Application of Non-linear Error Correction Models to the demand for Money in Tunisia

Sami Khedhiri
University of Wollongong in Dubai

Boudhina Riadh
University of Tunis

Application of Non-linear Error Correction Models to the demand for Money in Tunisia

Abstract

Non linear error correction models (NLECM) have been increasingly used recently in the econometric literature since they lead to more insightful results than the linear models in empirical studies. In this research we present alternative methods of dealing with non linearity in error correction models. We apply these models to the demand for money in Tunisia and our results show that we get better predictions than the usual linear models. It is also shown that the speed of adjustment towards the long-run equilibrium is faster than what we get using linear error correction models.

Keywords

Application, Non, linear, Error, Correction, Models, demand, for, Money, Tunisia

Disciplines

Business | Social and Behavioral Sciences

Publication Details

Khedhiri, S. & Riadh, B. (2005). Application of Non-linear Error Correction Models to the demand for Money in Tunisia. Middle East Business and Economic Review, 17 (2), 20-39.

The Middle East Business and Economic Review. Vol.17, No.2 (December 2005)

Application of Non Linear Error Correction Models to the Demand for Money in Tunisia

Sami Khedhiri
University of Wollongong in Dunai-UAE
Riadh Boudhina
University of Tunis, Tunisia

Abstract

Non linear error correction models (NLECM) have been increasingly used recently in the econometric literature since they lead to more insightful results than the linear models in empirical studies. In this research we present alternative methods of dealing with non linearity in error correction models. We apply these models to the demand for money in Tunisia and our results show that we get better predictions than the usual linear models. It is also shown that the speed of adjustment towards the long-run equilibrium is faster than what we get using linear error correction models.

Key words: Asymetric adjustment, cointegration, non linear models, long-run equilibruim.

I Introduction

The analysis of dynamic behavior of macroeconomic relationships is one of the main issues in time series modelling. Most of the relations investigated in empirical research between economic variables are linear because first they are easy to handle, and second we can obtain efficient estimation of the linear models. A very good exemple is the Granger representation theorem (Engle *et al.* 1987) which shows that cointegrated series have a linear error correction model representation that reconciles temporel horizons. Therefore, error correction models and cointegration give a new dimension to dynamic modelling and provide sound theorical basis to interpret the long run and the short run properties of time series. Nevertheless, the representation of linear ECM is based on restrictive conditions, in such the long run equilibrum is symetric and also the adjustment needs to be a constant proportion of the previous equilibrum error.

In fact there are theoretical reasons justyfying the limits of linearity. In several papers there was an attempt to test for the existence of asymetric relations between economic variables. Ball and Mankiw (1994) suggest that when we have costs associated with price changes we get an asymetric price adjustment. Krane (1994) developed a model of stock management in which the differences in cost of

possession stocks and the cost of stock depletion are explained by asymetric inventory adjustment. Gale (1996) proves in his model for investment that the profit rate is not the same during growth and recession periods, and therefore this rate is asymetric. This leads to the conclusion that the condition for a unique equilibrum, which is the main caracteristic of the linear error correction model, is not satisfied since asymetric adjustment requires the existence of multiple equilibria. The idea is that asymetric adjustment may be represented in a non linear ECM.
In this paper we show the existence of non-linearity in the error correction models and we suggest different methods to handle it. Section 2 presents non linear error correction models and the different methods of their estimation. In section 3 we show forecast methods with non linear ECM. An empirical study of the demand for money in Tunisia using NLECM is carried on in the last section of this paper, along with some concluding remarks.

II Review of NL Error Correction models

a) The Granger- Lee Approach

The Granger and Lee (1989) solution to the non linearity in ECM consists of decomposing the error correction term as follows:

:

$$Z_{t-1}^{+}=\begin{cases} Z_{t-1} & \text{if} \quad Z_{t-1}>0 \\ 0 & \text{otherwise} \end{cases}$$

$$Z_{t-1}^{-}=\begin{cases} Z_{t-1} & \text{if} \quad Z_{t-1}<0 \\ 0 & \text{otherwise} \end{cases}$$

$Z_t = (Y_t - \alpha X_t)$ denotes the equilibrum error. We may write the non linear ECM :

$$\Delta Y_t = \alpha + \sum_{i=1}^{p} \beta_i \Delta X_{t-i} + \sum_{i=1}^{q} \lambda_i \Delta Y_{t-i} + \delta_1 Z_{t-1}^{+} + \delta_2 Z_{t-1}^{-} + \varepsilon_t \qquad (*)$$

Using Monte Carlo simulation experiments, some authors like Cook, Holly and Turnu showed that these models have a modest performance especially with small size data in which case they turn out to be not powerful.

b) The Polynomial Approach

In this method we assume that the non linear function may be approximated by :

$$f(Z) = \sum_{i=1}^{p} a_i Z^i$$

Escribano (1997) used this approach to study the demand for money in the UK. The idea of using a polynomial function to approximate the non linear term in ECM is intuitive but the interpretation of the a_i coffecients is not straightforward.

c) The Escribano Pfann Approach

Escribao and Pfann suggest an asymetric partition of the error correction termas follows:

$$Z_{t-1}^{+}=\begin{cases} Z_{t-1} & \text{if} \quad \Delta Z_{t-1}>0 \\ 0 & otherwise \end{cases}$$

$$Z_{t-1}^{-}=\begin{cases} Z_{t-1} & \text{if} \quad \Delta Z_{t-1}<0 \\ 0 & \text{otherwise} \end{cases}$$

The novelty of this paper is to empirically use these models in order to analyze the effects of imposing unit root restrictions and cointegration restrictions on the I(1) variables in level and in first difference on the forecast accuracy. We therefore need to determine the forecast error variance in each representation that we consider. Therefore our goal is to study the implication of the different variable transformations (in level, in first difference, and the linear combinaison of cointegrated variables) and also the effects of imposing unit root and cointegration restrictions in each step of the estimation procedure. Next, we determine the rank of cointegration restrictions in the VAR representation. We present three specifications in terms of the number of these restrictions that are either imposed or estimated in each case. The technical detail is presented in Appendix C.

Next we focus on evaluating the forecast accuracy of the different methods. We define $\hat{x}_{t+h}= E[x_{t+h} / x_t]$ the h-step ahead forecast of X_t, then the forecast error term is given by:

$$e_{x,t+h} = x_{t+h} - \hat{x}_{t+h}$$

In order to evaluate the forecast accuracy, we use standard measures based on the square of the forecast error terms:

$$E\left[(x_{t+h} - \hat{x}_{t+h})^2\right] = E\left[e_{t+h}^2\right] = \{[e_{t+h}]\}^2 + V[e_{t+h}].$$

The forecast error variance of X_t is given by:

$$V_{x,h} = V\left[e_{x,t+h}\right] = V\left[e_{x,t+h} / x_t\right] = \sum_{i=0}^{h-1} A^s \Omega A^{s'}$$

If we use a system of variables in first difference then the h- step ahead forecast of ΔX_t is :

$\hat{\Delta}x_{t+h} = E[\Delta x_{t+h} / x_t] = A^{h-1}(\alpha\beta' x_t + \psi)$, and the forecast error variance of ΔX_t is given by:

$$V_{\Delta x,h} = V\left[e_{\Delta x,t+h}\right] = V\left[e_{\Delta x,t+h} / x_t\right] = \sum_{i=0}^{h-2} A^S \alpha\beta' \Omega \beta \alpha' A^{s'} + \Omega$$

III Empirical investigation: The Demand for Money in Tunisia

a) *Introduction and background*

The structural adjustment program (SAP) in 1988, marks an important turning point in the tunisian monetary policy. Part of SAP aimed directly at reducing inflation and containing domestic demand. In order to achieve this goal the monetary authority used the technique of monetary targeting. However this monetary policy can only be efficient if the demand for money is stable.

We will focus on a simple model of the demand for money where the quantity demanded (m) is proportional to price level (p), income (y) and the interest rate (r). All variables are in logarithm.

$$lm_t = \beta_0 + \beta_1 lp_t + \beta_2 ly_t + \beta_3 lr_t + \varepsilon_t$$

b) *Tests for cointegration and estimation of linear ECM*

Our ADF unit root tests show that all the level variables are non stationary (α=5%). We reject the unit root null hypothesis for the variables in first difference. This means that all the series are I(1).

We start with the estimation of the long run relation using Johansen (1988) maximum likelihood method. The cointegrated equation is specified with an intercept and the series in level have a linear deterministic trend. Next we determine r the number of cointegrated relations between the variables. The Trace test shows that r is equal to 1 at both the 5% and the 1% significance levels. However, as indicated in table 3, the maximum eigenvalue test shows that r is equal to 2 at 5%, but that there is no cointegration between the series at the 1% significance level. We therefore compute an information criteria test and we find that the value of r is equal to1.

We now move to the estimation of the linear error correction model. The Engle and Granger (1987) representation theorem states that cointegrated variables admit an error correction representation as follows:

$$\Delta Y_t = \alpha + \sum_{i=1}^{p} \beta_i \Delta X_{t-i} + \sum_{i=1}^{q} \lambda_i \Delta Y_{t-i} + \delta Z_{t-1} + \varepsilon_t$$

The specification of this model depends essentielly on the lag lengh l. In our study we perform likelihood ratio test statistics to find the parcimonious ECM specification. The results are reported in Table 1. In particular it is shown that the money demand ECM equation is given by:

Δlm_t = - 0.049 $\tilde{e}_{t-1}$ – 0.291Δlm_{t-1} + 1.016 Δly_{t-1} - 0.194 Δlp_{t-1} + 0.308Δlr_{t-1} + 0.026,
where the lag lengh l is equal to1.

Following a standard VAR analysis, we compute Granger causality test statistics for each equation in the vector error correction model. The results from table 2 show that past values of the price level variable, *lp*, are not significant in the VECM equations. However this VAR result should be cautiously interpreted because in a system of cointegrated variables we need further to test the zero restriction for the adjustment coefficient in the *lp* ECM equation in order for Granger causality to make sense. Table 4a shows that this restriction is binding for r =1, and thus we reject the hypothesis that *lp* is weakly exogenous.
We also test the hypothesis that money supply increases when nominal income decreases and vice versa. This means that in the cointegrating vector the coefficients of *ly* and *lp* are equal and the *lr* coefficient is zero. From the results reported in table 4b it shown that this hypothesis is accepted.

c) *Non Linear ECM*

We begin with the assessment of the Granger-Lee method. We apply this method to the annual tunisian data. Tthe GLS estimated equation is given in table 5a. We also found that the coefficients δ_1 and δ_2 in the NLECM equation (*) are jointly significant.
Unlike the polynomial approach, the nice thing about this type of non linear models is that we can give an insightful economic interpretation to the decomposed equilibrium error. Recall that in the Granger- Lee approach the error term Z_t may be either positive or negative.

If $Z_t > 0$ then we have: $(lm_t - \beta_0 - \beta_1 lp_t - \beta_2 ly_t - \beta_3 lr_t) > 0$

$$\Leftrightarrow (lm_t - \beta_1 lp_t - \beta_2 ly_t) > \beta_0 + \beta_3 lr_t$$

From the quantity theory of money $(mv = py)$

$$l\left(\frac{1}{v}\right) = lm - lp - ly$$

Imposing the restriction $\beta_1 = \beta_2 = 1$, one gets $v < e^{-\beta_0 + \beta_1 lr_t}$

Finally, take $r_t \in \left[r_{t,\min}, r_{t,\max}\right]$, we get

$$v \in \left[e^{-\beta_0 + \beta_1 lrt,\min}; e^{-\beta_0 + \beta_1 lrt,\max}\right]$$

In our study the corresponding values of $v_t \in [0.8;1.7]$. We find that if $v_t \in [0.8;1.7]$ then the speed of adjustment coefficient is 0.1, wheras if v_t, the speed of monetary transaction is greater then 1.7, then the speed of adjustment towards equilibrium is 0.12. we conclude that the speed of adjustment and the speed of monetary transactions move in the same direction.
We now present our results concerning *the Escribano- Pfann approach.* The results of generalized least squares estimation are given in table 5b. Recall that the idea is that if the error correction model is better specified non-linearly then we may represent the error correction term by two components as in (2.13). In our study we test the hypothesis that the two coefficients are equal. The p-value of the F-test

statistic is 0.064. It is reasonable to argue that the coefficients are statistically different and thus we continue our analysis based on the hypothesis of asymmetry.

Notice that $\Delta Z_t > 0$ mean $\Delta l\left(\frac{1}{v}\right) > \Delta(\beta_0 + \beta_3 lr_t)$ and suppose that $\Delta\beta_3 ln$ is zero, therefore $\Delta l\left(\frac{1}{v_t}\right) > 0$ which show that $v_t < v_{t-1}$. This means that if the speed of monetary transaction decreases across time periods then the equilibrium adjustment speed is 0.093. If however v_t increases between two time periods, the adjustment speed is 0.15. We therefore come to the same conclusion as with the Granger- Lee approach, the speed of adjustment is faster if monetary transactions are at a higher speed.

VI. Interpretation and concluding remarks:

In time series analysis a researcher's goal is to determine a modelling strategy that best predicts economic variables. We now go on to a comparative analysis between the linear error correction mpodel and the two forms of non linear ECM in termes of their forecast accuracy of money demand (*lm*). Our procedure is as follows:

- First we estimate the three models from 1885:01 to 1998:4.
- Next, for each model we compute the forecast series of *lm* using post sample observations from 1999:01 to 2002:4.
- Then we compare the three forecasts based on their root mean square error (RMSE), mean absolute error (MAE), and mean absolute percent error (MAPE), as shown in figure 1.

We present our findings in the table below:

Forecast measure	Linear ECM	Granger-Lee NLECM	Escribano-Pfann NLECM
RMSE	0.2043	0.2146	0.2020
MAE	0.1757	0.1815	0.1738
MAPE	1.8227	1.8823	1.8030

From the table we can see that forecast errors are lowest for the Escribano-Pfann non linear error correction model which outperforms, in terms of prediction, the linear ECM. This is not surprising since we already found that the equilibrium error can be decomposed in two statistically differents terms. The coefficient of speed of adjustment towards the long run equilibrium moves in the same direction as the speed of monetary transactions. This result is not revealed in standard linear modelling for which the adjustment coefficient is constant. Figure 2 shows the forecast series compared to the actual money demand series.

In this paper we present non linear error correction models as an alternative to standard VAR and ECM. Specifically, we consider two approaches to handle non linearity in the error correction term based on its asymetric partition.

We use these models to study the demand for money in Tunisia. Our results show that the non linear error correction model of Escribano-Pfann outperforms the linear

ECM in terms of prediction. Intuitively, in the NLECM the asymmetric partition of the error correction term yields two asymetric situations. In the first case we have a relatively fast adjustment and a high speed of money transactions (v > 1.7) which characterize regulated economies. However in the second case we have a slow adjustment towards equilibrium and also a relatively slow speed of money transactions which basically describe market economies. Early estimate of the speed (v) of money transactions in Tunisia was 2.2 which explains why some authors claim that the tunisian economy is state controlled. However the results of our study based on non linear modelling prove that a more accurate estimate of the speed v is less than 1.7, showing that the Tunisian economy is in transition toward a market economy. Thus, from the partition of the adjustment coefficient in the ECM representation by the two non linear approaches, we can see how the speed of adjustment to the long-run equilibrium and the speed of money transactions are related. It is shown that ignoring non linearity in the error correction models leads to less accurate prediction of money demand.

References:

- Alfonso Novales, (1990) “Solving non linear rational expectation models: A stochastic equilibrium model of interest rates”, *Econometrica*: 58(1), 93-111.
- Ball, L. and Mankiw, G., (1994) “Asymmetric price adjustment and economic fluctuation”, *Economic Journal* 104: 247-261.
- Brana, S. et Casals, M., (1997) « La monnaie». *Ed. Dunod, Paris.*
- Enders S.W., (1995) “Applied Econometric time series”. *Wiley Edition.*
- Engle R.F, Granger C.W.J, (1987) “Cointegration and error correction representation, estimation and testing”, *Econometrica*: 55(2),251-276.
- Escribano, A., (1996) ”Non linear Error correction: The case of Money Demand in the U.K (1878-1970)”, *Working paper 96-55-24 Universidad Carlos III de Madrid.*
- Escribano, A. and Mira, S., (1997) ”Non linear error correction, asymmetric adjustment and cointegration”. *Economic Modelling*, 15(2), 197-216.
- Gale, D.,(1996) ”Delay and Cycle”. *Review of Economic Studies* 63(2),169-198.
- Granger, C.W.J.,Lee,T.H (1989) “Investigation of Production, sales and non-symetric error correction models”. *Journal of Applied Econometrics*, Vol. 4,145-159.
- Holly, S. and Stannett, M., (1995) ”Are there asymmetric in U.K consumption? A time series Analysis”. *Applied Economics*, 27, 267-272.
- Johansen, S., (1995). ”Likelihood-based inference in cointegrated vector autoregressive models”, *Oxford University Press.*
- Khedhiri.S., (1993). ”Testing for stability in time series models with stationary and non-stationary processes”. *Ph.D dissertation, University of Southern California*, USA .

Krane, S.D., (1994). “The distinction between inventory holding and stock out costs: implication for target inventories, asymmetric adjustment and the effect of aggregation on production smoothing”, *International Economic Review*, 35,117-136.

- Michael P. Clements and David.F.Hendry, (1995). "Forecasting in cointegrated Systems", *Journal of applied Econometris*: 10,127-146.
- Pfann,G.A.,(1993) "Asymmetric adjustment costs in labour demand models" *Review of Economic Studies*:60,397-412.
- Steven, C., Sean, H. and Paul. T.,(1990) "The power of tests for non-linearity: the case of Granger-Lee asymmetry". *Economic Letters*:62,155-159.

Appendix A:

Table 1 : Estimation of linear ECM

Vector Error Correction Estimates
Date: 09/23/03 Time: 09:28
Sample(adjusted): 1985:3 2002:4
Included observations: 70 after adjusting endpoints
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1			
LM(-1)	1.000000			
LY(-1)	0.122949 (2.20925) [0.05565]			
LP(-1)	-1.307785 (2.12861) [-0.61438]			
LR(-1)	1.743302 (0.32415) [5.37814]			
C	-7.312155			
Error Correction:	**D(LM)**	**D(LY)**	**D(LP)**	**D(LR)**
CointEq1	-0.049108 (0.01758) [-2.79273]	0.012097 (0.00400) [3.02130]	0.007762 (0.00969) [0.80090]	-0.025410 (0.01673) [-1.51890]
D(LM(-1))	-0.291504 (0.11196) [-2.60359]	0.047764 (0.02549) [1.87351]	0.062703 (0.06171) [1.01610]	-0.365293 (0.10652) [-3.42940]
D(LY(-1))	1.016389 (0.52400) [1.93968]	-0.266072 (0.11932) [-2.22996]	0.232590 (0.28881) [0.80534]	0.289320 (0.49852) [0.58036]

D(LP(-1))	-0.194403	0.054392	-0.156236	0.120878
	(0.22941)	(0.05224)	(0.12644)	(0.21826)
	[-0.84739]	[1.04122]	[-1.23562]	[0.55383]
D(LR(-1))	0.308107	0.046582	0.078009	0.449457
	(0.10484)	(0.02387)	(0.05778)	(0.09974)
	[2.93888]	[1.95132]	[1.35004]	[4.50628]
C	0.026382	0.012368	0.009741	-0.000348
	(0.00740)	(0.00169)	(0.00408)	(0.00704)
	[3.56378]	[7.33716]	[2.38741]	[-0.04935]
R-squared	0.221923	0.269300	0.094078	0.313905
Adj. R-squared	0.161136	0.212214	0.023303	0.260303
Sum sq. resids	0.085628	0.004440	0.026012	0.077503
S.E. equation	0.036578	0.008329	0.020160	0.034799
F-statistic	3.650810	4.717456	1.329255	5.856298
Log likelihood	135.3927	238.9722	177.0935	138.8822
Akaike AIC	-3.696935	-6.656349	-4.888384	-3.796633
Schwarz SC	-3.504207	-6.463620	-4.695656	-3.603905
Mean dependent	0.025901	0.011063	0.011627	-0.007808
S.D. dependent	0.039937	0.009384	0.020399	0.040461

Determinant Residual Covariance	3.92E-14
Log Likelihood	695.6994
Log Likelihood (d.f. adjusted)	683.1537
Akaike Information Criteria	-18.71868
Schwarz Criteria	-17.81928

Table 2 : Granger causality tests

VEC Pairwise Granger Causality/Block Exogeneity Wald Tests
Date: 09/25/03 Time: 12:44
Sample: 1985:1 2002:4
Included observations: 70

Dependent variable: D(LM)

Exclude	Chi-sq	Df	Prob.
D(LY)	3.762345	1	0.0524
D(LP)	0.718072	1	0.3968
D(LR)	8.637033	1	0.0033
All	10.96892	3	0.0119

Dependent variable: D(LY)

Exclude	Chi-sq	Df	Prob.
D(LM)	3.510043	1	0.0610
D(LP)	1.084140	1	0.2978
D(LR)	3.807637	1	0.0510
All	9.959435	3	0.0189

Dependent variable: D(LP)

Exclude	Chi-sq	Df	Prob.
D(LM)	1.032467	1	0.3096
D(LY)	0.648575	1	0.4206
D(LR)	1.822602	1	0.1770
All	3.870824	3	0.2758

Dependent variable: D(LR)

Exclude	Chi-sq	Df	Prob.
D(LM)	11.76078	1	0.0006
D(LY)	0.336817	1	0.5617
D(LP)	0.306731	1	0.5797
All	12.48132	3	0.0059

Table 3 : Johansen Cointegration Tests

Unrestricted Cointegration Rank Test

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	5 Percent Critical Value	1 Percent Critical Value
None **	0.500819	85.09091	53.12	60.16
At most 1 *	0.260651	36.45583	34.91	41.07
At most 2	0.142222	15.31682	19.96	24.60
At most 3	0.063309	4.578150	9.24	12.97

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Trace test indicates 2 cointegrating equation(s) at the 5% level
Trace test indicates 1 cointegrating equation(s) at the 1% level

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	5 Percent Critical Value	1 Percent Critical Value
None **	0.500819	48.63508	28.14	33.24
At most 1	0.260651	21.13900	22.00	26.81
At most 2	0.142222	10.73867	15.67	20.20
At most 3	0.063309	4.578150	9.24	12.97

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Max-eigenvalue test indicates 1 cointegrating equation(s) at both 5% and 1% levels

Unrestricted Cointegrating Coefficients (normalized by b'*S11*b=I):

LM	LY	LP	LR	C
-0.584049	0.787425	-0.057560	0.967554	1.478239
5.027398	-0.862767	-5.180076	7.272611	-33.17165
2.546959	-44.70803	39.72186	0.737845	-0.274192
7.757145	-8.732466	-8.764471	-0.456640	10.70567

Unrestricted Adjustment Coefficients (alpha):

D(LM)	0.015678	-0.016481	0.001039	-0.003210
D(LY)	0.007511	0.001284	0.000192	0.000141
D(LP)	0.006201	0.000228	-0.006617	0.001142
D(LR)	-0.000638	-0.006193	0.001996	0.007845

1 Cointegrating Equation(s):		Log likelihood	687.0229	
Normalized cointegrating coefficients (std.err. in parentheses)				
LM	LY	LP	LR	C
1.000000	-1.348218	0.098553	-1.656632	-2.531019

	(9.35596)	(9.01446)	(1.37273)	(8.41617)

Adjustment coefficients (std.err. in parentheses)

D(LM)	-0.009157
	(0.00277)
D(LY)	-0.004387
	(0.00058)
D(LP)	-0.003622
	(0.00139)
D(LR)	0.000373
	(0.00245)

2 Cointegrating Equation(s): Log likelihood 697.5924

Normalized cointegrating coefficients (std.err. in parentheses)

LM	LY	LP	LR	C
1.000000	0.000000	-1.195028	1.899215	-7.191387
		(0.33953)	(0.36393)	(2.13855)
0.000000	1.000000	-0.959474	2.637442	-3.456688
		(1.09814)	(1.17708)	(6.91679)

Adjustment coefficients (std.err. in parentheses)

D(LM)	-0.092016	0.026565
	(0.02162)	(0.00499)
D(LY)	0.002070	0.004806
	(0.00492)	(0.00114)
D(LP)	-0.002475	0.004686
	(0.01203)	(0.00278)
D(LR)	-0.030762	0.004841
	(0.02091)	(0.00483)

3 Cointegrating Equation(s): Log likelihood 702.9617

Normalized cointegrating coefficients (std.err. in parentheses)

LM	LY	LP	LR	C
1.000000	0.000000	0.000000	-1039.052	1241.229
			(332.399)	(700.881)
0.000000	1.000000	0.000000	-833.1309	998.8864
			(266.198)	(561.292)
0.000000	0.000000	1.000000	-871.0689	1044.679
			(278.298)	(586.806)

Adjustment coefficients (std.err. in parentheses)

D(LM)	-0.089370	-0.019867	0.125727

	(0.02419)	(0.19094)	(0.17102)
D(LY)	0.002560	-0.003795	0.000557
	(0.00551)	(0.04349)	(0.03895)
D(LP)	-0.019328	0.300520	-0.264379
	(0.01263)	(0.09972)	(0.08932)
D(LR)	-0.025677	-0.084417	0.111420
	(0.02336)	(0.18443)	(0.16519)

Table 4a : Hypothesis Test of Cointegration (H_0: *lp* is weakly exogenous)

Restrictions:

a(3,1)=0

Tests of cointegration restrictions:

Hypothesized No. of CE(s)	Restricted Log-likehood	LR Statistic	Degrees of Freedom	Probability
1	684.1171	5.811618	1	0.015921
2	697.5924	NA	NA	NA
3	702.9617	NA	NA	NA

NA indicates restriction not binding.

1 Cointegrating Equation(s): Convergence achieved after 13 iterations.

Restricted cointegrating coefficients (not all coefficients are identified)

LM	LY	LP	LR	C
-0.401980	-5.473170	5.869207	1.008921	1.464847

Table 4b :Hypothesis Tests of Cointegration (H_0: feed-back rule)

Restrictions:

b(1,2)=b(1,3)
b(1,4)=0

Tests of cointegration restrictions:

Hypothesized No. of CE(s)	Restricted Log-likehood	LR Statistic	Degrees of Freedom	Probability
1	686.7449	0.555909	2	0.757331
2	697.5922	0.000458	1	0.982919
3	702.9617	NA	NA	NA

NA indicates restriction not binding.

1 Cointegrating Equation(s): Convergence achieved after 4 iterations.

Restricted cointegrating coefficients (not all coefficients are identified)

LM	LY	LP	LR	C
-1.270572	0.748456	0.748456	0.000000	6.185508

Table 5a: Non linear ECM estimation (Granger-Lee method)

Dependent Variable: D(LM)
Method: Least Squares
Date: 10/12/03 Time: 11:02
Sample(adjusted): 1985:3 2002:4
Included observations: 70 after adjusting endpoints
White Heteroskedasticity-Consistent Standard Errors & Covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.028579	0.008810	3.243776	0.0019
D(LM(-1))	-0.222207	0.136450	-1.628487	0.1084
D(LY(-1))	0.623270	0.479795	1.299033	0.1987
D(LP(-1))	-0.222444	0.238235	-0.933716	0.3540
D(LR(-1))	0.305180	0.070899	4.304418	0.0001
ZP	-0.107583	0.062428	-1.723313	0.0897
ZN	-0.120059	0.108680	-1.104695	0.2735

R-squared	0.196175	Mean dependent var	0.025901
Adjusted R-squared	0.119620	S.D. dependent var	0.039937
S.E. of regression	0.037472	Akaike info criterion	-3.635808
Sum squared resid	0.088461	Schwarz criterion	-3.410958
Log likelihood	134.2533	F-statistic	2.562538
Durbin-Watson stat	2.018878	Prob(F-statistic)	0.027670

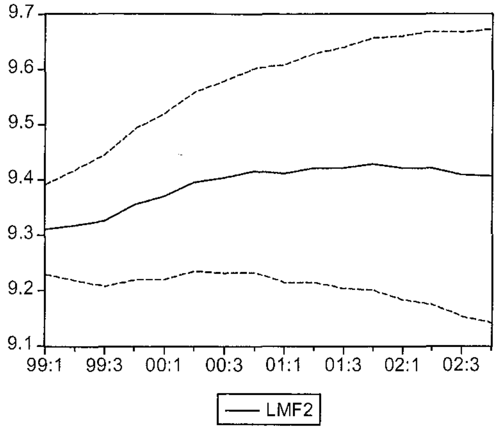
Table 5b: Non Linear ECM estimation (Escribano-Pfann method)

Dependent Variable: D(LM)
Method: Least Squares
Date: 10/15/03 Time: 12:09
Sample(adjusted): 1985:3 2002:4
Included observations: 70 after adjusting endpoints
White Heteroskedasticity-Consistent Standard Errors & Covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.027740	0.005531	5.015848	0.0000
D(LM(-1))	-0.208293	0.138232	-1.506836	0.1369
D(LY(-1))	0.663608	0.455192	1.457864	0.1498
D(LP(-1))	-0.229188	0.225777	-1.015109	0.3139
D(LR(-1))	0.320677	0.069115	4.639772	0.0000
WP	-0.093446	0.056493	-1.654122	0.1031
WN	-0.157422	0.071461	-2.202907	0.0313

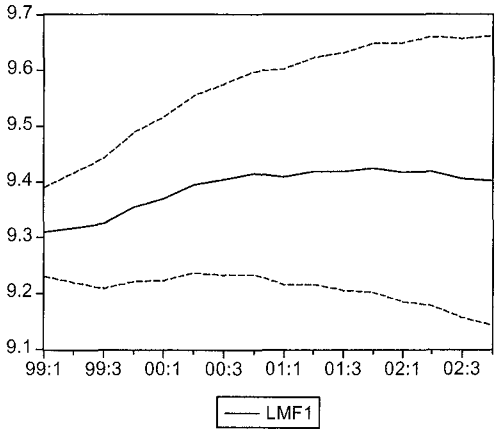
R-squared	0.200976	Mean dependent var	0.025901
Adjusted R-squared	0.124879	S.D. dependent var	0.039937
S.E. of regression	0.037360	Akaike info criterion	-3.641799
Sum squared resid	0.087933	Schwarz criterion	-3.416949
Log likelihood	134.4630	F-statistic	2.641037
Durbin-Watson stat	2.004909	Prob(F-statistic)	0.023872

Figure 1: Forecast and forecast errors of Money Demand with alternative methods



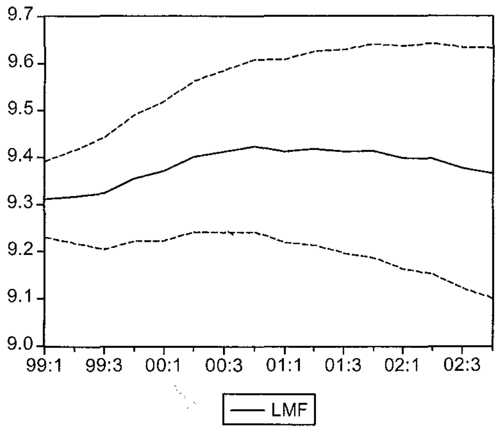
Forecast of LM using Escribano-Pfann method
Actual: LM
Forecast sample: 1999:1 2002:4
Included observations: 16

Root Mean Squared Error	0.202019
Mean Absolute Error	0.173832
Mean Abs. Percent Error	1.803005
Theil Inequality Coefficient	0.010662
Bias Proportion	0.689117
Variance Proportion	0.286971
Covariance Proportion	0.023912



Forecast of LM using Linear ECM
Actual: LM
Forecast sample: 1999:1 2002:4
Included observations: 16

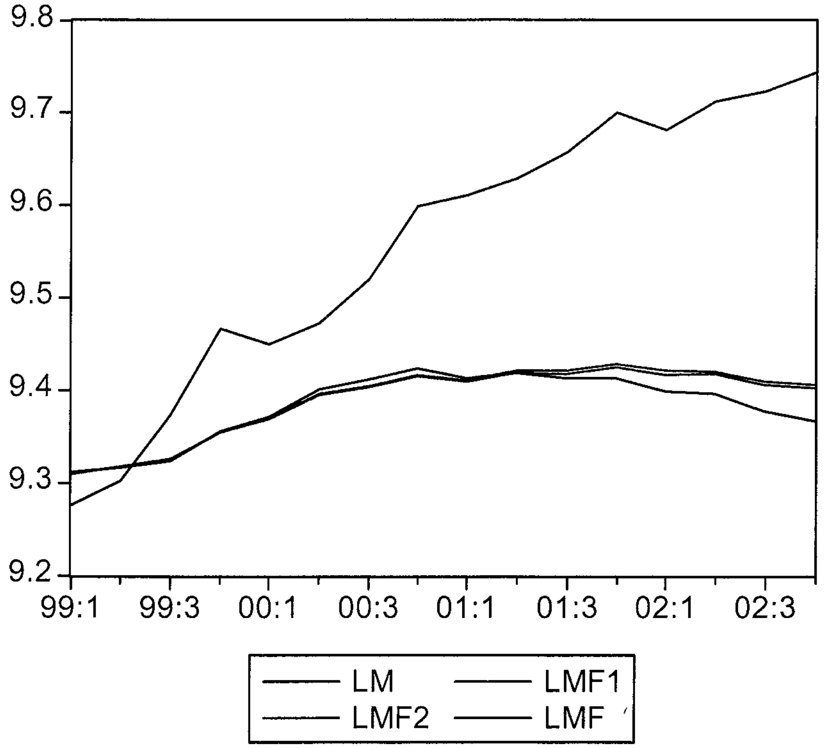
Root Mean Squared Error	0.204357
Mean Absolute Error	0.175737
Mean Abs. Percent Error	1.822722
Theil Inequality Coefficient	0.010786
Bias Proportion	0.689373
Variance Proportion	0.285208
Covariance Proportion	0.025419



Forecast of LM using Granger-Lee NLECM
Actual: LM
Forecast sample: 1999:1 2002:4
Included observations: 16

Root Mean Squared Error	0.214664
Mean Absolute Error	0.181559
Mean Abs. Percent Error	1.882344
Theil Inequality Coefficient	0.011334
Bias Proportion	0.667435
Variance Proportion	0.266420
Covariance Proportion	0.066145

Figure 2: Actual and forecast money demand series



Appendix B:

Let $W_t = (Y_t, X_t)$ be an (N, 1) vector of variables, where Y_t is a scalar and X_t is (N-1,1) vector. We can express the probability density function of W_t in terms of the conditional densities as follows:

$$D(W_t / W_{t-1},\ldots.W_0, \theta) = D(Y_t/X_t, W_{t-1},\ldots W_0,\theta_1).D(X_t/W_{t-1},\ldots W_0,\theta_2) \qquad (1)$$

Following Escribano and Mira (1997), if the parameter vector of interest Ψ is a function of θ_1, which means $\Psi=f(\theta 1)$, and if X_t is weakly exogenous with respect to Ψ, then statistical inference may be conducted based on conditional densities without any loss of information. In particular, we are interested in the conditional mean of Y_t, $E(Y_t/X_t,W_{t-1},\ldots W_0,\theta_1)$.

Let ε_t be a martingale difference sequence with constant variance σ^2_ε :
$\varepsilon_t = Y_t - E(Y_t / X_t,W_{t-1},\ldots.W_0,\theta_1)$.
The conditional mean of Y_t may be expressed in terms of an autoregressive distributed finite lag function with the following non linear term :

$E(Y_t/X_t,X_{t-1},W_{t-1},\ldots\theta_1)=-\Phi^1(B)Y_{t-1}-\Psi(B)X_t-g(Y_{t-1}-\alpha X_{t-1})$ (2)

This suggests that the equation of Y_t may be written as follows :

$\Phi(B)Y_t+\Psi(B)X_t=-g(Y_{t-1}-\alpha X_{t-1})+\varepsilon_t$ (3)

where $\Phi(B)$ is a finite lag polynomial, $\Phi(0)=1$ and $\Psi(B)$ is a (1x(N-1)) finite lag polynomial vector. We further assume that $\Phi(0)$ is (1, N-1) with some non-zero elements.

If $\Phi(B)$ and $\Psi(B)$ have unit roots then Y_t and X_t are I(1) and we have:

$$\phi(B)=\phi(1)+\phi^*(B)(1-B) \quad (4)$$

$$\psi(B)=\psi(1)+\psi^*(B)(1-B) \quad (5)$$

where $\phi^*(B)$ and $\psi^*(B)$ have all roots outside the unit circle.

Using equation (2.4) we obtain:

$$\begin{aligned}\phi(B)&=\phi(1)B+\left(\phi^*(B)+\phi(1)\right)(1-B)\\&=\phi(1)B+\phi^{**}(B)(1-B)\end{aligned} \quad (6)$$

Equation (2.5) allows us to write :

$$\begin{aligned}\psi(B)&=\psi(1)B+\left(\psi^*(B)+\psi(1)\right)(1-B)\\&=\psi(1)B+\psi^{**}(B)(1-B)\end{aligned} \quad (7)$$

Substituting equations (2.6) and (2.7) in (2.3) we get :

$$\phi^{**}(B)(1-B)Y_t+\psi^{**}(B)X_t=-\phi(1)Y_{t-1}-\psi(1)X_{t-1}-g(Y_{t-1}-\alpha X_{t-1})+\varepsilon_t$$
(8)

Let $\phi(1)=\delta\alpha_1, \psi(1)=\delta\alpha_2$, and divide equation (2.8) by α_1, we find :

$$\phi_\alpha(B)(1-B)Y_t+\psi_\alpha(1-B)X_t=-\delta(Y_{t-1}-\alpha X_{t-1})-g_\alpha(Y_{t-1}-\alpha X_{t-1})+\varepsilon_{\alpha t}$$
(9),

where $\phi_\alpha(B)=\frac{1}{\alpha_1}\phi^{**}(B)$

and $\psi_\alpha(B)=\frac{1}{\alpha_1}\psi^{**}(B)$,

$$g_\alpha(.)=\frac{1}{\alpha_1}g(.),$$

$$\varepsilon_{\alpha t}=\frac{1}{\alpha_1}\varepsilon_t.$$

The representation of NLECM is given by equation (9). Note that with $g_\alpha(.)=0$, we have the usual linear ECM. Now if we allow $Z_{t-1}=Y_{t-1}-\alpha X_{t-1}$ and $f(Z_{t-1})=-\delta(Y_{t-1}-\alpha X_{t-1})-g_\alpha(.)$, we may state the following result: The system of variables (X_t, Y_t) has a non linear ECM representation if the first difference of the variables are written as:

$$\Delta Y_t = \sum_{i=0}^{r} b_i \Delta X_{t-i} + \sum_{i=1}^{p} a_i \Delta Y_{t-i} + f(Z_{t-1}, \gamma) + \varepsilon_{yt} \qquad (10)$$

$$\Delta X_t = \varepsilon_{xt} \qquad (11)$$

Note the following :

(i) ε_{yt} is the martingale difference sequence with zero mean and a variance equals to σ^2, and ε_{xt} is I(0) with constant variance and covariance terms.

(ii) The roots of the equation $\left| 1 - a_1 B - a_2 B^2 - \ldots . a_p B^p \right| = 0$, are all outside the unit circle.

(iii) $f(Z, \gamma)$ is a continuous and differentiable function with the following stability condition: $-2 < \dfrac{df(Z_{t-1}, \gamma)}{dZ_{t-1}} < 0$.

Appendix C:

Let $X_t = (x_t, y_t)'$ be a vector given by :

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_{t-1} \\ y_{t-1} \end{pmatrix} + \psi \begin{pmatrix} D_{1t} \\ D_{2t} \end{pmatrix} + \begin{pmatrix} V_{1t} \\ V_{2t} \end{pmatrix}$$

$$X_t = AX_{t-1} + \psi D_t + V_t \qquad (12)$$

with $V_t \sim N(0, \Omega)$ for all $t = 1,2,\ldots..T$, X_0 is the fixed initial value and D_t is the matrix of deterministic components.

Rewrite (12) as follows:

$$\begin{pmatrix} \Delta x_t \\ \Delta y_t \end{pmatrix} = \begin{pmatrix} a_{11} - 1 & a_{12} \\ a_{21} & a_{22} - 1 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ y_{t-1} \end{pmatrix} + \psi \begin{pmatrix} D_{1t} \\ D_{2t} \end{pmatrix} + \begin{pmatrix} V_{1t} \\ V_{2t} \end{pmatrix}$$

$$\Delta X_t = \Pi X_{t-1} + \psi D_t + V_t \qquad (13)$$

under the condition ψ =0, equation (13) becomes :

$$Z_t = \begin{pmatrix} \Delta X_t \\ \\ \beta' X_{t-1} \end{pmatrix} = \begin{pmatrix} \Pi & \alpha \\ \\ \beta' & I_n \end{pmatrix} \begin{pmatrix} X_{t-1} \\ \\ \beta' X_{t-2} \end{pmatrix} + \begin{pmatrix} V_t \\ \\ 0 \end{pmatrix} \tag{14}$$

Note that the mean of ΔX_t is obtained from:

$$\begin{aligned} E(\Delta X_t) &= E(\alpha\beta' X_{t-1} + \psi + V_t) \\ &= (I - \alpha(\beta'\alpha)^{-1}\beta')\psi, \end{aligned}$$

which concludes our analytical findings.