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Numerical Modelling of the Propagation of Ocean Waves

A thesis submitted in partial fulfilment of the requirements
for the award of the degree

DOCTOR OF PHILOSOPHY

from

UNIVERSITY OF WOLLONGONG

by

Huan-Wen Liu
BSc, MSc

School of Mathematics and Applied Statistics
2001

Declaration

In accordance with the regulations of the University of Wollongong, I hereby state that the work described herein is my own original work, except where due references are made, and has not been submitted for a degree at any other universities or institutions.

Huan-Wen Liu

Date:

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Abstract

This thesis considers the refraction and diffraction of both linear and nonlinear waves. In the first part, a linear numerical model based on the dual reciprocity boundary element method (DRBEM) is presented for the study of combined diffraction and refraction of linear waves. This model is more general than that presented by Zhu (1993a) in the sense that areas or coastlines where the water depth is zero can be successfully dealt with. Our comparison study shows that the new model is very accurate for the propagation of long waves such as tsunamis. Moreover, it is numerically very efficient in comparison with models based on finite elements or differences. Using the new model, the interaction between the diffractive and refractive effects is examined.

In the second part, a numerical model is developed by expanding the Boussinesq equations using a perturbation method and the DRBEM. Based on the assumption that the incident waves are harmonic, the time-dependent nonlinear Boussinesq equations are transformed into three time-independent linear equations, where no approximation for the seabed slope is made. Then the first-order solution η_0 is found as the solution of the linear shallow-water equation. The first-order solution is then used in the governing equations at second-order. By employing a transformation, all the third-order and the fourth-order partial derivatives of η_0 in the right-hand sides are

removed, resulting in the minimization of any errors which occur in approximating these derivatives. To validate the new model, the wave run-ups of weakly-nonlinear waves scattered by islands are found. Thirteen cases of run-ups around a vertical cylindrical island are considered and it is found that the nonlinear and dispersive contributions of the new model are significant and a much better comparison with experimental results is obtained than for the linear diffraction theory. The combined wave diffraction and refraction by a conical island is also modelled and discussed. Our model is found to be more accurate than other nonlinear models as the dispersive effects have been included, but is also more computationally efficient since there is no time marching and the spatial dimensionality of the numerical calculation has been reduced by one with the adoption of the DRBEM.

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Symbols and abbreviations

α	coefficient vector
α_j	coefficients
$\alpha(\xi)$	internal angle of the boundary at point ξ
γ	Euler's constant ≈ 0.57721
Γ	boundary
Γ_i	coastline of an island
Γ_o	toe of a conical island
Γ_B	artificial boundary
δ	Dirac delta function
ϵ	wave nonlinearity, small parameter in the perturbation solution
η	free surface elevation without time-dependence
η^I	free surface elevation of incident waves without time-dependence
η_0	free surface elevation of 1st-order without time-dependence
η_1	free surface elevation of 2nd-order fundamental frequency without time-dependence
η_2	free surface elevation of 2nd-order double frequency without time-dependence

η_0^s	free surface elevation of 1st-order scattered waves without time-dependence
η_1^s	free surface elevation of 2nd-order scattered waves at fundamental frequency without time-dependence
η_2^s	free surface elevation of 2nd-order scattered waves at double frequency without time-dependence
ϕ	velocity potential without time-dependence
ϕ^I	velocity potential of incident waves without time-dependence
ϕ^s	velocity potential of scattered waves without time-dependence
ϕ^*	$= H_0^{(1)}$, Hankel function of first kind and order zero
ϕ_β	$= (c^2 + \ \mathbf{x} - \mathbf{x}_j\ ^2)^{\beta/2}$, the family of multiquadrics
$\hat{\phi}_j$	particular solution of inhomogenous Helmholtz equation
Φ	velocity potential with time-dependence
θ	polar coordinate
ω	wave frequency
Ω	two-dimensional domain
ξ	source point
ζ	free surface elevation with time-dependence
ζ^s	free surface elevation of scattered waves with time-dependence
ζ_{inc}	free surface elevation of incident waves with time-dependence
ζ_{phys}	physical surface elevation, real part of ζ
a	radius of shoreline circle
A	amplitude of incident waves

b	radius of island toe
C	phase velocity of waves
C_o	phase velocity of waves in outer region Ω_o
C_g	group velocity of waves
$C_{g,o}$	group velocity of waves in outer region Ω_o
f_j	radial basis function, interpolation function
g	gravitational acceleration
h	water depth (varying)
h_i	shoreline water depth
h_o	constant water depth in outer regin
H	wave height
$H_0^{(1)}$	Hankel function of first kind and order zero
i	imaginary unit, subscript
J_n	Bessel function of the first kind and order n
k	wave number, $k = 2\pi/L$
l	number of collocation points in domain
L	wave length
m	number of points on Γ_o
n	number of points on Γ_i
\mathbf{n}	outward normal unit vector of the inner domain Ω_i
\mathbf{n}'	outward normal unit vector of the inner domain Ω_o

p	nonuniform pressure distribution applied to the free surface
P	pressure
q	directional derivatives of η along the direction \mathbf{n}
r, θ, z	cylindrical coordinates with origin at the centre of a cylinder
Re	real part of a complex quantity
t	time
T	wave period
$\bar{\mathbf{u}}$	depth-averaged horizontal velocity vector
$\bar{\mathbf{u}}_0$	1st-order depth-averaged horizontal velocity vector
$\bar{\mathbf{u}}_1$	depth-averaged horizontal velocity vector of 2nd-order at fundamental frequency
$\bar{\mathbf{u}}_2$	depth-averaged horizontal velocity vector of 2nd-order at double frequency
x, y, z	rectangular coordinates
\mathbf{x}	$= (x, y)$
ATPS	augmented thin plate spline
BEM	boundary element method
DRBEM	dual reciprocity boundary element model
GDRBEM	general dual reciprocity boundary elements model
MSL	mean surface level
MSWE	the mild-slope wave equation

PDRBEM	perturbation dual reciprocity boundary elements model
RBF	radial basis function
TPS	thin plate spline