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## Frameworks for measuring the performance of the maintenance system in a capital intensive organisation

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**FRAMEWORKS FOR MEASURING THE PERFORMANCE OF THE  
MAINTENANCE SYSTEM IN A CAPITAL INTENSIVE ORGANISATION**

**A thesis submitted in fulfilment of the requirements  
for the award of the degree  
of**

**DOCTOR OF PHILOSOPHY**

**from**

**UNIVERSITY OF WOLLONGONG**

**by**

**RICHARD DWIGHT, BE**

**DEPARTMENT OF MECHANICAL ENGINEERING  
1999**

## **Dedication**

*To an endangered pursuit: an unbiased quest for knowledge*

## **Abstract**

The measurement of performance can be viewed at vastly different levels of complexity. Performance can be defined as the difference between what is deemed possible to achieve relative to the organisation's goals and what is actually achieved. This implies that the standard of performance must be in terms of the organisation's goals and what is possible. The measurement must connect actions undertaken in a period of interest with their immediate and long-term outcomes. The concept of measuring changes in value to determine performance is thus proposed, where value is normally expressed as the net present value of future cash flows.

Traditional measures have been found to ignore this fundamental requirement, leading to an inability to account for lags between harmful actions and their effects. This link is critical to pursuing logical approaches to the management of the maintenance system. It is missing from all currently advocated maintenance performance measurement approaches. The use of ratios to measure performance, whilst promising a catalogue of measures to be used and benchmarked, does not allow appropriate standards of performance to be determined.

Key to the measurement of performance is the establishment of a standard based on established goals. The subjective nature of the process of goal determination has been found to limit the possibility for objectivity in performance measurement. Similarly the perceived level of control of the subject system over the organisation's goals establishes the boundaries of the search for performance measures.

The measurement of performance in terms of changes in value accommodates these concerns. It also introduces some difficulties and constraints on measurement approaches. Determination of future operating requirements, changes in use, failure behaviour given a particular set of actions, and the organisations perception of risk are some of the variables that prove difficult to quantify. Each has a significant impact on the definition of the standard of performance. Nonetheless decisions, and so decision quality, depend on estimates of these future events. Performance measurement must be concerned with likely outcomes of current decisions and so with prediction of future outcomes.

Although the fundamental basis for performance measurement has been identified, practical approaches to its determination must be found. Based on the establishment of fundamental equations for value-based performance measurement, frameworks for two new measurement approaches have been established and their practical application explored. The systems-auditing approach concentrates on the alignment of maintenance system activities with the organisation's goals. The relative performance of these activities is established as a surrogate for performance in terms of value. The approach calls for the comparison of the current approaches to individual functions with alternatives. This introduces the search for alternatives as an integral part of performance measurement. It fits with a quest for improvement.

A second new approach to performance measurement has been labelled the event-analysis approach. It focuses on understanding the impact of specific actions on the

value of the organisation. Alternate actions are considered in terms comparing future values. The standards set are the best known actions given the situations arising during the period. Both new approaches focus on a standard of performance related to what could be achieved as opposed to achievement of a budget or benchmark.

The new approaches have been applied, allowing establishment of their nature and applicability. They establish a high scientific rigour to the measurement of the performance of management systems, including in particular maintenance management systems. Both new approaches will benefit from detailed modelling of the maintenance process.

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Definitions and Corresponding Symbols Uses

Term	Symbol	Definition	Section
action		An expending of the organisation’s resources. “A state or process of doing something. (Oxford Dictionary) The basic types of maintenance action are: inspection, servicing, replacement, repair. Other action types are planning, preparation and evaluation.	
action set, best	$A^*$	The best known response to the known output requirements for the system	4.5 and 5
activity		In general the identification of an activity, often also termed a function, is an undertaking of an organisation as part of its overall system of providing a product or service. The conducting of the activity may or may not be the responsibility or role of a particular person or functional group.	
audit		A methodical review of the activities or processes that make up a system of work, focusing on the intrinsic capability of the activities as opposed to, but not necessarily excluding, attempts to measure inputs and outputs of the system.	
cash flow	$CF$	The monetary value of transactions made during the period	4.1
coefficient of influence of the maintenance system	$CIMS$	The total influence of the maintenance system on the organisation’s success	
coefficient of influence of a system activity	$C_{rel}SA_k$	The overall correlation between the performance of a particular system activity, or element, $SA_k$ , and the organisation’s success,	5.1
condition		The state of a technical system relative to its ability to perform its required function.	

discount rate	$\beta_j$	The rate at which real cash flows are converted to a value at the time of interest. It reflects the weighted cost of capital which in turn represents the acceptable level of risk.	
effectiveness	$\varepsilon_I, \varepsilon_O$	The value generated by an input action, $I$ or output action, $O$ , divided by the actual direct amount of resources expended. It will normally be expressed in monetary terms. Note: The expected increased future production and reduced maintenance cost is relative to the expected value if the expected outcome of the action had not been achieved at that point of time.	4.4.1
efficiency	$\eta_I, \eta_O$	The actual resources, normally expressed as a cost, consumed in conducting an input or output action divided by some standard amount, being the lowest cost thought achievable for the task.	4.4.1
event	$F_i$	When referring to a technical system it is something that happens to a technical system that causes a change in its performance. This could be a failure, change in condition or a result of a maintenance action. It is an action that is observable from the perspective of the technical system, an action applied to the technical system. Alternatively, the occurrence of circumstances that cause a reduction in system potential output	6.3.1
ex-ante		This is used to define an implied evaluation of income based on future wealth or intangible assets. ie. “subjective goodwill - and not...the gain in monetary and physical assets on hand.” Barton(1974,672)	
expense	$E_i$	The expense associated with action or event $i$ ,	4.1

ex-post		This is used to define an evaluation of income based on tangible assets. It is taken here to indicate that the performance measurement activity considers the occurrence of external events that occurred during the period under review were known about. They do not affect performance per se.	
failure attribute	$FA_i$	The attribute of an action and its associated technical system failure that has a potential impact on the organisation's success	
failure behaviour		The characteristics of the mechanism of failure exhibited by a technical system or of a component of a technical system. Characteristics include the hazard and reliability functions and failure-predicting-property characteristics.	
function		This is strictly an activity allocated to a particular person or group of people eg. The function of the planning group is to carry out the activities of work identification and work planning. It is also used to refer to system output requirements or reasons for being as in 'intended function': eg. The function of the device is to propel a certain object 500 metres at a speed of at least 500 km/h.	
input	$I$	Normally an action aimed at increasing the value of the technical or maintenance system.	
input, future best known	$I^*$	The anticipated, normally best known, future input actions from time, $t$ on.	4.4.1
life-cycle		The phrase pointing to the phases in the life, typically of a system or technical system. The phases are the acquisition, use and disposal phases, although these can be further broken down depending on the purpose of the categorisation. It may be more grammatically correct simply to replace the term 'life-cycle' by 'phases in the life'.	
maintenance concept		The set of maintenance rules and specified maintenance actions for a technical system. Note: It is often referred to as maintenance	

		plans or strategies.	
maintenance problem		The circumstance requiring the design and management of a maintenance system	
maintenance system		those activities in an organisation that apply resources to control actual technical system behaviour, given the operations undertaken and the configuration of the technical system, with the aim of achieving the goals of the organisation	
maintenance system management		Commonly referred to as maintenance management:  The activities associated with attempts to influence the performance of the maintenance system	
organisation's success factor		The attributes of the organisation that determine its success in meeting its goals. This particularly includes the drivers of cost and value of its products.	
output expected	$\hat{O}$	the expected future outputs	4.4.1
performance		the level to which a goal is attained	
performance measurement		the quantitative or qualitative measure or indicator that communicates performance	
primal event		basic failure-mode	
revenue	$R_i$	the revenue associated with action or event $i$	4.1
system element		A specific component of the system. It identifies the actual sub-system that performs a particular activity.	
technical system		Software and hardware elements of the total system. Often referred to as the equipment, the plant, the physical assets, .  It explicitly excludes the maintenance system and the operations system which are other important elements of the total system as identified by Blanchard (1990)	

value		expected return from future activities, utilising the current means of operation	
value realised	$V_r$	is the value realised. This is equivalent to the cash flow, CF, during the period $(t-1, t)$ .	4.1
value realised, best	$V_r^*$	the estimated best possible change in value in the period $t-1, t$ , calculated ex-post	4.1
value lost	$V_l$	is the future value lost and represents the loss of future cash flow in real terms compared with the known best value change, during the period $(t-1, t)$ .	4.1

## Nomenclature

	Term	Section	Equation No.
$A^*$	action set, best	4.5 and 5	
$CF$	cash flow	4.1	Equation 4-7
$CIMS$	coefficient of influence of the maintenance system		
$C_{rel}FA_i$	the relative contribution of failure attribute $i$ to the overall organisation's success		
$C_{rel}FA_iOG_j$	the relative contribution of failure attribute $i$ to organisation goal $j$ .	5.1	Equation 5-3
$C_{rel}OG_j$	the relative contribution of organisation goal $j$ to the overall organisation's success		Equation 5-3
$C_{rel}SA_k$	coefficient of influence of a system activity	5.1	Equation 5-2
$C_{rel}SA_kFA_i$	the relative contribution of system activity $k$ to failure attribute $i$ .	5.1	
$E_i$	expense	4.1	Equation 4-7
$EA(A)$	the standard cost, or expense, of activities associated with action set A	6.3.2	
$EA_i(t_f)$	the standard cost, or expense, of activities required to transfer from one action set to another		
$EF(A)$	the standard cost, or expense, of an event associated with action set A, it is a probability function times an expected cash-flow	6.3.2	
$F_i$	event $i$	6.3.1	
$FA_i$	failure attribute		
$I$	input		
$I^*$	input, future best known	4.4.1	Equation 4-12

$\hat{O}$	output expected	4.4.1	Equation 4-12
$PSA_k$	the performance in each maintenance system activity	5.1	Equation 5-1
$R_i$	revenue	4.1	Equation 4-7
$SPSA_k$	standard of performance for system activity $k$ .	5.1	Equation 5-1
$t$	the point in time identifying the end of the period for which a performance measurement is required.		
$t_t$	expected transition or lead time associated with establishing a new action set.		
$VA$	The value associated with a particular action set, A. The element of this that is normally relevant to the maintenance system is represented by the expenses associated with preparatory or preventive activities associated with the relevant action set: ie $EA(A)$		
$VF$	the value of events, normally failures, associated with the relevant action set. The relevant element of this is the expenses incurred by the event: ie $EF(A)$		
$\Delta VI_i$	Change in value of the system given input action $I_i$	4.3, 4.4.1	Equation 4-10 Equation 4-12
$V_i$	value lost	4.1	Equation 4-1
$\Delta VO_i$	Change in value of the system given output action $O_i$	4.3, 4.4.1	Equation 4-9 Equation 4-11
$V_r^*$	the best known value that can be realised, during the period $(t-1, t)$ .	4.1	Equation 4-1
$V_t^*(t)$	the best available future value at time $t$ obtained by considering the opportunities existing at time $t$ (ie. as viewed at the time) identified by the subscript.	4.1	Equation 4-2
$V_{t-1}^*(t)$	the best available future value at time $t$ obtained by considering the opportunities existing at time $t-1$ .	4.1	Equation 4-2

$V_{t-1}^*(t-1)$	the best available future value at time $t-1$ obtained by considering the opportunities existing at time $t-1$ .	4.1	Equation 4-3
$V_r$	value realised	4.1	Equation 4-1
$V_r^*$	value realised, best	4.1	Equation 4-3