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A deformable model with cellular neural network

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Abstract

This paper presents a new methodology for deformable models by drawing an analogy between cellular neural network (CNN) and elastic deformation. The potential energy stored in an elastic body as a result of a deformation caused by an external force is propagated among mass points by the local connectivity of cells and the CNN dynamics. An improved CNN model is developed for propagating the energy generated by the external force on the object surface. A method is presented to derive the internal forces from the potential energy distribution established by the CNN. The methodology proposed in this paper can not only deal with large-range deformation, but it can also accommodate both isotropic and anisotropic materials by simply modifying capacitors of cells. Examples are presented to demonstrate the efficacy of the proposed methodology.

Keywords: deformation, deformable objects, CNN, analogous systems

1. Introduction

Virtual reality based surgery simulation is expected to provide benefits in many aspects of surgical procedure training and evaluation. To this end, a significant amount of research efforts have been dedicated to simulating the behaviours of deformable objects. In general, most of the existing methods for deformable modelling are fully built on a linear elastic model to describe the deformation, while the behaviours of deformable objects such as human tissues and organs are extremely nonlinear [1, 2]. The common deformation methods, such as mass-spring, FEM and BEM, are mainly based on linear elastic models because of the simplicity of linear elastic models and also because linear elastic models allow reduced runtime computation. However, linear elastic models cannot accommodate the large-range geometric deformation and the displacements are only allowed to be less than 10% of the deformable object size [1, 3]. Although few methods based on the nonlinear elastic model can handle the large-range deformation [4], the use of quadric strains generally requires a very expensive computation for real-time simulation. In addition, extra work often needs to be performed for anisotropic deformation.

This paper presents a new methodology for deformable object simulation by drawing an analogy between cellular neural network (CNN) and elastic deformation. The deformation is formulated as a dynamic CNN. The potential energy stored due to a deformation caused by an external force is calculated and treated as energy injected into the system, as described by the energy conservation law. An improved CNN model is developed for propagating the energy generated by the external force among mass points through the local connectivity of cells and the CNN dynamics. A method is presented to derive the internal forces from the potential energy distribution established by the CNN. The methodology can not only deal with large-range deformations, it can also accommodate both isotropic and anisotropic materials easily through simply modifying the capacitors of cells.

There are several investigations that combine neural network with deformable modelling [5, 6]. However, in these methods, neural networks are mainly used to determine the parameters of mass-spring models. To the best of our knowledge, this study is the first to directly use neural network techniques to

model deformable objects under externally applied loads.

2. CNN Model Design

2.1 CNN Analogy

The deformation of deformable objects is formulated as the activity of CNN. The following main features of the CNN make the CNN very suitable for describing the behaviours of deformable objects:

- The activity of a CNN depends on time. The time-continuous dynamics provides the description of space-time physical processes and has a fast response to variations of the external input. This feature is very suitable for real-time deformation accommodating the variable external force.
- A CNN has both local and global dynamics. The significant feature of a CNN is the local connectivity of cells, i.e. any cell in a CNN is connected only to its neighbor cells. Adjacent cells directly interact with each other. Cells not directly connected to each other have indirect effect because of the propagation effects of the continuous-time dynamics of a CNN. The activity of a cell can be propagated to others through the local connectivity of cells and the CNN global dynamics. The feature of local and global dynamics is suited to perform local and global deformations.
- Given the initial state and the external input, the activity of a CNN is determined by the local connectivity of cells. The local connectivity is similar to the internal force since the deformation is determined by the internal force under the given external force and the rest state.

In the proposed CNN analogy, the deformation of deformable objects is treated as the activity of a CNN. The object surface is treated as a CNN by using a number of locally connected cells. The energy generated by the external force is treated as the current source of the contacted cell. The energy is propagated among mass points through the local connectivity of cells and the CNN dynamics. The local connectivity of cells is treated as the internal force. Therefore, such a CNN with the current source, the local connectivity and the activity can be seen as a communication medium among an external force, internal forces and deformation.

2.2 CNN Architecture

The CNN model is designed as a 2D CNN in theory, while it is constructed on the 3D object surface. Therefore, the CNN model is actually one between 2D and 3D.

A CNN model can be applied to different grid types. Without loss of generality, we consider a CNN on a rectangular grid. Each node on the grid is occupied by a cell. The dynamics of the CNN is described by the following equation.

$$C\frac{dv_{xij}(t)}{dt} = -\frac{1}{R_x}v_{xij}(t) + \sum_{(k,l)\in N_r(i,j)} A(i,j;k,l)v_{ykl}(t) + \sum_{(k,l)\in N_r(i,j)} B(i,j;k,l)v_{ukl} + I_{ij} \quad (1)$$

where (i,j) denotes a cell on the i^{th} row and the j^{th} column. C is the capacitance of a linear capacitor. R_x is the resistance of a linear resistor and I_{ij} is the independent current source. A is the feedback template and B is the control template, whose values depend only on the relative positions of cells *(i,j)* and cells *(k,l)*. The $v_{uij}(t)$, $v_{xij}(t)$ and $v_{yij}(t)$ are the input, state and output of the cell *(i,j)* at the time t, respectively.

3. Construction of CNN Model

3.1 Formulation of Current Source

When a deformable object is deformed under an external force, there is a displacement observed. Therefore, the deformation can be regarded to be generated by the work done by the external force. According to the energy conservation law, the work done by the external force can be transformed into an equivalent electric energy at the contact point. Therefore, the current source I is

$$I = \frac{\vec{F} \bullet \vec{S}}{A_F} \quad (2)$$

where $\vec{F}$ is the external force, $\vec{S}$ is the displacement and A_F is the area on which the external force is applied.

If the external force is applied to a point or the area that the force is applied on is small, the current I may be regarded as the elastic strain energy density at the contact point:

$$I = \frac{1}{2}\sigma\varepsilon = \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\sigma_{ij}\varepsilon_{ij} \quad (3)$$

where σ is the stress tensor and ε is the strain tensor at the contact point. The commonly used and simple strain tensor is linear Cauchy strain tensor described by:

$$\varepsilon = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{4}$$

A straight forward approach that leads to a linear relationship between these two tensors is provided by the Hooke's law:

$$\sigma = C\varepsilon \tag{5}$$

From Eq. (5), we can deduce the displacement from the given external force and the current I can be subsequently obtained.

In most of CNN applications, the current source of each cell has the same value. For our purpose, the energy generated by the external force is treated as a current source and propagated to other mass points along the object surface. Therefore, the obtained current source is set only at the contact cell of the external force, and the current source values of other cells are set to zero.

3.2 Local Connectivity of Cells

The local connectivity of cells is obtained from the local interaction of a diffusion-type partial differential equation (PDE). The inherent analogies between the CNN dynamic equation and the PDE have been made evident in the CNN literature [7, 8]. Both the CNN and the PDE describe the time-continuous dynamics and have the same property of local interactions. In addition, both the dynamic CNN and the dynamic PDE have the same equilibrium.

The diffusion-type PDE is the well known heat equation from physics

$$k\nabla^2\varphi(u,w,t) = \frac{\partial\varphi(u,w,t)}{\partial t} \tag{6}$$

where ∇^2 is the Laplace operator applied to the potential $\varphi(u,w,t)$ with coordinates u and w at time t, and k is a constant. In order to obtain the CNN parameters, Eq. (6) has to be discretized on the 3D object surface. The discretization schemes are different due to the different grid types. The following discusses the discretizations on a rectangular grid and a triangular grid.

3.2.1 PDE discretization on a rectangular grid

The discretization on a rectangular grid is straightforward. The PDE can be discretized by a finite-difference scheme. For the point $\vec{P}_{i,j}$ shown in Fig. 1, the equation discretized by a finite-difference scheme is shown in Eq. (7).

$$\begin{aligned}
&\frac{2\varphi_{i+1,j}(n\Delta t)}{\Delta u_i(\Delta u_{i-1}+\Delta u_i)} + \frac{2\varphi_{i-1,j}(n\Delta t)}{\Delta u_{i-1}(\Delta u_{i-1}+\Delta u_i)} \\
&+ \frac{2\varphi_{i,j+1}(n\Delta t)}{\Delta w_j(\Delta w_{j-1}+\Delta w_j)} + \frac{2\varphi_{i,j-1}(n\Delta t)}{\Delta w_{j-1}(\Delta w_{j-1}+\Delta w_j)} \\
&- \frac{2\varphi_{i,j}(n\Delta t)}{\Delta u_{i-1}\Delta u_i} - \frac{2\varphi_{i,j}(n\Delta t)}{\Delta w_{j-1}\Delta w_j} \\
&= \frac{1}{k}\times\frac{\varphi_{i,j}((n+1)\Delta t)-\varphi_{i,j}(n\Delta t)}{\Delta t}
\end{aligned} \tag{7}$$

$$\Delta u_{i-1} = \left\|\overrightarrow{P_{i-1,j}P_{i,j}}\right\| \quad \Delta u_i = \left\|\overrightarrow{P_{i,j}P_{i+1,j}}\right\|$$

$$\Delta w_{j-1} = \left\|\overrightarrow{P_{i,j-1}P_{i,j}}\right\| \quad \Delta w_j = \left\|\overrightarrow{P_{i,j}P_{i,j+1}}\right\|$$

where Δt is a constant time step, $\varphi_{i,j}(n\Delta t)$ is the potential at point $\vec{P}_{i,j}$ at the time $n\Delta t$, $\left\|\overrightarrow{P_{i-1,j}P_{i,j}}\right\|$ and other similar terms represent the magnitudes of the vector $\overrightarrow{P_{i-1,j}P_{i,j}}$ and other similar vectors.

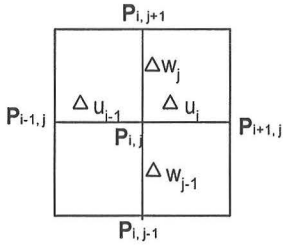

Figure 1 : PDE discretization on a rectangular grid

3.2.2 PDE discretization on a triangular grid

For the discretization on a triangular net, a finite volume method [9] is used to discretize the PDE at each node. One straightforward finite volume method is Voronoi diagram [10], which derives the discretized equation at each node from the energy conservation law. Fig. 2 shows the finite volume constructed by the Voronoi scheme. The finite volume of point $\vec{P}_0$ is constructed by connecting each intersection points between the perpendicular centerlines of each edges adjacent to point $\vec{P}_0$.

The discretization equation at point $\vec{P}_0$ is shown in Eq. (8).

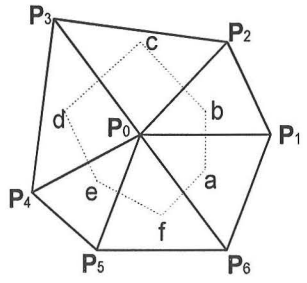

Figure 2 : Voronoi scheme for PDE discretization on a triangular net

$$\begin{aligned}&\frac{L_{ab}}{SL_{01}}[(\varphi_1(n\Delta t)-\varphi_0(n\Delta t)]+\frac{L_{bc}}{SL_{02}}[\varphi_2(n\Delta t)-\varphi_0(n\Delta t)]\\&+\frac{L_{cd}}{SL_{03}}[\varphi_3(n\Delta t)-\varphi_0(n\Delta t)]+\frac{L_{de}}{SL_{04}}[\varphi_4(n\Delta t)-\varphi_0(n\Delta t)]\\&+\frac{L_{ef}}{SL_{05}}(\varphi_5(n\Delta t)-\varphi_0(n\Delta t)]+\frac{L_{fa}}{SL_{06}}(\varphi_6(n\Delta t)-\varphi_0(n\Delta t)]\\&=\frac{1}{k}\times\frac{\varphi_0((n+1)\Delta t)-\varphi_0(n\Delta t)}{\Delta t}\end{aligned} \tag{8}$$

where $L_{nm}=\left\|\overrightarrow{P_mP_n}\right\|$ is the distance between two points $\vec{P}_m$ and $\vec{P}_n$, and S is the measure of the finite volume.

3.2.3 Formulation for local connectivity of cells

The local interaction of the PDE can be seen obviously from the discretized PDE. The analogy between the CNN and the discretized PDE can be extracted by letting $v_{yij}(n\Delta t)=v_{xij}(n\Delta t)$ and associating the state of a CNN cell $v_{xij}(n\Delta t)$ with the PDE potential $\varphi_{ij}(n\Delta t)$ in the case of a rectangular grid or $\varphi_i(n\Delta t)$ in the case of a triangular grid. Based on the analogy between the discretized PDE and the CNN, the local connectivity of cells can be obtained directly from the local interaction of the PDE. For example, with reference to the local interaction of the PDE shown in Eq. (7), the local connectivity of cells can be obtained as Eq. (9). With reference to Eq. (9), the local connectivity of cells of the CNN on a triangular grid can also be obtained from the local interaction of the PDE discretized on a triangular grid.

$$B=0 \tag{9}$$

$$A=\begin{pmatrix}0 & \dfrac{2}{\Delta u_{i-1}(\Delta u_{i-1}+\Delta u_i)} & 0\\ \dfrac{2}{\Delta w_{j-1}(\Delta w_{j-1}+\Delta w_j)} & \dfrac{1}{R_x}-\dfrac{2}{\Delta u_{i-1}\Delta u_i}-\dfrac{2}{\Delta w_{j-1}\Delta w_j} & \dfrac{2}{\Delta w_j(\Delta w_{j-1}+\Delta w_j)}\\ 0 & \dfrac{2}{\Delta u_i(\Delta u_{i-1}+\Delta u_i)} & 0\end{pmatrix}$$

The solution of the PDE usually needs initial values and boundary conditions. The initial values and boundary conditions are also incorporated in the CNN model. The initial values of the PDE can be directly associated with the initial state of CNN. The simplest boundary condition is the Dirichlet boundary condition, i.e. the given boundary values. The Dirichlet boundary condition can be realized by using some fixed-state cells.

4. Internal force derivation and model dynamics

4.1 Internal Force Derivation

Potential functions provide an efficient method of describing internal forces based on point positions. For a potential function ϕ, the force exerted on a point $\vec{P}_i$ is due to the gradient of the potential energy ϕ with respect to the change in position, as described below.

$$\begin{aligned}&\vec{f}=-\nabla_{\vec{P}_i}\phi\\&\nabla_{\vec{P}_i}\phi=\left(\frac{\partial\phi}{\partial\vec{P}_{ix}},\frac{\partial\phi}{\partial\vec{P}_{iy}},\frac{\partial\phi}{\partial\vec{P}_{iz}}\right)\end{aligned} \tag{10}$$

The potential field developed by the activity of the CNN describes the energy distribution on the object surface. The associated potential function is $\varphi(u,w,t)$. According to the energy conservation law, the potential at each point can be transformed into an equivalent elastic potential. Therefore, the internal elastic force at a point can be represented by the negative gradient of the potential.

$$f=-k\nabla_{\vec{P}_i}\varphi \tag{11}$$

The property of the local connectivity of cells is inherited by the internal force. An internal force is existed between any two connected points and the internal force at a point is derived from the connected neighbor points of this point. The force between any two connected points is calculated as follows:

For any two connected points $\vec{P}_i$ and $\vec{P}_j$, and assuming that the potentials at these two points are $\varphi_{\vec{P}_i}$ and $\varphi_{\vec{P}_j}$, the potential at any point $\vec{P}$ on the edge between these two points is regarded as a function of the distance between the point $\vec{P}_i$ and the point $\vec{P}$. Therefore, the following relationships can be written:

$$\begin{aligned} \varphi &= \varphi(l) \\ l &= \left\| \vec{P} - \vec{P}_i \right\| \end{aligned} \tag{12}$$

Thus,

$$\nabla_{\vec{P}_i}\varphi = \frac{d\varphi}{dl}\nabla_{\vec{P}_i} l = -\frac{\left|\varphi_{\vec{P}_j} - \varphi_{\vec{P}_i}\right|}{\left\|\vec{P}_j - \vec{P}_i\right\|}\overrightarrow{P_iP_j} \tag{13}$$

where $\overrightarrow{P_iP_j} = \dfrac{\vec{P}_j - \vec{P}_i}{\left\|\vec{P}_j - \vec{P}_i\right\|}$ and $\left|\varphi_{\vec{P}_j} - \varphi_{\vec{P}_i}\right|$ is the magnitude of the potential change between the point $\vec{P}_i$ and the point $\vec{P}_j$.

Therefore, the force between the point $\vec{P}_i$ and the point $\vec{P}_j$ may be written as:

$$\vec{f}_{ij} = k\frac{\left|\varphi_{\vec{P}_j} - \varphi_{\vec{P}_i}\right|}{\left\|\vec{P}_j - \vec{P}_i\right\|}\overrightarrow{P_iP_j} \tag{14}$$

The internal force $\vec{g}_i$ at a given point $\vec{P}_i$ is the sum of the internal forces from all connected neighbor points of point $\vec{P}_i$. Therefore, the following equation may be written:

$$\vec{g}_i = \sum_{j \in N(\vec{P}_i)} \vec{f}_{ij} \tag{15}$$

where $N(\vec{P}_i)$ is the connected neighbor points of $\vec{P}_i$, $\vec{f}_{ij}$ is the force between point $\vec{P}_i$ and its neighbor point $\vec{P}_j$.

4.2 Model Dynamics

When an external force is applied to a deformable object, the contact point of the external force is replaced with a new position. As a result, the other points not influenced by the external force are in an unstable state. The energy generated by the external force is propagated among mass points through the local connectivity of cells to establish a new equilibrium state by generating the corresponding internal forces. Based on the equilibrium state, the new position of each point is obtained. The dynamic behaviour is governed by the Lagrangian equation of motion of each node:

$$m_i\frac{d^2\vec{P}_i}{dt^2} + \gamma_i\frac{d\vec{P}_i}{dt} + \vec{g}_i = \vec{F}_i \tag{16}$$

where $\vec{P}_i$ is the position vector of the node i, m_i and γ_i are the mass and damping constant of the node i, respectively. $\vec{g}_i$ and $\vec{F}_i$ are the net internal force and the external force applied to the node i at time t, respectively. The equation can be solved by the explicit Euler integration scheme.

5. Implementation results and discussions

A prototype system has been implemented for interactive deformable simulation. Experiments are conducted to investigate isotropic deformation, anisotropic deformation and the non-linear load-deformation of the model.

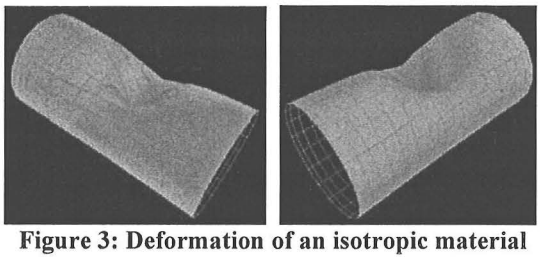

Figure 3: Deformation of an isotropic material

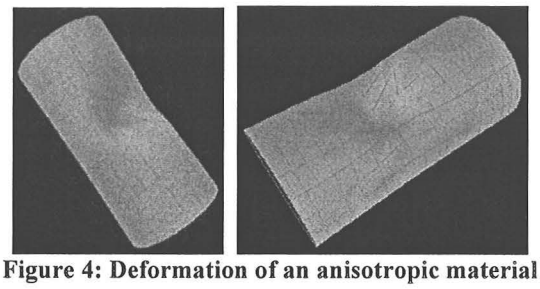

Figure 4: Deformation of an anisotropic material

Fig. 3 illustrates the deformations of an isotropic material with 900 mass points (capacitance=1.0, mass=1.0 and damping=2.0). The behaviour of anisotropic materials can also be simulated by the proposed methodology through simply setting different capacitances of cells. Fig. 4 illustrates the deformation of an anisotropic material with 900 mass points,

where the red lines (capacitance = 10.0) have a different capacitance value from the green lines (capacitance = 12.5). As shown in Fig. 4, some of the red lines are also deformed correspondingly during the deformation.

The proposed methodology has been tested to verify the non-linear relationship of load-deformation. Eight materials that are modelled with different damping constants (from right to left in Fig. 5, the damping is 1.0, 2.0, 4.0, 5.0, 6.0, 7.0, 9.0 and 10.0, respectively) are tested. The deformation is calculated when the force applied to the model is increased at a constant rate. The results in Fig. 5 demonstrate that deformation varies non-linearly with the applied force.

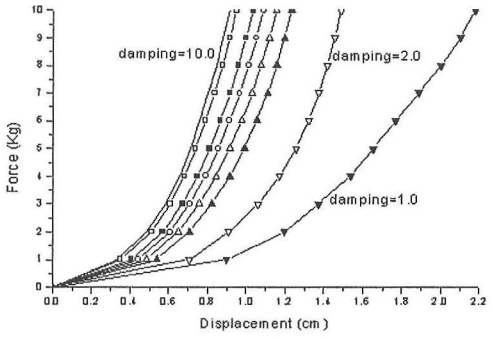

Figure 5: Non-linear load-deformation relationship

6. Conclusions

This paper presents a new methodology to mimic the behaviours of deformable objects by establishing an analogy between CNN and elastic deformation. The contribution of this paper focuses on establishing neural network based techniques to propagate the energy generated by the external force for extrapolating internal forces. An improved autonomous CNN model is developed for propagating the energy generated by the external force on the object surface. The improved CNN model provides a natural manner for energy propagation since the local connectivity of cells acts as the local interaction of the PDE. A method is presented for deriving the internal forces from the potential energy distribution. This proposed methodology can not only deal with large-range deformations due to the local connectivity of cells and the CNN dynamics, but also can accommodate both isotropic and anisotropic materials through simply modifying the capacitance constants of cells. Ongoing research mainly focuses on the integration of the virtual surgery environment with the Phantom haptic device for force feedback.

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