

# Activated Sludge Wastewater Treatment Process: Performance Comparison between a Two-reactor Cascade and a Single Tank

H.S. Sidhu<sup>1</sup>, S.D. Watt<sup>1</sup> and M.I. Nelson<sup>2</sup>

<sup>1</sup> School of Physical, Environmental and Mathematical Sciences  
University of New South Wales at Australian Defence Force Academy, Canberra, ACT 2600, AUSTRALIA

<sup>2</sup> School of Mathematics and Applied Statistics  
The University of Wollongong, Wollongong, NSW 2522, AUSTRALIA  
E-mail: [h.sidhu@adfa.edu.au](mailto:h.sidhu@adfa.edu.au)

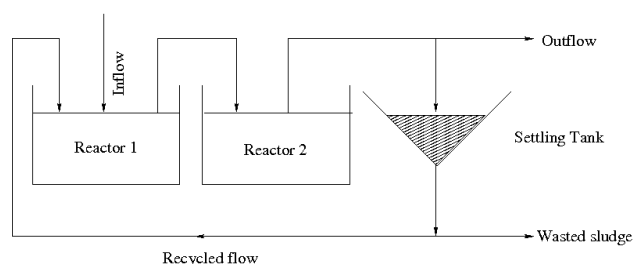
## Abstract

*We investigate a model for the treatment of wastewater in the activated sludge process. The process is based on the aeration of wastewater with flocculating biological growth, followed by the separation of treated wastewater from biological growth. Part of this growth is wasted, and the remainder returned to the system. The biochemical model consists of two types of bacteria, sludge bacteria and sewage bacteria, and two types of ciliated protozoa, free-swimming ciliates and ciliates attached to sludge flocs. A combination of steady-state analysis, path following techniques and numerical integration of the governing equations are used to study the dynamics of this system in a network of two coupled reactors arranged in a series. We compare the treatment efficiency for a single tank system with that of a two-reactor cascade. In the latter scenario the total residence time is fixed and the residence time in the first reactor is taken to be a design parameter. Process parameters that ensure optimal performance are discussed.*

## 1. INTRODUCTION

We consider wastewater management using an activated sludge process with recycle in a series of two reactors. A mathematical model of this process is given in Curds (1971). Figure 1 shows a schematic diagram of the process occurring in two well-mixed reactors. Curds analysed the wastewater process by solving the governing equations numerically to determine the stable steady-states of the system for one particular set of parameter values. The single well-mixed reactor, as well as the three and five reactor cascade were analysed, and Curds showed that the behaviour of the model was similar to those observed in full-

scale and experimental activated-sludge plants.



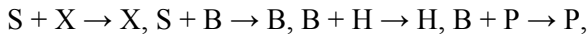
**Figure 1. A schematic diagram of a two-reactor activated sludge process with recycle**

Using the same model, Jianqiang & Ray (2000) analysed a well-mixed two-reactor system by solving the governing equations numerically. They mainly focussed on the use of natural oscillations to improve the efficiency in the system. By natural oscillations, it is meant that the process parameters are chosen so that the steady input of sewage into the first reactor generates self-sustained oscillations in its output, which then forces the second reactor. The attraction of this method is that it uses no external energy to generate the oscillations. These authors showed that by implementing such a strategy, the performance of the cascade can be improved.

In this paper, we analyse the model described by Curds (1971) using both bifurcation analysis and path-following methods to determine the optimal performance of the two-reactor cascade system. Using an efficiency measure of the activated sludge process we compare the performance of the single and two-reactor cascade. For a fixed total residence time in a cascade we treat the residence time in the first reactor as a design parameter and determine values that give the highest efficiency in the cascade. The optimal performance of the single reactor is used as a benchmark for comparison with the performance of a cascade.

## 2. MODEL EQUATIONS

To model the activated sludge treatment of wastewater, we use the system proposed by Curds (1971). The sewage is pumped into the first reactor at a constant rate and then pumped out of the reactor at the same rate. Apart from inflow and outflow of products, there are various biological reactions breaking down the sewage to bacteria and breaking down bacteria to protozoa, namely:



where S, X, B, H and P are the concentrations of the substrate, sludge bacteria, sewage bacteria, free-swimming protozoa and attached and crawling protozoa respectively. By assuming that both reactors are well stirred, the system can be modelled by the following equations:

### Reactor 1

$$V_1 \frac{dS_1}{dt} = F(S_0 - S_1) + aF(S_2 - S_1) - V_1 X_1 \frac{\mu_x S_1}{Y_x(K_x + S_1)} - V_1 B_1 \frac{\mu_B S_1}{Y_B(K_B + S_1)} \quad (1)$$

$$V_1 \frac{dX_1}{dt} = F(X_0 - X_1) + aF(bX_2 - X_1) + V_1 X_1 \frac{\mu_x S_1}{K_x + S_1} \quad (2)$$

$$V_1 \frac{dB_1}{dt} = F(B_0 - B_1) + aF(B_2 - B_1) + V_1 B_1 \frac{\mu_B S_1}{K_B + S_1} - V_1 H_1 \frac{\mu_H B_1}{Y_H(K_H + B_1)} - V_1 P_1 \frac{\mu_P B_1}{Y_P(K_P + B_1)} \quad (3)$$

$$V_1 \frac{dH_1}{dt} = F(H_0 - H_1) + aF(H_2 - H_1) + V_1 H_1 \frac{\mu_H B_1}{K_H + B_1} \quad (4)$$

$$V_1 \frac{dP_1}{dt} = F(P_0 - P_1) + aF(bP_2 - P_1) + V_1 P_1 \frac{\mu_P B_1}{K_P + B_1} \quad (5)$$

### Reactor 2

$$V_2 \frac{dS_2}{dt} = (1+a)F(S_1 - S_2) - V_2 X_2 \frac{\mu_x S_2}{Y_x(K_x + S_2)} - V_2 B_2 \frac{\mu_B S_2}{Y_B(K_B + S_2)} \quad (6)$$

$$V_2 \frac{dX_2}{dt} = (1+a)F(X_1 - X_2) + V_2 X_2 \frac{\mu_x S_2}{K_x + S_2} \quad (7)$$

$$V_2 \frac{dB_2}{dt} = (1+a)F(B_1 - B_2) + V_2 B_2 \frac{\mu_B S_2}{K_B + S_2} - V_2 H_2 \frac{\mu_H B_2}{Y_H(K_H + B_2)} - V_2 P_2 \frac{\mu_P B_2}{Y_P(K_P + B_2)} \quad (8)$$

$$V_2 \frac{dH_2}{dt} = (1+a)F(H_1 - H_2) + V_2 H_2 \frac{\mu_H B_2}{K_H + B_2} \quad (9)$$

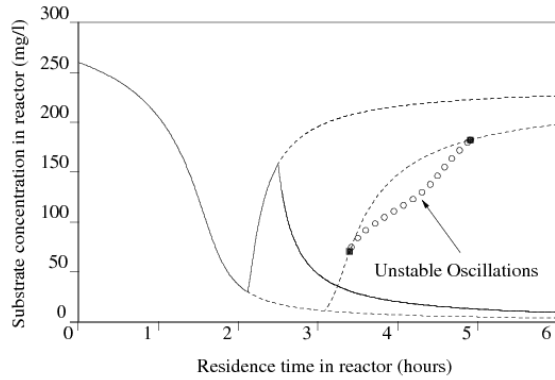
$$V_2 \frac{dP_2}{dt} = (1+a)F(P_1 - P_2) + V_2 P_2 \frac{\mu_P B_2}{K_P + B_2} \quad (10)$$

The terms that appear in equations (1) – (10) are defined in the nomenclature. We follow Curds (1971) and assume that there are neither sludge bacteria nor protozoa in the inflow (ie.  $X_0=H_0=P_0=0$ ) and that the biological reactions are the same for all types of protozoa ( $\mu_H=\mu_P$ ,  $Y_H=Y_P$  and  $K_H=K_P$ ). We note that in the study by Jianqiang & Ray (2000), the authors assumed that the feed contained sludge bacteria as well as protozoa. The residence time in each reactor is defined as  $\tau_i=V_i/F$  and the total residence time is  $\tau_{total} = \tau_1 + \tau_2$ . The limits  $\tau_1=0$  and  $\tau_1=\tau_{total}$  represent the degenerate case in which the cascade “reduces” back to a single reactor. In general, unless otherwise stated, when we refer to the cascade we do not include these degenerate cases. In the analysis that follows, we will fix the total residence time ( $\tau_{total}$ ) in the cascade and take the residence time of the first reactor ( $\tau_1$ ) as the primary bifurcation parameter.

## 3. STEADY-STATE ANALYSIS

Steady state diagrams were obtained using the path-following software Auto (Doedel *et al*, 1998). In these, the standard representation is used; solid and dashed lines represent stable and unstable steady states respectively; squares are Hopf bifurcation points (i.e. points at which oscillatory solution branches can emanate from the steady-state solution), open circles represent unstable periodic orbits. For a periodic orbit the norm that is used is the integral over the period of the solution. Unless otherwise stated, we choose parameter values based on those given in earlier studies (Jianqiang & Ray, 2000 and Curds, 1971):  $S_0=260$  mg/l,  $B_0=30$  mg/l,  $K_X=15$  mg/l,  $K_B=10$  mg/l,  $K_H=K_P=12$  mg/l,  $\mu_X=0.3$  h<sup>-1</sup>,  $\mu_B=0.5$  h<sup>-1</sup>,  $\mu_H=\mu_P=0.35$  h<sup>-1</sup>,  $a=0.35$ ,  $b=1.9$  and  $Y_X=Y_B=Y_H=Y_P=0.5$ .

### 3.1. Single Reactor Analysis



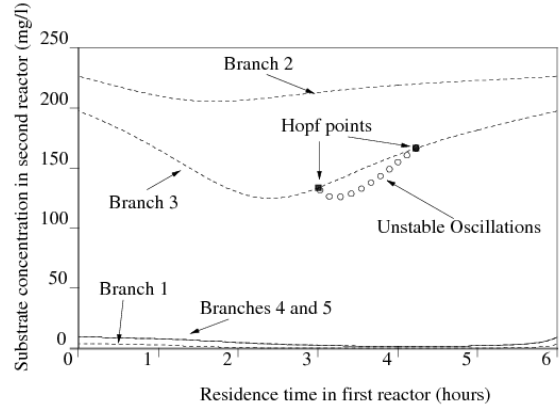
**Figure 2. Steady-state diagram for the single reactor showing the dependence of sewage concentration upon residence time**

The steady-state analysis for a single reactor system was carried out in Watt *et al* (2006). Here we shall just briefly highlight some of the relevant results from that analysis. The steady-state solutions are shown in figure 2. We can see that there are multiple steady-state solutions, but there is only ever one stable branch. We note that from figure 2 that as the residence time is increased, the (stable) substrate concentration initially decreases monotonically until  $\tau = 2.1$ , increases when  $2.1 < \tau < 2.5$ , and decreases again when  $\tau > 2.5$ . From this figure, we can also see that there are two Hopf points located at  $\tau = 3.4$  and  $\tau = 4.9$ . The oscillatory solution branch that exists between these two points is unstable. The relationship between this steady-state diagram and the corresponding performance of the single reactor will be discussed in section 4.1.

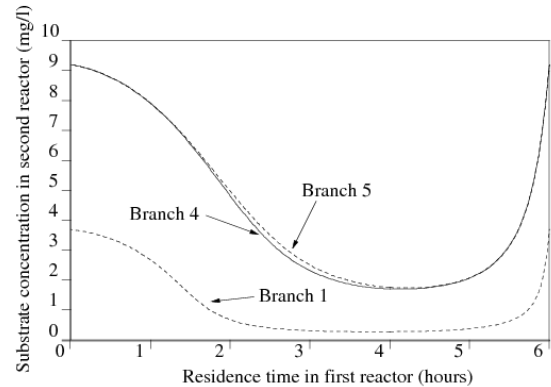
### 3.2. Two-Reactor Cascade Analysis

The steady-states of the two-reactor system can be found by setting the time derivatives of equations (1) – (10) to zero. It is possible to find the steady-states analytically for specific values of the parameters, and some can be found in general. However due to the large number of governing equations, this becomes tedious and does not offer much insight, although it is possible to show that the steady states come in combination of zero and non-zero values, which occur in pairs. In other words, if  $X_1=0$ , for example, then  $X_2=0$  as well. Similarly for  $H_i$  and  $P_i$  ( $i=1, 2$ ). We also found that  $S_i$  and  $B_i$  never vanish, as substrate and sewage bacteria are continuously flowing into the first reactor, however it is possible for  $X_i$ ,  $H_i$  and  $P_i$  to vanish in different combinations. This results in eight different classes of steady states. However, not all of these steady-states are physically

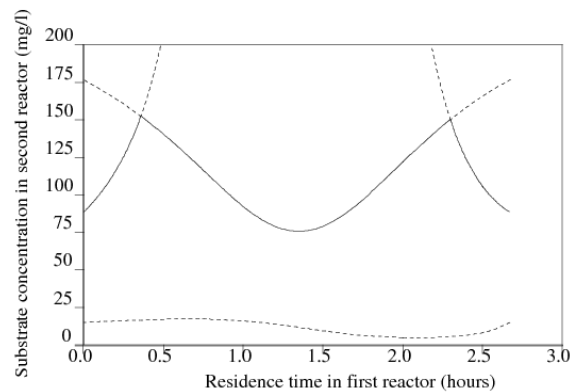
meaningful as concentrations must be real and non-negative. Figures 3a, b shows the steady-state diagrams of the substrate concentration leaving the cascade as function of the residence time of the first reactor ( $\tau_1$ ) with the total residence time ( $\tau_{\text{total}}$ ) fixed at 6 hours.



**Figure 3a. Steady-state diagram for the cascade with  $\tau_{\text{total}} = 6$  hours, showing the dependence of the sewage concentration flowing out of the second reactor ( $S_2$ ) as a function of the residence time in the first reactor ( $\tau_1$ )**



**Figure 3b. Enlarged view of Figure 3a showing Branches 1, 4 and 5**



**Figure 4. Steady-state solutions showing the sewage concentration flowing out of the second reactor as a function of the residence time in the first reactor. The total residence time is fixed at 2.67 hours**

From the above figures it is clear that when the total residence time was fixed at 6 hours, there is only one stable steady-state branch (Branch 4) that occur for low values of  $S$ . By changing the total residence time to 2.67 hours, as shown in figure 4, there is still only one stable steady-state for a given value of  $\tau_1$ , but this time the steady-state solution switches stability at  $\tau_1 = 0.36$  and  $\tau_1 = 2.3$ , and occur for values of  $S_2$  which are higher than those shown in figure 3.

#### 4. REACTOR PERFORMANCE

The main aim of this paper is to assess the performance of the two-reactor cascade system for the wastewater treatment by the activated sludge process. As stated in Sidhu & Nelson (2005), it is important to determine the best performance of a single reactor and then use this indicator as a benchmark for comparing with performances of the double-reactor system. Here we shall use the Treatment Efficiency (i.e. the amount of “purification” of the incoming sewage that occurs through the biochemical reactions) to gauge the performance of the cascade. Suppose the concentration of the incoming sewage is  $S_{in}$  and the concentration of the outgoing sewage is  $S_{out}$ , then the Treatment Efficiency (TE) is defined as

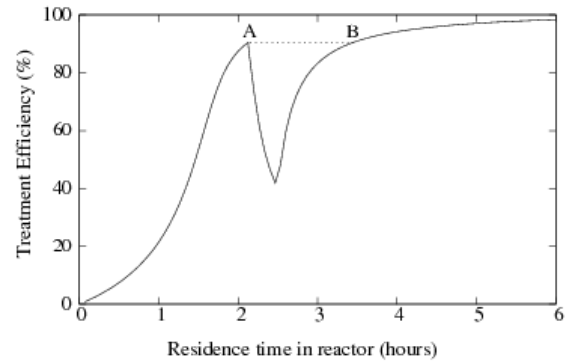
$$TE = \frac{S_{in} - S_{out}}{S_{in}} 100 \quad (11)$$

The two extreme values of TE are 0%, if the concentrations are unchanged (no conversion of the incoming sewage), and 100% (total conversion of the incoming substrate). In practice, the efficiency will fall somewhere between these two extremes.

##### 4.1. Single Reactor Efficiency

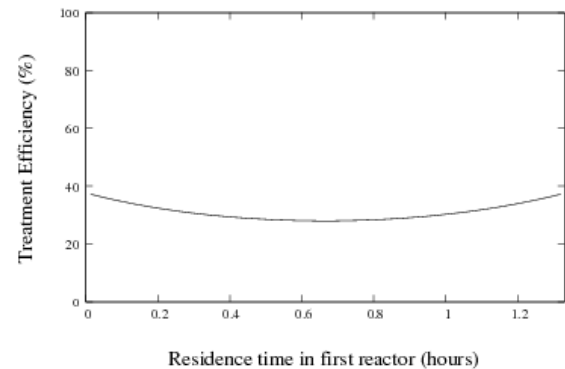
In figure 5 the treatment efficiency for a single reactor system is shown as a function of residence time. The corresponding steady-state results were shown in figure 2. Here it is clear that for  $0 < \tau < 2.1$  (Point A), the single reactor is optimised by operating it with the largest possible residence time in that range. For residence times between points A and B,  $2.1 < \tau < 3.33$  hours, the performance of the single reactor is optimised by operating at a smaller residence time of  $\tau = 2.1$  hours (Point A). We note that any slight deviation from this optimal value can result in a sudden drop in the reactor performance – very little robustness in the reactor performance. (The robustness at point B is better than point A, but the former occurs at larger residence time than point A.) However, for  $\tau >$

3.33 hours (Point B), we see that the single reactor performance is optimised by operating at the largest possible residence time. The drop and subsequent increase in the efficiency between points A and B, correspond to the increase in the substrate concentration  $S$  followed by a monotonic decrease which can be seen from the steady-state diagram figure 2.



**Figure 5. Treatment Efficiency of a single reactor system as a function of the residence time. Points A and B represent equal Treatment Efficiency of 88% for two different values of the residence time  $\tau$**

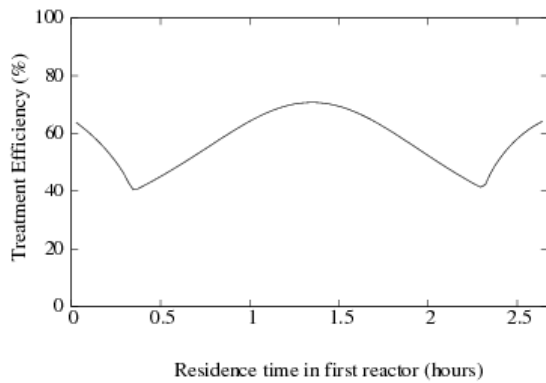
##### 4.2. Double-Reactor Cascade Efficiency



**Figure 6. The dependence of treatment efficiency of a two-reactor system upon the residence time in the first reactor. The total residence time is 1.33 hours**

When the total residence time is fixed at 1.33 hours in the cascade, from figure 6 the optimal efficiency occurs when the residence time in the first reactor is either equal to the total residence time, i.e.  $\tau_1 = \tau_{total}$  (no second reactor), or when  $\tau_1 = 0$  (no first reactor). These are the degenerate cases mentioned earlier when the cascade ‘reduces’ to a single reactor system. For such cases, the single reactor system always outperforms the cascade. However

in figure 7 (total residence time equals 2.67 hours), the optimal efficiency for the cascade system occurs at  $\tau_1 = 1.36$  hours (and  $\tau_2 = 1.31$  hours). Using these optimal values to design the cascade can result in a treatment efficiency of 71%. For a single reactor of residence time of 2.67 hours, the treatment efficiency is around 65%. From this analysis one would be tempted to conclude that the cascade is better than the single reactor. However, from our earlier discussion of figure 5, it is clear that the single reactor can be optimised if it was operated at a *lower* residence time of 2.1 hours (Point A in figure 5) rather than at 2.67 hours. The residence time corresponding to point A can result in a treatment efficiency of 88%, an improvement of 17% over the optimally designed cascade.

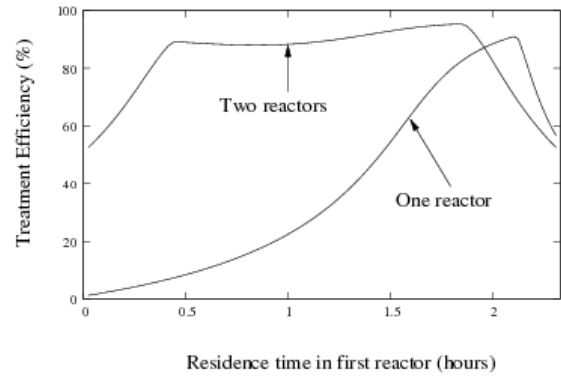


**Figure 7.** The dependence of treatment efficiency of a two-reactor system upon the residence time in the first reactor. The total residence time is 2.67 hours. The corresponding steady-state plot was shown in figure 4

#### 4.3. Comparing the Double-Reactor Cascade with the Single Reactor

Throughout our comparison we will ensure that the two configurations possess the same biochemical parameters. Figure 8 shows the treatment efficiency plots for the cascade when the total residence time is 2.33 hours, and for the single reactor when  $0 < \tau \leq 2.33$ . We can see that for this range of residence time, the best efficiency for the single reactor is 88% which occurs when the residence time is 2.1 hours (Point A of figure 5) and in a double-reactor, the best efficiency of 95% occurs when the residence time in the first reactor is 1.84 hours and the residence time in the second reactor is 0.49 hours. This gives an improvement of about 7% of using an optimally designed cascade over the best possible single-reactor performance when the residence time was fixed at 2.33 hours. Furthermore, we also note that to achieve this same level of efficiency of an

optimally designed cascade, a single-reactor system will have to operate with a residence time of 4.92 hours. This is more than double the total residence time of an optimally designed cascade, and hence under such circumstances a cascade can have a considerably greater throughput of wastewater. Comparing the residence times of each configuration to obtain equivalent treatment efficiencies is also an important practical consideration which is outside the scope of the present study, but will be analyzed in greater detail in the future.



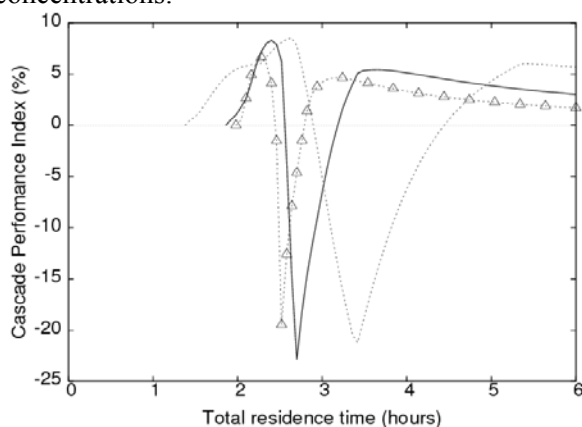
**Figure 8.** Treatment efficiencies for the single- and double-reactor cascade as a function of residence time in the first reactor when the total residence time is 2.33 hours

To simplify the comparison between the two configurations, we follow a similar approach outlined in Sidhu & Nelson (2005) and define the Cascade Performance Indicator (CPI) as

$$CPI = \frac{TE_2^{\max} - TE_1^{\max}}{TE_1^{\max}} 100 \quad (12)$$

where  $TE_1^{\max}$  is the optimal treatment efficiency in a single reactor system with a residence time no greater than the total residence time, and  $TE_2^{\max}$  is the optimal treatment efficiency in a double reactor system with a total residence time of  $\tau_{\text{total}}$ . Figure 9 shows the Cascade Performance Indicator of the double-reactor system over a single-reactor system as a function of the total residence time for three different values of feed sewage concentrations. For total residence times less than the values at which the curves begin, the “optimally” designed cascade corresponds to the degenerate case, i.e. when the cascade reduces to a single reactor (see the efficiency plot shown in figure 6). For these residence times, the single reactor *always* outperforms the cascade. For larger total residence times, there is a possibility when the optimally designed cascade outperforms the single reactor.

However, we can see that for each value of  $S_0$ , there is a region (corresponding to negative values in the CPI) where the optimised single reactor outperforms the best designed cascade. This region narrows for larger values of feed sewage concentrations.



**Figure 9.** The dependence of the Cascade Performance Indicator upon the total residence time for three different values of the feed substrate concentration  $S_0=100 \text{ mg l}^{-1}$  (dashed curve),  $S_0=260 \text{ mg l}^{-1}$  (bold curve) and  $S_0=500 \text{ mg l}^{-1}$  (dashed-triangle curve)

## 5. CONCLUSIONS

In this paper, we considered the treatment of wastewater by an activated sludge process with recycle in a series of two well-mixed reactors. We compared the performance of the single and double-reactor configurations. Figure 9 shows that clearly there are ranges of residence times when the single reactor outperforms an optimally designed cascade. We also noted that in some cases the improvement in the cascade performance may be small, however it was seen that to achieve the same level of efficiency as in the cascade, a single reactor would have to be operated for a significantly larger residence time. In such cases, an optimally designed cascade would have a considerably greater throughput of wastewater than the single reactor. Using the parameters outlined in this investigation, we did not find any circumstances in which stable periodic solutions were generated in the first reactor which could then be used to force the second tank - a strategy that Jianqiang & Ray (2000) claimed can improve the performance of the activated sludge model used in this study. This is most likely due to the absence of the sludge bacteria and protozoa at the inflow. Finally we would like to comment that although the model studied in this paper may be viewed as simplistic, the purpose of our work here is to illustrate the approach that should be used to

analyse such systems. We are currently in the process of examining a more realistic model.

## 6. ACKNOWLEDGMENTS

This work was supported by a grant from the Australian Research Council (DP0559177).

## 7. NOMENCLATURE

B – Sewage bacteria concentration (mg/l); F – Flow rate (l/hr); H – Concentration of free-swimming ciliates (mg/l);  $K_X$ ,  $K_B$ ,  $K_H$ ,  $K_P$  – saturation constants for the sludge bacteria, dispersed bacteria, free-swimming ciliates and attached ciliates respectively (mg/l); P – Concentration of attached ciliates; S – Substrate concentration (mg/l); V – Reactor volume (l); X – Concentration of sludge bacteria (mg/l);  $Y_X$ ,  $Y_B$ ,  $Y_H$ ,  $Y_P$  – Yield coefficient for the sludge bacteria, dispersed bacteria, free-swimming ciliates and attached ciliates respectively; a – volume ratio of recycled flow and sewage flow; b – concentration factor of sludge; t – time (hr);  $\mu_X$ ,  $\mu_B$ ,  $\mu_H$ ,  $\mu_P$  – maximum specific growth rate of the sludge bacteria, dispersed bacteria, free-swimming ciliates and attached ciliates respectively ( $\text{hr}^{-1}$ ). Subscripts 0, 1 and 2 refer to quantities at the feed (inlet), reactor 1 and reactor 2 respectively.

## 8. REFERENCES

- Curds, C.R. (1971), *Computer simulations of microbial population dynamics in the activated-sludge process*, Water Research, **5**, 1049-1066.
- Doedel, E.J., Fairgrieve, T.F., Sandstede, B., Champneys, A.R., Kuznetsov, Y.A. and Wang, X. (1998), *AUTO 97*.  
ftp.cs.concordia.ca/pub/doedel/auto
- Jianqiang, S. and Ray, A.K. (2000), *Performance improvement of activated sludge wastewater treatment by nonlinear natural oscillations*, Chem. Eng. Technol., **23**, 1115-1122.
- Sidhu, H.S. and Nelson, M.I. (2005), *Improving bioreactor performance : are two CSTBs always better than one?* Proceedings of the 33<sup>rd</sup> Annual Australasian Chemical Engineering Conference (CHEMECA 2005), Brisbane, ISBN 1864998326.
- Watt, S.D., Sidhu, H.S. and Nelson, M.I. (2006), *Analysis of a model for the treatment of wastewater by the activated sludge process*, ANZIAM J. (E), C15-C20.