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Abstract: - The problem of generalised minimum variance control of linear time-varying systems is studied in this paper. A generalised minimum variance controller is developed in a transfer operator framework for linear time-varying systems using a pseudocommutation technique and a more flexible cost functional that includes time-varying filters for the plant input and output.

Key-Words: - Adaptive control; minimum variance control; predictive control; time-varying systems.

1 Introduction

The generalised minimum variance controller (GMVC) developed by Clarke and Gawthrop [1], [2] is a very useful controller for stochastic adaptive control and have seen many applications. It was developed for linear time-invariant (LTI) systems and extends the minimum variance controller of Aström [3] by removing the minimum phase condition.

In our previous work the LTI GMVC has been extended to linear time-varying (LTV) systems using a left pseudocommutation technique. This technique requires that determinants of some left Sylvester matrices of the plant to be controlled are uniformly bounded away from zero [4]. This left pseudocommutability was later removed in the case of a simple LTV GMVC, where the cost functional is the sum of the output tracking error variance plus a squared current control variable weighted using a time-varying function [5].

In this paper we extend this LTV GMVC to the general case using a right pseudocommutation technique to allow a more flexible cost functional that is based on filtered plant input and output. Unlike the left pseudocommutation this right pseudocommutation will not only make the new GMVC more flexible for more complicated applications but also enable the new GMVC to control much more LTV systems by introduction of LTV filters into the cost functional.

The structure of the remainder of this paper is the following. Section 2 introduces the generalised minimum variance control cost functional and the LTV plants to be controlled. Section 3 develops the LTV GMVC and studies its closed-loop behaviour. A simulation example will be presented in Section 4. Section 5 concludes this paper.

2 Control objective

We consider the standard single-input and single output LTV systems described using a controlled autoregressive moving average (CARMA) model

$$\begin{aligned} &A(k,q^{-1})y(k+d) \\ &\qquad = B(k,q^{-1})u(k) + C(k,q^{-1})w(k+d), \end{aligned} \tag{1}$$

where $u(k)$ and $y(k)$ are the plant input and output and $d>0$ is the integral time delay between them, $w(k)$ is an independent Gaussian noise of zero mean and possibly time-varying variance. The variance is assumed to be uniformly bounded away from infinite. However, it is not necessary to know the function or the value of the variance. The LTV moving average operators (MAOs) in the CARMA model have the forms

$$\begin{aligned} A(k,q^{-1}) &= 1 + a_1(k)q^{-1} + \ldots + a_n(k)q^{-n} \\ B(k,q^{-1}) &= b_0(k) + b_1(k)q^{-1} + \ldots + b_m(k)q^{-m} \\ C(k,q^{-1}) &= 1 + c_1(k)q^{-1} + \ldots + c_h(k)q^{-h}, \end{aligned} \tag{2}$$

where q is the one-step-advance operator for both the MAO coefficients and the plant variables. The inverse of an MAO is defined as an autoregressive operator (ARO) [4].

It is assumed that

a) the LTV ARO, $C^{-1}(k,q^{-1})$ is exponentially stable,
b) all the coefficients of $A(k,q^{-1})$, $B(k,q^{-1})$ and $C(k,q^{-1})$ are uniformly bounded away from infinite and $b_0(k)$ is also uniformly bounded away from zero.

All the above assumptions are the same as those for our previous LTV GMVC [5] and are natural extensions of the assumptions made by the LTI GMVC from LTI systems for LTV plants.

Given a uniformly bounded reference, $s(k)$, the objective of our GMVC is to minimise the cost functional

$$J(k+d)=E\left\{\left|P(k,q^{-1})y(k+d)-Q(k,q^{-1})s(k)\right|^2 +\left|R(k,q^{-1})u(k)\right|^2\middle|D(k)\right\}, \tag{3}$$

where $D(k)=\{y(k), y(k-1), ..., u(k), u(k-1), ...\}$ is the set of input and output data up to and including time k, and the weighting LTV MAOs are chosen to have the forms

$$\begin{aligned} P(k,q^{-1}) &= 1+p_1(k)q^{-1}+...+p_{n_p}(k)q^{-n_p} \\ Q(k,q^{-1}) &= q_0(k)+q_1(k)q^{-1}+...+q_{n_q}(k)q^{-n_q} \\ R(k,q^{-1}) &= r_0(k)+r_1(k)q^{-1}+...+r_{n_r}(k)q^{-n_r}\,. \end{aligned} \tag{4}$$

It is further assumed that these MAOs are chosen such that all their coefficients are uniformly bounded away from infinite and $r_0(k)$ is also uniformly bounded away from zero. In addition, $P^{-1}(k,q^{-1})$ is chosen to be exponentially stable and the determinant of the right time-varying Sylvester matrices [4] for the LTV MAO pair $[A(k,q^{-1}), P(k,q^{-1})]$ is uniformly bounded away from zero. The above assumptions are not restrictive because the choice of the weighting MAOs is in our hands. It will be shown that the introduction of the general weighting LTV MAOs in the cost functional will make the LTV GMVC not only more flexible for more complicated control tasks but also capable of stabilising more LTV systems.

3 Generalised Minimum Variance Control

Without losing generality it is assumed that all initial conditions are independent of the process disturbance $\{w(k)\}$. Apply the right pseudocommutation

$$\tilde{A}(k,q^{-1})P(k,q^{-1})=\tilde{P}(k,q^{-1})A(k,q^{-1}) \tag{5}$$

we have the MAOs $\tilde{A}(k,q^{-1})$ and $\tilde{P}(k,q^{-1})$, and $\tilde{P}^{-1}(k,q^{-1})$ is exponentially stable [4]. Left multiplying by $\tilde{P}(k,q^{-1})$ on both sides of the CARMA model it becomes

$$\begin{aligned}\tilde{P}(k,q^{-1})A(k,q^{-1})y(k+d) &= \tilde{P}(k,q^{-1})B(k,q^{-1})u(k) \\ &+\tilde{P}(k,q^{-1})C(k,q^{-1})w(k+d)\;. \end{aligned} \tag{6}$$

Substituting (5) it follows that

$$\begin{aligned}\tilde{A}(k,q^{-1})P(k,q^{-1})y(k+d) &= \tilde{P}(k,q^{-1})B(k,q^{-1})u(k) \\ &+\tilde{P}(k,q^{-1})C(k,q^{-1})w(k+d)\;. \end{aligned} \tag{7}$$

Left dividing $\tilde{P}(k,q^{-1})C(k,q^{-1})$ using $\tilde{A}(k,q^{-1})$ we can separate the noise term in equation (7) using the equation

$$\begin{aligned}&\tilde{A}^{-1}(k,q^{-1})\tilde{P}(k,q^{-1})C(k,q^{-1}) = \\ &\qquad F(k,q^{-1})+\tilde{A}^{-1}(k,q^{-1})G(k,q^{-1})q^{-d}\,, \end{aligned} \tag{8}$$

where

$$\begin{aligned}&F(k,q^{-1}) = \\ &\qquad 1+f_1(k)q^{-1}+f_2(k)q^{-2}+...+f_{d-1}(k)q^{-d+1}\,. \end{aligned} \tag{9}$$

Substituting (8) into (7) we have

$$\begin{aligned}\tilde{A}(k,q^{-1})P(k,q^{-1})y(k+d) &= \tilde{P}(k,q^{-1})B(k,q^{-1})u(k) \\ &+\tilde{A}(k,q^{-1})F(k,q^{-1})w(k+d)+G(k,q^{-1})w(k)\,. \end{aligned} \tag{10}$$

GMVC Theorem

If the plant to be controlled is described by the CARMA model (1), where the inverse of the LTV MAOs $A^{-1}(k,q^{-1})$ and $C^{-1}(k,q^{-1})$ are exponentially stable, the LTV GMVC is given by

$$\begin{aligned}\hat{w}(k) = C^{-1}(k-d,q^{-1})\big[&A(k-d,q^{-1})y(k) \\ &-B(k-d,q^{-1})u(k-d)\big] \end{aligned} \tag{11a}$$

$$\begin{aligned}u(k) = T^{-1}(k,q^{-1})[&\tilde{A}(k,q^{-1})Q(k,q^{-1})s(k) \\ &-G(k,q^{-1})\hat{w}(k)]\,, \end{aligned} \tag{11b}$$

where

$$\begin{aligned}T(k,q^{-1}) = \tilde{A}(k,q^{-1})&\frac{r_0(k)}{b_0(k)}R(k,q^{-1}) \\ &+\tilde{P}(k,q^{-1})B(k,q^{-1})\,. \end{aligned} \tag{11c}$$

Proof

Letting

$$\psi(k+d)=P(k,q^{-1})y(k+d) \tag{12}$$

be the filtered plant output. Noting the exponential stability of $\tilde{A}^{-1}(k,q^{-1})$ [4] and left dividing both sides of (10) by $\tilde{A}(k,q^{-1})$ we have

$$\begin{aligned}&\psi(k+d)-F(k,q^{-1})w(k+d) \\ &\quad =\tilde{A}^{-1}(k,q^{-1})\big[\tilde{P}(k,q^{-1})B(k,q^{-1})u(k) \\ &\qquad\qquad +G(k,q^{-1})w(k)\big]. \end{aligned} \tag{13}$$

Letting

$$\hat{\psi}(k)=\psi(k)-F(k-d,q^{-1})w(k) \tag{14}$$

and substituting it into the cost functional (3) it follows that

$$J(k+d)=\left|\hat{\psi}(k+d)-Q(k,q^{-1})s(k)\right|^2 \\ +\left|R(k,q^{-1})u(k)\right|^2 \\ +E\left\{\left|F(k,q^{-1})w(k+d)\right|^2\Big|D(k)\right\}. \quad (15)$$

From (1), (2), (4), (12) and (14) we know that

$$\frac{\partial\hat{\psi}(k+d)}{\partial u(k)}=\frac{\partial y(k+d)}{\partial u(k)}=b_0(k). \quad (16)$$

Thus

$$\frac{\partial J(k+d)}{\partial u(k)}=2b_0(k)\hat{\psi}(k+d)-2b_0(k)Q(k,q^{-1})s(k) \\ +2r_0(k)R(k,q^{-1})u(k) \quad (17)$$

and

$$\frac{\partial^2 J(k+d)}{\partial u^2(k)}=2b_0^2(k)+2r_0^2(k). \quad (18)$$

It follows that the control signal $u(k)$, which minimises the cost functional (3), does exist and satisfies the following equation

$$\frac{r_0(k)}{b_0(k)}R(k,q^{-1})u(k)=Q(k,q^{-1})s(k)-\hat{\psi}(k+d). \quad (19)$$

Noting both, (13) and the exponential stability of $\tilde{A}^{-1}(k,q^{-1})$, it follows that

$$\frac{r_0(k)}{b_0(k)}R(k,q^{-1})u(k)=Q(k,q^{-1})s(k) \\ -\tilde{A}^{-1}(k,q^{-1})\left[\hat{P}(k,q^{-1})B(k,q^{-1})u(k) \\ +G(k,q^{-1})w(k)\right]. \quad (20)$$

Left multiplying by $\tilde{A}(k,q^{-1})$ on both sides we have

$$\left[\tilde{A}(k,q^{-1})\frac{r_0(k)}{b_0(k)}R(k,q^{-1}) \\ +\tilde{P}(k,q^{-1})B(k,q^{-1})\right]u(k) \quad (21) \\ =\tilde{A}(k,q^{-1})Q(k,q^{-1})s(k)-G(k,q^{-1})w(k).$$

Subtracting (1) and (11a) we have

$$C(k-d,q^{-1})\tilde{w}(k)=0\ , \quad (22)$$

where $\tilde{w}(k)=w(k)-\hat{w}(k)$ that will decay to zero exponentially because of the exponential stability of $C^{-1}(k,q^{-1})$. Replacing $w(k)$ in (21) using its estimate $\hat{w}(k)$ we have (11b). It can be further rewritten as

$$T(k,q^{-1})u(k)=\tilde{A}(k,q^{-1})Q(k,q^{-1})s(k) \\ +G(k,q^{-1})\tilde{w}(k)-G(k,q^{-1})w(k). \quad (23)$$

Noting (1), (22) and (23) we have the closed-loop system for the LTV GMVC as

$$\begin{bmatrix} C(k-d,q^{-1}) & 0 & 0 \\ -G(k,q^{-1}) & T(k,q^{-1}) & 0 \\ 0 & -B(k-d,q^{-1})q^{-d} & A(k-d,q^{-1}) \end{bmatrix}\begin{bmatrix}\tilde{w}(k)\\ u(k)\\ y(k)\end{bmatrix} \\ =\begin{bmatrix} 0 & 0 \\ -G(k,q^{-1}) & \tilde{A}(k,q^{-1})Q(k,q^{-1}) \\ C(k-d,q^{-1}) & 0\end{bmatrix}\begin{bmatrix}w(k)\\ s(k)\end{bmatrix}. \quad (24)$$

The left most matrix in the above equation is a lower triangular matrix with the inverse of its two diagonal MAOs exponentially stable. The closed-loop stability is, therefore, determined by the second diagonal MAO, $T(k,q^{-1})$. If the ARO of this MAO is exponentially stable then the closed-loop system will be exponentially stable. Noting (11c) and comparing with the result in [5] we know that the introduction of the weighting MAOs to replace the single time-varying weighting functions makes it much more likely for $T^{-1}(k,q^{-1})$ to be stabilised.

4 Simulation

The plant to be controlled has the form,

$$y(k+2)+a(k)y(k+1)=u(k)+b(k)u(k-1) \\ +w(k+2)+c(k)w(k+1), \quad (25)$$

where $w(k)$ is a white Gaussian noise with zero mean and unit variance. The time-varying parameters are

$$a(k)=\begin{cases}0.3(1-0.9e^{-k-2}) & 20i-2<k\le 20i+8\\ 0.3(0.9e^{-k-2}-11) & 20i-12<k\le 20i-2\end{cases} \quad (26a)$$

$$b(k)=2.3\{2+\sin[0.2\pi(k+2)]\} \quad (26b)$$

$$c(k)=\begin{cases}0.9\dfrac{k+2}{k+3} & 40i-17<k\le 40i+3\\ -0.9\dfrac{k+2}{k+3} & 40i+3<k\le 40i+23\end{cases} \quad (26c)$$

for i=0, 1, 2,... . In the CARMA model $B^{-1}(k,q^{-1})$ is exponentially unstable because the absolute value of $b(k)$ is uniformly greater than unit. However, $A^{-1}(k,q^{-1})$ and $C^{-1}(k,q^{-1})$ are exponentially stable because both $|b(k)|$ and $|c(k)|$ are uniformly less than unit. The weighting MAOs are chosen as

$$P(k,q^{-1}) = 1 + 0.04q^{-1}$$
$$Q(k,q^{-1}) = 5 \quad (27)$$
$$R(k,q^{-1}) = 30$$

The simulation result in Fig. 1 shows that our LTV GMVC is able to drive the CARMA model to track the square wave reference.

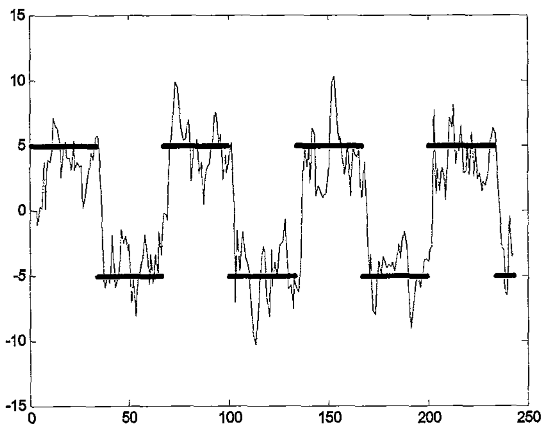

Fig. 1 GMVC Simulation results

5 Conclusion

An LTV GMVC has been developed based on a general cost functional. In this new LTV GMVC the left pseudocommutation for the plants itself is removed in order to avoid restriction of plants to be controlled. However, the right pseudocommutation is used for the introduction of the weighting LTV MAOs. This right pseudocommutation is not restrictive because the choice of the weighting MAOs is in our hands. The introduction of the three weighting MAOs makes the new GMVC not only capable to control more LTV plants but also applicable to more complicated control tasks.

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