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Shams Tamjeed Huque
University of Wollongong

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ANALYTICAL AND NUMERICAL INVESTIGATIONS INTO BELT CONVEYOR TRANSFERS

by

Shams Tamjeed Huque

Bachelor of Engineering (Mechanical), Graduate Certificate in Business

A thesis submitted in fulfilment of the
requirements for the award of the degree

Doctor of Philosophy

from

**University of Wollongong
Faculty of Engineering, School of Mechanical,
Materials & Mechatronic Engineering**

December 2004

Abstract

The mining industry is an immense field with granular flows (e.g. coal) occurring in numerous areas. Accordingly there are a significant number of problems that arise, with a great number requiring solutions that are difficult to achieve by conventional industrial means. The modelling of granular flow using the numerical technique known as Distinct Element Method (DEM) has great potential in industry, particularly for solving transfer point problems. The advantage of DEM for transfer applications is that an entire system can be simulated using the single numerical technique, as opposed to the existing situation where a myriad of design techniques are required (e.g. analytical solution for one component and graphical solution for another). DEM involves solving the equations of motion for the trajectory/rotation/orientation of each particle and modelling each collision between particles and between particles and boundary objects.

The research presented a comprehensive overview of all of the available analytical processes available to design chute system components, such as material trajectory calculations, impact plate models, and gravity flow chute aspects. To the author's knowledge, this was the first such review in the literature. A detailed comparison between the most common analytical design methods was conducted, recommendations for which method to use were established, and areas of weakness and further study were identified. It was found that: most areas apart from the prediction of the initial material discharge and trajectory were lacking in design method; often the few available design methods for chute components, such as impact plates and gravity flow chutes, were lengthy and often difficult to implement.

A computer code was developed during the course of the research to simulate bulk material using the Distinct Element Method (DEM). A background into DEM and its application to modelling material flow at transfer points was presented. One major drawback found in the recent transfer studies was the lack of quantification of the velocity distributions obtained using the DEM against existing analytical design theories. Contour coloured particulate simulations have also been recently produced by a number of companies (e.g. Overland Conveyor Company Inc.) however the flow

regimes observed from the relevant simulation screen captures were not adequately scrutinised. All the DEM mathematical formulation and numerical methods utilised for the current work were comprehensively described and relevant computational aspects were also detailed, such as the coding of a pre-processor and post-processor allowing animations of the DEM particles. A series of tests was conducted to gauge the validity of the computer code, and this produced satisfactory results.

The DEM code was also applied to simulate two separate transfers originally designed by The Gulf Group using their EasyFlowTM technology, and currently in operation in industry in Lithgow, Australia. By observing animation screen captures the current research confirmed the advantage of maintaining particle speed through the system when using curved chute elements. Quantitative DEM velocity data were compared to the velocities predicted by the most favourable analytical methods. It was found that DEM generally produced velocity regimes close to those of the analytical techniques. However it also provided the additional benefit of providing data on stream characteristics such as impact forces and velocities in the vicinity of the hood and spoon elements, which are difficult to examine in detail using analytical methods. An analysis of the micro dynamics of individual particles also identified that there are differing scales of contact during the flow through a chute. Although the analytical methods do not allow closer scrutiny of the flowing stream at the micro scale, they have the advantage of providing much faster solutions and are good for chute designs for free flowing material transfers.

Disclaimer

I, Shams T. Huque, declare that this thesis, submitted in fulfilment of the requirements for the award of Doctor of Philosophy, in the School of Mechanical, Materials & Mechatronic Engineering, Faculty of Engineering, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged, as defined by the University's policy on plagiarism, and that I may have received assistance from others on style, presentation and further formatting aspects. The document has not been submitted for qualifications at any other academic institution.

Shams T. Huque

16 December 2004

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Contents

Abstract	i
Disclaimer	iii
Acknowledgements	iv
Contents	vi
Appendices	xii
Tables	xiii
Figures	xiv
Nomenclature	xii

Chapter One

INTRODUCTION

1.1	A Current Challenge in Materials Handling	1
1.2	An Introduction to Transfer Chutes	2
1.3	Background and Objectives	3
1.4	Thesis Overview	4

Chapter Two

TRANSFER CHUTE LITERATURE OVERVIEW

2.1	Introduction	7
2.2	Attributes of Conveyor-to-Conveyor Transfers	7
2.3	Problems Occurring At Transfer Points	8
2.4	Material Discharge and Trajectory Techniques	9
2.4.1	Introduction to Material Discharge	9
2.4.2	Material Height Calculations	11
2.4.3	Method of Korzen	16
2.4.4	Method of Booth	19
2.4.5	Method of Golka	20
2.4.6	Method of Dunlop	21
2.4.7	Method of Goodyear	21
2.4.8	Method of M.H.E.A. (Early Version)	22

2.4.9	Method of C.E.M.A.	22
2.4.10	Method of M.H.E.A. (Updated Version)	23
2.4.11	Method of BTR	24
2.4.12	Method of BFGoodrich	25
2.4.13	Method of S-A 66	25
2.5	Material Impact and Flow – Upper Chute Element	25
2.5.1	Introduction to the Upper Chute Element	25
2.5.2	Cohesive Impact upon a Flat Plate	28
2.5.3	Non-Cohesive Impact upon a Flat Plate	30
2.5.4	Sliding Flow upon a Flat Plate	32
2.5.5	Impact upon a Curved Plate	32
2.6	Material Free Fall	37
2.6.1	Air Entrainment Overview	37
2.6.2	Air Resistance and Drag Overview	39
2.7	Material Impact and Flow – Lower Chute Element	41
2.7.1	Material Impact Aspects	41
2.7.2	Gravity Flow Chute Overview	43
2.7.3	Method of Roberts	46
2.7.3.1	<i>Straight Chutes</i>	46
2.7.3.2	<i>Curved Chutes</i>	47
2.7.3.3	<i>Lumped Parameter Model</i>	47
2.7.3.4	<i>Continuity of Flow</i>	49
2.7.3.5	<i>Drag Force</i>	50
2.7.3.6	<i>Equivalent Friction Coefficient</i>	50
2.7.3.7	<i>Stream Thickness Variation</i>	52
2.7.3.8	<i>Approximate Closed Form Solutions of Flow Equations</i>	53
2.7.4	Method of Korzen	56
2.7.4.1	<i>Methodology</i>	56
2.7.4.2	<i>Multi-Step Approximation Procedure</i>	58
2.8	Further Comments and Summary	60

Chapter Three

CHUTE DESIGN TECHNIQUE COMPARISONS

3.1	Introduction	61
3.2	Material Discharge and Trajectories	61
3.2.1	Overview of Trajectory Design Methods	61

3.2.2	Spreadsheet Setup	63
3.2.3	Comparisons for High-Speed Conveying Conditions	65
3.2.4	Comparisons for Slow-Speed Conveying Conditions	67
3.2.5	Material Discharge and Trajectory Summary	71
3.3	Material Impact and Flow – Upper Chute Element	72
3.3.1	Spreadsheet Setup	72
3.3.2	Comparisons for Impact upon a Flat Plate	73
3.3.3	Comparisons for Impact upon a Curved Plate	75
3.3.4	Materials Impact and Flow Summary – Upper Chute Element	77
3.4	Material Impact and Flow – Lower Chute Element	77
3.4.1	Material Impact Overview	77
3.4.2	Gravity Flow Chute Comparisons	79
3.4.3	Material Impact and Flow Summary – Lower Chute Element	82
3.5	Overall Comments and Summary	83

Chapter Four

DISTINCT ELEMENT METHOD (DEM)

4.1	Introduction	86
4.1.1	DEM Overview	87
4.1.2	DEM Background	88
4.1.3	Merits and Drawbacks of DEM	88
4.2	Applications of Distinct Element Method	90
4.2.1	DEM Applied to Transfer Chute Analysis	91
4.2.2	Summary and Proposed Area of Investigation	95
4.3	Mathematical Formulation for Distinct Element Method	96
4.3.1	Particle–Particle Definitions and Interactions	96
4.3.2	Particle-Boundary Definitions and Interactions	99
4.3.2.1	<i>Straight Line Boundaries</i>	100
4.3.2.2	<i>Curved Line Boundaries</i>	101
4.3.2.3	<i>Particle – Boundary Interactions</i>	104
4.3.3	Further Boundary Aspects	109
4.3.3.1	<i>Modelling Moving Boundaries</i>	109
4.3.3.2	<i>Periodic Boundaries</i>	109
4.3.4	Governing Equations	110
4.3.5	Modelling of Contact Forces	112

4.3.5.1	<i>Normal Inter-Particle Contacts</i>	112
4.3.5.2	<i>Tangential Inter-Particle Contacts</i>	118
4.3.5.3	<i>Implementing Tangential Force-Displacement Model</i>	122
4.4	Summary	131

Chapter Five

NUMERICAL METHODS AND COMPUTATION ASPECTS

5.1	Introduction	132
5.2	Numerical Methods	132
5.2.1	Background	132
5.2.2	Implicit, Explicit, and Implicit-Explicit Methods	133
5.2.3	Implementation of Numerical Method	134
5.3	Contact Detection Scheme	136
5.3.1	Particle – Particle Contacts	136
5.3.1.1	<i>Neighbourhood List Approach</i>	137
5.3.1.2	<i>Zoning / Boxing Algorithm</i>	138
5.3.1.3	<i>Recent Advances in Contact Detection</i>	140
5.3.2	Particle – Wall Contacts	140
5.3.3	Implementation of Particle – Particle Contact Detection	140
5.3.4	Implementation of Particle – Boundary Contact Detection	144
5.4	Selection of Critical Time Step	146
5.4.1	Time Step Selection in Literature	146
5.4.2	Selection of Critical Time Step for Current Work	150
5.5	Computation Aspects	151
5.5.1	Pre – Processing Module	151
5.5.2	DEM Calculation Module	154
5.5.3	Post – Processing Module	154

Chapter Six

QUALITATIVE TESTING OF DEM COMPUTER CODE

6.1	Introduction	158
6.2	Single Contact Tests	158
6.2.1	Normal contact between particles	160
6.2.2	Normal contact between particle and wall	160

6.2.3	Normal contact with rotation, particle – particle	162
6.2.4	Normal contact with rotation, particle – wall	162
6.3	Multiple Contact Tests	166
6.3.1	Influence of Normal and Tangential Stiffness	167
6.3.2	Influence of Coefficient of Restitution and Friction	171
6.4	System Stability Check	171
6.5	Summary	179

Chapter Seven

INTRODUCTION TO INDUSTRY CHUTE SYSTEMS

7.1	Introduction	181
7.2	Selection and Overview of Chute Systems	181
7.3	System Setup	182
7.3.1	DEM Processes	182
7.3.2	Analytical Processes	188
7.4	Parameter Selection	190
7.5	Animating the Particulate Flow	193
7.5.1	Software Set-Up	193
7.5.2	Problems Encountered and Solutions	194
7.6	Preliminary Observations and Comments	195
7.6.1	Boundary Set-up	195
7.6.2	Sensitivity to System and Material Parameters	196
7.6.3	Velocity Profile Set-up	199
7.6.4	Time to Reach Steady-State Condition	202
7.6.5	Influence of Particle Size Distribution	205
7.6.6	General Comments Regarding Analytical Set-Up	205
7.7	Summary	208

Chapter Eight

ANALYSIS OF INDUSTRY CHUTE SYSTEMS

8.1	Introduction	209
8.2	Analysis of Velocity Distributions using Contours	209
8.2.1	Hood-Spoon Transfer Chute	210
8.2.2	Single Hood Transfer Chute	214

8.3	Detailed Quantitative Analysis of Velocity Distributions	214
8.3.1	Hood-Spoon Transfer Chute	218
8.3.2	Single Hood Transfer Chute	223
8.4	Micro Dynamics of Discrete Particles	227
8.4.1	Hood-Spoon Transfer Chute	228
8.4.2	Single Hood Transfer Chute	231
8.4.3	Conclusions	233
8.5	Additional Quantitative Considerations	233
8.5.1	Elastic Potential Energies	240
8.5.2	Inter-Particle Forces	240
8.5.3	Particulate Torques	241
8.6	Further areas of consideration	241
8.6.1	Wear upon Chute and Conveyor Belt	242
8.6.2	Induced and Entrained Air Flow	242
8.6.3	Material Degradation	242
8.6.4	Chute Support Structure and Receiving Belt Aspects	243
8.7	Summary	243

Chapter Nine

CONCLUSIONS AND FUTURE WORK

9.1	Application of DEM in Industry	245
9.2	Remarks on Current DEM Work	245
9.3	Conclusions	247
9.4	Future Work	248

Chapter Ten

REFERENCES AND BIBLIOGRAPHY

10.1	References	252
10.2	Bibliography	278

Appendices

Appendix I

PROJECT GANTT CHARTS

I.1	Overview	II
-----	----------	----

Appendix II

EXPANDED IMPLEMENTATION OF TFD MODEL

II.1	Introduction	III1
II.2	Implementation	III2

Appendix III

EXAMPLES OF INPUT FILES

III.1	Parameter Input File	III1
III.2	Boundary Input File	III2

Appendix IV

ASSEMBLY DRAWINGS OF GULF TRANSFERS

IV.1	Overview	IV1
------	----------	-----

Appendix V

SCREEN CAPTURES OF ENTIRE CALCULATION SPACE

V.1	Overview	V1
-----	----------	----

Tables

Table 4.1	Formation of coordinates in text file containing boundary data
Table 5.1	Sorting results for cell structure and particle configurations as shown in Figure 5.3 (a)
Table 5.2	Sorting results for boundary configurations as shown in Figure 5.5
Table 5.3	Layout of text file produced by Display III TM module
Table 6.1	Common parameters used for the single contact tests
Table 6.2	General parameters used for the multiple contact tests
Table 7.1	Values used for the dimensions indicated in Figures 7.5 and 7.6
Table 7.2	Material properties and conveying conditions for each transfer
Table 7.3	Initial DEM parameters used for simulating each transfer system

Figures

- Figure 2.1 Schematic of conveyor-to-conveyor transfer
- Figure 2.2 Load cross-section area on a 2 idler belt
- Figure 2.3 Load cross-section area on a 3 idler belt
- Figure 2.4 Load cross-section area on a flat belt.
- Figure 2.5 Cross-section of troughed belt
- Figure 2.6 (a) Impact Plate (b) Rock box
- Figure 2.7 Schematic diagram of bulk solids behaviour upon impact with the rebound board/'Hood' section
- Figure 2.8 Cohesive impact upon a flat plate
- Figure 2.9 Non-cohesive impact upon a flat plate
- Figure 2.10 Impact upon a curved plate
- Figure 2.11 Defining the corrected angle of entry when examining impact upon a curved plate
- Figure 2.12 Inverted curved chute model (adapted from Roberts 2001)
- Figure 2.13 Schematic of a falling material stream involving air entrainment and fugitive dust generation. In addition to directing the flow, curved 'Hood' and 'Spoon' elements minimise dust emissions
- Figure 2.14 (a) Impact model proposed by Roberts (2004); (b) impact model proposed by Stuart-Dick & Royal (1991, 1992)
- Figure 2.15 Top and section views of material impact and flow upon V-shaped load out floor pate
- Figure 2.16 Modes of chute flow: (a) fast flow, ideal case; (b) fast flow, general case (adapted from Roberts 1998b)
- Figure 2.17 Chute flow model (adapted from Parbery & Roberts 1986). The dimensions in the figure have been exaggerated for clarity
- Figure 2.18 Pressure distributions around chute boundary and cross-section of flowing stream (adapted from Roberts & Scott 1981). The dimensions in the figure have been exaggerated for clarity

- Figure 2.19 Top view of the parameters needed for the design of a converging chute (adapted from Roberts 2004). The parameter $B = B_0 - 2stan\alpha$ represents the width of the elemental mass at any distance s from the chute entry.
- Figure 2.20 Conditions of motion of the stream of bulk material in a curved chute (adapted from Korzen 1984a)
- Figure 3.1 Separation angle α_d vs. Belt Velocity v_b for the major discharge prediction techniques
- Figure 3.2 Trajectories generated by the various methods at $v_b = 6 \text{ ms}^{-1}$
- Figure 3.3 Trajectories generated by methods S1, S2, S3 and S10 at $v_b = 1 \text{ ms}^{-1}$
- Figure 3.4 Trajectories generated by methods S4, S5, S6, and S10 at $v_b = 1 \text{ ms}^{-1}$
- Figure 3.5 Trajectories generated by methods S7, S8, S9 and S10 at $v_b = 1 \text{ ms}^{-1}$
- Figure 3.6 Analysis of material impact upon flat plate
- Figure 3.7 Analysis of material impact upon curved plate
- Figure 3.8 Ratio between particle velocity after impacting two half angles to particle velocity after one impact (adapted from Stuart-Dick & Royal 1991, 1992)
- Figure 3.9 Variation of horizontal and vertical components of velocity and total velocity along the chute
- Figure 3.10 Variation of cross-sectional area and stream thickness ratio along the chute
- Figure 4.1 Attributes of the various classes of discrete element methods (Bardet 1998)
- Figure 4.2 Definition of the quantities used for description of the impact
- Figure 4.3 Overlap between colliding particles with radii R_i and R_j
- Figure 4.4 A representation of a straight line in the system
- Figure 4.5 A representation of an arc in the system
- Figure 4.6 Overlap between a particle and a vertical line
- Figure 4.7 Overlap between a particle and a non-vertical line
- Figure 4.8 Overlap between a particle and an arc
- Figure 4.9 (a) An assembly of spherical particles with periodic boundaries at left and right hand sides; (b) Introduction of particle i' at left hand periodic boundary as particle i leaves right hand periodic boundary (adapted from Jensen et al. 1999).

-
- Figure 4.10 Schematic of partially-latching spring model (Walton & Braun 1986b)
- Figure 4.11 Schematic of force-displacement curve used to describe inelastic normal direction forces acting between two colliding spheres (adapted from Walton & Braun 1986b)
- Figure 4.12 Elastic-frictional contact: TFD curve for constant F_n and varying F_t showing hysteresis loop and residual displacement (adapted from Vu-Quoc et al. 2004)
- Figure 4.13 Direction change of tangential force (adapted from Vu-Quoc et al. 2000)
-
- Figure 5.1 Successive steps in the implementation of the leap-frog algorithm. The stored variables are in grey boxes (adapted from Allen & Tildesley 1987)
- Figure 5.2 Diagram illustrating the conventions chosen for the Verlet method
- Figure 5.3 Schematics of: (a) cell structure with arbitrary particle configurations, and (b) cell structure with coloured cells showing target cells to be searched
- Figure 5.4 (a) particle numbering at start of program (b) particle numbering during contact detection subroutine
- Figure 5.5 Searching through boundary contacts
- Figure 5.6 One-dimensional vibration system
- Figure 5.7 Flowchart of pre-processing module used to create input data files
- Figure 5.8 Flowchart for DEM calculation module
- Figure 5.9 Flowchart of post-processing module that creates the visualisations
-
- Figure 6.1 Normal contact between: (a) particle and wall; (b) particle and particle
- Figure 6.2 Normal contact with rotation between: (a) particle and wall; (b) particle and particle
- Figure 6.3 Vertical position (a) and normal force (b) for particle-particle contact with $\varepsilon = 0.3$ and $\varepsilon = 0.6$
- Figure 6.4 Vertical position (a) and normal force (b) for particle-wall contact with $\varepsilon = 0.3$ and $\varepsilon = 0.6$
- Figure 6.5 Angular position for particle-particle contact with (a) $\mu = 0.5$ and (b) $\mu = 0.9$, and overlap ratio $\xi = 0.1\%$, $\xi = 1.0\%$, and $\xi = 10.0\%$

- Figure 6.6 Angular velocity for particle-particle contact with (a) $\mu = 0.5$ and (b) $\mu = 0.9$, and overlap ratio $\xi = 0.1\%$, $\xi = 1.0\%$, and $\xi = 10.0\%$
- Figure 6.7 Friction force for particle-particle contact with (a) $\mu = 0.5$ and (b) $\mu = 0.9$, and overlap ratio $\xi = 0.1\%$, $\xi = 1.0\%$, and $\xi = 10.0\%$
- Figure 6.8 Angular position for particle-wall contact with (a) $\mu = 0.5$ and (b) $\mu = 0.9$, and overlap ratio $\xi = 0.1\%$, $\xi = 1.0\%$, and $\xi = 10.0\%$
- Figure 6.9 Angular velocity for particle-wall contact with (a) $\mu = 0.5$ and (b) $\mu = 0.9$, and overlap ratio $\xi = 0.1\%$, $\xi = 1.0\%$, and $\xi = 10.0\%$
- Figure 6.10 Friction force for particle-wall contact with (a) $\mu = 0.5$ and (b) $\mu = 0.9$, and overlap ratio $\xi = 0.1\%$, $\xi = 1.0\%$, and $\xi = 10.0\%$
- Figure 6.11 Hour-glass with $K_{n1} = K_t^0 = 1 \times 10^5 \text{ Nm}^{-1}$ at (a) $t = 0.00 \text{ s}$; (b) $t = 0.10 \text{ s}$
- Figure 6.12 Hour-glass with $K_{n1} = K_t^0 = 1 \times 10^7 \text{ Nm}^{-1}$ at (a) $t = 0.00 \text{ s}$; (b) $t = 0.10 \text{ s}$
- Figure 6.11 Hour-glass with $K_{n1} = K_t^0 = 1 \times 10^5 \text{ Nm}^{-1}$ at (c) $t = 0.20 \text{ s}$; (d) $t = 0.30 \text{ s}$
- Figure 6.12 Hour-glass with $K_{n1} = K_t^0 = 1 \times 10^7 \text{ Nm}^{-1}$ at (c) $t = 0.20 \text{ s}$; (d) $t = 0.30 \text{ s}$
- Figure 6.11 Hour-glass with $K_{n1} = K_t^0 = 1 \times 10^5 \text{ Nm}^{-1}$ at (e) $t = 0.40 \text{ s}$; (f) $t = 0.50 \text{ s}$
- Figure 6.12 Hour-glass with $K_{n1} = K_t^0 = 1 \times 10^7 \text{ Nm}^{-1}$ at (e) $t = 0.40 \text{ s}$; (f) $t = 0.50 \text{ s}$
- Figure 6.13 Hour-glass with $\varepsilon = 0.9$ and $\mu = 0.1$ at (a) $t = 0.00 \text{ s}$; (b) $t = 0.10 \text{ s}$
- Figure 6.14 Hour-glass with $\varepsilon = 0.1$ and $\mu = 0.9$ at (a) $t = 0.00 \text{ s}$; (b) $t = 0.10 \text{ s}$
- Figure 6.13 Hour-glass with $\varepsilon = 0.9$ and $\mu = 0.1$ at (c) $t = 0.20 \text{ s}$; (d) $t = 0.30 \text{ s}$
- Figure 6.14 Hour-glass with $\varepsilon = 0.1$ and $\mu = 0.9$ at (c) $t = 0.20 \text{ s}$; (d) $t = 0.30 \text{ s}$
- Figure 6.13 Hour-glass with $\varepsilon = 0.9$ and $\mu = 0.1$ at (e) $t = 0.40 \text{ s}$; (f) $t = 0.50 \text{ s}$
- Figure 6.14 Hour-glass with $\varepsilon = 0.1$ and $\mu = 0.9$ at (e) $t = 0.40 \text{ s}$; (f) $t = 0.50 \text{ s}$
- Figure 6.15 Distribution of particles within rectangular shaped boundary for numerical stability checking at times (a) $t = 0.0 \text{ s}$ (b) $t = 0.5 \text{ s}$ (c) $t = 1.0 \text{ s}$ (d) $t = 1.5 \text{ s}$ (e) $t = 2.0 \text{ s}$ (f) $t = 5.0 \text{ s}$
- Figure 6.16 Progressive readings of each of the four system energy components at each time step at time intervals of (a) $t = 0.0 \text{ s} - 0.5 \text{ s}$ (b) $t = 0.5 \text{ s} - 1.0 \text{ s}$ (c) $t = 1.0 \text{ s} - 1.5 \text{ s}$ (d) $t = 1.5 \text{ s} - 5.0 \text{ s}$
- Figure 6.17 Total energy of the system plus each individual energy component from $t = 0.0 \text{ s}$ to $t = 2.0 \text{ s}$

- Figure 7.1 Image depicting hood-spoon transfer chute system – view one
- Figure 7.2 Image depicting hood-spoon transfer chute system – view two
- Figure 7.3 Image depicting single hood transfer chute system – view one
- Figure 7.4 Image depicting single hood transfer chute system – view two
- Figure 7.5 A schematic of the first transfer to be examined, comprising a hood-spoon chute system. The heavy dotted lines represent the periodic boundaries.
- Figure 7.6 The second transfer to be examined is composed of a single hood to redirect material flow. The heavy dotted lines represent the periodic boundaries.
- Figure 7.7 Schematic showing the numbering of design areas for hood-spoon system
- Figure 7.8 Schematic showing the numbering of design areas for single hood system
- Figure 7.9 Particle size distributions for hood-spoon transfer chute and single hood transfer chute
- Figure 7.10 (a) Initial spoon location and (b) Final spoon location
- Figure 7.11 Average velocity components in x and y directions for first transfer with $\varepsilon = 0.2$ and $\varepsilon = 0.5$
- Figure 7.12 Average velocity components in x and y directions for second transfer with $\varepsilon = 0.2$ and $\varepsilon = 0.5$
- Figure 7.13 Average velocities in the x and y directions for $\Delta t = 1 \times 10^{-5}$ s and $\Delta t = 1 \times 10^{-6}$ s for the first transfer chute system comprising a hood and spoon
- Figure 7.14 Average velocities in the x and y directions for $\Delta t = 1 \times 10^{-5}$ s and $\Delta t = 1 \times 10^{-6}$ s for the first transfer chute system comprising a single hood
- Figure 7.15 Average velocities of all particles for transfer chute simulation comprising hood and spoon, from (a) $t = 0.00$ s to $t = 2.00$ s (b) $t = 2.00$ s to $t = 5.00$ s
- Figure 7.16 Average velocities of all particles for transfer chute simulation comprising single hood, from (a) $t = 0.00$ s to $t = 2.00$ s (b) $t = 2.00$ s to $t = 5.00$ s

- Figure 7.17 Kinetic energy in each transfer chute system from $t = 0.0$ to $t = 5.0$ s. The terms ‘old’ and ‘new’ in the legend refer to the earlier or latter periodic boundary locations used respectively for the first transfer system
- Figure 7.18 Screen captures at (a) $t = 2.0$ s, (b) $t = 3.0$ s, (c) $t = 4.0$ s, and (d) $t = 5.0$ s illustrating the particle size distribution for the first transfer
- Figure 7.19 Screen captures at (a) $t = 2.0$ s, (b) $t = 3.0$ s, (c) $t = 4.0$ s, and (d) $t = 5.0$ s illustrating the particle size distribution for the second transfer
- Figure 8.1 Screen captures that show the particulate speed distribution for the first transfer system at times of (a) $t = 2.0$ s, (b) $t = 3.0$ s, (c) $t = 4.0$ s, and (d) $t = 5.0$ s.
- Figure 8.2 Snapshots of the hood-spoon transfer system showing horizontal velocity components at times of (a) $t = 2.0$ s, (b) $t = 3.0$ s, (c) $t = 4.0$ s, and (d) $t = 5.0$ s.
- Figure 8.3 Snapshots of the hood-spoon transfer system showing vertical velocity components at times of (a) $t = 2.0$ s, (b) $t = 3.0$ s, (c) $t = 4.0$ s, and (d) $t = 5.0$ s.
- Figure 8.4 Screen captures that show the particulate speed distribution for the second transfer system at times of (a) $t = 2.0$ s, (b) $t = 3.0$ s, (c) $t = 4.0$ s, and (d) $t = 5.0$ s.
- Figure 8.5 Snapshots of the single hood transfer system showing horizontal velocity components at times of (a) $t = 2.0$ s, (b) $t = 3.0$ s, (c) $t = 4.0$ s, and (d) $t = 5.0$ s.
- Figure 8.6 Snapshots of the single hood transfer system showing vertical velocity components at times of (a) $t = 2.0$ s, (b) $t = 3.0$ s, (c) $t = 4.0$ s, and (d) $t = 5.0$ s.
- Figure 8.7 Particle position and horizontal & vertical components of velocity calculated using the analytical methods described in Section 7.3.2 for hood-spoon transfer chute. The numbers correspond to those shown in Figure 7.7.
- Figure 8.8 Snapshot of particle position, and horizontal and vertical components of velocity at (a) $t = 2.00$ s and (b) $t = 3.00$ s for hood-spoon transfer chute
- Figure 8.8 Snapshot of particle position, and horizontal and vertical components of velocity at (c) $t = 4.00$ s and (d) $t = 5.00$ s for hood-spoon transfer chute

- Figure 8.9 Particle position and horizontal & vertical components of velocity calculated using the analytical methods described in Section 7.3.2 for single hood transfer chute. The numbers correspond to those in Figure 7.8.
- Figure 8.10 Snapshot of particle position, and horizontal and vertical components of velocity at (a) $t = 2.00$ s and (b) $t = 3.00$ s for single hood transfer chute
- Figure 8.10 Snapshot of particle position, and horizontal and vertical components of velocity at (c) $t = 4.00$ s and (d) $t = 5.00$ s for single hood transfer chute
- Figure 8.11 Initial positions of selected particles in feeder for (a) hood-spoon transfer and (b) single hood transfer
- Figure 8.12 Two randomly selected particles from the hood-spoon DEM simulation with positions, and horizontal and vertical velocity components. The particle numbers examined are (a) $i = 26$ and (b) $i = 1116$
- Figure 8.13 Two randomly selected particles from the single hood DEM simulation with positions, and horizontal and vertical velocity components. The particle numbers examined are (a) $i = 377$ and (b) $i = 801$
- Figure 8.14 Screen captures that show the elastic potential energy (or strain energy) possessed by the particles for the first transfer system at times of (a) $t = 2.0$ s, (b) $t = 3.0$ s, (c) $t = 4.0$ s, and (d) $t = 5.0$ s.
- Figure 8.15 Screen captures that show the inter-particle forces (including gravity) possessed by the particles for the force transfer system at times of (a) $t = 2.0$ s, (b) $t = 3.0$ s, (c) $t = 4.0$ s, and (d) $t = 5.0$ s.
- Figure 8.16 Screen captures that show the torques possessed by the particles for the first transfer system at times of (a) $t = 2.0$ s, (b) $t = 3.0$ s, (c) $t = 4.0$ s, and (d) $t = 5.0$ s.
- Figure 8.17 Screen captures that show the elastic potential energy (or strain energy) possessed by the particles for the second transfer system at times of (a) $t = 2.0$ s, (b) $t = 3.0$ s, (c) $t = 4.0$ s, and (d) $t = 5.0$ s.
- Figure 8.18 Screen captures that show the inter-particle forces (including gravity) possessed by the particles for the second transfer system at times of (a) $t = 2.0$ s, (b) $t = 3.0$ s, (c) $t = 4.0$ s, and (d) $t = 5.0$ s.
- Figure 8.19 Screen captures that show the torques possessed by the particles for the second transfer system at times of (a) $t = 2.0$ s, (b) $t = 3.0$ s, (c) $t = 4.0$ s, and (d) $t = 5.0$ s.

Figure I.1 Initial Gantt Chart

Figure I.2 Final Gantt Chart

Figure II.1 Direction change of tangential force (adapted from Vu-Quoc et al. 2000)

Figure II.2 Decomposition of the incremental tangential displacement $\Delta\delta_t^N$ at time t^N (adapted from Vu-Quoc et al. (2000))

Figure IV.1 Image depicting hood-spoon transfer chute system

Figure IV.2 Image depicting hood-spoon transfer chute system

Figure IV.3 Image depicting hood-spoon transfer chute system

Figure IV.4 Image depicting hood-spoon transfer chute system

Figure IV.5 Image depicting hood-spoon transfer chute system

Figure IV.6 Image depicting single hood transfer chute system

Figure IV.7 Image depicting single hood transfer chute system

Figure IV.8 Image depicting single hood transfer chute system

Figure IV.9 Image depicting single hood transfer chute system

Figure IV.10 Assembly drawing for hood-spoon transfer chute

Figure IV.11 Assembly drawing for single hood transfer chute

Figure V.1 Capture of entire calculation space for first transfer taken at $t = 2.0$ s

Figure V.2 Capture of entire calculation space for first transfer taken at $t = 3.0$ s

Figure V.3 Capture of entire calculation space for first transfer taken at $t = 4.0$ s

Figure V.4 Capture of entire calculation space for first transfer taken at $t = 5.0$ s

Figure V.5 Capture of entire calculation space for second transfer taken at $t = 2.0$ s

Figure V.6 Capture of entire calculation space for second transfer taken at $t = 3.0$ s

Figure V.7 Capture of entire calculation space for second transfer taken at $t = 4.0$ s

Figure V.8 Capture of entire calculation space for second transfer taken at $t = 5.0$ s

Nomenclature

The author attempted to use symbols as close to common interpretations as possible in the thesis (for example, g is frequently used to represent gravitational acceleration and is therefore used similarly here). However due to this and the number of symbols required, some overlapping did occur. Therefore in the following nomenclature the symbol $\{\clubsuit\}$ represents the interpretation as used in Chapters Two and Three, and $\{\spadesuit\}$ represents the interpretation as used in Chapters Four and Five.

ARABIC LETTERS

a	$\{\clubsuit\}$ Acceleration along the tangent $\{= \ddot{s} = \dot{v}\}$ (ms^{-2}); $\{\spadesuit\}$ Index allowing for differing loading and unloading paths {NFD model}
A	Total cross-sectional area of bulk solid in flowing stream (m^2)
A_0	Initial cross-sectional area of the flowing stream at the point of entry of the chute (m^2)
$A_{1,2}$	Cross-sectional areas {rectangular portion, circular segment} of bulk solid in flowing stream (m^2)
A_a	Cross-sectional area of material stream at exit to ‘flow-round’ zone (m^2)
A_b	Area of trapezoidal {3 idler system} or triangular {2 idler system} area (m^2)
A_{BC}	Non-dimensional cross-sectional area factor
ac	Y-axis intercept of the perpendicular to the chord between successive points on the arc
A_i	Cross-sectional area of free-falling stream (m^2)
am	Gradient of the perpendicular to the chord between successive points on the arc
A_p	Cross-sectional area of material stream at entrance to ‘flow-round’ zone (m^2)
A_s	Area of segment (m^2)
A_T	Total area of material on the belt in the troughed portion (m^2)

a_w	Proportionality factor for air drag
$A(\kappa)$	Function that describes cross-sectional area of flow stream on impact plate (m ²)
b	{♣} Width of belt (m); {♠} Fixed parameter, often set to 1/3 to agree with Mindlin's frictional sphere theory {TFD model}
B	Width of rectangular chute (m)
B_0	Width of entry for converging chute (m)
b_s	Mean width of material stream on the belt prior to discharge (m)
b_t	Thickness of belt (m)
bw_2	Width of material on flattened belt {troughed belts only} (m)
c	{♣} Cohesive stress (kNm ²); {♠} Y-intercept of straight line
C	Constant of integration
$C_{1,2\&3}$	Constants used during calculation of the load cross-sectional area
C_{grav}	Distance from belt surface to centre of mass (m)
C_l	Inverse velocity Coulomb drag coefficient
C_s	Intergranular stress constant (s ² m ⁻²)
D	Horizontal distance from discharge point to impact point (m)
D_{base}	Base particle diameter (m)
d_{ij}	Sum of contacting sphere radii (m)
D_{max}	Maximum particle diameter (m)
D_{min}	Minimum particle diameter (m)
D_{mono}	Mono-sized particle (m)
dn	Displacement between particles (m)
D_{var}	Variance between particle sizes (m)
dx	Horizontal displacement difference between particles (m)
dy	Vertical displacement difference between particles (m)
E	Young's modulus (Nm ⁻²)
E_{ij}	Equivalent elastic modulus (Nm ⁻²)
E_T	Total energy of a particle (J)
$E_{1,2}$	Parameters in Equation (2.116)
f_0	Friction value of motion at the initial point of the chute
F_D	Drag force (N)
F_n	Normal force in Distinct Element Model (N)

F_N	Normal force in gravity flow chute theory (N)
F_n^{max}	Maximum force ever experienced by the contact (N)
F_t	Tangential force in Distinct Element Model (N)
F_t^*	Value of the tangential force F_t whenever the magnitude changes from increasing to decreasing, or vice versa (N)
F_v	Velocity dependent drag force (N)
f_φ	Friction value of motion at any angle φ around chute
F_μ	Coulomb frictional drag force (N)
F_mag_t	Magnitude of tangential force (N)
F_x_t	Horizontal component of tangential force (N)
$F_x_{t,u}$	Horizontal component of unit vector (N)
F_y_t	Vertical component of tangential force (N)
$F_y_{t,u}$	Vertical component of unit vector (N)
g	Acceleration due to gravity (ms^{-2})
G	Shear {or rigidity} modulus (Nm^{-2})
G_{ij}	Equivalent shear modulus (Nm^{-2})
h	Material drop height (m)
H	Flowing stream thickness (m)
H_0	Initial stream thickness (m)
$H_{1,2}$	Stream thickness {rectangular portion, circular segment} (m)
h_a	Thickness of material stream at exit of ‘flow-round’ zone (m)
h_b	Thickness of material on belt prior to discharge (m)
h_p	Thickness of material stream entering ‘flow-round’ zone (m)
h_φ	Stream thickness at any angle φ around curved chute (m)
I	Moment of inertia (kgm^2)
K	Constant of proportionality usually between 1.11 – 1.42
k_{EO}	Effective linear pressure gradient down the wall surface at zero velocity
k_i	Number of particles in contact with particle i
k_{max}	Largest inter-particle spring stiffness (Nm^{-1})
K_n	Some normal stiffness coefficient (Nm^{-1})
K_{n1}	Normal stiffness coefficients for the (loading stage) (Nm^{-1})
K_{n2}	Normal stiffness coefficients for the (unloading stage) (Nm^{-1})

K_t	Some tangential stiffness coefficient (Nm^{-1})
K_t^0	Initial tangential stiffness (Nm^{-1})
K_T	Effective incremental tangential stiffness (Nm^{-1})
k_v	Coefficient relating lateral pressure at the chute wall to the average normal pressure during flow
L	Distance between periodic boundaries (m)
L_{BC}	Contact perimeter of material burden on discharging belt (m)
m	Particle mass (kg) / gradient of straight line
\dot{m}	Mass flow rate of material (kgs^{-1})
m_{ij}	Effective mass of particles i and j acting in series (kg)
m_{min}	Mass of smallest particle in system (kg)
n	Parameter that is a function of the total number of particles in the system
N	Number of particle in system
N_{grid}	User defined term that specifies the maximum number of particles to be allowed in one cell
n_s	Number of time steps between searches
P_n	Pressure in normal direction (kPa)
Q_m	Flowrate (th^{-1})
r	Non-dimensional parameter representing ratio between outside and central idler contact
R	$\{\clubsuit\}$ Pulley radius; radius of curvature of curved chute (m); $\{\spadesuit\}$ Radius of sphere (m)
R_0	Radius of the conveying stream midpoint at the start of the chute (m)
r_1	Radius of interior sphere in Verlet neighbour list (m)
r_2	Radius of exterior sphere in Verlet neighbour list (m)
R_b	Distance from centre of discharge pulley to outer surface of belt (m)
R_c	Radius of curvature of discharge trajectory (m)
R_e	Distance from discharge pulley centre to material centre of mass (m)
R_{fz}	Radius of the ‘flow-round’ zone (m)
R_{ij}	Relative contact curvature (m)
R_m	Distance from centre of pulley to top of material upon belt (m)
R_{min}	Radius of smallest sized particle in the system (m)
R_p	Radius of curved impact plate (m)

s	Displacement along tangent (m)
S	{♣} Distance between end of ‘flow-round zone’ and bottom of the plate (m); {♠} An empirically determined model parameter
$S_{flowround}$	Portion of curved impact plate in contact with material stream (m)
S_p	Length of impact plate {flat or curved} (m)
s_v	Vertical fall distance (m)
t	Time (s)
U_{max}	Maximum particle velocity (ms^{-1})
v	Velocity {= \dot{s} } (ms^{-1})
v_0	{♣} Initial velocity of the flowing stream at the point of entry of the chute (ms^{-1}); {♠} Relative velocity of approach (ms^{-1})
$v_{0,S}$	Velocity of stream parallel to chute surface after impact (ms^{-1})
v_1^*	Velocity of stream before impact (ms^{-1})
v_2^*	Velocity of stream after the first deflection (ms^{-1})
v_3^*	Velocity of stream after second deflection (ms^{-1})
v_4^*	Velocity of stream after impact for a single deflection (ms^{-1})
v_a	Exit velocity of material leaving ‘flow-round’ zone (ms^{-1})
v_b	Conveyor belt velocity (ms^{-1})
v_c	Critical velocity (ms^{-1})
v_d	Discharge velocity (ms^{-1})
v_e	Exit velocity from bottom of flat impact plate (ms^{-1})
v_{f0}	Vertical component of bulk solid discharging velocity (ms^{-1})
v_i	Velocity of impact with the curved chute (ms^{-1})
v_I	Velocity of stream before impact (ms^{-1})
v_p	Material velocity at entrance to ‘flow-round’ zone (ms^{-1})
v_t	Tangential velocity; velocity of load stream centre (ms^{-1})
$v(\kappa)$	Velocity of stream at angle κ in ‘flow-round’ zone (ms^{-1})
$v(\psi)$	Discharge velocity at angle ψ (ms^{-1})
v_∞	Terminal velocity (ms^{-1})
x	General x-coordinate (m)
\dot{x}	Velocity in x-direction (ms^{-1})
\ddot{x}	Acceleration in x-direction (ms^{-2})

x_1	First x-coordinate of line / arc (m)
$x_{1,2,3,4}$	Four x-coordinates representing a boundary (m)
x_2	Second x-coordinate of line / arc (m)
x_3	Third x-coordinate of line / arc (m)
x_4	Fourth x-coordinate of line / arc (m)
X_c	X-coordinate of arc centre (m)
x_h	Height of material bed on belt (m)
X_{len}	Width of calculation space (m)
y	General y-coordinate (m)
\dot{y}	Velocity in y-direction (ms^{-1})
\ddot{y}	Acceleration in y-direction (ms^{-2})
y_1	First y-coordinate of line / arc (m)
$y_{1,2,3,4}$	Four y-coordinates representing a boundary (m)
y_2	Second y-coordinate of line / arc (m)
y_3	Third y-coordinate of line / arc (m)
y_4	Fourth y-coordinate of line / arc (m)
Y_c	Y-coordinate of arc centre (m)
Y_{len}	Height of calculation space (m)
$y(x)$	Function that describes the trajectory of free fall (m)
$z_{1,2,3,4}$	Four z-coordinates representing a boundary (m)

GREEK LETTERS

α	Angle of convergence for chute side walls ($^\circ$)
α_b	Conveyor belt inclination angle before discharge ($^\circ$)
α_d	Bulk solid stream discharge angle measured from the vertical ($^\circ$)
α_r	Angle at which material starts to slip on discharge pulley ($^\circ$)
β	Impact plate inclination angle ($^\circ$)
β_i	Angle of idler roll ($^\circ$)
β_v	Viscous drag coefficient (s^{-1})
Δm	Elementary mass of bulk solid (kg)

δ_n	Normal overlap {relative displacement of the centres of the two spheres} (m)
$\dot{\delta}_n$	Rate of change of the distance between centres of the colliding particles (ms^{-1})
δ_{n0}	Residual displacement after complete unloading {the value where the unloading curve goes to zero} (m)
δ_r	Residual tangential displacement (m)
Δr_x	Horizontal component of change in relative position vector (m)
Δr_y	Vertical component of change in relative position vector (m)
δ_t	Tangential overlap between particles (m)
Δt	Time step (s)
Δt_c	Critical time step (s)
$\Delta \delta_t$	Incremental tangential displacement (m)
$\Delta \delta x_t$	Horizontal component of relative surface displacement vector (m)
$\Delta \delta y_t$	Vertical component of relative surface displacement vector (m)
ε	Coefficient of restitution
ϕ	{♣} Wall friction angle used in gravity flow chute work $\{= \tan^{-1} \mu\}$ (°); {♠} Angle from horizontal {line} / angle from horizontal of the perpendicular to the chord between successive points {arc} (°)
Φ	Poisson's ratio (ν) dependent parameter for Rayleigh Wave speed critical time step determination
Φ_{ij}	Angle of the particle with reference to the arc during contact (°)
ϕ_w	Kinematic angle of wall friction between material and conveyor belt (°)
γ	Specific weight of the material being conveyed $\{= \rho g\}$ (kNm^{-3})
γ_1	Start angle of an arc (°)
γ_2	Finish angle of an arc (°)
γ_n	Damping constant
φ	Chute slope angle for Korzen's work $\{= 90 - \theta\}$ (°)
φ_0	Angle of chute to horizontal at impact (°)
κ	Angle of impact to horizontal {for flat plates}; angle the tangent to the end of the plate makes with the horizontal {for curved plates} (°)

λ	Angle of surcharge of material ($^{\circ}$)
λ_{bottom}	Angle tangent to end of curved plate makes with the vertical ($^{\circ}$)
μ	{♣} Coefficient of internal friction used in flat impact plate model {= $\tan\zeta$ }; coefficient of wall friction used in gravity flow chute work {= $\tan\phi$ }; {♠} Coefficient of friction
μ_E	Equivalent coefficient of friction
μ_k	Kinematic friction coefficient between material and belt {= $\tan\phi_w$ }
μ_s	Static friction coefficient
ν	Poisson's ratio
θ	{♣} Chute slope angle for Roberts' work {= dy/dx } ($^{\circ}$); {♠} General rotation (radians)
$\dot{\theta}$	Angular velocity (rads^{-1})
$\ddot{\theta}$	Angular acceleration (rads^{-2})
θ_1	Angle of incoming stream relative to chute surface ($^{\circ}$)
θ_2	Angle after impact of material stream relative to chute surface ($^{\circ}$)
θ_3	Angle of incoming stream relative to chute surface {for double deflection of material stream} ($^{\circ}$)
θ_a	Angle from horizontal made by incoming material stream to impact plate ($^{\circ}$)
θ_c	Corrected angle of entry of material on a curved impact plate ($^{\circ}$)
θ_{co}	Optimum cutoff angle for curved chute ($^{\circ}$)
θ_f	Limiting angle for maintenance of 'fast' flow ($^{\circ}$)
θ_i	Instantaneous angle of impact ($^{\circ}$)
θ_s	Angle opposite arc length $S_{flowround}$ ($^{\circ}$)
ρ	{♣} Bulk density (kgm^{-3}); {♠} Particle density (kgm^{-3})
σ_1	Normal stress corresponding to conditions on the belt prior to discharge (kPa)
σ_a	Adhesive stress (kPa)
τ	Shear stress (kPa)
ω	Angular velocity
ξ	{♣} Percentage admissible relative deviation for the estimation of the k-th value of v_a {impact plate model}; tolerated relative deviation for the

	estimation of the k-th value of $v(\varphi)$ {gravity flow chute model}; {♠}
	Percentage overlap or overlap ratio of two contacting particles
ψ	Wrap angle around discharge pulley (°)
ζ	Effective angle of internal friction (°)

SUBSCRIPTS

i	Particle number i
j	Particle / boundary number j
\parallel	Denotes parallel component
\perp	Denotes perpendicular component
old	Denotes previous time step

SUPERSCRIPTS

N	Time t^N
$N+1$	Time t^{N+1}
$N-1$	Time t^{N-1}
$N+1/2$	Time $t^{N+1/2}$
$N-1/2$	Time $t^{N-1/2}$
line	Represents line
arc	Represents arc

VECTOR QUANTITIES

\mathbf{F}_n	Normal contact force
\mathbf{F}_t	Tangential contact force
\mathbf{g}	Gravitational vector
\mathbf{i}	Denotes x-direction
\mathbf{j}	Denotes y-direction

\mathbf{k}	Denotes z-direction
$\hat{\mathbf{k}}_{ij}$	Unit vector in normal direction between particles
\mathbf{r}	Position vector for a particle
\mathbf{r}_{ij}	Relative position vector between two particles
\mathbf{R}	Radius vector
$\hat{\mathbf{t}}_{ij}$	Unit vector in the direction of the virgin loading
\mathbf{T}_{ij}	Torque
\mathbf{v}	Velocity vector for a particle
$\dot{\mathbf{x}}$	Velocity vector in x-direction
$\dot{\mathbf{y}}$	Velocity vector in y-direction
$\Delta\mathbf{r}_{ij}$	Change in the relative position vector during the last time step
$\Delta\boldsymbol{\delta}_\tau$	Relative surface displacement vector