

University of Wollongong - Research Online

Thesis Collection

Title: Directed graphs and k-graphs: topology of the path space and how it manifests in the associated C*-algebra

Author: Samuel Brendon Webster

Year: 2010

Repository DOI:

Copyright Warning

You may print or download ONE copy of this document for the purpose of your own research or study. The University does not authorise you to copy, communicate or otherwise make available electronically to any other person any copyright material contained on this site.

You are reminded of the following: This work is copyright. Apart from any use permitted under the Copyright Act 1968, no part of this work may be reproduced by any process, nor may any other exclusive right be exercised, without the permission of the author. Copyright owners are entitled to take legal action against persons who infringe their copyright. A reproduction of material that is protected by copyright may be a copyright infringement. A court may impose penalties and award damages in relation to offences and infringements relating to copyright material.

Higher penalties may apply, and higher damages may be awarded, for offences and infringements involving the conversion of material into digital or electronic form.

Unless otherwise indicated, the views expressed in this thesis are those of the author and do not necessarily represent the views of the University of Wollongong.

Research Online is the open access repository for the University of Wollongong. For further information contact the UOW Library: research-pubs@uow.edu.au

2010

Directed graphs and k-graphs: topology of the path space and how it manifests in the associated C^* -algebra

Samuel Brendon Webster
University of Wollongong

Follow this and additional works at: <https://ro.uow.edu.au/theses>

University of Wollongong

Copyright Warning

You may print or download ONE copy of this document for the purpose of your own research or study. The University does not authorise you to copy, communicate or otherwise make available electronically to any other person any copyright material contained on this site.

You are reminded of the following: This work is copyright. Apart from any use permitted under the Copyright Act 1968, no part of this work may be reproduced by any process, nor may any other exclusive right be exercised, without the permission of the author. Copyright owners are entitled to take legal action against persons who infringe their copyright. A reproduction of material that is protected by copyright may be a copyright infringement. A court may impose penalties and award damages in relation to offences and infringements relating to copyright material.

Higher penalties may apply, and higher damages may be awarded, for offences and infringements involving the conversion of material into digital or electronic form.

Unless otherwise indicated, the views expressed in this thesis are those of the author and do not necessarily represent the views of the University of Wollongong.

Recommended Citation

Webster, Samuel Brendon, Directed graphs and k-graphs: topology of the path space and how it manifests in the associated C^* -algebra, Doctor of Philosophy thesis, University of Wollongong. School of Mathematics and Applied Statistics, University of Wollongong, 2010. <https://ro.uow.edu.au/theses/3175>

NOTE

This online version of the thesis may have different page formatting and pagination from the paper copy held in the University of Wollongong Library.

UNIVERSITY OF WOLLONGONG

COPYRIGHT WARNING

You may print or download ONE copy of this document for the purpose of your own research or study. The University does not authorise you to copy, communicate or otherwise make available electronically to any other person any copyright material contained on this site. You are reminded of the following:

Copyright owners are entitled to take legal action against persons who infringe their copyright. A reproduction of material that is protected by copyright may be a copyright infringement. A court may impose penalties and award damages in relation to offences and infringements relating to copyright material. Higher penalties may apply, and higher damages may be awarded, for offences and infringements involving the conversion of material into digital or electronic form.

DIRECTED GRAPHS AND k -GRAPHS: TOPOLOGY OF
THE PATH SPACE AND HOW IT MANIFESTS IN THE
ASSOCIATED C^* -ALGEBRA

A thesis submitted in fulfilment of the requirements for the award of the degree

DOCTOR OF PHILOSOPHY IN MATHEMATICS

from

UNIVERSITY OF WOLLONGONG

by

SAMUEL B.G. WEBSTER, B.MATH (HONS)

2010

Declaration

I, Samuel B.G. Webster, declare that this thesis, submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the School of Mathematics and Applied Statistics, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. The document has not been submitted for qualifications at any other academic institution.

Samuel B.G. Webster

Acknowledgements

To my supervisors Professor Iain Raeburn and Doctor Aidan Sims, I extend my sincerest thanks: for their invaluable comments and suggestions for this thesis; and for their enthusiasm during my undergraduate years at the University of Newcastle, inspiring my interest in pure mathematics to this day. I'd also like to give a nod to the Universities of Newcastle and Wollongong: for their supply of red pens that were sacrificed by Iain in his comments and suggestions that have improved my writing to no end; and for supplying a stable internet connection to Aidan, facilitating lengthy discussions on the subtle nuances of dragon training.

Not directly related to my interest in maths, but affecting it in one way or another, I'd like to thank my girlfriend, Nikki, and my family for their support and encouragement. I'd like to thank my friends Darren and Marilyn for offering me a place to stay when I needed it, and for keeping me suitably distracted; whether it was in the form of trivia competitions, idle facebook chat or subtle encouragement to maintain my hobbies, I was able to reduce stress to an acceptable level approaching my thesis submission date.

Contents

Abstract	ix
Chapter 1. Introduction	1
1.1. Overview of the Thesis	7
Chapter 2. Directed Graphs	11
2.1. Topology	12
2.2. Desingularisation	16
2.3. Graph C^* -algebras	24
2.4. The Diagonal and the Spectrum	27
Chapter 3. Higher-Rank Graphs	37
3.1. Topology	43
3.2. Removing Sources	49
3.3. Topology of Path Spaces under Desourcification	71
3.4. High-Rank Graph C^* -algebras	81
3.5. The Diagonal and the Spectrum	85
3.6. From k -Coloured to Rank- k	91
Appendix A. C^* -algebras	97
Bibliography	99

Abstract

Directed graphs and their higher-rank analogues provide an intuitive framework for the analysis of a broad class of C^* -algebras which we call graph algebras. Kumjian, Pask, Raeburn and Renault built a groupoid \mathcal{G}_E from the infinite-path space of a locally finite directed graph E , and used the theory of groupoid C^* -algebras to define the graph C^* -algebra. Local finiteness was required so that \mathcal{G}_E was locally compact and r -discrete, with unit space homeomorphic to the infinite path space of E . Similarly, the higher-rank graphs of Kumjian and Pask were initially studied with similar restrictive hypotheses in order to use groupoid based analysis of their higher-rank C^* -algebras. In particular, the topology on the path space of a directed graph or higher-rank graph is crucial in the analysis of graph C^* -algebras.

Drinen and Tomforde defined a process called desingularisation which can be used to extend many results about the C^* -algebras of locally finite directed graphs to those of arbitrary directed graphs. Drinen and Tomforde construct from an arbitrary directed graph E a row-finite directed graph \widehat{E} with no sources such that $C^*(E)$ embeds in $C^*(\widehat{E})$ as a full corner. Subsequently, Farthing developed a partial desingularisation for higher-rank graphs, which constructs from a row-finite higher-rank graph Λ with sources a row-finite higher-rank graph $\widetilde{\Lambda}$ with no sources such that $C^*(\Lambda)$ embeds in $C^*(\widetilde{\Lambda})$ as a full corner.

In Chapter 2, we construct a topology on the path space of an arbitrary directed graph E and prove that it is locally compact and Hausdorff. We show that there is a homeomorphism ϕ_∞ from a subspace of the infinite-path space of the Drinen-Tomforde desingularisation \widehat{E} onto the boundary-path space ∂E of E . We then show that there is a commutative C^* -subalgebra D_E of $C^*(E)$ which is homeomorphic to the continuous functions on ∂E . Concluding our results on directed graphs, we show that the embedding of $C^*(E)$ in $C^*(\widehat{E})$ restricts to an embedding of D_E in $D_{\widehat{E}}$ which implements ϕ_∞ . In Chapter 3, we develop a modification of Farthing's desingularisation procedure for row-finite higher-rank graphs which contains cleaner proofs of her results. We use this modification to prove analogues for higher-rank graphs of the results from Chapter 2.