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Geometry of belief

Shiyan Li
University of Wollongong

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Geometry of Belief

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Shiyan Li

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Dedicated to

Xiaofan Li and Linghua Li

Declaration

This is to certify that the work reported in this thesis was done by the author, unless specified otherwise, and that no part of it has been submitted in a thesis to any other university or similar institution.

Shiyan Li
May 29, 2008

Abstract

Usually, the researchers of traditional belief change theories (e.g., AGM theory) assume that the knowledge of the agents which have the lower priorities should fully accept the knowledge of those higher priority ones in the process of belief revision. These kinds of theories are called prioritized belief change theories. On the contrary, in the discussion of non-prioritized belief change theories (e.g., Konieczny and Pino-Pérez's merging theory), the belief changes happen among the agents which have the same priorities. In this dissertation, we provide a new style of epistemic states and the belief change operations on this kind of epistemic states such that the prioritized or non-prioritized characteristics of belief change operators will be determined only by the properties of agents' knowledge.

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Chapter 1

Introduction

A rational agent has been defined as "for each possible percept sequence, a rational agent should select an action that is expected to maximize its performance measure, given the evidence provided by the percept sequence and whatever built-in knowledge the agent has" [47]. According to the definition, the intellection of a rational agent is mainly about acquiring, comparing, revising and merging the knowledge rationally. Therefore, the method of how the agents' knowledge would be represented and changed are two crucial aspects of modelling the agents' mental states. In this dissertation, we will utilize a new method of knowledge representation to carry the operations of belief change.

Inasmuch our research focuses on the agents' mental states, we simplify the models of agents to the models of information sources, i.e., different agents are regarded as independent information sources which provide their knowledge to the others. By "independent information sources", we mean that the knowledge included in one information source is not a partial knowledge of the relevant agent or a combination of the mental states of more than one agent, but belongs to exactly one agent and includes the entire content of this agent's knowledge. We hereby put a group of such independent information sources under our attention. Henceforward, when we mention the agents, we mean the independent information sources.

Since the early 1980's, belief change has been studied in both computer science and philosophy. In our opinion, the problems of belief change can be categorised into two branches based on the priorities of agents:

- The agents have different priorities: in traditional revision theory [1], it is one of the basic assumed premises in which the revision operation should be taken place. Traditionally, the knowledge of the agents which have the lower priorities should be revised by the knowledge of those higher priority ones, and the knowledge of the latter agents should be fully accepted in the result.
- The agents have the same priorities: this is the area of belief merging operations [29] or so-called non-prioritised belief revision [24], i.e., there is no necessary that the knowledge of one agent should be fully accepted by the others.

Therefore, one important problem is that how do the priorities of agents are assigned. In this dissertation, we will introduce a new knowledge representation in which the priorities of agents will be assigned fairly and rationally. Then the above two cases can be treated as one unified process such that we can use only one belief change operator to deal with all situations, i.e., we will banish the borderline between the belief change operations on prioritised and non-prioritised agents. In other words, the prioritised or non-prioritised characteristics of belief change operators will be determined only by the properties of agents' knowledge.

1.1 Priorities between Agents

"The passion of men for equality is ardent, insatiable, eternal, invincible" (de Tocqueville, 1860). In a narrow sense, this opinion is a slogan of egalitarianism in which it amounts to allocate equal rights to individual agents [43]. In a broad sense, it can be understood as that every agent is born without any congenital priorities, and the postnatal efforts of agents should be taken fairly. In other words, the agents can strive for higher priorities through their own hard work but not any god-given authorities. Since our agents are equal to the independent information sources, the ”hard work” means the efforts of deliberation. We assume that each agent will faithfully expose its efforts and history of deliberation.

Suppose that we are a group of rational people who are assigned to solve a specific problem. We will trust those who are called experts among us like this example:

Example 1.1.1. In a discussion of a specific subject, people always tend to accept the opinions of those who are the experts in this subject. Usually, the experts are those who have spent more work on some subjects than the other people (we admit that there exists some geniuses who can be professionals in some fields but need not to do much work). In other words, the experts have more mature points of view on some subjects than the other people. Therefore, we can say the more mature points of view have higher priorities than the less mature points of view.

From the above example, although we are born equally, we trust some people more than the others in the end. In other words, some people's knowledge get higher priorities than the others' following the deepening of their deliberation, and so do the agents. We have another example from which can explain that we sometimes trust something which looks like not that authoritative:

Example 1.1.2. When we try to solve some primitive arithmetical problems, we generally trust a calculator more than our brains. Seemingly, the calculator is just a machine which absolutely does not think more than human brains. On the contrary, the calculator should be treated as an expert of primitive arithmetical problems because it is a product of the efforts of its researchers, designers and workers. In other words, we trust the calculator because it is a convergency of the knowledge of its researchers, designers and workers.

Therefore, a group of rational agents can be prioritized based on their explicite (the example 1.1.1) or implicite (the example 1.1.2) degrees of deliberation. In other words, agents' knowledge states should imply the priorities of the relevant agents.

The agents begin to acquire and process the knowledge along with putting them in operation. In the life time of an agent, its priority is adjusted continually through the change of its knowledge. Hence, we can develop a method to embed the agent's priority into its knowledge representation.

1.2 Smoothness of Belief Change

We call the representations of agents' knowledge states (or belief states) plus their embeded information of priorities the epistemic states of agents. By reason of banishing the borderline between the belief change operations on prioritised and non-prioritised agents, it is necessary to build some belief change operators which can produce different results based on the priorities of the input epistemic states, e.g., if we can smoothly change the priority of one input epistemic state x from extremely low to extremely high, then the result of revision operation should be smoothly modified to be closer and closer to x, till the result completely accept x.

Example 1.2.1. Assume that Bob is a student who focuses on the subject of knowledge representation, and he decides to take a bachelor's degree first, then continue to a master's degree, and finally achieve the PhD. Thus, in the process of his study, his knowledge of his subject is improved from a layman level to a professional level bit by bit. Obviously, by following this improvement, Bob's opinions on the subject of knowledge representation get more and more creditability among the academia. Therefore, when Bob becomes the greatest name on knowledge representation, almost everyone tends to accept points of view from Bob on the subject.

Because we treat the prioritised and non-prioritised belief change as one unified process, it is unpractical to evaluate the result of belief change by testing whether the result entail some specific information, e.g., the (Success) postulate in AGM framework. Fortunately, this problem can be addressed by introduce the concept of distance between the epistemic states. The shorter the distance between two epistemic states is, the closer are those two epistemic states. Then the mission of belief revision operators are to close up the gap between two epistemic states (like the bridge building analogy in [7]).

The problem of how to design the distance functions is complex in our circumstance. The agents' priorities and the content of knowledge should be considered simultaneously while we try to calculate the distance between two epistemic states.

1.3 Organization of the Thesis

In the following chapters, we will first provide a brief background introduction of the area of belief change in Chapter 2. Then the framework of a specific kind of epistemic states, which are embeded with the priorities of information sources, will be proposed in Chapter 3. Chapter 4 will introduce an instance, which is based on n-dimensional analytic geometry [21], of our epistemic states and the belief change operations on them. The thesis will be concluded in Chapter 5. In the end, some proposed future work will be listed in the last chapter.

Chapter 2

Background

The discussion of belief change among contemporary researchers can be retrospected to the end of 1960's [33], and some of Isssac Levi's works [34, 35, 36, 37, 38] have motivated many present research topics on belief change. Since the early 1980's, the AGM theory [1] has graduated into the most popular theory of belief change. In [14, 17], we can find the beginning of the adherence of success (Section 2.2.1, (AGM∔2)) by which the prioritized belief change has been separated. Then, in 1990's, many works on non-prioritized belief change [41, 45, 48] ,which argue on the necessity of success, have been published (see [24] for more details). And the binary merging operators in [7] and the multiple merging operators in [26, 28, 29, 30, 31] are some remarkable works which have been done recently in the non-prioritized belief change field. For fusing the prioritized and non-prioritized belief change into one unified process, some background information will be provided in this chapter.

2.1 Preliminaries

When we mention logic in this thesis, we mean classical logic. Suppose $\mathcal{L}$ is a finitely generated propositional language which is closed under the usual propositional connectives (conjunction: $\wedge$, disjunction: $\vee$, negation: $\neg$). Throughout this dissertation, we define $W = \{w_1, w_2, \ldots, w_n\}$ the finite set of all possible worlds and $W \neq \varnothing$. And we also define $\mathcal{B}$ the set of all subsets of W, i.e., $\mathcal{B} = 2^W$. Then, any proposition $A \in \mathcal{B}$, i.e. that a proposition can be any subset of W. Obviously, we have $A \wedge B = A \cap B$, $A \vee B = A \cup B$ and $\neg A = \overline{A}$ ($\overline{A}$ means the complementary set of A) for all $A, B \in \mathcal{B}$. A belief set $\mathcal{K} \subseteq \mathcal{B}$ is a consistent and deductively closed set of propositions, i.e., $\bigcap \mathcal{K} \neq \varnothing$, and $A \in \mathcal{K}$ iff $\bigcap \mathcal{K} \subseteq A$. We also defined $\mathcal{C} = \bigcap \mathcal{K}$ the net content of the belief set $\mathcal{K}$. Thus, the belief set can be represented also by $\mathcal{K} = \{A | \mathcal{C} \subseteq A\}$. $\mathbb{R}$ and $\mathbb{I}$ are respectively the set of all real numbers and the set of all integers.

A preorder r on a set E is defined as a reflexive and transitive relation. r is total if for all $e, e' \in E$ ere' or $e're$. Moreover, $\min_r(E) = \{e | ere' \text{ for all } e', \text{ and } e, e' \in E\}$, $\max_r(E) = \{e | e're \text{ for all } e', \text{ and } e, e' \in E\}$. Usually, the output of min or max is a set. We define $\min(E) = \min_r(E)$ and $\max(E) = \max_r(E)$ for short if $E \subseteq \mathbb{R}$ and $r = \leq$. In this situation, we define the output of min or max is a value, e.g., $\min(\overline{\mathbb{I}^-}) = \{0\} = 0$.

Let $V = \langle v_1, v_2, \ldots, v_n \rangle$ and $U = \langle u_1, u_2, \ldots, u_n \rangle$ be two n-dimensional vectors in the Euclidean n-space. Then we have $V + U = \langle v_1 + u_1, v_2 + u_2, \ldots, v_n + u_n \rangle$. If $k \in \mathbb{R}$, then $k \cdot V = \langle k \cdot v_1, k \cdot v_2, \ldots, k \cdot v_n \rangle$. Moreover, we define $\min(V) = \min(\{v_1, v_2, \ldots, v_n\})$ and $\max(V) = \max(\{v_1, v_2, \ldots, v_n\})$.

A multi-set is the set in which the elements are not necessarily different. We denote a multi-set as $X = [x_1, x_2, \ldots]$. Two multi-sets $X = X'$ iff for every element x of X there is a unique element x' (position-wise) of X' such that $x = x'$ and for every element x' of X' there is a unique element x (position-wise) of X such that $x' = x$. $\sqcup$ is the union of multi-sets, i.e., $X \sqcup X' = [x_1, x_2, \ldots, x'_1, x'_2, \ldots] if X = [x_1, x_2, \ldots] and X' = [x'_1, x'_2, \ldots]$. The union of n same multi-sets is denoted as $X^n = X \sqcup X \sqcup \ldots \sqcup X$. If $MX = [X_1, X_2, \ldots]$ is a multi-set of multi-sets, then $\bigsqcup MX = X_1 \sqcup X_2 \sqcup \ldots$ which means the union of all elements in MX.

2.2 AGM Belief Change and Its Ramifications

The AGM theory has included the postulates of belief change operators and the method of how to distinguish the importances of the objects of belief. Spohn's theory of ordinal conditional functions has quantitatively embeded epistemic entrenchment into epistemic states. Since we tend to embed the agents' priorities into epistemic states, these two theories are immensely important for us.

2.2.1 AGM Belief Revision

In AGM theory, belief revision operators are defined by a set of postulates which are called AGM postulates. The most direct motivation for AGM postulates is that we want to make a minimal change when we try to revise our old beliefs by some new sentences [1], i.e., we want to keep our old beliefs as much as possible.

Suppose $\mathcal{K}$ is a belief set and A, B are two sentences. A basic revision operator $\dot{+}$ is characterized by the following postulates:

(AGM$\dot{+}$1) $\mathcal{K}\dot{+}A = \{B | \bigcap(\mathcal{K} \dot{+} A) \subseteq B\}$. (Closure)

(AGM$\dot{+}$2) $\mathcal{C}_{\mathcal{K}\dot{+}A} \subseteq A$. (Success)

(AGM$\dot{+}$3) $\mathcal{C}_{\mathcal{K}} \cap A \subseteq \mathcal{C}_{\mathcal{K}\dot{+}A}$. (Inclusion)

(AGM$\dot{+}$4) If $\mathcal{C}_{\mathcal{K}} \cap A \neq \varnothing$, then $\mathcal{C}_{\mathcal{K}\dot{+}A} \subseteq \mathcal{C}_{\mathcal{K}} \cap A$. (Vacuity)

(AGM$\dot{+}$5) $\mathcal{C}_{\mathcal{K}\dot{+}A} = \varnothing$ iff $A = \varnothing$. (Consistency)

(AGM$\dot{+}$6) If $A = B$, then $\mathcal{C}_{\mathcal{K}\dot{+}A} = \mathcal{C}_{\mathcal{K}\dot{+}B}$. (Extensionality)

(AGM$\dot{+}$1) and (AGM$\dot{+}$5) ensure that the outputs of the revision operator are the consistent belief sets except the input sentences are inconsistent. The success property of $\dot{+}$ is described by (AGM$\dot{+}$2), i.e., the input sentence A should always be accepted by the output belief set $\mathcal{K}$. Therefore, (AGM$\dot{+}$2) is also a property which defines the AGM revision operators as the prioritized operators. Then (AGM$\dot{+}$3) and (AGM$\dot{+}$4) denote that if the input sentence A is consistent with the old belief set $\mathcal{K}$, then the output should be exactly the conjunction of A and $\mathcal{K}$. Finally, (AGM$\dot{+}$6) means that if we try to revise a belief set by two semantically tantamount inputs, then the results should be identical. If an operator satisfies (AGM$\dot{+}$1)-(AGM$\dot{+}$6), then we call this operator the basic revision operator.

The following two postulates are called the supplementary postulates of basic revision operators:

(AGM$\dot{+}$7) $\mathcal{C}_{\mathcal{K}\dot{+}A} \cap B \subseteq \mathcal{C}_{\mathcal{K}\dot{+}(A\cap B)}$. (Superexpansion)

(AGM$\dot{+}$8) If $\mathcal{C}_{\mathcal{K}\dot{+}\mathcal{A}} \cap B \neq \varnothing$, then $\mathcal{C}_{\mathcal{K}\dot{+}(A\cap B)} \subseteq \mathcal{C}_{\mathcal{K}\dot{+}A} \cap B$. (Subexpansion)

These two postulates ensure that if the result of the revision of a belief set $\mathcal{K}$ is $\mathcal{K} \dot{+} A$ and $\mathcal{K} \dot{+} A$ is consistent with another sentence B, then the revision of $\mathcal{K}$ by $A \cap B$ is identical with the conjunction of $\mathcal{K} \dot{+} A$ and B.

AGM theory also includes the set of postulates for contraction operators; however we do not want to list them here because they will not be refered in the rest chapters. A contraction operator is denoted $\dot{-}$ such that $\mathcal{K}\dot{-}A$ means to contract the sentence A from the belief set $\mathcal{K}$. In addition, the revision operator $\dot{+}$ and the contraction operator $\dot{-}$ satisfy the Levi Identity and the Harper Identity:

(LI) $\mathcal{C}_{\mathcal{K}} \dot{+} A = (\mathcal{C}_{\mathcal{K}}\dot{-}\overline{A}) \cap A$. (Levi Identity)

(HI) $\mathcal{C}_{\mathcal{K}}\dot{-}A = (\mathcal{C}_{\mathcal{K}} \dot{+} \overline{A}) \cap \mathcal{C}_{\mathcal{K}}$. (Harper Identity)

Thus we can define a revision operator by a contraction operator and vice versa.

2.2.2 Epistemic Entrenchment

If we have two choices of contraction when we intend to contract some information from a belief set $\mathcal{K}$, i.e., we must give up the sentence A or B to form a revision or a contraction, then we need to know that which one of A and B will bring us the lowest losing of information after it has been given up [16]. Thus, the idea of epistemic entrenchment has been motivated in [15, 18], hence the sentences with the lowest degree of entrenchment should always be given up first.

Suppose we have two sentences A and B and a belief set $\mathcal{K}$. $A \preccurlyeq B$ means that B is at least as epistemically entrenched as A. We have the following postulates for the epistemic entrenchment $\preccurlyeq$:

(EE1) If $A \preccurlyeq B$ and $B \preccurlyeq C$, then $A \preccurlyeq C$. (Transitivity)

(EE2) If $A \subseteq B$, then $A \preccurlyeq B$. (Dominance)

(EE3) $A \preccurlyeq A \cap B$ or $B \preccurlyeq A \cap B$ for any A and B. (Conjunctiveness)

(EE4) When $\mathcal{K} \neq \varnothing$, $A \notin \mathcal{K}$ iff $A \preccurlyeq B$ for all B. (Minimality)

(EE5) If $B \preccurlyeq A$ for all B, then $A = W$. (Maximality)

The justification of (EE1) is trivial. (EE2) means that if we want to remove a sentence from a belief set, then we must remove all the logically stronger sentences first. (EE3) ensures that one needs only to remove A or B if the deletion of $A \cap B$ is required. And also, (EE3) reveals that the epistemic entrenchment $\preccurlyeq$ is a total pre-order. The rational for (EE4) is that all sentences which are not in $\mathcal{K}$ should hold the same degree of entrenchment and the less degree of entrenchment than the sentences which are in $\mathcal{K}$. And (EE5) states that the tautology should have the top degree of entrenchment. By the way, there are some methods to measure the information quantities of the sentences in logic systems, e.g., [39]. Maybe, they can be used to form some diverse kinds of epistemic entrenchment.

2.2.3 Ordinal Conditional Functions

Wolfgang Spohn has developed a very different but useful knowledge representation, which is called the ordinal conditional function, and its dynamic mechanisms in [49]. The ordinal conditional functions generalize the possible worlds models [20, 25] such that the epistemic states include not only the content of sentences, or, propositions, but also the degrees of belief. In other words, an ordinal conditional function implies a belief set and the epistemic entrenchment on this set [16].

According to Spohn's theory, an ordinal conditional function κ is a function from W into the set of ordinals which is the set of all nonnegative integers $\overline{\mathbb{I}^-}$, i.e., $\kappa : W \rightarrow \overline{\mathbb{I}^-}$, such that there is at least one element $w \in W$ which satisfies $\kappa(w) = 0$. Each ordinal conditional function implies a plausibility grading of the possible worlds. Moreover, for each proposition $A \in \mathcal{B}$, the ordinal of A is defined as the minimal ordinal of the possible worlds included in A, i.e., $\kappa(A) = \min(\{\kappa(w)|w \in A\})$. Thus, we have the following two properties of the ordinal conditional function $\kappa(w)$:

(κ1) $\kappa(A) = 0$ or $\kappa(\overline{A}) = 0$ for all $A \in \mathcal{B}$.

(κ2) $\kappa(A \cup B) = \min(\{\kappa(A), \kappa(B)\})$ for all $A, B \in \mathcal{B}$.

By taking each ordinal conditional function $\kappa(w)$ as a deterministic (not probabilistic) epistemic state, the net content of $\kappa(w)$ includes all possible worlds which have the ordinal 0, i.e., $\mathcal{C}_\kappa = \{w|\kappa(w) = 0\}$. Then A is believed or disbelieved in κ iff, respectively, $\kappa(\overline{A}) > 0$ or $\kappa(A) > 0$. We say that A is neutral in κ iff $\kappa(A) = 0$ and $\kappa(\overline{A}) = 0$. Moreover, the firmness α of a proposition A relative to κ is defined as:

$$\alpha_A^\kappa = \begin{cases} \kappa(\overline{A}) & \text{if } \kappa(A) = 0 \\ -\kappa(A) & \text{if } \kappa(A) > 0 \end{cases}$$

Thus, A is believed or disbelieved in κ iff, respectively, $\alpha_A^\kappa > 0$ or $\alpha_A^\kappa < 0$. And A is neutral in κ iff $\alpha_A^\kappa = 0$. Based on the concepts of epistemic entrenchment in Section 2.2.2, $B \preccurlyeq A$ iff $\kappa(A) < \kappa(B)$ for all $A, B \in \mathcal{B}$.

The belief change on ordinal conditional functions is called the conditionalization of the ordinal conditional functions. Let α be an ordinal, $A \in \mathcal{B}$ and κ an ordinal conditional function. The A, α-conditionalization of κ is defined as:

$$\kappa_{A,\alpha}(w) = \begin{cases} -\kappa(A) + \kappa(w) & \text{if } w \in A \\ \alpha - \kappa(\overline{A}) + \kappa(w) & \text{if } w \in \overline{A} \end{cases}$$

$\kappa_{A,\alpha}$ is also an ordinal conditional function.

Therefore, the A, α-conditionalization of κ intends to shift the ordinal of A to be 0 and the ordinal of $\overline{A}$ to be α. In other words, the A, α-conditionalization of κ means the epistemic state κ is revised by A with the parameter α, and A is believed in $\kappa_{A,\alpha}$ with firmness α. It can be shown that this kind of belief revision is a generalized version of AGM revision [16].

2.2.4 Distance Based Revision

Another interesting branch of AGM theory is the so-called distance based revision which has been introduced in [3, 32], and then the infinite version in [4]. We mention this theory just because a sort of distance functions will be introduced in the following chapters. In the distance based revision theory, a distance functions is actually a pseudo-distance (with triangle inequality omitted) between any two models. By comparing it with our theory, the readers will find that our distance functions are facing a much more complex situation in which we should measure the distance between any two epistemic states. Therefore, our distance functions should have the ability to provide a synthetical measurement of the distances between belief contents and the distances between agents' priorities.

2.3 Non-Prioritized Belief Change

In fact, the so-called non-prioritized belief change is not entirely that the belief change happens between the non-prioritized agents. Usually, when a belief change theory denies (AGM$\dot{+}$2), we classify it into the non-prioritized belief change. Richard Booth's belief negotiation models provide us the confidence to build a framework to weld the prioritized and non-prioritized belief change. And, Sébastien Konieczny and Ramón Pino-Pérez's postulates of belief merging operators can be an appropriate start point to build our own framework.

2.3.1 Negotiation-Style Framwork for Revision

Richard Booth has made a good analogy of non-prioritized belief change on two belief sets in 2001: If $\mathcal{K}, \mathcal{K}' \subseteq \mathcal{B}$ and $\mathcal{C}_{\mathcal{K}} \cap \mathcal{C}_{\mathcal{K}'} = \varnothing$, then there is an imaginary gap between A and B. The duty of belief revision operations is to build a bridge which straddle on this gap. Therefore, the problem of belief revision is then converted into the problem of how to build this bridge. The suggestion of "non-prioritized belief change builders" is to build the bridge from both sides, i.e., $\mathcal{K}$ and $\mathcal{K}'$, simultaneously.

A framework of belief negotiation models for non-prioritized belief change has been introduced in [7]. The framework assumes that the equally prioritized owners of $\mathcal{K}$ and $\mathcal{K}'$, while $\mathcal{K}, \mathcal{K}' \subseteq \mathcal{B}$, do some kind of negotiation during the process of belief revision that in each round of negotiation, the loser makes some kind of concession on its belief set, i.e., expands its net content, until the intersection of the net contents of two owners is not empty, i.e., $\mathcal{C}_{\mathcal{K}_n} \cap \mathcal{C}_{\mathcal{K}'_n} \neq \varnothing$ in which $\mathcal{K}_n$ and $\mathcal{K}'_n$ are respectively the belief sets of the owner of $\mathcal{K}$ and the owner of $\mathcal{K}'$ after the nth round of negotiation. And then the result of belief revision should be $\mathcal{C}_{\mathcal{K}_n} \cap \mathcal{C}_{\mathcal{K}'_n}$. After this work, several variation of the theory has been developed. In [26], the framework of belief negotiation models has been used to develop a new family of belief merging operators. And [46] has introduced a new version of negotiation-style framwork on possibilistic logic.

We will not list the formalization of belief negotiation models, because we will not refer the detail information of this framework in the rest of thesis. A very important conclusion for us is that the belief negotiation models will degenerate to the AGM models when one of the negotiators always be the winner. This conclusion proofs that the prioritized belief change and the non-prioritized belief change can be studied in the same framework.

2.3.2 Belief Merging

Sébastien Konieczny and Ramón Pino-Pérez have proposed a framework of belief merging operators which tend to combine multiple non-prioritized pieces of information at the same time in [29]. And an anamorphic framework is introduced in the subsequent papers [28, 30, 31]. Recently, the relation between the belief merging operators and Richard Booth's belief negotiation models has also been revealed in [26]. Here, we will only care about the primitive framework.

A knowledge base is the net content of a belief set $\mathcal{K}$, i.e., $\mathcal{C}_{\mathcal{K}}$. Different with the belief negotiation models, the belief merging operators are not binary operators, but they take the multi-sets of knowledge bases as the inputs. We call such multi-sets the knowledge sets which are denoted by $\mathcal{E} = [\mathcal{C}_{\mathcal{K}_1}, \mathcal{C}_{\mathcal{K}_2}, \ldots, \mathcal{C}_{\mathcal{K}_n}]$. Moreover, $\bigcap \mathcal{E} = \mathcal{C}_{\mathcal{K}_1} \cap \mathcal{C}_{\mathcal{K}_2} \cap \ldots \cap \mathcal{C}_{\mathcal{K}_n}$.

The merging operator Δ should satisfies the following postulates:

(BM1) $\Delta(\mathcal{E}) \neq \varnothing$.

(BM2) If $\bigcap \mathcal{E} \neq \varnothing$, then $\Delta(\mathcal{E}) = \bigcap \mathcal{E}$.

(BM3) If $\mathcal{E}' = \mathcal{E}$, then $\Delta(\mathcal{E}') = \Delta(\mathcal{E}')$.

(BM4) If $\mathcal{C}_{\mathcal{K}} \cap \mathcal{C}_{\mathcal{K}'} = \varnothing$, then $\mathcal{C}_{\mathcal{K}} \nsubseteq \Delta([\mathcal{C}_{\mathcal{K}}] \sqcup [\mathcal{C}_{\mathcal{K}'}])$.

(BM5) $\Delta(\mathcal{E}) \cap \Delta(\mathcal{E}') \subseteq \Delta(\mathcal{E} \sqcup \mathcal{E}')$.

(BM6) If $\Delta(\mathcal{E}) \cap \Delta(\mathcal{E}') \neq \varnothing$, then $\Delta(\mathcal{E} \sqcup \mathcal{E}) \subseteq \Delta(\mathcal{E}) \cap \Delta(\mathcal{E}')$.

(BM1) ensures that the outputs of Δ should always be the useful information. The rationale of (BM2) is that if the knowledge set is consistent, then the result of merging should entails all knowledge bases in this knowledge set. For the same reason with (AGM$\dotplus$6), we need (BM3). (BM4) means fairness, i.e., if two knowledge bases are inconsistent, then the result of merging should not totally accept any one of these two knowledge bases. Thus, this postulate strictly define the neutralism of Δ. (BM5) and (BM6) are similar with (AGM$\dotplus$3) and (AGM$\dotplus$4). They together ensures that if the mergings of two knowledge sets get the consistent results r and r', then the result of the merging of the union of these two knowledge sets should be identical with the conjunction of r and r'.

Moreover, we have the following supplementary postulates:

(BM-Majority) $\forall \mathcal{C}_{\mathcal{K}}\ \exists n\ \Delta(\mathcal{E} \sqcup [\mathcal{C}_{\mathcal{K}}]^n) \subseteq \mathcal{C}_{\mathcal{K}}$.

(BM-Arbitration) $\forall \mathcal{C}_{\mathcal{K}'}\ \exists \mathcal{C}_{\mathcal{K}}\ \mathcal{C}_{\mathcal{K}'} \nsubseteq \mathcal{C}_{\mathcal{K}}\ \forall n\ \Delta([\mathcal{C}_{\mathcal{K}'}] \sqcup [\mathcal{C}_{\mathcal{K}}]^n) = \Delta([\mathcal{C}_{\mathcal{K}'}] \sqcup [\mathcal{C}_{\mathcal{K}}])$.

We call Δ the majority operator if it satisfies (BM-Majority) which means that the result of merging should obey the majority rule. And then, Δ is called the arbitration operator if it satisfies (BM-Arbitration) by which it is ensured that the result of merging is irrelevant with the occurrence of the same knowledge bases in the input knowledge set. The inconsistency of these two postulates and (BM4) have been proved in [29]. Then it has been proved that (BM-Majority) is consistent with a revised version of (BM-Arbitration) in [31]. We will not care these two postulates too much in this thesis.

In [29], it has been proved that the occurrence of (BM4) has destroyed many useful properties of the merging operators, e.g., iteration, associativity, and etc. Obviously, (BM4) is too strong for us ([42] has the same opinion).

There are two branches of this belief merging theory we are interest in: Thomas Meyer's combination operations [42], and Sébastien Konieczny, Jérôme Lang and Pierre Marquis's distance based merging theory [27, 28]. Because of the proposed structure of epistemic states in the following chapters, we need some sort of quantitative methods to capture the dynamic properties of our epistemic states. Therefore, we should better start from the investigation of some existing insteresting methods.

2.3.3 Combination Operations

For applying the belief merging theory on epistemic states instead of knowledge bases, Thomas Meyer has developed a class of combination operations in 2001 [42]. In this theory, similar with Spohn's ordinal conditional functions, it is defined that an epistemic state is a function $\Phi : W \rightarrow \overline{\mathbb{I}^-}$. So for each epistemic state Φ, $\mathcal{C}_\Phi = \{w | w \in W \text{ and } \Phi(w) = 0\}$. An epistemic list $\mathcal{E}$ is defined as a finite multi-set of epistemic states, i.e., $\mathcal{E} = [\Phi_1, \Phi_2, \ldots, \Phi_n]$. Moreover, $\mathcal{ME}$ is a finite multi-set of epistemic lists.

A combination operator (or operation) Π is a function from the set of all non-empty epistemic lists to the set of all epistemic states. And, for all $w, w' \in W$, the function Π has the following properties:

(CO0) $\Pi([\Phi]) = \Phi(w) - \min(\{\Phi(w) | w \in W\})$.

(CO1) $\exists w \in W$ such that $\Pi(\mathcal{E})(w) = 0$.

(CO2) If $\Phi(w) = \Phi'(w) \forall \Phi, \Phi' \in \mathcal{E}$ and $s^{\mathcal{E}}_{\leq}(w) \sqsubset_{lex} s^{\mathcal{E}}_{\leq}(w')$, then $\Pi(\mathcal{E})(w) < \Pi(\mathcal{E})(w')$.

(CO3) If $\Phi(w) \leq \Phi(w') \forall \Phi \in \mathcal{E}$, then $\Pi(\mathcal{E})(w) \leq \Pi(\mathcal{E})(w')$.

(CO4) If $\Pi(\mathcal{E})(w) \leq \Pi(\mathcal{E})(w')$, then $\Phi(w) \leq \Phi(w')$ for some $\Phi \in \mathcal{E}$.

(CO0) is called the normalization process. It intend to keep the epistemic states' net contents consistent. In our opinion, (CO0) is an aftertreatment of combination operators, which can easily harm the properties of combination operators. (CO1) is a generalised (BM1), and it follows trivially from (CO0). (CO2) means that when w is assigned the same value by all epistemic states in an epistemic list $\mathcal{E}$, if the minimal value of the values of w' assigned by all epistemic states in $\mathcal{E}$ is greater than all of the values of w, then w' should be also assigned greater value than w in the combination result (please refer [42] if you want to know more detail about the expression $s^{\mathcal{E}}_{\leq}(w) \sqsubset_{lex} s^{\mathcal{E}}_{\leq}(w')$). Thus, (CO2) is a generalisation of (BM2). The rationale of (CO3) is that if the order of two possible worlds are same in all epistemic states in an epistemic list, then this order should be held in the result of combination. In our opinion, (CO2) is a special case of (CO3). In another direction, (CO4) says that if a specified order between two possible worlds occurs in the result of combining an epistemic list $\mathcal{E}$, then we can find such order occurs in at least one epistemic state in $\mathcal{E}$. (CO4) can be a different form of Pareto Principle in social choice theory [2].

(CO3) and (CO4) can be further generalized by the following postulates:

(CO5) If $\Pi(\mathcal{E})(w) = \Pi(\mathcal{E})(w') \forall \mathcal{E} \in \mathcal{ME}$, then $\Pi(\bigsqcup \mathcal{ME})(w) \leq \Pi(\bigsqcup \mathcal{ME})(w')$.

(CO6) If $\Pi(\bigsqcup \mathcal{ME})(w) \leq \Pi(\bigsqcup \mathcal{ME})(w')$, then $\Pi(\mathcal{E})(w) = \Pi(\mathcal{E})(w')$ for some $\mathcal{E} \in \mathcal{ME}$.

(CO5) and (CO6) generalize (CO3) and (CO4) respectively. And it has been proved that (CO1) and (CO5) imply (BM5) [2].

Moreover, from (CO3) the following postulate follows:

(CO-Unit) If $\Phi(w) = \Phi(w') \forall \Phi \in \mathcal{E}$, then $\Pi(\mathcal{E})(w) = \Pi(\mathcal{E})(w')$.

Then, by generalizing (BM-Majority) and (BM-Arbitration), the following postulates are introduced:

(CO-Majority) $\exists n$ such that $\forall w, w' \in W$, $\Phi(w) \leq \Phi(w')$ if $\Pi(\mathcal{E} \sqcup \Phi^n)(w) \leq \Pi(\mathcal{E} \sqcup \Phi^n)(w')$.

(CO-Arbitration) $\forall n$ $\Pi(\mathcal{E} \sqcup [\Phi])(w) \leq \Pi(\mathcal{E} \sqcup [\Phi])(w')$ iff $\Pi(\mathcal{E} \sqcup \Phi^n)(w) \leq \Pi(\mathcal{E} \sqcup \Phi^n)(w')$.

It has been pointed out in [31] that (CO-Arbitration) is not a real arbitration property, and it just degenerates the epistemic lists from the multi-sets to the usual sets.

Moreover, another postulate has been given in this theory:

(CO-Comm) If $\mathcal{E} = \mathcal{E}'$, then $\Pi(\mathcal{E}) = \Pi(\mathcal{E}')$.

It has been proved that if a combination operator satisfies (CO-Comm), then it can not satisfy (CO-Majority) and (CO-Arbitration) simultaneously [42].

2.3.4 Distance Based Merging

In 2002, Sébastien Konieczny, Jérôme Lang and Pierre Marquis have introduced a framework of belief merging in [27], which is general enough to used to describe many existing operators. From the combination of different distance functions and aggregation functions, many different merging operators can be built.

A set of possible worlds $\mathcal{C}_{IC}$ is provided as the domain of merging results, i.e., the results of merging operators should be a subset of $\mathcal{C}_{IC}$. This domain is called integrity constraint (we also call IC the integrity constraint). The idea of distance based merging of a knowledge set (we use the concept of knowledge set in Section 2.3.2) is about to find those possible worlds in the integrity constraint, which are the closest possible worlds with the knowledge set. Therefore, the main problem of distance based merging theory is about how to measure the distance between possible worlds and knowledge sets.

It is defined that a distance is a function which takes two possible worlds as inputs and gives a nonnegative number as the quantity of distance, i.e., $pd : W \times W \rightarrow \overline{\mathbb{I}^-}$. For all $w, w' \in W$ and $A \in \mathcal{B}$, a distance function pd is supposed to have the following properties:

(DBM-pd0) $\forall w, w' \in W \; pd(w, w') = pd(w', w)$.

(DBM-pd1) $pd(w, w') = 0$ iff $w = w'$.

(DBM-pd2) $pd(w, A) = \min\{pd(w, w') | w' \in A\}$.

Obviously, pd is a pseudo-distance discussed in Section 2.2.4. Recently, in [12], Daniel Eckert and Gabriella Pigozzi have used an almost same version of distance functions despite that they have extended the range of d to the set of nonnegative real numbers, i.e., $pd : W \times W \rightarrow \overline{\mathbb{R}^-}$.

Moreover, an aggregation function a is a function with variable number of inputs, i.e., $a : \overline{\mathbb{I}^-} \times \overline{\mathbb{I}^-} \times \ldots \times \overline{\mathbb{I}^-} \rightarrow \overline{\mathbb{I}^-}$, which satisfies the following properties:

(DBM-a0) If $x \leq y$, then $a(x_1, \ldots, x, \ldots, x_n) \leq a(x_1, \ldots, y, \ldots, x_n)$.

(DBM-a1) $a(x_1, \ldots, x_n) = 0$ iff $x_1 = \ldots = x_n = 0$.

(DBM-a2) $a(x) = x$.

Then let pd be a distance function and a and g be two aggregation functions. The merging operator of any knowledge set $\mathcal{E}$ under any integrity constraint IC, $\Delta_{IC}^{pd,a,g}(\mathcal{E})$, is defined as follow:

$$\mathcal{C}_{\Delta_{IC}^{pd,a,g}(\mathcal{E})} = \{w \in \mathcal{C}_{IC} | pd(w, \mathcal{E}) \text{ is minimal } \}$$

where

$$pd(w, \mathcal{E}) = g(pd(w, \mathcal{C}_{\mathcal{K}_1}), \ldots, pd(w, \mathcal{C}_{\mathcal{K}_n}))$$

and for every $A_{i,1}, \ldots, A_{i,n_i} \in \mathcal{B}$ and $\mathcal{C}_{\mathcal{K}_i} \subseteq A_{i,1}, \ldots, A_{i,n_i}$

$$pd(w, \mathcal{C}_{\mathcal{K}_i}) = a(pd(w, A_{i,1}), \ldots, pd(w, A_{i,n_i})).$$

As the above, $\Delta_{IC}^{pd,a,g}(\mathcal{E})$ uses two aggregation functions a and g to "transfer" the distance function of two possible worlds to the distance function of one possible world and a knowledge set. In this way, the problem of how to measure the distance between possible worlds and knowledge sets has been solved.

Chapter 3

Belief Change with Embeded Priorities

For the sake of embedding the priorities into the models of agents' belief states, we classfy the belief states by putting them into a hierarchy of generalisation:

(Class 1) The models of belief states consist of the sentences of some specified formal languages or the sets of possible worlds directly, e.g., the belief sets or the knowledge sets in [1], the belief bases in [13, 22, 23, 40, 44] and the possible worlds models in [7, 20, 25].

(Class 2) The information of epistemic entrenchment is embeded into this class of models, e.g., the ordinal conditional functions in [49].

(Class 3) The information of priorities between different belief states is embeded into this class of models. We call this kind of belief states the epistemic states.

The models of (Class 1) has the lowest generalized level in our hierarchy. They only include the primitive exertions of some formal languages from which we can simply assign the truth values to assertions. The (Class 2) models generalize the models of (Class 1) by grading sentences of the formal languages. Thus, we can distinguish different importances of assertions. Our discussion will focus on the models of (Class 3) which generalize the (Class 2) models, and we can tell the importances of information sources through them.

From another point of view, different (Class 1) belief states can be distinguished by different contents of beliefs, e.g., different belief sets have different sets of sentences. And two (Class 2) belief states are identical iff the contents of beliefs and the entrenchment information of beliefs of two belief states are identical. Finally, the identical (Class 3) belief states are those which have the same priorities except the same contents and the same entrenchments.

In the following sections, we will propose an abstract framework for the (Class 3) belief states and a special kind of belief change operators, which is called the belief combination operators, on them.

3.1 Groundwork of Epistemic States

Before we construct the (Class 3) models, we must choose a (Class 2) model which can be generalised to include the information of the agents' priorities. There are two ways to impart the entrenchment information. One is the qualitative way in which an pre-order is used to represent the entrenchments directly. The other one is the quantitative way which endue the belief sets with the entrenchment information by assigning each sentence a numerica value. Technically, the quantitative way is a generalisation of the qualitative way, however, we still classfy them into the same generalized level, i.e., they all are the (Class 2) belief states.

3.1.1 Belief States

In this chapter, for increasing the possibilities of generalising the agents' belief states, we will employ a qualitative method to build the groundwork of our (Class 3) belief states (or epistemic states). The other reason for this is that we intend to focus on the qualitative properties of belief states.

Definition 3.1.1. A belief state $\preccurlyeq$ is a total pre-order on W. Then $\mathcal{C}_{\preccurlyeq} = \min_{\preccurlyeq}(W)$ is the net content of $\preccurlyeq$. Moreover, let $A, B \in \mathcal{B}$ and $i, j \in \{1, 2, \ldots, n\}$, if $w_i \in \min_{\preccurlyeq}(A)$ and $w_j \in \min_{\preccurlyeq}(B)$, then we denote $B \preccurlyeq A$ iff $w_i \preccurlyeq w_j$, i.e., a proposition is more plausible than another iff its models are more plausible than the other's. We define the set of all possible total pre-orders on W as Ξ.

Thus, each total pre-order on W implies a belief set and an epistemic entrenchment which satisfies the postulate (EE1)-(EE5) [16]. Then A is believed in the belief state $\preccurlyeq$ iff $\mathcal{C}_{\preccurlyeq} \subseteq A$, i.e., $\overline{A} \preccurlyeq \mathcal{C}_{\preccurlyeq}$ and $\mathcal{C}_{\preccurlyeq} \not\preccurlyeq \overline{A}$.

Since we have the qualitative definition of belief states, there are many methods which can be employed to make the quantitative representations of them, e.g., the ordinal conditional functions. We will do this in the next chapter.

3.2 Epistemic States with Embeded Priorities

Suppose we have a set of agents $\mathcal{A} = \{a, a', \cdots\}$ as the set of information sources, and $S, S', \cdots$ are the epistemic states of agents $a, a', \cdots$ respectively. $S, S', \cdots \in \mathcal{S}$, while the set of all possible epistemic states is denoted by $\mathcal{S}$.

3.2.1 Components of Epistemic States

As discussed before, an epistemic state should be imparted with a belief state and a priority. In this chapter, we do not care the technical detail of the constructions of epistemic states, but only the abstruct structure. By assuming each epistemic state as an integrator of one belief state and one priority, we have the following defination:

Definition 3.2.1. Suppose $\mathcal{P} \subseteq \overline{\mathbb{R}^-}$ and $\mathcal{P}$ is continuing, we define two extraction functions of epistemic states as follow:

Belief State Extraction : $e_W : \mathcal{S} \to \Xi$, which extract the relative belief state of the input epistemic state.

Priority Extraction : $e_{\mathcal{P}} : \mathcal{S} \to \mathcal{P}$, which extract the relative priority of the input epistemic state.

We denote $\preccurlyeq_S = e_W(S)$, $p_S = e_{\mathcal{P}}(S)$ and $\mathcal{C}_S = \mathcal{C}_{\preccurlyeq_S}$ for convenient, in which $\preccurlyeq_S \in \Xi$ and $p \in \mathcal{P}$.

In Definition 3.2.1, we point out that the set of priorities is a subset $\mathcal{P}$ of real numbers. By $\mathcal{P}$ is continuing, we mean that for all $p, p' \in \mathcal{P}$ and $p < p'$, we have $p'' \in \mathcal{P}$ such that $p < p''$ and $p'' < p'$. The rationale for the requirement of continuity is that we assume that for any pair of epistemic states with different priorities, there is always the third epistemic state in which its priority is in between of them. For convenient, we will use $\mathcal{P}$ as our all-purpose number set.

3.2.2 Distance Functions

In Section 1.2, we have pointed out that the distance between epistemic states is hard to measure. The measurement of this kind of distances involves the measurements of the distances between the belief states and between the priorities. So we should synthetically consider the construction of distance function of belief states and priorities. Here, we provide an abstract framework of the distance function of epistemic states because the technical details of epistemic states have not been revealed yet.

Definition 3.2.2. The distance function $d : \mathcal{S} \times \mathcal{S} \to \mathcal{P}$ is a function which satisfies the following properties:

(d0) $d(S, S') = d(S', S)$,

(d1) $d(S, S') = 0$ iff $S = S'$,

(d2) $d(S, S') \leq d(S, S'') + d(S', S'')$, for all $S, S' \in \mathcal{S}$.

(d0) is the property of symmetry of d which stipulates that the distance function should measure the distance of two epistemic states regardless of the direction of measurement. By (d1), it implies that if the distance of two epistemic states is zero, then the two epistemic states are identical. The triangle inequality (d2) ensures that the shortest path from one epistemic state to another is the most direct path.

For the purpose of exposing the relationship between the distance functions and the priorities, we have the following supplementary property for d:

(d3) $d(S, S') \leq d(S, S'')$ if $|p_S - p_{S'}| \leq |p_S - p_{S''}|$ and the belief states of S, S', S'' are identical for all $S, S', S'' \in \mathcal{S}$.

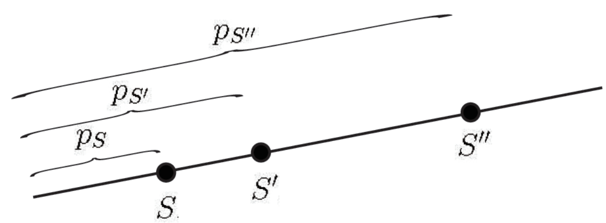

Figure 3.1: Intuition of (d3)

This property says that if the belief states of three epistemic states are identical, then the shortest distance is the distance between the two epistemic states which have the minimal difference between their priorities. It reveals that the relationship between the epistemic states, which have the same belief states, is like the points on a straight line (Figure 3.1). It is an important clue for building the actual epistemic states in the next chapter.

Different with the distance functions in [3, 32], d is not a pseudo-distance but a real distance with triangle inequality like the distance between matrixes in [19]. Therefore, the postulates imply a family of total quantitative instances of epistemic states.

3.2.3 Concession Functions

By the reason of analysing the dynamic properties of epistemic states, we propose a framework of concession functions which is intended to weaken some epistemic states to coincide with some other epistemic states, i.e., the gap between two epistemic states can be filled through such concession functions.

Definition 3.2.3. The concession function of epistemic states $c : \mathcal{S} \times \mathcal{S} \to \mathcal{S}$ is a function which satisfies the following properties:

(c0) $\preccurlyeq_S \subseteq \preccurlyeq_{c(S,S')}$,

(c1) $\mathcal{C}_{c(S,S')} \cap \mathcal{C}_{S'} \neq \varnothing$,

(c2) If $\mathcal{C}_S \cap \mathcal{C}_{S'} \neq \varnothing$, then $c(S,S') = S$,

(c3) $p_{c(S,S')} = p_S$,

for all $S, S' \in \mathcal{S}$.

Intuitively, as in figure 3.2, $c(S,S')$ means the result of the concession of the epistemic state S toward S', i.e., S and S' are respectively the source and the target of the conceded epistemic state $c(S,S')$. In other words, $c(S,S')$ "pushes" the epistemic state S toward S'. The justification of (c0) is that the gradings of the conceded epistemic states should not be more than their respective original epistemic states'.

Theorem 3.2.1. *The property (c0) implies the following property:*

(c4) $\mathcal{C}_S \subseteq \mathcal{C}_{c(S,S')}$ *for all* $S, S' \in \mathcal{S}$,

i.e., the conceded epistemic states are entailed by their respective original epistemic states.

Proof. Because $\mathcal{C}_S = \mathcal{C}_{\preccurlyeq_S} = \min_{\preccurlyeq_S}(W)$ and $\mathcal{C}_{c(S,S')} = \mathcal{C}_{\preccurlyeq_{c(S,S')}} = \min_{\preccurlyeq_{c(S,S')}}(W)$. From (c0), it follows that $\min_{\preccurlyeq_S}(W) \subseteq \min_{\preccurlyeq_{c(S,S')}}(W)$. Thus, for all $S, S' \in \mathcal{S}$, we have $\mathcal{C}_S \subseteq \mathcal{C}_{c(S,S')}$. □

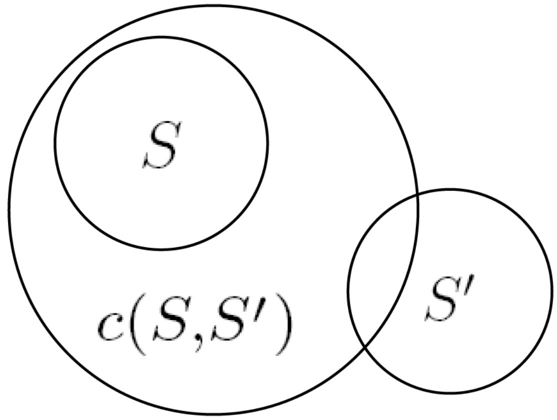

Figure 3.2: Concession Function

Therefore, (c0) and (c4) guarantees the safety of concession, i.e., the net contents of the conceded epistemic states do not exceed the scope of the belief sets of their respective original epistemic states.

(c1) and (c2) guarantees the success and the vacuity of concession, i.e., $c(S, S')$ pushes S toward S', untill the epistemic states S is consistent with S' or there is no necessity to push at the beginning. These two properties are the essence of concession functions due to the motivations of concession functions are revealed by them. In the end, the property (c3) makes that the concession functions do not affect the priorities of epistemic states. This property is attached because we want to simplify the behaviors of concession functions. However, the necessity of (c3) is still arguable.

We have another property which is caused by the properties (c2):

Theorem 3.2.2. *If the concession function c satisfies (c2), then c also satisfies the following property:*

(c5) $c(S, S) = S$ *for all* $S \in \mathcal{S}$.

Proof. The result trivially follows from (c2) and $S \cap S \neq \varnothing$. $\square$

The rationale of (c5) is that the concession of an epistemic state toward itself should be itself, i.e., there should be no concession while we try to concede one epistemic state toward itself.

To sum up this section, an abstract framework of epistemic states has been presented. We have also introduced the concepts of the method of measuring distance between two epistemic states and the concepts of concession functions. Based on these foundations, we can begin to build the general framework of belief combination operators.

3.3 A General Framework of Belief Combination

One of the central idea of this thesis is to build a kind of belief change operators which is sensitive to the priorities of epistemic states. We call this kind of operators the belief combination operators (or functions). As the epistemic states are a snapshot of the agents' belief states and priorities, the belief functions should merge the belief states and priorities simultaneously.

3.3.1 Combination Functions

A set of postulates is employed to represent the properties of combination functions. We assume the combination function f which is a binay operator since it can be implemented more easily and more efficiently. Thus, the formal definition of f is as follow.

Definition 3.3.1. A combination function $f : \mathcal{S} \times \mathcal{S} \to \mathcal{S}$ is a function which satisfies the following properties:

(f0) $f(S, S') \in \mathcal{S}$,

(f1) $f(S, S') = f(S', S)$,

(f2) If $S' = S''$, then $f(S, S') = f(S, S'')$,

(f3) $f(S, f(S', S'')) = f(f(S, S'), S'')$,

(f4) If $w_i \preccurlyeq_S w_j$ and $w_i \preccurlyeq_{S'} w_j$, then $w_i \preccurlyeq_{f(S,S')} w_j$,

(f5) If $p_S \leq p_{S'}$, then $d(S', f(S, S')) \leq d(S, f(S, S'))$,

for all $S, S', S'' \in \mathcal{S}$.

The first four postulates (f0), (f1), (f2) and (f3) focus on the mathematical properties of f that the closure, the commutativity, the extensionality and the associativity are incured by the four postulates respectively. (f0) ensures that the result of combining two epistemic states should be still an epistemic state. (f1) makes the combination functions to be the direction senseless functions. Then (f2) incurs that the semantically identity epistemic states should be taken without bias. Associativity, which is incured by (f3), is a useful property since it would ignore the order of combination. Intuitively, if the order of two possible worlds are same in two epitemic states, then this order should be maintained in the result epitemic state of combination functions, thus (f4) implies this property. (f5) reveals that the result of combination should be closer to the epistemic state which has the higher priority. Moreover, if the priority of one epistemic state is high enough, then the distance between this epistemic state and the result of combination may be close enough such that (f5) is degenerated to (AGM$\dotplus$2), i.e., the (Success) postulate.

We call the postulates (f0)-(f5) the basic postulates of combination functions. In these basic postulates, there is no postulate which is similar with the postulate (AGM$\dotplus$2) in AGM framework or the postulate (BM4) in belief merging framework. Therefore, the basic postulates of combination functions form a set of constraints which is mathematically strong and epistemicly weak. Therefore, (f0)-(f5) seem to be good at building the combination functions on the quantitative epistemic states.

3.3.2 Supplementary Properties of Combination Functions

From the postulate (f4), some properties can be deduced. The most interesting pair of priorities may be the inclusion postulate and the vacuity postulate. In other words, (f4) is a generalised version of (AGM$\dotplus$3) and (AGM$\dotplus$4). If the net contents of two epistemic states are consistent, then the net content of the combined epistemic state is identical with the intersection of them, more precisely we have the following:

Theorem 3.3.1. *If the function f satisfies the postulate (f4), then it has the following properties:*

(f6) $\mathcal{C}_S \cap \mathcal{C}_{S'} \subseteq \mathcal{C}_{f(S,S')}$,

(f7) *If* $\mathcal{C}_S \cap \mathcal{C}_{S'} \neq \varnothing$, *then* $\mathcal{C}_{f(S,S')} \subseteq \mathcal{C}_S \cap \mathcal{C}_{S'}$,

for all $S, S' \in \mathcal{S}$.

Proof. Obviously, $\mathcal{C}_S = \mathcal{C}_{\preccurlyeq_S} = \min_{\preccurlyeq_S}(W)$, $\mathcal{C}_{S'} = \mathcal{C}_{\preccurlyeq_{S'}} = \min_{\preccurlyeq_{S'}}(W)$ and $\mathcal{C}_{f(S,S')} = \mathcal{C}_{\preccurlyeq_{f(S,S')}} = \min_{\preccurlyeq_{f(S,S')}}(W)$. If $\min_{\preccurlyeq_S}(W) \cap \min_{\preccurlyeq_{S'}}(W) = \varnothing$, then it is trivial that $\min_{\preccurlyeq_S}(W) \cap \min_{\preccurlyeq_{S'}}(W) \subseteq \min_{\preccurlyeq_{f(S,S')}}(W)$. And if $\min_{\preccurlyeq_S}(W) \cap \min_{\preccurlyeq_{S'}}(W) \neq \varnothing$, then there exists $i \in \{1, 2, \ldots, n\}$ such that $w_i \in \min_{\preccurlyeq_S}(W) \cap \min_{\preccurlyeq_{S'}}(W)$ and $w_i \preccurlyeq_S w_j, w_i \preccurlyeq_{S'} w_j$ for all $j \in \{1, 2, \ldots, n\}$ and $j \neq i$. It follows from (f4) that also $w_i \preccurlyeq_{f(S,S')} w_j$ for all $j \in \{1, 2, \ldots, n\}$ and $j \neq i$. Then $\min_{\preccurlyeq_S}(W) \cap \min_{\preccurlyeq_{S'}}(W) = \min_{\preccurlyeq_{f(S,S')}}(W)$ iff $\min_{\preccurlyeq_S}(W) \cap \min_{\preccurlyeq_{S'}}(W) \neq \varnothing$, from which the result follows. □

Furthermore we can get two other postulates which are similar with (AGM$\dotplus$7) and (AGM$\dotplus$8):

Theorem 3.3.2. *If the function f satisfies the postulates (f3) and (f4), then it has the following properties:*

(f8) $\mathcal{C}_{f(S,S')} \cap \mathcal{C}_{S''} \subseteq \mathcal{C}_{f(S,f(S',S''))}$,

(f9) *If* $\mathcal{C}_{f(S,S')} \cap \mathcal{C}_{S''} \neq \varnothing$, *then* $\mathcal{C}_{f(S,f(S',S''))} \subseteq \mathcal{C}_{f(S,S')} \cap \mathcal{C}_{S''}$, *for all* $S, S', S'' \in \mathcal{S}$.

Proof. When $\mathcal{C}_{f(S,S')} \cap \mathcal{C}_{S''} = \varnothing$, then it is trivial that $\mathcal{C}_{f(S,S')} \cap \mathcal{C}_{S''} \subseteq \mathcal{C}_{f(S,f(S',S''))}$. And if $\mathcal{C}_{f(S,S')} \cap \mathcal{C}_{S''} \neq \varnothing$, then it follows from Theorem 3.3.1 that $\mathcal{C}_{f(f(S,S'),S'')} = \mathcal{C}_{f(S,S')} \cap \mathcal{C}_{S''}$. From (f3), we have $\mathcal{C}_{f(S,f(S',S''))} = \mathcal{C}_{f(f(S,S'),S'')}$, from which we have $\mathcal{C}_{f(S,f(S',S''))} = \mathcal{C}_{f(f(S,S'),S'')}$. □

Obviously, (f8) and (f9) are weaker than (AGM∔7) and (AGM∔8).

Apparently, if we combine the conceded epistemic state with its target, then the result of combination should be the intersection of the conceded epistemic state and its target.

Theorem 3.3.3. *If the concession function c satisfies the postulates (c1) and the combination function f satisfies the postulates (f4), then f has the following property:*

(f10) $\mathcal{C}_{c(S,S')} \cap \mathcal{C}_{S'} = \mathcal{C}_{f(c(S,S'),S')}$ *for all* $S, S' \in \mathcal{S}$.

Proof. If the combination function f satisfies the postulates (f4), then f also satisfies (f6) and (f7). From (c1), it follows that $\mathcal{C}_{c(S,S')} \cap \mathcal{C}_{S'} \neq \varnothing$. Then $\mathcal{C}_{c(S,S')} \cap \mathcal{C}_{S'} = \mathcal{C}_{f(c(S,S'),S')}$ for all $S, S' \in \mathcal{S}$ which is from (f6) and (f7). □

The relationship between the concession function c and the combination function f is revealed by this theorem.

3.4 Summary

The concepts of epistemic states with embeded priorities, i.e., the (Class 3) models, have been introduced in this chapter. Each of this kind of epistemic states consists of a belief state, which is presented with a total preorder on W, and a priority, which is presented with a number. However, this is just an abstract framework of epistemic states. Then the frameworks of distance functions and concession functions have been built. We thus can measure and manipulate the epistemic states on purpose. The postulate (d2) reveals the clue of the technical detail of epistemic states which will be introduced in Chapter 4.

Then we have studied the basic postulates and the induced postulates by them. It is remarkable that (f0)-(f5) are adept at the combination functions of epistemic states in which the belief states are presented in some quantitative methods. Thus we will use such a method in Chapter 4.

Chapter 4

Epistemic Vectors

We are going to build the actual epistemic states in this chapter based on the abstract frameworks which have been introduced in Chapter 3, and the belief states of epistemic states will utilize a quantitative method to impart the total pre-orders on the set of all possible worlds W. Furthermore, we will carefully design this quantitative method in order to embed the priorities into the epistemic states.

4.1 Generalised Ordinal Conditional Functions

Based on Chapter 3, the most direct way to build (Class 3) models is to generalise the (Class 2) models. So we first generalise the framework of ordinal conditional functions as an attempt to build epistemic states. By generalising the ordinal conditional functions, we expect to impart the information of priorities into it.

4.1.1 Extended OCF

Thus, we define the extended ordinal conditional functions in the following way:

Definition 4.1.1. An extended ordinal conditional function (EOCF) κe is a function from W into the set of all nonnegative real numbers $\overline{\mathbb{R}^{-}}$, i.e., $\kappa e : W \to \overline{\mathbb{R}^{-}}$. Moreover, we define that for all $A \in \mathcal{B} \setminus \{\varnothing\}$, $\kappa e(A) = \min(\{\kappa e(w) | w \in A\})$. We also define that the priority of the EOCF κe as $p_{\kappa e} = \min(\{\kappa e(w) | w \in A\})$. And the set of all possible epistemic vectors is denoted $\mathcal{V}$.

In the above definition, an EOCF assigns a nonnegative real number to each possible world and each proposition. We can say each EOCF is also an epistemic state. In the state κe, $\mathcal{C}_{\kappa e} = \{w|\kappa e(w) = \min(\{\kappa e(w)|w \in A\})\}$ is the net content. In other words, the net content of κe is the set of the possible worlds which make that κe gets the minimal value. Therefore, the proposition A is believed in the state κe iff $\kappa e(A) = \min(\{\kappa e(W)|w \in W\})$ and $\kappa e(\overline{A}) > \min(\{\kappa e(W)|w \in W\})$.

Definition 4.1.2. The extended firmness αe of a proposition A relative to κe is defined as:

$$\alpha e = \begin{cases} \kappa e(\overline{A}) & \text{if } \kappa e(A) = \min(\{\kappa e(W)|w \in W\}) \\ -\kappa e(A) & \text{if } \kappa e(A) > \min(\{\kappa e(W)|w \in W\}) \end{cases}$$

From definition 4.1.2, if the firmness αe of a proposition A in the state κe is a positive, a negative or 0, then, respectively, A is believed, disbelieved or neutral with firmness αe in the state κe.

The next two definitions introduce a kind of belief change on EOCF through generalising the belief change on OCF.

Definition 4.1.3. Suppose κe is an EOCF and $A \in \mathcal{B} \setminus \{\varnothing\}$. Then $\kappa e(w|A) = -\kappa e(A) + \kappa e(w) + \min(\{\kappa e(W)|w \in W\})$ for all $w \in A$. We also define $\kappa e(B|A) = -\kappa e(A) + \kappa e(A \cap B) + \min(\{\kappa e(W)|w \in W\})$ for $B \in \mathcal{B}$ with $A \cap B \neq \varnothing$.

Thus, $\kappa e(w|A)$ is intended to shift EOCF value for A to be the minimal value of κe.

Definition 4.1.4. Let κe be an EOCF, $A \in \mathcal{B} \setminus \{\varnothing\}$, and $\alpha e \in \overline{\mathbb{R}^-}$. Then $\kappa e_{A,\alpha e}$ is an EOCF which satifies the following equation:

$$\kappa e_{A,\alpha e} = \begin{cases} \kappa e(w|A) & \text{if } w \in A \\ \alpha e + \kappa e(w|\overline{A}) & \text{if } w \in \overline{A} \end{cases}$$

$\kappa e_{A,\alpha e}$ is called the $A, \alpha e$-conditionalization of κe.

The above definition of the $A, \alpha e$-conditionalization of κe is the central idea of the dynamics of epistemic states. It implies that the epistemic state κe is revised by a proposition A. Thus, the essence of the belief change on EOCF is a kind of prioritised belief revision.

4.1.2 Problems with EOCF

So far, we have tried to employ EOCFs as our epistemic states. But there are some problems which cause EOCFs to be some unsuitable carriers of epistemic states. First of all, the definition of the priorities of EOCFs is farfetched and unintuitionistic, i.e., the priorities are abruptly defined as the minimal values of EOCFs. Secondly, the belief change of EOCF is hard to fit the framework in Definition 3.3.1.

However, EOCF is still a good start point to build our epistemic states. In the next section, we will introduce a new representation of epistemic states, which is different with EOCF, but enlightened by it.

4.2 Modeling Epistemic States with Epistemic Vectors

Holding the purpose of modeling the belief states and grading the priorities of epistemic states together, we should provide a unified and coherent framework in which the degrees of deliberation and the contents of epistemic states will be expressed and associated properly. By assuming each possible world can be treated as the independent element of thinking, i.e. the evaluation of one possible world will not affect the evaluation of the other one, we can measure the attitudes of agents on each possible world separately. Obviously, an agent can have three different kinds of attitude on each possible world: believing, disbelieving or neutral, hence our measurements must be endued with the ability to evaluate all three kinds of attitudes. On the other hand, the measurements should also expose the priorities of agents which not depends on any oracle but the epistemic states of agents, e.g., in a group of people, usually, the one who spent the most resources(time, physical force, etc.) to think about a problem has the most important opinion in the discussion of this problem.

4.2.1 Epistemic Vectors as Epistemic States

By assuming the epistemic states as points in geometry, from (d3), it follows that the epistemic states which have the identical belief states are the points on the same straight line. Consequently, we can safely assume that the epistemic states which have different belief states with each other should on different lines. It seems that these properties have close association with the vectors in the same quadrant. Thus, we may treat the epistemic states as vectors in some sort of space. The above ideas arouse the following discussion.

Intuitively, we may define the possible world w_i as the i-th dimensional axis of the Euclidean n-space $\mathbb{R}^n$. If there are totally n possible worlds in W, i.e., $|W| = n$, then we define such space the n-dimensional epistemic space. So each point of the epistemic space has a coordinate measured by n possible worlds and the i-th element of the coordinate is measured by w_i.

Definition 4.2.1. An epistemic vector $V = \langle v_1, v_2, \ldots, v_n\rangle$ is a n-dimensional vector in the Euclidean n-space $\mathbb{R}^n$ where $n \in \mathbb{I}^+$ and $n = |W|$. Moreover, we call $|v_i|$ the maturity of V on w_i and $\epsilon_i = \frac{v_i}{|v_i|}$ (we define $\frac{v_i}{|v_i|} = 0$ iff $v_i = 0$) the signal of faith (SOF) of V on w_i for all $i \in [1, 2, \ldots, n]$.

In the epistemic vector V, w_i is believed or disbelieved iff, respectively, SOF $\epsilon_i = +1$ or $\epsilon_i = -1$; w_i is taken neutrally by V when $\epsilon_i = 0$. The maturity of V on w_i, i.e. v_i, coule be explained as the depth of thinking of V on w_i. In other worlds, with more maturity of V on w_i, it takes more cost to change the SOF ϵ_i. For example, suppose $V = \langle \ldots, v_i, \ldots, v_j, \ldots \rangle$, if $v_i = 4$ and $v_j = -6$, then the agent believes w_i with the maturity 4 and disbelieves w_j with the maturity 6. Thus, the total maturity of V is a measurement of stability of the epistemic state implied by V. We define the length of V as the total maturity of V.

Definition 4.2.2. We define the length of the vector $|V|$ the total maturity of the epistemic vector V, i.e., $|V| = \sqrt{\sum_{i=1}^{n} v_i^2}$. See Figure 4.1 for an example.

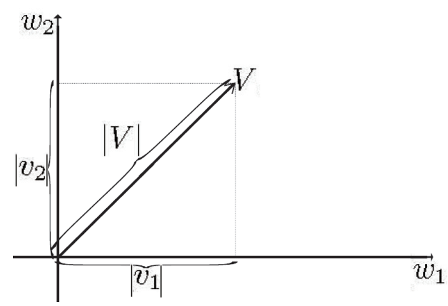

Figure 4.1: Total Maturity of V

Intuitively, the total maturity of V is a complex of an agent's deliberation on all possible worlds. So we define the total maturity of V as the priority of V, i.e., $p_V = |V|$.

4.2.2 Belief States of Epistemic Vectors

Sometime we need focus on the contents of epistemic vectors and thus we should get rid of the information of the priorities of such epistemic vectors. Different with the method of normalization of OCF in [42], we will decrease or increase the elements of the epistemic vector V pro rata in our process of normalization. From another point of view, we will get rid of the length of V, for the length of V is the information of the priority of V. Therefore the normalized epistemic vector V should be the norm of V (Figure 4.2).

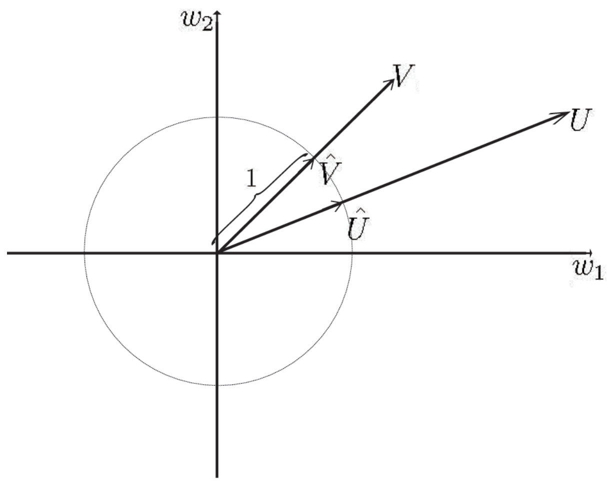

Figure 4.2: Normalized Epistemic States

Definition 4.2.3. The normalized epistemic vector $\hat{V}$ of V is defined by:

$$\begin{aligned}\hat{V} &= \frac{V}{|V|} \quad (|V| \neq 0)\\ &= \langle v_1/\sqrt{\sum_{i=1}^{n} v_i^2}, v_2/\sqrt{\sum_{i=1}^{n} v_i^2}, \ldots, v_n/\sqrt{\sum_{i=1}^{n} v_i^2}\rangle\end{aligned}$$

Hence $\hat{V}$ is the belief state of V. Apparently, when we try to focus on the belief states of a group of epistemic vectors, we should convert them to have the same priority, i.e. to make them to have the same maturity. For example, by Definition 4.2.3, $\{k \cdot \hat{V}_1, k \cdot \hat{V}_2, \ldots\}$, where $k \in (R)$, is called *coequalized group* of $\{V_1, V_2, \ldots\}$.

4.2.3 Plausible Orders of Epistemic Vectors

Since we have the definition of the priorities of epistemic vectors, we have also extracted the belief states from epistemic vectors. By taking the possible worlds as the axises of epistemic space, we can intuitively define that the distance between an epistemic vector and an axis (or a possible world) is the degree of the epistemic vector liking or disliking the possible world.

Definition 4.2.4. The plausibility ρ_i of the possible world w_i for the epistemic vector V is the distance between the vector $V_{axis(i)} = \langle 0, 0, \ldots, v_i, \ldots, 0\rangle$ and V. Then we have:

$$\begin{aligned}\rho_i &= \sqrt{v_1^2 + v_2^2 + \ldots + v_{i-1}^2 + v_{i+1}^2 + \ldots + v_n^2} \\ &= \sqrt{\sum_{j=1}^{n} v_j^2 - v_i^2}\end{aligned}$$

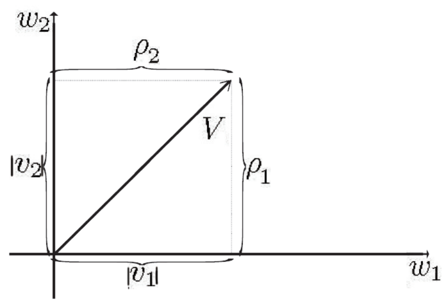

Figure 4.3: Plausibility of V

The plausibility of the possible world w_i in V is the distance between w_i and V (Figure 4.3). Simply, from V, the closest possible worlds with V are the most plausible possible worlds for V. However, that is not correct. An epistemic vector expresses not only the degrees of belief but also the degrees of disbelief. Therefore, for V, if $\epsilon_i = +1$, then ρ_i is the plausibility of believing w_i, i.e., the closer distance means the higher plausibility of believing, or the "grading of disbelief" [49] of w_i; if $\epsilon_i = -1$, then ρ_i is the plausibility of disbelieving w_i, i.e., the closer distance means the higher plausibility of disbelieving, or the grading of dis-disbelief of w_i. In other words, the plausibility of the possible world w_i in V is the distance between V and the affirmation or denial of w_i.

Obviously, there is a close relationship between the maturities and the plausibilities as follow:

Theorem 4.2.1. *Let $V = \langle v_1, v_2, \ldots, v_n \rangle$ be an epistemic vector. $\rho_i \leq \rho_j$ iff $|v_i| \geq |v_j|$ for all i, $j \in \{1, 2, \ldots, n\}$.*

Proof. Obviously, we have $\sqrt{\sum_{j=1}^{n} v_j^2 - v_i^2} = \sqrt{|V|^2 - |v_i|^2}$ for all $i \in \{1, 2, \ldots, n\}$. And $\sqrt{|V|^2 - |v_i|^2} \leq \sqrt{|V|^2 - |v_j|^2}$ iff $|v_i| \geq |v_j|$ for all $i, j \in \{1, 2, \ldots, n\}$. Therefore, from Definition 4.2.4 it follows that $\rho_i \leq \rho_j$ iff $|v_i| \geq |v_j|$ for all $i, j \in \{1, 2, \ldots, n\}$. □

Thus, there is an inverse ratio between the plausibility ρ_i and the i-th factor v_i of the epistemic vector V for any $i \in \{1, 2, \ldots, n\}$.

Next, the plausible orders of epistemic vectors are introduced as follow:

Definition 4.2.5. The plausible order $\preccurlyeq$ on W implied by an epistemic vector $V = \langle v_1, v_2, \ldots, v_n \rangle$ is defined by:

1. when $\epsilon_i = \epsilon_j$, $w_i \preccurlyeq w_j$ iff $\epsilon_i \rho_i \leq \epsilon_j \rho_j$,

2. when $\epsilon_i \neq \epsilon_j$, $w_i \preccurlyeq w_j$ iff $\epsilon_j \rho_j < \epsilon_i \rho_i$

for all $i, j \in \{1, 2, \ldots, n\}$. Moreover, we define:

- $w_i \sim w_j$ iff $w_i \preccurlyeq w_j$ and $w_j \preccurlyeq w_i$,

- $w_i \prec w_j$ iff $w_i \preccurlyeq w_j$ and $w_i \nsim w_j$,

- $w_i \succ w_j$ iff $w_j \preccurlyeq w_i$ and $w_j \nsim w_i$

for all $i, j \in \{1, 2, \ldots, n\}$. Refer table 4.1.

We denote the plausible order of V as $\preccurlyeq_V$. $w_i \preccurlyeq_V w_j$ means that the possible world w_i is at least as plausible as the possible world w_j. Obviously, $\preccurlyeq_V$ is a total pre-order on W. Hence $\preccurlyeq_V$ is the qualitative aspect of the belief state of V.

There is a tight relationship between the plausible order $\preccurlyeq_V$ and the maturities of V, more precisely we have the following:

Theorem 4.2.2. *The plausible order $\preccurlyeq_V$ on W implied by an epistemic vector $V = \langle v_1, v_2, \ldots, v_n \rangle$ can be also got by: $w_i \preccurlyeq_V w_j$ iff $v_j \leq v_i$ for all $i, j \in \{1, 2, \ldots, n\}$.*

ϵ_i	ϵ_i	ρ_i		ρ_j	w_i		w_j
			$>$			$\succ$	
$+1$	$+1$		$=$			$\sim$	
			$<$			$\prec$	
			$>$			$\prec$	
$+1$	-1		$=$			$\prec$	
			$<$			$\prec$	
			$>$			$\succ$	
-1	$+1$		$=$			$\succ$	
			$<$			$\succ$	
			$>$			$\prec$	
-1	-1		$=$			$\sim$	
			$<$			$\succ$	
$+1$	0		$<$			$\prec$	
0	$+1$		$>$			$\succ$	
-1	0		$<$			$\succ$	
0	-1		$>$			$\prec$	
0	0		$=$			$\sim$	

Table 4.1: Plausible Order of W

Proof. We have $\epsilon_i\rho_i \leq \epsilon_j\rho_j$ iff $\epsilon_i|v_i| \geq \epsilon_j|v_j|$ for all $i,\ j \in \{1, 2, \ldots, n\}$ based on Theorem 4.2.1. So from Definition 4.2.5 it follows that:

1. when $\epsilon_i = \epsilon_j$, $w_i \preccurlyeq_V w_j$ iff $\epsilon_i|v_i| \geq \epsilon_j|v_j|$,

2. when $\epsilon_i \neq \epsilon_j$, $w_i \preccurlyeq_V w_j$ iff $\epsilon_i|v_i| > \epsilon_j|v_j|$

for all $i,\ j \in \{1, 2, \ldots, n\}$. And $\epsilon_i|v_i| = v_i$ by Definition 4.2.1. So $w_i \preccurlyeq_V w_j$ iff $v_j \leq v_i$ for all $i,\ j \in \{1, 2, \ldots, n\}$. $\square$

Trivially, we can get the net content through the following methods:

Definition 4.2.6. The net content $\mathcal{C}_V$ of an epistemic vector $V = \langle v_1, v_2, \ldots, v_n \rangle$ is defined by $\mathcal{C}_V = \min_{\preccurlyeq_V}(W) = \{w_i | w_i \preccurlyeq_V w_j \text{ for all } j, \text{ and } i, j \in \{1, 2, \ldots, n\}\}$.

Theorem 4.2.3. *Let $V = \langle v_1, v_2, \ldots, v_n \rangle$ be an epistemic vector. $w_i \in \mathcal{C}_V$ iff $v_i = \max(\{v_1, v_2, \ldots, v_n\})$, $i \in \{1, 2, \ldots, n\}$.*

Proof. Let $i \in \{1, 2, \ldots, n\}$, then $v_i = \max(\{v_1, v_2, \ldots, v_n\})$ iff $v_j \leq v_i$ for all $j \in \{1, 2, \ldots, n\}$. And from Theorem 4.2.2, $w_i \preccurlyeq_V w_j$ for all $j \in \{1, 2, \ldots, n\}$ iff $v_j \leq v_i$ for all $j \in \{1, 2, \ldots, n\}$. Thus $w_i \in \varphi(V)$ iff $v_i = \max(\{v_1, v_2, \ldots, v_n\})$, $i \in \{1, 2, \ldots, n\}$ by Definition 4.2.6. □

Example 4.2.1. Suppose a language $\mathcal{L}$ includes only 3 different propositional variables, which means that there are totally 8 possible worlds in W, i.e., $W = \{w_1, w_2, \ldots, w_8\}$. Assume that an epistemic vector in this language is $V = \langle -6, 3, 9, 6, 9, 5, 2, 1 \rangle$. Thus, the maturity of V is $|V| \approx 16.5227$. Because $v_1 = -6$, $v_2 = 3$ and $v_3 = 9$, so the agent disbelieves w_1 with a plausibility 15.3948, believes w_2 with a plausibility 16.2481 and believes w_3 with a plausibility 13.8564, and so we can get all plausibilities of w_4-w_8. Moreover, it is obvious that w_1-w_8 satisfy $w_3 \sim w_5 \prec w_4 \prec w_6 \prec w_2 \prec w_7 \prec w_8 \prec w_1$. Therefore the net content of V is $\mathcal{C}_V = \{w_3, w_5\}$.

So far, the complete structure of epistemic states has been constructed. We define these epistemic vectors as the instances of our abstract epistemic states, i.e., for all $v \in \mathcal{V}$, $|V|$, $\hat{V}$ and $\mathcal{C}_V$ are respectively the priority, the belief state and the net content of the epistemic state V.

4.2.4 Direction Vectors of Epistemic Vectors

By Theorem 4.2.2, the plausible order of an epistemic vector V can be decided by the proportionality of all elements in V, i.e., the direction of V. In other words, the direction of V implies the belief state of the epistemic vector V, and the maturity (length) of V implies the priority of V. Therefore, an epistemic vector can be also represented by the pair of its direction Θ_V and its length $|V|$, i.e., $V = \langle \Theta_V, |V| \rangle$. The direction of V is defined by the direction cosines as the following definition:

Definition 4.2.7. A direction vector of an epistemic vector V is denoted by $\Theta_V = \langle \alpha_1, \alpha_2, \ldots, \alpha_n \rangle$. And for all $i \in \{1, 2, \ldots, n\}$, we have

$$\alpha_i = \frac{v_i}{|V|} = \frac{v_i}{\sqrt{\sum_{j=1}^{n} v_j^2}}.$$

If the angle between V and the possible world w_i is θ_i, then α_i is the cosine of θ_i, i.e., $\alpha_i = \cos(\theta_i)$. Thus, $V = \langle \Theta_V, |V| \rangle = \Theta_V \cdot |V| = \langle \alpha_1 \cdot |V|, \alpha_1 \cdot |V|, \ldots, \alpha_1 \cdot |V| \rangle$. Obviously, the smaller the angle between V and w_i is, the more the possible world w_i is believed in V.

There are two purposes for the concept of direction vectors being aroused. Firstly, an epistemic vector's belief state can be changed by changing its direction vector, hence its priority is kept. Secondly, the direction cosines of V can be close related to the possibilistic theory or the bipolar possibilistic theory [5]. Intuitively, the direction cosine $\Theta_V(w_i) = \alpha_i$ can be a posibility distribution on W, because $-1 \leq \alpha_i \leq 1$. We can define that $\alpha_i = -1$ means that the chance of w_i being disbelieved is 100 percent, and contrarily, $\alpha_i = 1$ means that the chance of w_i being believed is 100 percent. We do not want to talk more about the relationship between our theory and the possibilistic theory because of the limited problem scope in this thesis. Please refer [11] if more detail explanations of possibilistic logic are necessary.

4.2.5 Distances of Epistemic Vectors

For measuring the distance between two epistemic vectors, we choose our distance function as an Euclidean distance function because it is the most traditional distance function in Euclidean spaces. The second reason is of the importance of Euclidean distance functions which satify many useful properties [8].

Definition 4.2.8. Let $V = \langle v_1, v_2, \ldots, v_n\rangle$ and $U = \langle u_1, u_2, \ldots, u_n\rangle$ be two epistemic vectors. Then the distance between V and U can be calculated as follow:

$$d(V, U) = \sqrt{\sum_{i=1}^{n}(v_i - u_i)^2}$$

Theorem 4.2.4. *The distance function of epistemic vectors $d : \mathcal{V} \times \mathcal{V} \to \overline{\mathbb{R}^-}$, which is intrduced in Definition 4.2.8, satisfies the postulates (d0)-(d3).*

Proof. Because $d(V, U)$ is the Euclidean distance, the provements of (d0), (d1) and (d2) are trivial. For (d3), let $V = \langle v_1, v_2, \ldots, v_n\rangle$, $U = \langle u_1, u_2, \ldots, u_n\rangle$ and $Y = \langle y_1, y_2, \ldots, y_n\rangle$ be three epistemic vectors for which $\hat{V} = \hat{U} = \hat{Y}$. Hence there exists $k_1, k_2 \in \mathbb{R}$ such that $U = k_1 \cdot V$ and $Y = k_2 \cdot V$. Then $p_V = \sqrt{\sum_{i=1}^{n} v_i^2}$, $p_U = k_1 \cdot \sqrt{\sum_{i=1}^{n} v_i^2}$ and $p_Y = k_2 \cdot \sqrt{\sum_{i=1}^{n} v_i^2}$. Suppose $|p_V - p_U| \leq |p_V - p_Y|$. It follows that $|1 - k_1| \leq |1 - k_2|$. And we have $d(V, U) = \sqrt{\sum_{i=1}^{n}(v_i - u_i)^2} = |1 - k_1| \cdot \sqrt{\sum_{i=1}^{n} v_i^2}$ and $d(V, Y) = \sqrt{\sum_{i=1}^{n}(v_i - y_i)^2} = |1 - k_2| \cdot \sqrt{\sum_{i=1}^{n} v_i^2}$. Thus, $d(V, U) \leq d(V, Y)$, from which the result follows. □

So the distance function in Definition 4.2.8 satisfies all postulates of distance functions. From another point of view, the consistency of (d0)-(d3) has been proved by this result.

4.3 Dynamic Epistemic Vectors

Last section focuses on the static structure of epistemic vectors, and now our attention turns to the dynamics of epistemic vectors. The characteristics of concession functions and combination functions will be studied after we give the definitions of them.

4.3.1 Concessions of Epistemic Vectors

Intuitively, assume that the tail of the vector V is anchored on the origin of the n-dimensional epistemic space, and the length of V is fixed. For conceding an epistemic vector V toward U, the most direct way is to push V toward U. In the process of pushing, the vector V rotates around the origin, and the angle between V and U will be continuely decreased. The above process is complex to be expressed formally because the rotation in a high dimensional space is tricky. Thus, we provide another proposal which is much simple and easy to implement.

Definition 4.3.1. Let $V = \langle v_1, v_2, \ldots, v_n \rangle$ and $U = \langle u_1, u_2, \ldots, u_n \rangle$ be two epistemic vectors. The concession function of epistemic vectors

$$c(V,U) = \frac{|V|}{|V + \Lambda|} \cdot (V + \Lambda)$$

in which $\Lambda = \langle \lambda_1, \lambda_2, \ldots, \lambda_n \rangle$ and

$$\lambda_i = \begin{cases} 0 & \text{if } w_i \notin \min_{\preccurlyeq_V}(\min_{\preccurlyeq_U}(W)) \\ \max(V) - v_i & \text{if } w_i \in \min_{\preccurlyeq_V}(\min_{\preccurlyeq_U}(W)), \text{ and if } \lambda_1 = \ldots = \lambda_{i-1} = 0 \text{ for } i \neq 1 \end{cases}$$

for all $i \in \{1, 2, \ldots, n\}$.

The purpose of function $c(V,U)$ is to promote exactly one possible world, which is one in the net content of U and it is simultaneously the nearest possible world with V's net content, to be one of the most plausible possible worlds of V if $\mathcal{C}_V \cap \mathcal{C}_U = \varnothing$.

Example 4.3.1. Suppose there are two epistemic vectors $V = \langle -6, 3, 9, 6, 9, 5, 2, 1 \rangle$ and $U = \langle 20, 1, 7, 8, -4, 3, -2, 6 \rangle$. When we concede V towards U we have $c(V,U) = \langle 9, 3, 9, 6, 9, 5, 2, 1 \rangle$, and from another direction, when U is conceded towards V the result is $c(U,V) = \langle 20, 1, 20, 8, -4, 3, -2, 6 \rangle$.

Theorem 4.3.1. *The concession function of epistemic vectors in Definition 4.3.1 satisfies the postulates (c0)-(c5).*

Proof. Let $V = \langle v_1, v_2, \ldots, v_n\rangle$ and $U = \langle u_1, u_2, \ldots, u_n\rangle$ be two epistemic vectors. For (c0), suppose λ_i is the only nonzero element in Λ, hence $v_i + \lambda_i = \max(V)$. So $\preccurlyeq_{V+\Lambda} = \preccurlyeq_V + w_i \preccurlyeq w_1, w_i \preccurlyeq w_2, \ldots, w_i \preccurlyeq w_n$, i.e., $\preccurlyeq_V \subseteq \preccurlyeq_{V+\Lambda}$. Because we have $\preccurlyeq_{c(V,U)} = \preccurlyeq_{V+\Lambda}$, so it follows that $\preccurlyeq_V \subseteq \preccurlyeq_{c(V,U)}$. For (c1) and (c2), from Definition 4.3.1, if $\mathcal{C}_V \cap \mathcal{C}_U = \varnothing$, then suppose λ_i is the only nonzero element in Λ, hence we have $w_i \in \mathcal{C}_{c(V,U)} \cap \mathcal{C}_U \neq \varnothing$. If $\mathcal{C}_V \cap \mathcal{C}_U \neq \varnothing$, then $\Lambda = \langle 0, 0, \ldots, 0\rangle$. Thus, it follows that $c(V,U) = 1 \cdot V = V$. The proof of (c3) is trivial. (c4) and (c5) follows from (c0) and (c2) respectively. □

So the postulates of concession functions are consistent.

4.3.2 Addition as Combination Function

The combination of forces in classical physics is an interesting and practical analogy of the combination of epistemic vectors: the combination of epistemic vectors is like the situation that a group agents try to use their forces, i.e., their knowledge, to drag the origin. Thus, this metaphor sparks us to use the addition operator as the combination function of epistemic vectors.

Definition 4.3.2. Let $V = \langle v_1, v_2, \ldots, v_n\rangle$ and $U = \langle u_1, u_2, \ldots, u_n\rangle$ be two epistemic vectors. The combination function of epistemic vectors $f(V,U) = V + U$.

Please refer Figure 4.4 for an example of the combination function.

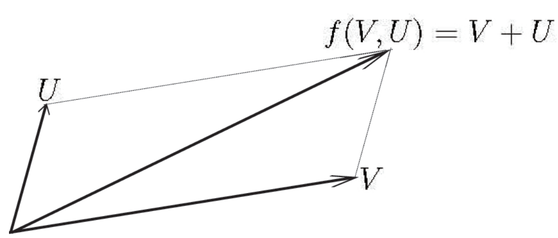

Figure 4.4: Combination of Two Epistemic States

Theorem 4.3.2. *The combination function of epistemic vectors in definition 4.3.2 satifies the postulates (f0)-(f10).*

Proof. The proof of (f0)-(f3) is trivial. Let $V = \langle v_1, v_2, \ldots, v_n\rangle$ and $U = \langle u_1, u_2, \ldots, u_n\rangle$ be two epistemic vectors. For (f4), assume $w_i \preccurlyeq_V w_j$ and $w_i \preccurlyeq_U w_j$. From Theorem 4.2.2, it follows that $v_j \leq v_i$ and $u_j \leq u_i$. Trivially, $v_j + u_j \leq v_i + u_i$, hence $w_i \preccurlyeq_{V+U} w_j$. For (f5), $d(U, f(V,U)) = \sqrt{\sum_{i=1}^{n}(u_i - (v_i + u_i))^2} = \sqrt{\sum_{i=1}^{n} v_i^2} = p_V$ and $d(V, f(V,U)) = \sqrt{\sum_{i=1}^{n} u_i^2} = p_U$. Thus, $d(U, f(V,U)) \leq d(V, f(V,U))$ when $p_V \leq p_U$. (f6)-(f10) follows from (f3), (f4) and (c1) as in Chapter 3. □

Thus, we have also proved the consistency of (f0)-(f10).

4.3.3 Belief Combination on Epistemic Vectors

Finally, we can show that the situation in Example 1.2.1 may happen in our framework:

Lemma 4.3.3. *Let $V = \langle v_1, v_2, \ldots, v_n\rangle$ and $U = \langle u_1, u_2, \ldots, u_n\rangle$ be two epistemic vectors. If $v_i \geq v_j$ and $u_i < u_j$, then there exists $k \geq 0$ such that $v_i + k\cdot u_i = v_j + k \cdot u_j$, $i, j \in \{1, 2, \ldots, n\}$ and $k \in \mathbb{R}$.*

Proof. Obviously, when $v_i \geq v_j$, $u_i < u_j$ and $k = \frac{v_i - v_j}{u_j - u_i}$, we have $k \geq 0$ and $v_i + k \cdot u_i = v_j + k \cdot u_j$. □

Lemma 4.3.3 implies that the difference of statuses between two possible worlds of an epistemic vector can be evened by another suitable epistemic vector.

Lemma 4.3.4. *Let $V = \langle v_1, v_2, \ldots, v_n\rangle$ and $U = \langle u_1, u_2, \ldots, u_n\rangle$ be two epistemic vectors. If $v_i \geq v_j$ and $u_i < u_j$, then there exists $\xi \geq 0$ such that $v_i + k \cdot u_i < v_j + k \cdot u_j$ for all $k > \xi$, $i, j \in \{1, 2, \ldots, n\}$ and $k, \xi \in \mathbb{R}$.*

Proof. Let $v_i \geq v_j$, $u_i < u_j$ and $\xi = \frac{v_i - v_j}{u_j - u_i}$. From Lemma 4.3.3, we have $v_i + \xi \cdot u_i = v_j + \xi \cdot u_j$. Assume that $\delta \in \mathbb{R}$ and $\delta > 0$ such that $k = \xi + \delta$. It follows that

$$v_i + k \cdot u_i = v_i + (\xi + \delta) \cdot u_i = v_i + \xi \cdot u_i + \delta \cdot u_i$$

and

$$v_j + k \cdot u_j = v_j + (\xi + \delta) \cdot u_j = v_j + \xi \cdot u_j + \delta \cdot u_j$$

Consequently, $v_i + k \cdot u_i < v_j + k \cdot u_j$. □

Lemma 4.3.5. *Let* $V = \langle v_1, v_2, \dots, v_n \rangle$ *and* $U = \langle u_1, u_2, \dots, u_n \rangle$ *be two epistemic vectors. If* $w_i \preccurlyeq_V w_j$ *and* $w_j \prec_U w_i$*, then there exists* $\xi \geq 0$ *such that* $w_j \prec_{V+k \cdot U} w_i$ *for all* $k > \xi$*,* $i, j \in \{1, 2, \dots, n\}$ *and* $k, \xi \in \mathbb{R}$*.*

Proof. Let $w_i \preccurlyeq_V w_j$ and $w_j \prec_U w_i$. It follows that $v_i \geq v_j$ and $u_i < u_j$ by Theorem 4.2.2. From Lemma 4.3.4, then there exists $\xi \geq 0$ such that $v_i + k \cdot u_i < v_j + k \cdot u_j$ for all $k > \xi$, $i, j \in \{1, 2, \dots, n\}$ and $k, \xi \in \mathbb{R}$, from which the result follows by, again, Theorem 4.2.2. □

Lemma 4.3.4 and Lemma 4.3.5 together show that the statuses of two possible worlds of an epistemic vector can be reversed by another suitable epistemic vector.

Theorem 4.3.6. *Let* $V = \langle v_1, v_2, \dots, v_n \rangle$ *and* $U = \langle u_1, u_2, \dots, u_n \rangle$ *be two epistemic vectors. There exists* $\xi \geq 0$ *such that* $\mathcal{C}_{V+k \cdot U} \subseteq \mathcal{C}_U$ *for all* $k > \xi$*,* $k, \xi \in \mathbb{R}$*.*

Proof. Suppose $w \in \mathcal{C}_U$, hence $w \prec_U w'$ for all $w' \notin \mathcal{C}_U$. From Lemma 4.3.5, there exists $\xi \in \mathbb{R}$ such that $w \prec_{V+\xi \cdot U} w'$ for all $w' \neq w$ or $w \in \mathcal{C}_{V+\xi \cdot U}$, from which the result follows. □

Therefore, the combination of two epistemic vectors can totally accept the epistemic vector whose priority is high enough. Moreover, Theorem 4.3.6 further reveals that our combination function will satisfy the (Success) postulate in AGM theory when the difference of the maturities of two epistemic vectors becomes extremely high.

4.4 Related Works with Belief Change on Epistemic Vectors

Now, we have introduced the complete idea of epistemic vectors and provided some dynamic mechanisms on this kind of epistemic states. In this section, we will compare our epistemic vector theory with some existing works.

There are two theories which attract our attention: Thomas Meyer's combination operations (Section 2.3.3), and Sébastien Konieczny, Jérôme Lang and Pierre Marquis's distance based merging theory (Section 2.3.4). At first glance, Thomas Meyer's combination operators are similar with the combinations of epistemic vectors, i.e., they are all operations of combining the quantified pre-orders. Then it seems that the combination operators on epistemic vectors can be instantialized by distance based merging framework. Therefore, our discussion will focus on these two theories.

4.4.1 Comparing with Combination Operations

An epistemic vector V and a Thomas Meyer's epistemic state Φ are all functions from the set of all possible worlds to the set of numbers, i.e., $V : W \to \mathbb{R}$ and $\Phi : W \to \overline{\mathbb{I}^-}$. Obviously, V has more power of expression, e.g., V can assign negtive numbers to possible worlds to represent the degrees of dislike. One important thing is that V will never get into the inconsistency while Φ has such chance, i.e., when $\Phi(w) \neq 0$ for all $w \in W$ (the ancestors of Φ - ordinal conditional functions - have the mechanism to prevent the inconsistency, refer Section 2.2.3).

The above characteristic of Φ then arouses (CO0) which is actually an embeded operator in a combination operator. It is easy to get that the combination functions of epistemic vectors satisfy (CO2), (CO3) and (CO5). Moreover, the postulate (f5) stipulate the direction of combination, which does not occur in Thomas Meyer's theory. V can be prioritized by its own content, hence the direction of combination functions are mainly controled by the properties of epistemic vectors. In combination operation theory, there is no problem to build "free directional operators" (i.e., the operators which do not force the results of combination to include or exclude something) because (BM4) has been abandoned. Unfortunately, combination operation theory has just provided some very weak postulates to direct the combination results.

Let us observe a combination operation Π_Σ which is very similar to our combination function of epistemic vectors: let $\mathcal{E} = [\Phi_1, \Phi_2, \ldots, \Phi_n]$ be a finite epistemic list. Let $\Phi^{\mathcal{E}}_{\Sigma}(w) = \sum_{i=1}^{n} \Phi_i(w)$. Then $\Pi_\Sigma(\mathcal{E})(w) = \Phi^{\mathcal{E}}_{\Sigma}(w) - \min(\{\Phi^{\mathcal{E}}_{\Sigma}(w) | w \in W\})$. It has been proved in [42] that Π_Σ satisfies (CO0)-(CO6), (CO-Comm), (CO-Majority) and (BM6), and it does not satisfy (CO-Arbitration) and (BM4).

Based on our postulates of combination functions, it is easy to prove that Π_Σ satisfies (f0)-(f2), (f4), (f6) and (f7), and it does not satisfy (f3), (f5) and (f8)-(f10).

4.4.2 Comparing with Distance Based Merging

The most important thing is that in distance based merging theory, from the geometrical point of view, the possible worlds can be treated a points in space, hence there is the concept of the distances between possible worlds. But in our theory, possible worlds are a bunch of measures which are used to locate the positions of points (Figure 4.5). If we must define the distances between possible worlds in epistemic vectors theory, then the distance between any two possible worlds should be zero because the n possible worlds are n orthogonal straight lines which are jointed at the origin. Therefore, (DBM-pd2) is not feasible for the distances between epistemic vectors.

We can revise (DBM-pd2) to make the postulates of distance functions in distance based merging theory to adapt the concepts of epistemic vectors.

(DBM-pd2-ev) $pd(w_i, V) = \sqrt{\sum_{j=1}^{n} v_j^2 - v_i^2}$, for all $w_i \in W$ and any epistemic vector V.

4.4.3 Other Related Works

Similar with epistemic vectors theory, the bipolar representation of knowledge introduced in [5] is an interesting knowledge representation in which both possible and

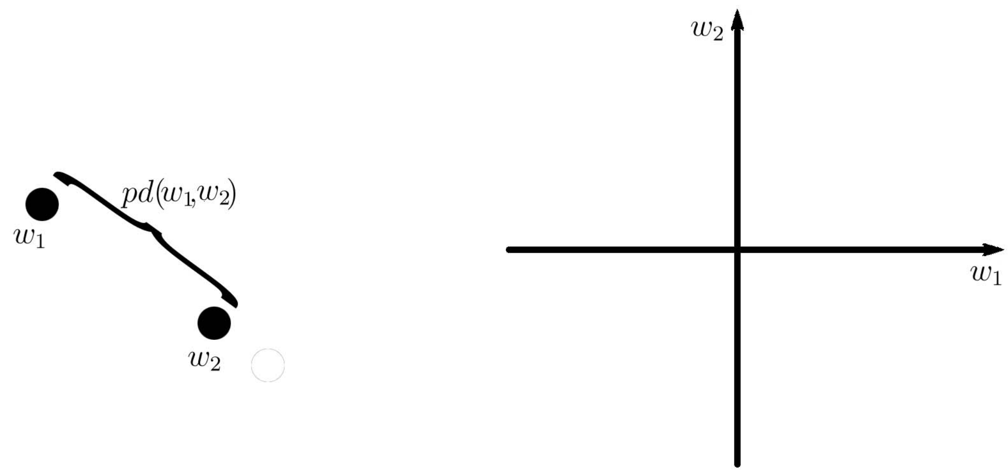

Figure 4.5: Possible Worlds in Distance Based Merging Theory (Left) and Epistemic Vectors Theory (Right)

negative preferences can be expressed. Epistemic vectors utilize the signals of faith to indicate agents' agreements or disagreements of belief, and [5] uses sets of goals and rejections to capture the similar function. Although the whole story of epistemic vectors is in the scope of deterministic epistemology so far, similar with its kin - ordinal conditional functions theory - it has a close relationship with probability theory. On the theory of possibilistic knowledge combination, we can find some useful conclusions from [6] and [10], and also a theory between possibilistic logic and classical logic [9]. Epistemic vectors may be further developed into the field of possibilistic theory.

4.5 Summary

We have tried to use EOCF to fit the framework proposed in Chapter 3 at the beginning of this chapter. Since the problems of EOCF have been pointed out, we turned to a new model of epistemic states. Then, the concepts of epistemic vectors have been introduced. By employing the knowledge of high dimensional analytic geometry, the problem of fusing the belief states and the priorities has been solved properly. After constructing the dynamic mechanism of epistemic vectors, we showed that the combination function on epistemic vectors fulfilled requirements in our proposal. In the end, we have compared the epistemic vectors theory with the combination operations theory and the distance based merging theory.

Chapter 5

Conclusion and Future Work

It seems that the content of the study of belief change consists of two aspects: the representation of agents' beliefs and the dynamics of agents' beliefs. The former investigates the static properties of knowledge, and the latter reveals the rules of knowledge change. Therefore, almost every method of belief change has been studied in both static and dynamic way and so did we.

In this thesis, a framework of epistemic states has been proposed in the first place to constrain the scope in which the belief changes would happen. In Chapter 3, a (Class 3) framework which consists of an abstract representation of belief states and priorities has been introduced. Then we established a framework to measure the distance between two epistemic states. Remarkablely, different with the concepts of distances in distance based revision [3, 32, 48], d is a kind of real distances but not a pseudo-distance. Thus, this kind of distance functions is ready for a totally quantitative method of knowledge representation. Next, the framework of concession functions has been provided for relaxing the restrictions of the belief states implied by the relevant epistemic states, i.e., contracting some sentences from the belief states implied by the relevant epistemic states. The significance of concession functions is to provide a method for changing an epistemic state toward another one as a target.

In Chapter 3, (d3) and (c3) have revealed separability of the belief states and the priorities of epistemic states. In other words, in our proposed frameworks, we have the ability to change the belief state of an epistemic state without affecting its priority, and vice versa. Furthermore, the belief state and the priority of one epistemic state are two interrelated independent components.

In the end of Chapter 3, a set of postulates of combination functions has been provided. Before proposing any postulates, our first assumption is that the combination functions should be binary. This assumption has aroused the motivation of (f1)-(f3) which ensure f's ability of iteration as a binary operator. (f4) has revealed some potential relations between our combination functions and the AGM revision operators. Then (f5) has ensured the direction of belief change.

Following the discussion of Chapter 3, Chapter 4 has proposed a detailed structure of epistemic states and an detailed method of the belief combinations on such epistemic states. Through the epistemic state and the combination function in Chapter 4, we have also proved the consistency of all postulates proposed in Chapter 3.

The proposed epistemic states has been called epistemic vectors in this dissertation because we have utilized the n-dimensional vectors to bear the properties of epistemic states. Thus, the knowledge representation by epistemic vectors is a totally quantitative method of knowledge representations. Naturally, it has inherited many characteristics of normal n-dimensional vectors, but we have only focused on those characteristics which are related to our topic.

Firstly, the definition of epistemic vectors has been established in Chapter 4. And then, the mechanisms of the integration of belief states and priorities have been introduced. Based on these definitions, each epistemic vector implies two different total pre-orders: the total pre-order of all possible worlds W and the total pre-order of all epistemic states (or epistemic vectors) $\mathcal{S}$ which imply respectively the entrenchment of beliefs and the importances of agents.

Finally, the addition of epistemic vectors has been taken as the combination operation of epistemic vectors. Moreover, we have proved this combination operator which satisfies our purpose of this thesis: to banish the borderline between the belief change operations on prioritised and non-prioritised agents.

One important problem which has not been discussed is the meaning of the zero epistemic vector $V = \langle v_1, v_2, \ldots, v_n \rangle$ in which $v_1 = v_2 = \ldots = v_n = 0$. For convenience, the zero epistemic vector is denoted by $\mathbf{0}$. Obviously, $V + \mathbf{0} = V$, $c(V, \mathbf{0}) = V$ and $c(\mathbf{0}, V) = \mathbf{0}$ for all V. Moreover, $\mathbf{0}$ has no norm because the maturity of $\mathbf{0}$ is 0, i.e., $|\mathbf{0}| = 0$. Therefore, $\mathbf{0}$ is a tautology with the mature 0. In other words, $\mathbf{0}$ is an epistemic state on which an agent has no ideal about anything, i.e., a "baby" agent. The problem is whether it is correct to allow the combination of two non-zero epistemic vectors to be $\mathbf{0}$. If it is necessary, for all $V \neq \mathbf{0}$ and $U \neq \mathbf{0}$ such that $f(V, U) = \mathbf{0}$, we may use the concession function to "shake" the result out of the zero epistemic vector, i.e., $f(c(V, U), U)$ or $f(V, c(U, V))$.

Furthermore, by treating the zero epistemic vector as the epistemic state of a "baby" agent, we can use $\mathbf{0}$ as a start point of an agent's life scope. In other words, each agent is born with the zero epistemic vector, and since then, it continuously acquire knowledge from the outer world to build up its own epistemic state. By utilizing the concepts of epistemic vectors, we can give a function $l : T \to \mathcal{V}$, in which T is the set of time and $\mathcal{V}$ is the set of all epistemic vectors, to describe the change of the agent's epistemic state in its whole life, i.e., $l(t) = V$ means the agent's epistemic vectors is V at time t and $l(0) = \mathbf{0}$. Therefore, this gives us the chance to use the mathematical analysis to analyse the epistemic behaviors of agents or groups of agents.

Our epistemic vectors are vectors in a finite dimensional space. Another direction of the study of epistemic vectors may be the epistemic vectors in a infinite dimensional space, hence the research will be taken into the territory of functional analysis. Another interesting idea is that if every epistemic vector is taken as a point in a n-dimensional space, then we may connect the belief change with the pattern recognition. These all will be left to future work.

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