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## Evaluation and updating of slope reliability (with particular reference to optimization and probabilistic analysis)

Shu Zhang  
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# **EVALUATION AND UPDATING OF SLOPE RELIABILITY**

**(with particular reference to optimization and probabilistic analysis)**

A thesis submitted in fulfilment of the requirements for the award of  
the degree

**Doctor of Philosophy**

from

**THE UNIVERSITY OF WOLLONGONG**



by

**SHU ZHANG, B.E. (Hons)**

**DEPARTMENT OF CIVIL AND  
MINING ENGINEERING**

**January, 1990**

## DECLARATION

I, Shu Zhang, declare that the work described in this thesis has not been submitted for a degree to any university or such institution except where specifically indicated. During the research work, the following published or to be published papers are based on this thesis.

Zhang, S. and Chowdhury, R.N. (1989) Identification of critical slope failure surfaces with critical tension cracks. *30th U.S. Symposium on Rock Mechanics* , Morgantown, U.S.A., pp.185-192.

Zhang, S. and Chowdhury, R.N. (1989) Computing techniques for identifying critical failure surfaces in slopes. *Symposium on Computer Systems in the Australian Mining Industry* , Wollongong, Australia, pp.126-132.

Zhang, S. and Chowdhury, R.N. (1990) Optimization Calculations in Back-analysis of Slope Failures. (under review)

Chowdhury, R.N. and Zhang, S. (1988) Prediction of critical slip surfaces. *Proc. 5th Australia - New Zealand Conference on Geomechanics*, Sydney, Australia, pp.451-455.

Chowdhury, R.N. and Zhang, S. (1989) Bayesian updating for open pit slopes. *2nd large Open Pit Conference*, Latrobe Valley, Australia, pp.9-12.

Chowdhury, R.N. and Zhang, S. (1990) Some aspects of convergence related to limit equilibrium slope stability models. *Canadian Geotech. Journal*. (in press)

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## ABSTRACT

The research work detailed in this thesis is mainly aimed at two aspects, i.e. (a). assessments of slope reliability considering both deterministic and probabilistic approaches, and (b). updating of slope reliability by reducing the contribution of systematic errors. Currently, the determination of critical slip surfaces within a probabilistic framework and the influence of systematic errors on calculated reliability are areas requiring further development. This research addresses these problems and provides appropriate and systematic approaches in these areas.

Four commonly used deterministic models have been employed in this research. Difficulties in convergence associated with the original Janbu generalized method have been addressed by the proposed modified Janbu method. The convergence of the modified Janbu generalized method developed in this thesis is generally rapid. Based on previously published examples, it is found that the calculation results are in good agreement with those based on other reliable methods. An extra effort made by the writer on the modified method is the determination of the position and depth of tension cracks. The effects of tension cracks on slope stability have been considered during the search for the critical slip surfaces. Critical slip surfaces identified without considering the possible formation of tension cracks are found to be different from those identified when cracks are considered as part of the search process.

Reliability framework for the consideration of inherent variability of geotechnical parameters has been developed on the basis of the adopted deterministic methods and the proposed probabilistic approaches. Two solution techniques for calculating moments of functions of random variables have been employed (i.e. first order second moment approximation and Rosenblueth point estimate method). Three probabilistic models (i.e. lumped parameter model, local average process model

and multi-layer model) have been developed for assessing slope reliability. Spatial variation of shear strength parameters of earth materials has been incorporated in the last two models.

The simplex reflection technique has been used for the determination of critical slip surfaces based either on the factor of safety or on the reliability index. Application of the technique to the search for the critical slip circles has been enhanced. Its extension to the identification of the critical slip surfaces of arbitrary shape has been implemented. Based on the proposed optimization approach, this research work back-analysed some documented failure case histories by searching for the critical slip surfaces and comparing the calculated critical surfaces with the observed failure surfaces. In the back-analysis studies, it is found that only one pair of values of cohesion  $c$  and friction angle  $\phi$  can be regarded as reasonable. This is the combination which leads to coincidence or approximate coincidence between the observed failure surface and the optimized critical failure surface.

Based on the developed models and approaches, the case and example studies have allowed a comprehensive analysis of the contributions of different uncertainties to the location of critical slip surfaces and the evaluation of slope reliability. The difference between conventional and reliability based critical slip surfaces have been explored. It is found that a conventional critical slip surface is not always close to the corresponding reliability based critical slip surface. Under certain conditions the reliability based critical slip surfaces are much deeper than the conventional critical slip surfaces; in other situations conventionally determined critical surfaces are deeper.

In addition to the inherent variability of shear strength parameters and pore water pressure and the intrinsic random measurement error, there are two systematic errors involved in the evaluation of slope stability, i.e. statistical estimation error

and measurement bias. These systematic errors do not follow the averaging rule and can not be expected to cancel out. They introduce uncertainties in the estimation of the statistical parameters (mean and standard deviation) and thus the evaluation of slope stability. However, these errors are not the inherent characteristics of the natural world and can be reduced on the basis of additional information. In the case of slope stability study, this information could be the performance of a soil or rock slope. The performance (e.g. failure or success) could be considered associated with each stage of construction or operation. An approach based on Bayes' theorem has been developed in this thesis to update slope reliability. The performance of a slope is regarded as a full scale field test based on this approach. The approach provides new prospects for the application of probability theory to slope stability analysis. It can be employed for a major project at any stage of its construction or operation. The approach can result in an enhanced appreciation of risk and reliability and may lead to significant cost savings for a project. The application of this approach has been demonstrated for modeling, analysis and updating of slope stability for both failed and survived slopes.



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# PART ONE.

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## NOTATION

$\alpha$	Inclination of slice
$\alpha_s$	Reflection coefficient in simplex method
$b$	Width of slice
$\beta$	Slope inclination
$\beta_r$	Reliability index
$\beta_{rm}$	Minimum reliability index
$\beta_s$	Contraction coefficient in simplex method
$c$	Cohesion

$C$	Cohesion as a random variable
$c'$	Effective cohesion
$\bar{c}$ or $\overline{c}$	Sample mean of cohesion
$\overline{C}$ or $\overline{C}$	Sample mean of $c$ as a random variable
$COV[x, y]$	Covariance between $x$ and $y$
$D_t$	Depth of tension crack
$E$	Normal force on slice interface
$\Delta E$	Increment of $E$
$E(. )$	Mean value of $(. )$
$f_0$	Correction factor for Janbu's simplified method
$F$	Factor of safety
$F_m$	Minimum factor of safety
$\gamma$	Density of earth mass
$\gamma_s$	Expansion coefficient in simplex method
$H$	Total height of slope
$l$	Length of slice base
$L_s$	Length of the initial simplex side
$L_t$	Horizontal distance from slope crest to tension crack location
$\mu_x$	Population mean value of $x$
$N(\mu, \sigma)$	Normal distribution with mean $\mu$ and standard deviation $\sigma$
$P$	Normal force on slice base
$P_f$	Probability of failure
$P_m$	Maximum probability of failure
$P_{sv}$	Probability of survival
$P_{i\pm}$	Coefficients of point concentrations or weighting factors in Rosenblueth's method
$\theta_c$	Correlation distance parameter for $c$
$\theta_t$	Correlation distance parameter for $\tan\phi$
$r_u$	Pore water pressure ratio

$\rho[x, y]$	Correlation coefficient between x and y
s	Shear force on slice base
$s_c$	Sample standard deviation of c
$s_t$	Sample standard deviation of $\tan\phi$
$S_c$	Sample standard deviation of c as a random variable
$S_t$	Sample standard deviation of $\tan\phi$ as a random variable
SM	Safety margin
$\sigma_x$	Population standard deviation of x
T	Shear force on slice interface
$\Delta T$	Increment of T
t	$\tan\phi$ as a random variable
$\bar{t}$ or $\overline{t}$	Sample mean of $\tan\phi$ as a random variable
$t_i$	$\tan\phi_i$
$\tan\phi$	Internal friction
$\tan\phi'$	Effective internal friction
$\overline{\tan\phi}$	Sample mean of $\tan\phi$
u	Pore water pressure at the base of slice
$\text{Var}[.]$	Variance of [.]
$V_c$	Global coefficient of variation of cohesion
$V_{cp}$	Point coefficient of variation of cohesion
$V_t$	Global coefficient of variation of $\tan\phi$
$V_{tp}$	Point coefficient of variation of $\tan\phi$
$v_x$	Skewness coefficient of x
w	Weight of slice
$W_{Fm}$	Weight of sliding mass involved by critical slip surface of $F_m$
$W_{pm}$	Weight of sliding mass involved by critical slip surface of $\beta_{tm}$
$\zeta_c$	Reduction factor for $V_{cp}$
$\zeta_t$	Reduction factor for $V_{tp}$