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Comparing worked examples and problem-solving methods in teaching mathematics to ESL students at tertiary level

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Comparing Worked Examples and Problem-Solving Methods in Teaching Mathematics to ESL Students at Tertiary Level

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DECLARATION

In accordance with the regulation of the University of Wollongong, I, Ali Algrani declare that this thesis, submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy at the School of Mathematics and Applied Statistics, University of Wollongong, is my own original work unless otherwise referenced or acknowledged. It has not been submitted for qualifications at any other academic institution.

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ABSTRACT

This study explores two challenges faced by tertiary ESL students learning mathematics, the complexity of learning the mathematics discipline together with the challenge of learning in a second language. It draws together the theoretical knowledge as to how bi-lingual students learn, using strategies such as code-switching, borrowing, multimodal approach, an understanding of the problems experienced by students learning mathematics with the Cognitive Load Theory (CLT) providing a basis for understanding the function of working memory.

Drawing on the theories regarding cognitive load, the use of different approaches to teaching to reduce the cognitive load of students learning tertiary mathematics in English as a second language is examined. To facilitate generalisation of results, the effectiveness of teaching strategies is compared through two cases studies wherein mathematics is taught to ESL students in two vastly different contexts. The first case study at the King Abdul-Aziz University (KAU), Saudi Arabia, involves 198 male students taught in their home country in English, which is not their first language. The second case study at University of Wollongong College (UOWC), Australia, involves a mix of 74 students comprising both domestic students and international students taught in a second language, English.

In terms of design, the second case study replicates the first. In each case study data was collected over three iterations involving different groups of students undertaking a tertiary mathematics subject, each with a curriculum covering the topics, Functions, Exponents, Quadratic Equations, Logarithms, Geometry, and an Introduction to Statistics. In the first iteration baseline data was gathered using student questionnaires, lecturer interviews and examination of teaching materials regarding students' experiences of learning via the methods, worked examples and problems-solving and student results achieved on topic tests. Following baseline data gathering, in the second iteration, a different cohort of students were taught the six topics wherein the teaching methods alternated between worked examples and problem-solving techniques, resulting in for three topics taught with each method. For the third iteration which involved a third cohort of students, the teaching strategies implemented in the second iteration were swapped for four of the topics and then faded worked examples were introduced as the method of teaching for the remaining two topics, one previously taught with problem-solving and one with worked examples.

The principal finding from both case studies was that worked examples which direct the attention of a learner to the problem stated, and show the steps required in solving a particular type of problem, facilitated learning. For both case studies, the performance of ESL students was improved by the use of worked examples. In the KAU case study, over the three phases a greater proportion of students indicated that

having worked examples (80%) improved their study than did problem-solving (20%). At UOWC, over the three phases a greater proportion of students indicated that having worked examples (72%) improved their work than did problem-solving (26%).

This improvement in learning is consistent with cognitive load theory that suggests a reduction in cognitive load should generally make learning easier. Seventy percent of KAU students surveyed and fifty-six percent of international students in the Australian case study indicated that learning mathematics was preferable through the use of worked examples.

In terms of perceived learning outcomes it was found that for both cases studies there is an improved attitude toward studying mathematics, 'increases my confidence about solving more problems', 'liking mathematics more' and 'reduces anxiety'. In the KAU case study, worked examples was found to enhance Quicker to study, Improved my review of mathematics notes and lab work, Easier to learn mathematics, Requires less mental effort, Makes mathematics learning more interesting. In UOWC case study, worked examples were found to enhance mathematics understanding, Increases my confidence about solving more problems, liking mathematics more and reduces anxiety. Students like to learn mathematics with worked examples more so than problem-solving even though they agreed that problem-solving increases their confidence in learning mathematics. Also, students have positive experiences in terms of learning outcomes with worked examples.

With respect to the use of faded worked examples, for both case studies, marks were significantly higher for the topic Geometry when taught with faded worked examples rather than worked examples. One could have expected that students in 2012 should have experienced higher cognitive load for this topic, however, faded worked examples increased their confidence which resulted in an increase in their marks (mean difference FWE-WE=-4.96, $p=.000$) for KAU. Marks were significantly higher for the topic Introduction to Statistics taught with faded worked examples rather than problem-solving. Students in 2012 should have experienced lower cognitive load for this topic. This was confirmed (mean difference FWE-PS=-4.58, $p<.0005$) for KAU and also for UOWC (mean difference FWE –PS=-4.35, $p<.0005$).

Moreover, language of teaching mathematics has an impact on students learning if they learn in their second language. At KAU students' ability to learn mathematics in English was seen to be lower than their ability to learn in mathematics and was also seen to decline with (67%) of students perceiving their ability to be fair/very good in 2010, declining to two percent of students in 2012. As for the ability to learn mathematics when it comes to learning mathematics in English, UOWC students' perceived ability is relatively constant in each cohort, with (68%) of students perceiving their ability to be fair/very good in 2010, and a comparable (69%) of students in 2012.

Therefore, worked examples would be preferable to problem-solving and faded worked examples in terms of lowering the cognitive load which results from a language barrier and the difficulty of learning mathematics. This may explain that over the three phases a greater proportion of students indicated that having worked examples improved their mathematics studying than did problem-solving and faded worked examples.

In conclusion, it is important for teachers to find ways to teach mathematics effectively for their students to learn and understand their subjects. These findings support an increase in the use of worked examples for students who are learning mathematics in a second language. Implementation of the worked example pedagogy in teaching mathematics should facilitate learning for those students learning mathematics in a second language. Further examination of the use of faded worked examples as a scaffold to problem-solving is recommended as performance in both case studies improved in the topics Geometry and Introduction to Statistics. So, it is important for teachers to find ways to teach mathematics effectively. This remains a realistic challenge for the Saudi and Australian governments and indeed other governments to give teachers the required training in hybrid pedagogy.

PUBLICATIONS

The following conference papers have been raised from this thesis so far.

Conference papers

Algarni, A.; Birrell, C. & Porter, A. (2012), Evaluating the Use of Worked Examples and Problem-Solving Methods in Teaching Mathematics for ESL Students at the Tertiary Level (Australia context), Proceedings of the Fifth Annual ASEARC Conference - Looking to the future - Program and Proceedings, 2 - 3 February 2012, University of Wollongong.

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1 INTRODUCTION TO THE STUDY

"Mathematics education begins in language, it advances and stumbles because of language, and its outcomes are often assessed in language" *(Durkin, 1991, p. 3)*

1.1 Introduction

Historically the learning process of many students in mathematical subjects worldwide differs, often with poor academic outcomes when taught to students in English when this is not their first language. The ability to read, learn, understand and apply mathematics in a second language is obviously influenced by a variety of language skills (Cossio, 1977).

The reality of teaching for many mathematics teachers in countries throughout the world is that they are teaching mathematics to students in English when it is the students' second language. In countries such as Malaysia and Saudi Arabia where English is not the first language of the country, students at University may be required to learn mathematics in English, which is typically their second language (Heng & Tang, 2003; Alsaeed, 2008). Along with finding mathematics difficult to learn and having problems in tertiary studies due to learning in a second language, there are also issues arising from the pedagogy of how mathematics is taught. One such explanation of these issues is the resulting Cognitive Load of the student. Cognitive Load Theory (CLT) is one of the more significant theories explaining how working memory functions during learning. CLT gives an instructional designer, or teacher, a better understanding about how to design material suitable for learning.

Cognitive Load Theory (CLT) is defined as:

> "A universal set of instructional principles and evidence-based guidelines that offer the most efficient methods to design and deliver instructional environment in ways that best utilise the limited capacity of working memory" (Clark et al., 2006, p.346).

The basic process of understanding new knowledge involves reconstruction of existing schemas otherwise defined as organised patterns of thought or behaviour (Paas et al., 2003), to generate new higher-order schemas, which contain the new knowledge. Therefore for less skilled learners the process of acquiring a new task or solving a complex task would involve processing the elements or components of knowledge or task as units into a number of low-order schemas. These low-order schemas are then combined to form higher-order schemas (Chandler & Sweller, 1992). Once the schema has been constructed for a complex task, all the related interactions are incorporated into the schema and this schema is treated as a single element by the working memory (WM), reducing the overall load of WM.

In dealing with mathematics instruction, Cognitive Load Theory assumes that schemas can be obtained more easily and rapidly by using a worked examples method compared with a problem-solving method (Sweller & Cooper, 1985; Cooper & Sweller, 1987; Sweller 1988). Sweller (1989) argued that studying worked examples facilitates solving mathematics problems more than problem-solving methods. A worked example is basically a solved problem with a shown step-by-step solution (NCTM, 2008). Problem-solving may be defined as engaging in a task for which the solution is not known in advance. It involves a mental process which requires higher-order cognitive processing (NCTM, 2008). In teaching mathematical topics, the focus is on teaching through problem-solving contexts and enquiry-oriented environments, where students are required to formulate questions and to find appropriate solutions to the questions and issues (Paas et al., 2003). Problem-solving and enquiry-oriented learning are characterised by the teacher as "helping students construct a deep understanding of mathematical ideas and processes by engaging them in doing mathematics: creating, conjecturing, exploring, testing, and verifying" (Lester et al., 1994, p.394). Recently, a number of educational advantages and benefits of worked examples compared to problem-solving have been demonstrated (Sweller et al., 2011).

Nowadays research has shown that worked examples have many advantages (Plass et al., 2010; Sweller et al., 2011; Clark et al., 2011) over problem-solving and following from this indications are that worked examples may help students by way of reducing cognitive load (Cooper & Sweller, 1987; Pass, 1992; Cooper, 1998; Trafton & Reiser, 1993; Tuovinen & Sweller, 1999; Van Gog et al., 2006). Nathan *et al.* (1994) and Chung & Tam (2005b) have suggested that worked examples could help English as a Second Language (ESL) students when they learn tertiary level mathematics. In this thesis, questions addressed are: How do students experience the use of worked examples and problem-solving approaches when attempting to achieve the learning and teaching objectives of their tertiary mathematics subjects? Is there a difference in learning performance through worked examples compared to learning through problem-solving approaches? Are there benefits in terms of greater confidence for example that can be attributed to either worked examples or problem-solving? Can faded worked examples be used to scaffold from worked examples to problem-solving in term of performance, confidence, or other attributes?

There are however many different learning outcomes. While better performance, say better test results, or time to learn or confidence may be attributable to one method, the issue is more complicated if other outcomes are positively associated with alternative methods. Many researchers argue that problem-solving builds skills and gives students all that they need for the work place whereas learning from worked examples does not qualify them and make them ready to solve problems in real life (Pass, 1992; Trafton & Reiser, 1993; Tuovinen & Sweller, 1999; Van Gog et al., 2006). As a result, this thesis examines of how to scaffold from worked examples to problem-solving.

1.2 Statement of the problem

One challenge facing tertiary level mathematics teachers is how to teach ESL students. This challenge is due to the difficulties faced by students learning mathematics in a foreign language in addition to the often found difficulties associated with students having inadequate mathematics skills. The combination of both of these factors is likely to increase cognitive load on learners (Paas et al., 2003).

Until the late 1970s mathematics research in education was predominantly quantitative in nature, when researchers and professionals in the field noticed there was a need for more qualitative research, especially in the area of language and learning of mathematics. By the 1990s, mathematics educators were becoming aware of the difficulties of second language learners (Lean et al., 1990; MacGregor & Moore, 1991) and the importance of language in the mathematics curriculum with various researchers identifying language features that influence mathematics teaching and learning (Cummins, 1981; Lean et al., 1990; Ellerton & Clements, 1991; Clarkson & Galbraith, 1992; Ellerton & Clarkson, 1996). As a result of these studies, there has been a view that the language features of word problems, textbooks, and tests affect understanding and consequently, performance. Further, if a student lacks understanding of a mathematics problem, there may be a decrease in their confidence in solving the problem. Mousley and Marks (1991) encouraged more language-sensitive approaches in mathematics classrooms with teachers who model and actively teach the language of mathematics. They addressed various levels of discourse that occur in mathematics classrooms such as reading, talking, and writing of text that may be written, spoken, in computer or in calculator software. In their view, a student's interaction with texts, tests and computers is dependent on language. The role of student discussion and writing in effective learning of the language of mathematics has been emphasised (Moschkovich, 1999).

While there has been a substantial amount of research in mathematics education at the school level (Grouws, 1992), the amount at the tertiary level was still modest (Seldon & Seldon, 1999; Varughese, 2009) with very few of the studies linking language to mathematical learning. One such study by Varughese and Glencross (1996) conducted in a South African university found that first year students whose first language was Xhosa, were being taught mathematics in English. They had difficulty with mathematical terms such as 'multiple', 'integer', and 'perimeter' due to the difficulty of the English language. The results of the study suggested that English as a second language students at the tertiary level could have difficulties with the language of mathematics. In more recent years, researchers have continued looking into the correlation between language proficiency and performance in mathematics courses, assessing it at the secondary and tertiary level (Setati, 2003a;

Barwell et al., 2007; Varughese, 2009). The impetus for such studies has been furthered by the enormous increase in the number of second language learners in most developed nations. Political and/or social changes along with globalisation have brought students together in groups in mainstream universities from all corners of the globe. Globalisation has resulted in an increase in new migrants and expatriate workers in many countries. Similarly, the current state of unrest in many parts of the world has led to an increase in refugees, which has contributed to the number of second language learners in host countries (Setati, 2003b). This continuation in interest of language proficiency and performance in mathematics courses is associated with increasing numbers of students from non-English speaking backgrounds in tertiary education and the large contributions they make to universities, economies and society. The contribution and growth in students from non-English speaking backgrounds is evident in the Australian education system (TEQSA, 2011). Consequently, there has been "a growing interest in language requirements for tertiary study and in the provision of programs that will assist students in their studies" (Barton & Beville-Barton, 2004, p.1). For example, a leading journal has devoted an entire issue to multilingual issues in mathematics education at all educational levels (Educational Studies in Mathematics, 2007, Volume 64, Issue 2). Three reasons that justify this focus are: increasing movement of people across international borders for work, education, opportunity, or peace; the rise of indigenous and minority movements bringing minority language groups into mainstream classrooms; and the growing interest and emphasis on culture-specific contexts such as code-switching or ethno-mathematics (Barwell et al., 2007).

While the earliest study comparing the use of worked examples and problem-solving in teaching mathematics was in 1990, there has been little attention paid to the use of worked examples with ESL students learning mathematics (Lewis, 2007). Early studies (Renkl, 1997; Aleven & Koedinger, 2002; Atkinson et al., 2003) examining how learners interpret worked examples found that some type of learner guidance must be included with the worked examples to increase their effectiveness. Other studies on worked examples (Cooper & Sweller, 1987; Paas & Van Merrienboer, 1993; Nathan et al., 1994; Sweller 1999; Tuovinen & Sweller, 1999; Chung & Tam, 2005a; Grobe & Renkl, 2006) focused on algebra problems showing that worked

examples are superior to practice problems because they reduce cognitive load. In this thesis special attention is paid to the use of worked examples and problem-solving over a range of mathematical topics with students who study mathematics in a second language.

1.3 Study contexts

Through a multiple case study approach, this thesis examines learning outcomes for ESL students using worked examples and problem-solving in two different contexts, Australia and Saudi Arabia. Both studies compare the use of worked examples and problem-solving in mathematics with ESL students. The aim is to identify effective strategies for teaching mathematics to students learning in a second language at tertiary level and in particular to explore the effectiveness of using a combination of worked examples and problem-solving techniques.

The first case study is carried out at King Abdul-Aziz University, Saudi Arabia. Saudi Arabia's decision to join the World Trade Organisation (WTO) in 2005 resulted in an economic boom for multinational corporations. However, for those seeking to find work, employability is heavily reliant on their capacity to communicate effectively in languages other than Arabic. Consequently, the Saudi government implemented legislation mandating that universities in Saudi Arabia, such as Abdul-Aziz University, instructing English. This was to ensure that graduates are equipped with the language and educational skills to compete in an employment market that has become more globalised. While the students study in the context of their country of birth, the challenge for teachers and students alike is that they seek to teach and learn respectively, a difficult discipline such as mathematics, in a second language (Shaabi, 2010).

The second case study is undertaken at the UOWC, Australia. The students attending the college are drawn primarily from overseas. They speak different languages such as Chinese, Japanese, and Arabic. These students have moved from their home country to study in another country, Australia. They potentially come from diverse backgrounds, but in each case the student is leaning mathematics in English, which is not their first language.

Potentially the comparison between the two contexts may suggest common or quite different approaches for teaching tertiary mathematics to students for whom English is their second language. Findings in common from such diverse environments will suggest that results are generalisable, certainly more broadly generalisable than an individual case study would suggest. Differences would need to be examined to determine if they are due to other varying factors between the educational systems.

1.4 Research questions

The specific questions regarding ESL students addressed by this research include:

1. How do students experience the use of worked examples and problem-solving approaches in terms of attributes such as anxiety, ease of learning, understanding, enjoyment, mental effort, speed of learning and confidence when attempting to achieve the learning and teaching objectives of their tertiary mathematics subjects?

2. Are there benefits in terms of greater confidence or other attributes such as reduced anxiety when learning that can be attributed to either worked examples or problem-solving?

3. Is there a difference in performance when learning through worked examples compared to learning through problem-solving approaches?

4. Can faded worked examples be used to scaffold from worked examples to problem-solving in terms of performance, confidence or other attributes?

1.5 Theoretical bases

This thesis draws on Cognitive Load Theory as it pertains to teaching ESL students and to the manner in which students learn. A student's second language acquisition has a great bearing on how they will manage to learn mathematics in a tertiary level classroom. According to Barton (2005) a student can become bilingual to varying degrees. Barton (2005) identified two main types of bilingualism. The first is 'colloquially bilingual', where a student's language is sufficient in the English language to carry out daily conversations and to function in terms of daily tasks. The second is 'cognitively bilingual'. In this instance the ESL student would be classified

as academically bilingual and is capable of learning at higher cognitive levels in their second language. A student that is cognitively bilingual will be more capable of absorbing mathematical material than the colloquially bilingual student who in performing daily tasks is operating with a lower cognitive load.

Cognitive Load Theory (CLT) proposes that the nature of information to be learned and the manner in which it is presented interact to place a total load on the working memory (WM) of the learner. Cognitive load researchers have distinguished three different components of cognitive load which are based on the source of the elements contributing to that load. The first component is cognitive load imposed by the nature of the information being learned and this is referred to as 'Intrinsic cognitive load' (ICL). The second component, 'Extraneous cognitive load' (ECL), is concerned with the teaching methodologies adopted to present the materials to a learner. 'Germane cognitive load" (GCL) is the cognitive load effort made by learners during the process of studying to build and automate schemas. It is argued that the load from these three components is additive, and it is the total cognitive load that is critical for learning and should not exceed the limits of working memory capacity (Sweller et al., 1998).

The primary purpose of CLT is to explain how to manage and maintain the cognitive load inside working memory (WM), which is supposed to have limited capacity and duration. Manipulating ICL, which is related to the material's complexity, can be fulfilled with two tactics: isolating the material factors (Pollock et al., 2002; Ayres, 2006; Sweller & Sweller, 2006) and utilising a pre-training, stage allowing learners to construct prior-knowledge or sub-schemas to assist them to deal with the complexity of the material (Mayer, 2002; Mayer, 2003; Clarke et al., 2005). These methods are based on the assumption that learners' previous knowledge is a key element, especially when handling material that has high interactivity between elements of the materials to be learned. The literature on Cognitive Load Theory (CLT) and its role in improving learning outcomes is further reviewed in Chapter 3. The discussion as guided by CLT examines the design of materials to eliminate common problems such as the split-attention effect (Sweller, 2003) in teaching mathematics.

1.6 Structure of the thesis

This thesis is presented in seven chapters. Chapter 1 provides an introduction and background to the main study and a statement of the problem and the significance of the study. It introduces the research questions, theoretical bases and initial definitions for the terms used in the study. A summary of literature in teaching and learning mathematics in the ESL context is presented in Chapter 2. It also provides the historical perspectives on mathematics education research, the inclusion of language as a focus, and recent research related to language difficulties in learning mathematics. Chapter3discussing aspects of cognitive load theory and its role in improving learning outcomes. It also includes discussion as to how teachers or lecturers can manage cognitive load during the mathematics learning process especially for ESL students. Chapter 4 focuses on worked examples and their advantages in teaching mathematics to ESL students. It includes a discussion on how to redesign examples so they are more effective for students undertaking higher mathematics education. The definition of problem-solving and the role of schema and guidance during problem-solving are also examined in chapter 4 along with the impact on learning through problem-solving at a tertiary level. Chapter 5 addresses the methodological issues and documentation of the theoretical frameworks that inform the selection of an appropriate research design and procedures for the study. While in Chapter 6 the context and outcomes of the two case studies comparing the use of worked examples and problem-solving techniques with ESL students is presented. The first case study examines learning for students in their home country when studying mathematics in a second language, English, at King Abdul-Aziz University, Saudi Arabia, Jeddah. The second case study is in the context of international students at UOWC, Australia who are studying mathematics in English which is a second language. In the final chapter, a summary and discussion of the results from the cases studies is provided, followed by a discussion of possible implications for future researchers.

2 TEACHING AND LEARNING MATHEMATICS IN THE ESL CONTEXT

Mathematics cannot be learned without being understood

it is not a matter of formulae being committed to memory

but of acquiring a capacity for systematic thought

(Hilton, 1986, p.3)

2.1 Introduction

Globalisation has placed a growing importance on both speaking and listening to the English language. Prior research indicates that many international students from Asia, studying in Australia, face serious learning difficulties, lacking confidence in both speaking and taking a proactive role in the classroom (Sawir, 2005). Globalisation, which is the tendency for world-wide convergence in education and other sectors (Held et al., 1999), is changing the environment in which English is learned as a second language (ESL). Economic and cultural globalisation includes the globalisation of language, and in particular the spreading role of English as a universal global language (Crystal, 2003).

It is English that stands at the very centre of the global language system. It has become the lingua franca par excellence and continues to entrench this dominance in a self-reinforcing process. It has become the central language of communication in business, politics, administration, science and academia, as well as being the dominant language of globalised advertising and popular culture (Held et al., 1999).

Challenges facing ESL students studying mathematics include the complexity of learning English and in particular the language associated with learning mathematics. Cognitive load theory (CLT) has made a significant contribution to the understanding of the learning process, particularly in relation to the complexity of material to be learned. Issues surrounding the learning processes of ESL students have come to light with the increased understanding of the function of working memory, the

susceptibility of working memory overload, and the impact such overload has on the development of schemas or the organisation of information elements and the automation process, whereby “learned material can be processed automatically without conscious effort allowing attention to be directed elsewhere” (Sweller, 1994, p.304). A by-product of CLT and the subsequent research it has motivated is the understanding educators have of the impact lesson plans and material design have on the way in which ESL students learn. By educating teachers/lecturer about the various types of cognitive load, and their ability to manage Extraneous cognitive load and perhaps even Germane Cognitive load by making simple modifications to the teaching materials, strategies can be implemented to reduce cognitive load for ESL students. One such potential strategy for use with ESL students is to move away from the problem-solving approach in teaching mathematics toward an approach based more heavily on worked examples.

Researchers have for some years been involved in examining the impact of language on learning mathematical concepts and mathematical cognition, while educators and policy makers have been using such information to adapt the mode of teaching (Latu, 2005), assessment (Ellerton & Clarkson, 1996), and the composition of multilingual classrooms (Lesh, 2002). Given the magnitude of the ESL student population and the implications for education this chapter highlights the importance of further research on language and the impact of language on ESL students’ ability to understand mathematics.

2.2 The progression of mathematics educational research

Mathematics education has come a long way since the late 1960’s and early 1970’s. The evolution of research in mathematics has followed the tide of societal change moving from simplistic models of cause and effect, to what it can be seen today, where mathematics education research is addressing the entire complex system of education and learning. This need for change has been driven by the fact that researchers and educators alike have concluded that quantitative experimental research alone is not conducive to understanding the interactions within a complex system of individuals, educators, teaching materials and learning environments (Lesh, 2002). The shift in research has resulted in the implementation of qualitative

based research protocols, the emergence of language as a research focus, and the interaction of language and mathematical cognition.

2.2.1 Implementation of qualitative based research protocols

The awareness brought about by Erlwanger's dissertation (1975) regarding the human interaction in mathematics classrooms, was a catalyst in motivating researchers to give prominence to qualitative, pheno-menographic, and ethno-graphic paradigms (English, 2002). In the late 1970's alternative modes of research began to surface. Researchers were investigating the best approach to teaching and learning mathematics in classrooms, with the classical use of inferential statistical procedures abandoned and a more qualitative approach adopted (Ellerton & Clarkson, 1996). This dominance on qualitative method continues, with Park and Bae (2011) finding for example that of 710 articles on research into mathematics education published in six journals from 1995 to 2010, only (21%) were purely quantitative in nature, while (50%) were qualitative and (29%) used mixed methods.

As a result of the shift in paradigm it became common to see researchers in more active participatory roles rather than gathering observations from afar. For example, Newman (1983a) used post-test interviews to investigate the errors made in mathematical problem-solving. A more pheno-menographic approach, allowed for a more comprehensive understanding of the learning process in mathematics (Clements, 1980; Watson, 1980). The paradigm shift to a more interpretive paradigm in mathematics allowed researchers to fill in gaps in understanding that relate to human experience and understanding. Such trends also saw researchers increasingly become participants in the research settings rather than just being outside observers. Park and Bae (2011) called attention to the actual teaching process in the classroom during this crucial stage in the evolution of mathematics education. With this shift in thinking among educational researchers around the world in this century, it is recognised that individual and cultural constraints integral to mathematics education now need to be taken into account.

2.2.2 Emergence of language in learning as a research priority

An important goal of research is to look beyond the immediate and find new ways of thinking about problems and potential solutions rather than quick fix solutions or answers to specific questions (Lesh & Lovitts, 2000). English (2002) identifies four catalysts for a shift in research priorities namely, national and international mathematics testing, influences from social, cultural, economic, and political factors, increased sophistication and availability of technology, and increased globalisation of mathematics education and research. Shifts in emphasis in mathematics teaching from teacher-centered, formalistic approaches to student-centered, heuristic approaches (Schoenfeld, 2007), advancement in technologies (Niss, 1999), and the emergence of ethno-mathematics linking mathematics and culture (Gerdes, 1996), were all factors that contributed to this change in priorities. One of the priorities that emerged was the recognition of the importance of language in mathematics learning. This has been brought about by two major factors: the emphasis on problem-solving, heuristic approaches, and the enormous increase in the numbers of second language learners in most developed nations.

Research into the first factor has investigated numerous aspects of mathematical problem-solving in mathematics education. For example, studies have looked into problem-solving in mathematics education policy and promise (Otten, 2010), problem-solving as a key feature of doing (and learning) mathematics (e.g., Carpenter et al., 1993; Coxford, 1997; Lappan et al., 1995), the importance of problem-solving in the curriculum (Schoenfeld, 2002; 2007; Lester et al., 1994; Santos-Trigo, 2007), the role of meta-cognition problem-solving (Siemon, 1993; Lesh & Zawojewski, 2007), and the role of comprehension in problem-solving (Cummins et al., 1988; Muis, 2004). It has emerged from all these studies that problem-solving is important for learning mathematics with understanding, and meta-cognition and comprehension are important in problem-solving. In my view, and as an ESL learner, both meta-cognition and comprehension are reliant on language proficiency, and consequently point to the importance of language in the learning of mathematics.

The second factor leading to language as a research priority is the increased numbers of second language learners in classrooms all over the world. This increase in numbers is attributed to political or social changes that have brought students from minority groups into mainstream classrooms (Setati, 2007). For example, political changes in South Africa saw the end of segregated 'black' and 'white' education, and social reforms in other countries provided educational opportunities for language minorities, resulting in multilingual classrooms (Varughese, 2009). Globalisation resulted in an increase in new migrants and expatriate workers in many countries and the current state of unrest in many parts of the world has led to an increase in refugees which has contributed to the number of second language learners in host countries. Another recent trend is the influx of international students to tertiary institutions in developed countries (Setati, 2007). Educational opportunities which were initially provided to foreign students with varying intentions such as specialisation in a discipline, or as assistance to politically or economically unstable countries, or enhancement of diplomatic ties, have now become a multi-billion dollar 'service export' marketed and promoted by educational institutions and governments alike (Setati, 2003a).

2.2.3 Increasing number of ESL students in tertiary education

Even though the number of ESL student is increasing in countries such as the United States and Australia most educators have not paid enough attention to the way ESL students learn mathematics, especially at the tertiary level (Varughese, 2009). According to the data released by the U.S. Institute of International Education (IIE) (2011), international tertiary student enrolments have been on an increase since the 1960's (refer Table 2.1). Figures released by Australian Education International (AEI) (2011) indicate similar results. Astonishingly, since 2007, Australia and the United States combined, accounted for more than a million international students in tertiary education each year.

Table 2. 1 International tertiary student enrolments in Australia and the US

Year	Australia	United States	Year	Australia	United States
1960	-	48,486	2004	325,356	572,509
1970	-	134,959	2005	346,079	565,039
1980	8,777	286,343	2006	383,818	564,766
1990	-	386,851	2007	455,185	582,984
2000	188,277	514,723	2008	543,898	623,805
2001	233,408	547,867	2009	582,234	645,632
2002	274,877	582,996	2010	632,597	691,352
2003	307,956	586,323	2011	676,632	712,362

Source: Data released by IIE and AEI, 2011

With countries such as the UK (UKCISA, 2013), Canada (AUCC, 2013), and New Zealand (MBIE, 2013) also having large cohorts of international students, and countries such as South Africa (DHET, 2010), and India (SI, 2012) with multilingual populations, potentially millions of students are learning mathematics in a language that is not their first language and these numbers are suggestive of the magnitude of problems for teachers and students alike.

2.2.4 Language and mathematical cognition

In the late 70s and early 80s much of the research conducted related to the components of language such as syntax, technical vocabulary, and grammar and its use in teaching, learning and assessing mathematics (Haylock & Thangata, 2007). Studies conducted in recent years focus on the role of syntax (Park & Bae, 2011), and sentence structure (Varughese, 2009), in the mathematics curriculum. One such study (Varughese, 2009) suggested that the syntactic complexity of written mathematical problems influences students' ability to solve mathematics problems. Although it was a preliminary study, it prompted questions regarding the language of standardised tests of mathematical performance and arithmetic texts that are commonly utilised at the upper elementary and junior high school level. Research led by Park *et al.* (2011) has confirmed the importance of reading, writing, vocabulary and symbolism of mathematics.

A multilingual context for researching the effects of language in learning and teaching mathematics began to emerge in the mid-1980s, when the interest in language was no longer limited to the dominant language of teachers in the

classroom. Researchers and educators began looking at the effects of language and cognition in migrants, the language of minority students in classrooms and at strategies for teaching such students. For example, a study by Mestre (1986) identified error patterns and areas of difficulty in problem-solving tasks for a group of Hispanic technical college students, and advocated the integration of the teaching of language skills with the teaching of problem-solving skills. This study was followed the research by Mousley and Marks (1991) which encouraged more language-sensitive approaches in mathematics classrooms by teachers who model and actively teach the language of mathematics. They addressed various methods of discourse that occur within mathematics classrooms such as talking, reading, and writing. In recent years there is also the use of different computer software, smart board technologies, and tablets, which have all transformed the medium on how mathematics is taught and learnt.

Further research was undertaken in the last decade as international student numbers began to increase rapidly in tertiary classrooms. Understanding of the role of language in mathematics continues to grow, with increased research being conducted in dissecting the relationship between language proficiency and performance in mathematics in higher education. The language of mathematics is often seen to be dominated by understanding and working with numbers, symbols and equations. For example a study conducted by Setati (2007) in a South African university found that first year students had difficulty with mathematical terms such as ‘multiple’, ‘integer’, and ‘perimeter’. The focus on working with numbers, symbols and equations is quite appropriate when discussing elementary level mathematics and even more so at the tertiary level, when as a student advances to university level mathematics, and are introduced to highly sophisticated concepts, which can be difficult for students to grasp if their language skills are not proficient (Barton, 2005). For example higher-level mathematics often demands students focus on proofs that require clear and concise expression of ideas to convey an argument and this is dependent upon both their ability to understand the mathematical concepts and the ability to communicate concisely. While the language of mathematics appears to be the abstract language of numbers and symbols, the core of mathematics is still very much rooted in broader language. Therefore, students who struggle with

language, such as ESL students, will have difficulties in reading and interpreting mathematical problems (Setati, 2007). In accord with this topic of research, this study is based on the premise that second language learners at the tertiary level are likely to have difficulties with the language of mathematics and for this reason it is important to consider strategies for teaching and learning.

2.3 ESL strategies for teaching and learning mathematics

Teachers need to be aware of a variety of strategies used by ESL students and also aware that they can facilitate or hinder learning. These include code-switching, borrowing, and multimodal approaches.

2.3.1 Code-switching

Code-switching is a product of bilingualism. It is a term used to describe the process by which an individual switches from one language to another. "Code-switching is a verbal skill requiring a large degree of linguistic competence in more than one language, rather than a defect arising from insufficient knowledge of one or the other rather than presenting deviant behaviour, [it] is actually a suggestive indicator of degree of bilingual competence" (Poplack, 2000, p.6).

Research suggests code-switching is common in ESL students (Clarkson & Galbraith, 1992; Kern, 1994) as ESL students attempt to compensate for the deficiency (MacGregor, 1991; Setati, 1998) in their second language with code-switching. The impact of bilingualism on learning mathematics is not clear cut with some studies suggesting it hinders learning (Simon & Tzur, 2004) and more recent studies suggesting it is of benefit (Weisman et al., 2007), that it reflects linguistic competence (Varughese, 2009) and facilitates learning (Park & Bae, 2011) .

The benefits of code-switching spans all age-groups (Nilep, 2006), meaning children as young as three have the capacity to code-switch to enrich their understanding of the world that surrounds them, just as adults can implement it to communicate and learn. For example, in a recent study by Clarkson (2007), bilingual Year 4 Vietnamese students in Australia were observed to have an advantage in learning mathematical concepts (such as congruent and similar, discrete and continuous, explicit and implicit, bar chart) as they had greater linguistic skills than their

monolingual peers. Their ability to code-switch from English to Vietnamese, resulting in increased understand and in turn led to an increased level of confidence when students attempted difficult mathematical problems.

Whether or not code-switching is beneficial or not appears to depend on language competence. For example, the observation in a Type B (refer Table 2.5) classroom by Kazima (2007) of bilingual Malawi students in relation to the vocabulary used in teaching probability highlighted some of the failures of code-switching for ESL students in the learning of mathematics. The students operated in both Chichewa and English, relying heavily on the former. It appeared that when the students encountered English probability vocabulary, they interpreted the words into Chichewa, did the thinking in Chichewa, and then translated their responses into English. One specific example of the effect of Chichewa on students' understanding of English probability words was observed on students' responses to the words 'likely' and 'unlikely'. Since in Chichewa 'likely' is understood as 'not unlikely', students might have difficulties with phrases such as 'not very likely' and 'equally likely' because in Chichewa they sound like 'not very not unlikely' and 'equally not unlikely' respectively. Similarly in a study of Year 12 Pasifka mathematics students in a Type B classroom in New Zealand it was found that language difficulties hindered students' ability to understand complex word problems. This was attributed to the fact that word problems require the student to "read the statement, think, analyse and carry out the appropriate computation" (Latu, 2005, p.489). Such a process requires the student to be able to draw meaning from the text, which in this case would be in a second language. Many other studies have identified similar sources of confusion arising from code-switching (Setati, 2007; Varughese, 2009; Park & Bae, 2011). This kind of confusion does not exist for English monolingual students, although other kinds of confusion might arise (Kazima, 2007, p.187).

2.3.2 Borrowing

Borrowing is not the same as 'Code-switching'. According to Grosjean (2001), borrowing uses a word or short phrase from the minority language and adapts it in form and sound into the majority language. Translating to another language can result in a change of meaning due to the linguistic differences between the two

languages (Kern, 1994), and this can have a detrimental effect on understanding. Setati (2005) stated that:

> [b]orrowing refers to the insertion of single words or short phrase into a sentence in another language. As learners engage in exploratory talk and this occurs largely in their main language, mathematical English in mixed into their speech. For example, words like 'equals' and 'sum' become part of a conversation in the learners' main language (e.g. Setswana). Borrowing is different from integration because in integration the borrowed words have been linguistically transformed from one language and have become part of the other language (p.73).

Borrowing concepts from other disciplines is a productive approach for the integration of language into the study of mathematics learning. For examples, "the distinction between 'national' languages such as Spanish, English or 'social' languages such as mathematical or academic discourses is useful in clarifying what we mean when we use the term 'language'.

2.3.3 A multimodal approach

A multimodal approach to teaching mathematics to ESL students has been noted to be one of the most effective means of instruction. A multimodal approach to teaching mathematics requires the educator to understand the learning needs of the students in their class. In a post-secondary environment, ESL students in mathematics classrooms will have varying levels of expertise with the subject matter. The greatest challenge for teachers is to not simply teach the material, but to ensure the ESL student is progressing in mathematics as it is taught in English. This requires that lessons for ESL students be specifically designed with their need for language acquisition but also for mathematical content (Di Pietro & Ern, 2012).

Educators using multimodal approaches to teaching ESL students, have various strategies to draw on. The focus of this section is to highlight some of the most commonly used strategies in teaching mathematics to ESL students. A summary of strategies is provided in Table 2.3.

Table 2. 2 Strategies in teaching mathematics to ESL students

Language Focused Strategies	**Summary**	**Source**
1. Teach Vocabulary using demonstration.	Implement the use of concrete objects in hands-on activities to make sense of abstract mathematics concepts.	Wiersman, 2005.
2. Relate mathematics problems and vocabulary to existing knowledge.	Build the learning of new material on existing knowledge and vocabulary. This makes teaching more manageable as educators are able to assess where students' understanding is deficient and tailor teaching strategies to meet these needs.	Ron, 1999.
3. Apply problems to daily life situations.	Apply mathematical concepts to real life situations, giving students practical applications for an otherwise abstract problem.	Wiersman, 2005.
4. Use manipulative to make problems concrete.	Use concrete examples along with commercial manipulative (patterns, visual images, etc.). These increase the enjoyment students get in learning mathematics as it takes away from the mundane worksheet and textbook work.	Torres-Velasquez & Lobo, 2005.
5. Encourage drawings to translate visual word problems.	Especially effective with beginning level English students, as visual representation assists in the processing of information from textual or auditory form to visual.	Van Garderen, 2004.
6. Encourage students to think aloud when solving word problems and have students give oral explanations of their thinking.	Have students verbalise the problem-solving process, so it becomes a meta-cognitive task that not only assists the student in the process, but also gives the teacher insight in to how the student is dissecting and answering the problem.	Simon & Tzur, 2004.
7. Have students write original word problems to exchange with classmates.	Reinforce phonetic skills of reading and writing.	Wiersman, 2005.
8. Explain directions clearly, and repeat key terms.	ESL students' issues lie in four critical areas of language in mathematics: "vocabulary skills, syntax, semantics, and discourse".	Toh *et al.*, 2010.

Table 2. 3 Strategies in teaching mathematics to ESL students continued

Mathematics focused Strategies	Summary	Source
1. Awareness that not all mathematics notations are necessarily universal.	Become aware of possible discrepancies in notations. For example notations such as 1.834 can be interpreted as 1,834.	Wiersman, 2005.
2. Group students heterogeneously during cooperative learning.	Use a cooperative learning strategy which lets ESL students know that the learning needs of all students in the class are variable, not just theirs.	Di Pietro & Ern, 2012.
3. Make interdisciplinary connections to what students are learning in mathematics.	Relate mathematics concepts to other subjects.	Wiersman, 2005.
4. Make cultural connections for students when teaching mathematics.	Embrace the various cultures within the classroom, and adapt lessons to use various elements of culture within the lesson.	Simon & Tzur, 2004.
5. Rewrite word problems in simple terms.	This act of rewriting the problem demonstrates students understanding of key term and concepts.	Wiersman, 2005.
6. Concretise mathematics concepts with Total Physical Response.	TPR (Total Physical Response) is an approach to learning a second language. In mathematics educators and students are physically engaged in the learning process, perhaps by participating in activity.	Torres-Velasquez & Lobo, 2005.
7. Create word bank charts and hang them in the classroom for viewing.	Key word bank (site words), can be reviewed and mounted in English and perhaps have a panel with space to write in the native language	Wiersman, 2005.
8. Take internet field trips and use mathematics software	Use software and internet sites to aid in various stages of language acquisition and of mathematics concepts.	Boero *et al.*, 2002.
9. Use of literature to teach mathematics and develop language.	Use literature to link students understanding and ability to communicate such understanding toward mathematics.	Wiersman, 2005.
10. Using a multimodal teaching style.	Adopting more technology laden techniques in the classroom that will stimulate the students in more ways than reading from a book or listening to a lecture. Use software, internet sites, books, models, etc.	Phillipson, 2005.

The strategies outlined are all recommendations that have been made as part of a changing pedagogical approach to teaching mathematics to ESL students. The needs based, student centered approach to teaching these students is considered critical for their success, especially in the instance of heterogeneous student classrooms with various language levels and backgrounds. By implementing strategies of inclusion and multimodal techniques, language acquisition can occur more rapidly as expression is encouraged and meaning is derived from the tasks accomplished within the classroom.

2.4 Conversational vs. academic bilingualism

According to Barton and Neville-Barton (2005) the ability of a person to speak a language for day-to-day purposes, referred to as conversational proficiency, will not necessarily result in academic proficiency (refer Figure 2.1). This is because without academic proficiency their language competency is not sufficient to take in new ideas and material, as this requires them to make language associations with the new material in the second language. Barwell (2005) indicated that conversational proficiency is achievable much earlier than academic proficiency, with cognitive framework adaptations and developments needed in order to reach that stage of academic proficiency. ESL students' performance may be partially explained by Cummins theory of threshold hypnosis. This theory states, "a minimum threshold in language proficiency must be passed before a second-language speaker can reap any benefits from language". It also states that, "in order to gain proficiency in a second language, the learner must also have passed a certain age-appropriate level of competence in his or her first language" (Franson, 2009, p.23). This was exemplified in a South African study that considered the performance of calculus students (Gerber et al., 2005). The students were divided into two groups: the first group received instruction in their first language, Afrikaans; and the second group was instructed in English. It was established that although the Afrikaans language is similar to English, the students receiving instruction in English did not perform as well as those in the Afrikaans class. The results of this test supported the notion that although a student may be conversationally proficient in a second language, in this case English, it does not mean they will have academic proficiency. All the students involved in the research were conversationally proficient in English but they still

lacked the cognitive ability to process information in English, leading to poorer learning outcomes.

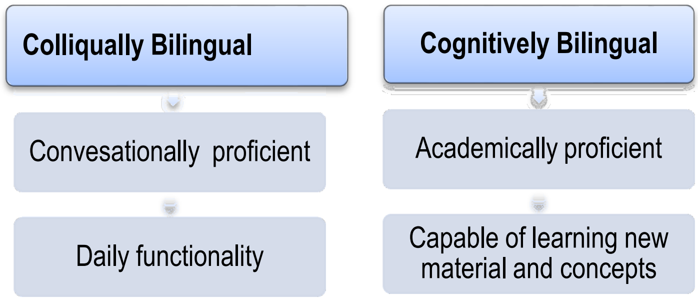


Figure 2.1 Bilingualism branches and learning proficiency
(Source: The National Centre for Academic Transformation (NATE), 2010)

The study by Gerber *et al.* (2005), established students could be colloquially bilingual but not necessarily cognitively bilingual. The important distinction between the two has varying implications for ESL students. A series of studies in New Zealand (Barton, 2005) examined the issues surrounding the performance of ESL students when various syntaxes (methods of wording the problem) were used in test questions. The students were then asked to report on their understanding of the questions. The study was conducted with ESL students studying mathematics at the secondary level and tertiary undergraduate level. The findings of this research indicated that as students progress in their degree the logical complexity of mathematics increases. Therefore academic proficiency is less likely to occur, if they have not established the cognitive framework early enough in order to process the complexity of language and symbols in their third year of undergraduate studying mathematics in English.

2.5 Language of texts and assessments

Sound literacy is a fundamental component of mathematical learning, as it is the foundation on which one learns and is assessed in the classroom. Numerous studies have been carried out in relation to language, literacy and academic performance in mathematics (Ron, 1999; Boero et al, 2002; Simon & Tzur, 2004; Torres-Velasquez & Lobo, 2005; Di Pietro & Ern, 2012). Mousley and Marks (1991) stated a student's ability to correctly solve mathematical problems is heavily dependent on their ability to read and interpret the questions. This notion was supported by Birmingham and

Haunty (2013) with their findings indicating that linguistic structures of mathematical word problems are likely to affect a student's performance. This could be attributed to the more cognitive load due to the student's language background, or they may not have had the opportunity to develop academic proficiency in the English language.

Reusser *et al.* (1988) and Kintsch *et al.* (2006), found that students in a classroom setting were more likely to make solution errors in word problems due to miscomprehension of abstract or ambiguous language. In support of the "linguistic development" they, like Lean *et al.* (1990), have indicated that such students may have more cognitive load when processing word problems.

Language used in mathematical assessments also, has a great impact on students studying in a second language. ESL students at any level of mathematics will struggle to make connections with unfamiliar material in a problem task due to language and cognitive deficiencies (Cooper & Sweller, 1987; Prins, 1998; Verschaffel, 2000). Questions may be raised regarding language use in texts and assessments, and how to assess students in multilingual classrooms, in a more accurate manner so that it would not advantage one group over another due to language impediments. This poses a concern for not only the ESL students but for educators as well. However, the issue is not straight forward. When evaluated more closely, the impact that textual language has on an ESL student's ability to achieve academically can be compounded positively or negatively depending on the classroom setting (either Type A or B) in which they find themselves (Varughese, 2009).

2.6 Student writing in mathematics

Writing is another component of language that is important to mathematics learning at the tertiary level as most learning and assessment tasks at this level are written. In my view, mathematical writing requires cognitive proficiency and academic literacy, both of which are required in some form at all levels but particularly so at the tertiary level. This view is supported by literature on student writing in mathematics. Being proficient in academic literacy requires knowledge of the type of language that is predominantly used in classrooms and is related to learning (Dougherty, 1996;

Kershner et al., 2006). Students from different cultures write in culturally different ways (Di Pietro & Ern, 2012), and this can be problematic when students are writing in an education system not based on their own culture. Moreover, Di Pietro and Ern (2012) concluded that mathematical learning is dependent on a student's ability to write and also, have shown that writing in mathematics is part of the learning process as it is used as a means of learning, evaluating and assessing a student's ability to process, understand and explain mathematical concepts. Such a skill is not developed without practice, so the more often students are encouraged to write down solutions, problems, and articulate their thinking in writing, the more likely they are to continually improve academically as a result of developing another area of their cognitive framework.

2.7 Mathematical learning difficulties related to language

The highly specialised language of mathematics impedes understanding for many students. For some students the technical vocabulary is problematic, whereas for others the formal tone and rule bound structure of mathematical language is difficult. For ESL students it is more likely that they will struggle with both aspects of mathematical language. The following sections examine the difficulties identified in mathematical language as well as theories that may increase our understanding of the challenges faced by ESL students.

2.7.1 Language difficulties defined

ESL teachers have long understood the relationship between language and learning. This has not been the case with mathematics teachers (Adler, 2001). With the up and coming research into the area of language and mathematics, the importance of language and its use in mathematics is taking hold. Language as a means of communication in mathematics is just as strong as it is in any other subject such as literature. However the challenges that face ESL students are due to the complexities that come with the way in which language is used in mathematics and its impact on the their learning. This section explores how vocabulary, syntax and structure can impact an ESL student's ability to understand mathematics.

The vocabulary of mathematics is technical, and at times words can have dual meanings in mathematics and in everyday language. For example the term 'independent' in everyday language is likely to mean that a person takes their own path, or does things differently to others, whereas for events to be 'independent' in probability the profiles of how those events occur is the same under independence. There are many such instances, where are dual meanings of mathematics language in everyday English. Words such as 'plane', 'volume' have vastly different meanings in mathematics than in everyday English.

Another issue is that the technical vocabulary of mathematics is often related to terms that are isolated in a sense to the mathematics classroom (Haylock & Thangata, 2007). For example, 'square root', the likelihood of the use of this term outside the classroom is rare and hence adds to the preconception that mathematics language is only for the classroom.

One source of ambiguity for ESL students is the often placed miscues in word problems. Miscues occur when the language used is meant to be used in a manner other than how it has been used when the miscue arose (Di Pietro & Ern, 2012). For example, Kate has $40, which is $30 dollars more than she had last week. How much money did Kate have last week? In this example the 'more than' statement used is commonly attributed to addition, but in this case is used for a subtraction problem. Such miscues will often confuse ESL students. In such instances, the use of exercises to ask them to rewrite a problem in their own words, or to draw a diagram or even verbalise their understanding, will help the teacher in identifying possible miscues.

Another language difficulty that ESL students are commonly encountering is issues with syntax. For example (refer Figure 2.2), the way in which problems and examples are worded, can cause a lot of ESL students strife, as they find it difficult to decipher what is actually being asked of them in the problem. Teachers should try to eliminate as much ambiguity as possible in the creation of word problems and examples whilst teaching ESL students.

The Laundry Problem

Sandy’s family does its laundry at a coin-operated Laundromat. It costs \$1.25 per load to use the washing machines and 25¢ per load to use the dryers for 10 minutes. Sandy’s family has 5 loads of laundry to do and each load will need to be in a dryer for 30 minutes. Which expression will give Sandy’s family the total cost of doing these loads of laundry?

A. (\$1.25 + \$0.25) × 3 × 5
B. [\$1.25 + (3 × \$0.25)] × 5
C. [(3 × \$1.25) + \$0.25] × 5
D. 3 × (\$1.25 + \$0.25) × 5

Figure 2.2 Mathematics Problem

Students need to be taught to seek out the structure of word problems to go beyond the narrative, in search of the mathematical question. By doing so students are able to isolate mathematical language and identify the variables and what is being asked. However this process of deciphering the meaning of mathematical language is only going to come about with extensive practice and exposure. The more diligent a teacher is in adopting different strategies to teach the language of mathematics to ESL students, that is by using multimodal teaching strategies and intensive student evaluation and observation, the more relaxed and adept the student can be in learning the language of English as well as the unfamiliar language of mathematics. The difficulty with such examples is that ESL students as in Figure 2.2 is that students would have to be, at a minimum, conversationally proficient in the English language to understand the aim of the problem, identify the variables of importance and follow through with a solution to the problem.

Numerous studies as categorised and presented in Table 2.4 attempt to decipher the cause of language difficulties in mathematics and systematically classify them. Such information is crucial for mathematics teachers at all levels, if they are to avoid making learning more difficult especially in today’s climate of multicultural classrooms.

Table 2. 4 Language difficulties in learning mathematics

Difficulties	Examples	Source
Distinguishing between pairs of similar words that refer to related concepts.	Congruent & similar, discrete & continuous, explicit & implicit, bar chart, histogram, convex & concave, row & column, hundreds & hundredths.	Reyes, 2000; Tupas, 2001.
Multiple usages or meanings of words.	Square, round, base, inverse Vertex, tangent	Tupas, 2001.
Words shared between English and mathematics with different meanings.	Right, event, power, volume, log.	Moschkovich, 2005.
Words shared by science and mathematics with different technical meanings.	Element, cell, tree, solution, radical, image.	Martin-Rhee, 2008.
Ambiguity in use of closely related or interchangeable words.	Depth, thickness, length, breadth, width, height.	Marian, 2006.
Grasping mathematical phrases with precise meanings.	Then, if and only if at most, at least, not more.	Emmorey *et al.*, 2008.
Words used only in mathematics.	Quotient, hypotenuse, isosceles, asymptote.	Bialystok, 2010.
Use of proposition affecting meaning.	The water rose from 5 cm The water level rose by The water level rose to	Bernardo, 2005 a, b.
Distinguishing between the use of articles and indefinite pronouns.	The, some, all, a, an.	Lee, 2005.

2.8 Learning issues in different types of classroom environments

The analysis of literature in this section focuses on the key language issues for learning mathematics explored through a template classifying language and mathematics related findings according to the type of classroom context. The literature reviewed relates to the two types of multilingual classrooms, those where the national language is English (Type A) and those where the national language is other than English (Type B), refer to Table 2.5.

Table 2. 5 Categories of Multilingual Classrooms

'Type A'	**'Type B'**
ESL students are from other non-English speaking countries (international) or from families that were recent immigrants from non-English speaking countries.	Students study in their home country, whose official language is not English, but where the prescribed language of study is English.
The teacher does not (or is less likely) to speak the student's first language.	The teacher speaks the student's first language.
The language of instruction is in English.	The language of instruction is variable and dependent on the student's language and understanding.

'Type A' multilingual classrooms would be commonly seen in countries such as Australia and the United States. In both of these countries, lessons would be conducted in English where the teacher is less likely to speak the language of the student (in context of ESL students). 'Type B' classrooms would be common in countries like South Africa, India and Malaysia, where the prescribed language of tuition is not the same as the dominant spoken language. In the 'Type B' classroom there is a high probability that the teacher speaks the student's first language, and the language of instruction is variable depending upon the students' language understanding.

Classrooms of students surveyed for the purpose of this study at UOWC, would be categorised as 'Type A'; whereas, classrooms of students surveyed at King Abdul-Aziz University (KAU) in Saudi Arabia would be categorised as 'Type B'. Although both context have been classed as multicultural, the dynamics in the learning and

teaching environment differ greatly, which allows for great insight into understanding how language and learning are interrelated.

Students in each type of classroom will engage in learning strategies such as code-switching or borrowing that may help or hinder different aspects of their learning. Similarly their academic learning skills may position them to learn successfully or lead to failure. Mediators of students' ability to learn engaging in strategies such as code-switching and borrowing include their proficiency in academic language.

Students in 'Type A' classrooms grapple with understanding textual assessments even more so than students in a 'Type B' classroom due to the lack of teacher support available. For example, ESL students in a 'Type A' classroom will find problems understanding mathematical language compounded as they are struggling with additional language and cognitive load. For ESL students in a 'Type B' classroom, mathematical language has a cognitive burden in the sense that the mathematical concepts and vocabulary is new, but the language barrier of English is somewhat softened with the availability of linguistic support that is offered from teachers, peers and available resources.

Adler (2001) attributes some of the difficulties faced by ESL students in multicultural classrooms, especially in 'Type A' classrooms in countries like Australia and the United States, to having a teacher less likely to speak or understand the student's first language. Without this knowledge of the students' language, it is extremely difficult for educators to understand the confusion experienced by ESL students in their attempts to make sense of material in the English language.

How students experience learning in these two types of classrooms may also differ. The critical analysis of textual material for ESL students is somewhat compromised in a 'Type A' classroom whereas in a 'Type B' classroom the teacher is likely to speak the same language as the student and therefore may be able to give more meaning by extending the students understanding of the topic/lesson using their primary language. This thought process is simplified for ESL students and understanding is improved if they are proficient enough in the second language to utilise code-switching effectively.

To recall in a ‘Type B’ setting, students are more likely to achieve cognitive bilingualism as they have the advantage of an educator who understands their primary language, and therefore has the ability to teach material to create the necessary learning pathways to establish the frameworks required for cognitive bilingualism. On the other hand, in a ‘Type A’ classroom, less support is provided for the ESL student. Hence, a student who may be colloquially bilingual in a ‘Type A’ classroom setting will still encounter difficulties in learning mathematics due to the lack of an established cognitive load to support the academic process of learning.

In ‘Type A’ classrooms, support is limited to the capacity of the teachers understanding of the students linguistic problems. For students situated in ‘Type B’ classrooms, assistance is available by educators to tackle the linguistic challenges related to learning mathematics. Some possible differences are provided in Table 2.6.

Table 2. 6 Multicultural classroom learning and assessment analysis

	'Type A' Classroom- National Language English	**'Type B' Classroom- National Language Not English**
Code-Switching	Ability to code-switch is beneficial to enriching understanding. Code-switching is often confused with borrowing.	Students are to be proficient in both languages in order to be able to code-switch effectively otherwise it is 'borrowing'.
Borrowing	Students run the risk of misunderstanding as they are substituting terms to fill the information gap.	Teachers are likely to pick up on the potential confusions that might be made in borrowing words/vocabulary from one language and applying it directly to another.
Colloquial/Cognitive Bilingualism	More likely to be colloquially bilingual, due to lack of student – educator linguistic support.	ESL students are more likely to achieve cognitive bilingualism based on student-educator linguistic support network.
Language of texts and assignments	Students have more difficulty in comprehension, processing, and application of textual material due to ambiguous language development.	Classroom support by educators and peers benefits the students in terms of ability to make sense of literature and assignment objectives.
Writing	Exposure to second language writing enables better expression, format and presentation in future.	Reliance on support framework impedes on writing development.
Syntax Complexities	Students are assisted to full extent by educator, as well as self-reliance measures to sort out complexities using other means.	More easily deciphered with educator assistance.
Age	As a consensus cognitive bilingualism is achievable in both 'Type A' and 'Type B' ESL students. However the younger the student the quicker the uptake of a second language. Secondly, students immersed in a 'Type A' environment will develop language proficiency at a much faster rate than those in a Type B' setting.	

2.9 Conclusion

Language is without a doubt, crucial to a student's understanding of mathematical concepts and their ability to solve problems. This is especially so in multilingual classrooms, where bilingual students are faced with a double task of learning the

language and mathematics at the same time. For many students who have a good understanding of their second language, they are able to code-switch effectively to increase their understanding of the problem. On the other hand, code-switching is less effective for students who have a weak understanding of the second language. In these instances the student is more likely to be borrowing linguistic concepts from the primary language possibly creating confusion, and misunderstanding. Such misinterpretation in readings, instruction and assessments can adversely impact students' academic performance. In 'Type B' classrooms, like those in King Abdul-Aziz University (KAU) in Saudi Arabia, students are more likely to fair better in learning mathematics than ESL students in 'Type A' classrooms, like those at UOWC in Australia. This is presuming the teacher in the 'Type B' classroom has better insight due to the sharing of the first language of the linguistic challenges that the student may be facing and hence is potentially better able to shape lesson plans and assessment to curb such challenges.

Research into the field of language and mathematics has also drawn on the bilingual research into the neuroscience of how learning is undertaken in the bilingual mind. Researchers such as Adler (2001) have furthered our understanding by highlighting that bilingualism is more than just the capacity to speak another language, (colloquial Bilingualism). In order for students to be able to learn new concepts and make intellectual connections for problem-solving, they need to be cognitively bilingual as well.

This chapter has presented a discussion regarding the load that may results from the language with ESL students. The next chapter examines the load that results from learning mathematics itself and how it can be managed or reduced.

3 COGNITIVE LOAD THEORY AND LEARNING MATHEMATICS

Cognitive load theory suggests that learning happens

Best under conditions that are aligned human

cognitive architecture (Sweller, 1988).

3.1 Introduction

There are many challenges facing ESL students particularly those who are not proficient in the language of instruction or with adequate academic language. Consider the difference between having to study a subject in one's native language versus trying to study a subject in a foreign language. The cognitive load is much higher in the second instance because the brain must work to translate the language while simultaneously trying to understand the new information (Pass et al., 2004). Learning in a second language complicates the learning that is to take place, in this context mathematical learning. Understanding the learning process involves understanding models of how learning takes place and this is intertwined with how information is processed and remembered, that is, how different memory systems work. In addition to understanding the complexity of learning as it relates to ESL students, there are also issues regarding how to best design materials for efficient learning as different teaching approaches will have differential impact on the memory system and hence cognitive processing abilities.

In this chapter learning and memory systems will initially be explored, and based on this, the implications of cognitive load theory will be examined in relation to ESL students learning mathematics.

3.2 Memory and learning

Learning is the process by which one acquires new, or modifies existing, knowledge and skills (Webster, 2012). The learning process is irrefutably linked to memory

(Alloway, 2010). It is proposed that learning takes place in working memory (WM) where new information or knowledge is rehearsed (Sweller, 2006). When WM and Long Term Memory (LTM) work together knowledge then is stored in long term memory and the learner gains expertise in a domain (Khateeb, 2008). As illustrated in Figure 3.1 WM, also known as short term memory, is where information is synthesised. Due to the limited storage capacity of WM, information that is not rehearsed, practised, or reviewed is lost, whereas rehearsed information continues on to LTM. Long term memory is essentially the storage facility of the human brain. For ESL students engaged in learning, and resorting to code-switching between languages, the possibility of 'memory overload' is apparent. The focus of this section is to review the learning process and its reliance on memory.

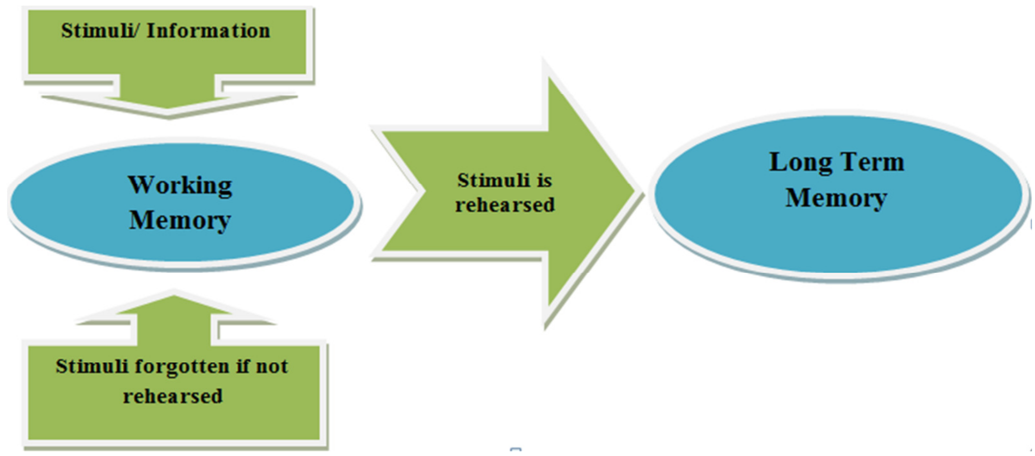


Figure 3.1 The learning process model
(Adapted from Waugh and Norman Model, 1965, p. 93)

3.2.1 Working Memory

'Working memory' was the term coined by Galanter and Pribam (1996) in a book entitled "Plans & the Structure of Behaviour". Other names for working memory include primary memory, Short Term Memory (STM) and immediate memory (Klatzky, 1975). The distinction between working memory and long term memory is represented in Figure 3.1, where the first stage of the learning process is exposure to information or some stimulus. This new information will have to be rehearsed to form synaptic pathways in the brain to allow for transfer from short term memory to long term memory. Information that is not rehearsed will be forced out of working

memory, due to capacity constraints. Long-term memory is more permanent so once knowledge is stored, there is no longer the necessity to rehearse it (Houston, 1981).

Working Memory (WM) is considered to be the temporary storage unit of the brain. It is where information is processed and stored for a short period of time. According to Atkinson and Shiffrin it is the point at which "decisions are made, problems are solved and information flow is directed" (1971, p.3). Understanding the role of working memory has evolved with time. Khateeb (2008) suggested the meaning of working memory has developed through seven stages, as indicated in Table 3.1. Today it is understood that working memory has multiple components, all of which are a part of a larger system that manages and maintains information in the short term and performs complex tasks, such as learning (Baddeley, 2012).

Table 3. 1 The stages of working memory development (Khateeb, 2008)

Stage 1	Working memory as contemplation	WM came to the attention of philosophers in the 17th century. John Locke characterised what he called 'idea in view' versus the 'storehouse of ideas' which is equivalent to the distinction between short and long term memory now in use.
Stage 2	Working memory as a primary memory	This is an expression used by Waugh and Norman (1965) who maintained that the primary memory is of limited capacity. They claimed that rehearsal is critical to keeping information in the primary memory and then taking it to the secondary memory or long-term memory.
Stage 3	Working memory as a short-term memory	Up to this stage, working memory was considered as passive information storage and not an active information processing. Referring to the short-term memory as both a storage and control process, is an innovation attributed to Atkinson and Shiffrin (1968). They suggested the limited capacity character of the working memory and argued for a flexible system that carries out both storage and processing of information.
Stage 4	Working memory as a processor	Craig and Lockhart (1972) gave a new meaning to working memory, thus regarding it as a kind of cognitive processing, not as a separate entity. That is to say, working memory, according to these authors is a process, not a fixed part of human cognitive structure.
Stage 5	Working memory as a constraint on language comprehension	Logie (1996) regarded this stage as the stage of highly developed language learning. This stage was originally proposed by Daneman and Carpenter (1980) who created a task with a view of measuring the working memory capacity.
Stage 6	Working memory as activation, attention, and expertise	In this stage, the working memory is considered as an independent entity characterized by little attention, capacity and activation (Cowan *et al.*, 1993). However, such little capacity and activation expand with expertise. This assumption was the main part of a model proposed by Ericsson and Kintsch (1995). The main idea of the model is that the capacity of working memory is larger when it operates in one's field of expertise.
Stage 7	Working memory as multiple components	In this view working memory takes the form of a workplace, not as a pathway. The most important research about this stage was conducted by researchers such as Baddeley and Hitch (1974) and Baddeley (1990).

3.2.1.1 Capacity of working memory

The notion of memory span has been studied since the late 1800's. Baddeley (1990) discovered that human memory can absorb short lists of works of seven items or less in just one reading. However, as the list grew longer, so did the learning time. These

findings were supported by Miller's (1956) paper entitled *'The Magical Number Seven Plus or Minus Two'*, where it was proposed that memory span is in the range of five to nine items. In the 1970s, Simon (1974) discovered that a person's ability to remember is also affected by the complexity of the information presented. Simon (1974) also discovered that the number of items that can be remembered immediately after listening or reading is seven 1-syllable words, seven 2- syllable words, and six 3-syllable words. In this regard, Baddeley and Hitch (2000) put forward that memory span varies with the nature and the complexity of the information that is to be stored.

In evaluating the capacity of working memory in relation to ESL students learning mathematics, it is apparent that there is a struggle for these students to process information optimally as they are not only trying to register a second language, but are also trying to retain mathematical concepts. They are likely to get 'information overload', with too much information for the WM to process at any given time. Retention is low due to the complexity and dual nature of the information they are trying to process. For example, in this thesis, international students from UOWC were instructed in English and therefore were not only grappling with the understanding of terminology of a second language, but were also struggling to learn the mathematical principles being taught.

Working memory is not only limited in capacity but also in the duration of storage of information. It has been suggested that information can be retained in the WM for a range of 0-60 seconds before it is lost (Baddeley & Hitch, 2000). The length of time information is kept in WM is dependent on rehearsal/review, which keeps information in an active state for longer (Baddeley & Hitch, 2000). In the case of the ESL students, information is often coming in at a rate that does not allow for rehearsal and therefore much of it 'spills over' and is lost.

3.2.1.2 Working memory model

Waugh and Norman's (1965) original memory model was elaborated upon by Baddeley and Hitch (2000). The Baddeley and Hitch model divides the processes of the working memory into three main areas that are controlled by the central executive, which is responsible for the intentional control of working memory and its transfer into long-term memory. These three sub-systems of WM's central executive,

as described by Baddeley and Hitch (2000) are: the phonological loop, episodic buffer, and the visuo-spatial sketchpad. This model (Figure 3.2) consisted of three main components:

1. Phonological loop: It is proposed that it contains a temporary, passive storage sub-system (Baddeley, 1998). Also, Khateeb (2008) suggested that the phonological loop plays a significant role in the development of language in learning a second language.

2. Episodic buffer: This sub-system is assumed to be a temporary memory and limited capacity storage sub-system.

3. Visuo-spatial sketch pad: This sub-system is assumed to be more complicated than the phonological loop (Baddeley, 1996). It is claimed that the Visuo-spatial sketch pad is responsible for temporary maintenance and manipulation of Visuo-spatial information (Baddeley, 2012).

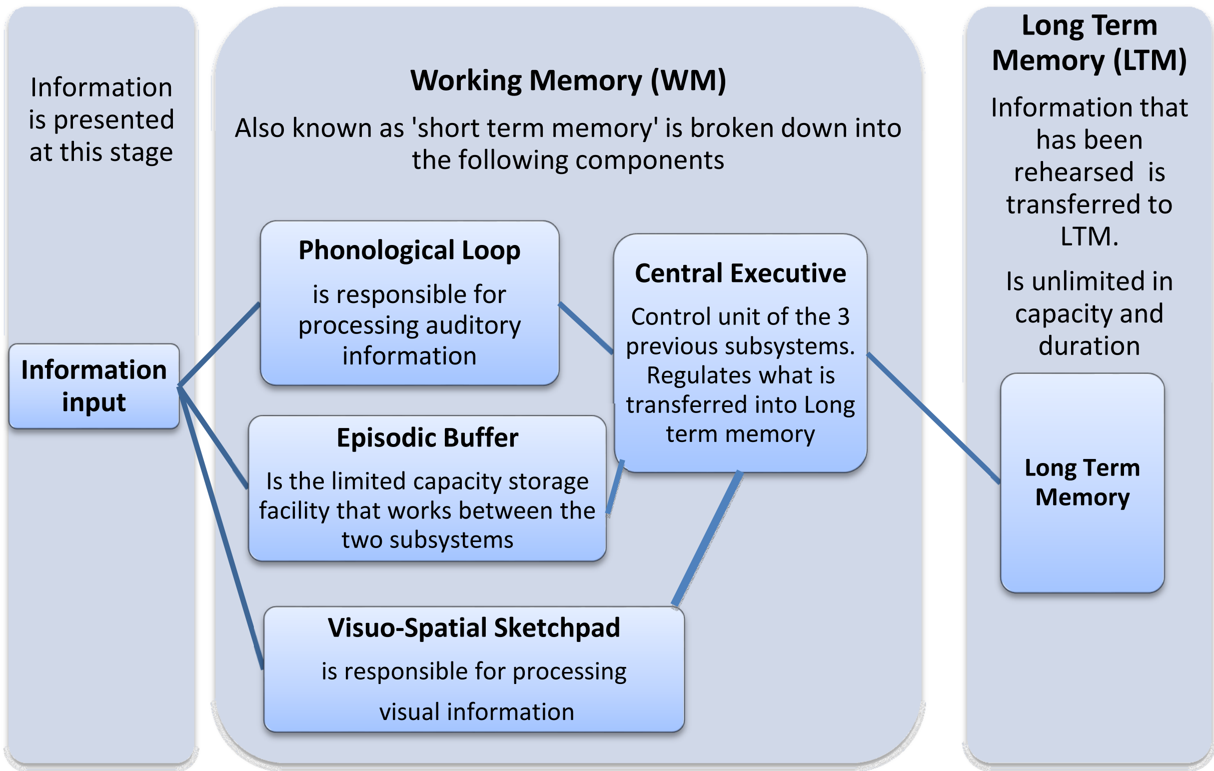


Figure 3.2 Memory system summary models

Kemps *et al.* (2000) claimed that the central executive is an intentional control system that is limited in its resources. They asserted it was responsible for three functions:

1. Transmission of information from short term memory to long term memory;
2. Selection of information to be stored;
3. Coordination of the other three sub-systems of the working memory system.

While these sub-systems: the phonological loop, episodic buffer, and visuo-spatial sketch pad are responsible for the automatic and impermanent translation of stimuli into language and visual semantics, it is the executive memory that selects elements to be stored, and transmits them to long term memory.

3.2.2 Long Term Memory

Long Term Memory (LTM) is the storage facility for permanent memory storage. Information makes its way to the long term storage from the short term or working memory by rehearsal or practise. Although the LTM has long been used as a major component in all the first memory models, its significance in controlling human cognitive activities was not stressed until De Groot (1965). Subsequently Chase and Simon (1973) discovered, in a study of a chess masters, that memory capacity and expert performance, which are a product of experience-based knowledge that can be recalled quickly and consistently and then deployed. Other studies of expertise in various domains have found similar results regarding the role of prior knowledge in performance (e.g. Beilock et al., 2002). Atkinson and Shiffrin (1971) maintained that the longer an item is kept in the short term storage by means of rehearsal, the more probable that it will be riveted in the long term storage. The long term memory is thought to be unlimited both in terms of capacity and duration. That is to say, it can hold a huge amount of information that can also last for a very long period of time. Thus two characteristics of long term memory were presumed: that its storage is of unlimited capacity (Newell & Simon, 1972; Baddeley, 1986), and information kept there last for long periods of time (Cowan, 1988) and can easily be recalled when adequate stimulus is provided (Baddeley, 2010). Baddeley (1992; 2006; 2007; 2010)

claimed that long term memory keeps information for ever, but it becomes less accessible as time goes by.

In summary, the human brain functions on two memory types, working memory (WM) (also known as short term memory) and long term memory (LTM). In regards to memory storage, working memory is of limited capacity, whereas long term memory has a large unlimited storage capacity. According to Miller's rule of working memory, we can only effectively process 7 ± 2 items at any given time. Once this 'memory span' (Klatzky, 1975) has been exceeded, our thinking and learning processes are diminished (Jan et al., 2010; Sweller et al., 2011; Ayres & Paas, 2012). Therefore the learning process is a cooperative effort of both long term memory and working memory (Sweller, 2011).

Schemas are central to overcoming the limited capacity of working memory. A schema is defined as a coordinated combination of cognitive functions and physical actions (Khateeb, 2008). Schemas also hold and organise previous experiences and can be affected by personal prior-knowledge (Baddeley, 1998). Similarly, Baddeley (2012) stated that schema allow the organisation of an individual's experiences and emphasised the significant role of personal prior-knowledge when constructing and arranging new schema. Cognitive load theory considers a schema as a major factor that determines how expert a learner is in a particular domain; the greater the number of schema that individuals hold in their long term memory and the more complicated these schema are, then the more expert they are (Sweller, 2003). However, utilising schema depends on having these schema automated so they can be processed unconsciously in working memory, i.e. with no need to invest cognitive load to activate them. Anderson (1996; 2005) and Sweller (2003; 2004; 2006b) agreed that schema can be automated as a function of time and effort in practicing. Automation can significantly reduce cognitive load and as a result facilitate learning in terms of time and effort (Kotovsky et al., 1985).

3.3 The Basics of Cognitive Load Theory

Cognitive Load Theory (CLT) is defined as:

> "A universal set of instructional principles and evidence-based guidelines that offer the most efficient methods to design and deliver instructional environments in ways that best utilise the limited capacity of working memory" (Clark et al., 2006, p.342).

Human cognitive architecture (e.g. WM, Schemas, LTM, etc.) provides a base for cognitive load theory. Recently, that theory has become one of the most influential theories in instructional psychology with applications in various areas of education. The fundamental assumption of this theory is that for instructional methods to be effective, instructional designers need to take human cognitive architecture into account. It also emphasises the necessity for instructional techniques to be designed in alignment with the basic operational principles of the human cognitive system (Chandler & Sweller, 1991; 1996; Sweller, 1988; 1989; 1993; 1994; 2003; 2004; 2006b; Sweller & Chandler, 1991; 1994; Sweller & Sweller, 2006).

3.3.1 Formation of schemas

As discussed earlier, the learning process is a cumulative one in which we are continually adding to existing (knowledge) schemas. Schemas can be described as being "chunks" of information relating to specific subject matter. Schemas are how individuals organise and make sense of the information or stimuli to which they are constantly exposed (Klatsky, 1975). In processing new information, the learner can either create new schemas, which are low order and are not associated with existing knowledge, or they will build upon existing schemas to create higher order schemas, which are more sophisticated and comprehensive. As the learner progresses to increasingly more complex tasks, all the related interactions along the learning path are incorporated in a higher level schema (Sweller & Chandler, 1994). These complex high level schemas are presented as a single unit (chunk) in working memory, therefore reducing the overall load on working memory. An example of how low level and high level schemas operate within working memory to process language based information is depicted in Figure 3.3.

Low Level Schemas **K I T E**	
In this example, working memory would register a cognitive load of 4, as it is processing each of the letters individually. The low level schema of ‘letter recognition’ has not culminated in word formation.	On the other hand in this example, working memory would register 1, as it is processing the letters K, I, T, E, as the word KITE. This higher level schema is more complex as it goes beyond letter identification to word formation, as well as all information relating to kites would be processed along with this schema.

Figure 3.3 Processing of low level and high level schemas in working memory.

Figure 3.4 is a simple geometric example, presenting three different triangles. An individual with low level schema formation would register a cognitive load of three (three lines) in processing this example. However, an individual with high level schema would register a cognitive load of one (a triangle). The difference in how the brain processes the information of this example can be attributed to geometric knowledge and basic understanding of the shape and their properties. An individual with high level schema would group the images as part of the ‘Triangles’ schema, classifying the images based on basic properties of triangle: three sides, and angles sum to 180°.

Triangles

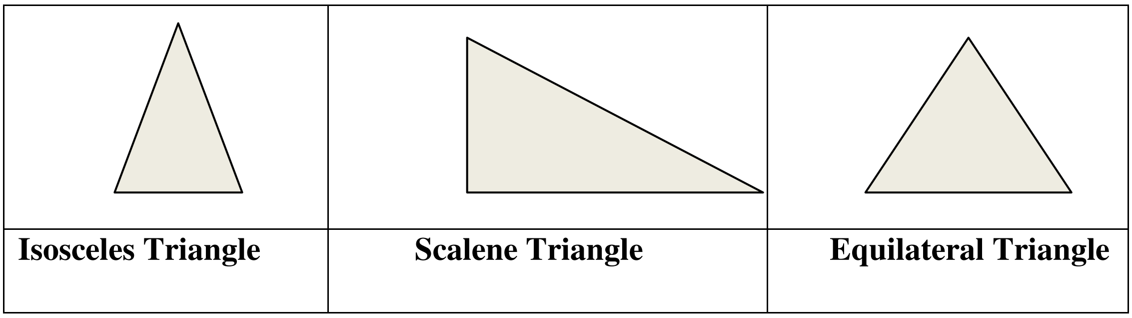


Figure 3.4 Cognitive processing of objects with low and high level schemas

3.3.2 Cognitive load theory in relation to learning and memory

CLT builds on the models of working memory, and can be applied to the major issues in teaching and learning mathematics. For students that work in a second language, cognitive load comes from two sources: the work of language translation and the new information (Corbin & Strauss, 1990). The new information, learning of mathematical principles at a tertiary level is built upon pre-existing mathematical knowledge. CLT recognises that the Intrinsic cognitive load imposed by the nature of the new information being learned, in this case the learning of mathematics is high due to the high interactivity between each separate piece of information or schema (Clark et al., 2006).

ESL students may have high level schemas available to process cognitively complex concepts, in this thesis mathematical concepts, however they may face language impediments that may hinder the retrieval of schemas. For instance, if the student was reading the word KITE as K, I, T, E, they would fail to recognise it as a word in itself, and therefore fail to be able to retrieve it as a high level schema from long term memory. Figure 3.5 diagrammatically represents the process that ESL students undergo in the formation of new schemas.

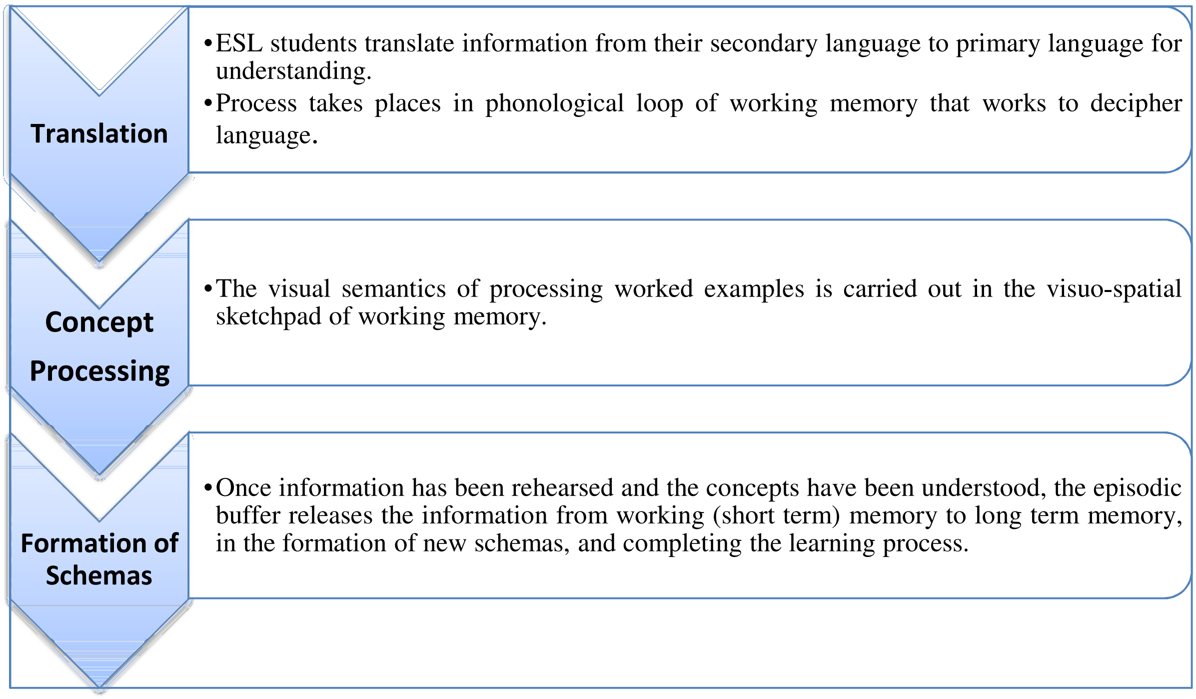


Figure 3.5 The learning process for ESL student

CLT is intrinsically tied to working memory and long-term memory. As indicated earlier, the working memory has limited storage capacity, and as a result in order to maintain optimal learning conditions, working memory can only process 7± 2 items (Ayres & Paas, 2012) at any given time. When WM attempts to process more, the episodic buffer goes into cognitive overload and the learning process begins to impair. Cognitive load theory advocates the minimisation of the waste of mental resources and the organisation of materials to maximise learning outcomes. According to Kirshner (2002), CLT deals with the limitations of working memory and its interaction with unlimited long term memory. Chandler and Sweller (1991) and Cooper (1998) argued that the total amount of mental activity that working memory must deal with simultaneously is considered an individual's cognitive load.

3.3.3 The educator's role in optimising learning by reducing cognitive load

CLT deals with the limitation of working memory. The basic hypothesis of CLT, and human cognitive architecture or 'the manner in which cognitive structures are organised' (Sweller, 2003, p. 219) suggests teachers could improve learning

outcomes by placing fewer demands on working memory (Kirshner, 2002). CLT directs focus to the number of interactions between elements in a particular task, and the fact that the number of interactions in learning can vary from individual to individual. Generally, the main concern for the teacher or lecturer is to develop the most appropriate method for the organisation or presentation of information to students, and this involves taking into consideration a number of factors relating to cognitive load. Sweller (1999) noted that the limitations imposed by the working memory are important in the learning process; and as such, the learning process should be analysed from a cognitive load perspective.

CLT explains how cognitive resources are allocated during the completion of worked examples and problem-solving processes (Chandler & Sweller, 1991; Sweller, 1993; 1999; Sweller & Chandler, 1994). CLT confirms that schema construction and automation are the key elements of learning and that working memory has limitations in processing. The limitations of the working memory are well documented by Sweller and his colleagues (2006); however the common approaches to teaching and training such as problem-solving and co-operative learning rarely consider limitations when designing material for mathematical learning, let alone for ESL students or non-native language students. The cognitive load required in conventional methods of teaching often exceeds the working memory capacity of many students and thus obstructs their studies (Sweller, 1991). One crucial aspect of planning for effective teaching involves being aware of the limitation of working memory and as this thesis proposes the higher cognitive load experienced by students learning new materials in a foreign language compared to students learning in their mother tongue. Cognitive load in the case of the ESL students will be much higher because their working memory must also focus on the task of translation. CLT suggests a rethinking of traditional teaching methodologies.

3.4 Categories of Cognitive Load

Understanding how to design for optimal learning involves designing with awareness of the three distinct forms of cognitive load, (refer Table 3.2) as defined in Cognitive Load Theory:

I. Intrinsic Cognitive Load (ICL)

II. Extraneous Cognitive Load (ECL)

III. Germane Cognitive Load (GCL)

Chinnappan and Chandler (2005) argued that given the impact on mathematical learning, lessons prepared based on the relationship between the three types of cognitive loads can greatly improve learning and retention. This section will discuss in greater detail the definitions of these three main types of cognitive load, the impact of each on learning and proposed methods to minimise cognitive load (CL).

Table 3. 2The three types of cognitive load

Intrinsic Cognitive Load (ICL)	**Extraneous Cognitive Load (ECL)**	**Germane Cognitive Load (GCL)**
• CL imposed on a learner as a result of the complexity of the material being studied. • Not a result of pedagogy or materials used by educators. • Has an inverse relationship with learning.	• CL imposed on learners due to ineffective teaching strategy. • Can be minimised by educators with careful planning and the use of more effective teaching methods. • Has an inverse relationship with learning.	• Mental effort required to create new schemas or add to low level schemas in the learning process. • Has a positive relationship with learning.

3.4.1 Intrinsic Cognitive Load (ICL)

Intrinsic cognitive load is “the mental work resulting from the complexity of the content being studied by the learner” (Clark et al., 2006, p.322). Sweller (1993) suggested that Intrinsic Cognitive Load (ICL) is a result of the complexity of the material being studied and was not a result of teaching methods. The ICL experienced by learners will vary greatly as it is dependent upon the ability of learners to draw on existing schemas stored in long-term memory. A greater number of higher order schemas available for retrieval will equate to a decreased ICL.

The process of learning mathematics has a high ICL because mathematics involves many elements to be processed simultaneously and this requires a high level of skill, see for example the problem in Figure 3.6. There is a high degree of interactivity between all the elements involved in solving the equation. The example outlines how concepts such as number recognition, letter identification, rules of addition and subtraction, division and multiplication are all required in solving for *y*. In a low level schema where students have not understood the rules relating to each of the above mentioned skills, the working memory will process them as individual concepts such as addition, subtraction, multiplication and division. Rather than processing the problem as an algebraic equation with all rules and methods required to solve them effectively. When they have more developed schemas a student whose mathematical ability and understanding is high, will have a low ICL, and therefore are able to draw on the necessary information to answer such problems, more so than students whose mathematical ability is low.

Solve the following equation

If $x = 2$, solve for y

$6x + 3y = 18$

$6(2) + 3y = 18$

$12 + 3y = 18$

$3y = 18 - 12$

$y = 6/3$

$y= 2$

Mathematics has a high level of interactivity. This example demonstrates how concepts such as number recognition, letter identification, the rules of addition, subtraction, division and multiplication all play a significant part in solving for *y*. Without the understanding of each concept, one would not be able to solve for *y*.

Figure 3.6 Example of interactivity of mathematical concepts

Mathematics, in comparison to other subjects, has higher levels of ICL due to the complex nature of the material. In tertiary level mathematics, the ability to retrieve

higher order schemas is vital as they are the building blocks for new material to be learnt. As it is subject specific, ICL cannot be reduced by teachers.

3.4.2 Extraneous Cognitive Load (ECL)

Extraneous Cognitive Load (ECL) is the additional, mental effort exerted on working memory due to poorly designed teaching methods (Quilici & Mayer, 1996). ECL is often described as 'the irrelevant load' because it arises through the way educators structure and present information (Chandler & Sweller, 1992; Renkl et al., 1998). ECL and ICL have an inverse relationship with learning. Nevertheless, through careful lesson planning, material selection and presentation of subject matter educators can reduce ECL (Chandler & Sweller, 1992; Chandler & Sweller, 1991; Van Merrienboer & Sweller, 2005; Ayres & Paas, 2012). When ECL is reduced there is more room in working memory, to more efficiently process other functional types of cognitive load.

3.4.3 Germane Cognitive Load (GCL)

The learning process requires the creation of new schemas and/or the building of more complex schemas, which adds to the load of working memory. This additional load attributed to the formation, processing and automation of schemas is the Germane Cognitive Load (GCL) (Clark et al., 2006; Plass et al., 2010). GCL is inherent in activities that are directly related and contribute to understanding, such as in mathematical schema development and schemas of automation. The process of constructing schemas also leads to benefits through motivating learners to connect concrete worked examples with abstract knowledge for every problem category. For example, an animation explicitly shows the transition from a concrete representation of the problem statement to the abstract representation necessary to construct a problem. The use of animation can be motivating as it allows learners a more flexible 'adaptive use of the animations' depending on whether they needed them or not. That is, learners with comprehension difficulties might decide to retrieve an animation, whilst those without difficulties could simply read the text-based worked examples. Students who learned to solve algebra problems using worked examples, combined with animations, have been found to be far more successful than those who only had

available the problem-solving approach (Atkinson et al., 2003; Chi et al., 1989; Gerjets et al., 2010).

For the ESL mathematics student, the mental effort taken to comprehend a lesson in a second language is part of their Germane load, as the student works to create schemas in the understanding of language and mathematics (refer Figure 3.7). There are many studies concerned with the implementation and manipulation of GCL (Berthold & Renkl, 2009; Gerjets et al., 2004; Renkl et al., 2004). Manipulations often involve changes within classrooms designed to redirect students' attention and cognitive resources in the creation of schemas and schema automation. Educators are facilitating such change in modifying visual and auditory presentations of mathematics, (refer Table 3.3) in the hope of reducing GCL and ECL (Seufert & Brunken, 2006; Seufert et al., 2007). The combination of methods may help to reduce ECL and enhance GCL which should in turn increase the performance and learning outcomes for learners and especially ESL students.

Multimedia teaching strategies are used by some teachers in teaching mathematics to ESL students. There is research to show that multimedia strategies can be beneficial in the learning process (Arnett, 1995). However, there is a risk that the use of multimedia technologies if they are not used effectively may actually result in an increase in the germane cognitive load in some students whereby the student's visual and incidental processing channels are overloaded by material and stimuli (Nikolova, 2002). Table 3.3 describes some of the overload scenarios that students may encounter when multimedia is used in the teaching of ESL students.

Table 3. 3 Scenarios of overloaded germane cognitive load and corrective methods (Nikolova, 2002)

Overloading Scenario	Method of cognitive load reduction	Effect on learning for ESL student
Essential Visual Processing Overload. The unit of working memory is overloaded with visual stimuli, resulting in increased cognitive load.	Redistribution: Redistribute some information (stimuli) from being visual to auditory to reduce the load on visual channel.	There is transfer in the mode of information (the modality effect), with better absorption of the material when information is presented in narration rather than just visually presented as text on screen.
Essential Dual Channel Overload. The visual and auditory channels are overloaded with the processing of material.	Segmenting: Break down the lesson into smaller units, and give students time between each unit to process the material.	When segmentation is practiced, a student is more capable of absorbing material presented, and therefore more effective in creating learning schemas for future lessons.
Essential Processing & Incidental Processing Overload. This is due to extraneous material. When one or both the essential and incidental channels of processing is overloaded by extraneous material.	Weeding: Reduce and restrict the material presented based on absolute importance to the topic of study. Signalling: Provide cues on how to process the information.	The removal of extraneous material from the lesson, allows information to coalesce with existing schemas, allowing for better retrieval, this is known as a coherence effect. Information processing and retrieval is facilitated by the use of cues by means of signalling.
Essential processing plus incidental processing due to confusing presentation. This occurs when one or both channels are overloaded due to the confusing presentation of material in multimedia.	Aligning: Ensure that relevant terms are placed near corresponding graphics to reduce the need for visual scanning. Redundancy removal: teachers should avoid presenting identical information in both spoken and textual formats.	As terms/words are placed near the corresponding graphics spatial contiguity is improved. Information processing is facilitated with reduced level of redundancy.
Essential processing plus representational holding. When one or both channels are overloaded by essential processing and representational holding.	Synchronising: in presenting both visual and corresponding auditory information together, it reduces the need to hold representation in working memory. Individualising: Teachers have to assess students' ability to retain mental images whilst listening to lesson.	Temporal contiguity effect: there is better transfer and retrieval of information when visual and auditory information correlate and are presented together. Spatial ability effect: By understanding which students are high and low spatial learners, the teacher can adapt the lesson accordingly to meet the learning needs of their students.

For ESL mathematics students, the functioning of germane cognitive load is different to non-ESL students because GCL works in two different ways: firstly to create understanding of new materials, and secondly, to comprehend the language of instruction which requires translation from English to the student's own language through which GCL is increased (refer Figure 3.7).

Germane Cognitive Load

GCL = GCL(M) + GCL(L)

Where M = Mathematics , L = Language

Figure 3.7 Germane Cognitive Load Function

3.4.4 Total Cognitive Load

Total cognitive load may be expressed as a function: it is the sum of the three different cognitive loads as shown in Figure 3.8.

Total Cognitive Load (CL) =ICL + ECL + GCL

Figure 3.8 Total cognitive load function

From these two relationships it can be concluded that total GCL is higher for an ESL student and this in turn increases the student's overall cognitive load. If total cognitive load is less than the working memory capacity of an individual learning will happen comfortably, whereas if the total is higher than the individual's working memory capacity, they will be overloaded and learning will be less than optimal. For ESL students, optimising cognitive load is important as they already use additional working memory capacity to comprehend language.

In summary, it can be considered that the three cognitive load types are added together to determine the total amount of cognitive load imposed on working memory. Working memory becomes overloaded if this total cognitive load imposed exceeds working memory capacity. Total cognitive load can be reduced if the ECL is reduced and thereby the resources of working memory will be freed. The total amount of cognitive load can affect the learning of new materials but it is particularly relevant to those students learning in their second language as they have to cope with the added GCL required to translate language.

3.5 Measuring Cognitive Load

Three techniques have been developed to measure cognitive load. These are:

i. Students' self-rating through questionnaires, where, students report how much mental effort they believe a set task has required (Brunken et al., 2003; Windell et al., 2007);

ii. Physiological measurement of bodily responses to tasks, for example heart rate variation (Haapalainen et al. 2010; Huang et al., 2009); and,

iii. Performance on a secondary simpler task, that must be performed simultaneously to the educational problem, for example, touching a screen at a given colour cue (Paas et al., 2003; Klingner, 2010).

Seven subjective rating scales were developed by Paas *et al.* (2003), from 1 (very low mental effort) to 7 (very high mental effort) to assess subjective cognitive load and specifically the mental load from different types of instructions (after each problem during training and testing). Tind all-Ford (1997) concluded that this measure can be a useful indicator of differences in working memory and cognitive load imposed by different instructional design techniques. Bryman (1992; 2006) further emphasised that such a measure will not only indicate a difference in task difficulty and working memory load, associated with instructional format and element interactivity, but also the total number of elements the learner must assimilate whether they are interacting or not. Their subjective rating results revealed that the cognitive load was higher under visual/visual than audio/visual conditions when the material was high in element interactivity. Based on this subjective load

scale, Paas and Van Merrienboer (2006) proposed an efficiency scale (high-instructional efficiency, low-instructional efficiency) that takes into account both mental load and a performance score.

There are some difficulties with cognitive load measurement because of the relative nature of the measurements and the difficulty interpreting which type of cognitive load (GCL, ICL or ECL) is affecting experiments. Recent experiments by Cierniak *et al.* (2009) addressed these problems by focusing on students rating the difficulty of tasks on a scale to reflect the three areas of cognitive load.

3.6 Cognitive Load Theory Effects

CLT has identified a number of methods or ways that cognitive load is impacted by aspects of lesson design, materials, procedures and teaching methods. All these methods are concerned with re-developing teaching or redesigning the materials so they become more effective. This section presents five of the main issues that can be addressed so as to enhance working memory to obtain a variety of improved learning outcomes. These issues pertain to:

i. Worked examples vs. problem-solving;

ii. Element interactivity;

iii. The modality effect;

iv. The redundancy effect; and,

v. The split-attention effect.

The next section demonstrates the main strategies for reducing student cognitive load by addressing these issues.

3.6.1 Worked example vs. problem-solving

Worked examples is a technique used to reduce cognitive load through the demonstration of each step of a task or of solving a problem (Sweller, 1988). That is, worked examples effectively show students each step in attaining a solution, and making explicit what the step entails (refer Figure 3.9).

Solve for x $$2x - 6 = 14 - 3x$$	
Steps	
1. Isolate the variable	$2x - 6 = 14 - 3x$
a) Add $3x$ to both sides	$2x + 3x - 6 = 14 - 3x + 3x$ $5x - 6 = 14$
b) Add 6 to both sides	$5x - 6 + 6 = 14 + 6$ $5x = 20$
2. Group	
a) Divide both sides by 5	$5x/5 = 20/5$
b) Simplify	$x = 4$
3. Check your solution	
a) Substitute x=4 into the left hand side (LHS) of the original equation	$2(4) - 6 = 14 - 3(4)$ LHS = $2(4) - 6$ $= 8 - 6$ $=2$
b) Substitute x=4 into right hand side (RHS) of original equation.	RHS = $14 - 3(4)$ $=14-12$ $= 2$
c) Check LHS=RHS	So LHS = RHS
4. Final answer	$x = 4$

Figure 3.9 A worked example from Math132

Figure 3.9 and 3.10 illustrate the difference between a worked example and a problem solving task. The worked example (refer Figure 3.9) is set out to show and make explicit each step to reach a solution. The problem-solving task (refer Figure

3.10) requires the learner to firstly interpret the extrinsic language of the problem and then solve the problem. Worked examples have been put forward as one technique designed to reduce the interference with learning that is caused by some forms of problem-solving techniques. Worked examples describe the problem statement and all the necessary steps to solve the problem. Unlike solving problems through the research process, in worked examples the attention of a learner is directed to the problem and it has statements explaining each step required to solve the particular problem.

Solve the following problems using simultaneous equations:

1. Nicky is 3 times as old as her daughter Casey. The sum of their ages is 48 years. Find the age of each.
2. The sum of two numbers is 1 and their difference is 5. Find the numbers.
3. A CD-single and a CD cost \$28.50. Three CD's and 2 CD singles cost \$78.25 find the cost of each.
4. A boat can travel downstream at full speed of 14 km/h. On the return trip, the boat travels upstream at full speed of 6 km/h. What is the water speed of the boat and the stream's current?
5. The owner of a coffee store wants to make a blend of coffee that is worth \$5.40/kg from two types of coffee worth \$5.00/kg and \$7.00/kg. How many kilograms of each must be used to make 10 kilograms of the blend?

Figure 3.10 An example of a problem-solving task from Math132

The effectiveness of the two approaches can be examined in terms of several learning outcomes including performance, motivation, speed of acquisition and transferability.

3.6.1.1 Performance

In terms of performance for different mathematical topics, several studies have found the use of worked examples is more effective than problem-solving (Trafton & Reiser, 1993). The ECL from worked examples has the potential to be smaller than that from the problem-solving methods and this occurs when learning is enhanced by

studying worked examples rather than by trying to solve the original problems. It is a form of direct instruction where students are shown complete worked examples instead of being asked to work out the solution steps by themselves. The most effective format is for learners to study a worked example and then immediately after wards, try to solve a problem with similar features. This example-problem pair format is repeated over a number of iterations building to a complete set of problems that students need to work through in order to master the new materials. Learning with the aid of worked examples can be more effective in terms of reducing cognitive load than learning from problem-solving alone (Sweller, 1988; Jelsma et al., 1990; Clark et al., 2006; Ayres & Paas, 2012).

3.6.1.2 Motivation

However, Sweller and Cooper's (1985) investigation into the use of worked examples as an instructional technique identified one possible limitation; they may not provide sufficient motivation for learners, as learners have the potential to just read worked examples and not use them as an effective way to develop the knowledge within the example. To overcome this limitation they used the worked examples with same problem exercises. It was expected that a learner would develop schemas that could be constructed more readily from studying the examples and then solving the same sort of problems; rather than a learner just being provided with simple problems to study.

3.6.1.3 Speed of acquisition

Sweller and Cooper (1985) identified that those who learned from worked examples were able to do the problems faster during a test when compared to those who only learned through problem-solving techniques. Moreover, recent studies (Cooper & Sweller, 1987; Clark et al., 2006; Kalyuga et al., 2012) have drawn similar conclusions: when students learn using worked examples, they can solve a subsequent the mathematics problems faster than problem-solving.

3.6.1.4 Transfer

Sweller and Cooper (1983; 1994) found that the use of the worked example approach is also effective in terms of improving transferability of knowledge to

new/similar areas. This compared favourably to using the problem-solving method. Further experiments with worked examples were conducted to evaluate if they could be used for the transfer of knowledge with similar and dissimilar problems allowing schema automation to occur. This indicated that knowledge from studying worked examples was useful when solving similar problems but not for dissimilar problems. However with prolonged use of worked examples they were beneficial in the transfer of knowledge to dissimilar problems as well (Sweller & Cooper, 1985).

3.6.1.5 Schema automation

Kotovsky *et al.* (1985) emphasised that extra practise time is necessary for schema automation and is a key factor in knowledge transfer when learning from worked examples. Zhu and Simon (1991) also confirmed from their experiments that learners acquired schemas and achieved automation more efficiently through the use of worked examples than through problem-solving methods. They found that a three-year mathematics course could be completed in two years by students who were taught using worked examples; their analysis of protocols noted that students were more engaged in learning using worked examples. Therefore when using worked examples in a learning environment it is important to consider the amount of time that a learner is given to develop their knowledge for this method to be successful in improving learning outcomes.

3.6.2 Issues for design of teaching

As mentioned earlier cognitive load is made up of intrinsic, extraneous, and germane cognitive loads. To promote efficient and effective learning, educators have to ensure that they are keeping the cognitive load to a minimum to ensure that working memory is not overloaded. The essential difference between the three different types of cognitive load is that ICL results from the main properties of information and these cannot be controlled, while the ECL and GCL can be controlled through improved teaching methods. Increasing or decreasing ECL and GCL load, has a close association with the presentation of instructional materials.

In order to increase learning efficiency, educators have to focus on reducing extraneous cognitive load as much as possible, as it has an inverse effect on the

formation of schemas and hence a negative effect on learning. Some recommendations include reviewing the instructional material selected for the course of study (Clark et al., 2006). This relates to the actual material that is being employed during the lesson, such as, textbooks, handouts, and multimedia. The teacher should assess all selected material to ensure the language, syntax, and visual semantics of the material are in logical order or layout within the lesson and are not organised in a manner that might add extraneous cognitive load.

Furthermore, in this thesis it is proposed that instructional design should be reviewed to move away from the traditional teaching of mathematics, to adopting a more structured worked examples approach for novice level mathematic students. An example of a structured worked example is demonstrated in Figure 3.9 where steps in finding the solution are broken down and explained. The merits of using worked examples with ESL students should not only be beneficial in decreasing ECL but also to help increase GCL. Furthermore Kalyuga *et al.*(2003), argued one way to minimise unnecessary ECL is to provide guidance or explanation as a substitute for the yet to be acquired schemas.

When designing work for students, CLT studies have identified several effects that need to be considered by teachers. These include element interactivity, the modality effect, the redundancy effect, the split-attention effect and the use of text and diagrams.

3.6.2.1 Element Interactivity

During the process of learning, the three types of cognitive load have the potential to impact on a learner’s working memory (Corbin & Strauss, 1990) and it is important for a learner to process a specific amount of information over a period of time. However, after instructions are given, the most crucial factor is the complexity of the information a learner has received (Pollock et al., 2002). Sweller and Chandler (1994) state that instructional content is made up of component parts or “elements”, and if a relationship between them exists, the elements may “interact”, leading to complexity of the instruction. This phenomenon has been described as “element interactivity” (Sweller & Chandler, 1994).

Element interactivity describes types of tasks that require various elements of knowledge to interact in the memory of the learner. Some tasks require more element interactivity and some require less resulting in increased or decreased ICL. ICL cannot be controlled by lesson design, or teaching methods because it comes from the subject being taught. The teacher or lesson designer can facilitate learning by taking ICL into account and deleting some of the interacting elements (Quilici & Mayer, 1996).

When processing unorganised and novel information, the capacity of working memory is limited because elements have to be reorganised and this continues to increase linearly, with some possible combinations increasing exponentially (Van Merrienboer & Sweller, 2005). The intrinsic structure of information is regarded by Sweller and Chandler (1994) as "unalterable" and thus needs to be managed as part of the learning process. Sweller and colleagues argue that when the cognitive load of instructions reaches a high level, the ICL of the instructions should be artificially reduced by instructional designers. This process can take place when a lesson is divided into small pieces and the ICL of the whole lesson is therefore reduced. Pollock *et al.* (2002) were the first researchers to develop the method of dividing the presentation of material to reduce ICL. The small pieces are described as "sub-schemas" (Clark et al., 2006).

When a lesson is divided into sub-schemas, learning is promoted at the expense of understanding. However, Sweller argues that a learner would not be able to understand the full schema and would not effectively learn the information (Sweller, 2006). It is important to note that CLT researchers were not the first researchers to suggest the division of instructional materials into their individual components. This phenomenon was first recognised by Gagne in the 1960s (Adler, 1996; Gagne, 1968; Gagne & Paradise, 1961; Quilici & Mayer, 1996). Researchers believed that it is not very challenging to understand low element interactivity material. Sweller and colleagues state that element interactivity can be learned serially instead of simultaneously, and this will not impose heavy irrelevant load on working memory (Sweller et al., 1998). When learners process individual elements of instructions serially instead of simultaneously, processing of these instructions becomes possible

since individual sub-schemas will be recombined and the whole problem will be eventually understood.

However, high element interactivity involves many elements interacting simultaneously, and imposes a heavy load on working memory. Sweller (1999) demonstrated that a learner may learn with greater understanding when elements are connected and interact with each other. This understanding applies only when material with high element interactivity needs to be processed. Sweller (1994) suggests that element interactivity cannot be measured independently of the learner as the elements are affected by the knowledge of each individual. As a novice develops their skills, schemas are acquired, and the process of learning and reading for example, starts to become automated. The elements previously processed individually now can be processed as a single element, freeing up the resources of working memory. As for reading, this is vital because when working memory no longer has to devote all resources to decoding and comprehension then understanding can take place. Cognitive load associated with learning differs between learners just as the elements that need to be learned vary from low to high element interactivity.

3.6.2.2 The modality effect

Working memory has separate visual and auditory channel capacities (Allport et al., 1972; Baddeley, 1992; Brooks, 1968; Frick, 1984; Levin & Divine-Hawkins, 1974; Marcus et al., 1996; Paivo, 1990; Penny, 1989). There is now a large body of evidence that supports the modality effect as an available method in reducing the cognitive load of a learner. When designing learning systems with integrated instructional design techniques, greater learning outcomes can be achieved through using instructional material presented with both auditory and visual ways to allow for the brain's dual-modality functioning.

Through using dual-modality presentation techniques a variety of efficiencies can be attained For example, the effectiveness of working memory can be increased when some of the material is presented in auditory form (e.g. textual instructions) and others in visual form (e.g. diagrams). Reinwein (2012) found that people could attend to and repeat continuous auditory speech while simultaneously processing unrelated visual scenes or sight text whilst playing the piano. Further research indicated that

individuals had better recall if they were presented with a number of words in visual form rather than auditory form while they were shadowing an auditory speech. Penny (1989) found that if auditory and visual information are processed separately, the subjects presented with this mixture prefer to recall the information in the order it is presented whereas when people are asked to perform two tasks concurrently, performance is better if the two tasks are presented in different modalities rather than in the same mode. These findings all support the enhancement of working memory using dual-mode as opposed to single mode.

CLT predicts that enhancements to learning due to the modality effect will not occur just because a dual mode such as audio/visual is used for a presentation. For instance, if matching information for auditory to visual modes requires extensive searching of schemas, then the cognitive load has the potential to exceed the learners working memory capacity and the modality effect is diminished. This emphasises the need to introduce further visual referents, which simplify the matching of auditory and visual material before this method can achieve the potential to reduce cognitive load significantly. Jeung *et al.* (1997) found that if the visual search effort was low, then the standard audio/visual format resulted in superior learning to a visual only format. This finding confirmed that the effectiveness of audio/visual instructions depends on the cognitive load imposed by visual search. Whereas, Tindall-Ford's (1997) findings from various experiments in the discipline of electrical engineering were consistent with Mousavi *et al.*(1995) who found that low element interactivity materials with low ICL do not demonstrate the modality effect. Under conditions where the new materials being processed do not impose a heavy load on working memory, the increase in working memory capacity achieved by dual-mode presentation is shown to be negligible.

3.6.2.3 The redundancy effect

Lecturers, teachers, and designers aiming to present information in various formats such as diagrams with text, may not always be effective in their methods as some of the information presented is often unnecessary or redundant. Redundancy can increase cognitive load and has the potential to interfere with the learning process. The interference caused by unwanted information in the learning process is known as

the “redundancy effect”. The effect can be identified where the same information is presented in different forms (Renkl et al., 1998). This is commonly seen in the field of mathematics, for example when multiple forms of information, tables of data, geometric images, graphs, charts, word examples and equations are presented.

Redundancy can be critical with some instructional formats, for example integrated formats can be effective at reducing cognitive load when dealing with multiple sources of information that cannot be understood in isolation. However, if multiple sources of information can be understood in isolation, it decreases the total cognitive load. Non-integrated formats of learning are where the learner identifies the one source of information that needs to be understood and ignores the other redundant sources of information. However, with an integrated structure the learner has to process all the sources of information simultaneously and redundant information in not easy to ignore. In these circumstances if the learner is not able to easily identify the redundant information, their cognitive load is increased and will be higher than for the non-integrated material.

The ability to discern what is redundant information comes with knowledge. A learner with a low level of expertise may require some additional information to understand a topic whereas for a learner with a high level of expertise that additional information would be unnecessary thus producing a redundancy effect (Toh et al., 2010). Research on the redundancy effect using higher education students’ original text with the one designed to increase the comprehension of the text, indicated that higher knowledge students learnt better from the original text (McNamara et al., 1996). The lower knowledge students however, learnt better from the modified version as this version contained additional information to help them understand the biology, which assisted in their learning. Since higher knowledge students learnt better from the original version the additional information in the modified version was redundant and hindered their learning. This also highlights one of the complexities in teaching when different types of students respond differently to the texts supplied. CLT emphasises the texts’ coherence and redundancy depends on the knowledge and experience of learners.

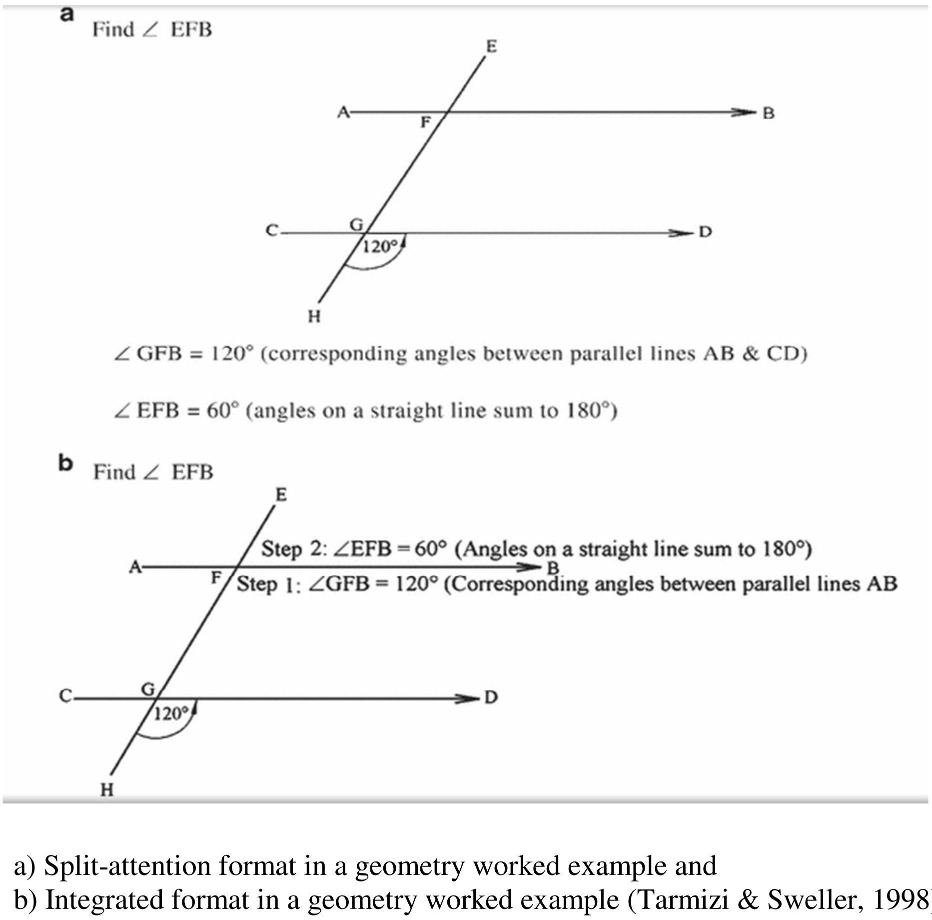
3.6.2.4 The Split-attention effect

Split-attention refers to the unnecessary splitting of attention that sometimes happens when a learner is trying to understand new material from multiple sources. For instance many materials consist of a diagram with textual information. Such materials require a learner to attend to both sources of information causing a split-attention effect. If the material can only be understood by mentally integrating multiple sources and the additional information does not assist in learning then a proportion of working memory is unnecessarily needed to integrate the multiple sources, which is then not available for the learning process. Split-attention happens when learners are required to split their attention between multiple integrated sources of physically or temporally disparate information, where each information source is necessary for understanding the materials (Ayres & Sweller, 2005). Researchers have successfully avoided split-attention by involving the strategy of physically integrating different information sources (Mayer, 1990; 2001; 2003; 2005; Clark et al. 2005). Having instructions in a mixed mode format has also been found to reduce the negative consequences. Musavi *et al.* (1995) also, found that learning mathematics could be improved by students using both visual/audio worked examples spending less time solving problems due to their increased working memory capacity.

Care needs to be taken with design, as the inappropriate design of worked examples can cause learners to split their attention, and hence fail to facilitate in schema construction. Consequently, the design does not help to improve learning outcomes (Tarmizi & Sweller, 1998; Ward & Sweller, 1990). Conventionally designed worked examples, with diagram and text, impose a cognitive load on the learner which may result in the acquisition process being as difficult as for problem-solvers. The split-attention effect diverts cognitive resources necessary for the construction of schemas to extraneous tasks such as mentally integrating the multiple sources of information. This extraneous task increases the cognitive load on working memory and hinders the learning process significantly. CLT explains that the presentation technique can result in high levels of ECL and this must be taken into consideration when developing integrated designs. To improve learning outcomes, teachers need to redesign instructional materials, reducing unnecessary split-attention.

3.6.2.5 Integrated text with diagram helps reducing split-attention

The purpose of an integrated design is to provide multiple sources of information including text and diagrams, where the related information is merged into a single unit of information (refer Figure 3.11). Split-attention occurs when learners have to mentally integrate two or more sources of information and each source of information is dependent on each other in order for the learners to understand the material. An integrated format does not require a learner to use their limited working memory resources to mentally integrate the material and thus reduces their cognitive load and facilitates learning. Several studies have found that the physical integration of words with the symbols and equation (Ward & Sweller, 1990) or with diagrams (Sweller et al., 1990; Tarmizi & Sweller, 1998) are not only easier for the learner to process, with fewer errors than for problem-solving, but they also decreased the time for solving other examples with better understanding.



a) Split-attention format in a geometry worked example and
b) Integrated format in a geometry worked example (Tarmizi & Sweller, 1998)

Figure 3.11 Split-attention versus integrated format

The working memory load imposed by the need to mentally integrate the disparate sources of information interferes with learning (Chandler & Sweller, 1991, Chandler & Sweller, 1992; Owens & Sweller, 2008; Sweller & Chandler, 1994; Tarmizi & Sweller, 1988). Sweller *et al.* (1990) pointed out that with integrated material the significance of the attention of a learner was directed in such a way that reduced their cognitive load (Chandler & Sweller 1991; Sweller & Chandler, 1994; Chandler & Sweller, 1996; Cerpa et al., 1996).

3.6.2.6 Pre-training

The primary purpose of CLT is to manage and maintain the cognitive load inside working memory, which is supposed to have limited capacity and duration. Manipulating ICL, related to the subject's complexity, can be fulfilled with two tactics: dealing with the material factors such as redundancy, split materials and modality (Ayres, 2006; Pollock et al., 2002; Sweller, 2006c) or utilising a pre-training stage allowing learners to construct prior-knowledge or sub-schemas to assist them to deal with the complexity of the material (Clarke et al., 2005; Mayer et al., 2002; Mayer & Moreno, 2003). GCL requires optimisation of what to build and automate relevant schemas to strengthen the learning process.

3.7 Worked example vs. problem-solving for ESL students

A reduction in cognitive load is likely to make the process of learning easier for ESL students. Sweller (2005) revealed that there was a degree of incompatibility between learning and problem-solving as per cognitive load theory. In using the problem-solving approach, a learner embarks on a cognitively laden task of searching through schemas before arriving at a solution to the problem. Therefore the search strategies may direct attention to different aspects of a problem and have the potential to use up all available working memory and reduce the effectiveness of the learning process. The process gets more difficult at a tertiary level, particularly for students with inadequate schema developed in the early years of mathematics education, as mathematics builds on earlier levels compared to lower levels of education, where there is less integration of mathematics elements. According to Campbell *et al.* (2007) the problem-solving method creates more cognitive load and increases the cognitive challenges for ESL students when solving mathematics problems on their

own because of the language barrier. The influence of the language on the solution process can clearly be seen using the problem in Figure 2.2 (chapter 2), as used by Campbell *et al.* (2007) where each of four of ESL students provided four different answers based on how they understood and interpreted the question, with “[l]ife experiences, language, cognitive processes, and knowledge of and the ability to apply mathematical content all interacting in the solution process” (p.6). This demonstrated the important role which language plays in increasing cognitive load on learners and shows that teachers and designers need to choose suitable methods for students with less extrinsic interference through language. This outcome suggested that in order to best manage the problem of language comprehension, worked examples may be most effective for students in the initial stages of concept development, and language familiarity.

3.8 Conclusion

Cognitive load theory (CLT) has made great contributions in the understanding of the learning process. ICL is connected with the topics to be learned. ECL is concerned with the teaching methodologies adopted to present the topics to a learner. GCL is linked with the cognitive load effort made by learners during the process of studying to build and automate schemas. The learner's overall cognitive load is the total of these three additive sources of cognitive load.

A by-product of CLT and the subsequent research is the understanding of the impact lesson plans and materials design have on the way in which students learn. Issues surrounding the mathematics learning process of ESL students have come to light with the increased understanding of the function of working memory, its susceptibility to overload, and the impact such overload has on the development of schemas and the automation process of students. Teachers/educators aware of the various types of cognitive loads, and their ability to manage Extraneous Cognitive Load and perhaps even Germane Cognitive Load, with simple modification in the materials used to present and assess students, should be able to improve the design of learning and hence outcomes. Educators should be able to implement strategies such as those summarised in Table 3.4, to reduce cognitive load in ESL students.

Table 3. 4 Possible strategies to reduce ECL and GCL in ESL students

	Extraneous Cognitive Load (ECL) Reduction Strategies	**Germane Cognitive Load (GCL) Reduction Strategies**
Instructional Material Selection	Attention to wording, syntax, grammar in handouts, visual material, assessments and worksheets. Provision of appropriate language level, and visual organisation.	Use of visual/ auditory material throughout the lesson. Use of multimedia. Dictionaries/translators for ESL students.
Instructional Design	Use of worked examples.	Allowing students time to rehearse new material. Use of a combination of worked examples, and exploratory problem-solving approaches to teaching mathematics.

This leads towards the question regarding ESL students dealt with in this thesis. How can educational providers reduce cognitive load for those students required to go through all the processes involved in mathematics when they also face difficulties with language, especially in tertiary level education? This is exactly the problem facing international students when they go to study in countries where the language of instruction differs from their own primary language. It is also the problem faced by students who are required to learn mathematics in a language other than their first language.

To address the question as to how to reduce cognitive load for ESL students learning mathematics, the work in the next chapter focuses on the use of problem-solving and worked examples for use with ESL students at the tertiary level.

4 FROM PROBLEM SOLVING TO WORKED EXAMPLES

"Mathematical problem solving is a thinking process in which Solver tries to make sense of a problem situation using Mathematical knowledge" (Lester & Kehle, 2003)

4.1 Introduction

Problem-solving has long been at the core of mathematics instruction with worked examples arising as a means to improve learning outcomes. However the debate as to the appropriate method for teaching continues. In this chapter the merits of problem-solving in mathematics education are compared to those of worked examples. This involves examining what distinguishes problem-solving and worked examples, the theoretical elements regarding how problem-solving takes place, the beneficial learning outcomes attributed to problem-solving, an analysis of cognitive load from the perspective of the worked example; examining how to structure worked examples to maximise efficiency and learning, the various types of worked examples, and theories supporting their functionality; and lastly, evaluation of the actual and perceived advantages of the worked example.

4.2 Problem-Solving

In mathematics education, the study of problem-solving has resulted in diverse interdisciplinary research. The last few decades have shown a burgeoning of ideas illuminating the field as extremely complex (Lesh & Zawojewski, 2007; Yee, 2012). Problem-solving has been construed in a variety of ways. In one sense the focus is on thinking skills and in another lesser sense it is defined in the teaching situations that are used to engender problem-solving. The difficulty is how to ensure that what is taught really is the true problem-solving skill. National Council of Teacher of

Mathematics (NCTM) states, "Problem-solving means engaging in a task for which the solution is not known in advance" (NCTM, 2012, p.5). Mathematical problem-solving is hard to define because it is often dependent upon the context of the problem. NCTM's definition has focused on the operation of problem-solving over the product. This agrees with Yee (2012):

> "Problem solving is a process. It is the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation. The process begins with the initial confrontation and concludes when an answer has been obtained and considered with regard to the initial conditions. The student must synthesize what he or she has learned and apply it to the new and different situation" (p. 21).

4.2.1 Problem-solving as advancing thinking skills

Problem-solving has generally been accepted as a means for advancing thinking skills (e.g. Schoenfeld, 1985). For example, in the NCTM Standards it is stated:

> "Problem-solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings. Solving problems is not only a goal of learning mathematics but also a major means of doing so" (NCTM, 2012, p.52).

It is a task that many students find difficult often due to the complexity of many cognitive tasks involved in solving the mathematical problem. This holds true even with those who have the prerequisite skills to solve complex problems (Sweller, 1988; Van Merrienboer, 1997).

4.2.2 Solving mathematics problems

In the context of this study, the definition of mathematics problems contains several elements. Problem-solving in the mathematics context requires the student to go beyond computing operations, to interpret and analyse the problem to arrive at a solution (Carpenter et al., 1993; Cawley & Miller, 1986; Passolunghi et al., 2005). It involves:

- Both single and multiple steps (Fuchs et al., 2004; Montague & Applegate, 1993); and

- Contextually straightforward problems as well as more complex forms that include irrelevant information (Fuchs & Fuchs, 2002; Passolunghi et al., 2005).

Different models can be used to describe problem-solving. For example one 5-stage model frames critical thinking and problem-solving skills with the following stages: *analysis*, *synthesis*, *evaluation*, *decision making* and *creative thinking* (Jonassen, 1997; Carson, 2007; Callejo & Vila, 2009; Winsor, 2010; Nardi, 2011). *Analysis* is when students determine how ideas are composed, and how they are related and interconnected with other ideas. Students are encouraged to discover assumptions and biases in order to uncover evidence. This involves them using their own opinions, ideas, chain of thoughts and personality. Thus, making the development of student analytical skills an important role in their overall problem-solving development. *Synthesis* involves the ability to put together the parts analysed with other information to create something original. Data or ideas are derived from a variety of sources. Individual students often have slightly conflicting ideas or views at the synthesis phase. Their personal life experiences, attitudes, backgrounds and interests all play a part in the chain of thoughts that go into synthesising a scenario in their own head. For example, a problem may ask, "what is the perimeter around a shape?" A student may see the shape as an "L", another may see it as an arrow, or even a building depending on their imagination at that moment in time. *Evaluation* is the step that represents the empowerment of the thinker over thought. This is when the student can either progress with their chain of thoughts to conclude on the scenario or they can doubt themselves and totally miss the point of the problem to be solved. *Decisions making, in this stage, students decide* to choose the right solutions after last stage when they evaluate the solutions for the problem. *Creative thinking* is the ability to breaking out of set rules or patterns to bring meaningful things or concepts or problem-solving skills into existence. Being creative helps students think differently, come up with new ways and methods of looking at situations, solve problems or meet needs.

The problem-solving process is difficult for many as it involves many complex processes at each of the problem-solving steps. In terms of this thesis, the focus on

language in Mayer's (1985) model of problem-solving (refer Figure 4.1) facilitates the consideration language has on ESL students' learning. In the model there are two key phases in the problem-solving process. The first is the problem representation phase, and the second is the problem solution phase. Phase one is comprised of two components: problem translation and problem integration. Problem translation requires the student to read and understand the problem in their own words. Problem integration is when the students create a visual representation of the problem using schematic information. The second phase also involves two components: solution planning, and solution execution. The student hypothesises or makes a plan to solve the problem and estimates a reasonable answer during the solution planning stage. The final stage of solution execution is when students compute or do the arithmetic to obtain the final answer.

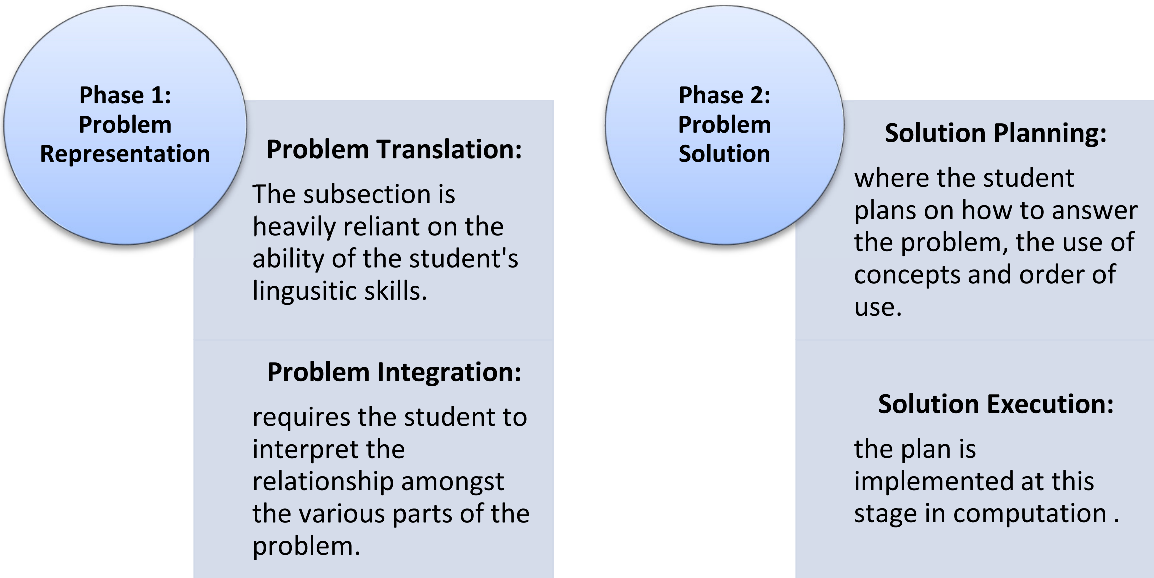


Figure 4.1 Illustration based on Mayer's model (1985) of problem-solving

Research undertaken on cognitive instruction by Montegue and colleagues (2003) has resulted in the validation of seven cognitive processes that are a part of successful reading problem-solving. According to the Montegue and Mayer model of problem-solving, illustrated in Figure 4.2, the seven cognitive processes: reading and

paraphrasing; visualising; hypothesising and estimating; computing and checking. These are activated in four sequential steps: 1-problem translation; 2-problem integration; 3-solution planning; and 4-solution execution. The model reflects both the phases and the process undertaken to carry out a successful problem-solving task.

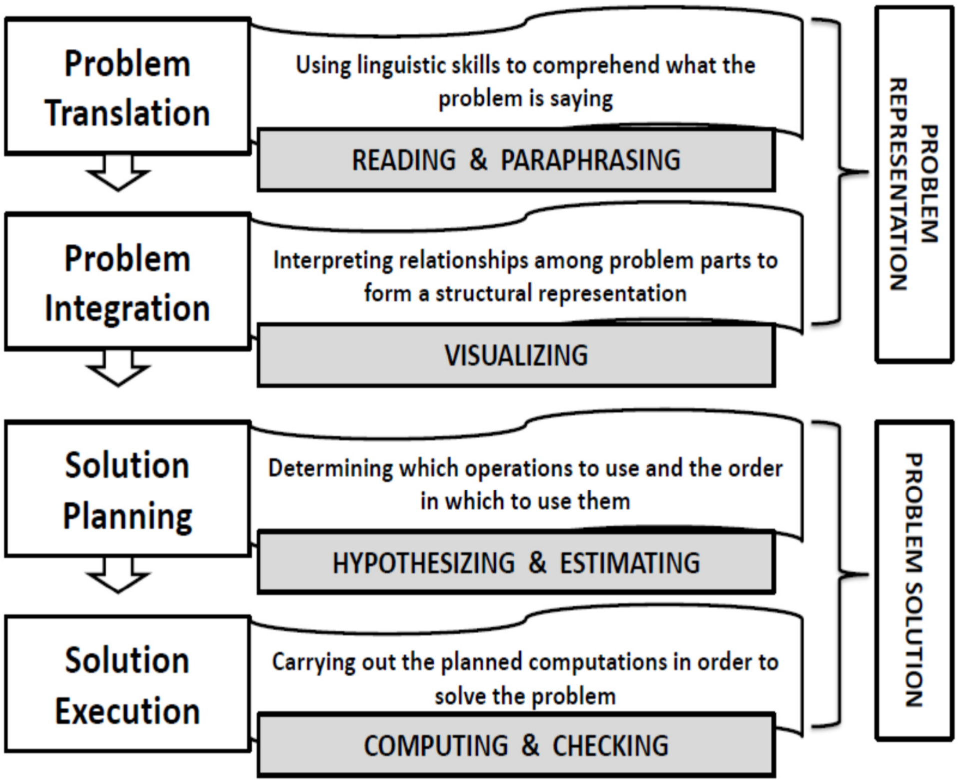


Figure 4.2 Conceptual framework of the mathematics problem-solving process. This figure illustrates an integration of the work of Mayer (1985) & Montague (2003)

4.2.2.1 Reading and Paraphrasing

The first stage in the problem-solving process as in (refer Figure 4.2) is particularly important in terms of ESL students as problem translation is where students are required to translate the problem into an understanding in their own words which involves reading and paraphrasing. The obvious importance of this stage of the problem-solving process goes without saying. In order for a student to effectively understand the problem asked, they need a firm understanding of the language in which the information is being relayed. In the context of this thesis, it is at this initial stage that ESL students can find the problem-solving process difficult. The language barrier may prevent ESL students from moving into the next stage of cognitive

processing. These students need to be able to paraphrase the problem (Krawec, 2010).

Paraphrasing has been used in the fields of mathematics and reading as a means to measure comprehension, as well as comprehension strategy (Nettles, 2006). This method of measurement is effective in reading, however in mathematics assessing comprehension is different. Students should be able to read a problem, decipher the meaning, as well as extract information that is relevant and redundant. This additional feature of redundancy is specific to mathematics and is generally non-existent in reading literature. There is a small body of research relating to students' ability to paraphrase and distinguish between relevant and irrelevant information, indicating that students with low ability have difficulties in distinguishing relevancy of information, particularly when it is numerical (Krawec, 2010). Measuring a student's ability to paraphrase early on, especially in the case of ESL students, would assist educators in tailoring their lessons to ensure that students who are at a low level of paraphrasing, can have the appropriate attention given to assist them in developing this skill, to ensure further success in the mathematical course content. Reading and paraphrasing is most important cognitive process for ESL students, which is the focus of this thesis.

4.2.3 Problem-solving schema

A learner can either solve a problem based on some kind of strategy for finding a solution or solve it based on the structure represented in a problem. The ability to successfully solve problems with proficient speed and accuracy is dependent upon the learner's ability to construct problem schema. Schema the cognitive framework or concept that helps organise and interpret information enables the learner to complete the problem-solving steps: understand the problem, process the information, determine the goal and establish the relevant sub-goals that will lead to the final goal, and execute strategies to achieve the goal within the parameters of rules and restraints (Chi et al., 1985).

Schema can be useful because they allow us to take shortcuts in interpreting the vast amount of information that is available in our environment (Lewis, 2007). Schema is based upon existing knowledge organised according to experiences and problem-

solving situations (Marshall, 1995). The quality of an individual's problem schema is determined by the accuracy of the knowledge, by the manner in which the different knowledge elements are connected, and by the number of connections (Marshall, 1995). Researchers have found that when learners are asked to sort a variety of problems based on similarity, experts categorise problems on the basis of underlying principles, whereas the learners used cover stories in the problem statement (Chi et al., 1981; Larkin et al., 1980; Silver, 1981). Experts' schema contains knowledge and experience that are applied to efficiently categorise and solve problems (Chi et al., 1981). This contrasts with learners' schema which are underdeveloped and incomplete, with limited knowledge and experience (Lavigne et al., 2008; Sabella, 1999). This lack of organised schema limits learners' ability to be successful in solving-problems.

Complex cognitive tasks, such as those involving mathematics require learners to reclassify existing schema, in addition to restructuring the problem (Van Merrienboer, 1997). This reorganisation process, increases the cognitive load during the problem-solving processes (Sweller, 1988). Much of existing cognitive science and schema theories stems from the early work of Newell (1990). The study of (Newell, 1990) has identified several strategies used by learners (novices) and experts to solve problems. The next two sub-sections present two of the main strategies: means-ends analysis and structure-based problem-solving.

4.2.3.1 Means-ends analysis

During means-ends analysis, a learner, typically a novice, will work iteratively backwards from the problem goal, by applying problem-solving operators, to achieve a sub-goal of the problem. Once this sub-goal has been reached the learner then reassesses the problem, and will continue to apply other problem-solving operators until the problem goal is reached (Larkin et al., 1980). Ward and Sweller (1990) highlighted the inherent cognitive load effects associated with using the means-end strategy to solve problems. Means-ends analysis involves taking actions to reduce the difference between a current state and a goal state. Due to the need to "simultaneously consider and make decisions about the current problem state, the goal state, differences between states, and problem-solving operators that can be used

to reduce such differences" (Ward & Sweller, 1990, p.3), this analysis takes up a large amount of working memory, making solving-problems difficult (Sweller, 2006).

Conventional teaching strategies for novice learners in mathematics will typically present the means-ends analysis strategy as a problem-solving strategy. According to Sweller (1988), the use of a means-ends analysis is considered to be an ineffective learning strategy as it increases Germane cognitive load with unnecessary search strategies, and the overwhelming focus on the diminishing difference. It was recommended that it be replaced with other learning strategies, such as worked examples (Sweller, 1988). Researchers have found the learners' cognitive loads were being unnecessarily overloaded using the means-ends analysis in problem-solving. Learners were reported to have been preoccupied with the diminishing differences between the problem and the goal states; and were left with little to no cognitive capacity to develop schemas (Chandler et al, 2001; Owen & Sweller, 1985; Sweller, 1988). As a result worked examples have been suggested as an alternative teaching strategy, to be presented to novice learners in place of the traditional means-ends approach.

The merits of worked examples are in the structural step-by-step solution. This provides learners with a framework for how to solve a particular type of problem (Atkinson et al, 2000). This approach reduces the extraneous cognitive load, as there is little extraneous material presented to the learner to saturate their processing in working memory. Furthermore, the use of worked examples has proven to assist in the formation of schema and facilitate learning, as was demonstrated in a study conducted by Zhu and Simon (1987). The study indicated that students who were instructed with the use of worked examples as opposed to the traditional means-ends approach, were able to complete a mathematics course in two years versus three, suggesting that the process of learning was facilitated by the use of worked examples. It was also noted that students were more likely to study the examples given rather than the written procedural instruction.

The traditional means-ends teaching strategies have come under strong criticism since the emergence of cognitive load theory and research, and the advance in

understanding of learning strategies (Sweller et al., 2011). Classrooms with novice learners and ESL learners stand to benefit academically with a shift from means-ends analysis to worked examples in the classroom.

4.2.3.2 Structure-based problem-solving techniques

Problem schema provides learners with short-cuts to problem-solving. Learners begin by categorising a problem based on the structure of the problem and then working toward a solution (Chi et al., 1981). Hence, students with appropriately developed schemata are able to bypass the 'searching the solution' stage to go directly to the stage, 'implementing the solution'. This is due to their ability to identify the key structures and principles required from similar types of problem statements, which enables them to construct a representation of the problem that activates their schema allowing them to implement solutions accordingly. This facilitates efficient and effective problem-solving based on the structure of problems. In solving-problems, students need to learn to understand and induce the schema for similar problem types in order to connect existing knowledge with newly acquired schema.

4.3 Facilitating problem-solving

Other techniques maybe used to facilitate problem-solving as alternatives to the standard four-step problem-solving methods such as those of Montegue and Mayer (1985). These include the use of graphic organisers, exploration, and visual representation.

4.3.1 Graphic organisers

One technique used to assist with problem-solving involves a Mathematical Graphic Organiser (MGO). A graphic organiser is a visual representation of content classification, mind mapping (Davies, 2011), concept development (Schoenfeld, 2009), flow charts (Leavitt, 2011) and relationship comparisons/Venn diagrams (Cai et al., 2013). Graphic organisers give teachers quick and efficient ways to diagnose students' level of skill and their individual abilities. They also provide a comfortable and familiar method to facilitate instruction.

Improving students' problem-solving abilities is a major goal of mathematics teachers. Specifically in relation to mathematical problem-solving, a strategy called the "Four corners and a diamond mathematics graphic organiser" was developed by Zollman (2009) (refer Figure 4.3) based on the four squares writing graphic organiser described by Gould and Gould (1999). The four square writing method is a formulaic writing approach originally designed to teach essay writing to children in a five paragraph, step-by-step approach. The graphic organiser portion of the method specifically assists students with thought, prewriting, and organising. The mathematical version has five areas. The first part asks –*What do you need to find?* This helps the student identify exactly what is asked of them and what is the point to solve. The second part asks – *What do you already know?* Students will then need to find all the given information that will assist them in completing the problem. Step three asks the student to *brainstorm* all their ideas about possible ways to solve the problem. Part four gives the student a place and opportunity to practice their possibilities and *try them out*. The last part is – That *explanations do you need to include in your response write-up? What mathematics did you learn by solving this problem?* This later part prompts the student to think about how to add all the finishing touches and then recheck their work for anything they may have missed. You could also say this is the reviewing before submission stage.

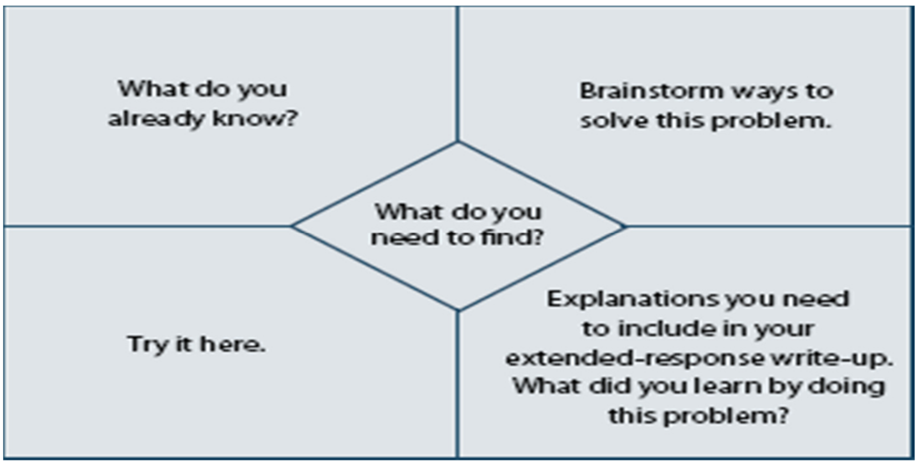


Figure 4.3 Four corners and a diamond mathematics graphic organiser (Zollman, 2009)

Research indicates that students who use graphic organisers to arrange their ideas improve their comprehension and communication skills (Lester, 2007). Using the graphics in whatever order or arrangement the student needs, enhances understanding of the problem in their own personal way Organisers help students clarify their thoughts with a physical image rather than just in their mind (Zollman, 2011). It assists to infer solutions to problems and then students are able to communicate their thinking strategies (Zollman, 2012).

Teachers have found the use of graphic organisers in mathematical problem-solving to be very efficient and effective for students; teachers saw that their less able students, whom normally would not have attempted problems, provided written partial solutions. The organisers appeared to help average-ability students organise their thinking strategies and help high-ability students improve their problem-solving communication skills (Zollman, 2006). The four corners and a diamond organiser provide students with an efficient and familiar method for writing and communication their thinking with logical arguments.

In summary, the use of reading and writing strategies (e.g. graphic organisers), may provide effects in students of all ability levels studying mathematics. From research (National Reading Panel, 2000; McREL, 2002; Zollman, 2006; 2009; 2011; 2012), it is known that graphic organisers work well with elementary students in the reading-writing process and, with the introduction of the four corners and a diamond method, also for mathematics. More specifically, when properly used, the four corners and a diamond organiser is an extremely useful instructional method, at least in the middle grades mathematics classroom where it was trialled. It helps students construct content knowledge, and to improve their mathematical communication skills. For most students, graphic organisers have multiple effects. It helps to connect parts of a problem solution to each other, assisting in the communication of ideas and thoughts, justifying findings then facilitating solving the problem. Not only does this tool assist students in their development directly, it also assists teachers in finding out quickly what the student’s strengths and weaknesses are in their problem-solving abilities enabling them to identify what students need and then tailor how the material is

taught to suit that student's needs. They also provide teachers with a familiar method to facilitate problem-solving.

The use of graphic organisers in accordance with the criteria for effective problem-solving provides for the effective development of problem-solving ability in students. To help students become successful problem-solvers, teachers need to come to terms with the fact that some students' problem-solving abilities develop slowly. This means that some students will require long term and sustained attention to problem-solving as an integral part of their mathematical studies. It is the teacher's/educator's responsibility to provide a problem-solving culture, consistent with practise in the classroom. Students need to be continually challenged, and engaged in activities to ensure they grasp the importance of problem-solving.

4.3.2 Exploration

Exploration is a form of discovery learning where, rather than an instructor setting goals, learners set their own goals and generate their own problems (Charney et al., 1990; Tuovinen & Sweller, 1999). Discovery learning is an instructional strategy used to build problem-solving skills. It is less direct than conventional instruction, leaving something for the learner to discover (Sweller et al., 2011). Charney *et al.* (1990) have shown that there is a significant benefit from exploration training; while problem-solving using the exploration method resulted in the longest exploration training time it produced the fastest and most accurate test performance. Exploration learning has the best results where independent goal setting is a critical feature of effective learning (Charney et al., 1990).

4.3.3 Visual representation

Understanding a mathematical problem is essential to solving it, and it is known that effective problem solvers use visualisation in order to comprehend the problem (Krawec, 2010). Visual representation can be defined as interpreting relationships among problem parts from a structural representation. Research (Hegarty & Kozhevnikov, 1999; Guoliang, 2003; Van Garderen & Montague, 2003, Krawec, 2010) has further divided visual representation into two categories: pictorial representations, which are primarily drawings of objects, and the second, schematic

representations, which are diagrams representing the spatial relationships among problem parts (Hegarty & Kozhevnikov, 1999). Analyses of students' visual representations have shown that successful problem solvers generally produce more schematic representations and thus have greater success solving-problems, whereas poor problem solvers more often rely on pictorial representations leading to inaccurate solutions.

4.4 Attributed learning outcomes

Ultimately the objective for any problem-solving approach is to maximise students' ability to effectively and efficiently solve mathematical problems. Specific learning outcomes highlighted in reference to problem-solving include functional logic, logical thinking, aesthetic appreciation leading to motivation to persevere in problem-solving, enhancement of students' confidence in their ability to attempt problems, increased usage of problem-solving strategies; positive attitudes toward mathematics, raised awareness as to the importance of strategic and systematic problem-solving approaches, the ability to arrive at the correct answer; increased ability to determine more than one way of finding a correct solution, and development of the ability to select and implement solution strategies. Comparisons between teaching methods often focus on one or two of these many possible outcomes.

4.4.1 Functional logic

Ockcroft (1982) espoused the view that problem-solving is a tool for daily living, and that the ability to problem-solve lies "at the heart of mathematics" (p.73), because it is the means by which mathematics can be applied to a variety of unfamiliar situations. Resnick (1987) also considered problem-solving to have practical everyday applications through helping develop the ability to adapt when the need arises in everyday life. According to Resnick (1998) problem-solving abilities contribute to the practical use of mathematics by helping people to develop the facility to be adaptable when, for instance, technology breaks down. It can also help people to transfer into new work environments at a time when most are likely to be faced with several career changes during a working lifetime. Resnick (1998) expressed the belief that "school should focus its efforts on preparing people to be

good adaptive learners, so that they can perform effectively when situations are unpredictable and task demands change" (p.18).

Problem-solving in mathematics is considered to be an essential skill of everyday life. The National Council of Teachers of Mathematics (NCTM, 2000) recommended that problem-solving be the focus of mathematics teaching because, they say, it encompasses skills and functions that are an important part of everyday life. The ability to problem-solve is highly regarded in our society as it indicative of one's ability to think laterally and on a higher level cognitively. Developing much needed skills by means of problem-solving is a mode to accessing existing schema, and an avenue to learning new concepts and skills (NCTM, 2008). More recently, the NCTM (2012) endorsed the recommendation that problem-solving should underlie all aspects of mathematics teaching in order to give students insight into the practical use and everyday application of mathematics in the world around them. The NTCM (2012) considered that issues in relation to environmental, situational and personal change can all be managed with the application of effective problem-solving skills.

4.4.2 Logical thinking

The importance of problem-solving as a means of developing logical thinking was described by Polya,

> "If education fails to contribute to the development of intelligence, it is obviously incomplete. Yet intelligence is essentially the ability to solve problems: everyday problems, personal problems..." (Polya, 1981, p.1).

Definitions of intelligence focus on practical intelligence that enables individuals to resolve issues in real situations that are difficult and problematic, inevitably leading to the acquisition of new knowledge (Garner, 1985; Muller, 2010). In recent years problem-solving skills have been further elaborated as enhancing logical reasoning in order to optimise functionality in everyday life (Engelman, 2011). Life decisions are more complex than being able to follow simplistic rules to obtain a correct answer; deductive reasoning is essential in the adaptation to situational changes. For these reasons problem-solving can be developed as a valuable skill in itself, a way of thinking (NCTM, 2010), rather than just as the means to an end of finding the correct answer.

4.4.3 Aesthetic values

The third value attributed to problem-solving relates to its aesthetic value. The aesthetic value of problem-solving is the exuberance attributed to the range of emotions associated with the various stages of the exploratory solution process. Mathematicians who successfully solve problems say that the experience of having done so contributes to an appreciation for the "power and beauty of mathematics" (NCTM, 1989, p.77; Chiawa , 2009), or the "joy of banging your head against a mathematical wall, and then discovering that there might be ways of either going around or over that wall" (Olkin & Schoenfeld, 1994, p.43; Burneche, 2010). However, the initial desire that motivates one to pursue a problem, and perseveres over time through the difficulty in finding a solution, is still not fully understood.

4.4.4 Confidence

Mathematical confidence is usually linked to level of mathematics achievement. If a student has been performing poorly in mathematics, they will usually develop the attitude they are "no good at maths". This fosters a negative attitude towards mathematics as well as instilling a poor level of confidence in the subject. According to Effandi and Normah (2009), by encouraging students to achieve a successful outcome in the use of problem-solving, students are instilled with a sense of achievement and attainment, which aids in the development of a positive attitude and increased self-confidence in the subject. This is vital to a student's success as it has been mentioned that "students' commitment in mathematics refers to students' motivation to learn mathematics, their confidence in their ability to succeed in mathematics and their emotional feelings about mathematics. "Students' commitment to mathematics plays a key role in the acquisition of mathematics skills and knowledge" (Effandi & Normah, 2009, p. 13).

4.4.5 Positive attitudes toward mathematics

According to Patton *et al.* (1997) the ability to solve a problem is a primary objective in learning mathematics, as it is an essential life skill. However, it is all too often that a negative attitude toward the learning of mathematics is seen especially in students in tertiary education. According to Ma and Kishnor (1997) a student's attitude

toward problem-solving can have a profound impact on their level of achievement. This finding has also been supported by Kargar *et al.* (2010); Hersh and John-Steiner (2010).

4.4.6 Awareness of strategic and systematic problem-solving approaches

Research also indicates that meta-cognitively aware learners are more strategic and perform better than unaware students (Pressley & Ghatala, 1990; Fernandez-Duque, et al., 2000). One explanation of this is that meta-cognitive awareness allows individuals to plan, sequence, and monitor their learning in a way that directly improves performance (Chatarajupalli, 2010).

4.5 Issues in teaching problem-solving

In teaching, the term "Problem-Solving" refers to mathematical tasks that have the potential to provide intellectual challenges for enhancing students' mathematical understanding and development. Solving a problem is the key outcome for any mathematical problem or question (Peters, 2012).

The teaching methods that are used to teach students how to problem-solve most effectively are key factors in the development of their problem-solving abilities. Problem-solving contexts are enquiry oriented environments where teachers are called upon to enrich the understanding of mathematical concepts and processes, allowing the student to lead the process of "creating, conjecturing, exploring, testing, and verifying" (Lester, 1994, p.154). According to Taplin (2006), characteristics of problem-solving approaches include the following:

- Inter-related relationships between teachers and students (Van Zoest et al., 1994; Konishi, 2010);
- Mathematical dialogue and consensus between students (Van Zoest et al., 1994; Zheng et al., 2013);
- Teachers providing just enough information to establish background/intent of the problem, and students clarifying, interpreting, and constructing one or more solution processes (Wood et al., 1991; Schoenfeld, 2009);

- Teachers accepting right/wrong answers in a non-evaluative way (Cobb et al., 1991; Das, 2013);
- Teachers guiding, coaching, asking insightful questions and sharing in the process of solving problems (Lester, 1994; Stella, 2012);
- Teachers knowing when it is appropriate to intervene, and when to step back and let the pupils make their own way (Lester, 1994; Sawyer, 2011);
- Students being encouraged to make generalisations about rules and concepts, a process which is central to mathematics (Evan & Lappin, 1994; Baker, 2010).

Not all purported "problem-solving" in mathematics adheres to these characteristics. Resnick (1987) illustrated the inconsistency in the school-based, algorithmic approach versus the "invented" strategy of problem-solving applied in practical life situations. This has also been supported study by Baker (2010). Schoenfeld and Olkin (1994) and Schoenfeld (2009) described their experience in early problem-solving courses where, students were directed to draw diagrams, analyse special cases analogies, specialise and generalise. They deemed this to be poly-type heuristics, whereas in recent years educators have been less focused on heuristics, putting more emphasis on fundamental principles such as, mathematical reasoning and proofs.

In problem-solving approaches, students require long term and sustained attention to ensure problem-solving is an integral part of their mathematical studies (Schoenfeld, 2009). It is the teacher's/educator's responsibility to provide a problem-solving culture, to teach problem-solving skills, and provide consistent practise in the classroom. Furthermore, teachers should ascertain what the students' strengths and weaknesses are in their problem-solving abilities.

As discussed in Chapter 3 researchers studying the effect of cognitive load on working memory, have been able to identify a myriad of effects which help explain problem-solving behaviour, including:

- The worked example effect (Cooper & Sweller, 1987; Sweller & Cooper, 1985; Sweller et al., 2011);
- The problem-solving effect (Van Merrienboer & Krammer, 1992; Sweller et al., 2011);
- Split-attention effect (Chandler & Sweller, 1992; Sweller et al., 2011; Paas & Ayres, 2012);
- The modality effect (Mayer, 1989; Mousavi et al., 1995; Penny, 1989; Sweller et al., 2011).

Teachers must be aware of these effects along with issues such as: knowing how, what and when to introduce problem-solving; how to select approaches to teach problem-solving; the selection of activities and determining how much guidance to give.

4.5.1 Knowing how, what and when

Knowing how and when to introduce problem-solving within the curriculum being taught, is the main challenge for teachers. Not only is how and when to teach problem-solving an issue for teachers, but also whether or not to teach problem solving as a topic on its own (Sweller et al., 2011). From all the evidence that has been collected over the past 30 years, there is little or no evidence that a student's problem solving abilities are enhanced by teaching problem-solving as a separate topic to learning mathematical concepts and procedures. Teachers also need to be aware of providing procedural information too early, as the early stages of schema acquisition can be adversely affected by procedural performance early on, as this adds to the cognitive load of working memory, which may already be in a complex learning environment (Sweller, 1988). Unfortunately, some learners are required to solve problems before they understand the problem schema, resulting in a search for the problem solution, without the engaging in schema acquisition (Sweller et al., 1998).

4.5.2 Selecting approaches to teaching problem-solving

A variety of approaches to problem-solving have been researched. The most common approach is: first teach the concepts and procedures, then assign the one-step "story" problems that are designed to provide practise, familiarity and a way for the student to relate to the problem and understand what needs to be solved. Lapan and Phillips (1998) developed a set of criteria and found them to be effective in teaching problem-solving and suggest that teachers might want to attend to these criteria in choosing, revising and designing problems. The criteria for teaching effective problem-solving include: 1) having useful mathematics embedded in the problem (Fox & Surtees, 2010); 2) for higher level thinking, involve problems with multiple solutions (Schukajlow, 2012); 3) identify what levels of ability students are at (Van Garderen et al., 2012); and, 4) identify what engages students and also what encourages them to do well (Schoenfeld, 2009).

4.5.3 Selecting problem-solving activities

A key discussion point is the types of problem-solving activities that should be given. In mathematics teaching, usually the first thing that comes to mind is a problem that evokes a conversation in the problem-solvers mind. The conception of problem-solving in these contexts is limited. Problems arise in that some "story" problems are not problematic enough and do not necessarily require high level thinking and analysis skills. These types of questions should be noted as exercises for students and not so much as problems. Generally, when researchers (Schoenfeld, 2009; Baker, 2010; Sawyer, 2011) refer to problem-solving, they are referring to mathematical tasks that have the potential to challenge individuals in order to enhance their skills. Tasks that are challenging problems can promote students' conceptual understanding, foster their ability to reason and communicate mathematically, and capture their interests and curiosity. Research recommends that students should be exposed to truly problematic tasks so that students can make mathematical sense and that sense making is practised. Regardless of the context, effective problem-solving tasks should be intriguing and contain a level of challenge that welcomes deliberation, hard work and critical thinking (Jonassen, 1997; Carson, 2007; Callejo, 2009).

4.5.4 How much guidance to give

In the classic text, *The Conditions of Learning*, "The discovery method is said to be liable to gross misinterpretation in practical learning situations" (Gagne, 1965, p. 165). Proponents of this technique argue for using a minimal amount of instruction, and unfortunately fall into the trap of providing problems "without prerequisite knowledge of principles and without guidance" (Gagne, 1965, p.165). Gagne (1965) stated that the process of searching and selection in problem-solving are all undertaken within the learners' nervous systems. The time spent in searching for a solution during the problem-solving approach, might not engage learning due to excessive cognitive load on working memory (Kirshner et al., 2006; Sweller 1988). Gagne (1965) describes problem-solving as the most complex form of learning, and hence recommends instructors provide additional guidance throughout the problem-solving process. The dichotomy between instruction and self-guidance is ever present with cognitive load researchers supporting the idea of learners teaching themselves, but introducing them first to worked examples, and as the required skills are gained, giving them the opportunity to practise on their own using a problem-solving method. Subsequent studies of expertise in various domains have found similar results regarding problem-solving (Carson, 2007; Callejo, 2009; Schoenfeld, 2009; Baker, 2010; Sawyer, 2011). The worked examples method has been favoured given the emergence of the expertise reversal effect (Kalyuga et al., 1998; Sweller et al., 2011; Paas & Ayers, 2012), which states that direct instruction is only useful during the earliest stages of learning as once the schema has been established, over instruction is no longer beneficial and actually commences to have a detrimental effect on learning. At this point it is suggested that learners transition from worked examples, to problem-solving (Renkl & Atkinson, 2000; Renkl et al., 2002; Paas & Ayers, 2012). Thus, it is not practise or discovery that cognitive load researchers are against. It is the timing of that practice which is under scrutiny.

Teaching problem-solving skills is highly desirable, and of major importance in teaching mathematics, however cognitive load theory suggests that, by clever and well thought out instructional design, through the use of worked examples, cognitive load can be decreased allowing for more efficient processing and formation of new schema (Sweller, 1993; Sweller, 2006; Tan L et al., 2010; Sweller et al., 2011; Ayres

& Paas, 2012). Worked examples (WE) is a technique for reducing cognitive load by demonstrating each step of a task or of solving a problem (Sweller, 1988; 2006; 2011). That is it effectively shows students each step in attaining a solution, making explicit what each step entails. Worked examples direct or guide the attention of a learner to the problem stated, and the steps required to solve the problem. This reduction in cognitive load should generally make learning easier. According to Jelsma *et al.*, (1990) learning from worked examples can be more effective in problem-solving than learning from solving the actual problem (Sweller & Cooper, 1985). Cooper and Sweller (1987) studied the use of worked examples and the problem-solving method for learning algebra. The study found that the use of worked examples improved the learner's ability to construct a method for solving an algebra problem and also improved their ability to transfer their knowledge to solving related algebra problems. Sweller and Cooper (1987) identified that those who learned from worked examples were able to complete the problems faster during a test compared to those who learned only through problem-solving.

Therefore, based on the above discussion, the purpose of this thesis is to compare the methods of problem-solving and worked examples from a cognitive load perspective, and to determine which is more effective for ESL students learning a variety of mathematics topics at the tertiary level.

4.6 Worked examples

This section demonstrates different aspects of worked examples including management processes and suggests how they can be used effectively for the purpose of minimising the cognitive load on ESL students when they learn mathematics in their second language. Also, this section presents the approaches of faded worked examples and guided worked examples and their applications, integrated worked examples and the associated learning outcomes such as near and far transfer. Drawing together these ideas the author has discussed issues associated with teaching with worked examples, namely the quantity of worked examples, sequencing of worked examples, and the structure of a worked example.

4.6.1 Terminology for worked examples

A worked example has been defined as follows:

> "A step-by-step demonstration of how to perform a task or solve a problem" (Ayres & Paas, 2012, p.2).

Other definitions are more expansive, involving explanations as to the purpose of the intermediate steps, such as:

> "A completed problem that displays the explanations to the intermediate steps as well as the resulting solution" (Crissman, 2006, p.6).

> "A worked example basically contains a problem with a procedure for solving the problem. It is a solved problem with a step-by-step solution that the learner needs to study" (Khateeb, 2008, p.54).

4.6.1.1 Demonstrations

Most mathematics teachers would be familiar with providing a demonstration of a solution to a problem. It involves showing the steps in the procedure to arrive at the solution, such as in Figure 4.4.

(i)
$$\begin{aligned} 3m + 4 &= 19 \\ 3m &= 19 - 4 \\ 3m &= 15 \\ m &= \frac{15}{3} = 5 \end{aligned}$$

Figure 4.4 Demonstrating the solution to a simple algebra problem.

4.6.1.2 Standard worked examples

Standard "worked examples" were simply worked-out examples unaccompanied by self-explanation prompts or instructional information explaining the solution process depicted in the worked example (refer Figure 4.5).

STUDY THIS	THEN DO THIS
(i) $3m + 4 = 19$	(i) $2n + 5 = 21$
$3m = 19 - 4$	
$3m = 15$	
$m = 15 = 5$	

Figure 4.5 An example of a standard worked example

4.6.1.3 Worked examples with explanation

Worked examples are problems that are solved using a procedural like approach, which enables a learner to follow a step-by-step process to the solution (Sweller et al., 1998). It differs from simply demonstrating the solution, to a question by articulating the steps involved in arriving at a solution such as the example in Figure 4.6.

Demonstration	Worked Example	
$3m + 4 = 19$	$3m+4 = 19$	
$3m = 19 - 4$	$3m = 19-4$	4 subtracted from each side
$3m = 15$	$3m = 15$	Simplify right hand side
$m = 15/3 = 5$	$m = 15/3=5$	Divide both sides by 3

Figure 4.6 Demonstration and worked example for a simple algebra problem.

4.6.2 Teaching with worked examples

There are several factors influencing the effectiveness of worked examples as a method of teaching. These include the quantity, sequencing, structure of examples; the split-attention effect, and the design of the worked example; the use of faded worked examples; and when to actually phase out the use of worked examples (Sweller et al., 2011).

4.6.2.1 Quantity of worked examples

There is debate as to whether multiple examples are required or not. Some researchers suggest the quantity of worked examples presented makes a significant impact on student learning (Cooper & Sweller, 1987; 1985; Gick & Holyoak, 1983;

Reed & Bolstand, 1991; Clarck et al., 2006; Paas & Ayers, 2012). In other words, the more worked examples presented the better the students understand the material. Sweller (2006) indicates that it is highly unlikely that one worked example would facilitate learning alone. Jitendra and Star (2011) noted that schema development most likely occurs with the presentation of two or more problems with multiple formats, rather than a single example of a problem format and that worked examples supports transfer by linking analogous solutions to problems and thus allowing transfer (Sweller et al., 2011). Figure 4.7 exemplifies this by providing three different worked examples of how to solve an equation.

Multiple examples:

Solve the following equations for the required variable:	
$3m+4 = 19$	
$3m = 19-4$	-4 on other side
$3m = 15$	Simplify right hand side
$m = 15/3$	÷ 3 on both sides
$m = 5$	Final solution
$2(x-5) = 12$	
$(x-5) = 12/2$	÷ 2 on both sides
$x-5 = 6$	Simplify right hand side
$x = 6+5$	-5 become + 5 on other side
$x = 11$	Final solution
$3-2f/5 = 1$	
$3-2f = 5$	Multiply 5 on both sides
$-2f = 5-3$	+3 become -3 in other side
$-2f = 2$	÷ 2 on both sides
$f = -1$	Final solution

Figure 4.7 Multiple worked examples for solving equations

It is suggested that the use of worked examples can aid in the generation of schema, which in turn, can lead to rule-use becoming automatic (Jitendra & Star, 2011). Alasraj (2012) extended the research on the issue of multiple examples to word problems, either simple or complex in nature, requiring students to alter different

elements within the equation. For example, the words rate, time, or tasks, with or without added procedures as illustrated in Figure 4.8.

$$\boxed{(\text{Rate } 1 \times \text{Time } 1) + (\text{Rate } 2 \times \text{Time } 2) =}$$

Figure 4.8 Time to solve a mathematics problem

The results supported the hypothesis that subjects receiving both complex and simple examples would demonstrate greater success than if receiving only a simple example, and if receiving the simple example plus procedures. The performance of the participants in the two-example condition provided evidence for the claim that multiple examples are needed to teach complex concepts (Alasraj, 2012).

4.6.2.2 Sequencing of worked examples

The dependence of novices on worked examples during problem-solving (Chi et al., 1982 ; 1989) has directed much of the research into pairing worked examples with problems to solve (Cooper & Sweller, 1987; Sweller & Cooper, 1985; Ward & Sweller, 1990). Due to research suggesting the need for examples to be interspersed with target problems (Pirolli, 1991; Sweller & Cooper, 1985; Trafton & Reiser, 1993), Trafton and Reiser (1993) sought to determine the optimal sequence of examples and problems in their study involving college undergraduate students in the domain of computer programming. They examined whether or not separating examples from target problems hindered learning by presenting subjects with one of four conditions:

i. Alternating-example where an example is immediately followed by the problem;
ii. Alternating-solve in which subjects solve an initial problem and then a hard problem;
iii. Blocked-example that presents all six examples and then all six problems; and,

iv. Blocked-solve requiring subjects to solve all examples first, followed by problems.

They found that the first option of alternating source examples with target problems produced overall better results, required less time during training and improved accuracy on post-tests more so than presenting blocked examples followed by a block of target problems (Trafton & Reiser, 1993).

4.6.2.3 Structure of worked examples

A well-structured worked example will detail each stage of the solution process as clearly as possible for all the students such as illustrated in Figure 4.4; and when combined with explanation (Ward & Sweller 1990; Sweller et al., 2011; Alasraj, 2012) can significantly reducing teaching time and effort (Paas & Ayers, 2012). Research has shown in many cases that worked examples have useful benefits, but there are still cases where they can actually have a negative effect on problem-solving. It has been suggested that the structure, or design of a worked example is a crucial component to it effectiveness (Catrambone & Holyoak, 1990; Mwangi & Sweller, 1998; Ward & Sweller, 1990; Zhu & Simon, 1987; Sweller, 2006; Sweller et al., 2012). Well-structured worked examples will involve the reduction of split-attention (Section 3.6.2.5) through integration of text and diagrams, visual and aural information; all of these are key to the successful design of worked examples (Atkinson et al., 2003; Alasraj, 2012).

4.6.2.4 Integration of text and diagrams

The placement and use of diagrams and the wording of worked examples is highly significant to the effectiveness of worked examples (Sweller, 1988; Sweller et al., 1998). In difficult topics such as mathematics, specifically in geometry, worked examples, more often than not, require the learner to simultaneously integrate textual information and a related diagram. Poorly integrated textual and diagrammatic presentation in worked examples, poses a risk of inflicting a heavy cognitive load on the learner (Mwangi & Sweller, 1998; Tarmizi & Sweller 1988; Ward & Sweller 1990; Alasraj, 2012).

4.6.2.5 Reducing the ‘split-attention effect’

The split-attention effect, is a term coined by Tarmizi and Sweller (1988) in their study of geometry based worked examples. The split-attention effect is a term used to describe what happens when students are required to split their attention among multiple sources of information, for example where students have to process information from both a diagram and written information presented separately (Alasraj, 2012). By having to split their attention between two sources of information, the student is exhausting working memory beyond capacity and is considerably increasing cognitive load. As a result the student has difficulties forming new schema, which greatly impedes learning (Sweller, 1988; Jitendra & Star, 2011). Those students who experience the split-attention effect present with poorer results than when provided with worked examples in classroom and laboratory settings (Clark et al. 2006). Hence, information integration into a single area is crucial in reducing the split attention effect.

Sweller *et al.* (1990), designed a set of experiments which aimed to investigate if schema could be built under conditions that required the learner to mentally integrate separate sources of information (Alasraj, 2012). The results drawn from this study indicated that problem-solving and worked examples approaches in coordinate geometry offered no benefit to solving-problems, however students working with the modified worked examples, in which the explanation and diagrams were integrated, required less time to process and performed better on both similar and transfer test problems (Sweller et al., 1990). In the second set of experiments, Chandler and Sweller (1991) extended their findings to Australian first-year trade apprentices learning numerical control machine programming, supporting the clear advantage of integrating sources of information in instructional material. From the findings, they were able to demonstrate that the alteration of instructional procedures can dramatically facilitate learning in both a formal educational setting, and the workplace.

4.6.2.6 Facilitating transfer

The concept of transfer states that, the more students practise, the more mathematical rules become automated hence leading to improvement in their ability to transfer

problems in which the same rules are used in a different context. In 1987, Cooper and Sweller found that more experienced students (refer Table 4.1) had problems with schema and performance, which sparked a real interest in the effect of worked examples on schema formation, and automation rule in terms of transfer. This finding is also supported by Clark *et al.* (2006).

Table 4. 1 Experimental of (Cooper & Sweller, 1987) outlines

Experiment 1		**Experiment 2**	
WE Group	**PS Group**	**WE Group**	**PS Group**
Both groups were given 4 problem formats. Problems were reviewed twice, before the next phase of experiment.		Students were given the 4 problem formats, then assessed on the following three variables: a) Period of acquisition (Short or Long) b) Problem category (Similar or Transfer) c) Student ability (High or Low)	
Test phase		Test phase	

Furthermore, the use of worked examples can facilitate transfer (Cooper & Sweller, 1987). Cooper and Sweller (1985) found that a group presented with worked examples required significantly less time for problem-solving during the acquisition phase. These finding were further supported in a later study conducted by Clark *et al.* (2006), where the students where subdivided into eight different groups based on a factorial design of three independent variables (refer Table 4.2):

- Ability (High vs. low),
- Practice (Short vs. Long acquisition), and
- Teaching method (WE vs. PS during acquisition).

Table 4. 2 Impact of ability, practice, and teaching method on test outcomes

Ability	**Short Acquisition**		**Long Acquisition**	
High	G	**G**	**G**	**G**
Low	G	**P**	**G**	**A**
	WE	**PS**	**WE**	**PS**
	Teaching Method			
Testing Outcomes	P = Poor A = Average G = Good			

The results revealed significant differences in test performance due to the length of the acquisition period and ability. It was also found that worked examples had little effect on transfer problems for low ability students in the short length of acquisition period. Positive effects on the transfer problem noticeably increased as the length of the acquisition period increased and to a lesser extent as the ability of the students increased. Although the impact of ability was less than that for the acquisition period, there was no effect for high ability students when comparing short verses long acquisition periods (Clark et al., 2006). Furthermore, the study reveals that for low ability students, better results are achieved for PS with longer acquisition time than worked examples with long or short acquisition times.

4.6.2.7 Faded worked examples approaches

A fading example (refer Figure 4.9) is one in which the last solution step is omitted first, with each subsequent example omitting the last step work (Renkl et al., 2002; Alasraj, 2012).

$3m+4 = 19$ $3m = 19-4$ $3m = 15$ $m = 15/3$ $m = 5$	This example has four steps
$3m = 15$ $m = 15/3$ $m = 5$	This example has removed 1 step from the original worked example
$m = 15/3$ $m = 5$	This example has removed 2 steps from the original worked example
$m = 15/3$	This example has removed 3 steps from the original worked example
$m = 5$	Final solution

Figure 4.9 Example of faded worked examples

Alasraj (2012) found novice learners benefit greatly from worked examples in the formation of schema and in the progression of learning (refer Figure 4.9), however, when the learner is no longer a novice and has started to gain expertise in the domain being taught, there is a need to modify the teaching method, integrating elements of problem-solving in the mode of fading out solutions in the worked examples. The ability to do this is facilitated by faded solutions, as the student is pushed to develop the skills to retrieve and apply from existing schema. This instructional model of fading solution steps proposes a smooth transition from complete worked out examples to independent problem-solving in which instructional support fades during the transition, a rationale that can be basically applied to example based learning (Renkl & Atkinson, 2002; Renkl et al., 2000, Alasraj, 2012).

The study by Renkl *et al.* (2000) involved college students learning mathematics and found that where a fading worked examples was used rather than problem-solving, students receiving the faded worked examples performed significantly better than the solving-problem approach. For transfer, which is most desirable in terms of eventually being able to solve problems rather than just following steps of worked examples, there was no significant difference between students with problem–

solving compared with those presented with the fading worked examples. Further research (Renkl et al., 2004b; Paas, 2012) has found university students who participated in the studies, receiving faded worked examples have been found to perform better than those receiving worked examples, and students receiving worked examples have been found to achieved greater success than those receiving problem-solving techniques. It was concluded that fading examples enables students to draw on the created schema (retrieval) and apply this existing knowledge to problems that may be similar to the rehearsed worked examples or they can transfer this knowledge and apply it as part of a more complex problem.

4.6.2.8 Phasing out worked examples as expertise increases

There are obvious advantages to using worked examples with low level learners due to the lack of schema-based knowledge in the targeted domain; without the added guidance they are unable to solve problems efficiently (Sweller et al., 2011). Similarly, (Kalyuga et al., 2001; Alasraj, 2012) found that in lessons that used both worked examples and problem-solving that worked examples became less necessary and learning improved as students gained expertise with the targeted material. The opposite was true when similar teaching methods were used for high level learners as they already have established schema on the targeted domain. By over guiding the learning process, the learner has conflicting and unnecessary repetition with existing schema, increasing cognitive load on working memory, thus having a reverse effect on the learning, which is known as expertise reversal. Therefore, existing schema can eliminate the worked example effect. Research (Clark et al, 2006; Paas et al., 2012; Sweller et al., 2011) recommendations regarding reducing or eliminating expertise reversal are as follows:

1. Entry level students should be given worked examples;
2. Students should be asked to participate in completing some of the steps in worked examples;
3. The series should be ended with a problem and students asked to look for the solutions; and,
4. The cover story of examples and practice should be varied when far transfer outcomes are desired.

Owens and Sweller (2008) found that being familiar with a topic enables learners to use their existing schemas instead of using the provided worked examples during the learning process. Further, following the study by Paas *et al.* (2012), regarding the correlation between learner levels of expertise and the level of learners benefit from worked examples, it is recommended that the use of worked examples needs to be phased out as learner expertise increases in a specific topic or domain.

4.7 The future of problem-solving and worked examples

Problem-solving techniques as a suggested mode of learning mathematics have been paramount. Recently, professional organisations such as the National Council of Teachers of Mathematics (NCTM, 2012) have recommended that the mathematics curriculum should be organised around problem-solving, focusing on:

1. Developing skills and the ability to apply these skills to unfamiliar situations;
2. Gathering, organising, interpreting and communicating information;
3. Formulating key questions, analysing and conceptualising problems, defining problems and goals, discovering patterns and similarities, seeking out appropriate data, experimenting, transferring skills and strategies to new situations;
4. Developing curiosity, confidence and open-mindedness (NCTM, 2012, p. 2-3).

Cognitive load theory assumes that the establishment of higher order learning is more efficiently achieved with the use of the worked examples method versus the problem-solving method (Cooper & Sweller, 1987; Sweller, 1989; Sweller & Cooper, 1985; Sweller, 2006; Khateeb, 2008). Worked examples have been demonstrated to be advantageous compared to problem-solving for many reasons. Sweller *et al.* (2011) outlined the benefits of worked examples, based on recent studies with a cognitive load theory perspective. They are as follows:

1. Worked examples require less working memory capacity than problem-solving as they remove the need to have to look for problem solutions;

2. Worked examples reduce unnecessary cognitive load. A worked example's effectiveness is dependent upon the extent of minimisation of the cognitive load;
3. As an in-class teaching method, worked examples provide a clearer illustration to finding mathematical solutions (Sweller, 2006).

Research has shown that worked examples have advantages over problem-solving even though their effectiveness are associated with several different variables. Variables include learners' ability to explain illustrated solutions to themselves (Chi et al., 1989; Renkl, 1997; 2002; Pass & Ayers, 2012) and the structure of learning material (i.e. worked problems) (Atkinson et al., 2003) along with previous domain knowledge (Tuovinen & Sweller, 1999; Sweller, 2006). Students who are offered worked examples before problem-solving, are more efficient and focused than students who are not given worked examples (Cooper & Sweller, 1987; Sweller & Cooper, 1985; Alasraj, 2012). Findings from various studies like those of Zhu and Simon (1987) regarding the efficiency of worked examples, have been extended to the classroom (Sweller et al., 2011) with findings that the students with worked examples performed better than those with problem-solving, with all students in the WE group scoring 'good' regardless of ability and acquisition length. Students in worked examples groups have been found to perform "better" in terms of:

- Higher assessment marks (Renkl & Atkinson, 2002);
- Higher post-test marks (Carroll, 1994);
- Fewer errors (Carroll, 1994);
- Fewer types of errors (Carroll, 1994);
- Less time required for acquisition of problem-solving skills (Carroll, 1994); with,
- Fewer requests for assistance from educators than the problem-solving group (Carroll, 1994).

Furthermore, the worked examples approach has been found to be more effective and is more advantageous to low level learners than the problem solving approach. This is because of the reduction of cognitive load on learners due to the limitation on working memory capacity (Paas & Van Merrienboer, 1993; 1994; Sweller, 2011).

The success of worked examples has been demonstrated across several domains including: algebra (Chung & Tam, 2005; Cooper & Sweller, 1987; Grobe & Renkl, 2006; Nathan et al., 1994; Sweller & Cooper, 1985), physics (Ward & Sweller, 1990; Ayres & Sweller, 2005), statistics (Pass, 1992), geometry (Paas & Van Merrienboer, 1993; Zhu & Simon, 1987), computer programming (Trafton & Reiser, 1993; Tuovinen & Sweller, 1999; Paas & Van Gog, 2006) and engineering (Van Gog et al., 2006).

4.7.1 The acquisition of cognitive skills

Acquiring the cognitive skills to be able to solve problems in the broadest sense of the term is the desired outcome for learning of mathematics. Worked examples are an efficient tool for initial acquisition of cognitive skills, but there are various opinions about their effectiveness in the transition from initial learning to later learning stages, in which more topic specific knowledge is required. For example, if a student is asked to solve a simple algebraic equation for *m*, then the worked examples approach to such a problem would be beneficial for the first 3-5 times. Afterward, when the student becomes proficient at solving for *m*, as they have understood the steps required to arrive at the solution, the teacher would need to challenge the student by phasing out some of the steps to allow for independent working toward the student using a problem-solving process as illustrated in Figure 4.10.

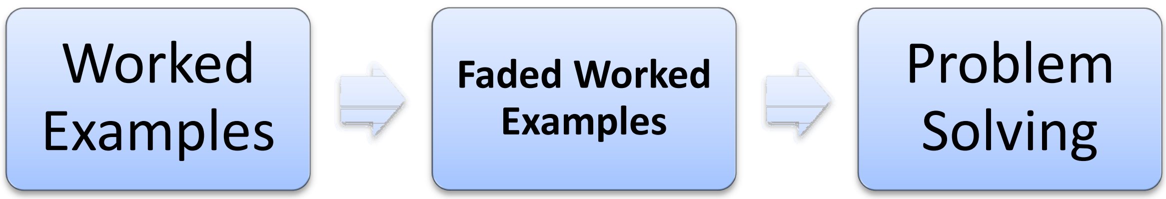


Figure 4.10 Scaffolding to problem-solving (Atkinson et al., 2003)

During the later stages of skill development, problem-solving may be more beneficial (Anderson et al., 1997; Kalyuga et al., 2001; Geary, 2012; Paas & Sweller, 2012; Youssef et al., 2012). Researchers have found the move from ‘worked examples’ to ‘problem-solving’ is a gradual process for undergraduate students

(Anderson & Fincham, 1994; Novick & Holyoak, 1991) which occurs over time with repeated examples in the area of concern (Reed & Bolstad, 1991).

4.8 Conclusion

In summary, worked examples function to reduce the cognitive load on working memory in the hope of decreasing time and increasing efficiency of the learning process. Extensive research into this field (Anderson et al., 1997; Kalyuga et al., 2001, Geary 2012; Paas & Sweller, 2012; Youssef et al., 2012) not only highlights the advantages of the approach, but also draws attention to several factors that would inherently limit the effectiveness of the approach, such as the expertise reversal effect, the structure of worked examples, the split-attention effect, and fading worked examples approach. The design and use of worked examples should adhere to recommended principles from the research including:

1. The integration of sources of information, such as diagrams, text, and aural information to avoid cognitive overload associated with split-attention (Paas & Van Gog, 2006);
2. Complex material accompanied by simultaneous aural information should provide visual cues to relevant elements of the example (Salden, 2009);
3. The inclusion of sub-goals can enhance learning, because sub-goal tasks within complex problems typically represent important conceptual ideas (Atkinson et al., 2003);
4. Fading solution steps offers a technique to move from worked examples to problem solving (Flores, 2011);
5. For the low ability students, better results are achieved for PS with longer acquisition time than shorter acquisition time (Clark et al., 2006).

Given the efficiency in mathematical learning promoted by the use of worked examples, it would seem prudent to present the worked examples approach to students learning in a language other than their primary language.

5 RESEARCH METHODOLOGY

"Both qualitative and quantitative methods may be used appropriately with any research paradigm"

(Guba & Lincoin, 1994)

5.1 Introduction

The importance of research design cannot be understated, as it is the framework by which methodology will achieve research objectives. In the early stages of the planning and design process, researchers need to be assiduous that information needs and objectives are being met, by selecting the most appropriate research method based on the needs of the study.

The focus of this study is to investigate how 'worked examples' and 'problem-solving' impact on the ability of ESL students to learn mathematical concepts. This chapter will explore the three phases of the research: the baseline data collection, and the first and second implementation of changes to the teaching. This will be carried out in six distinct sections as follows:

- Review of the research questions;
- Discussion on research paradigms and paradigm selection;
- Validating outcomes through triangulation;
- Research methodology including case study, action research, positioning of self, questionnaires and interview, and experimental design.
- Data collection tools including a discussion on data collection and analysis; and,
- Ethical considerations.

5.2 Research questions

The research questions target learning of ESL students studying tertiary level mathematics as studied in two contexts: Type A, where students are from other non-English speaking countries (international students in Australia) and Type B where students study in their home country, whose official language is not English, but where the prescribed language of study is English. The primary questions governing this research are:

1. How do students experience in terms of attributes such as anxiety, ease of learning, understanding, enjoyment, mental effort, speed of learning and confidence associated with the learning the use of worked examples and problem-solving approaches when attempting to achieve the learning and teaching objectives of their tertiary mathematics subjects?
2. Is there a difference in performance learning through worked examples compared to learning through problem-solving approaches?
3. Are there benefits in terms of greater confidence that can be attributed to either worked examples or problem-solving?
4. Can faded worked examples be used to scaffold from worked examples to problem-solving in term of confidence and performance?

The mathematics topics taught include functions, exponents, quadratic equations, logarithms, geometry, and introductory statistics.

5.3 Selection of research paradigm

All research design is undertaken within some paradigm or interpretative framework, which is guided by "a set of beliefs and feelings about the world and how it should be understood and studied" (Lincoln & Guba, 2000, p.65). Mertens (2009) listed five categories of those beliefs:

- *Ontology*: Converges on the question of “what is real?”
- *Epistemology*: The focus is on the relationship between the inquirer and the known.

- *Methodology*: Examines how we know the world, or gain knowledge of it.
- *Axiology*: role of value in the inquiry, positivists believe that the inquiry is value-free.
- *Generalisations*: positivists believe that time and context-free generalisation is possible.

Positivism is also known as a qualitative way of expressing results of a study. Qualitative method, is defined by Creswell (2003) as:

> Qualitative method is an approach to research methodology in which the inquirer often makes knowledge claims based on constructivist perspectives (i.e., the multiple meaning of individual experience, meanings socially and historical constructed, with an intent of developing a theory of pattern) or advocacy/participatory perspectives (i.e., political, issues-oriented, collaborative, or change oriented) or both. It also uses strategies of inquiry such as narrative, phenomenologies, ethnographies, grounded theory studies, or case studies (p.18).

This gave rise to the 'quantitative method'. Creswell (2003) defined the quantitative method as:

> Quantitative method is an approach to research in which the investigator primarily uses post-positive claims for developing knowledge (i.e., cause and effect thinking, reduction of specific variables, hypothesis and question, use of measurement and observation, and the test of theories), employ strategies of inquiry such as experiments and surveys, and collects data from predetermined instruments that yield statistical data (p.18).

Both quantitative and qualitative approaches have their own merits and pitfalls (Bryman, 2008). Despite the difference between the qualitative and quantitative approaches to research, there was a movement to reconcile and integrate the two methodologies in the 1980s. The concept of mixing different methods originated in 1959 and it is often referred to as a mixed method approach. This encouraged other researchers to use both 'qualitative' and 'quantitative' approaches in their studies (Creswell, 2003) (refer Table 5.1).

Table 5. 1.Three categories of research methods

Qualitative Research	Quantitative Research	Mixed Method
• A means of exploring and understanding social and human phenomena. • The research process generally comprised of questionnaires and interviews with participants. • Inductive analysis of data to form generalised meanings/conclusions.	• A means for testing objective theories by examining the relationship among variables. • Variables are tested and measured, using quantifiable means. • Numerical data is statistically analysed to form generalised meaning and conclusion.	• An approach to inquiry that combines or both qualitative and quantitative forms. • It involves philosophical assumptions, the use of qualitative and quantitative approaches, and the mixing of both approaches in a study. • A combination of both approaches so that the overall strength of a study is greater than either qualitative or quantitative research.

(Adapted from Creswell, 2003, p.4)

Creswell (2003) explained the mixed method design as having three strategies:

- *Sequential studies*: The researcher carries out two phases of the study; the first phase is qualitative in nature, the second quantitative, or vice versa.
- *Parallel/simultaneous studies*: The researcher conducts the qualitative and quantitative phases at the same time.
- *Equivalent status designs*: in order to best understand the phenomena of study, the researcher undertakes both the qualitative and quantitative methods equally. Then the data is assessed for the purpose of the study. This method is also known as the ‘concurrent triangulation strategy’.

The advantages to the mixed method approach in the use of mathematics education research are increasing its popularity for many reasons:

- The mixed method approach allows researchers to answer questions that could not have otherwise been answered on the basis of quantitative data alone;
- Mixed methods provide a more comprehensive understanding of the phenomena of study;
- It allows for research to address a wide range of exploratory questions from either the qualitative or quantitative approaches;

- Mixed method allows for more credible inferences;
- Mixed method allows for a wide assortment of views (Teddle & Tashakkori, 2009).

Approaches to qualitative, quantitative and mixed methods research, their underlying philosophy, and strategies for enquiry, methods and research practices are summarised in Table 5.2.

Table 5. 2 Qualitative, quantitative, and mixed methods approaches

Tendencies to use	Qualitative Approaches	Quantitative Approaches	Mixed Methods Approaches
Philosophical assumptions	• Constructivist/ advocacy/participatory knowledge claims	• Post-positivist knowledge claims	• Pragmatic knowledge claims
Strategies of inquiry	• Phenomenology, grounded theory, ethnography, case study, and narrative	• Surveys and experiments	• Sequential, concurrent, and transformative
Methods	• Open-ended questions, emerging approaches, text or image data	• Close-ended questions, predetermined approaches, numeric data	• Both open- and close-ended questions, both emerging and predetermined approaches, and both quantitative and qualitative data and analysis
Practices of research	• Position him- or herself • Collects participant meanings • Focuses on a single concept or phenomenon • Brings personal values into the study • Studies the context or setting of participants • Validates the accuracy of findings • Makes interpretations of the data • Creates an agenda for change or reform • Collaborates with the participants	• Tests or verifies theories or explanations • Identifies variables to study • Relates variables in questions or hypotheses • Uses standards of validity and reliability • Observes and measures information numerically • Uses unbiased approaches • Employs statistical procedures	• Collects both quantitative and qualitative data • Develops a rationale for mixing • Integrates the data at different stages of inquiry • Presents visual pictures of the procedures in the study • Employs the practices of both qualitative and quantitative research

(Adapted from Creswell, 2009, p.17)

The mixed methods approach can be used to describe this study, as such it uses a mixture of qualitative and quantitative approaches:

1) *Qualitative*: Data can be collected using a qualitative approach in the form of unstructured interviews, such as open-ended questions (Kervin, 2006). The purpose of such an approach is to gain insight into the attitudes, beliefs and motivation, that govern human behaviour (Kervin, 2006). For example, it allows researchers to delve into areas which cannot be quantified with numbers, such as details about experiences (Corbin & Strauss, 1990).The flexibility and open-ended nature of qualitative research is well suited to collecting and analysing data about student and lecturer experiences. This study involves the collection of qualitative data regarding student and lecturer experiences with two teaching methods, worked examples and problem-solving. As such, from a data measurement perspective the study is centered on pedagogical practice as well as educational outcomes. A qualitative approach suits this study as it allows research participants to provide in-depth responses regarding their experiences in learning mathematics through the problem-solving and worked example methods which are personal in nature and highly subjective as it is based on personal opinion and experience. Such variety and flexibility can give researchers a more thorough understanding of the research area. Data collection was in part in the form of surveys and closed and open-ended questions were used to gather both numerical and textual data.

2) *Quantitative*: Although the qualitative approach to data collection is advantageous, it alone was insufficient to meet the objectives of the study. While students' experiences of the teaching methods are important, the adequacy of the various pedagogical practices (worked examples versus problem-solving) implemented had to be assessed and hence quantified in terms of student achievement in the final examination, leading the research toward a more quantitative approach in terms of design and measurement. Within the case study approach the study was designed to ensure that the impact of the two teaching methods could be attributed to the method of teaching through worked examples and problem-solving techniques.

5.4 Triangulation

The origins of mixed method research techniques stems from pragmatism. The pragmatists combine qualitative and quantitative approaches to research (Merten,

2005). Theory and observations of phenomena, facts, and the application of inductive and deductive reasoning has supported the central notion of co-existence amongst the subjective and objective (Onwugbuzie et al., 2004). The concept of 'triangulation' emerged from the pragmatists, as a means of improving the credibility of the interpretation of findings (Merten, 2005). Some consider the use of the triangulation method, as a mixed method approach by which the qualitative and quantitative approaches are employed for a more comprehensive means of checking the validity of an interpretation (Bergman, 2008). In the broadest sense, the combined use of qualitative and quantitative methodologies offers a means to validate findings.

The concept of triangulation has also been articulated with much finer distinctions than simply the combined use of qualitative and quantitative methods. For example, Denzin (1977) proposed four types of triangulations as outlined in Figure 5.1:

- Data triangulation adopts the use of various sources of data to ensure the validity of the research.
- Investigator triangulation makes use of various researchers. The aim is to ensure the credibility of the data sources.
- The triangulation of theory aims to use multiple perspectives in the interpretation of the data. The various researchers review the data and assess the findings.
- Methodological triangulation is the use of various methods to evaluate a research problem. Researchers who have expertise in the specified area are enlisted with the final phase of the study in the empirical evaluation of data and observations.

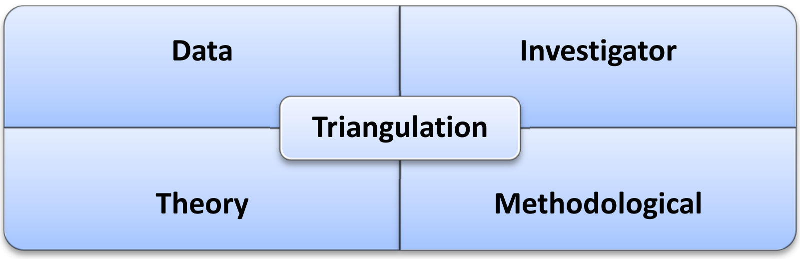


Figure 5.1 Denzin's Triangulation approach to mixed method research

The use of triangulation in a mixed method approach is by no means without faults, of bias, weakness and limitations. However, no approach is. For the purpose of this study the mixed method approach allowed for the collection of data regarding the learning of mathematics by ESL students in two different contexts in order to assess how the use of worked examples or problem-solving affected students' mathematical education. In accordance with the mixed methods approach, this study implements several methodologies and data collection tools. Data was generated from various evidentiary sources using both quantitative data from assessments and qualitative data, from surveys and interviews, as displayed in Table 5.3. The analysis of the information gathered also adopted mixed methods using a combination of statistical and theoretical analysis to develop a well-rounded understanding of the students learning experiences within the context of the study.

Table 5. 3 Approaches to data and evidence collection

Paradigm Assumptions	Pragmatism	
Strategies/ Methodologies	*Case study*	Each of the two case studies involved and related to the organisation of teaching/learning/assessments as experimentally manipulated within the closed systems of the individual subject and topic.
	Action research	Each of the case studies detailed in Chapter 6 involved three implementations, which refined both the interventions and the data collection protocols.
	Positioning of self	In each case study the researcher was involved with the participants.
	Experimental design	Each of the two case studies involved an initial observation phase (Baseline phase 1) in order to explore any underlying issues, design an appropriate course of action for problem resolution and/or to assess the needs and collect data to evaluate future outcomes. This was followed by two iterations involving the manipulation of teaching methods for different mathematics topics.
Data Collection Techniques	• Interviews • Questionnaires • Observations • Examination marks	
Data types	Quantitative and Qualitative	
Mode of analysis	Statistical and thematic	

(Adapted from Morris, 2008, p.28)

5.5 Research methodologies

Educational research is one instance where it is difficult to exert control over the many potential variables that can confound results. Hence the selection of research paradigm and methodological approach to this study is pragmatic, drawing upon approaches aligned with different philosophical assumptions as summarised earlier in Table 5.2. “Triangulation refers to a research strategy that involves approaching a

research question from two or more angles in order to converge and cross-validate findings from a number of sources" (Hewson, 2006, p.180) and as such this study seeks to validate findings using mixed methods approach to data collection. Data collected are deemed to be valid when the methods employed to extract the data are highly varied and therefore any inference or causal relationships established between students learning and teaching strategies has valid documentary evidence to support them (Mertens, 2009).

Results from case studies are often limited in their generalisability and ability to determine cause and effect. Well–designed quantitative studies in education are difficult to orchestrate due to the inability to randomly assign students to different treatment groups. With this in mind, it was decided to investigate the research questions triangulating from various perspectives using a range of methods in order to get a better insight into the effectiveness of worked examples and problem-solving with ESL students and through doing this to validate the findings. In this study combining qualitative and quantitative approaches the triangulation of findings drew on several approaches:

1. Case studies set in two different ESL contexts;
2. Positioning of the researcher to identify design of the study before engaging in action based research;
3. Experimentally designed study within each case study;
4. Quantitative data gathering regardingfinal marks within each case study.

5.5.1 Case studies

Case studies allow researchers to merge information from various sources, and not to rely solely on quantitative or qualitative data alone (Yin, 2003).The case study approach to researching entails a detailed investigation of a specific person, place, event or institution (Kervin, 2006). It is the preferred approach when there is little control over events and the focus of the study is on a phenomenon within a real life context (Yin, 2003), as it provides an in-depth analysis, and has the ability to report on real life events over a period of time (Merriam, 1998).

This research has involved two case studies described in more details in (Chapter 6): the first of which involves a tertiary mathematics subject, Math132, taught at King Abdul-Aziz University in Saudi Arabia; the second is a tertiary mathematics subject, Advanced Mathematics, taught at UOWC, Australia. Each case study involves a sequential set of data collections, baseline, and introduction of new learning designs (worked examples and problem-solving) and in the final iteration the introduction of faded worked examples. The teaching methods were experimentally manipulated with each student experiencing both problem-solving and worked example methods according to the topic to which the methods had been allocated. The topics receiving PS and WE were alternated in the final iteration. The effects on learning were compared within the sessions and changes across multiple sessions. The study involved the implementation of learning designs (worked examples versus problem-solving) developed for teaching and learning of mathematics, targeting ESL students. Students' improvements or lack thereof and their experience of the two methods were evaluated using both performance data and data from student questionnaires. Alexander (1999) suggested several case study methods to evaluate students' learning. These suggestions have been listed in Table 5.4. Suggestions implemented in this study have been marked with an asterix (*).

Table 5. 4 Alexander's (1999) Suggested Student Learning Evaluation

1. Comparison of the performance of students who used the different teaching method, with those who did not use it.	(*)
2. A comparative study with control and treatment group, and pre- and post-tests.	
3. Comparison of students' solutions to problems in examinations, with those of students from other universities.	
4. Pre- and post-tests combined with student interviews.	
5. Review of students responses in examinations.	(*)
6. Questionnaire concerning students' experience of the different teaching method as well as their reaction to it.	(*)
7. Questionnaires concerning students' perceptions of learning outcomes.	(*)
8. Questionnaires given to students before and after the implementations.	
9. Interviews with students about changes in their conceptions.	(*)
10. Focus groups.	
11. Expert reviews.	(*)
12. Observation of students' study of the course.	(*)
13. Assessment of content and retention of learning.	(*)

(Adapted from Alexander, 1999, pp. 179-180)

5.5.2 Action research

The study could also be considered to be action research. Action research generates knowledge and learning through participatory action, which then leads to personal/professional development. The evolution of action research, from its early introduction by Kurt Levin (1946), has seen it move into the mainstream, implemented as the forefront methodology used in studying the theory and practice of teaching and curriculum research development (Cohen et al., 2007). In the context of this study, each case study was conducted over a sequence of three classes. Figure 5.2 illustrates the O'Leary model of action research that was implemented across both case studies.

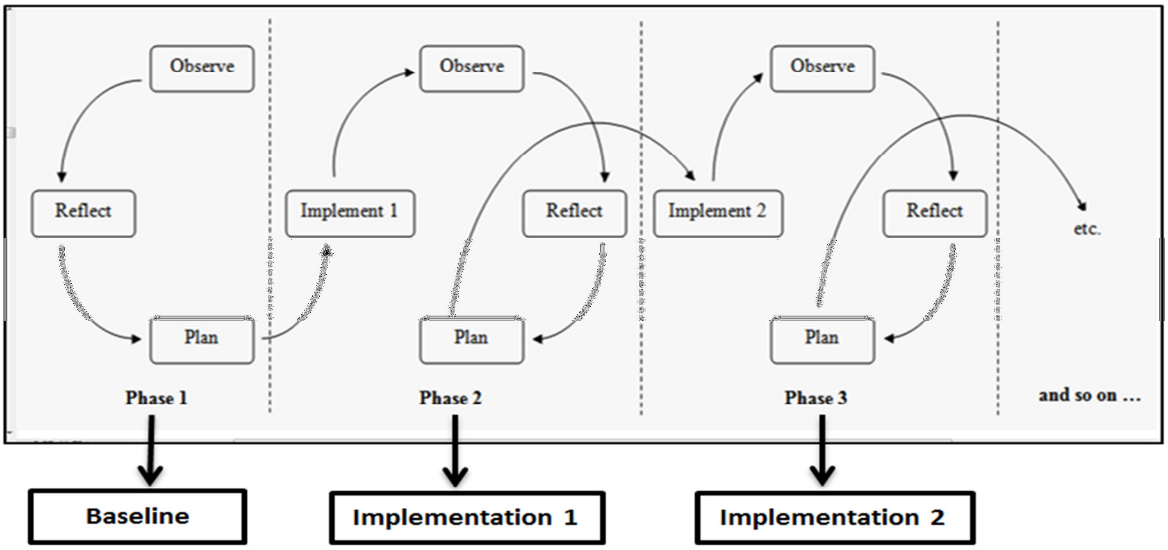


Figure 5.2 O'Leary's model of action research
(Adapted from Morris, 2008, p.31)

The collaborative nature of action research contradicts the traditional "objective" scientific methods of research, where implementation is detached from the research design process. O'Leary's (2005) model of action research stresses the cyclical nature of learning, an experiential learning approach, to change, where the goal is to continually refine the methods, data, and interpretations in light of the understanding developed in each earlier cycle. O'Leary describes this process, of continual change and improvement, as:

> … you learn, you do, you reflect, you learn how to do better, you do it better, you learn from that, do it better still, and so on and so forth. You work through a series of continuous improvement cycles that converge towards better situation understanding and improved action (2010, p.128).

Action research was used to improve teaching methodologies by continually assessing and reviewing practice in a collaborative setting between researchers and participants. Once more, diverging from traditional research strategies, this close collaborative setting can be perceived as a source of bias (Newton, 2006) resulting in what some may deem to be unconvincing justifications and conclusions and hence the need for positioning of oneself to help to identify potential for bias.

In this study baseline data was gathered prior to the manipulation of teaching methods and this acted as a reference point for comparison of the data collected throughout the remainder of the study. The pre-requisite to using baseline data, is that it be closely related to the proposed implementation. The baseline data was collected for both the Saudi Arabian Context (Type A) and the Australian context (Type B) prior to the first implementation of changed teaching methods, where topics were taught using either the worked examples approach or the problem-solving approach. Using baseline data is beneficial in this study as it allows the researcher to compare outcomes after implementation with outcomes prior to the start of the manipulation of teaching methods.

5.5.3 Positioning of self

As is appropriate in action research the researcher is appropriately placed as a ‘participant-observer’ throughout the course of the study. The position has a potential impact on the study and is well recognised in qualitative approaches, where the researcher is usually very involved with the participants in the study. Understanding this positioning is of importance in relation to the outcomes of the study, as the researchers personal “values, assumptions, beliefs, and biases” (Mertens, 2009, p.247) will be reflected in their observations, analysis and interpretation of the study.

Therefore, in this type of research there is considerable interest in “who the researcher is and what values, assumptions, beliefs, or biases her or she brings to the study” (Mertens, 2009, p.247). In the context of this study the researcher has a decade of experience as a mathematics educator, in Australia and Saudi Arabia. This experience has given the researcher insight into the learning strategies of students, particularly ESL students. Table 5.5 outlines the two case studies and the participatory roles the researcher had as part of each case study.

Table 5. 5 Positioning of Self

Position of Self: Participant – Observer	
Case Study 1	**Case Study 2**
King Abdul Aziz University , Saudi Arabia	**University of Wollongong College, Australia**
Researcher Modified notes. Observed students learning and understanding of various teaching methods. Tutor for Math 132 Interviewed staff – one lecturer and one administrative officer. Analysis of final marks. Gathered insight into how various teaching methods were performing with students.	**Researcher** Modified notes and teaching material for Advanced Math 1 & Math 2. Observed students learning in the classroom setting. Gave a presentation explaining WE and PS for students. Interviewed staff- 2 lecturers Analysis of final marks.

According to Mertens (2009), qualitative researchers, in particular those classified as participant observers, should take into consideration the environment as outlined in Table 5.6. Such consideration will give insight into the study that may go beyond what is directly expressed by participants, but may be relevant to the purpose of the study.

Table 5. 6 Additional considerations for researchers

• The program setting, that is the physical environment within which the program takes place;
• The human and social environment, that is, finding ways in which the people organise themselves into groups and subgroups;
• The program activities and participant behaviours;
• Informal interactions and unplanned activities;
• The native language of the program participants, that is, the observer should learn the exact language used by the participants to describe their experiences;
• Nonverbal communication;
• Unobtrusive measures; and
• Observing what does not happen, that is, the observer should take note of the things that are expected to happen but did not happen in the program.

(Adapted from Mertens, 2009, p.383)

5.5.4 Experimental design

The quantitative research approach is advantageous, as the research can be controlled, and replicated as need be, making it more commonly used in the fields of mathematics and science (Bryman, 2006). In order to determine the effectiveness of the two conditions, worked examples and problem-solving, an experimental within-subject design was implemented with all students learning all topics. The method for teaching topics was manipulated in terms of which method, worked examples, or problem-solving methods and in the second implementation faded worked examples was used. One of the advantages of such a within-subject design is that each student is effectively their own control, receiving both problem-solving and worked examples. In this way the ethical considerations typical of assigning a control group where there may be advantages or disadvantages to any individuals or groups based on their inclusion in either a control or an experimental group (De Vaus, 2006) are overcome.

In both case studies there were three cohorts of students (2010, 2011, and 2012) with three phases (Baseline, Implementation 1, and Implementation 2). Teaching is described as traditional, worked examples, problem-solving and faded worked examples. In each case students had a specified number of lectures hours (2 KSA and 4 UOWC), practical classes (1, KSA and 2 UOWC), homework sheets, and assignments. In 2010, students were taught traditionally, 2011 through worked

examples for three topics and problem-solving for three topics (refer Table 5.8), while in 2012 students were taught with worked examples (2 topics) problem-solving (2 topics) and faded worked examples (2 topics). The major difference between the two case studies was the time allocated to the subject, 3 hours per week for KAU and 6 hours per week for UOWC.

The *traditional method* basically means students were given theory and demonstrations in lectures, in practical classes they were given between 2 and 6 demonstrations per week with students completing the remaining problems-solving questions provided on a practical sheet, with homework an additional problems to solve and assessment conducted on each topic. They were given demonstrations followed by problems to complete in class and further problem to work at homework. The differences between the KAU and UOWC, The UOWC students do more problems than KAU students, with less homework. The breakdown of examples demonstrated, or set for completion in practical class, homework and assessment on each topic is provided in Table 5.7.

Table 5. 7 Baseline KAU 2010: number and type of examples demonstrated

Topic	Lecture Dem	Practical Dem	Homework PS	Assessment PS	Total
KAU: Functions	2	3	2	1	8
UOWC: Functions	4	6	-	2	12
KAU: Exponents	3	3	2	1	9
UOWC: Exponents	4	5	1	3	13
KAU: Quadratics Equations	3	3	3	2	11
UOWC: Quadratics Equations	5	6	-	2	13
KAU: Logarithms	2	4	3	2	11
UOWC: Logarithms	4	6	2	3	15
KAU: Geometry	3	2	2	1	8
UOWC: Geometry	5	4	1	2	12
KAU: Intro to Statistics	2	3	2	1	8
UOWC: Intro to Statistics	5	6	-	2	13

The *worked examples approach* meant in lectures the lecturer provided all theory and did intermixed theory with worked examples; in practical classes students were provided with a sheet of similar examples to those shown by the lecturer to solve

during the tutorial class to prepare them for assessments with similar questions, while in practical class, the tutor did worked examples the entire class explaining as they did to the students the theory of the topic. For homework students were give problems to solve. For assessment students were given problems to solve that were similar to what they have seen in the class. In the worked examples condition the lecturer must give instruction on how to correctly get to the final answer (see examples for worked examples in chapter 4 which would result in minimising the cognitive load on the students as described in chapter 3).

The *problem-solving approach* meant, in lectures the lecturer gave students theory and demonstrations as per the baseline. In practical classes students were provided with a sheet of problems to solve. They could ask questions of the lecturer but the lecturer did not formally demonstrate solutions or complete worked solutions (making explicit the steps). They were to solve problems independently by themselves a situation that should involve the highest cognitive load because of the difficulty of mathematics and the language as discussed in chapter 3. For homework they were given problems to solve as homework with solutions provided after the homework was completed. For assessment students were given different problems to solve.

The *faded worked examples* approach meant, in lectures students were given theory worked examples and then incomplete worked examples where students were asked to complete the steps to the final answer (missing one step). In practical classes they were provided with a sheet of worked examples with missing steps to complete solutions (missing 2 steps), for homework students were given problems to solve. For assessment they were given problems to solve. The faded worked example is a first step, to move the students from being dependent on their lecturer to providing them the complete worked solution to being independent which will push them forward to being problem solver by themselves. This method is to gradually build their confidence to be mathematical problem solvers which is the really aim of the tertiary studies. The cognitive load with faded worked examples approach should be less than for the problem-solving approach. The use of faded worked examples is thought to be a way to transition from worked examples to problem-solving.

The method of teaching in each phase can be characterised by the nature and number of examples or problems provided to students to be completed in both case studies. The provision of these are summarised in Table 5.8 and Table 5.9.

Table5. 8 Implementation 1-2011: Number WE and PS each case study

Topic	**Lecture WE**	**Lecture PS**	**Practical WE**	**Practical PS**	**Home PS**	**Assess PS**
KAU: Functions	3	0	2	0	2	1
UOWC: Functions	6	0	4	0	-	2
KAU: Exponents	0	3	0	3	2	1
UOWC: Exponents	0	4	0	5	1	3
KAU: Quadratics Equations	3	0	3	0	3	2
UOWC: Quadratics Equations	5	0	6	0	-	2
KAU: Logarithms	0	2	0	4	3	2
UOWC: Logarithms	0	4	0	6	2	3
KAU: Geometry	3	0	2	0	2	1
UOWC: Geometry	5	0	4	0	1	2
KAU: Intro to statistics	0	2	0	3	2	1
UOWC: Intro to statistics	0	5	0	6	-	2

*Appendix A

PS		WE	

Table 5. 9 Implementation 2-2012: Number WE, PS and FWE for studies

Topic	Lec[a] WE	Lec PS	Lec FWE	Prac[b] WE	Prac PS	Pra[c] FWE	Home[c] PS	Assess[d] PS
KAU: Functions	0	3	0	0	2	0	2	1
UOWC: Functions	0	6	0	0	4	0	-	2
KAU: Exponents	3	0	0	3	0	0	2	1
UOWC: Exponents	4	0	0	5	0	0	1	3
KAU: Quadratic Equations	0	3	0	0	3	0	3	2
UOWC: Quadratics Equations	0	5	0	0	6	0	-	2
KAU: Logarithms	2	0	0	4	0	0	3	2
UOWC: Logarithms	4	0	0	6	0	0	2	3
KAU: Geometry	0	0	3	0	0	2	2	1
UOWC: Geometry	0	0	5	0	0	4	1	2
KAU: Intro to statistics	0	0	2	0	0	3	2	1
UOWC: Intro to statistics	0	0	5	0	0	6	-	2

PS		WE		FWE	
a = Lecture, b = Practical, c = Homework, d = Assessment					

The manipulations of treatment conditions, whether topics were taught with worked examples, problem-solving or faded worked examples is summarised in Table 5.10, with subsequent analysis of the associated topic marks and the overall mark in the final examination. Having collected baseline data a between group comparison allows examination of whether or not the problem-solving and worked examples techniques are better than the traditional techniques employed. The design is weak in the sense that one cohort of students may be different to the other given that no randomisation has taken place in allocating students to groups, but is strong in that students were within a year there own control, completing all topics.

Table 5. 10 Teaching methods used for each implementation and topic

Phases	Year	1 Functions	2 Exponents	3 Quadratic Equations	4 Logarithms	5 Geometry	6 Intro to Statistics
Baseline	2010	Both (Demonstrations, PS)					
Implementation 1	2011	WE	PS	WE	PS	WE	PS
Implementation 2	2012	PS	WE	PS	WE	FEW	FWE

WE= worked examples, PS= problem-solving, FWE=Faded worked examples

For implementation 1 the topics taught by the two methods are alternated, with all students learning all topics and thus providing control, one cohort is not better or worse than the other. Under the design in implementation 1 the differences between problem-solving and worked example approaches could be due to the difficulty of the topics that received worked examples and problem-solving. To address this unavoidable flaw in design, in the second implementation the topics receiving worked examples and problem-solving were swapped. In this way both methods had the harder topics, both the easier ones.

In addition to the collection of examination marks and individual topic marks, the study collected data in each iteration, from questionnaires and interviews of staff evaluating their experience with using worked examples and problem-solving as teaching methods. The use of questionnaires and interviews as a method of investigation is widely applied in the social sciences (Gordon, 2000), and has also been used pervasively in educational and psychological research (Mertens, 2005).

The researcher employed two formats for the instruments: 1) Survey. 2) Interviews. The following sections describe their development in more detail.

5.5.5 Survey and Interviews

A survey is a method of data collection using questionnaire or interviews in this case from students in classes that had been selected to represent a population to which the findings of the data can be generalised (Fontana & Frey, 2000). The survey is one of the more commonly used methods in quantitative research, particularly in the social sciences (Crabtree & Miller, 1992; Creswell, 2003; Mertens, 2009). According to Glasow (2005), the use of a survey questionnaire is but one form of data collection that may be used in the effort of gathering information about characteristics, actions, opinions. Other methods of survey research can be in the form of interviews, content analysis and observations (Gordon, 2000). Utilising surveys is considered to be an efficient way of collecting data from a large number of respondents, accurately representing a whole population (Gordon, 2000), although in this case the questionnaires have been used within the confines of two case studies.

Questionnaires and interviews were used to collect data from students and the lecturers as illustrated in Figure 5.3.

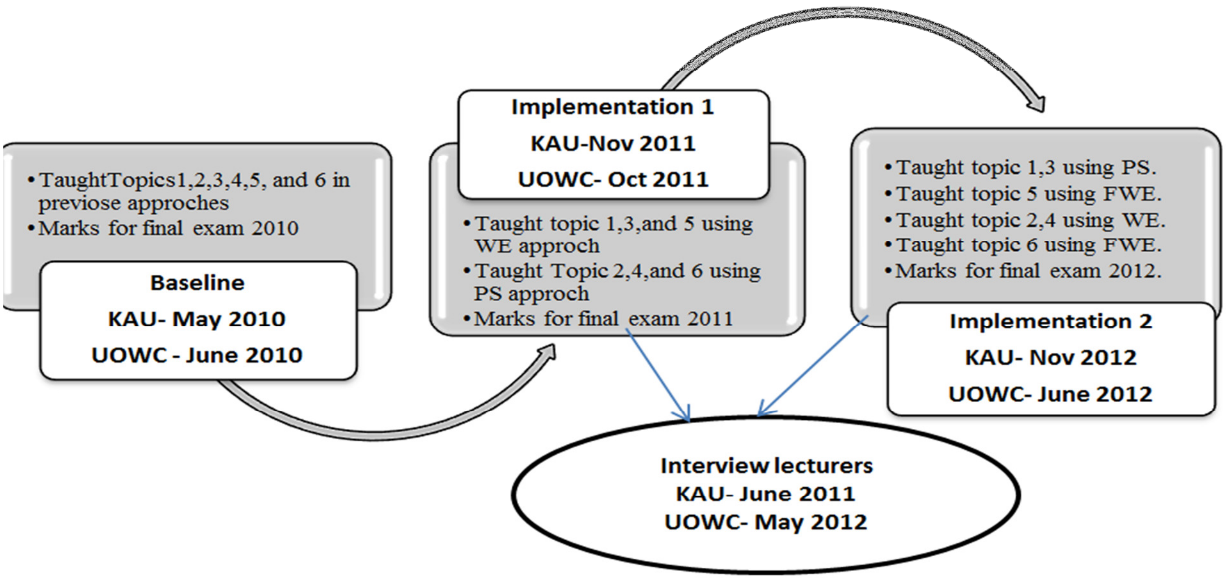


Figure 5.3 An overview of the three research stages for both case studies

The questionnaire (Section 5.6.2) used in this study included structured open and closed-ended written items, including measurement by Likert scales. Questionnaires were distributed to participants in both case studies on three separate occasions: at the end of the baseline implementation prior to any manipulation of teaching methods; after the redesign wherein topics were taught by either worked examples or problem-solving approaches in Implementation 1; and after the third iteration when students were taught by worked examples or problem-solving or faded worked examples in Implementation 2. Each implementation involved a different class of students and hence there are six different cohorts, three for each case study. There was no random sampling of students; the entire student cohorts were participants. The aim of the survey was to collect data on the effect of the design intervention on the teaching, learning, and assessment of participants throughout their mathematics courses. To increase the ability to generalise findings, data collection involved students from two entirely different ESL contexts.

5.6 Data collection tools

The study involved three primary data collection tools, staff interviews, student questionnaires and examination results.

5.6.1 Staff Interviews

According to Patton (1990), the main purpose of an interview (Appendix B) is to decipher what respondents feel, think, and believe. It is utilised to provide fundamental insights into motivations and factors that affect people's attitudes, preferences and behavioural patterns. The advantages of the interview technique is that it allows for immediate and direct interaction with the interviewee (Allan & Skinner, 1991). Interviewing is a favoured and effective method of obtaining primary data. Interviewing has the potential to provide more significant data from the research participants, because they are verbally articulating the information in response to the questions asked by the researcher. Bell's (2005) recommendations were to allow the respondent the freedom to talk about what they deemed important during the interview. Therefore, it can also promote more consistent understanding and clarification because the researcher can then engage in probing and deeper lines of questioning (Kervin, 2006). This view is supported by Burns' claim that that

"interviews are essential, as most case studies are about people and their activities … (and that interviewees) … provide important insight and identify other sources of evidence" (2000, p.267).

In this study two sets of interviews were conducted. One set of interviews was conducted with four students as part of the process of questionnaire validation with students. This pilot of the questionnaire was to test if the language is clear and questions are unambiguous (Section 5.6.2.1).

The second set of interviews was conducted with lecturers (n=4, 1 lecturer and 1administrator from KAU, both proficient English speakers, and 2 lecturers from UOWC, both native English speakers) to ascertain their experience teaching with worked examples and problem-solving approaches. The interview process with lecturers is outlined in Table 5.11.

Table 5. 11 Interview Process for Staff

• Interviews were conducted in English, in both countries.
• All participants were either native English speakers or were proficient in English as a second language.
• Each interviewee was asked seventeen open-ended questions relating to the study.
• The duration of each interview ranged from 30 minutes to 60 minutes.
• Interview participants were administrators or lecturers of the mathematics subjects.
• All lecturers and administrators were interviewed individually in their offices. All interviews with teachers and the administrator were digitally recorded.

The interview was developed to cover three important areas: prior experience teaching with worked examples and problem-solving; experience during the lecture series; suggested revisions after the lecture series. The staff members were told:

> The purpose of these interviews is to evaluate the effectiveness of the worked examples or problems-solving which are used for teaching mathematics at university level for ESL Students.

1) Prior experience

In this section the lecturers were asked to detail their experience in preparing lessons with worked examples and problem-solving (refer Table 5.12).

Table 5. 12 Structured interview, part 1

1. What is your experience with teaching using worked examples method?
2. What is your experience with teaching using problem-solving method?
3. How does the preparation time for the lecture with worked examples compare with problem-solving methods?
4. How does the preparation effort for lecture with worked examples compare with problem-solving methods?
5. Does the teaching method (WE or PS) effect the operation of your classes?

2) During the lecture series

In this section (refer Table 5.13) the lecturers were asked to give their opinions based on their experience in using worked examples and problem-solving during their classes.

Table 5. 13 Structured interviews of lecturers, part 2

6.	Have you had any discussion with students on their preferred method of study?
7.	Which way do you prefer to teach your students mathematics at university course? Why?
8.	What do you observe when students are taught with WE?
9.	What do you observe when students are taught with PS?

3) Suggested revisions after the lecture series

After having implemented the worked examples and problem-solving for different topics in their teaching of mathematics the staff were asked to discuss their experiences of the two methods (refer Table 5.14).

Table 5.14 Structured interview of lecturers, part 3

10. Can the worked examples method be improved to help your teaching of mathematics? If so, how?
11. Can the problem-solving method be improved to help your teaching of mathematics? If so, how?
12. What do you see as the main advantages of the worked examples method in teaching in your subject?
13. What do you see as the main advantages of the problem-solving method in teaching in your subject?
14. What do you see as the main disadvantages of the worked examples method in teaching in your subject?
15. What do you see as the main disadvantages of the problem-solving method in teaching in your subject?
16. Do you have any suggestions about using the worked examples method in teaching?
17. Do you have any suggestions about using the problem-solving method in teaching?

5.6.2 Questionnaire for students

According to Polit *et al.* (1991), a questionnaire is a tool for gathering self-reported information from respondents regarding their attitudes, knowledge, beliefs and feelings. There are numerous advantages to using a questionnaire as a mode of gathering data. Some of the key advantages are: firstly, the standardised format makes delivery easy; secondly, it is a quick and efficient way to collect data; and thirdly, it allows for surveying a large sample size in minimal time.

The method for gathering data regarding students' experiences of learning via the methods, worked examples, faded worked examples and problems-solving in this study is the questionnaire.

5.6.2.1 Questionnaire Development

The process of questionnaire development drew on:

- The examination of other researchers' questionnaires relevant in terms of measurement Students' self-rating through questionnaires, where, students report how much mental effort they believe a set task has required (Brünken et al., 2003; Wendell et al., 2007) to the present study.

- The questions were initially checked for wording, grammar, by two experts.
- The first administration of the questionnaire was in the form of an interview of four students who had English as a second language (i.e., for the purpose of a pilot test, to refine the question wording).

The survey (Appendix C) was the same for both cases studies (KAU & UOWC) and was developed into seven components:

1. Students' language capacity, comprehension, mathematical competence and background;
2. Evaluation of learning resources;
3. Experience of worked example and problem-solving teaching methods;
4. Overall impression of the mathematics course using problem-solving and worked examples in their learning objectives and confidence on topics;
5. Evaluation of using worked examples and problem-solving and in the final questionnaire faded worked examples used in this subject;
6. Demographics; and,
7. Self-report of their grades for first, last assignment, their expectations of the final exam.

The questionnaire was given to three classes of students in each country. In each case the questionnaire was completed at the end of the formal teaching session after having firstly provided students with the participant information sheet and consent form. When given the questionnaire to complete, administered to students during a break between lectures, they were then instructed as follows:

> The purpose of this questionnaire is to provide feedback that can assist in the development of these subjects for future students. Feedback of ALL students those who like the subject and those who do not like it, is essential in this process. You can let us know how to improve the subject so that you or future students can learn better.

The following sections provide details for each of the seven components of the questionnaire:

Section 1: students' language capacity and capacity in relation to mathematics

The main purpose in this component of the question (refer Table 5.15) is:

- To collect details on their first language, background, ability in mathematics and in English;
- To determine the students' perceived level of ability in mathematics (Alharbi, 2012);
- To determine the students perceived level ability in mathematics when learning in English.

Table 5.15 Questions regarding language ability

Q 1. Is your first language? 1. Arabic 2. English 3. Other language (Please specify)….. Q2. Is your background prior to university? 1. Mathematics 2. Science 3. Arts 4. Other (Please specify)………
Q3. How many years have you learnt mathematics using English? 1. Primary School 2. High school years 7-10 3. High school year 11 4. High school year 12
Q4. How would you describe your ability to do mathematics? 1. Very poor 2. Poor 3. Fair-good 4. Very Good
Q5. How would you describe your ability to do mathematics in English? 1. Very poor 2. Poor 3. Fair-good 4. Very Good

Section 2: Evaluation of learning resources

This set of questions drew on the work of Morris (2008) who examined the perceived usefulness of resources in terms of helping students to understand their work. The main purpose of this component is:

- To determine the perceived usefulness of the learning resources available to students;
- To identify which resources other than worked examples are preferred by the students;
- To allow an assessment of whether or not the learning environment is supportive of student learning (refer Table 5.16).

Table 5. 16 Questions regarding evaluation of learning resources

Q.6. How useful are your existing resources in helping you understand this subject	Rarely used this resource	Little use	Moderately useful	Extremely useful
a. Lecture	1	2	3	4
b. Work in Practical classes	1	2	3	4
c. Tutor in Practical classes	1	2	3	4
d. Practical Worksheets	1	2	3	4
e. Tutorial assignments	1	2	3	4
f. Lecture Handbook	1	2	3	4
g. Worked examples	1	2	3	4
h. Team learning or group work	1	2	3	4
i. Theory review in practical classes	1	2	3	4
j. Interaction with lecturer	1	2	3	4

Section 3: Provision of worked examples and problem-solving

Building on work Wendell *et al.* (2007) who explored rating scale techniques are based on the assumption that people are able to introspect on their cognitive processes and to report the amount of mental effort expended. Although self-ratings may appear questionable, it has been demonstrated that people are quite capable of giving a numerical indication of their perceived mental burden. Moreover, it has been demonstrated that such scales are sensitive to relatively small differences in cognitive load and that they are valid, reliable, and un intrusive, students' self-rating through questionnaires, where, students measuring how much effort they believe a set task has required. The main purpose for these questions was:

- To allow evaluation of the use of WE & PS approaches;

- To determine the differences in the provision of WE & PS from the student's perspective;

- To determine from a student perspective how the lecturers teach their students based on the average of how many WE or PS students complete per week (refer Table 5.17).

Table 5.17 Questions about the use of worked examples and problem-solving

Q7. On average how many problems have you completed per week in this subject class using worked examples as a guide? 1) 0 examples 2) 1-2 examples 3) 3-4 examples 4) 5 or more examples
Q8.On average how many problems have you solved per week without worked examples? 1) 0 examples 2) 1-2 examples 3) 3-4 examples 4) 5 or more examples
Q9. Does using the Worked Example approach improve your study for this subject? 0) No 1) Yes If your answer is "Yes". Please explain how? If "No" explain why not?
Q10. Does the Problem-solving approach without Worked examples improve your study for this subject? 0) No 1) Yes If your answer is "Yes". Please explain how? If "No" explain why not?
Q11. How would you prefer to study this subject? And why? And explain why you prefer this? 1) Worked examples 2) Problem-solving 3) Mix worked examples and problems-solving
Q12. Do you have any suggestions as to how the worked examples could be improved?
Q13. Do you have any suggestions as to how the problems-solving could be improved?
Q14. Is there a better way of setting the worked examples or problem-solving that would motivate you to learn more?

Section 4A: Satisfaction with worked examples and problem-solving learning resources

The main purpose for these questions was:

- To determine the level of satisfaction of students with the provision and use worked examples;
- To determine the level of satisfaction of students with the provision and use of problem-solving questions (refer Table 5.18).

Table 5. 18 Questions about satisfaction with learning resources for WE and PS

Q15.How satisfied were you with the (WE and PS) in:	**Not Satisfied**	**Slightly Satisfied**	**Somewhat Satisfied**	**Satisfied**	**Very Satisfied**	**Not Applicable**
a. (WE) provided to you before or during class.	1	2	3	4	5	6
b. (PS) provided to you before or during class.	1	2	3	4	5	6
c. The variety of (WE).	1	2	3	4	5	6
d. The variety of (PS).	1	2	3	4	5	6
e. The lesson in terms of them being (WE) easy to understand.	1	2	3	4	5	6
f. The lesson in terms of them being (PS) easy to understand.	1	2	3	4	5	6
g. The lesson in terms of them being (WE) interesting.	1	2	3	4	5	6
h. The lesson in terms of them being (PS) interesting.	1	2	3	4	5	6

Section 4B: Confidence in Topics

Building on the work of Morris (2008) who determined that student's self-ratings of confidence were associated with performance. It is this relationship between ratings of confidence and performance for particular topics that sets it apart from the next section on experience. The main purpose for these questions was:

- To determine the students' confidence on solving-problem with different topics as displayed at Table 5.19.

Table 5.19 Confidence in Topics

16. How confident are you now that you can solve problems in the following topics?	**Not at all**	**Might have a little difficulty**	**Moderately confident**	**Could do this**
Function	1	2	3	4
Exponents	1	2	3	4
Quadratic Equation	1	2	3	4
Logarithms	1	2	3	4
Geometry	1	2	3	4
Introduction to Statistics	1	2	3	4

Section 5: Students' experience of worked examples and problem-solving.

This work built on the work of Baharun (2009) that examined students' perceptions of learning outcomes, in particular enhancing understanding and anxiety. Preis and Biggs (2001) stated that negative experiences in mathematics may result in poor performance. Moreover, Arul *et al.* (found that students who have positive experience in mathematics strong mathematics background) were less anxious and performed well in mathematics. Mathematics anxiety hence appears to be the main reason for low performance in mathematics (Arul et al., 2004). The main purpose of these questions was:

- To determine the differences between students in terms of their perceptions of confidence, anxiety, mathematics being made interesting, easy to learn;
- To determine from the student perspective, the level of improvement of student's mathematics ability, if they used worked examples approach;

- To determine from the student perspective, the level of improvement of student's mathematics ability, if they used problem-solving approach (refer Table 5.20).

Brunken *et al.* (2003) found that students' self-rating through questionnaires, where, students report how much mental effort they believe a set task has required is way of measuring cognitive load in the given task. So, these questions also addressed ideas associated with cognitive load, the ease of learning and mental effort involved in learning.

Table 5. 20 Questions regarding the WE and PS (based on students' experiences)

Q17.I believed that:	Strongly Disagree	Disagree	Mildly Disagree	Neither Agree or Disagree	Mildly Agree	Agree	Strongly Agree
a) Using worked examples enhanced my understanding in the mathematics tasks.	1	2	3	4	5	6	7
b) Using problem-solving enhanced my understanding in the mathematics tasks.	1	2	3	4	5	6	7
c) Using worked examples made it quicker to study mathematics.	1	2	3	4	5	6	7
d) Using problem-solving made it quicker to study mathematics.	1	2	3	4	5	6	7
e) Having access to worked examples improved my review of mathematics notes and lab work.	1	2	3	4	5	6	7
f) Having access to problem-solving improved my review of mathematics notes and lab work.	1	2	3	4	5	6	7
g) Using worked examples it is much easier to learn than solving-problems in Mathematics lesson.	1	2	3	4	5	6	7
h) Using problem solving it is much easier to learn than worked examples in Mathematics lesson.	1	2	3	4	5	6	7
i) Worked examples increases my confidence about solving more problems in mathematics.	1	2	3	4	5	6	7
j) Problem-solving increases my confidence about solving more problems in mathematics.	1	2	3	4	5	6	7
k) Using worked examples to learn mathematics requires a lot of mental and learning effort.	1	2	3	4	5	6	7
l) Using problem-solving to learn mathematics requires a lot of mental and learning effort.	1	2	3	4	5	6	7
m) Using worked examples makes mathematics learning more interesting.	1	2	3	4	5	6	7
n) Using problem-solving makes mathematics learning more interesting.	1	2	3	4	5	6	7
o) I like to learn mathematics by using worked examples.	1	2	3	4	5	6	7
p) I like to learn mathematics by using problem-solving.	1	2	3	4	5	6	7
q) Using worked example helps reduce my anxiety when learning mathematics.	1	2	3	4	5	6	7
r) Using problem-solving helps reduce my anxiety when learning mathematics.	1	2	3	4	5	6	7

Section 6: Demography of students

In terms of equity in the provision of education it is useful to know whether or not educational practices are equally applicable to different groups of students. The main purpose of these questions was to:

- Allow for examination of differences based of details on students' background and gender (refer Table 5.21).

Table 5. 21 Questions regarding student's demography

Q20. Indicate your origin:
1) International student
2) Domestic student
Q21. Indicate your gender:
1) Male
0) Female

Section 7: Self- reports about students' expectation for their grades and confidence

According to Morris (2008) self-reports, or expectations about grades may be used as an indication of students perceived ability and or confidence.

The main purpose of these questions was:

- To determine their expectations for the final result;
- To determine their mark for first assignment;
- To determine their mark for the last assignment (refer Table 5.22).

Table 5. 22 Questions regarding self- reports of students' expectation for grades

Q 23.	a. What grade do you expect to get for this subject?	/100
	b. What mark did you get for first assignment?	/100
	c. What mark did you get for last assignment?	/100

5.6.3 Measurement of Performance

The structure of final exam paper was kept the same for the three years (2010, 2011, and 2012) that is for the duration of this case study, allowing the researcher to test the effectiveness of the methods over several topics. Within this format questions were kept similar. The similarity in the questions, over those three years, for testing "functions" topic is evident in Figure 5.4.

Question 1 Exam-2010 **10 Marks**

Solve the following simultaneous equations by using the elimination technique.

(i) $x + y = 10$, $x - y = 2$

(ii) $2x + 3y = 17$, $4x - 3y = 7$

(iii) $x + 2y = 8$, $x + y = 3$

(iv) $7x + 3y = 10$, $11x - 6y = 5$

(v) $2x - 3y = 4$, $3x + 2y = 19$

Question 1 Exam-2011 **10 Marks**

Solve the following simultaneous equations by using the elimination technique

(i) $2x - 5y = 2$, $x = 3y$

(ii) $x + 4y = 2$, $y = 5 - x$

(iii) $x - y = 1$, $2x + y = 14$

(iv) $x + y = 3y$, $-2x = 15$

(v) $7 - x = 2y$, $x + 5y = 4$

Question 1 Exam-2012 **10 Marks**

Find the solution of the following systems by using the elimination technique.

(i) $x + 4y = 2$, $y = 5 - x$

(ii) $x - y = 1$, $2x + y = 14$

(iii) $x + y = 10$, $x - y = 2$

(iv) $x + 2y = 8$, $x + y = 3$

(v) $2x + 3y = -6$, $x - 4y = -3$

Figure 5.4 Sample final exam question for the three years: 2010, 2011, and 2012.

5.7 Ethical considerations

Research does involve collecting data from people, about people (Punch, 2005).

> [Hence, ethics approval] is required in making an argument for a study as well as being an important topic in the [study]. Researchers need to protect their research participants; develop a trust with them; promote the integrity of research; guard against misconduct and impropriety that might reflect on their organizations or institutions; and cope with new, challenging problems (Isreal & Hay, 2006). (Cited in Creswell, 2009, p. 87).

With data collected in two different countries the processes for ethics approval differed.

5.7.1 Ethics approval processes in Saudi Arabia (KAU)

Action research and participant observation in a classroom situation involves careful planning to address potential ethical issues. Obtaining the approval from the Human Research Ethics Committee at the University of King Abdul-Aziz was the first step before participant recruitment could commence. As this research involved students, a formal application was made to the Human Research Ethics Committee of King Abdul-Aziz University; written permission was obtained from the Head of the KAU unit, and all ethical norms were carefully adhered to.

Firstly, participation was completely voluntary, and no one was pressured to participate. Students were informed that data collected would be used for research purposes. A plain language statement was issued at the outset to all potential participants as to the purpose and method of study. Every participant signed a consent form prior to taking part in the study. Secondly, participants' names and identities were to be kept confidential and pseudonyms used for each participant during the data analysis. Thirdly, it was to be explicitly explained to the participants that none of what they said would adversely affect them. Fourthly, it was explained to the participants that they had the right to withdraw, at any time, during the study without any consequences. Fifthly, regulations about conducting research in the institution where the KAU operations had to be followed.

5.7.2 Ethics approval processes in Australia (UOWC)

In Australia, research conducted in or by public institutions such as universities and Government Departments involving human participants, must be approved by an accredited Human Research Ethics Committee (HREC). People do not need to be physically involved to be considered participants; HREC approval is necessary for research ranging from examination of records containing personal information, to anonymous surveys and medical intervention.

Permission to undertake two case studies was provided by the University of Wollongong and the Illawarra Shoalhaven Local Health District (ISLHD) Social Sciences Human Research Ethics Committee (HREC). As a graduate student enrolled at this university approval was required for data collections in both contexts. As for the ethics process in Saudi Arabia, key components of this approval involved, providing a participant information sheet and informed consent. Questionnaires and audio recordings were organised in digital folders and stored for later access. More information may be found at (http://www.uow.edu.au/research/ethics/human/index.html).

5.7.3 Participants' Recruitment

To start data collection the researcher had to contact the institutions, the KAU and UOWC, to secure access to their premises. Despite the fact that the researcher was part of the teaching staff in the KAU, permission had to be obtained before commencing data collection. A request letter was sent to the department in charge of research within the institution to seek permission to conduct data collection in the KAU on the designated dates. The researcher's request to start data collection was approved promptly. Individual personal meetings with the staff and a meeting with students informed them about the aim of the research and the procedures for data collection. At the meeting, each participant was given a copy of the consent form detailing the objectives of the study; the participant's role and benefits; as well as participant's rights. Signatures on the consent forms were obtained from the participants before data collection commenced. The researcher had presented several workshops and seminars to all the academic staff of the mathematics department at King Abdul-Aziz University and at the University of Wollongong College, regarding

the strength and importance of the area of study. A prepared workshop for Math132 students (UOW) and Advanced Mathematics 1 & 2 (KAU) explained the differences between worked examples and problem-solving.

5.8 Student Data Collection

The two case studies were conducted with different groups of students enrolled in similar mathematics subjects. The researcher was a *participant teacher* in the KAU case study and designer for the UOWC case study. The researcher declared his involvement and interest in the studies and overtly discussed with other lecturers involved any values, assumptions or biases resulting from his espoused theories of teaching and learning of mathematics. Preparation for the modification of lecture notes began with an analysis of the teaching materials and agreement from each of the lecturer's. This analysis was undertaken to determine the quantity, quality and placement of different types of worked examples. That is, they are assessed in terms of whether or not cognitive load could be reduced by different structuring of material.

Data collection was conducted in the KAU and UOWC over a 2 year period. For KAU, the data collection started with baseline data collected in May 2010. Data was collected for the Implementation (1) in May-Nov 2011; and Implementation (2) in May–Nov 2012; for the classes finishing in November at KAU. The researcher started data collection at UOWC with the baseline data collected in June 2010 and follow-up data collected for the two Implementation (1) in Jun-Oct 2011; and Implementation (2) in Mar-Jun 2012 (refer Table 5.23).

The design of the study involved the collection of baseline data over all topics as traditionally taught. The first implementation involved alternating the manner in which topics were taught between worked examples and problem-solving, while in the second implementation faded worked examples were introduced, as detailed in Table 5.10. The same pattern of alternating the teaching methods for each topic was kept consistent for the two case studies.

Table 5.23 Timeline of data collection from students for both cases studies

Case Study 1 KAU Saudi Arabia	Analysis of Teaching	Case Study 2 UOWC Australia
Baseline Class 1 May 2010	Analysis of students' previous academic records Examine existing teaching strategies. Draft Lecture/Tutorial material based on assigned curriculum for student to learn mathematics using the prescribed learning strategies.	Baseline Class 1 Jun 2010
Nov 2010	Student marks over all topics on the final examination.	Jun 2010
Implementation 1 Class 2 May-Nov 2011	Topics taught to students using WE or PS methods (see Table 5.5)	Implementation 1 Class 2 Jun-Oct 2011
Nov 2011	Follow-up questionnaire Review student marks2011 Revise materials for next implementation	Oct 2011
Implementation 2 Class 3 May-Nov 2012	Topics taught by PS and WE swapped Introduction of faded worked examples	Implementation 2 Class 3 Mar-Jun 2012
Nov 2012	Follow up questionnaires Student marks in final exam	Jun 2012

For both cases studies (KAU & UOWC) the strategy for data collection was the same. The completing students from the first class (Baseline) were asked to complete the student questionnaire, comparing the effectiveness of worked examples and problem-solving in learning mathematics. This provided baseline data including students' experiences of worked examples and problem-solving against which changes in teaching methods in the following session of study could be assessed. After the baseline gathering of data, changes were made to the teaching methods alternating worked examples and problem-solving strategies for the different topics taught. After the next intake of students had completed the subject (Implementation 1), a follow-up data collection was taken. The same questionnaire was used as that in the baseline class, this time examining student experiences of working with the teaching materials (in this case new materials) was gathered. This allowed the identification of any other issues that needed to be addressed in order to improve the

teaching of the subject. Student marks for each topic taught and overall marks were also collected together with their final examination mark. These follow-up data allow a comparison with the original baseline experiences of respondents and performance on topics. The within subject design in the first experimental implementation allowed a comparison of performance on problem solving topics and worked example topics and a comparison with performance on topics under baseline teaching conditions. Issues were examined before the redesign for the second implementation.

Two issues arose during the analysis of the first implementation. When comparing outcomes for worked example methods and problem-solving it was considered possible that certain topics may better suit worked examples rather than problem-solving. This led to a swap in methods used to teach topics in the second implementation. The second issue that was highlighted was the possibility that students could benefit if faded worked examples were used to scaffold from worked examples to problem-solving, so two topics were modified to include faded worked examples (refer Table 5.9). This change allowed the comparison of three methods worked examples, faded worked examples and problem-solving on the same topic and the comparison of the methods in different topics.

5.9 Conclusion

This chapter has provided a detailed description of the research methodology of this study. There is discussion about strategies of enquiry such as exploratory research, survey research and action research. Furthermore, the chapter outlines the research questions and a briefing on the on research design and paradigm selection. As participants are a crucial part in the study, recruitment, and ethical consideration have been briefly discussed, as have the methods of data collection. Explanation of data analysis procedures and results are presented in the next chapter.

6 CASE STUDIES AND RESULTS

6.1 Introduction

The study seeks to explore different modes of teaching mathematics to students studying in a second language, and the effects of the varying approaches on student outcomes such as educational achievement and confidence in learning mathematics. Two case studies were used to examine the effectiveness of using worked examples versus the problem-solving approach for teaching mathematics to ESL students.

In order to be better able to generalise the results, the case studies have been undertaken in two vastly different cultural and educational settings. This is to show if findings from one context are applicable in another and hence to be able to generalise results of the study more widely than if one case study were used.

The first case study involves a subject called Math132 at King Abdul-Aziz University (KAU), Jeddah, Saudi Arabia. In the case study at KAU students are in their home country, but are taught in their second language English. The second case study involves a subject called Advanced Mathematics 1 & 2 at UOWC, New South Wales, Australia. In the second case study, the students are international students learning mathematics in a foreign country and learning in a second language, English.

In both case studies the design of the study was the same with a baseline data collection, followed by an intervention with the same mix of topics taught by problem-solving and worked examples in Implementation 1. Following the first implementation the method of teaching assigned to topics was swapped and a third method, namely faded worked examples replaced one topic previously taught with problem-solving and one previously taught with worked examples, as described previously in Figure 5.7.

6.2 Saudi Arabia, Context for Case Study 1

In this case study the students are studying at the King Abdul-Aziz University in Jeddah and the selection of this case is inspired by the recent focus on education in the Kingdom of Saudi Arabia. To understand directions taken in modern education in the Kingdom of Saudi Arabia one needs to be aware of the religious and social culture of the nation, the need for economic growth, the nature of the population to be educated, population, and the education system.

6.2.1 Social and religious influences

Islam was not only born in Saudi Arabia, but is very much still alive today amongst its citizens. Home to the two Holy Mosques of Makkah and Medina, Saudi society is strongly influenced by Islam (Al-Saggaf, 2004; Alebaikan, 2010; Oyaid, 2009). Islam is regarded as not only a faith, but a comprehensive way of life, encompassing all facets of one’s personal, social, economic, and spiritual life (Al-Munajjed, 1997). Thus, it is of no surprise that Islam plays an integral role in defining culture, social norms and traditions (Al-Saggaf, 2004). Old Arab traditions before Islam, such as the domestic role of women still infiltrate modern day society, (Al-Munajjed, 1997; Alhazmi, 2010). Segregation is a normal way of life in Saudi Arabia. The free mixing of the sexes is not permitted legally for religious reasons. Generally speaking men and women who are not married or related are discouraged from mixing. Society accommodates segregation in all areas of education, business, public transport, and even in social situations such as restaurants (Al-Munajjed, 1997; Alhazmi, 2010). Furthermore, women are obliged to cover their face and body (*hijab*) when outside their home, and encouraged to avoid idle talk with unrelated men (Alebaikan, 2010). Thus segregation of the genders influences all aspects of life in Saudi Arabia, including education (Alebaikan, 2010; Oyaid, 2009).

6.2.2 Economy

The Kingdom of Saudi Arabia is synonymous with oil, and rightly so, as nearly 80 percent of government revenue is generated from crude oil sales (Kasser, 2011).

Today, Saudi Arabia supplies 28 percent of oil to OPEC (Organisation of Petroleum Exporting Countries) nations, and is a supplier of oil to other nations, making oil the number one export and industry of the Saudi Arabian economy. Economic growth was measured at five percent in 2011, and the private sector growths was four percent (Kasser, 2011). Such promising growth figures are a direct result of the high consumption needs of OPEC nations, and Saudi Arabia's ability to supply oil. The reality of the matter is that the economy is so heavily reliant on global markets it is at the mercy of the market. Although, prosperity and growth are the tune of the present, the past oil price shock of the 70's and 80's, wreaked havoc on the Saudi economy, and foreshadowed what may lie ahead when the precious natural resource of oil is expended completely.

With globalisation pressures and the impending exhaustion of natural resources, the Saudi government has implemented economic diversification programs. Saudi Arabia's Ninth Development Plan (2010-2014) provides for a comprehensive socioeconomic vision to be achieved by 2024. The plan provides an overall framework for development up to 2014 (Kasser, 2011). Such programs have been designed to ensure that the young population will have promising job opportunities in various industries across the nation, and given educational opportunities the ability to compete in the global workforce.

6.2.3 Population

The Kingdom of Saudi Arabia is home to nearly 26 million people in 2011. According to a national demographic survey in 2011, Jeddah, the site of this study, is the largest region with a population of over six million (Central Department of Statistics and Information, 2011). A surge in migrants has kept population growth figures on the upswing in additional to the noticeable increase in birth rates. Table 6.1 presents a comparison of the latest estimates for 2011, compared with those for 1995.

Table 6.1 Comparison of demographic indicators from 1995 and 2011

Demographic Indicators	Year 1995	Year 2011
Population		
Midyear population (in thousands)	18,755	26,132
Expatriates within population (in thousands)	5,576	6,250
Growth rate (per cent)	1.5	2.9
Fertility		
Total fertility rate (births per woman)	2.3	5.0
Births (in thousands)	505	568
Mortality		
Life expectancy at birth (years)	71	74

Source: U.S. Census Bureau, International Data Base

The Saudi population is relatively young with over sixty percent of the population under the age of 25, and an amazing twenty-nine percent (7.7 million) under the age of 15 in 2010 (Ministry of Higher Education, 2010a). With globalisation, and a young population, education is at the forefront of government policy and planning for future direction of the nation.

6.2.4 Education in the Kingdom of Saudi Arabia

The Kingdom of Saudi Arabia is emerging as an international heavyweight, as it is wealthy, with a young and promising population (Alebaikan, 2010). The Saudi vision for the future requires a well-educated and highly informed citizenry to provide leaders and professionals capable of bringing Saudi Arabia to the global forefront. This is the basis for the significant changes to educational policy (Alhazmi, 2010).

Changes to the educational system in the Kingdom of Saudi Arabia have been profound moving from a long established approach to one that is relatively new and challenging given previous tradition. Over fourteen hundred years ago, the Prophet Mohammed united the Arabian Peninsula, and established organised modes of learning in the establishment of small local schools, "*maktabs*", and mosque based educational facilities called *"madrasahs"*(Asimov & Bosworth, 1998) that continued until 1970. After the formation of the Kingdom of Saudi Arabia (1970), educational institutes were established and government provided universal education was made freely available to all residents as recently as 1975.

One of the challenges impeding educational growth, especially for women, is the segregation policy. As part of the movement to educate all Saudis, the inclusion of girls was met with resistance from the religious elite, and performance based entry to higher education has only come to fruition in 1980 (Ministry of Higher Education, 2006).As there are segregated schools and universities for males and females, there are staffing issues at female only universities, due to the shortage of female lecturers (Oyaid, 2009). It has been possible only in recent years for male lecturers to teach females via technology and in particular through videoconferencing systems.

This case study is inspired by the recent focus on education and surge in educational achievement over the last 20 years in the Kingdom of Saudi Arabia, which can be partially attributed to the pedagogical change in learning from traditional rote learning to explorative, and experienced based learning. Of particular interest as a case study is the change in the language of instruction from Arabic to English that has occurred in some secondary schools and all tertiary institutions in 2008, across the country including King Abdul-Aziz University.

6.2.4.1 Saudi Arabian educational history with English

Societal values permeate the curricula substructure and provide "conditions that operate to sustain and facilitate [these values] in every school in every subject within the curriculum" (Jackson et al., 1993, p.15-16). Such conditions are rarely "explicitly acknowledged by either teachers or students" (Jackson et al., 1993, p.16). The language spoken in the Kingdom of Saudi Arabia is Arabic. However, the English curriculum and enactment thereof in Saudi Arabia provides a number of "enabling conditions" (Al-Mohanna, 2010) that facilitate the cultural and religious values of the society, a notion supported by Saudi English teachers as they expressed their concerns regarding language and culture, indicating that it was difficult to separate the two (Elyas, 2008a; b). The historical apprehension of teaching English in Saudi schools was a result of the status accorded to English in Saudi society. A feeling of apprehension was felt across the board with the teaching of any foreign language at schools. There were only a select few 'madrassa' that included English as part of their curriculum, and they only had a few hours a week for high school students

(Szyliowicz, 1973). Primary schools introduced English language in 2003, only after significant international pressure (Elyas, 2008a; b).

Educational reforms have been a long time coming, due to international and national concerns. The pressure of global political events post 9/11 stalled the educational reforms of Saudi Arabia (Elyas, 2010) resulting in national concern regarding graduates ability to compete in the global economy. Educators were pushing for further development programs to explore a range of pedagogical practices to better meet the needs of their students. This was particularly alarming when statements released by teachers claimed that both verbal and written communication in not only English but Arabic as well was substandard for a university graduate, and analytical skills were also weak. Such awareness indicated that practices and mindsets were starting to change.

With an increased number of educators training abroad, and foreign nationals teaching in Saudi schools, and universities, new practices and pedagogical approaches are beginning to make their way into the classrooms. However, the overzealous adoption of Western educational practices sometimes results in local Saudi teachers feeling marginalised and resentful (Elyas, 2010). To overcome such negative undercurrents, and to reinstate a sense of uniqueness and empowerment to Saudi teachers, a mix of teachers was sought, including some who have studied abroad (Al-Mohanna, 2010). Such a process of developing higher education is to ensure that teachers' skills are updated with techniques and methodologies taught in many western schools, and further that students will be encouraged to become critical learners that observe, evaluate and question. Hybrid English language pedagogy is interconnecting traditional Islamic approaches with applicable Western practices in an attempt to create an approach that is both culturally balanced and educationally efficient (Elyas, 2010).

6.2.4.2 Tertiary development

Higher education in Saudi Arabia effectively commenced in 1949, when the College of Islamic Jurisprudence was established in Makkah. It was the first college in Saudi Arabia, and later became the University of Umm Al-Qura (Ministry of Higher Education, 2010b). Riyadh University, now named King Saud University, is the

oldest university in Saudi Arabia, established in 1957 with nine lecturers and 21 students (Alebaikan, 2010). With limited resources and community disinterest the progress of higher education was slow, with limited institutions and programs. Students aspiring to pursue professional qualifications were granted scholarships to study abroad in Europe, or the United States of America, UK, and Australia.

In the early 1970s, the government focused on establishing higher education to create more qualified professionals to service the countries socio-economic needs (Alebaikan, 2010). The Ministry of Higher Education established in 1975, is responsible for universities and institutes of higher education. The responsibilities of the ministry included:

- Establishing tertiary institutions and drafting curricula to meet the country's needs;
- Providing policy and practices for tertiary institutions; and,
- Coordinating communications between tertiary institutions and the public sector to focus on the country's requirements (Ministry of Higher Education, 2010b).

The Ministry of Higher Education is responsible for all aspects of higher education including vocational and technical training schools. All Saudi universities are funded completely, with no tuition fees. Since 1993, all higher education students receive an allowance, currently between SR 700 to SR 1000 (Average $AU200) a month in a country where the average monthly wage is SR800 (Ministry of Higher Education, 2012). Data collected from various public and private universities and colleges shows a high rate of growth for male and female tertiary students over the period 2000 to 2009 particularly at the higher levels of study (refer Table 6.2). The growth is especially marked for female undergraduates, the numbers nearly doubling over the nine-year period. Policy changes are reflected in the number of students undertaking their master's and doctoral qualifications.

Table 6.2 Tertiary student growths, 2000-2009

Qualification	**2000**			**2009**			**Growth Percentage**	
	Males	Female	Total	Male	Female	Total	Male	Female
Diploma	19,783	18,469	38,252	26,173	23,494	49,667	32	27
Bachelor	143,925	180,498	324,423	214,303	387,062	601,365	49	214
Masters	979	637	1,616	2,849	1,811	4,660	291	284
PhD	49	129	178	498	249	747	1016	193

Source: Adapted from Ministry of Higher Education (2010a, p. 13)

6.2.4.3 Higher education policy

Segregated universities are still prominent with King Fahd University and the Islamic University admitting male students only, while Princess Nora bint Abdurrahman University only admits females. Other Universities provide segregated undergraduate and graduate degrees. As recently as 2005, the Ministry was focused on establishing specialised faculties providing work ready qualifications in areas such as health, engineering, and management (Ministry of Higher Education, 2010a; b).

As part of the Ministry's five year development plan, a comprehensive review of the last two plans was undertaken, with dramatic changes made to ensure that Saudi Arabia's educational policy was at the forefront of the Arab and Western world with overhauls to pedagogical practice (Ministry of Higher Education, 2010a; b). Resources and funding were poured into education, to ensure that the new higher education policy of English tuition introduced in 2008 was not only a reality but, a success for tertiary level students and graduates.

6.2.4.4 King Abdul-Aziz University

King Abdul-Aziz University was established in 1967 as a national university and to support students who live in Jeddah. The University opened its doors in 1968, providing only preparatory courses for the few enrolled students. In 1974, the University was included as part of the government universities.

King Abdul-Aziz University is spread across two main campuses, one for males the other for females. Both campuses were inaugurated in 1975, the same year the New

South Wales Parliament incorporated the University of Wollongong, site of the second case study, as an independent institution of higher learning. Each campus of King Abdul-Aziz University is fully equipped with all cultural, recreational, and athletic facilities. Libraries on both campuses are fitted with state of the art equipment to serve students and teaching staff. The university is continually improving its programs with the development of scientific and theoretical fields of study including specialisations such as Seas Sciences, Nuclear Engineering, Medical Engineering, and Mineralisation. Such specialisations will further the opportunities to Saudi students. Distance programs are available enabling education for all students.

In the last 40 years, King Abdul-Aziz University established itself to be among the most distinguished Universities of the Kingdom and around the world, ranking 340 in 2012 while UOW in the same year ranked 100 on the QS (World University Rankings are annual university rankings published by Quacquarelli Symonds). All the additional programs and specialisations and outreach facilities, have been designed to give the best education to the Saudi population in the hope that this will broaden their prospects in the global job market, and give the nation the edge it needs to keep pace with the rest of the world (Alebaikan, 2010).

6.2.4.4 Subject chosen, Math132

The subject used to test teaching innovations in this study was Math132. It is offered to undergraduate students through the Mathematics Department at the University of King Abdul-Aziz. Math132 was chosen, as the curriculum was similar to the second accessible subject in Case Study 2 at the UOWC. Math132 is a core requirement for all students who enter the Science Faculty. The subject is designed to give students basic mathematical knowledge, especially students who are interested in continuing their career or study path in mathematics. Math132 is structured so as to have one block of three hours per week in total class time. Two hours for a lecture, and a one-hour tutorial in each block. The subject is divided into six areas sequenced over thirteen weeks as follows: *functions*, *exponents*, *quadratic equations*, *logarithms*, *geometry* and *introduction to statistics*. Teaching weeks are followed by one week of study recess and one week for examinations. The students who take this subject

come from various levels of mathematical backgrounds. Following the policy of the university the Arabic speaking students are taught mathematics in English.

6.2.5 Statistical tests

Several simple statistical tests have been used on multiple occasions throughout the two case studies:

1. **Pearson's Chi-Square Test:**

A Chi-Square test is used to determine whether there is a significant difference between the expected frequencies and the observed frequencies in one or more categories. The data are typically represented in tabular form as a two-way or contingency table. It can be used to test:

a. For a difference in two proportions (an alternative to the test for differences in proportions);

b. For independence (Mwitondi, 2012).

Expected counts for the cell in the ith row and jth column is given by $E_{ij} = \frac{R_i \times C_j}{n}$.

R_i= ith row total

C_j= jth column total

n = grand total

The chi-square is the sum of the squared difference between observed (*O*) and the expected (*E*) counts (or the deviation, *d*), divided by the expected counts. When conducting a chi-square test it is assumed that the expected frequency in each cell is greater than five. In many instances in this analysis, two or more response categories are often combined to form one categories for example, (strongly Agree and Agree) to obtain expected counts that are five or more.

$$x^2 = \sum_{i,j} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

i=1,….r

j=1,….c

The null hypothesis is rejected when $x^2 \geq x^2{}_{\mathrm{df},\,\alpha}$

df=(r-1)(c-1)
r= number of rows
c= number of columns

When this assumption regarding the expected counts is violated a Fisher's exact test can be used. It is also assumed that data are nominal in type rather than ordered categorical data. When a significant chi-square result is determined for ordered data the decision holds, however when there is no significance detected analysing ordered categories using some form of ordinal modelling is more appropriate as in these instances chi-square does not take into account the shift of data across categories.

2. **Fisher's Exact Test:**

Fisher's exact test is a statistical significance test used in the analysis of contingency tables. Although in practice it is employed when sample sizes are small, it is valid for all sample sizes. When the response variable is recorded using counts, however, x^2 test may be employed. But when the number of observations obtained for analysis is small, thex^2test may produce misleading results. "A more appropriate form of analysis (when presented with a 2 x 2 contingency table) is to use Fisher's exact test" (Bower. 2003).

The test is useful for categorical data that result from classifying objects in two different ways; it is used to examine the significance of the association (contingency) between the two kinds of classification (Mehta, 2009).The probability (*p*) of observing a given set of frequencies a, b, c, and d in a 2 x 2 contingency table, given fixed row and column marginal totals and sample size *n*, is:

$$p = \frac{\binom{a+b}{a}\binom{c+d}{c}}{\binom{n}{a+c}} = \frac{(a+b)!\,(c+d)!\,(a+c)!\,(b+d)!}{a!\,b!\,c!\,d!\,n!}$$

When the 2x2 table is:

	Variable 2		
Variable 1	a	b	a+b
	c	d	c+d
	a+c	b+d	n

The null hypothesis for the test is that there is no association between the rows and columns of the 2 × 2 table, such that the probability of a subject being in a particular row is not influenced by being in a particular column. If the columns represent the study group and the rows represent the outcome then the null hypothesis could be interpreted as the probability of having a particular outcome not being influenced by the study group, and the test evaluates whether the two study groups differ in the proportions with each outcome. The null hypothesis is that there is no difference in the proportions between study groups. Evidence for rejecting the null hypothesis occurs for smaller the value of p, usually smaller than 0.05.

An important assumption of Fisher's exact test, is that the binary data are independent. If the proportions are correlated then more advanced techniques should be applied (Mehta, 2009).

An exact p-value is the exact probability of observing a table at least as extreme as the observed one, under the null hypothesis. However, in 2×2 tables this probability typically depends on one or more unknown parameters, such as the common success probability in comparing two binomials in the one margin fixed design (Mehta, 2009).

On occasion in the analysis of the case study data, when the numbers of subjects is low and several tests for differences in proportion would typically be used, on these occasions a sign test could be used to test.

3. Sign Test:

A sign test is a non-parametric or distribution free test. The test statistic is expected to follow a binomial distribution, the standard binomial test is used to calculate

significance (Kitchens, 2003). A sign test can be used when you have one sample of subjects with some measure repeated on each subject (e.g. before and after scores) and you cannot use a repeated measures t-test (because one of the assumptions of the t-test has been violated, but the assumptions of the sign test for the same data are not violated). The sign test assumes that the differences between the paired scores are independent (e.g. the difference between before and after scores for person 1 must be independent of the difference between before and after scores for person 2). The sign test simply computes a significance test of a hypothesised median value for a single data set. You can choose whether you want to use a one-tailed or two-tailed distribution based on your hypothesis; basically, do you want to test whether the median value of the data set is equal to some hypothesised value (H_0: $\eta = \eta_o$), or do you want to test whether it is greater (or lesser) than that value (H_0: ($\eta >_{\eta o}$) or H_0:($\eta < \eta_o$).The sign test simply computes whether there is a significant deviation from this assumption, and gives you a *p* value based on a binomial distribution. If you are only interested in whether the hypothesised value is greater or lesser than the sample median (H_0: ($\eta > \eta_o$) or H_0:($\eta < \eta_o$), the test uses the corresponding upper or lower tail of the distribution. The binomial probabilities are given by:

$$P(\mathrm{X} = \mathrm{r}) = \binom{n}{r} p^{\mathrm{r}} (1-p)^{\mathrm{n-r}}$$

Where,

n= Number of events
r = Number of successful events
p = Probability of success on a single trial
1-*p* = Probability of failure

4. McNemar’s Test:

Many tests required the comparison of related events, where a respondent had for example indicated how worked examples and problem-solving had impacted on various learning outcomes asked in two different questions (chapter 5 section 5) the responses of which are expected to be correlated. In these cases, a McNemar’s test was used. The McNemar test can be used to test if there is a statistically significant difference between two correlated proportions such as the case where the two

proportions are based on the same sample of students (Xuezheng & Zhao, 2008). The test is applied to a 2 × 2 contingency table, which tabulates the outcomes of two tests on a sample of *n* subjects. The null hypothesis of marginal homogeneity states that the two marginal probabilities for each outcome are the same. McNemar's test statistics is given by:

$$x^2 = \frac{(b-c)^2}{(b+c)}$$

The McNemar test has the following assumptions:

- **Randomness** - Sample members must be randomly drawn from the population.
- **Independence** - Within-group sample scores must be independent of each other.
- **Scaling** - the dependent measure (scores) must be nominal scale.
- **Expected Frequencies** - no expected frequencies should be less than 5.

Thus the null and alternative hypotheses are:

$$H_0: P_b = P_c$$

$$H_1: P_b \neq P_c$$

In reference to the 2x2 table in 2) *Fisher's Exact Test*, P_b and P_c are:

$$P_b = \frac{a+b}{n} \ \& \ P_c = \frac{a+c}{n}$$

Under the null hypothesis, with a sufficiently large number of discordant (cells b and c), x^2 has a chi-squared distribution with 1 degree of freedom. If either *b* or *c* is small ($b + c < 25$) then $x^2$is not well-approximated by the chi-squared distribution.

The binomial distribution can be used to obtain the exact distribution for an equivalent to the uncorrected form of McNemar's test statistic. In this formulation, *b* is compared to a binomial distribution with size parameter equal to $b + c$ and "probability of success" = ½, which is essentially the same as the

binomial sign test. For $b + c < 25$, the binomial calculation should be performed, and indeed, most software packages simply perform the binomial calculation in all cases, since the result then is an exact test in all cases. When comparing the resulting x^2 statistic to the right tail of the chi-squared distribution, the p-value that is found is two-sided, whereas to achieve a two-sided p-value in the case of the exact binomial test, the p-value of the extreme tail should be multiplied by 2 (Durkalski, 2009).

5. Multivariate analysis of variance:

Much of the analysis in this chapter is conducted with complex analytic techniques, such as MANOVA where different processes are used depending on outcomes at each step in the process. The general purpose of multivariate analysis of variance (MANOVA) is to determine whether multiple levels of independent variables on their own or in combination with one another have an effect on two or more dependent variables. In each case study there are multiple dependent variables, specifically test scores for the six topics taught. MANOVA requires that the dependent variables meet parametric requirements (Mwitondi, 2012). The assumptions are that the errors are independent, and normally distributed with a mean of zero and a common variance, and that the treatment effects are additive.

Due to the design of this study and the expectation that there would be interaction between baseline analyses and multiple follow-up analyses required, the implementation of the process is discussed further with the analysis of the case study data. Mwitondi (2012) describes the process of conducting a MANOVA and subsequent analyses as follows:

> To tease out higher level interactions in MANOVA, smaller ANOVA models which include only the independent variables which were significant can be used in separate analyses and followed by post hoc tests. Post-hoc and pre-planned comparisons compare all the possible paired combinations of the independent variable groups…The most frequently used pre-planned and post-hoc tests are Least Squares Difference (LSD), Scheffe, Bonferroni, and Tukey. The tests will give the mean difference between each group and a p-value to indicate whether the two groups differ significantly (Mwitondi, 2012, p.34).

In this study, Tukey's Honesty Significance Difference is used to undertake post-hoc tests and principally to report p-values adjusted for the number of tests undertaken. However the findings when significant in this study are also found to be significant when using alternative approaches such as LSD, Bonferroni and Scheffe tests. The Tukey's Honesty Significance Difference test:

> also corrects for multiple comparisons, but it considers the power of the study to detect differences between groups rather than just the number of tests being carried out i.e. it takes into account sample size as well as the number of tests being performed. This makes it preferable when there are a large number of groups being compared, since it reduces the chances of a Type I error occurring (Mwitondi, 2012, p.34).

Further information about LSD, Bonferroni and Scheffe tests can be found at (Mwitondi, 2012 section 5.3).

6.2.6 Results

As discussed in Chapter 5 there were three data collections in Saudi Arabia. The purpose for the first one was to gather baseline data and contextual information before introducing changes to pedagogy, namely teaching with worked examples and problem-solving in the first implementation. The baseline allowed a comparison of the difficulty of each topic when taught traditionally (students given theory followed by demonstrations then problem-solving). The first implementation of changed pedagogy allowed an examination of the worked examples approach and problem-solving and a baseline for subsequent changes. The second implementation involved the swapping of topics taught by problem-solving and worked examples and the introduction of faded worked examples for the last two topics allowing a comparison between the three approaches: worked examples, problem-solving and faded worked examples. The cohorts were compared in terms of students' perceived ability to learn mathematics, the ability to learn mathematics in English, the value of the learning resources provided, confidence with topics along with exploration of other learning outcomes and performance on each topic in the final examinations.

6.2.6.1 Participants

A total number of 198 students, enrolled in Math132 over three implementations, were involved in this study. Virtually all students were studying in their second language, English (refer Table 6.3). In terms of the study it was important that students were matched on key factors that could affect performance outcomes. The three cohorts were close in terms of size ranging from 64-68 students. Approximately 98 percent are from an Arabic background and approximately (95%) of the students are domestic students, again with each cohort similar on these factors. Due to the segregated culture of Saudi Arabia and that male and females are taught in separate campuses, no female students were selected to be a part of this study. The students that participated in the study came from varying high school strands with (61%) from Mathematics, (25%) from Science and (10%) from Arts for example for 2011; with a similar proportion in each year. Furthermore, in 2012, approximately (40%) have learned their mathematics in English since Year 11 and 37 percent since Year 12, again with similar proportions in each data collection.

Table 6.3 Distribution of KAU participants in this study

Stages	**Year**	**Number of Students**	**With English 2nd language %**	**From Maths Strand %**	**Learning English from Year 11 & 12 %**
Baseline	2010	66	99	56	76
Implementation1	2011	68	97	61	81
Implementation2	2012	64	98	62	77

6.2.6.2 Impact of language on student learning

There is a significant difference in students' perceived ability to learn mathematics across cohorts. In 2010, (83%) of students described their ability to learn Mathematics as fair/very good, but for 2012 this proportion is only (31%) of students (x^2=20.150, *df*=1, p<.0005) (refer Table 6.4).

When it comes to learning mathematics in English the perceived ability of students is also significantly different with (66%) of students perceiving their ability to be

fair/very good in 2010, declining to two percent of students in 2012 (x^2=82.149, *df*=1, p<.0005) (refer Table 6.4).

Table 6.4 Perceived ability to learn mathematics and mathematics in English

Stages	Ability to Learn Mathematics					Ability to learn Mathematics in English			
		Poor /Very poor		Fair/Very good		Poor/Very poor		Fair/Very good	
	N	n	%	n	%	n	%	n	%
2010	66	11	16.7	55	83.3	22	33.3	44	66.7
2011	68	20	29.4	48	70.6	60	88.2	8	11.8
2012	64	44	68.8	20	31.3	63	98.4	1	1.6
Total	198	75	37.8	123	61.8	145	73.2	53	26.8

The most striking impact of learning in a second language is apparent when comparing the students rating their mathematics ability versus the rating of the mathematics ability when learning in English (refer Table 6.5). Overall, 148 students (75%) rated their general mathematics ability higher than their ability to learn mathematics in English compared to (n=50, 25%) who rated it at the same or lower level.

Table 6. 5 Mathematics ability by mathematics ability in English

Mathematics Ability	Mathematics Ability in English				Total
	Very Poor	Poor	Fair	Very Good	
Very Poor	18	2	0	0	20
Poor	44	9	2	0	55
Fair	14	43	17	2	76
Very Good	7	8	32	0	47
Total	83	62	51	2	198

A chi-square test based on data classified as poor/very poor and fair/very good, found a significantly different proportion of students being confident in learning mathematics in English as compared to learning mathematics in general (x^2=35.775, *df*=1, p<.0005). Of the 123 students who considered their ability to do mathematics to be fair/good only (42%) considered their ability to do mathematics in English as fair or very good (refer Table 6.6).

Table 6. 6 Ability to learn mathematics by learning mathematics in English

Ability to learn mathematics	Ability to learn mathematics in English				Total	
	Very poor/Poor		Fair/ Very good			
	n	%	n	%	n	%
Very poor/Poor	73	97.3	2	2.7	75	100
Fair/Very good	72	58.5	51	41.5	123	100
Total	145	73.2	53	26.8	198	100

6.2.6.3 Value of learning resources

In order to assess the impact of the changes in attitudes towards the subject due to the changes in teaching methods, students in each year were asked to provide a rating of the perceived usefulness of all identified learning resources (refer Table 5.13 for original questions). In question 6 from Table 5.13, the frequency counts were joined for responses to ‘moderately useful’ and ‘extremely useful’. In 2011, the subject redesign led to the number of worked examples being increased in the topics: *Functions*, *Quadratic Equations* and *Geometry*. The remaining three topics were left with a problem-solving orientation. As is evident in Table 6.7, a Fisher’s Exact Test, used because of the small expected count size, revealed a significant difference in proportions (p=0.0075) with 53 students (80%) rating the work in practical classes as moderately or extremely useful in 2010 compared to 65students (96%) in 2011. Furthermore, there was significant difference in proportions (Fisher’s Exact Test, p=0.0087) with 50 students (76%) rating the worked examples as moderately or extremely useful in 2010 compared to 63 students (93%) in 2011.

In 2012, faded worked examples were introduced for topics *Geometry* and *Introduction to Statistics*, replacing worked examples and problem-solving respectively. The FWE were introduced to scaffold from worked examples to problem-solving because in the 2010 data problem-solving appeared to build student confidence (refer section 6.2.6.6). As illustrated in Table 6.7 using a Fisher’s Exact Test, no significant differences in the valuing of any resource were found between 2011 and 2012.

Table 6. 7 Perceived usefulness of resources for students learning 2010-2012

	Learning Resources	2010 Baseline N=66 n	%	P1*	2011 WE & PS N=68 n	%	P2^	2012 Introduce FWE N=64 n	%
1	Work in Practical Classes	53	80.3	0.0075	65	95.6	0.3145	58	90.6
2	Worked Examples	50	75.8	0.0087	63	92.6	0.0687	52	81.3
3	Practical Worksheets	49	74.2	1.0000	51	75.0	0.6873	50	78.1
4	Tutor in Practical Classes	48	72.7	0.7057	47	69.1	0.4345	49	76.5
5	Tutorial Assignments	47	71.2	0.3613	43	63.2	0.188	48	75.0
6	Team Learning or Group Work	40	60.6	0.8617	40	58.8	0.5936	41	64.1
7	Interaction With Lecturer	39	59.1	0.7298	38	55.8	0.4817	40	62.5
8	Theory Review in Prac Classes	32	48.5	0.8629	35	51.4	0.4888	37	57.8
9	Lecture Handbook	29	43.9	0.7320	32	47.0	0.602	34	53.1
10	Lecture	23	34.8	0.2221	31	45.5	0.6011	26	40.6

*P1 calculated using the Fishers exact test, two tailed comparing 2010 and 2011 data
^P2 calculated using the Fishers exact test, two tailed comparing 2011 and 2012 data

To the extent that worked examples would be considered preferable in terms of lowering cognitive load to faded worked examples, there is no significant preference for the teaching when more explicit worked examples were used (Fisher's exact test, p=0.068), a borderline result with (93%) of students rating the worked examples as 'moderately or extremely important' compared to the rating for worked examples in the year the faded examples with steps missing for one topic were introduced (81%). Perhaps it is that as most of students (n=48) in 2011 said "I feel that worked examples method is better than faded worked examples and problem-solving". Some of the 2011 and 2012 students expressed an appreciation of worked examples and reasons for why they prefer worked examples:

> *I love to learn math by worked examples.*
>
> *Worked examples reduce my stress when I learn math.*
>
> *Worked examples are helping me to prepare for the final exam because I have all the solutions for all problems.*
>
> *The FWE and materials used during this course were really wonderful, I am more comfortable with mathematics now.*

I had no idea about mathematics at all before I started this subject. I was extremely anxious at the start. However, worked examples helped and attending lectures assisted me greatly in completing this subject.

I don't like problem-solving because it increases my anxiety.

In 2011, when worked examples were increased, and 2012,when faded worked examples were introduced, the top three learning resources in terms of usefulness to students' understanding were: work in practical classes (students work most of WE's on the board), worked examples and practical worksheets (students work most of WE's on the sheets)(refer Table 6.7). In all three years the proportions of students valuing the lecture handbook or lectures as 'moderate or extremely useful' resourceswere low, between (35%) and (53%) of students.

6.2.6.4 Worked example and problem-solving usage comparisons

The results in this section are from Table 5.14 (chapter 5) question 7, (*On average how many problems have you completed per week in Math132 class using worked examples as a guide?*) and question 8, (*On average how many problems have you solved per week without worked examples?*) in questionnaire. In terms of outcomes, the proportion of students working on average five or more problems using worked examples is significantly different (x^2=4.661, *df*= 1, p=.031) changing from (15%) in 2010 to (31%) in 2011 and is not significantly differentfrom 2010 (x^2=.643, *df*=1, p=.423) changing to (38%) in 2012. In terms of problem-solving, (21%) students during the baseline period 2010 worked on average five or more problem-solving items per week, a significant difference (x^2=5.287, *df*= 1, p=.021) compared to only seven percent in 2011 and a borderline significant difference (x^2=3.817, *df*= 1, p=.051) comparing (21%) of students in 2010 to (16%) for the last implementation in 2012. The number of KAU students doing no problem-solving questions increased from (15%) in 2010 to (55%) in 2011 (x^2=22.15, *df*=1, p<.0005) and for worked examples it increased from nine percent to (21%) (x^2=3.48, *df*=1, p=.062). Whereas for UOWC students, there was no significant difference for the number of students doing no problem-solving questions

with (23%) in 2010 and (15%) in 2011, whereas no significant difference (x^2=2.62, *df*=1, p=.105) for the number completing no worked examples fell from (18%) in 2010 to four percent in 2011 (refer Table 6.8).

Table 6. 8 Average number of problems completed each week using WE or PS

No. Items	**2010**				**2011**				**2012**				**PS Total**		**WE Total**	
	PS		**WE**		**PS**		**WE**		**PS**		**WE**					
	n	**%**	**n**	**%**	**n**	**%**	**n**	**%**	**n**	**%**	**n**	**%**	**n**	**%**	**n**	**%**
0	10	15.2	6	9.1	37	54.5	14	20.6	10	15.6	12	18.8	57	28.8	32	16.2
1-2	15	22.7	26	39.4	13	19.1	12	17.6	14	21.9	15	23.4	42	21.2	53	26.8
3-4	27	40.9	24	36.4	13	19.1	21	30.9	28	43.8	13	20.3	68	34.3	58	29.3
5 or more	14	21.2	10	15.2	5	7.4	21	30.9	12	15.7	24	37.5	31	15.7	55	27.8
Total	66	100.	66	100	68	100.	68	100	64	100	64	100	198	100	198	100

Over the combined data for the three years a significantly different percentage (McNemar x^2=8.229, p=.006) of students (n=55, 28%) indicated that the average number of items that they solved per week using worked examples was five or more, whereas only (n=31, 16%) solved on average the same number using problem-solving in their lesson per week (refer Table 6.9).

Table 6. 9 Average number of problems using WE and by PS

	PS			
WE		< 5	≥ 5	
	<5	120	23	143
	≥ 5	47	8	55
		167	31	198

Over the combined data for the three years a significantly different proportion of students (McNemar x^2=109.10, p<.001) in response to the question 9, (*Does using worked example approach improve your study for Math132?*) and question 10, (*Does the problem-solving approach without worked examples improve your study for Math132?*) indicated that having worked examples (81%) improved their studying mathematics moreso than did problem-solving (20%). Table 6.10 shows the

combined data for the three years and Table 6.11 shows breakdown of the 'Yes' response by cohort for these two questions.

Table 6. 10 Problem-solving and worked example improves study

Improve Worked Examples	Improve Problem-solving		Total
	No	Yes	
No	32	6	38
Yes	126	34	160
Total	158	40	198

Table 6. 11 Numbers finding PS and WE improved their study

Stages	N	PS Improve		WE Improve	
		n	%	n	%
2010 Baseline	66	17	25.7	45	68.1
2011 (Introduce WE)	68	16	23.5	58	85.2
2012 (Introduce FWE)	64	7	10.9	57	89.0
Total	198	40	20.2	160	80.8

In response to the question 11 (refer Table 5.14), (*How would you prefer to study Math132? And explain why you prefer this?*), students' combined data overall years (refer Table 6.12) indicated that (n=140, 71%) of students preferred to study this subject with worked examples, (n=30, 15%) of students preferred to study with problem-solving, while (n=28, 14%) preferred a mixture of worked examples and problem-solving.

Table 6. 12 Preference problem-solving, worked examples or mix of approaches

Stages	N	Worked Examples		Problem-Solving		Mixed WE & PS	
		n	%	n	%	n	%
2010 Baseline	66	56	84.8	2	3.0	8	12.1
2011 (Introduce WE)	68	45	66.1	12	17.6	11	16.1
2012 (Introduce FWE)	64	39	60.9	16	25.0	9	14.0
Total	198	140	70.7	30	15.1	28	14.1

When the students were asked to explain their preferences, some students gave more than one reason (refer Table 6.13). The dominant reasons given by students for preferring worked examples were:

- Gives them a guide for solution (34%)
- Reduces anxiety (16%)
- Make mathematics more interesting (14%)
- Easy to understand (11%)

The primary reasons for preferring problem-solving related to:

- Increased confidence (43%)
- Builds critical thinking (23%)
- Preparation for solving real life problems (13%)
- Increased independence (10%)

Table 6. 13 Number of student's reasons for their learning preference 2010-2012

Reasons	**Preferred Method**					
	WE		PS		Mixed WE & PS	
	n	%	n	%	n	%
Give them guide for solution	48	34.2				
Reduced the anxiety of mathematics on the students	23	16.4				
Makes mathematics more interesting	20	14.0	1	3.3	2	7.1
Easy to understand	15	10.7				
Help them with revision subject materials	13	9.2				
More confidence			13	43.3	9	32.1
Easy to follow	9	6.4				
No wasting time looking for solution	7	5.0				
Build critical thinking			7	23.3		
Two ways for learning math					7	25.0
Quicker to learn	5	3.5				
Easy to Learn					3	10.7
Prepares me for solving real life problems			4	13.3		
More independent			3	10.0		
Feel math is challenge			2	6.6	4	14.2
Keeps you thinking differently					3	10.7
Total	140	100	30	100	28	100

A breakdown of the preferences for teaching method as related to students' ability to learn in English is provided in Table 6.14. There was no significant association (x^2=1.54, *df*=2, p= 0.47) between preferred method and perceived mathematics ability. For the students who prefer WE to study; (76%) considered themselves to

have weaker mathematical abilities, whereas of those who preferred problem-solving, (67%) considered their ability to learn mathematics in English as poor or very poor, and for those who preferred a mixture of WE and PS, (68%) considered their ability to learn mathematics in English as poor/very poor.

Table 6. 14 Mathematics ability in English against preferred method

Mathematics Ability in English	**Preferred Method**						**Total**	
	Worked examples (WE)		Problem-Solving (PS)		Mixed between (WE) & (PS)			
	n	%	n	%	n	%	n	%
Very Poor/Poor	106	75.7	20	66.7	19	67.9	145	73.2
Fair/Very Good	34	24.3	10	33.3	9	32.1	53	26.8
Total	140	100	30	100	28	100	198	100

6.2.6.5 Learning outcomes

There are many learning outcomes that can be examined when one compares teaching methods. Students were asked a series of questions (refer Q17, Table 5.17, Evaluation of using problem-solving and worked examples approaches in this subject was through 7-points likert scales) to indicate the extent to which worked examples and problem-solving assisted them with a variety of learning outcomes. The outcomes addressed are: enhances understanding; makes it quicker to study mathematics; improves my review of mathematics notes and lab work; easier to learn; increases my confidence about solving more problems; requires a lot of mental effort; makes learning more interesting; like to learn mathematics; and, reduces my anxiety. The results for the categories Mildly Agree, Agree, and Strongly Agree were combined as ‘Agree’ and compared to the combined categories of Strongly Disagree, Disagree, Mildly Disagree and Neither as ‘Disagree’. Table 6.15 shows the number of students in the combined category ‘Agree’ on the learning outcomes for WE and similarly for PS. This summary table overall years reveals that except for one of the items, worked examples are significantly more favourable than problem-solving. The one exception was for the learning outcome “increases my confidence” where there was a positive association with problem-solving with almost (89%) of students agreeing with the statement that problem-solving increased confidence compared to only (22%) who considered that worked examples improved confidence.

Table 6. 15 Comparison of learning outcomes

Learning Outcomes	McNemar's	Test	Worked Examples		Problem-Solving	
	x^2	p*	n	%	n	%
Enhances understanding	30.39	<0.0005	162	82	113	57
Quicker to Study	67.23	<0.0005	149	75	63	32
Improved my review of mathematics notes and lab work	40.96	<0.0005	179	90	127	64
Easier to Learn mathematics	24.20	<0.0005	149	75	105	53
Increases my confidence about solving more problems	131.02	<0.0005	43	22	176	89
+Requires a lot of mental effort	102.51	<0.0005	49	25	164	83
Makes mathematics learning more interesting	58.50	<0.0005	172	87	94	48
Like to learn mathematics	24.50	<0.0005	144	73	102	52
Reduces my anxiety	75	<0.0005	133	67	43	22

* Probability associated with test to assess for a change between approaches using McNemar's test
+ The reverse of the item is positive
All two way tables are appended in Appendix (D)

The results indicate that students thought worked examples helps achieve all of the different learning outcomes, more so than problem-solving except for increasing confidence. These included Enhances understanding, Quicker to Study, Improved my review of mathematics notes and lab work, Easier to Learn mathematics, Requires a less mental effort, Makes mathematics learning more interesting, Like to learn mathematics, Reduces my anxiety. Increased confidence was associated with problem-solving; the students indicated that problem-solving helps them better than worked examples. Students' comments supported this:

> *Problem-solving helps me a lot to understand mathematics.*
>
> *I don't like problem-solving but I think I need it.*
>
> *The benefit from problem-solving builds the independency.*
>
> *Problem-solving makes me more confident for solving mathematical problem.*
>
> *I find problem-solving is useful because it is preparing me for solving real life problems.*

6.2.6.6 Confidence in topics

In relation to each of the major topics, 198 students were asked a question of the form (*How confident are you now that you can solve problem on the following topics?* refer to Q16 Table 5.16). The response categories for this question were 'Not at all', 'Might have a little difficulty', 'Moderately confident', and 'Could do this'. The categories 'Moderately Confident' and 'Could do this' were combined to create a new category 'Confident'. The remaining two categories were classified as 'Not confident'. Table 6.16 reports the number of students whose responses were grouped into the new 'confident' category.

In terms of individual topics, students were more confident on the topic of *Functions* and *Exponents* than for other topics. To test for a difference in proportions between cohorts and results on the confidence on topics a two-tailed test Fisher's exact test was used. A Fisher's exact test revealed that there was a significant difference in the proportion of students perceiving themselves as confident in the topic *Functions*, with a higher percentage in 2011 (96%) than in 2010 (80%) (p=0.0075). More students also indicated that they were confident (p=0.0087) in topic of *Exponents* in 2011 (93%) than in 2010 (76%) (refer Table 6.16). Comparisons of 2011 and 2012 indicated no significant differences in confidence for any of the topics.

Table 6. 16 Confidence on topics in three phases (results for 'confident')

Topic	**2010 N=66**		**P1***	**2011 N=68**		**P2^**	**2012 N=64**	
	n	%		n	%		n	%
Functions	53	80.3	0.0075	65	95.6	0.3145	58	90.6
Exponents	50	75.7	0.0087	63	92.6	0.0687	52	81.2
Quadratic equations	49	74.2	1.0000	51	75.0	0.6873	50	78.1
Logarithms	48	72.7	0.7057	47	69.1	0.4345	49	76.5
Geometry	29	43.9	0.7320	32	47.0	0.7277	33	51.5
Introduction to Statistics	23	34.8	0.2221	31	45.5	0.6011	26	40.6

*P1 Fisher's Exact Test2-tiled comparison of 2010 and 2011
^P2 Fisher's Exact Test 2-tiled comparison of 2011 and 2012

`

6.2.6.7 Impact of changing teaching approaches on performance

To examine the impact of the teaching method changing is complicated. The process of the examination is as follows:

1. The first step involves gaining a sense of the design clarifying the means and standard deviations (refer Table 6.17) associated with each of the teaching techniques and topics followed by a formal examination of differences between means. This provides for easy reference in the subsequent discussion of results. Table 6.17 summarises the results obtained from each topic test where the maximum mark was ten. The coloured rows indicate the teaching method associated with the topic in the three cohorts. The highest observed mean across cohorts for topics *Functions*, *Quadratic Equations*, and *Logarithms* were attained where the teaching method was WE. *Exponents* was the only topic which used PS method and obtained the highest observed mean. For topics *Geometry* and *Introduction to Statistics*, the highest observed mean was obtained when the teaching method was FWE.

Table 6. 17 Final topic mean marks KAU 2010-2012

Topic	Year	N	Mean	S.D
Function	2010	66	6.08	2.433
	2011	68	8.06	1.370
	2012	64	4.52	2.330
Exponents	2010	66	6.82	2.067
	2011	68	8.34	1.334
	2012	64	5.80	2.457
Quadratic Equations	2010	66	4.85	2.579
	2011	68	8.81	1.069
	2012	64	5.39	2.524
Logarithms	2010	66	6.41	2.280
	2011	68	3.07	1.568
	2012	64	7.92	1.546
Geometry	2010	66	5.77	2.630
	2011	68	3.04	1.670
	2012	64	8.00	1.392
Introduction to Statistics	2010	66	6.68	2.432
	2011	68	2.63	1.647
	2012	64	7.22	1.759

Traditional		WE		PS		FWE	

A plot of the means for each topic is shown in Figure 6.1. The most obvious feature is the polarity of means for 2011 cohort into two distinct groups. In 2011, the means for topics *Functions*, *Exponents*, and *Quadratics Equations* are much higher than for topics *Logarithms*, *Geometry*, and *Introduction to Statistics*. These groupings do not exactly coincide with the different teaching techniques.

Figure 6.1 Means of topics for KAU over years 2010-2012

The movement of the topic means from year to year (shown by non-parallel lines) suggests that there will be an interaction between year (that is the different teaching methods) and topic. This will be examined more closely in the next step of the analysis.

Another plot of the same topic means. This time showing the topics on the horizontal axis is given Figure 6.2. This plot emphasises the different movement of the topic means for 2011 compared to 2010 and 2012.

Figure 6.2 Means of topics for KAU over years 2010-2012

2. The first analysis has six dependent variables (T1 to T6), the marks (0 lowest -10 highest) on each of the six topic tests, and year (2010, 2011, and 2012) as the proxy independent variable. In different years, different teaching methods were used for different topics allowing an examination of the impact of teaching method on topic marks. As the topic tests are given to the same students, the means derived from the same students are measured on each topic, and used to examine whether there is an interaction between year and topic. This initial analysis was carried out using a Repeated Measures ANOVA, with the first step an examination of assumptions and the second an examination of the multivariate tests.

Step 1: Examination of Assumptions

The first test, the Mauchly's test of sphericity, indicated that the assumption of sphericity (that the error covariance matrix of the normalised transformed dependent variables is proportional to an identity matrix) is violated (x^2=41.082, *df*=14, p<.0005) (refer Table 6.18). This indicates that there is sufficient correlation between the dependent variables (topic marks) to carry out a MANOVA.

Table 6. 18 Mauchly's test of sphericity[b]

Within subject effect	Epsilona						
	Mauchly's W	Approx. Chi-Square	df	Sig.	Greenhouse-Geisser	Huynh-Feldt	Lower-bound
Topic	.808	41.082	14	.000	.914	.948	.200

Tests the null hypothesis that the error covariance matrix of the normalised transformed dependent variables is proportional to an identity matrix.
a. May be used to adjust the degrees of freedom for the averaged tests of significance. Corrected tests are displayed in the Tests of Within-Subjects Effects table.
b. Design: Intercept + Year
Within Subjects Design: topic

MANOVA compares the groups whether the means differences between the groups (i.e. cohorts) on the combination of each of dependent variable is occurred. The advantage of using MANOVA is that 'controls' or adjusts for this increased risk of Type I error. MANOVA has a number of assumptions that must be met (Tabachnick & Fidell, 2001). The assumptions are:

1. *Sample size*: The sample should have more cases in each cell than dependent variable which will help with violations of some other assumptions (e.g. normality). In this case there were six dependent variables and 198 cases, suggesting that sample size was adequate, to provide some robustness least other assumptions be violated.
2. *Normality*: "Although the significance test MANOVA is based on the multivariate normal distribution, in practice it is reasonably robust to modest violations of normality (except where the violations are due to outliers)" (Pallant, 2005). As advised by Tabachnick and Fidell, (2001) checking for multivariate outliers involves comparing the maximum value of Mahalanobis distance, with the critical value given the number of dependent variables. In this case the critical value is 22.46 (Tabachnick & Fidell, 2001). The decision is that if the maximum value for Mahalanobis distance is less than critical value, then there are no substantial multivariate outliers. As the maximum value for Mahalanobis distance here is 21.226 (refer Table 6.19), this assumption is not violated, there are no substantial outliers.

Table 6. 19 Residual statistics

	Minimum	Maximum	Mean	Std. Deviation	N
Predicted Value	2009.72	2012.11	2010.99	0.437	198
Std. predicted value	-2.919	2.574	0.000	1.000	198
Standard Error of predicted value	0.730	0.234	0.128	0.027	198
Adjusted predicted value	2009.69	2012.12	2010.99	0.439	198
Residual	-1.594	1.584	0.000	0.685	198
Std. Residual	-2.291	2.278	0.000	0.985	198
Deleted Residual	-1.718	1.680	-0.001	0.713	198
Stud. Deleted Residual	-2.408	2.374	-0.002	1.009	198
Mahal. Distance	1.198	21.226	5.970	3.078	198
Cook's Distance	0.000	0.092	0.006	0.011	198
Centered Leverage value	0.006	0.108	0.030	0.016	198

3. *Outliers*: MANOVA is quite sensitive to univariate outliers (on each of the dependent variables i.e. topic marks) and multivariate outliers, where subjects have a strange combination of scores on the various dependent variables (topic marks). As advised by Tabachnick and Fidell, (2001) linear regression analyses with Topic as the dependent variable and Year as the independent variable were used to identify if there were any outliers. One outlier with a standardised residual greater than three was identified for the second topic, *Exponents* (refer Table 6.20). Re-analysis with that point omitted had no impact, therefore all results have continued to include the case.

Table 6. 20 Casewise diagostics[a]

Case Number	idV	Std. Residual	Topic 2	Predicted Value	Residual
177	177.00	-3.105	0	6.45	-6.450

Dependent Variable: Topic 2

4. *Linearity*: this assumption refers to the presence of a straight-line relationship between each pair of your dependent variables, in this case the six topics. A visual inspection of all two way scatterplots (15*3 plots= 45, topic 1 versus topic 2, topic 1 versus topic 3 etc. for each year) of topic marks provided no evidence of nonlinearity.

5. *Homogeneity*: This assumption is important only if researcher intending to perform a step down analysis. This approach is used when research have some theoretical or conceptual reason for ordering dependent variables. This was not applicable in this study as there was only one independent variable 'Year'.
6. *Multicollinearity and singularity*: MANOVA works best when the dependent variables are only moderately correlated. When dependent variables are highly correlated this is referred to multicollinearity. As seen in the table of correlations, Table 6.21 the correlations were all moderate ranging from 0.255 to 0.590 suggesting that multicollinearity was not problematic in this analysis.

Table 6. 21 Correlations between topics at KAU

		Topic 1	**Topic 2**	**Topic 3**	**Topic 4**	**Topic 5**	**Topic 6**
Topic 1	Pearson Correlation	1	.255**	.368**	-.465**	-.316**	-.454**
	Sig. (2-tailed)		.000	.000	.000	.000	.000
	N	198	198	198	198	198	198
Topic 2	Pearson Correlation	.255**	1	.256**	-.325**	-.319**	-.326**
	Sig. (2-tailed)	.000		.000	.000	.000	.000
	N	198	198	198	198	198	198
Topic3	Pearson Correlation	.368**	.256**	1	-.442**	-.345**	-.489**
	Sig. (2-tailed)	.000	.000		.000	.000	.000
	N	198	198	198	198	198	198
Topic 4	Pearson Correlation	-.465**	-.325**	-.442**	1	.505**	.590**
	Sig. (2-tailed)	.000	.000	.000		.000	.000
	N	198	198	198	198	198	198
Topic 5	Pearson Correlation	-.316**	-.319**	-.345**	.505**	1	.481**
	Sig. (2-tailed)	.000	.000	.000	.000		.000
	N	198	198	198	198	198	198
Topic 6	Pearson Correlation	-.454**	-.326**	-.489**	.590**	.481**	1
	Sig. (2-tailed)	.000	.000	.000	.000	.000	
	N	198	198	198	198	198	198

**. Correlation is significant at the 0.01 level (2-tailed).

7. *Homogeneity of variance-covariance matrices*: the test of this assumption is generated as part of MANOVA output. The test used assess this is Box's M Test Equality of covariance matrices as in Table 6.22. Tabachnick and Fidell (2001) indicate that "If the p-value is larger than 0.001 then you have *not* violated the assumption", however they also caution that the test is too conservative when sample sizes are large. In this case the assumption has been violated as the p-value is less than 0.0005. As the analyses to test for differences in year (teaching methods) will be further examined through univariate analyses because the evidence suggests that there are interactions, assumptions relate to those univariate analyses will be of greater importance.

Table 6. 22 Box's test of equality of covariance matrices[a]

Box's M	172.700
F	3.931
df1	42
df2	112368.662
Sig.	.000

Tests the null hypothesis that the observed covariance matrices of the dependent variables are equal across groups.
a. Design: Intercept + Year

Further examination of Levene's test for the equality of error variance over the three years indicates that for Topic 4 (p=0.065) and Topic 6 (p=0.091) variances are not statistically different over the three years (refer Table 6.23). To address this Tabachnick and Fidell, suggest that a more conservative *p*-value be used for example 0.001 rather than 0.05.

Table 6. 23 Levene's test of equality of error variances[a]

	F	df1	df2	Sig.
Topic 1 Functions	12.150	2	195	.000
Topic 2 Exponents	13.737	2	195	.000
Topic 3 Quadratic Equations	19.430	2	195	.000
Topic 4 Logarithms	2.769	2	195	.065
Topic 5 Geometry	12.928	2	195	.000
Topic 6 Introduction to Statistics	2.423	2	195	.091

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.
a. Design: Intercept + Year

Step 2: Multivariate tests

As the Sphericity assumption was violated the MANOVA was used, assumptions checked and the interaction between topic and year as suggested by the profile plots (Figures 6.1 & 6.2) was found to be significant using the Greenhouse-Geisser adjustment ($F_{9.142,891.352}$=88.9, p<0.0005) (refer Table 6.24).

Table 6. 24 KAU tests of within-subjects effects

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
Topic	Sphericity Assumed	303.861	5	60.772	15.033	.000
	Greenhouse-Geisser	303.861	4.571	66.475	15.033	.000
	Huynh-Feldt	303.861	4.742	64.075	15.033	.000
	Lower-bound	303.861	1.000	303.861	15.033	.000
Topic * Year	Sphericity Assumed	3592.618	10	359.262	88.869	.000
	Greenhouse-Geisser	3592.618	9.142	392.976	88.869	.000
	Huynh-Feldt	3592.618	9.485	378.787	88.869	.000
	Lower-bound	3592.618	2.000	1796.309	88.869	.000
Error (topic)	Sphericity Assumed	3941.545	975	4.043		
	Greenhouse-Geisser	3941.545	891.352	4.422		
	Huynh-Feldt	3941.545	924.742	4.262		
	Lower-bound	3941.545	195.000	20.213		

When there are interactions it is not appropriate to examine overall differences in year or topics as any differences may be masked by the interactions. The significant interaction means there is a different profile for what happens in different years, depending upon which topic you are looking at and this can be seen by reference to the two profile plots (refer Figure 6.1 & Figure 6.2). The profile plot shows the interaction between year and topic as the lines are not parallel, meaning that it is not appropriate to combine all the data from topics to test for an overall difference in years. An interaction was expected because of the nature of the design. The confirmation that these interactions exist support the next stage of analysis which examines separately what happens for each topic, and because of the multiple comparisons that ensue, Tukey HSD tests that control for the type error 1, adjusted for the number of tests undertaken will be used.

Due to the interaction the exact nature of the differences needs to be tested but it would appear that in the very least Topics 1, 2 & 3 have higher means in 2011 than 2010, whereas Topics 4, 5 & 6 have lower means in 2011. Conversely in 2012, the means for Topics 1, 2 & 3 appear lower than those for 2011, whereas means for Topics 4, 5 & 6 appear higher. Whether the differences in 2011 and 2012 are significant needs to be tested. The next step typically involves looking at whether or not there are differences in just the WE topics, or just the PS topics. Although it would appear there is at least one difference in each. So it appears reasonable to skip this step and move on to examine the comparisons between means undertaken for each separate topic rather combining worked examples and combining problem-solving.

Following the interaction identified in this study, two types of analyses were of interest:

a) Differences between the topics at each year, but in particular at the baseline, and;

b) Differences between years (teaching methods) for each of the topics.

a) Differences between topics:

To identify differences between topics a within-subjects analysis was undertaken with subsequent paired t-tests to identify which of the topic marks were different. The topic marks in the baseline (2010) need to be examined to see if there are differences between topics because in 2011 and 2012 the topics were taught with different methods. If there are differences between the topics in the baseline data collection, then it is difficult to determine whether or not there are differences between the problem-solving and worked example approaches in the first and second experimental implementations. In Figures 6.1 and 6.2 there appears to be differences in some topics in some years but not in others.

I. 2010-Baseline

To compare average topic marks the MANOVA follow-up post-hoc tests were used. To tease out higher level interactions in MANOVA, smaller ANOVA models as is appropriate when there are interactions which include only the independent variables which were significant can be used in separate analyses and followed by post-hoc tests. Univariate analyses (ANOVAs, refer Table 6.25) suggest that there are differences between years for all topics.

Table 6. 25 Univariate tests KAU 2010

Dependent Variable		Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Topic 1 Functions	Contrast	439.670	2	219.835	51.163	.000	.344
	Error	837.870	195	4.297			
Topic 2 Exponents	Contrast	212.976	2	106.488	30.123	.000	.236
	Error	689.353	195	3.535			
Topic 3 Quadratic Equations	Contrast	698.110	2	349.055	81.201	.000	.454
	Error	838.234	195	4.299			
Topic 4 Logarithms	Contrast	864.253	2	432.127	154.976	.000	.614
	Error	543.727	195	2.788			
Topic 5 Geometry	Contrast	809.784	2	404.892	108.306	.000	.526
	Error	728.989	195	3.738			
Topic 6 Introduction to Statistics	Contrast	910.309	2	455.155	128.006	.000	.568
	Error	693.368	195	3.556			

The F tests the effect of Year. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

Post-hoc and pre-planned comparisons compare all the possible paired combinations of the independent variable groups (Mwitondi, 2012). The descriptive statistics suggests when the topics were taught using the traditional approach 2010 (refer chapter 5) the means low are compared to two topics taught with the WE approach in 2011. Conversely two topics taught with PS in 2011 have lower means than when taught traditionally. Both these are in accord with cognitive load theory, however the

average mark for one topic taught with WE appears to have declined, while one topic taught with PS techniques seems to have performed better. In Table 6.26, the topics are arranged from least difficult (highest means) to most difficult (lowest means).

Table 6. 26 Descriptive statistics for KAU 2010

Topic	Topic Name	N	Mean*	Mean	Mean	Std. Deviation
2	Exponents	66	6.82			2.06
6	Introduction to Statistics	66	6.68	6.68		2.43
1	Functions	66	6.08	6.08		2.43
4	Logarithms	66	6.41	6.41		2.28
5	Geometry	66		5.77		2.63
3	Quadratic Equations	66			4.85	2.57

Valid N (list wise)
*Means are presented as homogenous subsets

This is confirmed with paired T-tests as reported in Table 6.27. In 2010, the four easiest topics, with no significant difference between them, are *Exponents*, *Introduction to statistics*, *Functions*, and *Logarithms*. In this sense they may be considered to be comparable in difficulty. *Quadratic equations* appears to be the most difficult topic, with the mean significantly different each of the other five topics means. In addition, the mean for *Geometry* is significant different to the mean for *Exponents*.

Table 6. 27 Outcome of paired-samples T-tests for KAU 2010 means

Topic Comparisons		**t**	**df**	**P-Value (2-tailed)**
Pair 1	Functions-Exponents	-1.814	65	.074
Pair 2	Functions-Quadratic equations	3.250	65	.002*
Pair 3	Functions-Logarithms	-0.723	65	.472
Pair 4	Functions-Geometry	0.790	65	.433
Pair 5	Functions-Intro to Stats	-1.328	65	.189
Pair 6	Exponents-Quadratic equations	4.602	65	.000*
Pair 7	Exponents-Logarithms	1.226	65	.225
Pair 8	Exponents-Geometry	2.359	65	.021*
Pair 9	Exponents-Intro to Stats	0.403	65	.688
Pair 10	Quadratic equations-Logarithms	-3.255	65	.002*
Pair 11	Quadratic equations-Geometry	-2.184	65	.033*
Pair 12	Quadratic equations-Intro to Stats	-3.927	65	.000*
Pair 13	Logarithms-Geometry	1.288	65	.202
Pair 14	Logarithms-Intro to Stats	-0.775	65	.441
Pair 15	Geometry-Intro to Stats	-1.951	65	.055

II. 2011-Implementation I

To compare average topic marks in 2011 a MANOVA (Multivariate Analysis of Variance) was initially used (Step 2) but a significant interaction meant that analysis of differences in marks needed to be undertaken separately for each topic and time period. That is, either pre-planned contrasts or post-hoc tests were needed to confirm impressions from Figure 6.1 and 6.2 that there are differences between the topics and that these differences were associated with the method used, worked examples versus problem-solving. The more conservative approach of post-hoc tests, adjusted for the number of tests was undertaken.

In 2010, the topic *Quadratic Equations* was the worst performing topic (lowest mean), whereas in 2011, taught with worked examples, it has the highest mean of all topics. Also noteworthy is *Introduction to Statistics* achieving the highest mean when taught in 2010 (traditional method) but when taught with (PS) the topic mean was the lowest in 2011 (refer Table 6.28).

Table 6. 28 Descriptive statistics for KAU 2011

Topics	Topic Name	N	Method	Mean*	Mean	Mean	Std. Deviation
3	Quadratic Equations	68	WE	8.81			1.06
2	Exponents	68	PS		8.34		1.33
1	Functions	68	WE		8.06		1.37
4	Logarithms	68	PS			3.07	1.56
5	Geometry	68	WE			3.04	1.67
6	Introduction to Statistics	68	PS			2.63	1.64

Valid N (list wise)
*Means are presented as homogeneous subsets

Differences are confirmed with paired T-tests as reported in Table 6.29. In 2011, *Functions*, *Quadratic Equations* and *Geometry* were all taught with worked examples and the remaining topics with problems-solving. *Quadratic equations*, now the best performing topic, is significantly different in average marks to all other topics. *Exponents* and *Functions* are not significantly different from one another and can be considered equivalent ranking second in average marks, and are each significantly different to all other topics. The last three topics: *Logarithms*, *Geometry*, and *Introduction to Statistics* appear to be equivalent in difficulty with no significant differences between them.

Table 6. 29 Outcomes of paired-samples T-tests for KAU 2011 means

Topics Comparisons		t	df	P-value (2-tailed)
Pair 1	Functions-Exponents	-1.250	67	.216
Pair 2	Functions-Quadratic equations	-3.555	67	.001 *
Pair 3	Functions-Logarithms	22.141	67	.000 *
Pair 4	Functions-Geometry	21.895	67	.000 *
Pair 5	Functions-Intro to Stats	20.584	67	.000 *
Pair 6	Exponents-Quadratic equations	-2.429	67	.018 *
Pair 7	Exponents-Logarithms	21.657	67	.000 *
Pair 8	Exponents-Geometry	22.965	67	.000 *
Pair 9	Exponents-Intro to Stats	22.930	67	.000 *
Pair 10	Quadratic equations-Logarithms	26.763	67	.000 *
Pair 11	Quadratic equations- Geometry	21.477	67	.000 *
Pair 12	Quadratic equations-Intro to Stats	26.165	67	.000 *
Pair 13	Logarithms-Geometry	0.105	67	.917
Pair 14	Logarithms-Intro to Stats	1.595	67	.116
Pair 15	Geometry-Intro to Stats	1.511	67	.136

III. 2012-Implementation II

Table 6.30 reports the summary statistics and teaching method in 2012 for six topics in descending order of mean. It was suggested in Figure 6.1 and 6.2 that there are differences between the topics in 2012 as the topic means differ especially. This is confirmed with paired T-tests as reported in Table 6.30.

Table 6. 30 Descriptive statistics for KAU 2012

Topic	Name	N	Method	Mean *	Mean	Mean	Mean	Std. Deviation
5	Geometry	64	FWE	8.00				1.39
4	Logarithms	64	WE	7.92				1.54
6	Introduction to Statistics	64	FWE		7.22			1.75
2	Exponents	64	WE			5.80		2.45
3	Quadratic Equations	64	PS			5.39		2.52
1	Functions	64	PS				4.52	2.33

Valid N (list wise)
*Means are presented as homogeneous subsets

In 2012, the easiest topics can be considered to be *Geometry* and *Logarithms* with no significant difference between these two topics (refer Table 6.31). *Introduction to Statistics* then follows being significantly different to *Exponents,* which is equivalent (not significantly different) in means to *Quadratic Equations*. *Introduction to Statistics* is also significantly different to the other four topics *Functions*, *Exponents*, *Logarithms*, and *Geometry*. *Functions* now taught using problem-solving has a significantly different mean, and is lower in marks compared to all other topics.

Table 6. 31 Outcomes of paired-samples T-tests for KAU 2012 means

	Topics Comparisons	t	df	P-Value (2-tailed)
Pair 1	Functions-Exponents	-2.717	63	.009*
Pair 2	Functions-Quadratic equations	-2.001	63	.050*
Pair 3	Functions-Logarithms	-9.347	63	.000*
Pair 4	Functions-Geometry	-11.291	63	.000*
Pair 5	Functions-Intro to Stats	-7.193	63	.000*
Pair 6	Exponents-Quad. Equations	0.875	63	.385
Pair 7	Exponents-Logarithms	-6.624	63	.000*
Pair 8	Exponents-Geometry	-6.192	63	.000*
Pair 9	Exponents-Intro to Stats	-3.570	63	.001*
Pair 10	Quadratic equations-Logarithms	-7.446	63	.000*
Pair 11	Quadratic equations-Geometry	-8.112	63	.000*
Pair 12	Quadratic equations-Intro to Stats	-4.658	63	.000*
Pair 13	Logarithms-Geometry	-0.313	63	.755
Pair 14	Logarithms-Intro to Stats	2.409	63	.019*
Pair 15	Geometry-Intro to Stats	2.555	63	.013*

b) Differences between teaching methods

This section presents the results from the post-hoc comparisons following MANOVA to compare the means between the topics in 2010, 2011 and 2012 wherein, according to the design, different topics were taught with different methods. Table 6.10 presents the topics and method used in teaching Math132. Univariate analyses (ANOVAs) indicated there were differences between the years for all topics (refer Table 6.25).

Tukey comparisons of means for each topic comparing outcomes for years are provided in Tables 6.32 noting that comparisons using Tukey and Least Significant differences, also lead to the same conclusions. These comparisons reveal that:

1. Students who were taught with the worked examples technique in 2011 performed significantly better than students in 2012 who were taught using problem-solving techniques, for the topics *Functions* (Mean difference WE-PS = 3.54, $p<.0005$) and *Quadratic Equations* (Mean difference WE-PS = 3.42, $p<.0005$).
2. Students who were taught with the worked example technique in 2012 for *Logarithms* performed better (mean difference PS-WE = -4.85, $p<.0005$) than students in 2011 who were taught using problem-solving techniques. An anomaly is that students taught *Exponents* with problems-solving technique in 2011 performed better than students taught via worked examples (mean difference PS-WE = 2.54, $p<.0005$).
3. For the topic *Geometry*, students in 2011 were taught with worked examples and those in 2012, taught with faded worked examples. Those taught with worked examples could be expected to experience lower cognitive load than those taught with faded worked examples, however students taught with faded worked examples performed better than those taught with worked examples (mean difference WE-FEW = -4.96, $p<.0005$).
4. For the topic, *Introduction to Statistics*, students in 2012 were taught with faded worked examples and students in 2011 were taught by problem-solving methods. Students in 2012,taught with FWE could be expected to have experienced lower cognitive load for this topic and this is what occurred in terms of better performance (mean difference PS-FEW = -4.58, $p<.0005$).

Table 6. 32 Mean difference in specific topics for KAU 2010-2012

Dependent Variable	**Year (I)**	**Year (J)**	**Mean Difference (I-J)**	**P-value**
Functions	2010	2011	-1.98	.000*
		2012	1.56	.000*
	2011	2010	1.98	.000*
		2012	3.54	.000*
Exponents	2010	2011	-1.52	.000*
		2012	1.02	.012*
	2011	2010	1.52	.000*
		2012	2.54	.000*
Quadratic Equations	2010	2011	-3.96	.000*
		2012	-0.54	.463
	2011	2010	3.96	.000*
		2012	3.42	.000*
Logarithms	2010	2011	3.34	.000*
		2012	-1.51	.000*
	2011	2010	-3.34	.000*
		2012	-4.85	.000*
Geometry	2010	2011	2.73	.000*
		2012	-2.23	.000*
	2011	2010	-2.73	.000*
		2012	-4.96	.000*
Introduction to Statistics	2010	2011	4.05	.000*
		2012	-0.54	.369
	2011	2010	-4.05	.000*
		2012	-4.58	.000*

*The mean difference is significant at 0.05 level
Based on Tukey LSD adjustments for the number of tests undertaken

Traditional		PS		WE		FWE	

6.2.6.8 Teacher Experiences

Interviews reveal positive attitudes from experienced lecturers towards using worked examples approach in their experience, but they are also pointing out the importance of the use of problem-solving and faded worked examples in mathematics. They said:

> *Students benefit a lot from using worked examples.*

Problem-solving method is essential for mathematics.

Preparing worked examples takes longer as I include all steps in the solution.

There is some effort put into preparing problem-solving method questions.

It is easy for students to follow the steps also if students are absent they have some worked examples to follow.

There are no disadvantages for worked examples.

The main advantages for problem-solving is that students need to use the strategies they have learnt in an independent way…… more practise ….. more confidence.

I prefer to use both WE and PS in my teaching.

My suggestion to use WE first, followed by PS.

6.2.7 Summary

Despite the preparatory classes in English that students undertake in their senior year, students' language skills are still very basic and for many students assistance should still provide in their first year mathematics courses.

Nevertheless, the purpose of this study was not to investigate the handicap created by the language barrier, as it has been well documented that where the student's first language is different from the predominant language of instruction, the student tends to benefit more if mathematics is taught in their first language (Ellerton & Clarkson, 1996; Setati, 2003). As this is not an option for students at KAU they should benefit greatly compared to the UOWC students from the ability to "code switch". Adler (2001) explain that code-switching refers to bilingual or multilingual settings; which simply means to switch between the language of learning and teaching and the learner's first language. The approach enables learners to harness their local

language as a learning resource as well as to increase their participation in classroom discourse. However, code-switching as a means to understanding mathematical concepts, also results in an increased cognitive load.

Worked examples are a technique designed to reduce cognitive load that is caused by some forms of problem-solving (Clark et al., 2006). With WE teaching techniques, details of the problem statement and all the necessary steps to solve the problem are described. Worked examples direct the attention of a learner to the problem stated, and the steps required to solve a particular type of problem. This reduction in cognitive load should generally make learning easier. While this statement was supported by 82 percent of students surveyed in the study indicating that learning was facilitated by the use of worked examples, there was one anomaly, the topic *Exponents* where the problem-solving group (2011) performed better than those with worked examples (2010).

This study indicated that the language of teaching mathematics has impacted on students learning if they learn in their second language. For example, (66%) perceiving their ability to be fair/very good in 2010, when they come to indicate their ability to learn mathematics in English, this percentage has declined to two percent of students in 2012. This difference is significantly different. Moreover, 148 students (75%) rated their general mathematics ability higher than their ability to learn mathematics in English (n=50, 25%). Therefore, using worked examples would be preferable in term of lowering cognitive load with reference to the language barrier and difficulty of mathematics (refer Figure 3.8) than using problem-solving and faded worked examples. This may explain that over the three phases a greater proportion of students indicated that having worked examples improved their worked examples than did problem-solving and faded worked examples.

The aim of any tertiary studies is to prepare students to have the ability to solve their own problems, so there is a need to build students' confidence through the teaching and learning process. As it has been clearly seen through this study, students love to learn mathematics with worked examples but it did not help them to become more confident in their mathematics. In the first Implementation, it appeared from student comments that students were more confident when using problem-solving

approaches. As a consequence the researcher suggested the use of faded worked examples produced better results than problem-solving and worked examples, further work is needed to explore the impact of faded worked examples on confidence.

6.3 University of Wollongong College, Context for Case Study 2

This case study was used to examine the effectiveness of using worked examples versus problem-solving approaches for teaching mathematics to ESL students in a second context and second country. The replication of the study was to enable the results to be generalised wider than would be otherwise permitted with a single case study. The intent was to identify to what extent the results were the same and different in the two contexts. The design of the study was identical to that conducted at King Abdul-Aziz University (KAU), Jeddah, Saudi Arabia but this was conducted at the University of Wollongong, College (UOWC), Australia. A within-subjects design was used to compare students' performance with the two teaching techniques. As there was the potential for the topics to vary in difficulty, in the second implementation, the teaching methods for four topics was alternated, allowing a between group comparison of the teaching methods. As for the first case study baseline data was collected and in the final implementation faded worked examples were introduced. The students differed in that students were international students, studying in a country and language other than their own, whereas in the first case study, the students were in their home country learning in a second language. The initial phase for this case study involved exploring the context, allowing the identification of similarities and differences in context.

6.3.1 Australia's changing demographics

Australia has been home to the indigenous population of Aboriginal and Torres Strait Islanders for centuries before James Cook from Britain discovered it in 1770 by. The British and Irish were the first settlers to arrive on the shores of Australia in 1788, docking at Port Stephens, on the coast of NSW. Over a century later, 'Australia' was officially born with the federation of the nation in 1901.

The Australian parliament passed the Immigration Restriction Act in 1901 (http://www.immi.gov.au/media/fact-sheets/08abolition.htm). This legislation was

“described as an Act 'to place certain restrictions on immigration” (www.immi.gov, 2012). This legislation was later known as the ‘White Only’ policy. In 1957, the act was revised to allow for non-European settlers to attain citizenship so long as they resided in Australia for over 15 years. Further reforms, such as the Migration Act 1958 abolished some of the stringent testing of migration. However, the changing point in the immigration policy was March 1966, when the ‘White Only’ policy was abolished and the borders where opened to non-European migrants (Australian immigration, 2012). According to the department of immigration, figures of migration for non- Europeans rose from 746 in 1966 to 2696 people in 1971, while the yearly migration figures for 1971 rose from 1498 to a staggering 6054 (DIC, 2012). Today Australia is home to nearly 22 million people (Australian Bureau of Statistics, 2012), from over 300 ethnic backgrounds, with over 300 languages and dialects. The five largest migrant groups are from the United Kingdom, New Zealand, Italy, Vietnam and China.

The department of immigration’s 2012-2013 immigration programs is expecting 190,000 migrants to enter Australia in 2013. The migration criterion by law is no longer racially based, but has been broken down into the following categories:

- 60,185 places for family migrants who are sponsored by family members already in Australia;
- 129,250 places for skilled migrants who gain entry essentially because of their work or business experience, business qualifications, skills or sponsorship ;
- 565 places for special eligibility migrants who are former permanent residents and have maintained close business, cultural or personal ties with Australia (http://www.immi.gov.au/media/fact-sheets /20 planning. htm).

The Australian demographic has transformed enormously over the last century from a nation that was (98%) white Anglo-Saxon of British and Irish decent, to today’s heterogeneous of cultures and races and ethnicities.

6.3.2 International students’ contribution to Australia

Australia continues to be a popular country for migrants for various reasons, such as safety, economic stability, geographic orientation, and without a doubt the tertiary education. The international student has been a part of the Australian educational

landscape since the post-World War II era (Atweh & Clarkson, 2001). Since the *Colombo Plan* for Cooperation (1949-1957) many schemes have come and gone, however the international students' interest in an Australian education remains (Atweh & Clarkson, 2001).

According to the Australian Bureau of Statistics (ABS), in 2010 there were approximately 617,000 international students in Australia (Australian Bureau of Statistics, 2012). These figures are down two percent from 2009 yet still (14%) higher than 2008 (ABS, 2012). According to ABS the top five countries of origin of international students are: the People's Republic of China (20%), India (11%), South Korea five percent, Brazil four percent, and Malaysia four percent (ABS, 2012). Although the international student numbers have declined slightly over the last year due to civil uprising and political changes in North Africa and the Middle East, the economic crisis in Europe and the slow recovery of the developed world from the GFC 2008 (Global Financial Crisis) Australia is still managing to attract a market share of six percent of the international tertiary students, which is comparable to the UK ten percent, Germany eight percent (refer Figure 6.3).

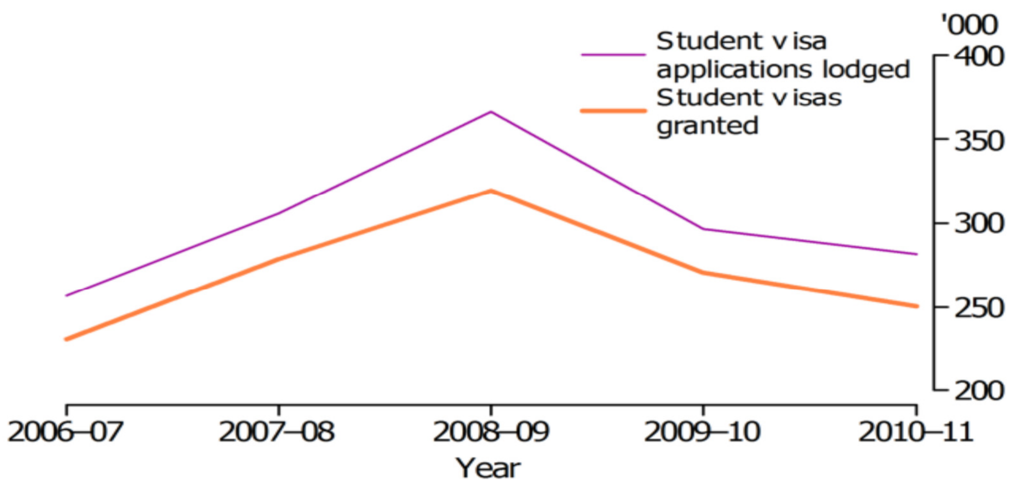


Figure 6.3 Australian student visas issued from 2006-2011
(Source: ABS website)

6.3.3 The Australian economy and International Students

The Australian economy is the thirteenth largest economy in the world, with a GDP of $1.57 trillion, it ranks number five in terms of GDP. Unemployment in 2012 is five percent, which is a modest figure when compared to eight percent in America (ABS, 2012).

Australia is rich in natural resources such as agricultural products (wheat and wool) and also rich in minerals such as gold, iron ore, natural gas and coal. Ironically, these sectors only account for three percent and five percent of GDP (http://www.tradingeconomics.com/united-states/unemployment-rate). The economic powerhouse of the Australian economy is the service sector, accounting for approximately (70%) of GDP. The sector includes tourism, financial services and education.

According to the Australian Bureau of Statistics the 617,000 international students in Australia in 2011, contributed an astounding $16.3 billion dollars to the economy. The contribution made by the international student is not isolated to the fees paid to the educational institutions. As they undertake their studies in Australia, they are renting accommodation, contributing to local communities and small businesses. They are using public transport, shopping in local retail outlets, are contributing to the tourism industry as they explore and travel the Australian landscape with friends and family. This was best exemplified by the 'Group of Eight' (Go8) in their e-article regarding the importance of international education in Australia (2012), as it states:

> International education provides significant economic benefits for Australia. It is both a major export industry and a source of domestic economic growth. Around one quarter of all higher education students in Australia are international students. International student fees more than cover costs of study and account for some 16% of total higher education revenue. Education related tourism (family and friends who visit international students) is just one of the many indirect benefits of international education to the Australian economy (p.3).

There is a need to recognise that international students play an important role in the Australian economy and the higher education system.

6.3.4 Education in Australia

Education is a responsibility that is shared among the states (Rudkin, 2005) although the Federal Government grants funding to both public and private schools. However in regards to tertiary education the funding that is provided by government bodies is supplemented by funding received by revenue generated by the charging of fees to students, more reliantly so on international students who pay on average three times what domestic students pay upfront (refer Table 6.33).

Table 6. 33 Fees for courses payable by domestic and international students

Program	Domestic Student*	International Student
Bachelor of Accounting	$ 9,792	$ 33,344
Bachelor of Engineering	$ 8,363	$ 33, 472
Bachelor of Nursing	$8,363	$24,096
Bachelor of Education	$5,868	$24,288

Domestic students offered a 10% discount if payment made before census date, or fee relief with HECS-HELP (Source: University of Melbourne 2013-2014 fees schedule)

The obvious financial/economic benefit gained by the universities as a result of the international student, has lead universities across Australia to campaign hard for the attention of the international students, and the agencies that represent them, to ensure that they are getting a fair piece of the international student market.

As the competition for international student numbers is fierce, especially as the numbers are down from previous years, universities are working to cater for the international students (ABS, 2012) by pouring resources into facilities and programs to ensure that their international students are not only comfortable but also successful on their campuses. One of the programs that have been rolled out on campuses such as that of the University of Wollongong College is their wide range of English courses available for international students wishing to continue their studies at University of Wollongong. Some of the English courses available to students at the University of Wollongong are as follows:

- IELTS test preparation
- IELTS testing
- Introductory English
- Academic English
- General English
- English for business
- English plus University
- Free conversation classes

6.3.4.1 Teaching mathematics to tertiary level ESL students in Australia

The approaches adopted by universities, such as the University of Wollongong College (UOWC), in the creation of programs to assist international students in learning English, has a great impact on the success rate of the student as they continue on in their specialised studies at university. A study conducted by the Australian Government (2008) found that (79%) of international students were studying at a tertiary level. Many of these are from non-English speaking backgrounds having to do additional English classes to ensure they meet the minimum International English Language Testing System (IELTS) score of 6. However, as discussed earlier, students who may be conversationally proficient in the English language may not be as proficient in an academic setting. This may be especially so in the case of ESL students studying mathematics.

The language of mathematics and the English language have many common terms that are not necessarily of the same meaning. For instance, the word plane, in a mathematical sense means “a flat two dimensional surface”, whereas a plane in the English language refers to an aircraft. While tertiary level ESL mathematics students are often faced with a daunting task of mastering a second language whilst learning mathematical concepts, research into the extent and effect of language in the learning of mathematics is very much in a primitive stage. As stated by Green and Brown (2005), “research is perhaps most appropriately carried out when there is uncertainty: when we recognise that we need to know more about a problem in order to solve it, or when we have identified a gap in our knowledge”(p.65). Tertiary level educators and university policy makers have realised that the success of their ESL mathematics students can only be improved with further understanding about how ESL students learn mathematics. Therefore, research funding into studies of language and

mathematics have begun to emerge (Bonnici, 2008) and appropriately so when one evaluates the gain that stands to be earned when ESL mathematics students are excelling in their fields, due to modification in pedagogy as well as the improvement in English language skills.

6.3.4.2 University of Wollongong College (UOWC)

The University of Wollongong College (UOWC) is situated in Wollongong, NSW, Australia, on the campus of the University of Wollongong (UOW). The college provides various university preparatory and bridging courses for students seeking to pursue further education at the UOW. The college is also an IELTS testing and preparation facility, as well as providing various English courses to students. The Advanced Mathematics 1 & 2 subject that is the focus of this case study is one of several subjects students will take in a course called Foundation Studies. The Foundation Studies program is specifically designed for international students as an accredited alternative to the Australian Year 12. However the program also provides Australian students with both the skills to succeed at UOW as well as guaranteed entry to UOW at successful completion of the program. Advanced Mathematics 1 & 2 subject provides minimum content of mathematics for students entering UOW degree courses in Mathematics, Engineering or IT. It is also meets the requirement for Commerce degrees in a number of universities other than UOW.

6.3.4.3 The Subject Chosen, Advanced Mathematics at UOWC

The subject surveyed for the purpose of this study was Advanced Mathematics 1 & 2. The subject was designed as part of a Foundation Program for international students wishing to pursue further education at a university level. Upon successfully completing the program students are awarded a certificate that can be credited towards their applicable degree. Specified credit can be granted for student passing Advanced Mathematics 1 & 2 such that they do not need to complete first level mathematics at the UOW.

This subject is taken over two 14 week study sessions (12 weeks of scheduled classes and 2 weeks study/examination period) and consists of 4 hours of lectures and two 2-hour tutorials per week. Students are also expected to spend at least one hour in

individual study and research for every hour of scheduled class time. The students come from various mathematics backgrounds, and the subject is designed to give the student a basic level of understanding of mathematics. The subject caters for over a hundred students per year from domestic and international backgrounds. In most recent years there has been a high concentration of students from the Middle East and subcontinent (India, Pakistan and Sri Lanka). The international students arriving at UOWC in their first year will usually undertake extensive English classes, before commencing the Advanced Mathematics subject. The students English language ability and mathematical abilities in these classrooms vary (due to the domestic and international student enrolment), and as a result educators are left with the difficult task of trying to teach mathematics effectively to many students who have English, the language of instruction, as a second language.

6.3.5 Results

The analysis of this second case study proceeds in the same manner as the case study in KAU. A baseline data collection followed by two implementations varying the teaching approaches in the same manner as the KAU case study. The same data collection tools, survey and interviews were used together with the final examination marks.

6.3.5.1 Participants

The classes in this study were somewhat smaller than in KAU classes (total n=198) with data collected from 74 students in total over the three data collections The breakdown of the 74 students studying Advanced Mathematics at UOWC is as illustrated in Table 6.34. A lower proportion of students were studying in a second language than in KAU. In 2010, (95%) of students were studying in their second language, English, but this declined to (65%) in 2011 and (54%) in 2012. In terms of the study it was important that students were matched on key factors that could affect performance outcomes. The three cohorts were close in terms of numbers ranging from 22-26 students. Over all these years approximately (30%) are from an Australian background and approximately (70%) of the students are international students, again with each cohort similar on other factors. While all the students in KAU were male, at UOWC both male (32%) and females (68%) were represented in

Advanced Mathematics 1 & 2 classes. The students that participated in the study came from varying high school strands with overall (42%) from Mathematics, (24%) from Science and (7%) from Arts, with a similar proportion in each year. Furthermore, approximately (41%) have learned their mathematics in English since Year 11 and (36%) since Year 12, again with similar proportions in each data collection.

Table 6. 34 Distribution of UOWC participants in this study

Stages	Year	Number of Students	With English 2nd language %	From Maths Strand %	Learning English from year11 &12 %
Baseline	2010	22	95.2	30	(41 & 36)
Implementation1	2011	26	65.4	58	(42 & 30)
Implementation2	2012	26	53.8	77	(34 & 46)

The analysis of the class data was not as hoped in 2011 and 2012 as the proportion of ESL students in the class dropped from (95%) in 2010 to (54%) in 2012. In this way, the analysis changed from to the analysis of a class with a majority of ESL students rather than ESL students in a foreign country. The number of ESL students was too small to effectively analyse them separately except for analyses regarding performance involving the within and between-subject design. In this instance sufficient analysis was undertaken to identify whether the results were the same or different when analysing the whole class as distinct from only the ESL students.

6.3.5.2 Impact of language on student learning

Unlike the KAU case study where perceived ability to learn mathematics declined, the UOWC students' perceived ability to learn mathematics has remained constant over the three years. In 2010, (86%) of students described their ability to learn Mathematics as fair/very good, but for 2012 this proportion was relatively constant at (84%) of students (x^2=.029, *df*=1, p=864) (refer Table 6.35).

Similarly when it comes to learning mathematics in English, the UOWC students' perceived ability is similar in each cohort, with (68%) of students perceiving their

ability to be fair/very good in 2010, and (65%) in 2011, and a comparable (69%) of students in 2012 (x^2=.003, *df*=1, p=.957)(refer Table 6.35).

Table 6. 35 Perceived ability to learn mathematics and mathematics in English

Stages	Ability to Learn Mathematics					Ability to Learn Mathematics in English			
	N	Very Poor /Poor		Fair/Very Good		Very Poor /Poor		Fair/Very Good	
		n	%	n	%	n	%	n	%
2010	22	3	13.6	19	86.4	7	31.8	15	68.2
2011	26	4	15.4	22	84.6	9	34.6	17	65.4
2012	26	4	15.4	22	84.6	8	30.8	18	69.2
Total	74	11	14.9	63	85.1	24	32.4	50	67.6

As for the KAU case study the most striking impact of learning in a second language is apparent when comparing the students rating their mathematics ability versus the rating of the mathematics ability when learning in English (refer Table 6.36). Overall, 54 students (73%) rated their general mathematics ability higher than their ability to learn mathematics in English compared to (n=20, 27%) who rated it at the same or lower level (refer Table 6.36).

Table 6. 36 Mathematics ability and mathematics ability in English UOWC

Mathematics Ability	Mathematics Ability in English				Total
	Very Poor	Poor	Fair	Very Good	
Very Poor	1	2	0	0	3
Poor	5	1	2	0	8
Fair	2	11	12	2	27
Very Good	0	2	34	0	36
Total	8	16	48	2	74

A chi-square test based on data classified as poor/very poor and fair/very good, found a significant difference (x^2=14.380, *df*=1, p<.0005) in the proportion of students being confident in learning mathematics in English as compared to learning mathematics in general. Of the 63 students who considered their ability to do Mathematics to be fair/very good only 42 (76%) considered their ability to do Mathematics in English as fair or good (refer Table 6.37).

Table 6. 37 Ability to learn mathematics and to learn mathematics in English

Ability to learn mathematics	**Ability to learn mathematics in English**				**Total**	
	Very Poor/Poor		Fair/ Very Good			
	n	%	n	%	**n**	**%**
Very Poor/Poor	9	81.8	2	18.2	11	100
Fair/Very Good	15	23.8	48	76.2	63	100
Total	24	32.4	50	67.6	74	100

6.3.5.3 Value of learning resources

In order to assess the impact of the changes in attitudes towards the subject due to the changes in teaching methods students in each year were asked to provide a rating of the perceived usefulness of all identified learning resources (refer Table 5.13 for original questions). In question 6 from Table 5.13, the frequency counts were combined for responses to ‘moderately useful’ and ‘extremely useful’. In 2011, the subject redesign led to the number of worked examples being increased in the topics *Functions*, *Quadratic Equations* and *Geometry*. The remaining three topics were left with a problem-solving orientation. As is evident in Table 6.38, a Fisher’s Exact Test, used because of small expected count size, revealed a significant difference in proportions (p=0.0188) with 24 students (92%) rating the work in practical classes as moderately or extremely useful in 2011 compared to 16 students (61%) in 2012. Furthermore, there was significant difference in proportions (Fisher’s Exact Test, p=0.0128) with 14 students (54%) rating the lectures as moderately or extremely useful in 2012 compared to 23 students (88%) in 2011.

In 2012, faded worked examples were introduced for topics *Geometry* and *Introduction to Statistics,* replacing worked examples and problem-solving respectively. The FWE were introduced to scaffold from worked examples to problem-solving because in the analysis of the 2010 data, problem-solving appeared to build student confidence (Section 6.3.5.6). As illustrated in Table 6.38 using a Fisher’s Exact Test, no significant differences in the valuing of any other resources were found between 2011 and 2012.

Table 6. 38 Perceived usefulness of resources for students learning UOWC

Learning Resources		2010 Baseline N=22			2011 WE & PS N= 26			2012 Introduce FWE N=26	
		n	%	P1*	n	%	P2^	n	%
1	Worked Examples	18	81.8	0.7352	20	76.9	0.5414	17	65.3
2	Tutor in Practical Classes	16	72.7	0.2669	23	88.4	0.0975	17	65.3
3	Work in Practical Classes	15	68.1	0.0607	24	92.3	0.0188	16	61.5
4	Practical Worksheets	15	68.1	1.0000	18	69.2	0.7554	20	76.9
5	Tutorial Assignments	13	59.0	0.7761	14	53.8	0.3929	18	69.2
6	Lecture	14	63.6	0.0822	23	88.4	0.0128	14	53.8
7	Team Learning or Group Work	10	45.4	1.0000	11	42.3	0.0929	18	69.2
8	Theory Review in Prac Classes	11	50.0	0.3806	9	34.6	0.0512	17	65.3
9	Lecture Handbook	11	50.0	1.0000	13	50.0	0.2581	18	69.2
10	Interaction With Lecturer	9	40.9	0.5729	13	50.0	0.2581	18	69.2

*P1 calculated using the Fisher's Exact Test, two-tailed comparing 2010 and 2011 data
^P2 calculated using the Fisher's Exact Test, two-tailed comparing 2011 and 2012 data

6.3.5.4 Worked example and problem-solving usage comparisons

The results in this section are from Table 5.14 (chapter 5) question 7, (*On average how many problems have you completed per week in Advanced Mathematics 1 & 2 class using worked examples as a guide?*) and question 8, (*On average how many problems have you solved per week in Advanced Mathematics 1 & 2 without worked examples?*) in the questionnaire. In terms of outcomes, the proportion of students working on average five or more problems using worked examples is significantly different, (x^2=4.72, *df*= 1, p=.030) changing from five percent in 2010 to (23%) in 2011 and is not significantly differentfrom 2010 (x^2=1.23, *df*=1, p=.26) changing to (15%) in 2012. In terms of problem-solving, five percent students during the baseline period in 2010 worked on average five or more problem-solving items per week,a significant difference (x^2=6.53, *df*=1, p=.011) compared to (35%) in 2011 and no significant difference (x^2=3.81, *df*= 1, p=.173) comparing five percent of students in 2010 to (19%) for the last implementation in 2012 (refer Table 6.39).

Table 6. 39 Average number of problems completed each week using WE or PS

No. items	2010				2011				2012				PS Total		WE Total	
	PS		WE		PS		WE		PS		WE					
	n	%	n	%	n	%	n	%	n	%	n	%	n	%	n	%
0	5	22.7	4	18.2	4	15.4	1	3.8	1	3.8	2	7.7	10	13.5	7	9.5
1-2	3	13.6	11	50.0	7	26.9	8	30.8	7	26.9	9	34.6	17	23.0	28	37.8
3-4	13	59.1	6	27.3	6	23.1	11	42.3	13	50.0	11	42.3	32	43.2	28	37.8
5 or more	1	4.5	1	4.5	9	34.6	6	23.1	5	19.2	4	15.4	15	20.3	11	14.9
Total	22	100.	22	100	26	100	26	100	26	100	26	100	74	100	74	100

Over all three years, there is not a significantly different percentage (McNemar x^2=7.941, p=.242) of students (n=15, 20%) who indicated that the average number of problems that they solved per week was five or more through mathematics problems-solving, whereas only (n=11, 14%) used worked examples in their lessons per week (refer Table 6.40).

Table 6. 40 Average number of problems using WE by PS at UOWC

		PS		
WE		< 5	≥ 5	
	< 5	51	12	63
	≥ 5	8	3	11
		59	15	74

Over the combined data for the three years a significantly different proportion of students (McNemar x^2=32.163, p<.0005) in response to question 9, (*Does using worked example approach improve your study for Advanced Mathematics 1 & 2?*) and question 10, (*Does the problem-solving approach without worked examples improve your study for Advanced Mathematics 1 & 2?*) indicated that having worked examples (72%) improved their studying mathematics moreso than did problem-solving (26%). Table 6.41 shows the combined data for the three years and Table 6.42 shows breakdown of the 'Yes' response by cohort for these two questions. UOWC students' perception that worked examples improved study is similar to that of students at KAU.

Table 6. 41 Problem-solving and worked examples improve study UOWC

Improve WE	Improve Problem-solving		Total
	No	Yes	
No	6	15	21
Yes	49	4	53
Total	55	19	74

Table 6. 42 Number of students finding PS and WE improved their study

		PS Improve		WE Improve	
	N	n	%	n	%
2010 Baseline	22	6	27.2	18	81.8
2011 (Introduce WE)	26	8	30.7	16	61.5
2012 (Introduce FWE)	26	5	19.2	19	73.0
Total	74	19	25.6	53	71.6

In response to the question 11 (refer Table 5.14), (*How would you prefer to study Advanced Mathematics 1 & 2? And explain why you prefer this?*), students' combined data overall years (refer to Table 6.43) indicated that (n=42, 57%) of students preferred to study this subject with worked examples, (n=15, 20%) of students preferred to study with problem-solving, while (n=17, 23%) preferred a mixture of worked examples and problem-solving. In comparison with KAU, students regarding their preferences are similar to UOWC students' preference with the percentage of students preferring problem-solving lower than for worked examples.

Table 6. 43 Preference for PS, WE or mix of approaches at UOWC

Stages	N	PS		WE		Mixed	
		n	%	n	%	n	%
2010 Baseline	22	3	13.6	13	59.1	6	27.3
2011 (Introduce WE)	26	7	26.9	15	57.7	4	15.4
2012 (Introduce FEW)	26	5	19.2	14	53.8	7	26.9
Total	74	15	20.3	42	56.8	17	23.0

When the students were asked to explain their preference, some students gave more than one reason (refer Table 6.44). The dominant reasons given by students for preferring worked examples were:

- Reduces anxiety (45%)
- Quicker to learn (21%)
- Helps them with revision subject materials (19%)

As for KAU students, the primary reason for preferring problem-solving related to:

- More confidence (93%)

Table 6. 44 Number of student's reasons for their learning preference 2010-2012

Reasons	**Preferred Method**					
	WE		**PS**		**Mixed WE & PS**	
	n	**%**	**n**	**%**	**n**	**%**
Reduced the anxiety of mathematics on the students	19	45.2				
Help them with revision subject materials	8	19.2				
More confidence			14	93.0		
Easy to follow	6	14.2				
Build critical thinking					6	35.2
Quicker to learn	9	21.4				
Feel math is challenge					7	41.3
Keeps you thinking differently			1	7.0	4	23.5
Total	42	100	15	100	17	100

A breakdown of the preferences for teaching method as related to students' ability to learn in English is provided in Table 6.45. There was no significant association (x^2=.097, *df*=2, p=.755) between preferred method and perceived mathematics ability. For the students who prefer WE to study; (31%) considered themselves to have weaker mathematical abilities, whereas of those who preferred problem-solving, (27%) considered their ability to learn mathematics in English as poor or very poor, and for those who preferred a mixture of WE and PS, (41%) considered their ability to learn mathematics in English as poor/very poor. These findings were less pronounced than in the KAU case study because UOWC have a mixture of Australian and International students, where for example, (76%) of students who

preferred worked examples considered themselves of poor or very poor ability to learn mathematics in English.

Table 6. 45 Mathematics ability in English against preferred method

Mathematics Ability in English	**Preferred Method**						**Total**	
	Worked Examples (WE)		**Problem-Solving (PS)**		**Mixed between (WE) & (PS)**			
	n	**%**	**n**	**%**	**n**	**%**	**n**	**%**
Very Poor/Poor	13	31.0	4	26.7	7	41.2	24	32.4
Fair/Very Good	29	69.0	11	73.3	10	58.8	50	67.6
Total	42	100	15	100	17	100	74	100

6.3.5.5 Learning Outcomes

There are many learning outcomes that can be examined when one compares teaching methods. Students were asked a series of questions (refer Q17, Table 5.17, Evaluation of using problem-solving and worked examples approaches in this subject was through 7-point likert scales) to indicate the extent to which worked examples and problem-solving assisted them with a variety of learning outcomes. The outcomes addressed are: enhances understanding; makes it quicker to study mathematics; improves my review of mathematics notes and lab work; easier to learn; increases my confidence about solving more problems; requires a lot of mental effort; makes learning more interesting; like to learn mathematics; and, reduces my anxiety. The results for the categories Mildly Agree, Agree, and Strongly Agree were combined as ‘Agree’ and compared to the combined categories of Strongly Disagree, Disagree, Mildly Disagree and Neither as ‘Disagree’. Table 6.46 shows the number of students in the combined category ‘Agree’ on the learning outcomes for WE and similarly for PS. This summary table over all years reveals that while all except for one of the items, worked examples are more favourable than problem-solving, only two of these are individually significant, “likes to learn mathematics” and “reduces my anxiety”. The one exception was for the learning outcome “increases my confidence” which for the KAU case study showed a positive association with problem-solving whereas for students at UOWC there is no difference between PS and WE in increasing confidence. For UOWC students there was no significant

difference between the WE and PS in terms of mental effort required to learn, unlike the KAU study where PS was deemed to require more mental effort.

Table 6. 46 Comparison of learning outcomes UOWC

Learning Outcomes	McNemar's	Test	WE		PS	
	x^2	p*	n	%	n	%
Enhances understanding	1.64	0.286	51	68.9	45	60.8
Quicker to Study	0.92	0.442	42	56.8	37	50.0
Improved my review of mathematics notes and lab work	1.12	0.377	33	44.6	27	36.5
Easier to Learn mathematics	2.13	0.200	31	41.9	23	31.1
Increases my confidence about solving more problems	0.04	1.000	30	40.5	29	39.2
Requires a lot of mental effort	0.81	0.473	42	56.8	37	50.0
Makes mathematics learning more interesting	4.17	0.061	38	51.4	27	36.5
Likes to learn mathematics *	6.00	0.023	32	43.2	20	27.0
Reduce my anxiety *	4.84	0.043	25	33.8	14	18.9

* Probability associated with test to assess for a change between approaches using McNemar's test
All two way tables are appended in Appendix (E)

The results indicate that students thought worked examples helps achieve*, liking mathematics more* and *reduces anxiety* more so than problem-solving. Students' suggested in relation to problem-solving that:

> *Problem-solving increases my anxiety.*
>
> *Problem-solving makes mathematics a challenge.*
>
> *I need problem-solving for my future.*
>
> *Problem-solving makes me more confident in my learning of mathematics.*

6.3.5.6 Confidence in topics

Similar to the KAU case study, in relation to each major topic, 74 students in UOWC were asked a question of the form (*How confident are you now that you can solve problem on the following topics?* Refer to Q16 Table 5.16). The response categories for this question were 'Not at all', 'Might have a little difficulty', 'Moderately Confident', and 'Could do this'. The categories 'Moderately Confident' and 'Could

do this' were combined to create a category 'Confident'. The remaining categories were classified into the new 'Not confident' category.

In terms of individual topics, in 2010 students had the most confidence for topics *Functions* (91%) and *Logarithms* (91%), in 2011 students had most confidence in *Exponents* (80%) and in 2012 students confidence was greatest for the topics *Introduction to Statistics* (92%) and *Quadratic Equations* (81%).

To test for a difference in proportions between cohorts and outcomes for the confidence on topics a two-tailed test Fisher's Exact Test was used. A Fisher's Exact Test revealed that there was a significant difference in the proportion of students perceiving themselves as confident in the topic *Logarithms*, with a lower percentage in 2011 (65%) than in 2010 (91%) (p=0.0454). Comparisons of 2011 and 2012 indicated significant differences in confidence for three of the topics. More students indicated that they were confident in 2012 in the topic of *Introduction to Statistics* (p=0.0002) (92%) than in 2011 (42%) and in the topic of *Quadratic Equations* (p=0.0095), (81%) and 2011 (42%), whereas confidence declined for the topic *Geometry* from 2011 (77%) to 2012 (42%), (refer Table 6.47).

Table 6. 47 Confidence on topics 2010-2012 UOWC

Topic	**2010 N=22**			**2011 N=26**			**2012 N=26**	
	n	**%**	**P1***	**n**	**%**	**P2^**	**n**	**%**
Functions	20	90.9	0.2603	20	76.9	0.3675	16	61.5
Exponents	18	81.8	1.0000	21	80.0	0.0746	14	53.8
Quadratic equations	5	22.7	0.2212	11	42.3	0.0095	21	80.8
Logarithms	20	90.9	0.0454	17	65.4	0.7645	19	73.1
Geometry	12	54.5	0.1310	20	76.9	0.0227	11	42.3
Introduction to Statistics	5	22.7	0.2212	11	42.3	0.0002	24	92.3

*P1 Fisher's Exact Test comparison of 2010 and 2011
^P2 Fisher's Exact Test comparison of 2011 and 2012

6.3.5.7 Impact of changing teaching approaches on performance

To examine the impact of the teaching method changing is complicated. The process of the examination is as follows:

1. The first step involves gaining a sense of the design clarifying the means and standard deviations (refer Table 6.48) associated with each of the teaching techniques and topics followed by a formal examination of differences between means. This provides for easy reference in the subsequent discussion of results. Table 6.48 summarises the results obtained from each topic test where the maximum mark was ten. The coloured rows indicate the teaching method associated with the topic in the three cohorts. As for KAU the highest observed mean across cohorts for topics *Functions*, *Quadratic Equations, Logarithms* were attained where the teaching method was WE. No topic taught with PS has the highest mean. As for KAU, the topic *Introduction to Statistics*, the highest observed mean was obtained when the teaching method was FWE. For the topic *Exponents*, KAU had the highest means when taught with PS whereas for UOWC the traditional teaching produced the best mean. For *Geometry* KAU had the best results with the FWE approach whereas for UOW it was with the WE approach.

Table 6. 48 Final topic mean marks UOWC 2010-2012

Topic	Year	N	Mean	S.D
Functions	2010	22	5.09	2.389
	2011	26	8.42	1.447
	2012	26	4.58	2.266
Exponents	2010	22	6.77	1.950
	2011	26	3.19	1.524
	2012	26	5.96	2.630
Quadratic Equations	2010	22	4.45	2.405
	2011	26	8.54	1.272
	2012	26	5.77	2.286
Logarithms	2010	22	7.32	1.492
	2011	26	2.65	1.548
	2012	26	8.42	1.332
Geometry	2010	22	4.86	2.376
	2011	26	8.69	1.225
	2012	26	8.08	1.468
Introduction to Statistics	2010	22	7.00	2.370
	2011	26	2.96	1.907
	2012	26	7.31	1.619

Traditional		PS		WE		FWE	

A plot of means for each topic is shown in Figure 6.4. The most obvious feature is the polarity of means for 2011 cohort into two distinct groups. In 2011, the means for topic *Functions*, *Quadratics Equations* and *Geometry all taught* through the WE approach are much higher than for topics *Exponents*, *Logarithms*, and *Introduction to Statistics* due to different teaching techniques, all taught with the PS approach.

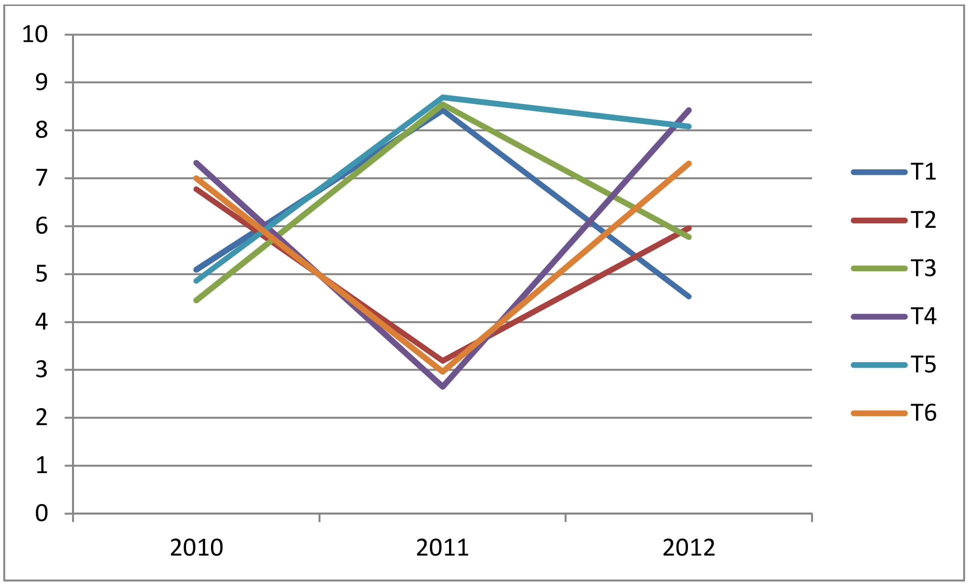


Figure 6.4 Means of topics for the UOWC class 2010-2012

The movement of the topic means from year to year (shown by non-parallel lines) suggests that there will be an interaction between year (that is the different teaching methods) and topic. This will be examined more closely in the next step of the analysis.

Another plot of the same topic means, this time showing the topics on the horizontal axis is given Figure 6.5. This plot emphasises the movement of the topic means within each cohort.

Figure 6.5 Means of topics for the UOWC class 2010-2012

2. The first analysis has six dependent variables (T1 to T6), the marks (0 lowest - 10 highest) on each of the six topic tests, and year (2010, 2011, and 2012) as the proxy independent variable. As for the KAU case study indifferent years, different teaching methods were used for different topics allowing an examination of the impact of teaching method on topic marks. As the topic tests are given to the same students, the means derived from the same students are measured on each topic, and used to examine whether there is an interaction between year and topic. This initial analysis was carried out using a Repeated Measures ANOVA, with the first step an examination of assumptions and the second an examination of the multivariate tests.

Step 1: Examination of Assumptions

The first test, the Mauchly's test of sphericity, indicated that the assumption of sphericity (that the error covariance matrix of the normalised transformed dependent variables is proportional to an identity matrix) is violated (χ^2=23.366, *df*=14, p=.055) (refer Table 6.49). While the result is borderline significant, in keeping with the analysis for the KAU case study the results suggests that there is

sufficient correlation between the dependent variables (topic marks) to carry out a MANOVA.

Table 6. 49 Mauchly's test of sphericity[b]

Within Subjects Effect	Epsilona						
	Mauchly's W	Approx. Chi-Square	df	Sig	Greenhouse-Geisser	Huynh-Feldt	Lower-bound
Topic	.713	23.366	14	.055	.906	1.000	.200

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.
a. May be used to adjust the degrees of freedom for the averaged tests of significance. Corrected tests are displayed in the Tests of Within-Subjects Effects table.
b. Design: Intercept + Year
Within Subjects Design: topic

MANOVA compares whether the means differences between the groups (i.e. cohorts) on the combination of each of dependent variable has occurred. The advantages of using MANOVA is that it 'controls' or adjusts for this increased risk of Type I error, when carrying out multiple tests. MANOVA has a number of assumptions that must be met (Tabachnick & Fidell, 2001). The assumptions are:

1. *Sample size*: The sample should have more cases in each cell than the dependent variable as this helps with violations of some other assumptions (e.g. normality). In this case there were six dependent variables and 74 cases, suggesting that sample size was adequate, to provide some robustness least other assumptions be violated.
2. *Normality*: "Although the significance test MANOVA is based on the multivariate normal distribution, in practice it is reasonably robust to modest violations of normality (except where the violations are due to outliers)" (Pallant, 2005). As advised by Tabachnick and Fidell, (2001) checking multivariate outliers involve comparing the maximum value of Mahalanobis distance, with the critical value given the number of dependent variables. In this case the critical value is 22.46 (Tabachnick & Fidell, 2001). The decision is that if the maximum value for Mahalanobis distance is less than critical value, then there are no substantial multivariate outliers. As the maximum value for

Mahalanobis distance here is 14.494 (refer Table 6.50), this assumption is not violated, there are no substantial outliers.

Table 6. 50 Residuals statistics[a]

	Minimum	Maximum	Mean	Std. Deviation	N
Predicted Value	2009.47	2012.06	2011.05	.552	74
Std. Predicted Value	-2.877	1.829	.000	1.000	74
Standard Error of Predicted Value	.096	.284	.186	.038	74
Adjusted Predicted Value	2009.33	2012.08	2011.06	.554	74
Residual	-1.355	1.437	.000	.592	74
Std. Residual	-2.195	2.327	.000	.958	74
Stud. Residual	-2.296	2.437	-.002	1.005	74
Deleted Residual	-1.539	1.576	-.002	.651	74
Stud. Deleted Residual	-2.374	2.534	-.003	1.019	74
Mahal. Distance	.772	14.494	5.919	2.831	74
Cook's Distance	.000	.144	.015	.022	74
Centered Leverage Value	.011	.199	.081	.039	74

3. *Outliers*: MANOVA is quite sensitive to univariate outliers (on each of the dependent variables i.e. topics marks) and multivariate outliers, where subjects have a strange combination of scores on the various dependent variables (topics marks). As advised by Tabachnick & Fidell, (2001) linear regression analyses with topic as the dependent variable and Year as the independent variable were used to identify if there were any outliers. No standardised residuals greater than three were identified for any of the topics.
4. *Linearity*: this assumption refers to the presence of a straight-line relationship between each pair of your dependent variables, in this case the six topics. A visual inspection of all two way scatterplots (15*3 Plots= 45, topic 1 versus topic 2, topic 1 versus topic 3 etc. for each year) of topic marks provided no evidence of nonlinearity.
5. *Homogeneity*: This assumption is important only if the researcher is intending to perform a step down analysis. This approach is used when research have some theoretical or conceptual reason for ordering dependent variables. This was not applicable in this study as there was only one independent variable ‘Year’.

6. *Multicollinearity and singularity*: MANOVA works best when the dependent variables are only moderately correlated. When dependent variables are highly correlated this is referred to multicollinearity. As seen in the table of correlations, Table 6.51 the correlations were all moderate ranging from 0.222 to 0.665 suggesting that multicollinearity was not problematic in this analysis.

Table 6. 51 Correlations between topics at UOWC

		Topic 1	Topic 2	Topic 3	Topic 4	Topic 5	Topic 6
Topic 1	Pearson Correlation	1	-.374**	.432**	-.578**	.323**	-.378**
	Sig. (2-tailed)		.001	.000	.000	.005	.001
	N	74	74	74	74	74	74
Topic 2	Pearson Correlation	-.374**	1	-.366**	.489**	-.288*	.541**
	Sig. (2-tailed)	.001		.001	.000	.013	.000
	N	74	74	74	74	74	74
Topic 3	Pearson Correlation	.432**	-.366**	1	-.437**	.316**	-.468**
	Sig. (2-tailed)	.000	.001		.000	.006	.000
	N	74	74	74	74	74	74
Topic 4	Pearson Correlation	-.578**	.489**	-.437**	1	-.222	.665**
	Sig. (2-tailed)	.000	.000	.000		.057	.000
	N	74	74	74	74	74	74
Topic 5	Pearson Correlation	.323**	-.288*	.316**	-.222	1	-.315**
	Sig. (2-tailed)	.005	.013	.006	.057		.006
	N	74	74	74	74	74	74
Topic 6	Pearson Correlation	-.378**	.541**	-.468**	.665**	-.315**	1
	Sig. (2-tailed)	.001	.000	.000	.000	.006	
	N	74	74	74	74	74	74

** Correlation is significant at the 0.01 level (2-tailed).
* Correlation is significant at the 0.05 level (2-tailed).

7. *Homogeneity of variance-covariance matrices*: The test of this assumption is generated as part of MANOVA output. The test used to assess this is Box's M Test Equality of Covariance Matrices as in Table 6.52. Tabachnick and Fidell, (2001) indicate that "If the p-value is larger than 0.001 then you have *not* violated the assumption", however they also caution that the test is too

conservative when sample sizes are large. In this case the assumption has been violated as the p-value is less than 0.0005. As the analyses to test for differences in year (teaching methods) will be further examined through univariate analyses because the evidence suggests that there are interactions, assumptions relate to those univariate analyses will be of greater importance.

Table 6. 52 Box's Test of equality of covariance matrices[a]

Box's M	92.153
F	1.924
df1	42
df2	14222.434
Sig.	.000

Tests the null hypothesis that the observed covariance matrices of the dependent variables are equal across groups.
a. Design: Intercept + Year

Further examination of Levene’s test for the equality of error variance over the three years indicates that variances are not equal over the three years for the topics *Functions*, *Quadratic Equations* and *Geometry* (refer Table 6.53). To address this Tabachnick and Fidell, suggest that a more conservative p-value be used for example, 0.001 rather than .05.

Table 6. 53 Levene's test of equality of error variances[a]

	F	**df1**	**df2**	**Sig.**
Topic 1 Functions	3.493	2	71	.036
Topic 2 Exponents	2.941	2	71	.059
Topic 3 Quadratic Equations	5.461	2	71	.006
Topic 4 Logarithms	.778	2	71	.463
Topic 5 Geometry	3.414	2	71	.038
Topic 6 Introduction to Statistics	.908	2	71	.408

Tests the null hypothesis that the error variance of the dependent variable across groups.
a. Design: Intercept + Year

Step 2: Multivariate tests

As the Sphericity assumption was violated (borderline) the MANOVA was used, assumptions checked and the interaction between topic and year as suggested by the

profile plots (Figures 6.4 & 6.5 was found to be significant using the Greenhouse-Geisser adjustment ($F_{9.061,321.672}$=44.453, p<0.0005) (refer Table 6.39).

Table 6. 54 Tests of within-subjects effects UOWC

Source		Type Sum of Squares	df	Mean Square	F	Sig.
Topic	Sphericity Assumed	147.554	5	29.511	8.690	.000
	Greenhouse-Geisser	147.554	4.531	32.568	8.690	.000
	Huynh-Feldt	147.554	5.000	29.511	8.690	.000
	Lower-bound	147.554	1.000	147.554	8.690	.004
Topic * Year	Sphericity Assumed	1509.655	10	150.965	44.453	.000
	Greenhouse-Geisser	1509.655	9.061	166.607	44.453	.000
	Huynh-Feldt	1509.655	10.000	150.965	44.453	.000
	Lower-bound	1509.655	2.000	754.827	44.453	.000
Error (topic)	Sphericity Assumed	1205.615	355	3.396		
	Greenhouse-Geisser	1205.615	321.672	3.748		
	Huynh-Feldt	1205.615	355.000	3.396		
	Lower-bound	1205.615	71.000	16.980		

When there are interactions it is not appropriate to examine overall differences in year or topics as any differences may be masked by the interactions. The significant interaction means there is a different profile for what happens in different years, depending upon which topic you are looking at and this can be seen by reference to the two profile plots (refer Figure 6.4 & Figure 6.5). The profile plot shows the interaction between year and topic as the lines are not parallel, meaning that it is not appropriate to combine all the data from topics to test for an overall difference in years. An interaction was expected because of the nature of the design. The confirmation that these interactions exist support the next stage of analysis which examines separately what happens for each topic, and because of the multiple comparisons that ensue, Tukey HSD tests that control for the Type I Error, adjusted for the number of tests undertaken will be used.

Due to the interaction the exact nature of the differences needs to be tested but it would appear that in the very least Topics 1, 3 & 5 have higher means in 2011 than 2010, whereas Topics 2, 4 & 6 have lower means in 2011. Conversely in 2012, the means for Topics 1, 3 & 5 appear lower than those for 2011, whereas means for

Topics 2, 4 & 6 appear higher. Whether the differences in 2011 and 2012 are significant needs to be tested. The next step typically involves looking at whether or not there are differences in just the WE topics, or just the PS topics. It would be possible to average over all WE topics and PS topics, however to keep the analysis the same as for KAU this is not done and an examination of the comparisons between means undertaken for each separate topic rather combining worked examples and combining problem-solving.

Following the interactions identified in this case study, two types of analyses were of interest:

a) Differences between the topics at each year level, but in particular at the baseline; and

b) Differences between years (teaching methods) for each of the topics.

a. Differences between topics:

To identify differences between topics a within subjects analysis was undertaken with subsequent paired t-tests to identify which of the topics were different. The baseline (2010) need to be examined marks to see if there are differences between topics because in 2011 and 2012 the topics were taught with different methods. If there are differences between the topics in the baseline data collection, then it is difficult to determine whether or not there are differences between the problem-solving and worked example approaches in the first and second experimental implementations. In Figures 6.4 and 6.5 there appears to be differences in some topics in some years but not in others.

I. 2010-Baseline

To compare average topic marks a MANOVA and follow-up post-hoc tests were used. To tease out higher level interactions in MANOVA, smaller ANOVA models as is appropriate when there are interactions which include only the independent variable which was significant can be used in separate analyses and followed by post-hoc tests. Univariate analyses (ANOVAs, refer Table 6.55) suggest that there are differences between years for all topics.

Table 6. 55 Univariate tests UOWC

Dependent Variable		Sum of Squares	df	Mean Square	F	Sig.
Topic 1 Functions	Contrast	223.003	2	111.502	26.344	.000
	Error	300.510	71	4.233		
Topic 2 Exponents	Contrast	174.231	2	87.115	19.897	.000
	Error	310.864	71	4.378		
Topic 3 Quadratics Equations	Contrast	212.333	2	106.167	25.768	.000
	Error	292.531	71	4.120		
Topic 4 Logarithms	Contrast	481.659	2	240.829	113.235	.000
	Error	151.003	71	2.127		
Topic 5 Geometry	Contrast	196.579	2	98.289	33.235	.000
	Error	209.976	71	2.957		
Topic 6 Intro to Statistics	Contrast	299.351	2	149.676	38.714	.000
	Error	274.500	71	3.866		

The F tests the effect of Year. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

Post-hoc and pre-planned comparisons compare all the possible paired combinations of the independent variable groups (Mwitondi, 2012). The descriptive statistics suggests when the topics were taught using the traditional approach 2010 (refer chapter 5) the means are low compared to those for the WE approach taught in 2011, except for topic 2 (*Exponents)*. Conversely the means for topics taught with problem-solving all appeared lower than when taught in 2010 except for topic 2 (*Exponents)*. Both of these changes are in accord with cognitive load theory that suggests that PS results in higher cognitive load than worked examples. In Table 6.56, the topics are arranged from least difficult (highest means) to most difficult (lowest means).

Table 6. 56 Descriptive statistics for UOWC 2010

Topic	Topics Name	N	Mean *	Mean	Std. Deviation
4	Logarithms	22	7.32		1.49
6	Introduction to statistics	22	7.00		2.37
2	Exponents	22	6.77		1.95
1	Functions	22		5.09	2.39
5	Geometry	22		4.86	2.38
3	Quadratic Equation	22		4.45	2.41

Valid N (list wise)
*Means are presented as homogeneous subsets

This is confirmed with paired T-tests as reported in Table 6.57. As for the KAU case study, in 2010 there is no significant difference between the easiest topic *Logarithms*, *Introduction to Statistics* and *Exponents*, although for KAU *Functions* was also similar in difficulty. At UOWC *Functions* is the next easiest topic, and equivalent in difficulty to *Geometry*, both of which are significantly different from *Exponents* (higher). As for KAU *Quadratic Equations* appears to be the most difficult topic, with the means different to the closest topic *Geometry*.

Table 6. 57 Outcomes of paired samples T-tests for UOWC 2010 means

Topics Comparisons		t	df	p- value (2-tailed)
Pair 1	Functions-Exponents	-3.713	21	.001*
Pair 2	Functions-Quad. Equations	1.022	21	.318
Pair 3	Functions-Logarithms	-3.590	21	.002*
Pair 4	Functions-Geometry	0.364	21	.719
Pair 5	Functions-Intro to Stats	-3.398	21	.003*
Pair 6	Exponents-Quad. Equations	3.769	21	.001*
Pair 7	Exponents-Logarithms	-0.940	21	.358
Pair 8	Exponents-Geometry	2.947	21	.008
Pair 9	Exponents-Intro to Stats	-0.479	21	.637
Pair 10	Quad. Equations-Logarithms	-4.798	21	.000*
Pair 11	Quad. Equations-Geometry	-0.483	21	.634
Pair 12	Quad. Equations-Intro To Stats	-3.450	21	.002*
Pair 13	Logarithms-Geometry	4.710	21	.000*
Pair 14	Logarithms-Intro to Stats	0.576	21	.570
Pair 15	Geometry-Intro to Stats	-2.889	21	.009*

II. 2011-Implementation I

To compare average topic marks in 2011 a MANOVA (Multivariate Analysis of Variance) was initially used (Step 2) but a significant interaction meant that analysis of differences in marks needed to be undertaken separately for each topic and time period. That is, either pre-planned contrasts or post-hoc tests were needed to confirm impressions from Figure 6.5 and 6.6 that there are differences between the topics and that these differences were associated with the method used, worked examples versus problem-solving. The more conservative approach of post-hoc tests, adjusted for the number of tests was undertaken.

In 2010, the topic *Quadratic Equations* was the worst performing topic (lowest mean) in both case studies, whereas in 2011, taught with worked examples, it has the second highest mean of all topics at UOWC and highest at KAU. Also noteworthy is *Introduction to Statistics* achieving effectively the equal highest mean when taught in 2010 (traditional method) but when taught with PS the topic mean, as for KAU, is one of the lowest in 2011 (refer Table 6.58).

Table 6. 58 Descriptive statistics for UOWC 2011

Topic	Topic Name	N	Method	Mean*	Mean	Std. Deviation
5	Geometry	26	WE	8.69		1.225
3	Quadratics Equations	26	WE	8.54		1.272
1	Functions	26	WE	8.42		1.447
2	Exponents	26	PS		3.19	1.524
6	Intro to statistics	26	PS		2.96	1.907
4	Logarithms	26	PS		2.65	1.548

Valid N (list wise)

*Means are presented as homogeneous subsets

Differences are confirmed with paired T-tests as reported in Table 6.59. In 2011, *Functions*, *Quadratic Equations* and *Geometry* were taught with worked examples and the remaining topics with problems-solving. *Geometry* now the best performing topic but is not significantly different in average marks to all other topics. *Quadratics Equations* and *Functions* that can be considered equivalent ranking second in average marks are not significantly different to all other topics. The last three topics: *Exponents*, *Logarithms*, *Introduction to Statistics* and, all taught with problem-solving, appear to be equivalent in difficulty with no significant differences between them.

Table 6. 59 Outcomes of paired samples T-tests for UOWC 2011 means

Topics comparisons		t	df	P-value (2-tailed)
Pair 1	Functions-Exponents	12.143	25	.000*
Pair 2	Functions-Quad. Equations	-0.282	25	.780
Pair 3	Functions-Logarithms	13.176	25	.000*
Pair 4	Functions-Geometry	-0.0689	25	.497
Pair 5	Functions-Intro to Stats	13.248	25	.000*
Pair 6	Exponents-Quad. Equations	-15.250	25	.000*
Pair 7	Exponents-Logarithms	1.383	25	.179
Pair 8	Exponents-Geometry	-18.655	25	.000*
Pair 9	Exponents-Intro to Stats	0.550	25	.587
Pair 10	Quad. Equations-Logarithms	14.527	25	.000*
Pair 11	Quad. Equations-Geometry	-0.464	25	.646
Pair 12	Quad. Equations-Intro to Stats	12.177	25	.000*
Pair 13	Logarithms-Geometry	-18.539	25	.000*
Pair 14	Logarithms-Intro to Stats	-0.712	25	.483
Pair 15	Geometry-Intro to Stats	15.837	25	.000*

III. 2012-Implementation II

Table 6.60 reports the summary statistics and teaching method in 2012 for six topics in descending order of mean. It was suggested in Figure 6.5 and 6.6 that the there are differences between the topics in 2012. This is confirmed with examination of the means in Table 6.60 and paired T-tests as reported in Table 6.61.

Table 6. 60 Descriptive statistics for UOWC 2012

Topic	Topic name	N	Method	Mean*	Mean	Mean	Std. Deviation
4	Logarithms	26	WE	8.42			1.332
5	Geometry	26	FWE	8.08	8.08		1.468
6	Intro to Statistics	26	FWE		7.31		1.619
2	Exponents	26	WE			5.96	2.630
3	Quadratics Equations	26	PS			5.77	2.286
1	Functions	26	PS			4.58	2.266

N (list wise)
*Means are presented as homogeneous subsets

In 2012, the ordering of topic means at UOWC is the same as for KAU. The easiest topic can be considered to be *Logarithms* (WE) and then *Geometry* (FWE) and *Introduction to Statistics* (FWE). These in turn are significantly different to *Exponents* (WE), which is equivalent in marks to *Quadratic Equations* (PS). *Functions* now taught using problem-solving is not significantly different (and lower in marks) than all other topics. This is confirmed with paired T-tests as reported in Table 6.61.

Table 6. 61 Outcomes of paired samples T-tests for 2012 means

Topics comparisons		t	df	P-value (2-tailed)
Pair 1	Functions-Exponents	-1.765	25	.090
Pair 2	Functions-Quad. Equations	-1.900	25	.069
Pair 3	Functions-Logarithms	-7.767	25	.000*
Pair 4	Functions-Geometry	-7.323	25	.000*
Pair 5	Functions-Intro to Stats	-4.725	25	.000*
Pair 6	Exponents-Quad. Equations	0.271	25	.789
Pair 7	Exponents-Logarithms	-4.411	25	.000*
Pair 8	Exponents-Geometry	-3.462	25	.002*
Pair 9	Exponents-Intro to Stats	-2.235	25	.035*
Pair 10	Quad. Equations-Logarithms	-6.703	25	.000*
Pair 11	Quad. Equations-Geometry	-4.779	25	.000*
Pair 12	Quad. Equations-Intro to Stats	-2.678	25	.013*
Pair 13	Logarithms-Geometry	0.858	25	.399
Pair 14	Logarithms-Intro to Stats	2.633	25	.014*
Pair 15	Geometry-Intro to Stats	1.467	25	.155

b. Differences between teaching methods:

This section presents the results from the post-hoc comparisons following MANOVA to compare the means between the topics in 2010, 2011 and 2012. Table 6.62 presents the topics and method used in teaching Advanced Mathematics 1 & 2. Tukey comparisons of means for each topic comparing outcomes for years are provided in Tables 6.62, noting that Scheffe, Tukey and Least Significant differences, also lead to the same conclusions. These comparisons reveal that:

1. As expected students who were taught with worked example techniques in 2011 performed significantly better than students in 2012 who were taught using problem-solving techniques, for the topics *Functions* (Mean difference WE-PS=3.85, $p<.0005$) and *Quadratic Equations* (Mean difference WE-PS= 2.77, $p<.0005$).
2. As expected students who were taught with worked example techniques in 2012 for *Logarithms* (Mean difference PS-WE=-5.77, $p<.0005$) and *Exponents* (Mean difference PS-WE=-2.77, $p<.0005$) performed better in 2012 than students in 2011 who were taught using problem-solving techniques.
3. For the topic *Geometry* there was no significant difference in mean marks (Mean difference FWE-WE=0.62, $p=.201$) for those students in 2011 who were taught with worked examples when compared to students in 2012 who were taught with faded worked examples.
4. For the topic, *Introduction to Statistics*, those taught via faded worked examples in 2012 had higher mean marks (Mean difference PS-FWE=-4.35, $p<.0005$) than those taught by problem-solving techniques in 2011. Theory would suggest that this is because students in 2012 taught with faded worked examples would have experienced lower cognitive load for this topic.

Given the classes at UOWC had a large domestic student composition in 2011 and 2012, analysis was undertaken to examine topic marks with ESL students only. This was possible because of the within-subject design with respect to marks. As no conclusions are altered when examining the ESL only students, these results are not reported and the analyses reported in accord with earlier analysis include all data.

Table 6. 62 Mean difference in specific topics UOWC 2010-2012

Dependent Variable	**Year (I)**	**Year (J)**	**Mean Difference (I-J)**	**Sig.**
Function	2010	2011	-3.33	.000*
		2012	0.51	.391
	2011	2010	3.33	.000*
		2012	3.85	.000*
Exponents	2010	2011	3.58	.000*
		2012	0.81	.185
	2011	2010	-3.58	.000*
		2012	-2.77	.000*
Quadratic Equations	2010	2011	-4.08	.000*
		2012	-1.31	.029+
	2011	2010	4.08	.000*
		2012	2.77	.000*
Logarithms	2010	2011	4.66	.000*
		2012	-1.10	.011+
	2011	2010	-4.66	.000*
		2012	-5.77	.000*
Geometry	2010	2011	-3.83	.000*
		2012	-3.21	.000*
	2011	2010	3.83	.000*
		2012	.062	.201
Introduction to statistics	2010	2011	4.04	.000*
		2012	-0.31	.591
	2011	2010	-4.04	.000*
		2012	-4.35	.000*

*Based on Tukey adjustments for the number of tests undertaken
+ These tests were not significant for the topic *Quadratic Equations* and Logarithms with the only ESL student group between the baseline 2010 and 2012. These changes do not impact on conclusions.

Traditional		WE		PS		FWE	

6.3.5.8 Teacher Experiences

Interviews reveal positive attitudes from experienced lecturers towards the worked examples approach given their experience but they also pointed out the importance of problem-solving and faded worked examples in mathematics. They said:

> *Students use worked examples as a guide for solving equivalent math problems.*

Students don't like the problem-solving method. They do prefer worked examples but it is hard for me to do all topics using worked examples.

Preparing worked examples take longer than problem-solving and faded worked examples.

Students need to do maths problem by themselves.

I think students like to start with faded worked examples instead of problem-solving.

The main disadvantage of worked examples is that students become lazy waiting for the teacher help.

The main advantage for problem-solving is that student depends on themselvess to do research for solutions.

Always, I start my lesson with worked examples then give students examples with incomplete steps then give them a problem to solve without any help.

I feel students in worked examples lessons are more active than in problem-solving.

6.3.6 Summary

This section presented the results from data conducted at UOWC for 74 students studying Advanced Mathematics 1 & 2. Based on the results drawn from the participants' responses to the survey questions, it has been seen that students' perceived ability to learn mathematics has remained constant over the three years. In 2010, (86%) of students found their ability to learn Mathematics as fair or very good, and this proportion remained relatively constant at (84%) of students in 2012.Similarly when it comes to learning mathematics in English UOWC students' perceived ability is similar in each cohort, with (68%) of students perceiving their ability to be fair or very good in 2010, and a comparable (69%) of students in 2012.

Over all three years a higher percentage of students (n=15, 20%) indicated that the average number of problems solved that they solved per week was five or more through mathematics problems-solving, whereas only (n=11, 14%) used worked

examples in their lesson per week. Moreover, over the three phases a greater proportion of students indicated that having worked examples improved their work than did problem-solving.

It is now known that the UOWC students surveyed preferred to study their subject with worked examples instead of problem-solving. Fifty-seven percent of students preferred the use of worked examples in their studies, twenty percent preferred the use of problem-solving, while twenty-three percent of students preferred a combination of the two methods. More students have agreed that worked examples have positive effects on their way of learning mathematics when English is their second language. Not only did students prefer to use worked examples, their performance in examinations when worked examples were used was also better except for one topic (*Exponents*) compared with 2010. When students were taught with problem-solving their results declined except for one topic (*Quadratics Equations*) compared with 2010. One explanation for these findings is cognitive load theory that suggests Cognitive Load will be lower when the germane cognitive load and intrinsic cognitive load are lower.

In accord with cognitive load theory, for all three topics students' performance when taught with worked examples was better than when taught with problem-solving. Topics taught with worked example techniques in 2011 had better marks than topics in 2012 when they were taught using problem-solving techniques. Students who were taught with worked example techniques in 2012 for *Logarithms* and *Exponents* performed better in 2012 than students in 2011 who were taught using problem-solving techniques. These findings are in accord with cognitive load theory.

Faded worked examples could be this way as it has been seen through this study the students performed better in topic 5, (*Geometry*) and topic 6 (*Introduction to Statistics*) those taught via faded worked examples in 2012. For the KAU case study there was a significant difference between 2011 and 2012 in topic 6 (*Introduction to Statistics*) as students obtained higher mean marks when taught with FWE than those taught by problem-solving techniques in 2011, as students in 2012 should have experienced lower cognitive load for this topic. Similarly, for the UOWC classes, *Introduction to Statistics* was taught via faded worked examples in 2012 had higher

mean marks than those taught by problem-solving technique in 2011. Not just that Faded worked examples also, build confident on students as it was evident that students indicated they are more confident with topics *Geometry* and *Introduction to Statistics* when the faded worked examples were introduced to them in 2012 than in 2010 and 2011. A Fisher's Exact Test confirmed the significant difference between the confidence on both topics respectively *Geometry* (0.0227), *Introduction to Statistics* (0.0002). Also, a sign test revealed the probability that the confidence would be higher overall topics was (0.0044) and this was also true comparing 2010 (traditional) and 2012 (Introduction of Faded worked examples).

6.4 Conclusion of the Chapter

This chapter presents the results from two case studies in two different contexts. The two case studies both highlighted the importance for ESL students with their difficulties in learning mathematics at tertiary level of using techniques that reduce cognitive load so as to make learning easier. With a few exceptions the findings for one case study are similar to the other, even though the contexts are quite different. In the final chapter the results will be drawn to together to identify where findings are in accord between the cases and where there are discrepancies.

7 CONCLUSION AND DISCUSSION

7.1 Introduction

Challenges facing ESL students include the complexity of mathematical learning and in particular learning in a second language while the challenge for teachers includes how to best design material for efficient learning. Cognitive Load Theory (CLT) has made a great contribution to the understanding of the learning process as demonstrated in this thesis wherein the effectiveness of using worked examples versus problem-solving approaches has been explored.

Issues surrounding the learning process of ESL students has come to light with the increased understanding of the function of working memory and the susceptibility with overload, and the impact such overload has on the development of schemas and the automation process in students, particularly tertiary level ESL students. A by-product of CLT and the subsequent research have motivated educators to understand the impact lesson plans and materials design have on the way in which ESL students learn. By educating teachers/educators about the various types of cognitive loads, and their ability to manage extraneous cognitive load and perhaps even germane cognitive load with simple modification to the materials used to present and assess students, it is possible to improve learning in a variety of ways. Strategies can be implemented by educators to reduce cognitive load in ESL students have meant a move away from a means-end analysis of teaching toward a more structured worked examples approach.

The purpose of this concluding chapter is to bring together the findings from the two case studies, to triangulate upon answers to the questions addressed. It involves examination of the similarities and differences in outcomes between the two case studies in finer detail and hence to report from the study on what is most generalisable and what areas require further clarification. It seeks to do this by examining the similarities and differences in outcomes, and as it do draws attention to differences within the cases that may contribute to any discrepant findings.

7.2 Similarities and differences between contexts

The context within which students study or the baseline from which they start can sometimes limit or enhance the possibility for change to be measured. Certainly finding that outcomes hold over different contexts are supportive of the results being generalisable, more so than would occur with a single case study. In this section the contexts within which learning took place are compared.

7.2.1 Ability to learn Mathematics

Students in KAU revealed a decline over the three years in their perceived ability as fair/very good to learn mathematics from (83%) 2010 of students to (31%) in 2012, even though there was no discernible change in the composition of the class. The students studying at UOWC, remained constant in their perceived ability to learn mathematics with (86%) of students reporting a fair/very good ability to learn in mathematics in 2010 and (85%) in 2012, despite there being a change in composition of the classes. It would appear the international students abroad at UOWC were slightly higher in their perceived ability to learn mathematics in general.

7.2.2 Ability to learn mathematics in English

At KAU students' ability to learn mathematics in English was seen to be lower than their ability to learn in mathematics. Students perceived ability to learn mathematics in English was also seen to decline with (67%) of students perceiving their ability to be fair/very good in 2010, declining to two percent of students in 2012. As for the ability to learn mathematics when it comes to learning mathematics in English UOWC students' perceived ability is relatively constant over the three cohorts, with (68%) of students perceiving their ability to be fair/very good in 2010, and a comparable (69%) of students in 2012.

7.2.3 Value of learning resources

The initial valuing of resources by students at KAU and UOWC was similar. The valuing of resources in terms of being moderately or extremely useful ranged from 35percent of students (lectures) to (80%) of students (work in practical classes) at KAU, whereas UOWC students ratings were from (41%) (Interaction with lecturer) to (82%) (Worked examples). No one resource was highly regarded by either groups

of students and only (50%) or fewer students valued three of the KAU resources (lectures, lecture handbook, theory review in practical classes) whereas only (50%) or fewer valued as moderately or extremely useful four of the resources as UOWC (team learning or group work, interaction with lecturer, theory review in practical classes, lecture handbook) as in Table 7.1.

Table 7. 1 Usefulness of resources for students learning at KAU & UOWC 2010

Learning Resources		Rank KAU Resource	2010 KAU Baseline N=66		2010 UOWC Baseline N=22		Rank UOWC Resource
			n	**%**	**n**	**%**	
1	Work in Practical Classes	1	53	80.3	15	68.1	3.5
2	Worked Examples	2	50	75.8	18	81.8	1.0
3	Practical Worksheets	3	49	74.2	15	68.1	3.5
4	Tutor in Practical Classes	4	48	72.7	16	72.7	2.0
5	Tutorial Assignments	5	47	71.2	13	59.0	6.0
6	Team Learning or Group Work	6	40	60.6	10	45.4	9.0
7	Interaction With Lecturer	7	39	59.1	9	40.9	10
8	Theory Review in Practical Classes	8	32	48.5	11	50.0	7.5
9	Lecture Handbook	9	29	43.9	11	50.0	7.5
10	Lecture	10	23	34.8	14	63.6	5.0

In 2011 when three of six topics had worked examples introduced and three were taught by problem-solving, the valuing of work in practical classes increased significantly from (80%) of students to (96%) of students at KAU and similarly the valuing of worked examples increased from (76%) to (93%). Students at UOWC on the other hand showed no significant differences in valuing of resources from one year to the next.

There were suggestions early in the study that problem-solving appeared to build student confidence and as a consequence FWEs were introduced replacing one topic for worked examples and one for problem-solving respectively for both KAU and UOWC to trial scaffolding from worked examples to problem-solving. No significant differences in the valuing of resources were found between 2011 and 2012 for KAU students. For UOWC students a significant difference in proportions of students finding work in practical classes useful with a declines from (92%) of students rating

the work in practical classes as moderately or extremely useful in 2011 compared to (62%) students in 2012. The proportion of students at UOWC valuing the lectures was also significantly different and showing a decline from (88%) in 2011 to (54%) in 2012.

7.2.4 Worked example and problem-solving comparisons

In the baseline phase, 2010, the number of students attempting five or more problem-solving questions (21%) and worked examples (15%) in KAU appears to be higher than in UOWC for problem-solving five percent, and WE five and those attempting no WE, nine percent at KAU and (18%) at UOWC (refer Table 7.2) despite students at UOWC being offered more examples (refer Table 5.8 & Table 5.9).

Table 7. 2 Completing 5 or more WE and PS questions AT KAU & UOWC 2010

No. items	KAU 2010				UOWC 2010			
	PS		WE		PS		WE	
	n	%	n	%	n	%	n	%
0	10	15.2	6	9.1	5	22.7	4	18.2
1-2	15	22.7	26	39.4	3	13.6	11	50.0
3-4	27	40.9	24	36.4	13	59.1	6	27.3
5 or more	14	21.2	10	15.2	1	4.5	1	4.5
Total	66	100.	66	100	22	100.	22	100

In the KAU case study the number of students completing 5 or more problems-solving questions fell significantly from (21%) in 2010 to seven percent in 2011whereas the number of students completing 5 or more worked examples per week significantly increased from (15%) in 2010 to (31%) in 2011. At UOWC the number of students completing 5 or more problem-solving questions in contrast to KAU significantly increased from five percent during 2010 to (35%) in 2011 while the number of students completing 5 or more worked examples significantly increased between from four percent 2010 to (23%) in 2011. The number of KAU students doing no problem-solving questions increased from (15%) in 2010 to (55%) in 2011and for worked examples increased from nine percent to (21%), whereas for UOWC students the number of students doing no problem-solving questions appeared constant (23%) in 2010 and (15%) in 2011, whereas the number completing

no worked examples fell from (18%) in 2010 to four percent in 2011 (refer Table 7.3).

Table 7. 3 Completing 5 or more WE and PS questions AT KAU & UOWC 2011

No. items	KAU 2011				UOWC 2011			
	PS		WE		PS		WE	
	n	%	n	%	n	%	n	%
0	37	54.5	14	20.6	4	15.4	1	3.8
1-2	13	19.1	12	17.6	7	26.9	8	30.8
3-4	13	19.1	21	30.9	6	23.1	11	42.3
5 or more	5	7.4	21	30.9	9	34.6	6	23.1
Total	68	100.	68	100	26	100	26	100

In 2012 faded worked examples were introduced for two topics, replacing one topic taught by problems-solving topic and one topic taught with worked examples. In the KAU study there were significant changes in the number of students completing 5 or more worked examples or problems-solving questions between 2011 and 2012. In 2012 (16%) of students completed five or more PS questions and (38%) completed 5 or more WE. At UOWC, there was no significant difference in the number of students completing five or more worked examples or five or more problems-solving questions completed in 2011compared to 2012, with (19%) completeing five or more problem-solving questions and only (15%) completing five or more worked examples, less than half that of KAU (refer Table 7.4). So, It would appear that KAU students became less engaged with doing worked examples in 2011 than they were in 2012, wheras UOWC seem to be more engaged.

Table 7. 4 Completing 5 or more WE and PS questions at KAU & UOWC 2012

No. items	KAU 2012 PS		KAU 2012 WE		UOWC 2012 PS		UOWC 2012 WE	
	n	%	n	%	n	%	n	%
0	10	15.6	12	18.8	1	3.8	2	7.7
1-2	14	21.9	15	23.4	7	26.9	9	34.6
3-4	28	43.8	13	20.3	13	50.0	11	42.3
5 or more	12	15.7	24	37.5	5	19.2	4	15.4
Total	64	100	64	100	26	100	26	100

7.2.5 Confidence in topics

In the baseline 2010, KAU students were highly confident, with over (75%) indicating that they could do problems for the topics *Functions* (80%) and *Exponents* (76%), whereas UOWC students in 2010 were highly confident for three topics *Functions* (91%), *Exponents* (82%), and *Logarithms* (91%). KAU had two topics where students were low in confidence *Geometry* (43%) and *Introduction to Statistics* (34%) whereas UOWC has two topics where students were extremely low in confidence *Quadratic Equations* (23%) and *Introduction to Statistics* (23%).

Table 7. 5 Comparing confidence between KAU & UOWC 2010

Topic Name	Rank	KAU 2010 N=66 %	Rank	UOWC 2010 N=22 %
Functions	1	80.3	1	90.9
Exponents	2	75.7	1	81.8
Quadratic Equations	3	74.2	2	22.7
Logarithms	4	72.7	1	90.9
Geometry	5	43.9	2	54.5
Introduction to Statistics	6	34.8	3	22.7

When taught with the worked examples approach in the proportion of KAU students perceiving themselves as a confident in the topic *Functions*, increased from (80%) in 2010 to (95%) in 2011. Whereas there was a significant difference in the proportion of UOWC students perceiving themselves as confident in the topic *Logarithms*, with (91%) confident in 2010 declining when taught by problem-solving approach to (65%) in 2011.

In 2012 students were introduced to faded worked examples for two topics, there were no changes in confidence at KAU. At UOWC in 2011 when students were taught *Introduction to Statistics* with the problem-solving approach (42%) indicated that they were confident whereas in 2012 when taught with FWE (92%) indicated they were confident they could do the problems. There was also a change in confidence for the topic *Quadratics Equations*, in 2011 when taught by worked examples (42%) of students were confident whereas in 2012 when taught by the problem-solving approach (81%) reported being confident doing problems in that topic areas. Also, significant change for UOW was for *Geometry* (77%) in 2011 to (42%) in 2012.

7.2.6 Difficulty of topics

In Table 7.6, topics are ranked such that if there were no significant difference in means marks they are given the same rank, that is they are considered to be the same in terms of difficulty.This ranking is done separately for each case study. In the baseline phase for KAU the easiest topics were *Exponents*, *Introduction to Statistics*, *Functions* and *Logarithms*. For UOWC three of these same topics were ranked as easiest topics*Logarithms*, *Introduction to Statistics* and *Exponents.* For KAU the middle topic is *Geometry* and the most difficult is *Quadratic Equations*, whereas for UOWC, *Functions* is the next easiest topic, and equivalent in difficulty to *Geometry*. *Quadratic Equations* appear to be the most difficult topic. Viewed in this manner there is a striking similarity between the KAU students and the UOWC students in terms of the difficulty with the various mathematics topics.

Table 7. 6 Comparing the difficulty of topics between KAU & UOWC 2010

Topic Name	**KAU N=66**			**UOWC N=22**		
	Rank	Mean	Std. Deviation	Rank	Mean	Std. Deviation
Exponents	1	6.82	2.06	1	6.77	1.95
Intro to Statistics	1	6.68	2.43	1	7.00	2.37
Logarithms	1	6.41	2.28	2	7.32	1.49
Functions	1	6.08	2.43	1	5.09	2.39
Geometry	2	5.77	2.63	2	4.86	2.38
Quadratic Equations	3	4.85	2.57	2	4.45	2.41

Most difficult		Middle difficult		Easy	

7.3 Finding answers for the research question

In conclusion the usefulness of WE, PS and FWE is addressed in terms of learning outcomes, with a special focus on confidence and performance.

7.3.1 Learning Outcomes

- **Question one:** How do students experience in terms attributes such as anxiety, ease of learning, understanding, enjoyment, mental effort, speed of learning and confidence associated with the learning the use of worked examples and problem-solving approaches when attempting to achieve the learning and teaching objectives of their tertiary mathematics subjects?

Over the three phases in case study of KAU a greater proportion of students indicated that having worked examples (80%) improved their studying mathematics than did problem-solving (20%). In UOWC over the three phases a greater proportion of students indicated that having worked examples (72%) improved their studying mathematics than did problem-solving (26%). In terms of perceived learning outcomes significant differences between worked examples and problem-solving were found:

- In the KAU case study, worked examples more so than problems-solving was found to enhance understanding, quicker to study, improved my review of mathematics notes and lab work, easier to learn mathematics, requires a less mental effort, makes mathematics learning more interesting, likes to learn mathematics and reduces anxiety.
- In the UOWC case study worked examples more so than problems solving were found to cause students to likes to learn mathematics more and to reduce anxiety.
- In the KAU case study, problem-solving was seen to increase my confidence about solving more problems, but not so for the UOWC case study.

For UOWC all other learning outcomes, while not significant were in the direction that worked examples were more likely to achieve the outcomes than problem-solving. Both case studies revealed that students like to learn mathematic with worked examples more so than problem-solving even though at least KAU students

agreed that problem-solving increases their confidence in learning mathematics. Students appear to have more positive experiences in terms of learning outcomes when worked examples rather than problem-solving is used.

The difference in learning outcomes may be due to the KAU students being more engaged in terms of completing more worked examples and more problem-solving question than the UOWC students, although would need to be investigated further. Upon completion of this thesis, there is an overall consensus that ESL students, irrespective of the country of study, have found based on their experience terms of perceived learning outcomes of using worked examples and problem-solving that it is advantageous in terms of learning mathematics for them to have worked examples introduced into the teaching regime.

Student sentiments or comments regarding the use of worked examples was synonymous with reduction in anxiety, “problem-solving increases my anxiety”, “worked examples reduces my stress when I learn math”. Students perceived problem-solving to be akin to analytical skill development, critical thinking and challenge “problem-solving makes mathematics a challenge” in the subject.

7.3.2 Confidence

- **Question two:** Are there benefits in terms of greater confidence that a student can successfully complete problems that can be attributed to either worked examples or problem-solving?

Confidence in learning is thought to be a subconscious tool the most effective learners use to keep them driven in pursuit of an end result (Nordin et al, 2012). If the learner has no confidence in their ability to reach their end goal, they will forfeit early in the process and fail. Therefore, building the students confidence in their ability to understand not only language but also mathematics is of great importance. Students have cited that they had an increased level of confidence considering their confidence to “do” problems in each topics area.

In 2011 when problem-solving was used for KAU students confidence changed significantly for two topics, *Functions* and *Exponents,* the first taught with WE and the second with PS whereas for UOWC confidence decreased for one topic,

Logarithms when taught with PS as in Table 7.7. There was a significant difference in the proportion of KAU students perceiving themselves as a confident in the topic *Functions*, with higher percentage in 2011 (95%) than 2010 (80%). Whereas there was a significant difference in the proportion of UOWC students perceiving themselves as confident in the topic *Logarithms*, with lower percentage in 2011 (65%) than in 2010 (91%).

Table 7. 7 Comparing confidence between KAU & UOWC in 2011

Topic Name	Teaching Method	Rank	KAU 2011 n	KAU 2011 %	Rank	UOWC 2011 n	UOWC 2011 %
Functions	WE	1	65	95.6	1	20	76.9
Exponents	PS	2	63	92.6	1	21	80.0
Quadratic Equations	WE	3	51	75.0	2	11	42.3
Logarithms	PS	4	47	69.1	1	17	65.4
Geometry	WE	5	32	47.0	1	20	76.9
Introduction to Statistics	PS	6	31	45.5	2	11	42.3

In 2012 when faded worked examples used for KAU there was no significant change in confidence from 2011, whereas for UOWC there was a increase in confidence for the topics *Geometry* and *Introduction to Statistics* and a decrease in confidence for topic *Quadratics Equations*. More UOWC student indicated that they were confident in 2012 than 2011in two topic *Introduction to Statistics* (92%), *Quadratics Equations* (81%) as in Table 7.8.

Table 7. 8 Comparing confidence between KAU & UOWC in 2012

Topic Name	Teaching Method	Rank	KAU 2012 n	KAU 2012 %	Rank	UOWC 2012 n	UOWC 2012 %
Functions	PS	1	58	90.6	1	16	61.5
Exponents	WE	2	52	81.2	3	14	53.8
Quadratic Equations	PS	3	50	78.1	1	21	80.8
Logarithms	WE	4	49	76.5	1	19	73.1
Geometry	FWE	5	33	51.5	2	11	42.3
Introduction to Statistics	FWE	6	26	40.6	1	24	92.3

The pattern of change in confidence is illustrated in Table 7.9. Where confidence did change for both KAU and UOWC, it changed in the same method.

Table 7. 9 Comparing Confidence between KAU & UOWC over all years

Change in Method	Topics	KAU Confidence	UOWC Confidence
Traditional to WE	Functions	UP	SAME
	Quadratic Equations	SAME	SAME
	Geometry	SAME	SAME
Traditional to PS	Exponents	UP	SAME
	Logarithms	SAME	DOWN
	Introduction to Statistics	SAME	SAME
WE to PS	Functions	SAME	SAME
	Quadratics Equations	SAME	UP
PS to WE	Exponents	SAME	SAME
	Logarithms	SAME	SAME
WE to FWE	Geometry	SAME	DOWN
PS to FWE	Introduction to Statistics	SAME	UP

Different outcomes		Same outcomes	

Table 7.9 revealed that there is no clear impact of WE, PS or FWE on confidence. Findings in relation to confidence are not the same for KAU and UOW, nor are the findings the same for topics which have experienced the same changes, for example for traditional to worked examples when method is changed from traditional to worked examples, there is changed in confidence between KAU and UOWC. For examples, *Functions* the confidents went up in KAU and in UOWC stayed same. Furthermore, when method changed from traditional to problem-solving, there was also changed in confidence. For example, *Logarithms* in UOWC case goes down. Also, *Exponents* went up at KAU. When method is changed from worked examples to problem-solving *Quadratics Equations* went up at UOWC. Furthermore, when method is changed from problem-solving to worked examples no change in confident has found. Moreover, when method has changed from worked examples to problem-solving *Geometry* the confidence went down. Also, when method is changed from problem-solving, the confidence went up in the *Introduction to Statistics*.

Introduced as a way of scaffolding from worked examples to problem-solving one could expect that the use of faded worked examples could have an impact on the confidence in being able to solve problems in the topic areas where the technique

was used. Findings suggest impact of faded worked examples on confidence is not clear cut.

7.3.3 Performance

- **Question three:** Is there a difference in performance learning through worked examples compared to learning through problem-solving approaches? and,
- **Question four:** Can faded worked examples be used to scaffold from worked examples to problem-solving in terms of performance and confidence?

Based on cognitive load theory it was expected that topic based examination results would improve when students moved from traditional or problem-solving to worked examples and decline when the mover was from worked examples to problems-solving. In accord with their performance, it was found that KAU and UOWC students in response to the experimental conditions were different in some aspects, but similar in others.

- **2010-2011**

In 2010 students were taught with traditional techniques and the outcomes for KAU and UOWC students were similar in terms of the difficulty of topics (refer Table 7.6). In 2011, as reported in Table 7.8, *Functions*, *Quadratics Equations* and *Geometry* were taught with worked examples while the remaining topics *Exponents*, *Logarithms* and *Introduction to Statistics* were taught with problems-solving, for both the KAU and UOWC case studies.

For KAU in 2011, students responded to the change in technique and performance. The outcomes for the topics taught alternate according to teaching method, the top, *Quadratic Equations* taught with WE, the next *Exponents* taught with problem-solving, then *Functions* (WE), *Logarithms* (PS), *Geometry* (WE) and *Introduction to Statistics* (PS). For UOWC the pattern of outcomes is different to KAU with the three topics taught by worked examples as having the highest marks and the three lowest mean marks all taught with problem-solving. This pattern is what one would expect under cognitive load theory, assuming topics were of equivalent difficulty.

For KAU *Quadratic Equations* now taught with worked examples is the best performing topic significantly different in average marks to *Exponents*, *Logarithms*, and *Introduction to Statistics*. It is notable that *Quadratics Equations* in 2010 was the worst performing topic, whereas now taught with worked examples it is the top ranked subject. *Introduction to statistics*, formerly the second top subject is when taught with problems-solving the worst.

UOWC also responded to the change in teaching technique, but in contrast to KAU, the performance in topics for conforms to what is expected if cognitive load is lowered when topics are taught by worked examples. *Geometry*, *Quadratic Equations*, and *Functions,* all taught with worked examples are the best performing in terms of mean marks. *Geometry* the best performing topic is significantly different in average marks to *Functions*, *Exponents*, *Quadratics Equations*, followed by *Quadratic Equations* and *Functions* that are equivalent in difficulty. For UOWC the last three topics, also equivalent in difficulty with no significant differences between them were taught with problem-solving. Notable is that *Introduction to statistics*, formerly the second top subject is when taught with problems-solving one of the worst for UOWC.

Table 7. 10 Comparing the difficulty of topics between KAU & UOWC 2011

Topic	**Method**	**KAU N=68**			**Topic**	**Method**	**UOWC N=26**		
		Rank	**Mean**	**S.D**			**Rank**	**Mean**	**S.D**
Quadratic Equations	WE	1	8.81	1.06	Geometry	WE	1	8.69	1.22
Exponents	PS	2	8.34	1.33	Quadratics Equations	WE	2	8.54	1.27
Functions	WE	2	8.06	1.37	Functions	WE	2	8.42	1.44
Logarithms	PS	3	3.07	1.56	Exponents	PS	3	3.19	1.54
Geometry	WE	3	3.04	1.67	Intro to Statistics	PS	3	2.95	1.52
Intro to Statistics	PS	3	2.63	1.64	Logarithms	PS	3	2.65	1.90

WE		PS	

From the perspective discussed in Chapter 6, the patterns in Table 7.10 also reflect the hypothesised changes given cognitive load theory, except in two instances, for KAU. The change from traditional to WE for *Geometry* according to cognitive load

theory was expected to be high performance as the traditional method was similar to a problems-solving approach, whereas the change expected when changing to PS for *Exponents* was to be low performance as the problem-solving method was similar to traditional method.

In 2012 the methods used to teach topics were swapped, and the real test of cognitive load theory ensued; students were taught two topics with worked examples *Exponents*, and *Logarithms* whereas *Quadratics Equations* and *Functions* topics were now taught with problem-solving. A new technique faded worked examples was introduced to teach *Geometry* and *Introduction to Statistics*. Faded worked examples were introduced to KAU students appeared to link problem-solving with greater confidence, although this was not the case for UOWC students. Theoretically the disadvantage in using a problem-solving approach is that problem-solving questions increase the cognitive load on students because of the intrinsic cognitive load in learning mathematics and this is additional to the cognitive load due to the language capability of ESL students. The introduction of worked examples was to address a need to find a way which would build students' confidence but to reduce cognitive load of problem-solving, but recognising that it may increase the cognitive load when compared to worked examples.

When these changes were made in 2012, the pattern in outcomes for KAU and UOWC were similar in that the four top topics were taught either by worked examples or faded worked examples, the bottom two topics taught by problem-solving approaches. The pattern of performance outcomes for both cases studies fit with cognitive load theory in terms of worked examples (including faded worked examples) resulting in better performance than problem-solving.

From the perspective discussed in Chapter 6 changes were discussed in terms of the experimental condition that was applied the comparative outcomes between 2011 and 2012 can be summarised as follows:

1. The pattern of outcomes were the same for KAU students and UOWC students in that students who were taught with the worked examples technique in 2011 performed significantly better than students in 2012 who were taught using problem-solving techniques, for the topics *Functions* and *Quadratic Equations*
2. Students at both KAU and UOW who were taught with the worked example technique in 2012 for *Logarithms* performed better than students who in 2011 were taught using problem-solving techniques. A difference emerged between the two case studies, with an anomaly with KAU students taught *Exponents* with problems-solving technique in 2011 performed better than students taught via worked examples, contrary to what one would expect from cognitive load theory whereas UOWC in accord with cognitive load theory students performed better in 2012 than students in 2011 who were taught using problem-solving techniques.
3. For the topic *Geometry*, students in 2011 were taught with worked examples and those in 2012, taught with faded worked examples. Those taught with worked examples could be expected to experience lower cognitive load than those taught with faded worked examples, however for KAU students taught with faded worked examples performed better than those taught with worked examples while for UOWC students *Geometry* there was no significant difference in mean marks for those students in 2011 who were taught with worked examples when compared to students in 2012 who were taught with faded worked examples.

4. For the topic, *Introduction to Statistics*, students in 2012 were taught with faded worked examples and students in 2011 were taught by problem-solving methods. Students could be expected to have experienced lower cognitive load for this topic and for this to translate into better performance. For both KAU and UOWC students in 2012, taught with FWE had significantly better performance than when taught with problem-solving.

Table 7. 11 Comparing the difficulty of topics between KAU & UOWC 2012

Topic	Method	KAU N=64			Topic	Method	UOWC N=26		
		Rank	Mean	S.D			Rank	Mean	S.D
Geometry	FWE	1	8.00	1..39	Logarithms	WE	1	8.42	1.33
Logarithms	WE	2	7.92	1.54	Geometry	FWE	1	8.08	1.46
Intro to Statistics	FWE	2	7.22	1.75	Intro to Statistics	FWE	2	7.31	1.61
Exponents	WE	3	5.80	2.45	Exponents	PS	3	5.96	2.63
Quadratic Equations	PS	3	5.39	2.52	Quadratics Equations	PS	3	5.77	2.28
Functions	PS	3	4.52	2.33	Functions	PS	3	4.58	2.26

WE		PS		FWE	

From the perspective discussed in Chapter 6, the patterns in Table 7.11 also reflect the hypothesised changes given cognitive load theory, except in two instances, for KAU. The change from problem-solving to WE for *Logarithms and Exponents* according to cognitive load theory was expected to be high performance. For UOWC the change from problem-solving to worked examples for *Exponents* according to CLT was expected to be high performance. Whereas the change expected when changing to faded worked examples for *Geometry* and *Introduction to Statistics* was to be high performance.

7.4 Summary of findings that call for action

"Many first year students experience difficulties with their transition to University" (Aminifar, 2007). Despite the preparatory classes they undertake in their senior year, ESL students' language skills are still very basic and for many provide nominal assistance in their first year mathematics courses. The majority of students found it difficult to learn mathematics in their second language as seen, and found the language has negative impact on students understanding.

Nevertheless, the purpose of this study was not to investigate the handicap created by the language barrier, as it has been well documented that where the students' first language is different from the predominant language of instruction, students tend to benefit more if mathematics is taught in their first language (Ellerton & Clarkson,

1996; Setati, 2003). Adler (2001) explain that code-switching refers to bilingual or multilingual settings; which simply mean to switch between the language of learning and teaching and the learner's first language. The approach enables learners to harness their local language as a learning resource as well as to increase their participation in classroom discourse. However, code-switching as a means to understanding mathematical concepts, also results in an increased cognitive load.

Worked examples is a technique designed to reduce cognitive load that is associated with some forms of problem solving (Clark et al., 2006). With WE teaching techniques, details of the problem statement and all the necessary steps to solve the problem are described. Worked examples direct the attention of a learner to the problem stated, and the steps required to solve a particular type of problem. This reduction in cognitive load should generally make learning easier.

This study indicated that the language of teaching mathematics has impact on students learning if they learn in their second language while (85%) of students from KAU rated their ability to general mathematics fair/very good only (68%) rated their ability to learn mathematics in English as fair/very good, similarly at UOWC (62%) of students rated their general mathematics ability as fair/very good whereas only (27%) rated their ability to learn mathematics in English as fair/very good.

Under these circumstances, using the worked examples approach appears to be an important strategy in term of lowering cognitive load that arises from both language barriers and the difficulty of mathematics more so than problem solving and faded worked examples in the cognitive load perspective. This may explain that over the three phases a greater proportion of students indicated that having worked examples improved their mathematics study than did problem-solving and faded worked examples.

It is now known that the UOWC students surveyed preferred to study their subject with worked examples instead of problem-solving. Fifty-six percent of students preferred the use of worked examples in their studies, twenty percent preferred the use of problem-solving, while twenty-three percent of students preferred a combination of the two methods. More students have agreed that worked examples

have positive effect on their way of learning mathematics when English is their second language. Not only did students prefer to use worked examples, their performance in examinations when worked examples were used instead of problem-solving was for all but one topic, *Exponents*, in one case study KAU better. One explanation for the better performance is that when problem-solving is used as suggested by cognitive load theory, cognitive load will be lower when the GCL and ICL are lower and this occurs with the used of worked examples. In the case of *Exponents* this was in the baseline phase for KAU the easiest topic, perhaps it had lower intrinsic load, and therefore did not respond

7.5 Implications for policy makers

In order to allow for change to progress for the benefit of the ESL student and the universities alike, there needs to be active communication among the two key players: the policy makers, and the educators. The policy makers at all stages of the educational roadmap from the university chancellor to Department of Immigrations, should be well versed regarding the academic criteria in the selection process of the universities, and also the programs in place to assist ESL students on their educational pathway in Australia.

Upon completing the research for this study, policy makers and educators need to work together to devise a method to assess whether students are academically proficient in the English language in order to continue on at a tertiary level and keep pace with domestic English speaking students or they need to ensure that effective teaching strategies are in place that work to reduce cognitive load.

In many cases students may be conversationally proficient but lack the language skills to perform academically at a tertiary level in English. To ensure the success of their international students, it is the role of policy makers to ensure there are academic programs in place such as the UOWC free conversation classes for ESL students. Programs such as these go beyond the classroom and allow for students to interact on a day-to-day level with academic language. There are also preparatory classes available at some of the universities in academic English that will enable students to grasp the linguistic differences in everyday English and academic English. Peer support groups, and multilingual study groups are also excellent in the

aide of ESL students. While these programs are offered there also needs to be attention paid to the strategies used to teach mathematics.

Moreover, academics should be required to undertake professional development courses to train them with educational techniques, which will enable them to assess and interpret difficulties experienced by their ESL students. This is due to the fact that many lecturers and tertiary level educators are not trained as educators but as experts in the field of study in which they teach and this is especially true in the field of mathematics.

7.6 Implications for lecturers

There is a stark difference between the teacher in a classroom and the lecturer at a university. The most noted is the fact that most university lecturers are not trained teachers, although at least at UOW there is a requirement for staff to complete a university teaching and learning program. Therefore, the obligation felt by most teachers as professionals to address the needs of all their students in class (Kersaint et al., 2009) is not so much as sentiment felt by lecturers in mathematics course. In large campus universities all over Australia, the reality is that the lecturer will not personally know each student as is the case in school. Therefore the individual needs of ESL students can only come from pedagogical changes in teaching methodology. It would be unrealistic to expect a university lecturer to avoid technical language or contextual words that may otherwise detract from the quality of the mathematics course being taught. As other studies have suggested, lecturers can assist through developing scaffolds through the teaching methods that assist the ESL student in developing academic and language skills as they progress through their mathematics courses (Ferrari, 2004). For language effective means of scaffolding in the maths classroom is the use of peer conversational exercises that encourage dialogue amongst students, as language is socially derived and constructed and therefore is also developed and dispensed by social means (Ferrari, 2004). In other words, by encouraging students to discuss and justify procedures with their peers, the process of language development is also encouraged (Sideman, 1997). In terms of this study in the mathematics discipline appropriate scaffolds to problem-solving include the use of worked examples and faded worked examples.

The notion of meeting the educational needs of each of your students is something that each lecturer would love to aspire to. However, simply due to the logistics, such an ambition would be virtually impossible in large lecture halls. Therefore, lecturers should have an understanding of the general demographic of the students in their classes. For instance, if they were teaching Advanced Mathematics at UOWC, by reviewing enrolment records, they would whether their class is (98%) ESL or as in the case of UOW in the latter two years composed of (42%) ESL students and the remainder domestic. Although, as results from the UOWC case study would tend to suggest, domestic English speaking students are also likely to benefit from changes to pedagogy.

Nevertheless, an educator should firstly asses the class they are instructing and ensure that their teaching methods are at a pace that is easily followed by the majority of the class, and ensure that the resources and materials used in the classroom are designed to encourage learning and not overload working memory with redundant information.

7.6.1 Selecting textbooks

In today's classrooms textbooks are still an integral part of the teaching resources used by educators. As students progress in their education path they will be required to read and understand textual content from books for the purpose of content understanding, assessments, and assignments. In order for students to do this effectively, there needs to be a basic level of understanding of the language of presentation. For ESL students their ability to grasp not only the mathematical concepts but to understand the intricate language used within the text itself may be a challenge.

The results of this study indicate that teachers and educators are to take considerable care in the selection of prescribed texts, reading resources. ESL students will require books that are simpler in language and that will provide plenty of worked examples and less in problem-solving and more in faded worked examples, also, avoiding split attention and redundancy to keep cognitive load low.

With advent of the technological change and its infiltration into the fabric of our everyday life, there is a new classroom emerging with digital literacies and new multimodal learning techniques. The humble text book will still be present, but perhaps not for long as it will be replaced with digitised software that will enable educators to present material to students using multimodal communication strategies that go beyond the written language of text books to the use of visual semantics, auditory learning, and other modes of visual learning (Winch et al., 2010). Educators will then have the option of supplementing such technologies with textbooks to further enhance their teaching strategies for ESL students.

7.6.2 Course design

The manner in which the educators choose to design their course should be consciously planned to ensure they are accommodating the needs of the growing number of ESL students in the tertiary classroom. This is not suggesting that the one should be doing away with technical language and some high level language, rather it is to recommend that content of curricula not be altered, rather pedagogical practice should be modified.

In accord with cognitive learning theory which appears to have been supported in this study, one would further presume that adopting teaching strategies, graphics, images, text, simulators that engage the ESL learner in multimodal learning, will work towards developing their understanding of complex topics, as they are being exposed to learning from a more diverse sensory level than just text and images. This concept of multimodal learning was introduced in the 1990s by the New London Group (2002) as they discussed the merits of multimodal learning as being: “written language, sound, images, are mixed together. Students can learn in a more holistic multimodal way when text is accompanied by audio and visual content” (pp.406-407).

7.6.3 Writing assessment tasks

In as much as cognitive load theory has predicted improvement in performance outcomes it is likely that there is another imperative namely that the lecturer familiarises students with key terms in order to prepare ESL students for assessment

tasks to ensure that students are not overwhelmed with vocabulary in assessments and tasks. There are various ways in which this can be accomplished. For example, students can be given a vocabulary sheet to assist students with common terms that will be used in the unit being taught. Students can familiarise themselves with terms, words and/or phrases. The use of passive voice, comparative phrases and other complicated linguistic features should be introduced gradually to reduce student anxiety or alternatively avoided in the interests of clear communication. While the aim of the classroom and tutorials is to expose ESL students to the mathematics vocabulary, during the assessment problems should be asked in a simple, clear and unambiguous manner.

7.7 Recommendations for further research

One of the primary goals of this study was to further our understanding on how learning takes place among ESL students in mathematics, as well as examining the most effective means of teaching ESL students mathematics. Despite the success of this study, there were some anomalies in the findings and there is much research that needs to be done to further our understanding of language and its effect on learning for ESL students, as well as in the field of pedagogy for mathematics and ESL students. To clarify the inconsistencies in findings the following are recommendations for further research. More work needs to be undertaken to:

- Gain a better understanding of the use of faded worked examples as a strategy, FWE appears to have an impact in terms of improved performance. Confirming this with a larger cohort of students taught in mathematics in their first language, over a greater range of topics may establish that the use of worked examples, followed by faded worked examples bridging to problem-solving is a better technique for all students.
- Examine of which of the techniques or combination of techniques engages students in the completion of examples.
- Understand the impact of teaching techniques on confidence and in turn the relationship that confidence has with performance.

- Further examination as to the impact of the teaching methods on topics of intrinsically different levels of difficulty, as distinct to topics poorly taught or not optimised for teaching would be useful. A study that follows a specific group of ESL students through their journey in tertiary level mathematics, monitoring the identified difficulties established by the participants, over the duration of their studies at that university would be useful in identifying whether the teaching method is appropriate at higher levels of mathematics.

To extend generalisability it would be useful to:

- Replicate the study in various campuses and perhaps different nations to determine whether ESL students are experiencing the same difficulties elsewhere.
- At KAU, only male participants were permitted to partake in the study. If further research where to be undertaken, I would recommend the study extend the sample size in evaluating the language of gender groups. The trends observed in language and gender groups of this sample could be further investigated if the study was extended to a larger sample of students. For instance.
- Of great interest to many would be the understanding how ESL students think or write mathematically in their first language, and the manner in which it affects their ability to think and write mathematically in their primary language.

7.8 Final Word

As societies change with the influx of migrants and international students, so do our classrooms. As the classroom demographic continues to change to a more multicultural and multilingual classroom, it is imperative that educators and policy makers have an understanding as to the challenges that face ESL students in the mathematics classroom.

Educators, as a result of cognitive load theory, have learnt they can play a pivotal role in reducing the total cognitive load imposed on students by ensuring that the material and resources used are laid out and presented in a manner in which reduces the extraneous and intrinsic cognitive load on the student. This type of effect can be attributed to the low cognitive load demonstrations have on working memory and therefore the ease in which it is transferred to long term memory in the formation and development of schemas. The pedagogical methods tested in this study in teaching mathematics, compared the use of worked example versus the problem-solving approach for groups of ESL students. Based on the data, students surveyed in both case studies preferred to learn mathematics using worked examples and performed better with this approach. The teaching methodology at this level of study appears apt for both ESL students and students who are taught in their first language.

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8 APPENDIX A SAMPLE PRAC WORK AND HOMEWORK

Appendix A

KAU Practical class WE

Examples FUNC-C2 continued

Given $f(x)=2x+1$ and $g(x)=x^2+2x$, find:

(i) $f(a+2)$

$f(x)=2x+1$

$f(a+2)=2(a+2)+1$ • replace x with $a+2$ in the expression x^2+2x

$=2a+4+1$ • expand and simplify

$=2a+5$

(ii) $g(a^2)$

$g(x)=x^2+2x$ • replace x with a^2 in the expression x^2+2x

$g(a^2)=(a^2)^2+2(a^2)$ • expand and simplify

$=a^4+2a^2$

Homework KAU

Functions

Exercise FUNC-D1
Determine the following, given $f(x) = 3 - 2x$ **and** $g(x) = x^2 + 1$

(i) $f(3) - g(3)$

(ii) $f(g(0))$

Exponents Practical PS KAU

Problem-solving EXPO-

1. Use the multiplication rule of powers to simplify the following.

(i) $p^{3} \times p^{2}$ (xi) $a^{5} \times a^{7} \times a^{3}$

(ii) $3^{4} \times 3^{5}$ (xii) $y^{3} \times y^{6} \times y^{5}$

Practical Class KAU

Examples QUAD **Square of a Sum**

Expand the following using the rule:

(i) $(x\ 2)^2$

$x^2\ 2\ 2\ x\ 2^2$

$x^2\ 4x\ 4$

(ii) $(x\ 7)^2$

$x^2\ 2\ 7\ x\ 7^2$

$x^2\ 14\ x\ 49$

(iii) $(2\ a\ 4)^2$

$(2\ a)^2\ 2\ 2\ a\ 4\ 4^2$

$4\ a^2\ 16\ a\ 16$

Homework KAU

QUAD

Expand the following using the rules:

(i) $a\ 3^2$

(ii) $y\ 5^2$

(iii) $z\ 1^2$

Logarithms KAU

LOGS-

1. Rewrite each of the following statements in logarithmic notation.

   (i) 5^3 125

   (ii) 10^2 100

   (iii) 3^2 9

   (iv) 10^1 0.1

2. Rewrite the following in exponential form and solve where possible.

   (i) $\log_4 16$ 2

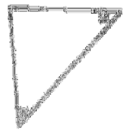
   (ii) $\log_2 32$ 5

   (iii) $\log_5 125$ 3

Examples GEOM-C1

1. Classify each of the following triangles according to both their side lengths and their angles.

(i) 59°



- isosceles
- acute angled

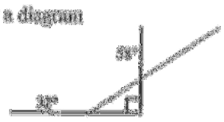
(ii)

- scalene
- obtuse angled

(iii) Triangle ABC where $\angle A = 32°$, $\angle B = 58°$ and $\angle C = 90°$

- draw a diagram

58°



- scalene
- right angled

2. Find the missing value in the following diagrams. Give reasons.

(i)

$a = 180° - 81° - 54°$
$= 45°$

- triangle angles add to180°

(ii)

$b = 70°$

- isosceles triangle

$c = 180° - 70° - 70°$
$= 40°$

- triangle angles add to180°

(iii)

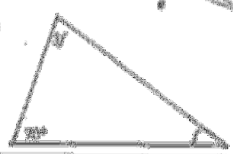
$d = 180° - 160° - 10°$
$= 10°$
$\therefore$ isosceles triangle

- triangle angles add to180°
- equal angles

$e = 15cm$

- isosceles triangle

(iv)



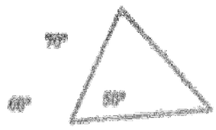
$180° = 70° + 3f + f$
$180 = 70 + 4f$
$110 = 4f$
$27.5° = f$

- triangle angles add to 180°
- solve for f

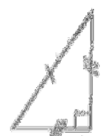
Exercises GEOM-C1

1. Classify each of the following triangles according to both their side lengths and their angles.

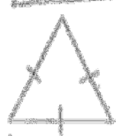
(i)



(iv)

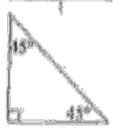


(ii)



(v)

(iii)



GEOM 9

Exercise STSD-G1

1. The weights of cattle have a mean of $434kg$ and standard deviation of $69kg$. What percentage of cattle will weigh between $330.5kg$ and $537.5kg$?
2. The age of pensioners residing in a retirement village has a mean of 74 years and standard deviation of 4.5 years. What is the age range of pensioners that contains at least 89% of the residents?
3. It was found that for a batch of soft drink bottles, the mean content was $994ml$. If 75% of the bottles contained between $898ml$ and $1090ml$, what was the standard deviation for the soft drink batch?
4. On a test the mean is 50 marks and standard deviation 11. At most, what percentage of the results will be less than 17 and greater than 83 marks?

Homework KAU

1. A soft-drink filling machine uses cans with a maximum capacity of $340ml$. The machine is set to output soft drink with a mean capacity of $330ml$. It has been found that due to machine error the amount outputted varies with a standard deviation of $8ml$ and the amount outputted is normally distributed.
   (a) What proportion of cans will have between 330 ml and $340ml$ of soft drink?
   (b) What percentage of cans will have between $325ml$ and $340ml$?
   (c) What percentage of cans will overflow?
   (d) If the smallest 5% of drinks must be rejected, what is the smallest amount which will be accepted?

2. (a) If a set of data has a mean of 76 and a standard deviation of 28.8, what is the interval that should contain at least 75% of the data?
   (a) A data set has a mean of 827 and a standard deviation of 98. At least what percentage of values should lie been 582 and 1072?
   (b) A set of data has a mean of 468. If 89% of the data values lie between 336 and 600, what is the standard deviation for the data set?

UOWC Practical Class
WE

Examples FUNC-

Given $f(x)=x^2$ and $g(x)=2x+1$, then $\quad f(3)=3^2=9$ and $g(3)=2\times 3+1=7$

(i) $(f+g)(x)=x^2+2x+1$

$(f+g)(3)=3^2+2\times 3+1$
$=9+6+1=16$

$f(3)+g(3)=9+7$
$=16=(f+g)(x)$

(ii) $(f-g)(x)=x^2-2x-1$

$(f-g)(3)=3^2-2\times 3-1$
$=9-6-1=2$

$f(3)-g(3)=9-7$
$=2=(f-g)(x)$

(iii) $(f\times g)(x)=x^2(2x+1)$
$=2x^3+x^2$

$(f\times g)(3)=2\times 3^3+3^2$
$=54+9=63$

$f(3)\times g(3)=9\times 7$
$=63=(f\times g)(x)$

(iv) $(fg)(x)=f(x)\times g(x)$
$=(3-2x)(x^2+1)$
$=3x^2+3-2x^3-2x$
$=-2x^3+3x^2-2x+3$

$f(-1)=3-2(-1)$
$=5$

OR $\quad g(-1)=(-1)^2+1$
$=2$

Practical Class UOWC

EXPO-

1. **Use the power of a product rule to write the following as a product of powers. Simplify where possible.**

   (i) $(pq)^3$

   (ii) $(3^2 r^4)^3$

   (iii) $(2 \times 10^3)^2$

   (iv) $(2s^2 + a^3)^3$

   (v) $(6p^3 t)^{-2}$

Homework EXPO

Evaluate the following.

(i) 16^2

Examples QUAD-E1

Determine the factors for the following factorisations:

(i) $x^2 \quad 2x \quad 15$

c negative: factors +, –
b positive: largest +

15, 1: 15 1 14
5, 3 : 5 3 2

factors are 5 and –3

(ii) $x^2 7x \quad 12$

c positive: factors same
b negative: both –

12, 1: 12 1 13
6, 2 : 6 2 8
4, 3 : 4 3 7

factors are – 4 and – 3

(iii) $x^2 7x \quad 18$

c negative: factors +, –
b negative: largest –

18,1 : 18 1 17
9, 2 : 9 2 7
6,3 : 6 3 3

factors are – 9 and 2

The factors of *c* that add to *b* are then placed in each of the brackets to complete the factorisation.

Examples QUAD-E2

Factorise the following quadratics:

(i) $a^2 \quad 6a \quad 8$ factors of 8 that add to 6

$a \; 4 \; a \; 2$

4 2 8, 4 2 6

(ii) $b^2 \quad 3b \quad 18$ factors of –18 that add to +3

$b \; 6 \; b \; 3$

6 3 18, 6 3 3

(iii) $y^2 \quad 5y \quad 6$ factors of 6 that add to –5

$y \; 3 \; y \; 2$

3 2 6, 3 3 5

(iv) $x^2 \quad 3x \quad 10$ factors of –10 that add to –3

$x \; 5 \; x \; 2$

5 2 10, 5 2 3

Practical UOWC

Examples LOGS-G1
Use logarithmic identities to simplify the following.

1. (i) $\log_8 8 \quad 1$
   (ii) $\log 10 \quad 1 \qquad \log 10 \quad \log_{10} 10$
   (iii) $\ln e \quad 1 \qquad \ln e \quad \log_e e$

2. (i) $\log_8 1 \quad 0$
   (ii) $\ln 1 \quad 0$

3. (i) $\log_3 3^4 \quad 4$
   (ii) $\ln e^6 \quad 6$
   (iii) $\log 100$
   $\log 10^2 \quad 2$
   (iv) $e^{\ln x} \quad x$
   (v) $a^{\log_a x} x$

Homework UOWC

Examples LOGS-G2
Use logarithmic properties to rewrite the following.

(i) $\log 10x \quad \log 10 \quad \log x$ $\qquad \log_a xy \quad \log_a x \quad \log_a y$
$1 \quad \log x$ $\qquad \log_a b \quad 1$

(ii) $\log \frac{4}{5} \quad \log 4 \quad \log 5$ $\qquad \log_a \frac{x}{y} \quad \log_a x \quad \log_a y$

(iii) $\log_2 x^3 \quad 3\log_2 x$ $\qquad \log_a x^n \quad n \log_a x$
$\underline{\log 100} \quad 2$ $\qquad$ change of base rule

(iv) $\log_2 100 \quad \log 2 \quad \log 2$ $\qquad \log 100 \quad \log 10^2 \quad 2$

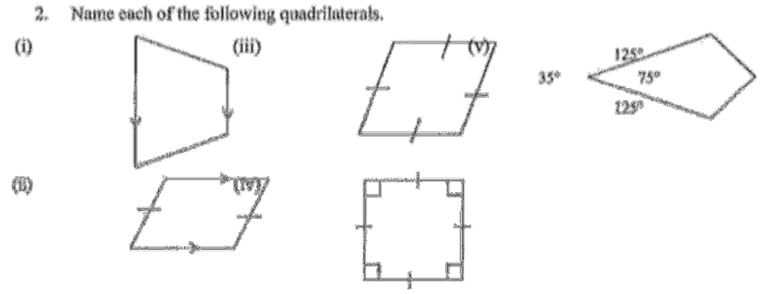
(v) $\log_3 6 \quad \frac{\log 6}{\log 3}$ or $\frac{\ln 6}{\ln 3}$ $\qquad$ use log or ln for calculations

Exercises GEOM-C2

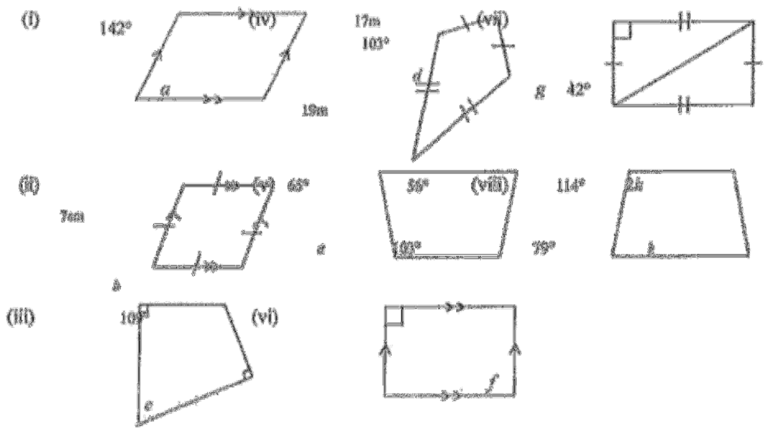
1. Copy and complete the following table ticking the appropriate boxes.

Quadrilateral Type	Equal Sides		Equal angles		Opposite sides parallel
	All	Opposite	All	Opposite	
Parallelogram					
Square					
Rectangle					
Rhombus					
Trapezium					

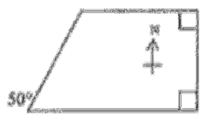
2. Name each of the following quadrilaterals.



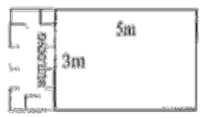
3. Find the missing values for the following.



4. A pool is being built in the shape of a trapezium. The eastern end of the pool meets the parallel sides at right angles and the angle between the western and southern edges of the pool is 50°. What is the angle between the northern and western edges of the pool?



5. The rectangular block that borders the building shown below is being fenced. What length of fencing will be required if the building is used for one side of the fence?



9 APPENDIX B INTERVIEW OF LECTURERS

University of Wollongong

Interview of Lecturers
Title of project:
Comparing Worked Examples and Problem-Solving Methods in Teaching Mathematics to ESL Students at Tertiary Level
By: Ali Algarni

The purpose of this interview is to evaluate the effectiveness of the worked examples or problems-solving based on lecturers' experiences with the implementation which are used for teaching MATH132 in Saudi context or Advanced Mathematics1 & 2 in Australia context for ESL Students.

Subject:______________ **Lecturer name:________________**

Date:______________

The following are samples as questioning will be directed by lecturer responses:

1) Were there any students who were enrolled in the subject but did not attend any classes? If so, how many?

2) What is your experience with teaching using worked examples method?

3) What is your experience with teaching using problem-solving method?

4) How does the preparation time for the lecture with worked examples compare with problem-solving methods?

5) How does the preparation effort for the lecture with worked examples compare with problem-solving methods?

6) Does the teaching method (WE or PS) effect the scheduling of your classes?

7) Have you had any discussion with students on their preferred method of study? If so can you summarise their comments?

8) What do you observe when students are taught with WE?

9) What do you observe when students are taught with PS?

10) Can worked examples method be improved to help your teaching of mathematics? If so, How?

11) Can Problem-solving method be improved to help your teaching of mathematics? If so, How?

12) What do you see the main advantages of worked examples method in teaching in your subject?

13) What do you see the main disadvantages of worked examples method in teaching in your subject?

14) What do you see the main advantages of problem-solving method in teaching in your subject?

15) What do you see the main disadvantages of problem-solving method in teaching in your subject?
16) Which way do you prefer to teach your students mathematics at university course?
 And why for example. (WE, SP, Mix between them)?

17) Do you have any suggestions about using worked example method in teaching?

18) Do you have any suggestions about using problem-solving method in teaching?

19) Is there anything else that you would like to tell me about your experience teaching your subject?

10 APPENDIX C SURVEY

University of Wollongong

Survey

Instructions:

The primary purpose of this survey is to provide feedback that can assist in the development of this subject for the future students. Feed back of ALL Students those who like subject and those who do not like it is essential in this process. You can let us know how to improve the subject so that you or future students can learn better.

Your cooperation in completing this Survey is greatly appreciated. **Your responses are anonymous**

Section 1: Background (Please circle the correct answer)

Q1. Is your first language:

1.Arabic

2.English

3. Other language (Please specify)…………….

Q2. Is Background prior to university?

1. Mathematics
2. Science
3. Arts
4. Other (please specify) ……………….

Q3. How many years have you learnt mathematics using English?

1. Primary school
2. High school years 7-10
3. High school year 11
4. High school year 12

Q4. How would you describe your ability to do mathematics?

1. Very poor
2. Poor
3. Fair-good
4. Very Good

Q5. How would you describe your ability to do mathematics in English?

1. Very poor
2. Poor
3. Fair-good
4. Very Good

<u>Section 2: Usefulness of Learning Resource:</u>

Q.6 How useful are your existing resources in helping you understand in this subject	Rarely used this resource	Little use	Moderately useful	Extremely useful
a. lecture	1	2	3	4
b. Work in Practical classes	1	2	3	4
c. Tutor in Practical classes	1	2	3	4
d. Practical Worksheets	1	2	3	4
e. Tutorial assignments	1	2	3	4
f. Lecture Handbook	1	2	3	4
g. Worked solutions for prac tasks, midterms and exams	1	2	3	4
h. Team learning or group work	1	2	3	4
i. Theory review in practical classes	1	2	3	4
j. Interaction with lecturer	1	2	3	4

<u>Section 3: Use of worked examples& problem solving:</u>

Q7. On average how many problems have you completed per week in class using worked examples as a guide?

1. 0 examples

2.1-2 examples

3.3-4 examples

4. 5 or more examples

Q8.On average how many problems have you solved per week without worked examples?

1. 0 examples

2.1-2 examples

3.3-4 examples

4. 5 or more examples

Q9. Does using Worked Example approach improve your study?

0. No

1. Yes

If your answer" Yes". Please explain how? If no explain, why not?

Q10. Does the Problem solving approach without Worked examples improve your study for math132?

0. No
1. Yes

If your answer" Yes". Please explain how? If no explain, why not?

Q11. How would you prefer to study math132? And ,why? And explain why you prefer this?

1 worked examples
2. Problem Solving
3. Mix worked examples and problems-solving

Q12. Do you have any suggestions as to how the worked examples could be improved?

Q13. Do you have any suggestions as to how the problems solving could be improved?

Q14. Is there a better way of setting the worked examples or problem-solving that would motivate you to learn more?

Section 4A: Usefulness of learning Resources for Worked Examples (WE) and Problem sS0lving (circle the correct response)

Q15.How satisfied were you with the (WE and PS)in:	Not Satisfied	Slightly Satisfied	Somewhat Satisfied	Satisfied	Very Satisfied	Not Applicable
a. (WE) provided to you before or during class.	1	2	3	4	5	6
b. (PS) provided to you before or during class.	1	2	3	4	5	6
c. the variety of (WE)	1	2	3	4	5	6
d. the variety of (PS)	1	2	3	4	5	6
e. the lesson in terms of them being (WE) easy to understand	1	2	3	4	5	6
f. the lesson in terms of them being (PS) easy to understand	1	2	3	4	5	6
g. the lesson in terms of them being (WE) interesting	1	2	3	4	5	6
h. the lesson in terms of them being (PS) interesting	1	2	3	4	5	6

Section 4B: Confidence on Topics

16. How confident are you now that you can solve problem on the following topics?	Not at all	Might have a difficulty	Moderately confident	Could do this
a. Function	1	2	3	4
b. Exponents	1	2	3	4
c. Quadratic Equation	1	2	3	4
d. Logarithms	1	2	3	4
e. Geometry	1	2	3	4
f. Introduction to Statistics	1	2	3	4

Section 5: Evaluation of using problem solving and worked examples approaches in this subject:

Q17.I believed that:	**Strongly Disagree**	**Disagree**	**Mildly Disagree**	**Neither Agree or**	**Mildly Agree**	**Agree**	**Strongly Agree**
a. Using worked examples enhanced my understanding in the mathematics tasks	1	2	3	4	5	6	7
b. Using problem solving enhanced my understanding in the mathematics tasks	1	2	3	4	5	6	7
c. Using worked examples made it quicker to study mathematics	1	2	3	4	5	6	7
d. Using problem solving made it quicker to study mathematics	1	2	3	4	5	6	7
e. Having access to worked examples improve my review of mathematics notes and lab work	1	2	3	4	5	6	7
f. Having access to problem solving improve my review of mathematics notes and lab work	1	2	3	4	5	6	7
g. Using worked examples it is much easier to learn than solving problems in Math lesions.	1	2	3	4	5	6	7
h. Using problem solving it is much easier to learn than solving problems in Math lesions	1	2	3	4	5	6	7
i. Worked examples increases my confidence about solving more problems in mathematics	1	2	3	4	5	6	7
j. Solving problem s increase my confidence about solving more problems in mathematics	1	2	3	4	5	6	7
k. Using worked examples to learn math requires a lot of mental and learning effort	1	2	3	4	5	6	7
l. Using problem solving to learn math requires a lot of mental and learning effort	1	2	3	4	5	6	7
m. Using worked examples makes mathematics learning more interesting	1	2	3	4	5	6	7
n. Using problem solving makes mathematics learning more interesting	1	2	3	4	5	6	7
o. I like to learn mathematics by using worked examples	1	2	3	4	5	6	7
p. I like to learn mathematics by using problems solving	1	2	3	4	5	6	7
q. Using worked example helps reduce my anxiety when learning mathematics	1	2	3	4	5	6	7
r. Using problem solving helps reduce my anxiety when learning mathematics	1	2	3	4	5	6	7

Q18: What do you think you learn when learning to complete mathematics through problem-solving?

Q19: What do you think you learn when learning to complete mathematics by following worked examples?

Q.20: as you know you are learning mathematics in your second language, is there anything that would help you to learn mathematics better in English?

Section 6: Demographics

Q20. Indicate your origin:

1. I'm International student
2. Domestic student

Q21. Indicate your gender:

1. Male
0. Female

Section 7: Grades:

a. What grade do you expect to get for this subject?	/100
b. What mark did you get for first assignment?	/100
c. What mark did you get for last assignment?	/100

THANK YOU … for the thought, time, and effort you have

made into completing this survey

11 APPENDIX D MCNEMAR'S TEST KAU

WE Enhanced learning * PS Enhanced learning Crosstabulation

			PS Enhanced learning		Total
			Disagree	Agree	
WE Enhanced learning	Disagree	Count	21	15	36
		% within WE Enhanced transform	58.3%	41.7%	100.0%
		% within PS Enhanced	24.7%	13.3%	18.2%
	Agree	Count	64	98	162
		% within WE Enhanced transform	39.5%	60.5%	100.0%
		% within PS Enhanced	75.3%	86.7%	81.8%
Total		Count	85	113	198
		% within WE Enhanced transform	42.9%	57.1%	100.0%
		% within PS Enhanced	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	4.261[a]	1	.039		
Continuity Correction[b]	3.528	1	.060		
Likelihood Ratio	4.221	1	.040		
Fisher's Exact Test				.043	.031
Linear-by-Linear Association	4.240	1	.039		
McNemar Test				.000[c]	
N of Valid Cases	198				

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 15.45.

b. Computed only for a 2x2 table

c. Binomial distribution used

McNemar's Chi-Square = $(64-15)^2/(64+15)= 30.39$

WE Quicker to study * PS Quicker to study Crosstabulation

			PS Quicker to study		Total
			Disagree	Agree	
WE Quicker to study	Disagree	Count	37	12	49
		% within WE Quicker	75.5%	24.5%	100.0%
		% within PS Quicker	27.4%	19.0%	24.7%
	Agree	Count	98	51	149
		% within WE Quicker	65.8%	34.2%	100.0%
		% within PS Quicker	72.6%	81.0%	75.3%
Total		Count	135	63	198
		% within WE Quicker	68.2%	31.8%	100.0%
		% within PS Quicker	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	1.612[a]	1	.204		
Continuity Correction[b]	1.194	1	.274		
Likelihood Ratio	1.666	1	.197		
Fisher's Exact Test				.221	.137
Linear-by-Linear Association	1.604	1	.205		
McNemar Test				.000[c]	
N of Valid Cases	198				

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 15.59.

b. Computed only for a 2x2 table

c. Binomial distribution used.

McNemar's Chi-Square = $(98-12)^2/(98+12) = 67.23$

WE Improve my review * PS Improve my review Crosstabulation

			PS Improve my review		Total
			Disagree	Agree	
WE Improve my review	Disagree	Count	12	7	19
		% within WE Improve my review	63.2%	36.8%	100.0%
		% within PS Improve my review	16.9%	5.5%	9.6%
	Agree	Count	59	120	179
		% within WE Improve my review	33.0%	67.0%	100.0%
		% within PS Improve my review	83.1%	94.5%	90.4%
Total		Count	71	127	198
		% within WE Improve my review	35.9%	64.1%	100.0%
		% within PS Improve my review	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	6.810[a]	1	.009		
Continuity Correction[b]	5.560	1	.018		
Likelihood Ratio	6.485	1	.011		
Fisher's Exact Test				.012	.010
Linear-by-Linear Association	6.775	1	.009		
McNemar Test				.000[c]	
N of Valid Cases	198				

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 6.81.

b. Computed only for a 2x2 table

c. Binomial distribution used.

McNemar's Chi-Square = $(59-7)^2/(59+7) = 40.96$

WE Easier to learn* PS Easier to learn Crosstabulation

			PS Easier		Total
			Disagree	Agree	
WE Easier	Disagree	Count	31	18	49
		% within WE Easier	63.3%	36.7%	100.0%
		% within PS Easier	33.3%	17.1%	24.7%
	Agree	Count	62	87	149
		% within WE Easier	41.6%	58.4%	100.0%
		% within PS Easier	66.7%	82.9%	75.3%
Total		Count	93	105	198
		% within WE Easier	47.0%	53.0%	100.0%
		% within PS Easier	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	6.942[a]	1	.008		
Continuity Correction[b]	6.100	1	.014		
Likelihood Ratio	6.977	1	.008		
Fisher's Exact Test				.013	.007
Linear-by-Linear Association	6.907	1	.009		
McNemar Test				.000[c]	
N of Valid Cases	198				

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 23.02.

b. Computed only for a 2x2 table

c. Binomial distribution used.

McNemar's Chi-Square $= (62-18)^2/(62+18) = 24.2$

WE Increase my confidence * PS Increase my confidence Crosstabulation

			PS confident		Total
			Disagree	Agree	
WE confident	Disagree	Count	21	134	155
		% within WE confident	13.5%	86.5%	100.0%
		% within PS confident	95.5%	76.1%	78.3%
	Agree	Count	1	42	43
		% within WE confident	2.3%	97.7%	100.0%
		% within PS confident	4.5%	23.9%	21.7%
Total		Count	22	176	198
		% within WE confident	11.1%	88.9%	100.0%
		% within PS confident	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	4.293[a]	1	.038		
Continuity Correction[b]	3.232	1	.072		
Likelihood Ratio	5.668	1	.017		
Fisher's Exact Test				.051	.026
Linear-by-Linear Association	4.271	1	.039		
McNemar Test				.000[c]	
N of Valid Cases	198				

a. 1 cells (25.0%) have expected count less than 5. The minimum expected count is 4.78.

b. Computed only for a 2x2 table

c. Binomial distribution used.

McNemar's Chi-Square $=(1-134)^2/(1+134)= 131.02$

WE Requires a lot of mental work* PS Requires a lot of mental work Crosstabulation

			PS Requires a lot of mental work		Total
			Disagree	Agree	
WE Requires a lot of mental work	Disagree	Count	27	122	149
		% within WE Requires	18.1%	81.9%	100.0%
		% within PS Requires	79.4%	74.4%	75.3%
	Agree	Count	7	42	49
		% within WE Requires	14.3%	85.7%	100.0%
		% within PS Requires	20.6%	25.6%	24.7%
Total		Count	34	164	198
		% within WE Requires	17.2%	82.8%	100.0%
		% within PS Requires	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.381[a]	1	.537		
Continuity Correction[b]	.159	1	.690		
Likelihood Ratio	.394	1	.530		
Fisher's Exact Test				.664	.353
Linear-by-Linear Association	.379	1	.538		
McNemar Test				.000[c]	
N of Valid Cases	198				

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 8.41.

b. Computed only for a 2x2 table

c. Binomial distribution used.

McNemar's Chi-Square = $(7-122)^2/(7+122)= 102.51$

WE Makes maths more interesting * PS Interesting maths more interesting Crosstabulation

			PS makes maths more Interesting		Total
			Disagree	Agree	
WE makes maths more interesting	Disagree	Count	13	13	26
		% within WE interesting	50.0%	50.0%	100.0%
		% within PS Interesting	12.5%	13.8%	13.1%
	Agree	Count	91	81	172
		% within WE interesting	52.9%	47.1%	100.0%
		% within PS Interesting	87.5%	86.2%	86.9%
Total		Count	104	94	198
		% within WE interesting	52.5%	47.5%	100.0%
		% within PS Interesting	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.077[a]	1	.782		
Continuity Correction[b]	.004	1	.947		
Likelihood Ratio	.076	1	.782		
Fisher's Exact Test				.835	.473
Linear-by-Linear Association	.076	1	.783		
McNemar Test				.000[c]	
N of Valid Cases	198				

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 12.34.

b. Computed only for a 2x2 table

c. Binomial distribution used.

McNemar's Chi-Square $= (91-13)^2/(91+13) = 58.5$

WE likes to learn maths * PS likes to learn maths Crosstabulation

			PS Like to learn maths		Total
			Disagree	Agree	
WE like to learn maths	Disagree	Count	39	15	54
		% within WE like	72.2%	27.8%	100.0%
		% within PS like	40.6%	14.7%	27.3%
	Agree	Count	57	87	144
		% within WE like	39.6%	60.4%	100.0%
		% within PS like	59.4%	85.3%	72.7%
Total		Count	96	102	198
		% within WE like	48.5%	51.5%	100.0%
		% within PS like	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	16.750[a]	1	.000		
Continuity Correction[b]	15.469	1	.000		
Likelihood Ratio	17.163	1	.000		
Fisher's Exact Test				.000	.000
Linear-by-Linear Association	16.666	1	.000		
McNemar Test				.000[c]	
N of Valid Cases	198				

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 26.18.

b. Computed only for a 2x2 table

c. Binomial distribution used.

McNemar's Chi-Square = $(57-15)^2/(57+15) = 24.5$

WE Reduced anxiety * PS Reduced anxiety Crosstabulation

			PS Reduced		Total
			Disagree	Agree	
WE Reduced	Disagree	Count	56	9	65
		% within WE Reduced	86.2%	13.8%	100.0%
		% within PS Reduced	36.1%	20.9%	32.8%
	Agree	Count	99	34	133
		% within WE Reduced	74.4%	25.6%	100.0%
		% within PS Reduced	63.9%	79.1%	67.2%
Total		Count	155	43	198
		% within WE Reduced	78.3%	21.7%	100.0%
		% within PS Reduced	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	3.526[a]	1	.060		
Continuity Correction[b]	2.871	1	.090		
Likelihood Ratio	3.741	1	.053		
Fisher's Exact Test				.068	.042
Linear-by-Linear Association	3.508	1	.061		
McNemar Test				.000[c]	
N of Valid Cases	198				

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 14.12.

b. Computed only for a 2x2 table

c. Binomial distribution used.

McNemar's Chi-Square = $(99-9)^2/(99+9)= 75$

Average WE * Average PS Crosstabulation

			Average PS		Total
			1	2	
Average WE	1	Count	120	23	143
		% within Average WE	83.9%	16.1%	100.0%
		% within Average PS	71.9%	74.2%	72.2%
	2	Count	47	8	55
		% within Average WE	85.5%	14.5%	100.0%
		% within Average PS	28.1%	25.8%	27.8%
Total		Count	167	31	198
		% within Average WE	84.3%	15.7%	100.0%
		% within Average PS	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.071[a]	1	.790		
Continuity Correction[b]	.002	1	.961		
Likelihood Ratio	.072	1	.788		
Fisher's Exact Test				1.000	.490
Linear-by-Linear Association	.071	1	.790		
McNemar Test				.006[c]	
N of Valid Cases	198				

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 8.61.

b. Computed only for a 2x2 table

c. Binomial distribution used.

McNemar's Chi-Square $= (47-23)^2/(47+23) = 8.22$

Improve Worked Examples * Improve Problem Solving Crosstabulation

			Improve Problem-Solving		
			No	Yes	Total
Improve Worked Examples	No	Count	32	6	38
		% within Improve Worked Examples	84.2%	15.8%	100.0%
		% within Improve Problem-Solving	20.3%	15.0%	19.2%
	Yes	Count	126	34	160
		% within Improve Worked Examples	78.8%	21.3%	100.0%
		% within Improve Problem-Solving	79.7%	85.0%	80.8%
Total		Count	158	40	198
		% within Improve Worked Examples	79.8%	20.2%	100.0%
		% within Improve Problem-Solving	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.568[a]	1	.451		
Continuity Correction[b]	.280	1	.597		
Likelihood Ratio	.595	1	.441		
Fisher's Exact Test				.510	.306
Linear-by-Linear Association	.565	1	.452		
McNemar Test				.000[c]	
N of Valid Cases	198				

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 7.68.

b. Computed only for a 2x2 table

c. Binomial distribution used.

McNemar's Chi-Square = $(126-6)^2/(126+6) = 109.09$

12 APPENDIX E MCNEMAR'S TEST UOWC

Enhanced Understanding WE * Enhanced Understanding PS Crosstabulation

			Enhanced Understanding PS		Total
			Disagree	Agree	
Enhansed Understanding WE	Disagree	Count	15	8	23
		% within Enhanced Understanding WE	65.2%	34.8%	100.0%
		% within Enhanced Understanding PS	51.7%	17.8%	31.1%
	Agree	Count	14	37	51
		% within Enhanced Understanding WE	27.5%	72.5%	100.0%
		% within Enhanced Understanding PS	48.3%	82.2%	68.9%
Total		Count	29	45	74
		% within Enhanced Understanding WE	39.2%	60.8%	100.0%
		% within Enhanced Understanding PS	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	9.487[a]	1	.002		
Continuity Correction[b]	7.968	1	.005		
Likelihood Ratio	9.434	1	.002		
Fisher's Exact Test				.004	.002
Linear-by-Linear Association	9.359	1	.002		
McNemar Test				.286[c]	
N of Valid Cases	74				

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 9.01.
b. Computed only for a 2x2 table
c. Binomial distribution used.

McNemar's Chi-Square = $(14-8)^2/(14+8)=$ **1.63**

Quicker to study WE * Quicker to study PS Crosstabulation

			Quicker to study PS		
			Disagree	Agree	Total
Quicker to study WE	Disagree	Count	21	11	32
		% within Quicker to study WE	65.6%	34.4%	100.0%
		% within Quicker to study PS	56.8%	29.7%	43.2%
	Agree	Count	16	26	42
		% within Quicker to study WE	38.1%	61.9%	100.0%
		% within Quicker to study PS	43.2%	70.3%	56.8%
Total		Count	37	37	74
		% within Quicker to study WE	50.0%	50.0%	100.0%
		% within Quicker to study PS	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	5.506[a]	1	.019		
Continuity Correction[b]	4.460	1	.035		
Likelihood Ratio	5.582	1	.018		
Fisher's Exact Test				.034	.017
Linear-by-Linear Association	5.432	1	.020		
McNemar Test				.442[c]	
N of Valid Cases	74				

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 16.00.
b. Computed only for a 2x2 table
c. Binomial distribution used.

McNemar's Chi-Square= $(16-11)^2/(16+11)= 0.92$

Improved my review WE * Improved my review PS Crosstabulation

			Improved my rewiew PS		Total
			Disagree	Agree	
Improved my review WE	Disagree	Count	28	13	41
		% within Improved my review WE	68.3%	31.7%	100.0%
		% within Improved my review PS	59.6%	48.1%	55.4%
	Agree	Count	19	14	33
		% within Improved my review WE	57.6%	42.4%	100.0%
		% within Improved my review PS	40.4%	51.9%	44.6%
Total		Count	47	27	74
		% within Improved my review WE	63.5%	36.5%	100.0%
		% within Improved my review PS	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.906[a]	1	.341		
Continuity Correction[b]	.503	1	.478		
Likelihood Ratio	.905	1	.342		
Fisher's Exact Test				.467	.239
Linear-by-Linear Association	.894	1	.344		
McNemar Test				.377[c]	
N of Valid Cases	74				

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 12.04.
b. Computed only for a 2x2 table
c. Binomial distribution used.

McNemar's Test: Chi-square= $(19-13)^2/(19+13)= 1.12$

Easier to learn WE * Easier to learn PS Crosstabulation

			Easier to learn PS		Total
			Disagree	Agree	
Easier to learnWE	Disagree	Count	32	11	43
		% within Easier to learnWE	74.4%	25.6%	100.0%
		% within Easier to learn PS	62.7%	47.8%	58.1%
	Agree	Count	19	12	31
		% within Easier to learnWE	61.3%	38.7%	100.0%
		% within Easier to learn PS	37.3%	52.2%	41.9%
Total		Count	51	23	74
		% within Easier to learnWE	68.9%	31.1%	100.0%
		% within Easier to learn PS	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	1.449[a]	1	.229		
Continuity Correction[b]	.901	1	.342		
Likelihood Ratio	1.440	1	.230		
Fisher's Exact Test				.309	.171
Linear-by-Linear Association	1.430	1	.232		
McNemar Test				.200[c]	
N of Valid Cases	74				

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 9.64.
b. Computed only for a 2x2 table
c. Binomial distribution used.

McNemar's Test: Chi-square=$(19-11)^2/(19+11)= 2.13$

Increase my confidence WE * Increase my confidence PS Crosstabulation

			Increase my confidence PS		
			Disagree	Agree	Total
Increase my confidence WE	Disagree	Count	31	13	44
		% within Increase my confidence WE	70.5%	29.5%	100.0%
		% within Increase my confidence PS	68.9%	44.8%	59.5%
	Agree	Count	14	16	30
		% within Increase my confidence WE	46.7%	53.3%	100.0%
		% within Increase my confidence PS	31.1%	55.2%	40.5%
Total		Count	45	29	74
		% within Increase my confidence WE	60.8%	39.2%	100.0%
		% within Increase my confidence PS	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	4.236[a]	1	.040		
Continuity Correction[b]	3.296	1	.069		
Likelihood Ratio	4.231	1	.040		
Fisher's Exact Test				.053	.035
Linear-by-Linear Association	4.178	1	.041		
McNemar Test				1.000[c]	
N of Valid Cases	74				

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 11.76.
b. Computed only for a 2x2 table
c. Binomial distribution used.

McNemar's Test: Chi-square$=(14-13)^2/(14+13)= 0.037 = 0.04$

Requires a lot of mental work WE * Requires a lot of mental Work PS Crosstabulation

			Requires a lot of mental Work PS		
			Disagree	Agree	Total
Requires a lot of metal work WE	Disagree	Count	19	13	32
		% within Requires a lot of mental work WE	59.4%	40.6%	100.0%
		% within Requires a lot of mental Work PS	51.4%	35.1%	43.2%
	Agree	Count	18	24	42
		% within Requires a lot of mental work WE	42.9%	57.1%	100.0%
		% within Requires a lot of mental Work PS	48.6%	64.9%	56.8%
Total		Count	37	37	74
		% within Requires a lot of metal work WE	50.0%	50.0%	100.0%
		% within Requires a lot of mental Work PS	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	1.982[a]	1	.159		
Continuity Correction[b]	1.376	1	.241		
Likelihood Ratio	1.992	1	.158		
Fisher's Exact Test				.241	.120
Linear-by-Linear Association	1.955	1	.162		
McNemar Test				.473[c]	
N of Valid Cases	74				

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 16.00.
b. Computed only for a 2x2 table
c. Binomial distribution used.

McNemar's Test: Chi-square=$(18-13)^2/(18+13)= 0.80$

Make mathematics interesting WE * make mathematics interesting PS Crosstabulation

			make mathematics interesting PS		
			Disagree	Agree	Total
Make mathematics interesting WE	Disagree	Count	27	9	36
		% within Make mathematics interesting WE	75.0%	25.0%	100.0%
		% within make mathematics interesting PS	57.4%	33.3%	48.6%
	Agree	Count	20	18	38
		% within Make mathematics interesting WE	52.6%	47.4%	100.0%
		% within make mathematics interesting PS	42.6%	66.7%	51.4%
Total		Count	47	27	74
		% within Make mathematics interesting WE	63.5%	36.5%	100.0%
		% within make mathematics interesting PS	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	3.991[a]	1	.046		
Continuity Correction[b]	3.085	1	.079		
Likelihood Ratio	4.051	1	.044		
Fisher's Exact Test				.056	.039
Linear-by-Linear Association	3.937	1	.047		
McNemar Test				.061[c]	
N of Valid Cases	74				

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 13.14.
b. Computed only for a 2x2 table
c. Binomial distribution used.

McNemar's Test: Chi-square= $(20-9)^2/(20+9)=4.17$

like to lean mathematics PS * like to learn Mathematics Crosstabulation

			Like to learn Mathematics		
			Disagree	Agree	Total
like to learn mathematics PS	Disagree	Count	36	6	42
		% within like to learn mathematics PS	85.7%	14.3%	100.0%
		% within like to learn Mathematics	66.7%	30.0%	56.8%
	Agree	Count	18	14	32
		% within like to learn mathematics PS	56.3%	43.8%	100.0%
		% within like to learn Mathematics	33.3%	70.0%	43.2%
Total		Count	54	20	74
		% within like to learn mathematics PS	73.0%	27.0%	100.0%
		% within like to learn Mathematics	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	7.995[a]	1	.005		
Continuity Correction[b]	6.570	1	.010		
Likelihood Ratio	8.052	1	.005		
Fisher's Exact Test				.008	.005
Linear-by-Linear Association	7.887	1	.005		
McNemar Test				.023[c]	
N of Valid Cases	74				

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 8.65.
b. Computed only for a 2x2 table
c. Binomial distribution used.

McNemar's Test: Chi-square$=(18-6)^2/(18+6)=6$

Reduced Anxiety WE * Reduced anxiety PS Crosstabulation

			Reduced anxiety PS		Total
			Disagree	Agree	
Reduced Anxiety WE	Disagree	Count	42	7	49
		% within Reduced Anxiety WE	85.7%	14.3%	100.0%
		% within Reduced anxiety PS	70.0%	50.0%	66.2%
	Agree	Count	18	7	25
		% within Reduced Anxiety WE	72.0%	28.0%	100.0%
		% within Reduced anxiety PS	30.0%	50.0%	33.8%
Total		Count	60	14	74
		% within Reduced Anxiety WE	81.1%	18.9%	100.0%
		% within Reduced anxiety PS	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	2.030[a]	1	.154		
Continuity Correction[b]	1.234	1	.267		
Likelihood Ratio	1.948	1	.163		
Fisher's Exact Test				.211	.134
Linear-by-Linear Association	2.002	1	.157		
McNemar Test				.043[c]	
N of Valid Cases	74				

a. 1 cells (25.0%) have expected count less than 5. The minimum expected count is 4.73.
b. Computed only for a 2x2 table
c. Binomial distribution used.

McNemar's Test: Chi-square=$(18-7)^2/(18+7)$= 4.84

Average WE * Average PS Crosstabulation

			Average PS		Total
			1	2	
Average WE	1	Count	51	12	63
		% within Average WE	81.0%	19.0%	100.0%
		% within Average PS	86.4%	80.0%	85.1%
	2	Count	8	3	11
		% within Average WE	72.7%	27.3%	100.0%
		% within Average PS	13.6%	20.0%	14.9%
Total		Count	59	15	74
		% within Average WE	79.7%	20.3%	100.0%
		% within Average PS	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.392[a]	1	.531		
Continuity Correction[b]	.048	1	.826		
Likelihood Ratio	.369	1	.544		
Fisher's Exact Test				.684	.392
Linear-by-Linear Association	.387	1	.534		
McNemar Test				.503[c]	
N of Valid Cases	74				

a. 1 cells (25.0%) have expected count less than 5. The minimum expected count is 2.23.

b. Computed only for a 2x2 table

c. Binomial distribution used.

McNemar's Chi-Square = $(8-12)^2/(8+12) = 0.8$

Improve Worked Examples * Improve Problem-Solving Crosstabulation

			Improve Problem Solving		
			No	Yes	Total
Improve Worked Examples	No	Count	6	15	21
		% within Improve Worked Examples	28.6%	71.4%	100.0%
		% within Improve Problem-Solving	10.9%	78.9%	28.4%
	Yes	Count	49	4	53
		% within Improve Worked Examples	92.5%	7.5%	100.0%
		% within Improve Problem-Solving	89.1%	21.1%	71.6%
Total		Count	55	19	74
		% within Improve Worked Examples	74.3%	25.7%	100.0%
		% within Improve Problem-Solving	100.0%	100.0%	100.0%

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	32.163[a]	1	.000		
Continuity Correction[b]	28.903	1	.000		
Likelihood Ratio	30.817	1	.000		
Fisher's Exact Test				.000	.000
Linear-by-Linear Association	31.729	1	.000		
McNemar Test				.000[c]	
N of Valid Cases	74				

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 5.39.

b. Computed only for a 2x2 table

c. Binomial distribution used.

McNemar's Chi-Square $= (49-15)^2/(49+15) = 18.06$