that if I used a seemingly infinite process to generate pitch and derived my scales from the same infinity-suggesting process, there would at least be a conceptual agreement between harmonic and melodic generating structures in my work.

Creating the complete work took 6 weeks, 7 days a week, for about 4 hours a day. In that period I listened intently to all 264 scales. At only one or two times in that period did I hear a scale, or a melodic type that I thought was less than fascinating. The resources of the additive sequence generators for making both melody and harmony may not be infinite, but they are incredibly rich.

I recorded the piece onto 12 CDs. I also made MP3 files of the sections, and these are contained in Appendix 3.11.1. I hope to have a public showing of this work, in the form of a sound installation, at the De Havilland Gallery in Wollongong in June 2007.

On the CDs of the work, I enclosed the following program note, which I think encapsulates the spirit of the piece very well.

“Warren Burt: Proliferating Infinities (2006)

A 13 hour ambient environmental composition for harp samples using all of the Triangle Scales in “A Catalog of 12-Note scales made from Additive Sequences Obtained from the Diagonals of Number Triangles. (2004-2006)”

Each additive sequence scale can extend off into infinity, and each has an infinite number of MOS subsets.

Each number triangle can generate an infinite number of additive sequence scales.

And there can be an infinite number of number triangles.

There are an abundance of proliferating infinities in this system.

If our limited senses and abilities can only apprehend, and physically create, but the smallest corner of that universe, perhaps that will be glimpse enough of the infinities inherent therein.

There are 264 sections in this piece, each approximately 3 minutes long. In each section, the melody is produced by the same additive sequence, with the same seed, that generated its scale. Each set of 11 scales from each number triangle are edited together into a continuity which lasts just over a half-hour. There are 24 of these in the piece. The whole composition lasts around 13 hours. And this is but the smallest fraction of the potential of these scales and processes.

Like the many thousand of images of the Buddha that adorn the interior of Buddhist temples, showing that everyone can become a Buddha, every etude, every scale, is part of the greater infinity, and can be observed as such. Or, more aptly, like the multiple statues of Ganesha (or any other god) in a Hindu temple, each one revealing the different characteristics of the particular manifestation of the god, yet always recognizable, so these etudes highlight a few of the infinite facets of this harmonic and compositional system.

It is envisioned that this set of CDs will provide a kind of continually evolving sonic perfume with which selected environments (such as gardens) may be adorned.

The tuning system developed for this piece will be discussed in Section 2.11.2. The process of melody generation using additive sequences will be discussed in Section 2.11.3. The overall form of the piece, as well as other considerations of its implications, will be discussed in Section 2.11.4.

2.11.2 Tuning System

In Chapters 1.9 and 2.6 we examined scales based on additive sequences derived from the sums of the diagonals of Pascal's triangle. Pascal's triangle has 1s cascading down both sides of it, as was shown in Figure 7, Chapter 1.9. However, Figure 8, Chapter 1.9 showed that number triangles could be formed with two "seed values." That is, different numbers could run down the different sides of the triangle.

If a number triangle has only one seed, as in Pascal's (the 1-1 triangle), the triangle is symmetrical, and diagonals, whether left or right facing, will produce the same number series. However, if a triangle has two seeds, as in the 2-1 triangle, also known as the Lucas triangle, left and right facing diagonals having the same angle will produce different number series (Robbins 2005). These series will, however, have the same generating rule, as long as the opposite facing diagonals are indeed at the same angle. Figure 58, from Wilson 2002 shows this quite elegantly.

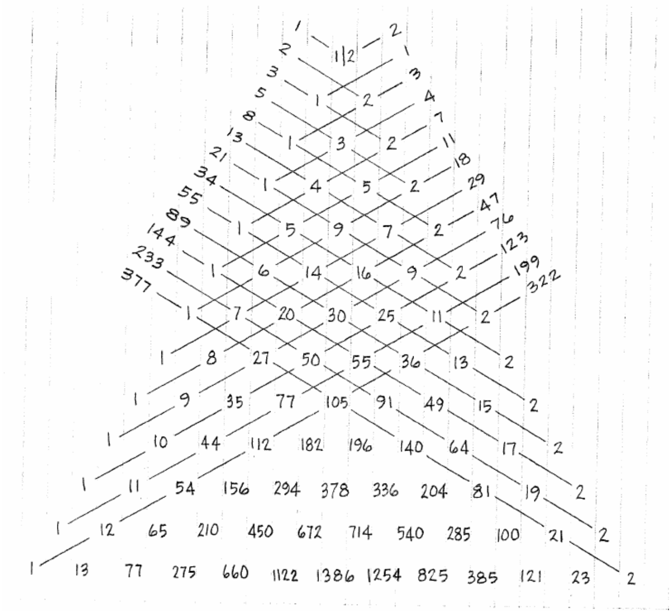


Figure 58: 2-1, or Lucas Triangle, showing different additive sequences on different sides of the diagram

Notice the series on the left-wing of the triangle is the Fibonacci Series. The series on the right-wing of the triangle, however, is the Lucas Series. Note that both series follow the same $A_n = A_{n-2} + A_{n-1}$ rule. That this rule is unvarying with diagonals of the given angle is shown by Figure 59, also from Wilson 2002.

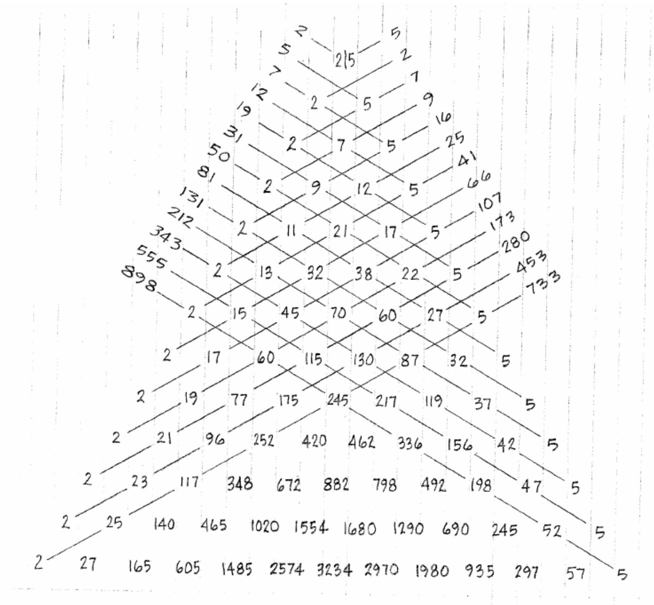


Figure 59: 2-5 Triangle, showing different additive sequences on different sides of the diagram, which follow same rule as additive sequences in Figure 58

Note that the number series on both the left- and right-wings of the triangle follow the same additive rule, $A_n = A_{n-2} + A_{n-1}$, as the sequences in the previous diagram. This is because the diagonals in both triangles are at the same angle. Any number triangle will produce a number series on each side with the same rule if the diagonals are at the same angle. Changing the number triangle only changes the seed values placed into the additive sequence rule, not the rule itself.

However, summing diagonals at a different angle to the one above produces additive sequences with different rules. Chapter 2.6, Figure 30, from Wilson 1997b, showed the first 31 of 192 rules produced by different angles on a number triangle. In *The MOSsy Slopes of Mt Meru*, discussed in Chapter 2.6, I used the first 11 of these rules to derive scales from Pascal's triangle. Now however, I wanted to use other number triangles. Table 76 reiterates these 11 rules and gives the limit-ratios for each sequence.

Table 76: Wilson's first 11 additive sequence rules and limits

Number	Sequence Rule	Limit
1	$A_n = A_{n-2} + A_{n-1}$	1.618033989
2	$B_n = B_{n-3} + B_{n-1}$	1.465571232
3	$C_n = C_{n-3} + C_{n-2}$	1.324717957
4	$D_n = D_{n-4} + D_{n-1}$	1.380277569
5	$E_n = E_{n-4} + E_{n-3}$	1.220744085
6	$F_n = F_{n-5} + F_{n-1}$	1.324717957
7	$G_n = G_{n-5} + G_{n-2}$	1.236505703
8	$H_n = H_{n-5} + H_{n-3}$	1.193859111
9	$I_n = I_{n-5} + I_{n-4}$	1. 167303978
10	$J_n = J_{n-6} + J_{n-1}$	1.285199033
11	$K_n = K_{n-6} + K_{n-5}$	1.134724138

The subscripts to these rules gave me the seeds for each new number triangle to investigate. That is, if Pascal's triangle has seeds of 1,1, then the subscripts of the first rule, the "2" and the "1" in the first A_n rule, would provide the seeds for the next triangle. Each of the sets of subscripts given here will generate a unique triangle, whereas number pairs not contained in this listing (for example 4, 2) will not generate a unique triangle, but one that will be related in some way to an already generated triangle.

This rule also applies to additive sequences themselves. Taking the first rule, seeds of 1, 1, generate the classic Fibonacci series. Seeds of 2, 1, generate the Lucas series. The series generated with 4, 2, as seeds is simply the Lucas series doubled, which, when turned into a musical scale based on using each number as a harmonic would simply generate the same scale as the Lucas series. However, each triangle, additive sequence and scale generated with the subscripts in the above list *is* unique.

Using the 11 sets of subscripts in the above rules, I generated 11 new number triangles in addition to the original Pascal's triangle. This gives 12 number triangles to work with. Each of the 11 rules could be used with each of the 12 triangles to generate scales. Table 77 gives the seeds values for each of the 12 triangles.

Table 77: Number seeds for 12 number triangles used to generate scales

Number	Triangle seeds
1	1, 1
2	2, 1
3	3, 1
4	3, 2
5	4, 1
6	4, 3
7	5, 1
8	5, 2
9	5, 3
10	5, 4
11	6, 1
12	6, 5

To find the 11 additive sequences generated on both the left and right wings of each of these triangles, I did not actually draw diagonal lines, but examined the structure of the triangles, and found which elements of the triangle would generate the first numbers of the additive series desired, and worked from there. These rules worked for both left-leaning and right-leaning diagonals, producing different number series on the left and right wings of the triangle. This was done in the following manner.

1. Assign each of the upper elements of the number triangle a place-number, as given in Figure 60.
2. Using the rules given in Table 78, make a sequence of numbers by applying the place-numbers in the rule to the upper elements of the triangle under consideration. Hyphenated numbers are place-numbers. The number 0 means that a 0 should be inserted into the sequence of numbers being made.
3. Use this sequence of numbers with the appropriate additive sequence rule. The desired additive sequence will be produced.

This procedure generates right-wing additive sequences. If one wants a left-wing sequence, one simply reverses the symmetry of the rules given in Table 78. That is, if the rule states that one should take the farthest-right element in the 2nd row, in reversed symmetry, one would simply take the farthest-left element in that same row.

```
        0-1
      1-1 1-2
    2-1 2-2 2-3
  3-1 3-2 3-3 3-4
4-1 4-2 4-3 4-4 4-5
```

Figure 60: Place-numbers for positions of digits in a number triangle

Table 78: Generalized rules for deriving right wing additive sequences from any number triangle

Number	Additive sequence rule	Use this rule on this string as a seed:
1	$A_n = A_{n-2} + A_{n-1}$	0-1, 1-1
2	$B_n = B_{n-3} + B_{n-1}$	0-1, 1-1, 2-1
3	$C_n = C_{n-3} + C_{n-2}$	0-1, 0, 1-1
4	$D_n = D_{n-4} + D_{n-1}$	0-1, 1-1, 2-1, 3-1
5	$E_n = E_{n-4} + E_{n-3}$	0-1, 0, 0, 1-1
6	$F_n = F_{n-5} + F_{n-1}$	0-1, 1-1, 2-1, 3-1, 4-1
7	$G_n = G_{n-5} + G_{n-2}$	0-1, 0, 1-1, 0, 2-1
8	$H_n = H_{n-5} + H_{n-3}$	0-1, 0, 0, 1-1, 0
9	$I_n = I_{n-5} + I_{n-4}$	0-1, 0, 0, 0, 1-1
10	$J_n = J_{n-6} + J_{n-1}$	0-1, 1-1, 2-1, 3-1, 4-1, 5-1
11	$K_n = K_{n-6} + K_{n-5}$	0-1, 0, 0, 0, 0, 1-1

Using these rules, from each of the 12 number triangles whose seeds are given in Table 77, we can generate 11 different right-wing number sequences, and 11 different left-wing sequences from each. In the case of Pascal's (1-1) triangle, the sequences on both wings of the triangle are identical. In all others, they are different. This gives a very large number of additive sequences with which to generate scales.

For this piece, I decided to limit myself to 12-note scales. I also made scales from the limit-ratios of each additive sequence rule, following both the Wilson/Finnamore and Burt methods, given in Chapter 2.7, for turning a limit-ratio into a generator. This generated a family of 276 (mostly) different 12-note scales. In *Proliferating Infinities*, I did not use the scales made with the Burt limits, meaning that only 264 scales were used in this piece.

To sum up, the process by which these scales were made was the following:

1) Make 12 number triangles with generating seeds from 1-1 to 6-5.

2) Derive additive sequences from rules A-K for each of these number triangles.

3) Make scales of harmonics from the first 12 unique elements of each additive sequence of each triangle.

4) Do this for both left and right diagonals, if necessary, generating both left-wing and right-wing scales.

5) Also list the order of generation of the scale degrees. Derive MOS subsets of between 5 and 12 notes for each scale.

6) Make 12-note "limit" scales, by taking the limit-ratio of the additive sequences as a generating interval, and stacking them and octave reducing. Use both Wilson/Finnamore and Burt methods for deriving generators. Take the MOS subsets of these scales as well.

Such a massive set of scales deserves much study. This is obviously a project for the future. However, here are some preliminary findings, based on a first examination:

A) In many of the scale families, the smaller MOS subsets are identical. That is, in say, the 4-1 triangle, nearly all of the MOS subsets of 5 degrees are the same, even if the 12-note scales aren't. This gives possibilities for modulation.

B) There are surprisingly few scales that duplicate each other exactly. There are, however, a number of scales which have only 1 or 2 degrees different than a neighbour.

C) Beginning at the 6-5 triangle, with the left-leaning diagonals, we begin to get scales with no 1/1 in them. That is, the additive sequence does not contain any numbers that are powers of 2. This generates scales without a fundamental. Looking just a little bit further out in the family of triangles and scales, it is speculated that there are whole areas of "atonal" scales, which will not contain lower numbered harmonics.

The complete catalogue of all 276 scales, listing ratios, cents, and their MOS subsets of between 5 and 12 notes, will be found in Appendix 3.11.2. *Scala* *.scl and

*.tun files for each scale are found in Appendix 3.11.3 The algorithmic process applied to the use of these scales in *Proliferating Infinities* will now be discussed in Section 2.11.3.

2.11.3 Algorithm

In Chapter 1.11, a series of additive sequence generators for *ArtWonk* was discussed. Initially, I made these generators to calculate the additive sequences described in Section 2.11.2, but quickly realised they would be useful information generators for musical and computer-animation control purposes and refined their use. They have since been incorporated into the distribution version of the software, along with the other function generators described in Chapter 1.11. Figure 61 shows some of these generators. How they operate will now be discussed.

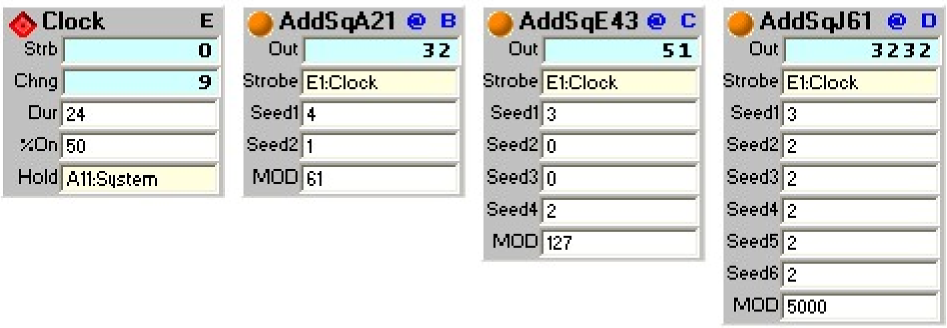


Figure 61: Additive sequence generators for *ArtWonk*

Figure 61 shows a clock module and 3 of the Additive Sequence Generators. The name of the module reflects its generating rule. “AddSqA21,” for example, generates numbers with $A_n = A_{n-2} + A_{n-1}$, while “AddSqE43” uses $E_n = E_{n-4} + E_{n-3}$. as its rule. All 3 modules shown here are driven by the clock module shown at the left of Figure 61. The top blue panel on each module is its output which is dragged to another module’s input for patching.

The white "Strobe" panel below each Output is the clock input. Every time a clock pulse is received here, a new number is generated. The "Seed" panels below that are inputs for the required number of seeds to start the equation. Once the seeds are input, and the equation is started, values advance in a bucket brigade fashion so that new numbers of the sequence are constantly being generated.

The "MOD" input is the number by which the sequence will be divided, modulo N. Low values here will produce short repeating cycles of numbers at the output. Higher values will generally produce longer length sequences. In this piece, the MOD value was set to 61, as I wanted to control pitch over a 5 octave range. This setting produced a 60 element sequence from the AddSqA21 modules, and extremely long sequences for all the other AddSq modules. I do not know the exact length of the sequences produced by the other AddSq modules for a MOD value of 61. I stopped looking for repetitions when each sequence was longer than 1500 elements. For the purposes of this piece, in which each section lasted three minutes, 1500 elements was more than long enough to ensure that there was no repetition heard at the chosen tempi.

In *Proliferating Infinities*, I used different means for making durations, choices of scales, velocities, melodic sequences, and the choices of tempi. Only the main melodic sequence and the scale itself were related through the use of the same additive sequence formula. Although I was tempted to see if I could use the same additive sequence to generate every aspect of the piece, I decided not to use that kind of extension of total serialism here. I've used the technique of deriving all or most parameter values for a piece from a single source many times in the past. In the works discussed in this thesis for example, six of the eleven works discussed use some kind of extension of total serialist techniques. Chapters 2.4, 2.5, 2.6, 2.8, 2.9, and 2.10 all discuss works in which this is the case. Here, however, I was interested in having different kinds of control for each parameter, shaping each to produce results that I felt were intrinsic to the character of the parameter being controlled.

Each three-minute section of the 13-hour composition *Proliferating Infinities* has the same structure; two contrapuntal melodies of harp samples tuned in a specific scale are controlled by the additive sequence generator with the same seed that generated the scale. The rules by which parameters were chosen are now described. The probabilities described below were produced by the "Probability" generator in *ArtWonk*, described in Chapter 2.8, which allows any probability curve, or set of values and their probabilities, to be specified and generated in real time.

Rhythm: There are two identical voices, differing only in their rhythm. **Voice One's** rhythm is a stream of 9, 18 and 36 pulse notes – 9 pulses happen 98% of the time, while 18 and 36 pulse notes occur 1% of the time each. **Voice Two's** rhythm consists of 69% 36 pulse length notes, 15% 48 pulse notes; 8% 18 pulse notes; 6% 60 pulse notes; and 2% 9 pulse notes. This produces an interesting composite rhythm. Voice 1 produces a steady pulse, with its infrequently occurring longer durations creating sub-phrases. **Voice Two**, however, has a more complex rhythm. 69% of the time it has durations of 36 pulses – these are four times as slow as the pulse of line one, giving a sense of "being in four" for most of the piece. Shorter durations of 18 pulses and 9 pulses occur 8% and 2% of the time, respectively in **Voice Two**. However, in the same voice, 15% of the time, notes of 48 pulses occur, while 6% of the time, durations of 60 pulses occur. Neither of these durations is a multiple of 9, and when these occur, the rhythm of **Voice Two** is "thrown off" the pulse of **Voice One**, producing quite abrupt disjunctions of the composite rhythm. There is still a very strong feeling of a driving pulse felt throughout the piece, but it is constantly being pushed and pulled by both the registral breakup, discussed below, and the composite rhythm.

Dynamics: The velocities for each voice are determined by their own random walk generator. It wanders between velocity levels 78 and 108, with a maximum change of +/- 10 levels between each new velocity. There is an equal probability of upward and downward change occurring. Therefore, amplitude is mostly steady, with slight crescendi and decrescendi at times. The small value for maximum change was used to produce the kind of involuntary dynamic variation a live performer might create.

Melody: The melodic line in both voices is produced by switching between different melodic sequence generators. The main generator, occurring 90% of the time, is an additive sequence generator whose seeds determine both the melodic contour and scale architecture of that section of the piece. The other sequences are all either ascending or descending scale passages, articulating the MOS subsets of the current section's scale. Scale / arpeggio passages were chosen because they are a salient feature of much harp music. They are either one of the "idiomatic" elements of harp writing, or are one of the biggest clichés of the genre. I found that I had to be very judicious with these. They are so tradition-laden that only a small amount was necessary to "flavor" the music with the idea of "scale-ness."

The melodic sequences are switched between very frequently. A switch to a new sequence (which might be the same sequence we've just left, if that sequence is picked twice in a row) occurs every 7, 11, 17, 19, 29 or 31 notes. The shorter numbers of notes occur more often than the longer ones. 7 and 11 note long sequences happen 31% of the time each, while 17 and 19 note long sequences happen 16% of the time each, and 29 and 31% note sequences happen only 3% of the time each.

However, while a new melodic sequence might be picked every 7 (say) notes, the probability that the sequence will be the additive sequence is 90%. There is, therefore, much switching to the same sequence. This guarantees that if one of the scale passage sequences is chosen, it will not hang around long – or at least the probability of it persisting for a long time is very small.

The scale passages, however, are just designed as deviations from the main continuity of the piece, which are the two five-octave-spanning melodies produced by the additive sequence generators. I found these very attractive and absolutely fascinating. Due to the wide pitch range, the melody is segmented into several interactive lines based on register.

High notes perceptually aggregate with high notes, low with low, and all sorts of melodic phrases and harmonic implications result from the "extraction" from a continuous stream that is produced by our ears. These "resultant phrases" are continually surprising. Not only are composite melodies produced, but the basic pulse of the piece is also continually offset by the continually changing different lengths of time the melody stays in each register.

This kind of auditory streaming is an old trick; characteristic of, for example, Bach solo partitas, and the solo instrumental works of Milton Babbitt. The most interesting investigation of this effect was Robert Erickson's *Loops* project, which I was privileged to observe for all of its 9 months (Erickson 1975). Breaking a wide-ranging melody up into different registers may be an old trick, but it was very useful here in allowing many of the characteristics of my new scales to be heard.

2.11.4 Process – Real Time Usage

The process outlined above chooses pitches, durations, and dynamics for a two-voice texture which explores the scales described in Section 2.11.2. Each of the 264 three-minute sections of the piece is made in this way. The sections are fixed into a particular order organized into a larger hierarchical structure in the following way:

A. **First level: The CD.** The largest level consists of 12 CDs. Each CD explores all the scales that come from a particular number triangle.

B. **Second level: The Large Piece (2 per CD)**. There are 2 Large Pieces on each CD. Each Large Piece is 33 minutes long. Each number triangle has left-wing and right-wing scales. Each of the 2 Large Pieces on a CD explores either all the left-wing scales or all the right-wing scales of its number triangle. There are 24 Large Pieces in the entire composition.

C. **Third level: The Sections.** Each set of left-wing or right-wing scales has 11 different scales. Each Large Piece is made up of 11 sections, each three minutes long, each using one of the 11 different scales. For example, on the CD devoted to the 4-3 triangle, each section of Large Piece 1 will explore right-wing scales A–K in alphabetical order, one per section, while each section of Large Piece 2 will explore left-wing scales A-K, also in alphabetical order, one per section. Again, there are 264 three-minute sections in the entire composition.

D. **Overall tempo changes.** Each three-minute section is in a particular tempo. Over the course of the 11 sections of a Large Piece, the tempo always gets slower. The basic clock pulse for the first section is always 20 ms. For each section, it gets 1 ms slower, until the basic clock pulse for the final section is 30ms.

E. **Moment to moment details.** Each section is produced by the *ArtWonk* patch described in Section 2.11.3. This patch controls two voices, picking pitches, durations and dynamics as described above. The same additive sequence generator is used for both voices, but **Voice Two** is, on average, ¼ the tempo of **Voice One**; the result, despite random interruptions from scale passages, is that each section is in the form of a pseudo-canon.

F. **Timbre:** The piece uses modified harp samples from S. Christian Collins *General User 1.4* public-domain set of instrumental samples (Collins 2000). These are played in *VSampler3* and are tuned in the additive sequence scales via the use of 12-note *Scala* files, which *Vsampler3* accepts easily. This is a very good harp timbre – clear, bright, and well recorded. The very pleasant nature of the harmonies in the piece is helped by the harp timbres. These have a long decay which sustains, but not at high amplitude, so that some of the dissonant intervals present in the scales (or even the unfamiliar ones, like 11/8) don't get a chance to grind on, as they would with sustained timbres. That being said, I have played these scales with many different timbres, and have found that if a timbre is fairly attractive on its own, most of these scales will sound attractive. I have not, to date, tried to construct timbres which mirror the structure of the scales, as proposed by William Sethares (Sethares 2005), to produce a greater match between timbre and tuning.

2.11.5 Thoughts after listening

While recording the piece over the six weeks of its making, I found the harmonic and metric modulations that occur at the joins between the sections very striking. The metric modulations are very noticeable – despite my doing everything possible to obscure the "joins" between the sections. Even though each change is slight (the tempo slowing down by 21/20, or 30/29, or some other superparticular ratio between them) the section transitions are really noticeable. This is because of the constant presence of pulse. Despite a seemingly infinite variety of micro-rhythms generated by the melodic architecture, and each section's slowing down process, pulse is so prevalent that any change in it is strongly perceived.

Harmonic changes are also felt, but a little more slowly. Still, within 10 seconds of entering a new scale, unless the difference between the two adjacent scales is small (and it usually isn't) it is quite obvious that a new harmonic world has been opened up.

I find I've written an extremely long piece, but so far, I have listened to a maximum of one 33 minute Large Piece at a time. It's beautiful and fascinating to hear all those patterns and tunings. However, it ***is*** hard work, and pushes the threshold of listening concentration. Perhaps with repeated hearings, my endurance will increase.

This music does not work as background for driving. This is not the humorous statement it may first appear. On a recent trip to Melbourne, we tried playing one of the CDs on the car stereo. After about 15 minutes we took it off – it was too soporific, and perhaps contradictorily, the pulse was too distracting, to be good as driving music – we felt like we were simultaneously being put to sleep, and pushed around by the music – not a good state to be in while driving.

The piece was meant to be an exploration of a possible infinity, but it takes great work to "last the distance." The attention to detail needed to have this "glimpse of infinity" is lost in a "background music" context. This piece, despite its enormous time demands, is unashamedly "foreground music." This does not bother me. No one ever said the conscious apprehension of infinity was going to be easy.

With 13 hours, I realise I've gone beyond reasonable expectations of just about any person's attention span. Most "infinite" or "ambient" pieces exist in excerpt or conceptually, or even by reputation alone. An exception to this is La Monte Young's "Dream House" pieces, which I've been privileged to visit many times, each time staying for as long as I could (Gann 1996). It is my hope that I can find some similar context for this piece, somewhere where it can "work" as well as Young's drones do in his environment.

Practicality, then, makes me think of this piece as living best in some kind of installation context. However, the installation, as currently practiced in the visual arts world, is an inappropriate venue for the kind of focused attention this work requires. As my wife Catherine said:

> It is ***NOT*** an art-gallery installation! It's an object of contemplation! After all that work, do you want people to walk through it with glasses of champagne?

Clearly, although the work is "environmental," it can not work as "ambience." When composing the work, I didn't treat it as background. Over the six weeks of it's composing, I listened very carefully to each section as it was generated. Now, when listening, I'm find I'm still very tuned into the details of its making. It may be a while before I can hear the piece in a more global manner. This kind of global listening, while not ignoring the details of the piece, may allow one to listen to the work for longer time spans, so that the larger scale harmonic changes from one number triangle to another might become evident.

A concluding anecdote: In October 2006, I showed my friend Chaitanya Kabir the process by which I was generating the harp etudes. He, a Just-Intonation theorist, and virtuoso performer of Indian music, said, "Well, when you find one that you think is really beautiful, play it to me." That was a good and normal reaction to the idea of a process generating a multitude of outputs from which the artist, using their taste, makes a selection.

But this piece is not about that. This piece is about listening unflinchingly to infinity – about hearing many many scales, and counterpoints made with the sequences that generated those scales, and listening for the differences between the scales, savouring the different harmonic qualities that exist, while realizing that an infinity of other flavours also exist inherent in the process.

2.12 Conclusions and Future Directions

2.12.1 Conclusions

It would be tempting, though disingenuous, to say at this point that there are no conclusions. Serious work was done; work needs to be heard seriously. However, in a time when parsimony seems to be the rule, I hope that this thesis will stand as a counter-example of abundance. I would like my work to be viewed as a gift to the profession, to the world; a gift not only of individual pieces, but also of models for kinds of music, and a wide variety of structural resources, and tunings.

I still remain engaged in the search for a context for this music. On-line presence will certainly be used and developed, but I feel an urgency for a communal place for this music to live, even if only with a few interested friends. It's only in person-to-person exchange that music lives for me. This of course is a contradiction, especially since several of these pieces: *The Animation of Lists, 18 New Fuguing Tunes for Henry Cowell, Someone Moved in a Room,* and *Proliferating Infinities*, are conceived of as recordings. This music *exists* on recordings, but *lives* in the situation of live presentation. This problem needs to be addressed on my personal, compositional, performative level, and also in the communities in which we live and work. I am not enamored of the ideal of the isolated listener engaging music via downloads as music's ultimate fate. It's one option, but it's not the whole story. Music as iPod fodder is not an ideal to which I can subscribe.

On listening to these works, they certainly prove to me that these methods of generating microtonal scales produce attractive and engaging harmonies and timbres. I'm also convinced that the kinds of algorithmic structuring I use produce work with engaging structure. This work has expanded my musical perceptions, but not as much as it might have. Perhaps I'm already at the state where I can listen to anything and find it meaningful. The quest now is not to understand how others listen and to make my work conform with that, the quest is to *help others attain the open-minded listening state.*

2.12.2 Future Directions

Some areas of work I would like to pursue:

- More exploration of the probability distributions I made for *ArtWonk* and the new function tools just added to the program in January 2007.
- More work with the Logos Foundation's robotic instruments, preferably at Logos in Gent, where I can physically engage with the instruments, and work with them in a truly interactive manner.
- More works that combine my "instrumentarium" – my harmonic canon, tuning forks, balloon gongs, microtonal guitars, repaired microtonal accordion, toy pianos, electro-acoustic percussion boards, etc. – in live performance works. More development of new microtonal acoustic instruments, such as the 72-tone guitar, and a set of metallophones tuned to Harry Partch's scale, both to be constructed in the first half of 2007. Although designed as recordings, my recent works *Illawarra Raga* and *Three Watermoods* point in the direction I would like these works to take.
- More work with interactive performance using alternate controllers. Adaptation of photocell-based controllers, along the lines of Percy Grainger's *Electric Eye Tone Tool*, to control microtonal pitch complexes.

- Continuing engagement with Ervin Wilson's work. His recent work with the Co-Prime Grid holds out some tantalizing clues to possible musical structures. It would be worthwhile to see where these clues lead.

And, on a more frivolous and personal note:

- More work with music that has nothing whatever to do with pitch organization. It's time to deal with texture, timbre, and sonic found objects once again.
- More environmental musical work. Less work which keeps one chained to a computer. *Less cyberculture, more permaculture!*
- I definitely want to do less academic-style writing, and more writing ala the "mind-blowing splatt" "laying it out straight with canary and sidemen" of Kenneth Gaburo's *LA* and Benjamin Boretz' *Language, as a Music* (Gaburo 1987, Boretz 1979).
- More time for walking in the woods, not being concerned with any music.
- More sleep.

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