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Fourth Order Geometric Evolution Equations

A thesis submitted in fulfilment of the requirements for the award of the degree

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by

Glen Edward Wheeler, B. Comp. Sci., B. Math. Adv. (Hons.) First Class

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Abstract. In this thesis the chief object of study are hypersurface flows of fourth order, with the speed of the flow varying from the Laplacian of the mean curvature, to the more general constrained flows which include a function of time in the speed, and satisfy various conditions. Our aim is to instigate a study of the regularity of these flows, answering questions of local and global existence, and some preliminary singularity analysis. Among our results are positive lower bounds for smooth and regular existence, classification of stationary solutions, interior estimates, and blowup asymptotics. Applying these results to a certain class of constrained surface diffusion flows, we obtain long time existence and exponential convergence to spheres for initial surfaces with small L^2 norm of tracefree curvature. We present one application of this theorem, using it to deduce the isoperimetric inequality with optimal constant for 2-surfaces satisfying the above smallness condition. The theorem can be thought of as a stability of spheres result, as the smallness condition is an averaged distance from a standard round sphere to the initial manifold in L^2. This strengthens a related earlier result specialised to surface diffusion flow where the distance is small in $C^{2,\alpha}$, obtained through a completely different method. The results throughout this thesis are new contributions for both surface diffusion flow, which has been considered by many authors, and the constrained flows, which have only recently been considered.

Certification

I, Glen Edward Wheeler, declare that this thesis "Fourth Order Geometric Evolution Equations" submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the School of Mathematics and Applied Statistics, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged below. The first publication from this thesis, "Lifespan Theorem for Constrained Surface Diffusion Flows" is under supervision of Dr. McCoy and Assoc. Prof. Williams with each contributing approximately 20%. It is convention to list authors alphabetically by last name regardless of contribution. Later work in the thesis was completed with significantly less input from supervisors and the second publication "Surface Diffusion Flow Near Spheres" is wholly my own work. The document has not been submitted for qualifications at any other academic institution.

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