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Solitary wave interaction and evolution

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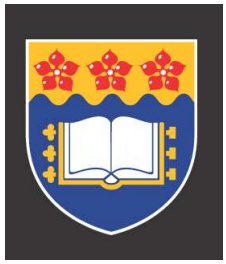
B.Sc., M.Sc. Mathematics

A thesis submitted in fulfilment of the requirements for the award of the degree of Doctor of Philosophy



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Dedicated to

My wife, Amir Ali, newborn Fatemeh Sadat,
my father and my mother.

In The Name of Allah

Abstract

Asymptotic theory is applied to examine solitary wave interaction for three higher-order model equations, which represent small perturbations to integrable equations. The higher-order equations considered are the higher-order Nonlinear Schrödinger equation and the focusing and defocusing higher-order Hirota equations. The asymptotic theory, which involves a transformation, allows the straightforward determination of parameter choices, for which the higher-order equations are asymptotically integrable, and of the higher-order phase and coordinate shifts due to the collision, in the asymptotically integrable cases. For the higher-order Hirota equations, direct soliton perturbation theory is also used, to determine the details of the evolving solitary waves; in particular analytical expressions are found for the solitary wave tails.

An important feature of the asymptotic and perturbation theories is that they allow cross-validation of the theoretical results and also allow families of asymptotic embedded solitons to be identified.

Numerical solutions of the governing equations are also obtained. For solitary wave interaction, asymptotically elastic and inelastic cases are considered. When the higher-order coefficients satisfy the appropriate algebraic relationship then the numerical results confirm the prediction of the asymptotic theory. Numerical solutions for evolving solitary waves are also used to confirm the results of the soliton perturbation theory.

Declaration

I, Sayed M. Hoseini, declare that this thesis, submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy, in the school of Mathematics and Applied Statistics, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. The document has not been submitted for qualifications at any other academic institution.

Sayed M. Hoseini

June 2007

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List of publications

The following publications have been published by the author during his studies.

[**1**] S. M. Hoseini and T. R. Marchant. Solitary wave interaction and evolution for a higher-order Hirota equation. *Wave Motion.*, 44:92-106, 2006.

[**2**] B. Whiten , M. McGuiness and S. M. Hoseini. Sustainable water management in the minerals industry. *Proceedings of the Mathematics-In-Industry-Study Group.*, ISBN 0-473-11068-7, 2006.

[**3**] S. M. Hoseini and T. R. Marchant. Solitary wave interaction for a higher-order nonlinear Schrödinger equation. *IMA Journal of Applied Mathematics*, 72:206-222, 2007.

[**4**] S. M. Hoseini and T. R. Marchant. Gray soliton interaction and evolution for a higher-order Hirota equation. to be submitted.

[**5**] S. M. Hoseini and T. R. Marchant. Soliton perturbation theory on a higher-order Hirota equation. submitted to *Mathematics and Computers in Simulation*, submitted, 2007.

Papers 1, 3, 4 and 5 are included in this thesis. Papers 1 and 5 are Chapter 3, paper 3 is Chapter 2 and paper 4 is Chapter 4.

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