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A spatial-temporal statistical approach to command and control problems in battle-space digitization

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Abstract

There are considerable difficulties in the integration, visualization, and overall management of battle-space information for the purpose of Command and Control (C2). One problem that we see as being important is the timely combination of digital information from multiple (possibly disparate) sources in a dynamically evolving environment. That is, there is a need to assimilate incoming data rapidly, so as to provide the battle commander with up-to-date knowledge about the battle-space and thereby to facilitate the command-decision process. In this paper, we present a spatial-temporal approach to obtaining accurate estimates of the constantly changing battlefield, based on noisy data from multiple sources. Specifically, we examine the danger posed to a theoretical warfighter in the combat theater. The danger-potential field generated by an enemy's weapons is defined in the spatial domain and is later extended to incorporate the temporal dimension. We propose that maps of fields of this sort are very effective decision tools for the battle commander. Kalman-filtering techniques are proposed to facilitate the rapid estimation of these danger-potential fields. Methods of displaying these predictions and the uncertainty associated with them are discussed. It is the quantification of uncertainty in C2 predictions that distinguishes our statistical approach from deterministic approaches. An application is given to a data set generated by an object-oriented combat-simulation program that we have developed.

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A Spatial-Temporal Statistical Approach to Command and Control Problems in Battle-Space Digitization

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ABSTRACT

There are considerable difficulties in the integration, visualization, and overall management of battle-space information for the purpose of Command and Control (C2). One problem that we see as being important is the timely combination of digital information from multiple (possibly disparate) sources in a dynamically evolving environment. That is, there is a need to assimilate incoming data rapidly, so as to provide the battle commander with up-to-date knowledge about the battle-space and thereby to facilitate the command-decision process. In this paper, we present a spatial-temporal approach to obtaining accurate estimates of the constantly changing battlefield, based on noisy data from multiple sources. Specifically, we examine the danger posed to a theoretical warfighter in the combat theater. The danger-potential field generated by an enemy's weapons is defined in the spatial domain and is later extended to incorporate the temporal dimension. We propose that maps of fields of this sort are very effective decision tools for the battle commander. Kalman-filtering techniques are proposed to facilitate the rapid estimation of these danger-potential fields. Methods of displaying these predictions and the uncertainty associated with them are discussed. It is the quantification of uncertainty in C2 predictions that distinguishes our statistical approach from deterministic approaches. An application is given to a data set generated by an object-oriented combat-simulation program that we have developed.

1. WHAT IS C2?

Command and Control (C2) and its intellectual brethren (C3, C4I, etc.) are umbrella terms used to describe a body of research and applications dedicated to preparing all branches of the armed forces for the "War After Next" [1]. Applications incorporated into the greater C2 framework include, but are not limited to, command applications, operations applications, intelligence applications, fire-support applications, logistics applications, and communications applications. C2 needs are shared by all branches of the armed services. Consequently, applications should be sufficiently flexible to work across services and across allied forces, as needed. In order to be applicable to future wars, applications need to be able to convert a flood of data from a wide variety of sources into information and knowledge in a timely manner.

In preparing for the Command Post of the Future, tools should be developed with the following three goals in mind: to rapidly visualize the battle-space, to rapidly analyze the battle-space, and to rapidly understand the battle-space. When visualizing the battle-space, the prototypical commander wants a variety of information, and emergency information needs to be highlighted so that time-critical decisions can be made. In addition, the location and status of friendly and enemy forces need to be available to the commander. In order to facilitate the rapid visualization and understanding of the battle-space, to provide the ability to receive and send information while mobile, and to begin converting that information into knowledge is critical to the battle commander. Facility for intelligent alerting and reporting, as well as customizable tactical display elements, need to be integrated into any C2 application.

Systems developed for C2 applications need to keep several general capabilities in mind. The military currently suffers, not from a lack of data but from a flood of data. Data arrive from multiple sources and possibly multiple nationalities. Data arrive in visual, verbal, and other sensor formats, and also from historical data bases. At the same time, there is a distinct deficiency of information and knowledge. C2 applications should aim at developing decision aids that can turn massive amounts of data into highly useful information and that reduce the number of viable options available to the commander at key decision-making junctures. Further, even though there are large amounts of data available, developed systems must be able to deal with missing and corrupted data. In addition to

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traditional causes of missing or corrupted data, military applications face the potential problem of hostile corruption. In a war, it should be anticipated that data could be actively intercepted, altered, and destroyed by unfriendly forces.

Given the wide variety of battle scenarios possible in future wars, systems developed for C2 need to be scaleable between large-scale operations and unit tactics. At the same time, these systems need to provide all war fighters with a common picture of any particular battle-space. Finally, and perhaps most crucially, information processing needs to be completed quickly. To the modern war fighter, time is of the essence and it is projected that speed of processing will become even more vital in the future.

Statisticians have a unique perspective on the challenges presented by C2 and thus have an opportunity to contribute to research and applications. Statistical techniques can help make the transition from the flood of data to meaningful information and knowledge for decision making. For example, consider the area of situational awareness. The statistician might consider probabilistic frameworks, scaleable algorithms, and sequential decision-making. In particular, one might examine methods of estimating and predicting the threat or danger to a region, posed by enemy constituents. Additional areas of interest are change and anomaly detection, measures of information and understanding, and decision theory. Statisticians should examine source data, developing methods to address confidence, accuracy, and completeness of such data. They might also consider the validity of information-processing results and the quality of database information. Other research areas should focus on the representation of uncertainty or confidence in statistical results and might also pursue algorithms that allow statistical inference to be decentralized. Note that, in addition to providing processing stability in a hostile environment, such decentralization should lead to increased inference speed through parallel, distributed processing.

In conclusion, Command and Control (C2) is a broad field, providing a wide variety of options for statistical research and applications. It is the specific goal of this paper to examine one aspect, namely optimal mapping of the regional danger posed by enemy constituents in the battle-space. In Section 2, we develop a quantification of C2 for statistical analysis and define the danger-potential field in space and time. In Section 3, we examine the data for C2 decisions and the possible degradations such data might suffer. We consider analysis of the data with regard to estimating danger-potential fields in Section 4, and give a brief example. Finally, in Section 5, we summarize our efforts and discuss future directions.

2. QUANTIFICATION OF COMMAND AND CONTROL (C2) FOR STATISTICAL ANALYSIS

Before considering C2 from a statistical point-of-view, it is important to examine the nature of the battle-space, D, and the data that might emerge from it, as well as the state process of interest. By the state process, or Y-process, we mean the underlying spatial-temporal process describing the battle, assuming perfect, noiseless, complete knowledge. It was decided that, regardless of the choice of Y, a flexible simulation tool would be developed to produce imperfect, noisy, incomplete data Z. To define the tool properly, a set of definitions and rules were established. In order to keep the structure as flexible as possible, we settled on the following hierarchical design for any battle-space object.

At the largest size, the battle-space was defined to be a meta-object. For a particular simulation experiment, the battle-space contains all the other objects in the hierarchy. The next smallest class of objects is made up of *constituents*. In general, a constituent is an object that cannot be further combined (in the scope of the experiment) with another object of that class in a meaningful way. Specific examples of constituents include the environment in which the battle occurs, tanks and other offensive objects, and radar towers and other passive sensing objects. It should be noted that for battles of a larger scale, a constituent might actually be a group of "smaller" objects. For example, a group of three tanks on patrol together might be considered a constituent in a battle scenario of broad enough extent. In the scale considered in this paper, single tanks are considered to be constituents. Constituents are made up of *elements*. Elements are a smaller class of objects that have a specific sort of activity assigned to them. A tank constituent, for example, might have one or more human elements, mobility elements, sensor elements, and weapon elements. Elements of the environment constituent might include terrain and weather. The smallest classes of objects in our hierarchy are *functions* and *parameters*. Functions and parameters work together to define specifically how elements perform their activities. Functions may have deterministic or random aspects associated with them, and parameters provide the necessary numeric arguments to functions. Again, for a movement element of a tank constituent, functions might include a deterministic 'head straight for your target' function ('goStraight' object) or a 'random walk toward your target' function ('goZigZag' object). Either of these functions might accept parameters such as maximum speed, mean angle, and the standard deviations on speed and angle. Other functions

Figure 1. Hierarchical Design of Battle-Space Object

meta-object
(battle-space)

↙ ↘

physical constituents (environment)	military constituents (offensive-type, observer-type)
↓	↓
physical elements (natural terrain, urban terrain, landmark, weather)	military elements (sensor, weapon, movement)
↙ ↘	↙ ↘
functions (contour) — parameters (altitude)	functions (goZigZag, coneError) — parameters (velocity, variance)

might include functions that describe targeting error ('coneError') or other scenario-specific activities. Functions need not necessarily be limited to acting on parameters. For instance, in more complicated models, elements of a given constituent might interact according to some element-level function. It is conceivable that even higher-level interactions or functions might also occur, but our analysis will focus only on the parameter-level functions for the time being. In the design of this hierarchy, additional object classes could be inserted for more complex simulations. For example, a *component* class could be inserted between *constituents* and *elements*. For more information on the battle-space simulation developed by our group at The Ohio State University, see [7].

It should be noted that one or more commanders may be associated with each level of this hierarchy. An overall battle commander would be associated with the battle-space at the meta-object level; at the military-constituent and military-element levels, a commander would receive orders from the overal commander but would also direct actions associated with that constituent or element. Even at the lowest level of the hierarchy, one can imagine a function commander who is responsible for performing the tasks described by the function and who follows orders from higher levels of the hierarchy.

At this point, a brief notational comment should be made. For the purposes of formulating the battle-space model and developing the analysis, constituents are notated as vectors. Offensive-type constituents, such as tanks, are notated $\mathbf{w}_k$, where $k = 1, 2, \ldots$. Observer-type constituents, such as radar towers, are notated $\mathbf{v}_i$, where $i = 1, 2, \ldots$. In initial work, this vector is interpreted strictly as the Cartesian coordinates of the constituent in question, but can be thought of more generally as the state of the constituent in question. Elements of a particular constituent are assumed to be located at the same location as their 'parent' constituent, although this assumption could be generalized if the constituents are distributed. When time is incorporated into the analysis, these vectors become functions of t. Specifically, in spatial-temporal-analysis settings, offensive-type constituents are notated $\mathbf{w}_k(t)$ and observer-type constituents are notated $\mathbf{v}_i(t)$. This notation refers to the location (or state) of the constituent in question at time t.

Consequently, all our analyses are done on continuous fields rather than on grids or lattices, although the grid is used for some numerical and visualization algorithms. For convenience, we have started with a region D that is a rectangle of fixed dimension. In this paper, we focus on simple, flat terrain elements, but later work will include the effects of more complex terrain elements.

To carry out a battle-space simulation, we need to define the number, position, and type of constituents that would exist within a battle. This could be done in advance or assigned randomly in an obvious way (e.g., Poisson distribution [2] for numbers of objects, constrained and scaled uniform distribution or Beta distribution [2] for location coordinates). For initial experiments, two general classes of constituents were settled on: weapons and observers. We shall discuss observers in greater detail later (see Section 3), where data collection and degradation is considered,

but we note here that observers are defined by functions and parameters associated with their viewing area and their error types.

Weapons, in general, have more complicated functions and parameters than do observers. For the simplest scenarios, only functions and parameters associated with movement and offensive elements need to be considered. For more complicated scenarios, defensive functions and parameters need to be defined as well. Movement parameters include such things as autoregressive parameters [3] and error types, assigned waypoints, and commands. Offensive functions and parameters include targeting functions (such as 'coneError') and the explosion parameters used below. Defensive parameters include amount of damage that an element can suffer and still be operational and the amount of resistance an element carries against damage. We recognize that many additional parameters might be of interest to the experimenter, and so provision for their inclusion later was incorporated into the design of the simulation.

For initial simulation experiments, waypoint parameters are purely deterministic, with a look to providing them later with a random function based on commander orders and affected by environmental parameters. One way to model this randomness is to consider the location-based error that arises from landmark-based orders. That is, a constituent might be instructed to move to a specific landmark element within the natural or urban environment constituent rather than to a specific set of coordinates. If the location of the landmark element is known with error, a level of uncertainty emerges.

Recognizing that maps are an intuitive way to present knowledge, we now focus on mapping some sort of summary of the Y-process based on imperfect, noisy, incomplete data, Z. As an example, consider the damage potential posed by an enemy unit or set of enemy units. This should be particularly interesting to a commander, and it exhibits a lot of space-time variability. From the damage potential, we wish to estimate a danger-potential field generated by a set of enemy units and to examine the uncertainties associated with it. We consider our study illustrative of the general problem of producing statistically optimal maps that evolve with the changing battle-space.

We see the damage potential of an enemy weapon as analogous to potential energy; that is, the damage done to a target can be thought of as being equal to the damage potential of a weapon element times an armor parameter that depends on the target's ability to protect itself. Thus, the damage potential of a given weapon element is equal to the damage it can do to any target constituent with a unit armor parameter.

The statistical distribution of damage is examined next. Clearly, a missile applying damage at a precise, confined location is unrealistic. As a result, all damage was considered to be of an explosive type, that is, affecting a continuous region and being a non-increasing function of distance from the impact point. The following formula describes one possible form of the damage potential at a distance r from the impact point:

$$\delta(r) = \begin{array}{ll} \alpha(1-(r/R)^{p_1})^{p_2}, & for \quad 0 < r < R, \\ 0, & else, \end{array} \tag{1}$$

where α, R, p_1, and p_2 are all explosion parameters defined for the weapon element in question. Under this definition, a single location in the battle-space can be affected by damage resulting from nearby impacts in the space, and the damage potential will vary with distance from the impact.

Before continuing, consider the following notational conventions. Let $\mathbf{w}_{kl}$ denote the lth location impacted by weapon $\mathbf{w}_k$, allowing a single weapon to possibly have multiple 'hits' in a short, specific time interval. For the purpose of this paper, attention will be focused on single-impact weapons with the impact denoted $\mathbf{w}_{k1}$. Further, denote $f(\mathbf{w}_{k1}|\mathbf{s}, \mathbf{w}_k)$ as the probability density function of an impact at location $\mathbf{w}_{k1}$, given the weapon is located at $\mathbf{w}_k$ and is aiming at location $\mathbf{s}$. Similarly, let $\mathbf{v}_{ij}$ denote the jth location observed by observer i in a specific time interval. Notice that enemy weapon constituents may not always be distinguishable by an observer and, thus, we do not know in general which weapon $\mathbf{v}_{ij}$ refers to. However, for the purpose of this paper, we will assume that the observe can identify the weapon without ambiguity, and we let $\mathbf{v}_{ik}$ denote the location observed by observer i for weapon k.

We define the *danger-potential* generated by a single weapon element at location $\mathbf{w}_k$, as the expected damage at any location $\mathbf{s}$:

$$g(\mathbf{s};\mathbf{w}_k) \equiv \int_{\mathbf{w}_{k1}} \delta(r_{\mathbf{s},\mathbf{w}_{k1}}) f(\mathbf{w}_{k1}|\mathbf{s},\mathbf{w}_k) d\mathbf{w}_{k1}; \qquad \mathbf{s} \in D. \tag{2}$$

The distribution, $f(\mathbf{w}_{k1}|\mathbf{s},\mathbf{w}_k)$, is based on the same radius-angle probability distribution given in Section 3. It should be noted that, given the parameters of a given weapon, $g(\mathbf{s};\mathbf{w}_k)$ can be computed ahead of time, thus reducing the amount of time it takes to generate a danger potential over D.

We further assume that danger is summable. That is, we define the danger potential to an object at a specific location, from a set of enemy weapon elements, as the sum of the individual danger potentials of each weapon element. Such a definition makes sense if the individual damage potentials are summable, as we now illustrate. The danger potential at $\mathbf{s}$ is,

$$\begin{aligned} g(\mathbf{s}) &= \int_{\mathbf{w}_{k1}} \sum_k \delta(r_{s,\mathbf{w}_{k1}}) f(\mathbf{w}_{k1}|\mathbf{s},\mathbf{w}_k) d\mathbf{w}_{k1} & (3)\\ &= \sum_k \int_{\mathbf{w}_{k1}} \delta(r_{s,\mathbf{w}_{k1}}) f(\mathbf{w}_{k1}|\mathbf{s},\mathbf{w}_k) d\mathbf{w}_{k1} & (4)\\ &= \sum_k g(\mathbf{s};\mathbf{w}_k), & (5) \end{aligned}$$

where $\sum_k \delta(r_{s,\mathbf{w}_{k1}})$ is the damage potential of the multiple weapons $\{\mathbf{w}_{k1}\}$.

Though danger potential was defined above in a purely spatial setting, there is a natural extension to the spatial-temporal setting. Consider an offensive constituent at location $\mathbf{w}_k(\tau)$, and time τ. Then the expected damage potential at location $\mathbf{s}$ and time $t > \tau$, not only depends on the probability of applying damage, but also on the probability of the weapon's location at unobserved times. That is, we wish to take an expectation on the targeting distribution as in (2), but the expectation of the damage at a given spatial-temporal location is now based on knowledge of the weapon's location at some prior time (τ). The resulting danger-potential field at $\mathbf{s}$ and t is:

$$\begin{aligned} g_t(\mathbf{s};\mathbf{w}_k(\tau)) &= \int_{\mathbf{w}_{k1}(t)} \delta(r_{\mathbf{s}(t),\mathbf{w}_{k1}(t)}) f(\mathbf{w}_{k1}(t)|\mathbf{s}(t),\mathbf{w}_k(t)) d\mathbf{w}_{k1}(t) & (6)\\ &= \int_{\mathbf{w}_{k1}(t)} \delta(r_{\mathbf{s}(t),\mathbf{w}_{k1}(t)}) [\int_{\mathbf{w}_k(t)} f(\mathbf{w}_{k1}(t),\mathbf{w}_k(t)|\mathbf{s}(t),\mathbf{w}_k(\tau)) d\mathbf{w}_k(t)] d\mathbf{w}_{k1}(t) & (7)\\ &= \int_{\mathbf{w}_{k1}(t)} \int_{\mathbf{w}_k(t)} \delta(r_{\mathbf{s}(t),\mathbf{w}_{k1}(t)}) f(\mathbf{w}_{k1}(t)|\mathbf{s}(t),\mathbf{w}_k(t)) h_1(\mathbf{w}_k(t)|\mathbf{w}_k(\tau)) d\mathbf{w}_k(t) d\mathbf{w}_{k1}(t) & (8)\\ &= \int_{\mathbf{w}_k(t)} g(\mathbf{s}(t);\mathbf{w}_k(t)) h_1(\mathbf{w}_k(t)|\mathbf{w}_k(\tau)) d\mathbf{w}_k(t)\,. & (9) \end{aligned}$$

The probability density function, $h_1(\mathbf{w}_k(t)|\mathbf{w}_k(\tau))$, represents the conditional probability that the weapon is at $\mathbf{w}_k(t)$ given that it was at spatial-temporal location $\mathbf{w}_k(\tau)$, which introduces a dynamic aspect to the analysis. The probability depends not only on the movement function and the parameters associated with the weapon element, but also on the evolution of the spatial-temporal battle-space. For instance, if the weapon element is damaged, its mobility may be affected. In the simple example discussed in this paper, the weapon cannot be damaged, so that part of the dynamic aspect may be ignored. Expanding this conceptualization, one might re-write (8) and (9) as:

$$\begin{aligned} g_t(\mathbf{s};\mathbf{W}) &= \sum_k \int_{\mathbf{w}_{k1}(t)} \int_{\mathbf{w}_k(t)} \delta(r_{s_t,w_{k1}(t)}) f(\mathbf{w}_{k1}(t)|\mathbf{s}(t),\mathbf{w}_k(t)) h_1(\mathbf{w}_k(t)|\mathbf{W}) d\mathbf{w}_k(t) d\mathbf{w}_{k1}(t) & (10)\\ &= \sum_k \int_{\mathbf{w}_k(t)} g(\mathbf{s}(t);\mathbf{w}_k(t)) h_1(\mathbf{w}_k(t)|\mathbf{W}) d\mathbf{w}_k(t) & (11) \end{aligned}$$

where $\mathbf{W}$ the set of all known information and $h_1(\mathbf{w}_k(t)|\mathbf{W})$ is the conditional probability density function describing the probability that a weapon is at a specific location in space-time, given knowledge of all weapons at various locations in space-time.

In the proposed conceptualization, data are primarily observed locations of military constituents. That is, at a given time point, each observer constituent provides a list of Cartesian coordinates representing its observed locations of other constituents. Note that these observed locations are almost certainly noisy and perhaps compromised through censoring or enemy interference. The nature of the error associated with these data is discussed further in Section 3.

Given the data, our goal in this article is then to estimate and map the true danger potential everywhere in the spatial-temporal region of interest. Information about the parameters of the weapon constituents and observer constituents is combined to generate estimates of the danger potential. An advantage of the spatial-temporal statistical approaches discussed in this paper is the flexibility of the questions that might be answered using the data. Simple questions about the location of a weapon or a set of weapons are not the only ones that are answerable. In addition, one might pose questions such as, "How often are enemy weapons within 10 miles of the border?" or "Does the danger posed to friendly regions appear to increase significantly over time?" Given the form of the danger-potential field, all of these questions are non-linear, and thus techniques such as Kalman filtering [5] will typically yield biased estimates. However, the development of covariance-matching maps may prove to be more successful in representing and answering these queries.

3. DATA FOR C2 DECISIONS

Once a simulation of the Y-process has been completed, it is time to consider the collection of data. Generating and running the simulation gives the experimenter access to the true battle information, but in real-world situations, the information is degraded in some manner. To reflect this, we degraded the simulated Y-process in several ways. Censoring might occur in both the time- and space-domains for various reasons. Terrain features might limit the observation region of an observer, and technical difficulties might prevent or delay observation of data. Location-based error was also deemed to be a fairly likely situation. Less likely, but worth considering, was false data provided by the enemy.

Location-based error is worth particular focus for at least two reasons. First, of all the possible degradations, it seemed to us to be appropriate to include it in all experiments. Additionally, in the battle-space definition, the radius-angle distributions primarily used for this error appear repeatedly. While standard bivariate normal error might be applied to any location and is indeed applied for satellite-type sensors, radar-type sensors and weapon-targeting applications within the battle-space simulation were given radius-angle error distributions. That is, the angle and distance between the observer and the observed, or the weapon and the target, are computed. It is assumed that these coordinates are reported with the following error structure. If the true angle is θ_0, then the reported angle θ is:

$$\theta = \theta_0 + \epsilon_\theta, \tag{12}$$

where ϵ_θ is distributed with mean 0 and variance σ_θ^2. If the true distance is r_0, it is assumed that the reported distance coordinate r is randomly distributed with mean $r_0/(1-\frac{1}{2}\sigma_\theta^2)$ and variance σ_r^2. Note that if r is unbiased for r_0 then it can be shown that the observations of the weapon location are biased. The choice of the mean value $r_0/(1-\frac{1}{2}\sigma_\theta^2)$, represents an adjustment for that bias; see Section 4.1. These coordinates are then converted back into the standard Cartesian coordinates used to describe all constituents in the battle-space.

4. ANALYSIS

4.1. Details of Analysis

For the purpose of this paper, the goal of the analysis is to estimate the true danger-potential field of a battle-space over time. Note that the deterministic danger-potential field is well defined if the true distribution of the impact location and the location of the firing weapon is known. Since the former is likely to be known from intelligence sources, our focus is on the latter. Consequently, we wish to estimate the true danger potential in the face of unknown weapon locations. We discuss two possible approaches, whose differences we will investigate in the future.

One approach is to apply Bayesian methods [4]. In this case, one might generate an ensemble of danger-potential fields through the weapon element's posterior location distribution. An alternative approach is to apply some sort of 'plug-in' estimate for the locations of the offensive constituent(s). Under this approach, the estimate might be achieved by replacing the true location of the weapon elements with estimates of the same, obtained from the data.

Regardless of the approach used, an estimate of the posterior probability distribution of the locations of the weapons, given the observations, would be useful. When dealing with the radius-angle errors described earlier, the distributions of the observations can be complicated to represent. However, second-degree Taylor approximations can be used to approximate the mean and the (2×2) covariance matrix of the observations with reasonable accuracy. We now give a calculation for the radius-angle error structure.

Figure 2. Notation Conventions

$\mathbf{v}_i, \mathbf{v}_i(t)$	the location (state) of observer i (at time t)
$\mathbf{v}_{ik}, \mathbf{v}_{ik}(t)$	the location of weapon k observed by observer i (at time t)
$\mathbf{w}_k, \mathbf{w}_k(t)$	the true location (state) of weapon k (at time t)
$\mathbf{w}_{kl}, \mathbf{w}_{kl}(t)$	the l^{th} location impacted by weapon k (at time t)
$\mathbf{s}, \mathbf{s}(t)$	a hypothetical location (at time t)
$r_{\mathbf{s}\mathbf{w}_k}$	the Euclidean distance, $\|\|\mathbf{w}_k - \mathbf{s}\|\|$, between weapon k at $\mathbf{w}_k$, and the target location $\mathbf{s}$
$r_{\mathbf{w}_k\mathbf{w}_{kl}}$	the Euclidean distance, $\|\|\mathbf{w}_{kl} - \mathbf{w}_k\|\|$, between weapon k at $\mathbf{w}_k$, and the impact location $\mathbf{w}_{kl}$
$r_{\mathbf{s}\mathbf{w}_{kl}}$	the Euclidean distance, $\|\|\mathbf{w}_{kl} - \mathbf{s}\|\|$, between the impact location $\mathbf{w}_{kl}$ and the target location $\mathbf{s}$
$\theta_{\mathbf{s}\mathbf{w}_k}$	the angle, $\arctan[(w_{ky} - s_y)/(w_{kx} - s_x)]$, between weapon k at $\mathbf{w}_k$, and the target location $\mathbf{s}$
$\theta_{\mathbf{w}_k\mathbf{w}_{kl}}$	the angle, $\arctan[(w_{kly} - w_{ky})/(w_{klx} - w_{kx})]$, between weapon k at $\mathbf{w}_k$, and the impact location $\mathbf{w}_{kl}$
$\theta_{\mathbf{s}\mathbf{w}_{kl}}$	the angle, $\arctan[(w_{kly} - s_y)/(w_{klx} - s_x)]$, between the impact location $\mathbf{w}_{kl}$, and the target location $\mathbf{s}$
etc.	

Before examining this approximation, the following facts are worth noting. Second-degree Taylor approximations show that $\cos(\theta)$ is approximately equal to $1 - \frac{1}{2}\theta^2$, and $\sin(\theta)$ is approximately equal to θ. Similarly, $\cos^2(\theta)$, $\sin^2(\theta)$ and $\cos(\theta)\sin(\theta)$ are each approximately equal to $1 - \theta^2$, θ^2, and θ respectively. Assuming the same distribution on the distance and angle as discussed above, consider the observation, $\mathbf{v}_{ik} \equiv (v_{ikx}, v_{iky})$. Write $\mathbf{v}_{ik}$ as $\mathbf{v}_i + \mathbf{\Delta}_{\mathbf{v}_{ik}}$, where $\mathbf{\Delta}_{\mathbf{v}_{ik}} = (\Delta_x, \Delta_y)$, $\Delta_x = r\cos(\theta)$, $\Delta_y = r\sin(\theta)$, and $\theta = \theta_{\mathbf{v}_i\mathbf{w}_k} + \epsilon_\theta$. Combining these equalities and an assumed independence of r and θ, we obtain

$$\begin{aligned} E[v_{ikx}] &= E[v_{ix} + r\cos(\theta_{\mathbf{v}_i\mathbf{w}_k} + \epsilon_\theta)] \\ &= v_{ix} + E[r]E[\cos(\theta_{\mathbf{v}_i\mathbf{w}_k} + \epsilon_\theta)] \\ &= v_{ix} + \frac{r_{\mathbf{v}_i\mathbf{w}_k}}{1 - \frac{1}{2}\sigma_\theta^2}\cos(\theta_{\mathbf{v}_i\mathbf{w}_k})E[\cos(\epsilon_\theta)] - \sin(\theta_{\mathbf{v}_i\mathbf{w}_k})E[\sin(\epsilon_\theta)]\,, \end{aligned} \tag{13}$$

where the last equality is given by the assumptions following equation (12). Applying the Taylor approximations, we obtain

$$E[v_{ikx}] \simeq v_{ix} + \frac{r_{\mathbf{v}_i\mathbf{w}_k}}{1 - \frac{1}{2}\sigma_\theta^2}[\cos(\theta_{\mathbf{v}_i\mathbf{w}_k})E[1 - \frac{1}{2}\epsilon_\theta^2] - r_{\mathbf{v}_i\mathbf{w}_k}\sin(\theta_{\mathbf{v}_i\mathbf{w}_k})E[\epsilon_\theta]] = v_{ix} + r_{\mathbf{v}_i\mathbf{w}_k}\cos(\theta_{\mathbf{v}_i\mathbf{w}_k}). \tag{14}$$

A similar approach can be used to compute the expected values of v_{iky}, v_{ikx}^2, v_{iky}^2 and $v_{ikx}v_{iky}$. With these expected values, the approximate expected value and covariance matrix of $\mathbf{v}_{ik}$ can be computed. After simplification, we obtain

$$E[\mathbf{v}_{ik}] \simeq \mathbf{w}_k \tag{15}$$

$$var[\mathbf{v}_{ik}] \;\simeq\; [\sigma_r^2 - r^2_{\mathbf{v}_i\mathbf{w}_k}(1-\gamma) - \sigma_\theta^2(\sigma_r^2 + r^2_{\mathbf{v}_i\mathbf{w}_k}\gamma)]\mathbf{p}\mathbf{p}' + [\sigma_r^2 + r^2_{\mathbf{v}_i\mathbf{w}_k}\gamma]\sigma_\theta^2\mathbf{q}\mathbf{q}'\,, \tag{16}$$

where $\mathbf{p} = (\cos\theta_{\mathbf{v}_i\mathbf{w}_k}, \sin\theta_{\mathbf{v}_i\mathbf{w}_k})'$, $\mathbf{q} = (\sin\theta_{\mathbf{v}_i\mathbf{w}_k}, -\cos\theta_{\mathbf{v}_i\mathbf{w}_k})'$, and $\gamma = (1 - \frac{1}{2}\sigma_\theta^2)^{-2}$. Note that the covariance matrix above involves the unknown angle $\theta_{\mathbf{v}_i\mathbf{w}_k}$; it is suggested that the observed angle $\theta_{\mathbf{v}_i\mathbf{v}_{ik}}$ be used in its place.

More generally, consider the following model for data obtained by observing the offensive constituent $\mathbf{w}_k$ from an observation constituent, $\mathbf{v}_i$. The resulting observation, $\mathbf{v}_{ik}$, can be expressed as:

$$\mathbf{v}_{ik} = \mathbf{w}_k + \varepsilon_{ik}, \tag{17}$$

where ε_{ik} is a random vector with mean zero and 2×2 covariance matrix depending on the type of error distribution associated with the observer, as well as the relative position of the observation constituent with respect to the offensive constituent. In the case of the observation constituents being radar sites, we assume a radius-angle error distribution and the covariance matrix of ε_{ik} is given by (16).

Thus, for observations on $\mathbf{w}_k$ from multiple observers, $\mathbf{v}_1, \ldots, \mathbf{v}_M$, the following model applies:

$$\begin{pmatrix} \mathbf{v}_{1k} \\ \vdots \\ \mathbf{v}_{Mk} \end{pmatrix} = G\mathbf{w}_k + \begin{pmatrix} \varepsilon_{1k} \\ \vdots \\ \varepsilon_{Mk} \end{pmatrix}, \tag{18}$$

where

$$G = \begin{bmatrix} I_2 \\ I_2 \\ \vdots \\ I_2 \end{bmatrix}, \tag{19}$$

and the ε_{ik} are independent (2×1) zero-mean error vectors with covariance matrix, Σ_i, as given by (16). Notice that G is a $(2M \times 2)$ matrix, $\mathbf{w}_k$ is a (2×1) vector, and the other vectors represented above are $(2M \times 1)$ vectors. The generalized least-squares estimate for $\mathbf{w}_k$ is then

$$\hat{\mathbf{w}}_k \;= (G'\Sigma^{-1}G)^{-1}G'\Sigma^{-1} \begin{bmatrix} \mathbf{v}_{1k} \\ \vdots \\ \mathbf{v}_{Mk} \end{bmatrix}, \tag{20}$$

where

$$\Sigma = \begin{bmatrix} \Sigma_1 & 0 & \cdots & 0 \\ 0 & \Sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma_M \end{bmatrix} \tag{21}$$

and recall that Σ_i is given by (16). This is a rather crude estimate, and we are currently exploring forecasting methods that incorporate past temporal information.

For example, consider the spatial-temporal danger potential defined in (11). Then the expected damage at time t given past observations, $\mathbf{V} = \{\mathbf{v}_{ik}(\tau) : \tau < t;\ k = 1, 2, \ldots\}$, for the enemy weapons, can be obtained as follows:

$$\hat{g}_t(\mathbf{s}; \mathbf{V}) = \int_{\mathbf{W}} g_t(\mathbf{s}; \mathbf{W}) f(\mathbf{W}|\mathbf{V}) d\mathbf{W} \tag{22}$$

$$= \int_{\mathbf{W}} [\sum_k \int_{\mathbf{w}_k(t)} g(\mathbf{s}_t; \mathbf{w}_k(t)) h_1(\mathbf{w}_k(t)|\mathbf{W}) d\mathbf{w}_k(t)] f(\mathbf{W}|\mathbf{V}) d\mathbf{W} \tag{23}$$

$$= \sum_k \int_{\mathbf{w}_k(t)} g(\mathbf{s}_t; \mathbf{w}_k(t)) [\int_{\mathbf{W}} h_1(\mathbf{w}_k(t)|\mathbf{W}) f(\mathbf{W}|\mathbf{V}) d\mathbf{W}] d\mathbf{w}_k(t) \tag{24}$$

$$= \sum_k \int_{\mathbf{w}_k(t)} g(\mathbf{s}_t; \mathbf{w}_k(t)) h(\mathbf{w}_k(t)|\mathbf{V}) d\mathbf{w}_k(t)\,, \tag{25}$$

where $h(\mathbf{w}_k(t)|\mathbf{V})$ is the conditional probability density function describing the probability that a weapon is at a specific location in space-time, given the observation of all weapons at various locations in the past. It should also be noted that at times where observations are not taken or are incomplete, the danger potential will tend to spread out as the uncertainty about the location of the weapons increases. The calculations leading to (25) require modification if $\mathbf{V}$ also contains current observations $\{\mathbf{v}_{ik}(t) : k = 1, 2, \ldots\}$; this results in a filtered (rather than a forecasted) space-time danger potential. These calculations will be reported on elsewhere.

4.2. Small Example

By way of an example, consider a small scenario involving tank manuevers. A 100-mile by 100-mile piece of border territory is observed by two radar stations positioned in the friendly southwest and southeast corners of the region. The region is of interest because two groups of mobile enemy weapons have been detected in the area; there are no friendly units in the region, so that the commander is watching for potential enemy invasion. The radar stations provide noisy location data with a radius-angle error distribution described earlier. The radar stations have a limited range but provide data at consistent time intervals without any missing data.

The enemy weapons include two tanks coming from the northwest corner of the region and three tanks coming from the northeast corner of the region. In the simulation, all of the weapons are assigned a series of waypoints, which represent their orders for the maneuvers. The groups move along two separate paths and appear to converge on a single location as the operation progresses. Waypoint parameters might be assigned to a mobile constituent as a tactical decision made by a battle commander, or in response to known terrain elements. In more complicated simulation experiments, weapon components might be given the ability to assign or alter their own waypoint parameters in response to newly observed data about the terrain element.

It is important to note that, for this example, the parameters for the weapons and observers are not intended to reflect real-world capabilities. Rather, we chose the parameters to illustrate how the danger-potential field can be mapped. Also, for simplicity, we show maps of the danger potential using the true weapon locations rather than the estimated location. See Section 4.1 for discussion of weapon-location and danger potential estimation.

While the overall danger-potential field generated by these vehicles could be examined as it evolves over time, attention will be focused on three time slices for the purpose of this demonstration. In particular, we shall map the danger potential at the start of the scenario, at 120 minutes, and at 240 minutes, where each of the images are presented in the same logarithmic grey scale. It should be noted that the mapping of danger potential was originally conceptualized in a white-to-red color scale but that, even in grey scale, the additive properties of danger are discernable. The shape of the danger-potential field for each tank was pre-computed, so that the overall map could be generated quickly using the additivity property described in (5).

In the first image (Figure 3) we see the danger-potential field at the beginning of the scenario. Note that the danger-potential fields of the individual tanks are still discernable in the upper right and upper left corners of the image.

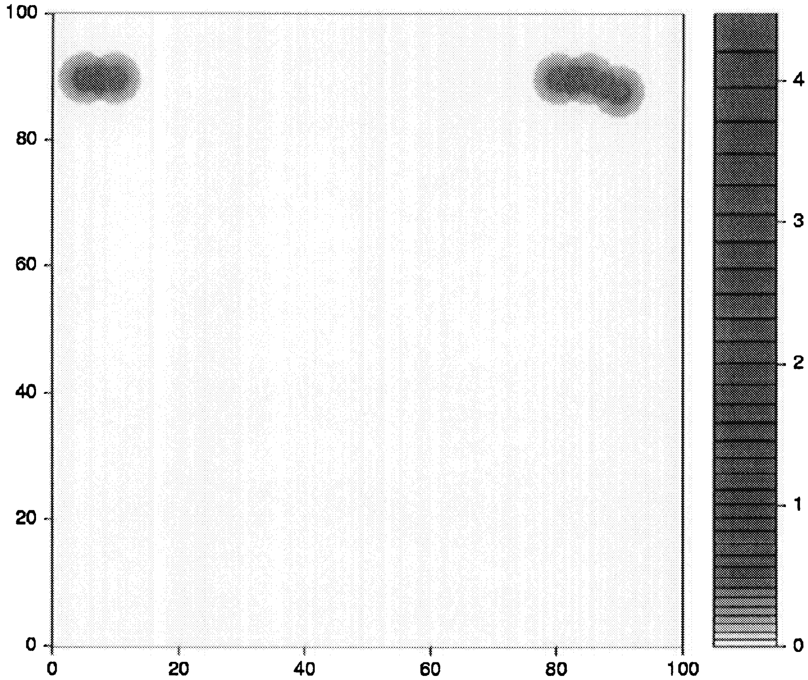

Figure 3: Danger-Potential Field at Beginning of Scenario

In Figure 4, we see the simulated battle scenario after two hours of observation. Note that each group of tanks has closed ranks. That is, the two tanks on the left and the three tanks on the right are traveling in close proximity. As a result, the danger-potential fields cover a smaller total area and have higher intensities over the regions they cover. However, the total danger integrated over the entire observed region has remained constant from the begining of the scenario until now.

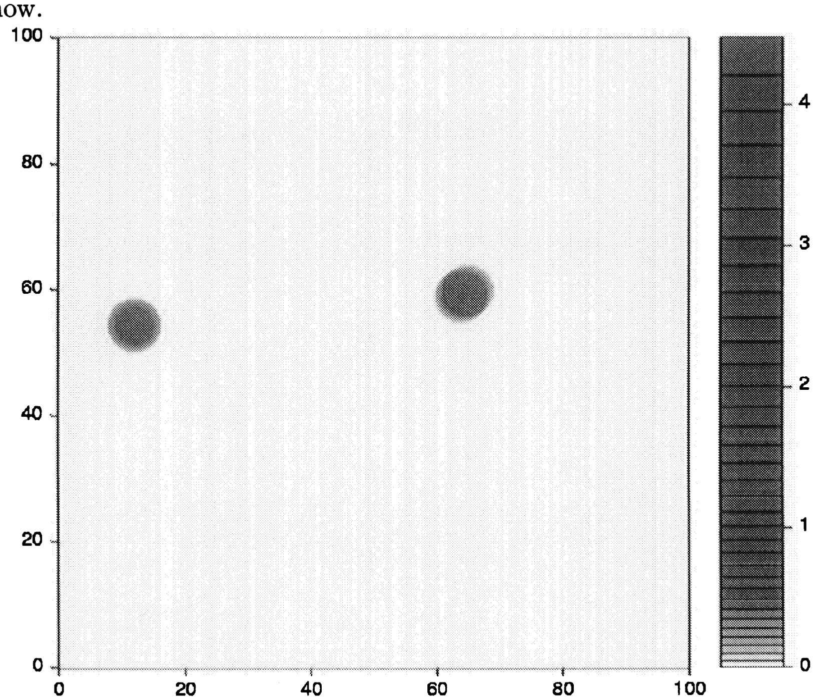

Figure 4: Danger-Potential Field at 120 Minutes

The final image (Figure 5) is a representation of the danger-potential field four hours into the scenario. Note that all of the tanks are now traveling in close proximity. This results in the highest danger potentials of the scenario, but over an even more limited total area than at the first or second time points under consideration.

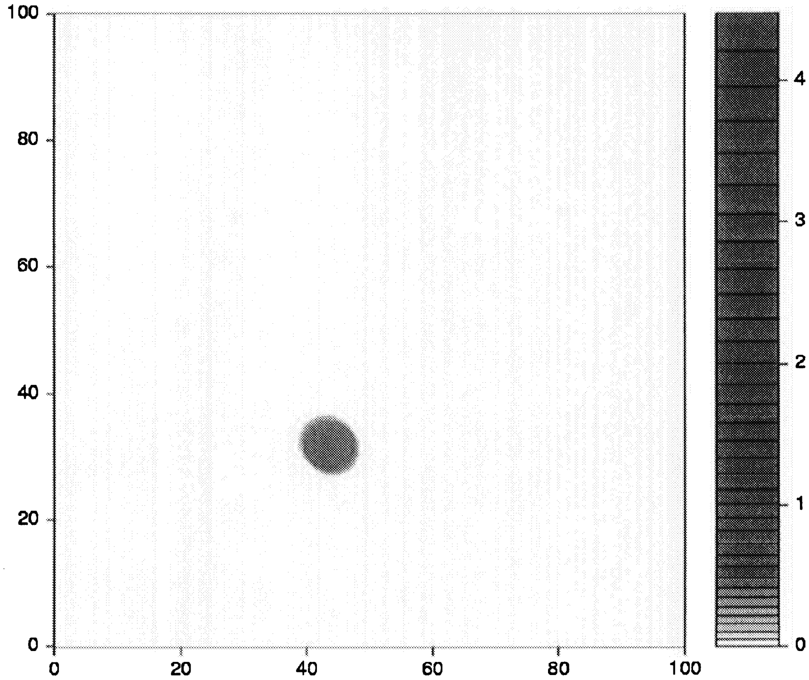

Figure 5: Danger-Potential Field at 240 Minutes

5. SUMMARY AND FUTURE DIRECTIONS

In examining the ways in which statistics can contribute meaningfully to problems in Command and Control, our research efforts at The Ohio State Unversity [6] are based on our belief that maps are an effective way of representing knowledge and uncertainty. We have developed a hierarchical design and notation for discussing the hypothetical battlespace, which has led to the postulation of danger potential as an information tool of interest to the battle commander.

The danger-potential field has a number of desirable properties. First, the fields are summable, so that the effect of additional weapons can be easily incorporated into the C2 statistical analysis. In addition, danger potential extends naturally to spatial-temporal fields. Since the form of the danger potential of a specific weapon can be precomputed and can be represented concisely as a function of the weapon location, a spatial-temporal picture of the danger-potential field can be developed quickly, assuming the location of the weapons are known.

The weapons' locations are often unknown and we are exploring two approaches to incorporating the location uncertainty into the danger-potential field: plug-in estimation and full Bayesian inference. Covariance-matching kriging methods are being considered for plug-in estimation and sequential-imputation methods are being considered for Bayesian inference.

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REFERENCES

1. Command and Control Applications Compendium 2000 (http://www.mcu.usmc.mil/ccss/ccsc/C2%20Compendium/default.htm)
2. DeGroot, M. H., *Probability and Statistics.* Addison-Wesley, Reading, 1989
3. Chatfield, C., *The Analysis of Time Series: An Introduction.* Chapman & Hall, London, 1989
4. Gelman, A., Carlin, J. B., Stern, H. S., Rubin, D. B., *Bayesian Data Analysis.* Chapman & Hall, London, 1995
5. Ramachandra, K.V., *Kalman Filtering Techniques for Radar Tracking.* Marcel Dekker, New York, 2000
6. Probability and Statistics in Command and Control (http://www.stat.ohio-state.edu/~C2/)
7. Cressie, N., Wendt, D., Johannesson, G., Mugglin, A., Hrafnkelsson, B., "A Spatial-Temporal Statistical Approach to Problems in Command and Control", in *Proceedings of U.S. Army Conference on Applied Statistics 2000*, ed. B. Bodt, forthcoming in 2001