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Keywords

message, passing, gallager, schedules, codes, comparison, under, convergence, simple, two

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A Simple Convergence Comparison of Gallager Codes Under Two Message-Passing Schedules

Sheng Tong and Xinmei Wang, *Member, IEEE*

Abstract—The convergence rate of iterative decoding of Gallager codes on the additive white Gaussian noise (AWGN) channel using the sum-product algorithm (SPA) under the flooding schedule (FS) is compared with that under the turbo-decoding schedule (TDS). Analyses using extrinsic information transfer (EXIT) charts show that TDS exhibits a much faster convergence behavior than FS.

Index Terms—Gallager codes, extrinsic information transfer (EXIT) chart, turbo-decoding schedule (TDS), flooding schedule (FS).

I. INTRODUCTION

SIMULATION results in [2] show that Gallager codes, or regular low-density parity-check (LDPC) codes, first invented by Gallager in 1960s [1], decoded using the sum-product algorithm (SPA) under the turbo-decoding schedule (TDS) exhibit a much faster convergence speed compared to the conventional flooding schedule (FS). This observation is favorable for the practical application of Gallager codes by providing lower decoding latency and reduced storage requirements [2].

In this letter, we apply the successful EXIT chart [3], [4] technique to investigate the convergence behaviors of Gallager codes on the AWGN channel decoded using SPA under both message-passing schedules. With the help of EXIT chart, the decoding trajectories under both schedules are visualized, from which we can easily find that TDS exhibits a much faster convergence behavior than FS.

II. GALLAGER CODES AND TWO MESSAGE-PASSING SCHEDULES

First, we will briefly review Gallager codes and the two message-passing schedules.

A. Gallager Codes

An (N, j, k) Gallager code C is defined by an $M(=Nj/k)$ by N parity-check matrix H , which consists of j submatrices H^i ($i = 1, 2, \dots, j$), each containing a single 1 in each column and k 1's in each row. The first submatrix H^1 arranges the k 1's of its i th row in columns $(i-1)k+1$ to ik . And the other submatrices H^i are pseudo-randomly permuted versions of H^1 . Every H^i defines a super code C^i , which can be viewed as the direct sum of N/k single parity-check codes (SPCCs) $(k, k-1, 2)$. Thus, C is the intersection of the super codes C^i 's [2].

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B. Flooding Schedule

In bipartite graphs FS can be described as a two-phase schedule [2]: in the first phase, every check node receives new messages from its neighboring variable nodes and updates the messages passed to these variable nodes; in the second phase, vice versa.

C. Turbo-Decoding Schedule

As stated in Section II-A, we could regard Gallager codes as a class of parallel concatenated codes, which allows us to decode Gallager codes in the turbo-decoding fashion by employing a soft-in soft-out (SISO) decoder for each super code and exchanging extrinsic information between the super codes. The iterative decoding procedure in the log-likelihood ratio (LLR) domain of (N, j, k) Gallager codes under TDS on the AWGN channel is detailed below. Let the channel be modeled by $y_n = x_n + n_n$, where $x_n \in \{\pm 1\}$ and $n_n \sim N(0, \sigma^2)$.

1) *Initialization*: For the n th variable node, initialize the extrinsic information E_n^i from the i th super code C^i and the a priori input Z_n by letting $E_n^i = 0$ and $Z_n = \ln\left(\frac{p(y_n|x_n=+1)}{p(y_n|x_n=-1)}\right)$, ($i=1,2,\dots,j$; $n=1,2,\dots,N$);

2) *Iteration Stage*:

For $i = 1$ to j , the decoder of the i th super code do

For $l = 1$ to N/k , the l th SPC decoder of the i th super code do

$$A_l(m) = \sum_{t \neq i} E_{l(m)}^t, \quad L_{l(m)} = Z_{l(m)} + A_{l(m)} \quad (1)$$

$$\tanh\left(\frac{E_{l(m)}^i}{2}\right) = \prod_{n \neq m} \tanh\left(\frac{L_{l(n)}}{2}\right) \quad (2)$$

End

End

where $l(m)$ denotes the m th variable node involved in the l th SPC.

3) *Decision*: At completion, make the decision according to the following rule.

$$\hat{x}_n = \begin{cases} +1, & \text{if } D_n = Z_n + \sum_{i=1}^j E_n^i > 0 \\ -1, & \text{otherwise} \end{cases}$$

III. EXIT CHARTS AND CONVERGENCE COMPARISON

Note that the concentration of the performance of Gallager codes around that of the cycle-free case regardless of message-passing schedules has been proved in [7]. Hence, we only consider the convergence rate of Gallager codes under both schedules. In the following discussions, the AWGN channel model and the BPSK signaling are assumed.

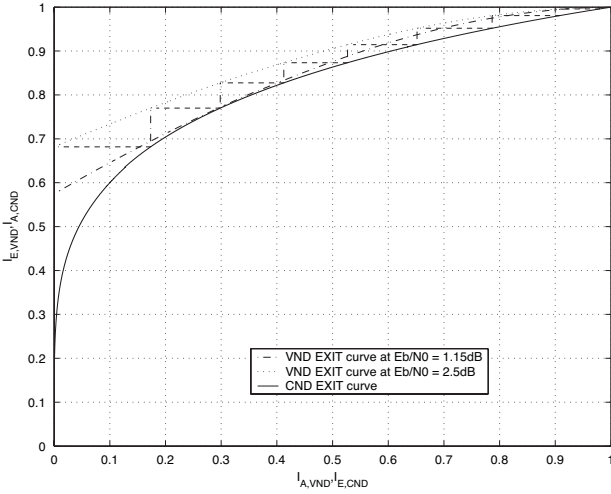


Fig. 1. EXIT chart of (3,6) Gallager codes under FS. The VND EXIT curve at $E_b/N_0 = 1.15$ dB fits to the CND EXIT curve and so the threshold of (3,6) Gallager codes is 1.15dB.

A. EXIT Charts

The basic idea for EXIT charts to predict the convergence behavior of the iterative decoder lies on considering the input/output mutual information of individual constituent decoders separately [3]. Denote random variables $X \in \{\pm 1\}$, A and E as the transmitted modulated bit, the *a priori* input and the extrinsic information from a constituent decoder. By the Gaussian assumption and consistency condition, A can be modeled as $A = \mu_A X + n_A$, $n_A \sim N(0, \sigma_A^2)$ and $\mu_A = \sigma_A^2/2$. Thus, the mutual information $I_A = I(X; A)$ between X and A is given by [3]

$$I_A(X; A) = \frac{1}{2} \sum_{x=\pm 1} \int_{-\infty}^{+\infty} p_A(\xi|X=x) \times \log_2 \frac{2 \cdot p_A(\xi|X=x)}{p_A(\xi|X=-1) + p_A(\xi|X=+1)} d\xi \quad (3)$$

A similar formula holds for the mutual information $I_E = I(X; E)$. With symmetric and consistency conditions, I_E can be closely approximated by the time average [6].

$$I_E(X; E) \approx 1 - \frac{1}{N} \sum_{n=1}^N \log_2(1 + e^{-x_n E_n}) \quad (4)$$

In all of the following EXIT charts, I_E 's are obtained using (4).

B. Convergence Comparison

Now, we start to investigate the convergence behaviors of Gallager codes under both schedules.

1) *EXIT Charts of Gallager Codes Under FS*: It is known that an LDPC code decoded under FS can be viewed as a serially concatenated code that is based on a mixture of inner repetition codes and a mixture of outer SPCCs [5]. Thus, the EXIT chart of (j, k) Gallager codes under FS can be realized with the EXIT curves of a $(j, 1, j)$ repetition code and a $(k, k-1, 2)$ SPCC. Here, we denote the check node decoder and variable node decoder as CND and VND respectively as used in [5]. Fig. 1 shows the EXIT chart of (3,6) Gallager codes decoded under FS.

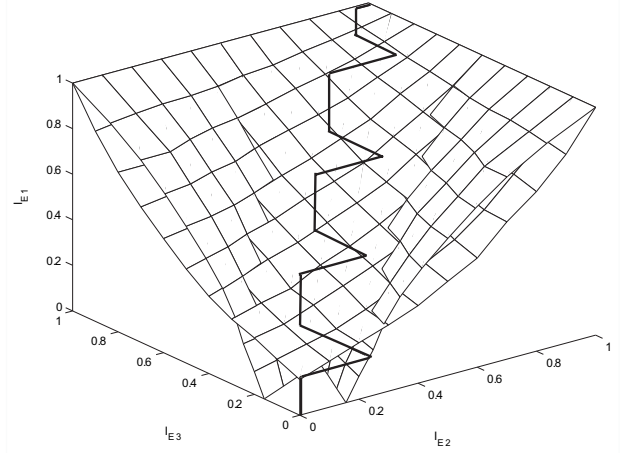


Fig. 2. EXIT chart of (3,6) Gallager codes under TDS at $E_b/N_0 = 2.5$ dB.

2) *EXIT Charts of Gallager Codes Under TDS*: Since it is difficult to draw j -dimensional graphs ($j > 3$), we only focus on $(3, k)$ Gallager codes. As stated in Section II-A, for an $(N, 3, k)$ Gallager code there are three super codes, each being a direct sum of N/k SPCCs $(k, k-1, 2)$. Hence, the EXIT charts of $(3, k)$ Gallager codes under TDS can be realized with the transfer characteristic of a $(k, k-1, 2)$ SPCC. Here, the extended EXIT chart technique [4] for three-dimensional turbo codes is adopted. Based on the simulation observation that for large interleavers the extrinsic information E^i 's from constituent decoders remain uncorrelated from each other, the *a priori* input at the decoder for C^1 is $A_1 = E_2 + E_3$. Following the notations of [3]–[5], the extrinsic output I_{E^1} is defined as a function of two variables, $I_{E^1} = T(I_{E^2}, I_{E^3})$, using the E_b/N_0 -value as a parameter. The EXIT chart for (3,6) Gallager codes under TDS and the corresponding decoding trajectory is shown in Fig. 2.

3) *Thresholds*: For long codes the performance of SPA regardless of the message-passing schedule approaches that of maximum likelihood decoding. Thus, Gallager codes under both schedules exhibit the same threshold. Fig. 1 also shows the transfer characteristic of (3,6) Gallager codes at the threshold ($E_b/N_0=1.15$ dB) under FS. It should be pointed out that by adopting the method proposed in [4] the threshold of (3,6) Gallager codes under TDS can also be determined with the same value $E_b/N_0=1.15$ dB.

4) *Convergence Comparison Under Two Schedules*: From Figs. 1 and 2, it can be easily seen that TDS really exhibits a much faster convergence speed than FS. For example, the BER performance of a randomly constructed (1008,3,6) Gallager codes decoded under both schedules is shown in Fig. 3(a). And the number of iterations required for convergence of the same code under both schedules is plotted versus E_b/N_0 in Fig. 3(b).

IV. CONCLUSION

The convergence rates of Gallager codes under FS and TDS have been compared by using EXIT charts, from which we can see that TDS exhibits a much faster convergence speed than FS. Simulation results verify this result.

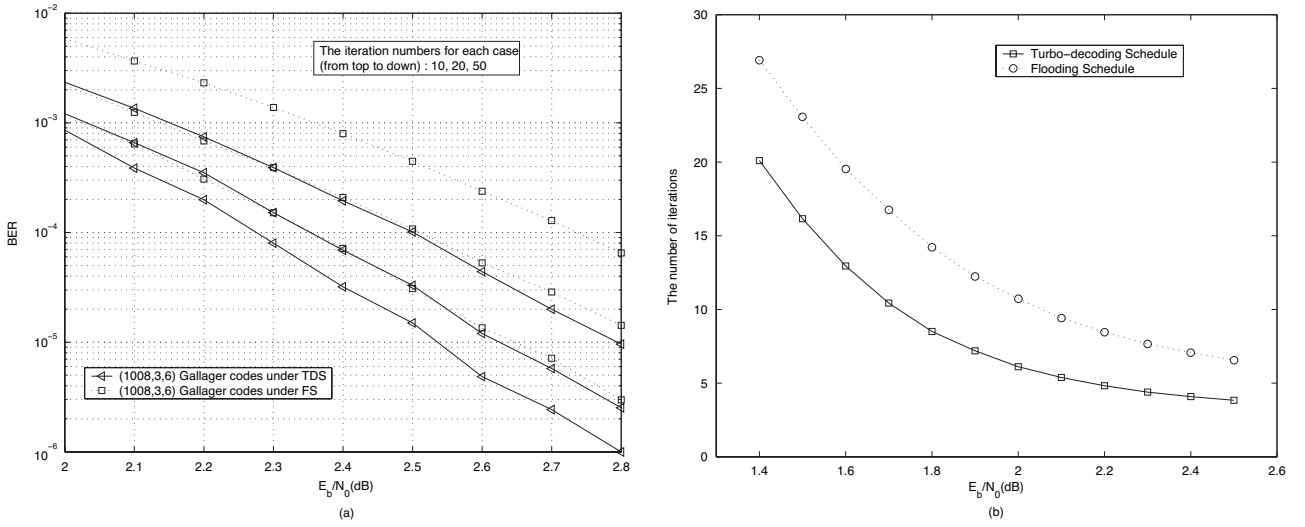


Fig. 3. (a) BER and (b) convergence rate vs. SNR for a (1008,3,6) Gallager code under FS and TDS.

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