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Pricing Volatility Derivatives with Stochastic Volatility

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CERTIFICATION

I, Guanghua Lian, declare that this thesis, submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the School of Mathematics and Applied Statistics, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. The document has not been submitted for qualifications at any other academic institution.

Guanghua Lian

March, 2010

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Abstract

Volatility derivatives are products where the volatility is the main underlying notion. These products are particularly important for market investors as they use them to have insight into the level of volatility to efficiently manage the market volatility risk. This thesis makes a contribution to literature by presenting a set of closed-form exact solutions for the pricing of volatility derivatives.

The first issue is the pricing of variance swaps, which is discussed in Chapter 2, 3, and 4. We first present an approach to solve the partial differential equation (PDE), based on the Heston (1993) two-factor stochastic volatility, to obtain closed-form exact solutions to price variance swaps with discrete sampling times. We then extend our approach to price forward-start variance swaps to obtain closed-form exact solutions. Finally, our approach is extended to price discretely-sampled variance by further including random jumps in the return and volatility processes. We show that our solutions can substantially improve the pricing accuracy in comparison with those approximations in literature. Our approach is also very versatile in terms of treating the pricing problem of variance swaps with different definitions of discretely-sampled realized variance in a highly unified way.

The second issue, which is covered in Chapter 5, and 6, is the pricing method for volatility swaps. Papers focusing on analytically pricing discretely-sampled volatility swaps are rare in literature, mainly due to the inherent difficulty associated with the nonlinearity in the pay-off function. We present a closed-form exact solution for the pricing of discretely-sampled volatility swaps, under the framework of Heston (1993) stochastic volatility model, based on the definition of the so-called average of realized volatility. Our closed-form exact solution for discretely-sampled volatility swaps can significantly reduce the computational time in obtaining numerical values for the discretely-sampled volatility swaps, and substantially improve the computational accuracy of discretely-sampled volatility swaps, comparing with the continuous sampling approximation. We also investigate the accuracy of the well-known convexity correction approximation in pricing volatility swaps. Through both theoretical analysis and numerical examples,

we show that the convexity correction approximation would result in significantly large errors on some specifical parameters. The validity condition of the convexity correction approximation and a new improved approximation are also presented.

The last issue, which is covered in Chapter 7 and 8, is the pricing of VIX futures and options. We derive closed-form exact solutions for the fair value of VIX futures and VIX options, under stochastic volatility model with simultaneous jumps in the asset price and volatility processes. As for the pricing of VIX futures, we show that our exact solution can substantially improve the pricing accuracy in comparison with the approximation in literature. We then demonstrate how to estimate model parameters, using the Markov Chain Monte Carlo (MCMC) method to analyze a set of coupled VIX and S&P500 data. We also conduct empirical studies to examine the performance of the four different stochastic volatility models with or without jumps. Our empirical studies show that the Heston stochastic volatility model can well capture the dynamics of S&P500 already and is a good candidate for the pricing of VIX futures. Incorporating jumps into the underlying price can indeed further improve the pricing the VIX futures. However, jumps added in the volatility process appear to add little improvement for pricing VIX futures. As for the pricing of VIX options, we point out the solution procedure of Lin & Chang (2009)'s pricing formula for VIX options is wrong, and alert the research community that this formula should not be further used. More importantly, we present a new closed-form pricing formula for VIX options and demonstrate its high efficiency in computing the numerical values of the price of a VIX option. The numerical examples show that results obtained from our formula consistently match up with those obtained from Monte Carlo simulation perfectly, verifying the correctness of our formula; while the results obtained from Lin & Chang (2009)'s pricing formula significantly differ from those from Monte Carlo simulation. Some other important and distinct properties of the VIX options (e.g., put-call parity, the hedging ratios) have also been discussed.

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