1-1-2007

Assessment of drucker-prager type plasticity models for predicting the behaviour of FRP-confined concrete

T Yu
Hong Kong Polytechnic University, taoy@uow.edu.au

J G. Teng
Hong Kong Polytechnic University

Y L. Wong
Hong Kong Polytechnic University

S L. Dong
Zhejiang University

Assessment of drucker-prager type plasticity models for predicting the behaviour of FRP-confined concrete

Abstract

This paper presents a critical assessment of the ability of existing Drucker-Prager (D-P) type plasticity models to predict the behaviour of confined concrete using both experimental observations and a recent analytical model. This assessment shows that for a D-P plasticity model to succeed in predicting the behaviour of FRP-confined and other passively-confined concrete, it needs to possess the following three features: (a) a yield criterion which includes the third deviatoric stress invariant, (b) a hardening/softening rule which is dependent on the confining pressure; and (c) a flow rule which is dependent on both the confining pressure and the rate of confinement increment. None of the existing D-P type models possesses all three features, so they cannot be expected to lead to accurate predictions for both actively-confined and passively-confined (e.g. FRP-confined) concrete.

Keywords

plasticity, concrete, type, confined, prager, drucker, assessment, frp, behaviour, predicting, models

Disciplines

Engineering | Science and Technology Studies

Publication Details

Yu, T., Teng, J. G., Wong, Y. L. & Dong, S. L. (2007). Assessment of drucker-prager type plasticity models for predicting the behaviour of FRP-confined concrete. In S. T. Smith (Eds.), Proceedings of the First Asia-Pacific Conference on FRP in Structures: APFIS 2007 (pp. 161-166). Hong Kong: Department of Civil Engineering, University of Hong Kong.

Asia-Pacific Conference on FRP in Structures (APFIS 2007)
S.T. Smith (ed)

ASSESSMENT OF DRUCKER-PRAGER TYPE PLASTICITY MODELS FOR PREDICTING THE BEHAVIOUR OF FRP-CONFINED CONCRETE

T. Yu [1], J.G. Teng [1*], Y.L. Wong [1], and S.L. Dong [2]

[1]Department of Civil and Structural Engineering,
The Hong Kong Polytechnic University, Hong Kong, China; cejgteng@polyu.edu.hk
[2]Department of Civil Engineering, Zhejiang University, Hangzhou 310027, China

ABSTRACT

This paper presents a critical assessment of the ability of existing Drucker-Prager (D-P) type plasticity models to predict the behaviour of confined concrete using both experimental observations and a recent analytical model. This assessment shows that for a D-P plasticity model to succeed in predicting the behaviour of FRP-confined and other passively-confined concrete, it needs to possess the following three features: (a) a yield criterion which includes the third deviatoric stress invariant, (b) a hardening/softening rule which is dependent on the confining pressure; and (c) a flow rule which is dependent on both the confining pressure and the rate of confinement increment. None of the existing D-P type models possesses all three features, so they cannot be expected to lead to accurate predictions for both actively-confined and passively-confined (e.g. FRP-confined) concrete.

KEYWORDS

Confinement, concrete, FRP, plasticity, finite elements, modelling.

INTRODUCTION

The finite element (FE) method has been employed by many researchers to predict the behaviour of FRP-confined concrete elements. The accuracy of such predictions depends on the reliability of the constitutive model adopted for the concrete. Many different concrete constitutive models have been employed in the analytical and FE modelling of FRP-confined concrete elements. These constitutive models include plasticity models (e.g. Karabinis and Kiousis 1994, 1996; Lan 1998; Fang 1999; Mirmiran et al. 2000; Shahawy et al. 2000; Mahfouz et al. 2001; Karabinis and Rousakis 2002; Oh 2002; Malvar et al. 2004; Parvin and Jamwal 2006; Tsionis and Pinto 2007; Rousakis et al. 2007) and plastic-damage models (e.g. Luccioni and Rougier 2005; Huang 2005; Yan and Pantelides 2006). Although some of the models (Luccioni and Rougier 2005; Huang 2005; Yan and Pantelides 2006) include damaged elasticity, a concrete plasticity model is common to all these constitutive models.

Although most of the existing papers on the FE modelling of FRP-confined concrete presented FE results which are in close agreement with test results in terms of overall responses such as axial stress-strain curves, such close agreement constitutes only the necessary but not the sufficient evidence for the accuracy and reliability of a constitutive model. A plasticity model contains three distinct components: the yield criterion, the hardening/softening rule and the flow rule, which all affect its performance in predicting the behaviour of FRP-confined concrete. Close predictions of test results may be achieved as a result of counteracting errors in the three components. The fact that rather different plasticity models lead to similarly good predictions is clearly an issue that needs to be clarified.

This paper aims to clarify the effects of the three key components of a Druker-Prager (D-P) type plasticity model on its performance in predicting the behaviour of FRP-confined concrete and to identify the key characteristics a D-P plasticity model must possess in order to provide close predictions of test results. The assessment is focused on the D-P type concrete plasticity models because they have been widely used (e.g. Karabinis and Kiousis 1994, 1996; Karabinis and Rousakis 2002; Rousakis et al. 2007; Oh 2002; Mirmiran et al. 2000; Lan 1998; Fang 1999; Mahfouz et al. 2001; Shahawy et al. 2000); the conclusions reached for such models are also relevant to other plasticity models. The paper starts with a summary of the characteristics of confined concrete to provide a basis for subsequent discussions. Existing D-P type plasticity models are then critically reviewed and assessed. To simplify the discussions, the paper is limited to concrete subjected to uniform confinement as found in circular

concrete sections, although the FE method is a more useful tool in studying the behaviour of non-uniformly confined concrete in non-circular sections.

BEHAVIOUR OF CONFINED CONCRETE

It has been well established (Richart et al. 1928; Mander et al. 1988; Candappa et al. 2001; Sfer et al. 2002) that actively-confined concrete (i.e. concrete subjected to a constant lateral confining pressure as the axial stress increases) has the following properties: (1) the peak axial stress of concrete and the corresponding strain increase with an increase in the confining pressure; (2) the axial stress-strain curve of concrete confined by a larger confining pressure has a more gradual descending branch; (3) corresponding to the same axial strain, the lateral expansion of concrete under a larger confining pressure is less; and (4) actively-confined concrete exhibits continuous volume dilation after volume compaction, and the volumetric strain of the transition point increases with an increase in the confining pressure; (5) the shear strength of concrete, which is directly related to the peak value of the second deviatoric stress invariant, increases with an increase in the confining pressure.

Extensive research on FRP-confined concrete has also been conducted (Teng et al. 2007). The main characteristics of FRP-confined concrete include: (1) an approximately bilinear axial stress-strain curve provided the FRP jacket is reasonably stiff, the first part of which differs only slightly from the curve of unconfined concrete while the second part of which depends on the circumferential stiffness and strength of the FRP jacket; (2) corresponding to the same axial strain, the lateral strain of concrete confined by a stiffer FRP jacket is less; and (3) the volumetric change depends significantly on the FRP jacket stiffness; a concrete cylinder confined by a weak FRP jacket exhibits continuously-increasing volume dilation after volume compaction while a concrete cylinder confined by a stiff FRP jacket exhibits continuous volume compaction (Yu et al. 2007). Teng et al. (2007) also concluded from test results that although the lateral strain-axial strain paths of actively-confined concrete and FRP-confined concrete are very different, the axial strain at a given lateral strain depends mainly, if not completely, on the current confinement ratio defined as the ratio between the lateral confining pressure and the strength of unconfined concrete.

Table 1. Summary of existing D-P type models for FRP-confined concrete

D-P type model	Yield criterion	Hardening rule		Flow rule
		Including strain hardening/ softening?	Related to the confining pressure?	
Fang (1999)	D-P	No	N/A	Associated flow rule
Mahfouz et al. (2001)	D-P	Yes	No	Associated flow rule
Lan (1998)	D-P	Yes	Yes	Associated flow rule
Karabinis and Kiousis (1994)	D-P	Yes	Yes	Non-associated flow rule with a constant dilation angle
Karabinis and Kiousis (1996); Rousakis et al. (2007)	Modified D-P with the third deviatoric stress invariant included	Yes	Yes	Non-associated flow rule with a constant dilation angle
Karabinis and Rousakis (2002)	D-P	Yes	Yes	Non-associated flow rule with a dilation angle varying with the plastic deformation
Mirmiran et al. (2000); Shahawy et al. (2000)	D-P	No	N/A	Non-associated flow rule with a constant dilation angle
Oh (2002)	D-P	Yes	Yes	Non-associated flow rule with a dilation angle varying with both the plastic deformation and the confining pressure

EXISTING DRUCKER-PRAGER (D-P) TYPE PLASTICITY MODELS

General

The Drucker-Prager (D-P) yield criterion has been widely adopted for the modelling of confined concrete because of its simplicity (involving only two parameters) and its capability to capture shear strength increases as a result of hydrostatic pressure increases, which is a unique property of concrete under confinement. When a plasticity model is based on the D-P yield criterion, it is referred to as a D-P type plasticity model. In this section, the ability of this type of models to simulate the behaviour of both actively-confined and FRP-confined concrete is discussed by examining the three key aspects mentioned above, namely, the yield criterion, the hardening/softening rule, and the flow rule. Various existing D-P type models, whose key characteristics are summarized in Table 1, are examined. In this paper, compressive stresses/strains are defined to be positive while tensile strains/stresses are defined to be negative, unless otherwise specified.

Yield Criterion

Observations

The D-P yield and failure surfaces are represented by an inclined line in the meridian plane and by a circle in the deviatoric plane (Figure 1). It has been noted by many researchers (e.g. Chen 1982; Lan 1998; Huang 2005) that the shear strength of concrete under equal biaxial compression (i.e. concrete subjected to equal stresses in two principal directions and a zero stress in the third principal direction) and that under triaxial compression are different, even when the first stress invariant of the two cases are the same. Based on the plasticity theory, it is known that the stress states of equal biaxial compression and triaxial compression correspond to different circumferential positions on the failure surface in the deviatoric plane (Figure 1). The shear strength ratio between these two cases (i.e. equal biaxial compression and triaxial compression) can be found from experimental results or empirical equations for the strengths of concrete under equal biaxial compression and triaxial compression. If the experimental results of Kupfer et al. (1969) are used for concrete under equal biaxial compression and the empirical equation proposed by Richart et al. (1928) is adopted for concrete under triaxial compression, this strength ratio is around 0.7 (Yu et al. 2007), much less than 1 as implied by the circular failure curve. Therefore, a failure surface (i.e. a special case of the yield surface) which aims at reflecting the experimental behaviour of concrete should account for the effect of the third deviatoric stress invariant and adopt a non-circular failure curve in the deviatoric plane. Figure 1 shows a possible shape of such a failure surface.

Assessment of existing models

Karabinis and Kiousis (1994), Mirmiran et al. (2000), Oh (2000), Shahawy et al. (2000), and Karabinis and Rousakis (2002) adopted the D-P yield criterion directly. Fang (1999) and Mahfouz et al. (2001) simulated concrete using a concrete model in ABAQUS (2004), which is known as the Smeared Crack Concrete Model and has a yield criterion which is the same as the D-P yield criterion for concrete in compression. Therefore, these models cannot provide accurate predictions for both the strength of concrete under equal biaxial compression and that of concrete under triaxial compression. It can also be deduced that the peak stress of concrete under non-uniform confinement cannot be accurately predicted by these models. The stress state of non-uniform confinement corresponds to a circumferential position between that of triaxial and that of equal biaxial compression in the deviatoric plane (Figure 1). Therefore, the peak stress of this case cannot be accurately predicted as the two extreme cases (i.e. equal biaxial compression and triaxial compression) cannot be both accurately defined by the same model.

In Karabinis and Kiousis (1996) and Rousakis et al. (2007), the third deviatoric stress invariant was included in the yield criterion, which causes the shape of the yield surface in the deviatoric plane to become non-circular. Karabinis and Kiousis (1996) and Rousakis et al. (2007) suggested the shear strength ratio to be approximately 0.5, which is lower than the experimental value (about 0.7) as discussed above.

Strain Hardening and Softening

Observations

In classical metal plasticity models, the hardening/softening function is only a function of the equivalent plastic strain. If this concept is adopted with the D-P yield criterion, the strain hardening/softening rule can be determined by a single uniaxial stress-strain curve of concrete, either with or without confinement. However, it has recently been noted (Lan 1998; Karabinis and Kiousis 1994; Oh 2002; Chen and Lan 2004; Huang 2005) that without considering the confining pressure, the hardening/softening rule cannot provide reasonable predictions of the enhanced ductility of confined concrete (e.g. a more gradual descending branch of the axial stress-strain curve of concrete confined by a larger confining pressure). Some of these authors (e.g. Lan 1998;

Oh 2002) have also proposed modified hardening/softening rules in which the confining pressure is taken as another parameter, and presented close predictions of test results.

Assessment of existing models

Fang (1999), Mirmiran et al. (2000), and Shahawy et al. (2000) adopted an elastic-perfectly plastic model, which is incapable of close predictions of the behaviour of confined concrete. Mahfouz et al. (2001) adopted a hardening/softening function with the equivalent plastic strain as the only parameter, which is also inadequate for confined concrete as explained above. Some researchers (e.g. Karabinis and Kiousis 1994; Lan 1998; Oh 2000) have included the confining pressure into the hardening/softening function. Karabinis and Kiousis (1994) employed a complicated equation for the hardening/softening function which reflects the dependence of this function on both the confining pressure and the plastic deformation. Lan (1998) included the effect of confining pressure based on a set of experimental stress-strain curves of actively-confined concrete with different confining pressures. Oh (2000) proposed a set of complicated equations to determine the hardening/softening function. These equations included six parameters and twenty subordinate parameters and were obtained from nonlinear regression analysis of data produced using an empirical axial stress-strain model developed by the same author for concrete under triaxial compression. Despite the complicated form, the main parameters controlling strain hardening/softening were still the plastic deformation and the confining pressure, for concrete with a given unconfined strength. The above three approaches are conceptually correct and can be expected to provide close predictions of the axial stress-strain behaviour of actively-confined concrete, if the material parameters are suitably selected. However, in order to closely predict the behaviour of FRP-confined concrete, an appropriate flow rule is another important issue, as discussed below.

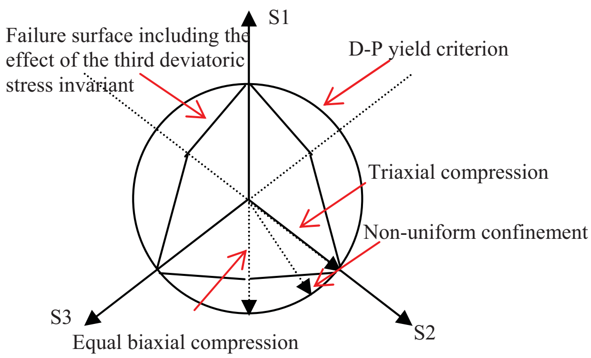

Figure 1 Failure surfaces in the deviatoric plane

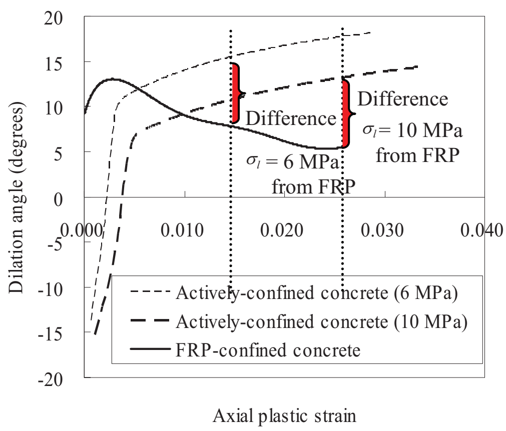

Figure 2 Comparison between actively-confined and FRP-confined concrete

Flow Rule

Observations

While the associated flow rule was adopted in some studies (e.g. Fang 1999; Mahfouz et al. 2001), this approach leads to overestimations of the expansion of confined concrete (e.g. Mirmiran et al. 2000; Oh 2002; Chen and Lan 2004; Huang 2005). As stated in the preceding section, experimental observations show that actively-confined concrete exhibits volume compaction followed by volume dilation, and furthermore, the volumetric strain of the transition point increases with an increase in the confining pressure. This phenomenon suggests that the dilation angle of confined concrete, which determines the direction of plastic deformation and thus determines the ratio of the lateral (or volume) plastic strain to the axial plastic strain for concrete under triaxial compression (Yu et al. 2007), should be related to both the plastic deformation and the confining pressure (Oh 2000). The dilation angle of actively-confined concrete can be calculated using an empirical transverse deformation model (e.g. Teng et al. 2007) which predicts the lateral strain from a given axial strain and a given confining pressure.

FRP-confined concrete is subjected to a passive confining pressure which increases with the axial strain. Although the axial stress of FRP-confined concrete at a given axial strain and confining pressure can be taken to be equal to that of actively-confined concrete with the same confining pressure and axial strain (Teng et al. 2007), the dilation angles of the two cases are different, as explained below using Teng et al. (2007)'s analysis-oriented stress-strain model. In Teng et al. (2007)'s model, the responses of the concrete core and the FRP jacket as well as their interaction are explicitly considered, so both the axial stress-strain and the lateral expansion behaviour of FRP-confined confined concrete can be predicted. Some results obtained from this model are shown in Figure 2, in which the horizontal axis is the axial plastic strain and the vertical axis is the dilation angle. The dark solid

curve in Figure 2 is for a concrete cylinder (concrete cylinder compressive strength = 39.6 MPa, diameter = 152.5 mm) confined by an FRP jacket with an elastic modulus of 80,100 MPa and a thickness of 0.34 mm. The two dashed curves in Figure 2 are for actively-confined concrete with the confining pressures being 6 MPa and 10 MPa respectively. The vertical dotted line on the left represents the axial plastic strain value of FRP-confined concrete when the confining pressure provided by the FRP jacket is equal to 6 MPa while the right vertical dotted line represents the axial plastic strain value when the confining pressure is 10 MPa. It is evident from Figure 2 that the dilation angle of actively-confined concrete is different from that of FRP-confined concrete even when both have the same axial plastic strain and confining pressure. The shaded regions in Figure 2 indicate the differences between actively-confined concrete and FRP-confined concrete in the dilation angle at the two key values of axial plastic strain. These results again suggest that a plasticity model, which relates the flow rule to the confining pressure and is capable of accurate predictions of the behaviour of actively-confined concrete, tends to overestimate the lateral dilation of FRP-confined concrete and in turn overestimate the axial stress-strain behaviour.

The difference in the dilation angle between actively-confined concrete and FRP-confined concrete can be explained as follows. For actively-confined concrete, the confining pressure is constant and is not related to the deformation of concrete. For FRP-confined concrete and concrete subjected to other forms of passive confinement, however, the confinement level varies with the deformation and at the same time controls the deformation. The dilation angle of passively-confined concrete thus needs to be related to not only the confining pressure but also the incremental ratio between the confining pressure and the lateral strain (i.e. the rate of confinement increment). For FRP-confined concrete, this ratio is directly related to the FRP jacket stiffness.

Assessment of existing models

Fang (1999), Lan (1998) and Mahfouz et al. (2001) adopted the associated flow rule which leads to an overestimation of the lateral expansion of confined concrete. Mirmiran et al. (2000) and Shahawy et al. (2000) explored the use of a non-associated flow rule with a constant dilation angle equal to zero for the modelling of FRP-confined concrete. The inability of such an assumption in predicting the dilation properties of FRP-confined concrete is evident from available test results, as a zero dilation angle corresponds to no volume change. Karabinis and Rousakis (2002) related the dilation angle to plastic deformation and assumed it to be a constant in their other studies (Karabinis and Kiousis 1994, 1996; Rousakis et al. 2007). However, Karabinis and Rousakis (2002) did not consider the variation of dilation angle with the confining pressure. This variation, however, is evident from numerous tests as explained earlier. Oh (2002) noted the complicated deformation properties of concrete and related the dilation angle to both plastic deformation and the confining pressure. Several equations were proposed to express the dilation angle based on an empirical transverse deformation model proposed by the same author and non-linear regression analysis. These equations, although complicated, reflect the variation of the dilation angle with both the confining pressure and plastic deformation. Therefore, it can be expected that the equations of Oh (2002) are capable of close predictions of the dilation behaviour of actively-confined concrete, provided that his empirical model has sufficient accuracy. However, Oh's (2000) equations cannot provide close predictions of FRP-confined concrete, as explained in the preceding subsection.

The above discussions indicate that the flow rules adopted by all existing D-P type models cannot be expected to provide close predictions for FRP-confined concrete. Given the dilation properties of passively-confined concrete as discussed above, to achieve reasonably accurate predictions, the flow rule should also include the rate of confinement increment as an important parameter, besides plastic deformation and the confining pressure.

CONCLUSIONS

From the discussions presented in this paper, it is clear that for a D-P type plasticity model to provide reasonably accurate predictions for both actively-confined and passively-confined concrete, its needs to possess the following three features: (1) a yield criterion that reflects the effect of the third deviatoric stress invariant; (2) a confinement-dependent hardening/softening rule; (3) a confinement-dependent non-associated flow rule, in which the dilation angle is related not only to the confining pressure but also to the rate of confinement increment. If the first feature is appropriately implemented, the strength of concrete under non-uniform confinement can be accurately predicted. If the second feature is appropriately implemented, the axial stress-strain curve of actively-confined concrete can be accurately predicted. Finally, if the third feature is appropriately implemented, the lateral deformation of both passively-confined and actively-confined concrete can be accurately predicted. None of the existing D-P type models includes all three features, so they cannot be expected to provide accurate predictions for both actively-confined and passively-confined (e.g. FRP-confined) concrete. Based on these conclusions, a modified concrete plasticity model including all three features has recently been developed by the authors and is presented in Yu et al. (2007).

ACKNOWLEDGMENTS

The authors are grateful for the financial support provided by The Hong Kong Polytechnic University (Project Codes: RG7E and BBZH) and the Natural Science Foundation of China (Project No. 50329802).

REFERENCES

ABAQUS (2004). ABAQUS Analysis User's Manual, version 6.5.

Candappa, D.C., Sanjayan, J.G. and Setung, S. (2001) "Complete triaxial stress-strain curves of high-strength concrete", Journal of Materials in Civil Engineering, ASCE, 13(3), 209-215.

Chen, W.F. (1982). Plasticity in Reinforced Concrete, McGraw-Hill Book Company, New York, N.Y.

Chen, W.F. and Lan, Y.M. (2004). "Finite element study of confined concrete", Proceedings of the International Symposium on Confined Concrete, Changsha, China.

Fang, J.H. (1999). Finite Element Analysis of FRP-Retrofitted RC Beams, Master Degree Thesis, Department of Civil Engineering, National Taiwan University (in Chinese).

Huang, Y.L. (2005). Analysis-oriented and Constitutive Modeling of FRP-Confined Concrete, Master Degree Thesis, Tsing-Hua University (in Chinese).

Karabinis, A.I. and Kiousis, P.D. (1994). "Effects of confinement on concrete columns: plasticity approach", Journal of Structural Engineering, ASCE, Vol. 120, No. 9, pp. 2747-2766.

Karabinis, A.I. and Kiousis, P.D. (1996). "Strength and ductility of rectangular concrete columns: a plasticity approach", Journal of Structural Engineering, ASCE, Vol. 122, No. 3, pp. 267-274.

Karabinis, A.I. and Rousakis, T.C. (2002). "Concrete confined by FRP material: a plasticity approach", Engineering Structures, Vol. 24, pp. 923-932.

Kupfer, H., Hilsdorf, H.K. and Rusch, H. (1969). "Behavior of concrete under biaxial stresses", ACI Journal, Vol. 66, pp. 656-666.

Lan, Y.M. (1998). Finite Element Study of Concrete Columns with Fiber Composite Jackets, PhD Dissertation, School of Civil Engineering, Purdue Universtiy.

Luccioni, B.M. and Rougier, V.C. (2005). "A plastic damage approach for confined concrete", Computers and Structures, Vol. 83, pp. 2238-2256.

Malvar, L.J., Morrill, K.B. and Crawford, J.E. (2004). "Numerical modeling of concrete confined by fiber-reinforced composites", Journal of Composites for Construction, ASCE, Vol. 8, No. 4, pp. 315-322.

Mander, J.B., Priestley, M.J.N. and Park, R. (1988) "Theoretical stress-strain model for confined concrete", Journal of Structural Engineering, ASCE, Vol. 114, No. 8, pp. 1804-1826.

Mahfouz, I., Rizk, T. and Sarkani, S. (2001). "An innovative FRP confining system for repairing rectangular columns", Construction and Materials Issues 2001, Proceedings of the Second Congress, ASCE and Construction Institute, pp. 16-25.

Mirmiran, A., Zagers, K. and Yuan, W.Q. (2000). "Nonlinear finite element modeling of concrete confined by fiber composites", Finite Elements in Analysis and Design, Vol. 35, pp. 79-96.

Oh, B. (2002). A Plasticity Model for Confined Concrete under Uniaxial Loading, PhD Thesis, Lehigh University.

Parvin, A. and Jamwal, A.S. (2006). "Performance of externally FRP reinforced columns for changes in angle and thickness of the wrap and concrete strength", Composite Structures, Vol. 73, pp. 451-457.

Richart, F.E., Brandtzaeg, A. and Brown, R.L. (1928). A Study of the Failure of Concrete under Combined Compressive Stresses, Bulletin No. 185, Engineering Experiment Station, University of Illinois, Urbana, U.S.A.

Rousakis, T.C., Karabinis, A.I. and Kiousis, P.D. (2007). "FRP-confined concrete members: Axial compression experiments and plasticity modeling", Engineering Structures, Vol. 29, pp. 1343-1353.

Sfer, D., Carol, I., Gettu, R. and Etse, G.. (2002). "Study of the behavior of concrete under triaxial compression", Journal of Engineering Mechanics, ASCE, Vol. 128, No. 2, pp. 156-163.

Shahawy, M., Mirmiran, A. and Beitelman, T. (2000). "Tests and modeling of carbon-wrapped concrete columns", Composites: Part B-Engineering, Vol. 31, Nos. 6 & 7, pp. 471-480.

Teng, J.G., Huang, Y.L., Lam. L. and Ye, L.P. (2007). "Theoretical model for fiber reinforced polymer-confined concrete", Journal of Composites for Construction, ASCE, Vol. 11, No. 2, pp. 201-210.

Tsionis, G. and Pinto, A. (2007). "Numerical analysis of RC bridge piers with rectangular hollow cross-section retrofitted with FRP jackets", Journal of Earthquake Engineering, Vol. 11, pp. 607-630.

Yan, Z. and Pantelides, C.P. (2006). "Fiber-reinforced polymer jacketed and shape-modified compression members: II---model", ACI Structural Journal, Vol. 103, No. 6, pp. 894-903.

Yu, T., Teng, J.G., Wong, Y.L. and Dong, S.L. (2007). "Finite Element Modelling of Confined Concrete-I: Drucker-Prager Type Plasticity Models", in preparation.