2006

Impulse control systems for servomechanisms with nonlinear friction

Stephen van Duin

University of Wollongong, svanduin@uow.edu.au

IMPULSE CONTROL SYSTEMS FOR SERVOMECHANISMS WITH NONLINEAR FRICTION

A thesis submitted in fulfilment of the requirements for the award of the degree

DOCTOR OF PHILOSOPHY

From

UNIVERSITY OF WOLLONGONG

By

STEPHEN VAN DUIN, BE (mech.) Hons.

2006

SCHOOL OF ELECTRICAL, COMPUTER AND TELECOMMUNICATIONS ENGINEERING

THESIS CERTIFICATION

I, Stephen van Duin declare that this thesis submitted in fulfilment of the requirements for the award of Doctor of Philosophy in the School of Electrical, Computer and Telecommunications Engineering, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. The document has not been submitted for qualification at any other academic institution.

Stephen van Duin

30th August 2006

TABLE OF CONTENTS

LIST OF FIGURES .... **v**

LIST OF TABLES .... **xii**

LIST OF ABBREVIATIONS .... **xiii**

ABSTRACT .... **xiv**

ACKNOWLEDGEMENTS .... **xv**

CHAPTER 1: INTRODUCTION **1**

1.1 BACKGROUND 1
1.2 FRICTION MODELS 4
1.2.1 Classical Friction Models 5
1.2.2 Dynamic Friction Modelling 10
1.2.3 Modified Bristle Model – a new model 13
1.2.4 Position Dependent Friction 15
1.3 FRICTION COMPENSATION 16
1.3.1 Problem Avoidance 18
1.3.2 Non-Model Based Friction Compensation 20
1.3.3 Model Based Friction Compensation 23
1.4 IMPULSE CONTROLLERS 24
1.5 DISCUSSION AND SUMMARY OF THE LITERATURE REVIEW 27
1.6 PROBLEM DEFINITION 29
1.7 OUTLINE OF THE THESIS 32

CHAPTER 2: EXPERIMENTAL EQUIPMENT **35**

2.1 INTRODUCTION 36
2.2 FRICTION TEST BED 37
2.2.1 Friction Mechanism 38
2.2.2 Control Interface and Digital Signal Processing 39
2.2.3 Control Scheme 40
2.2.4 Electrical Circuit Response 42
2.2.5 Mechanical System Parameters 43
2.2.5.1 Static Friction and Pre-Sliding Constant 44
2.2.5.2 Estimation of Inertia, Viscous Friction and Coulomb Friction 47
2.3 HIRATA ROBOT 51
2.3.1 Electrical Circuit Response and Control Scheme 52
2.3.2 Mechanical System Parameters 52
2.4 CONCLUSIONS 53

CHAPTER 3: MODELLING AND SIMULATION.. 54
3.1 INTRODUCTION.. 54
3.2 SYSTEM MODELLING 54
3.2.1 Static Modelling of the Friction Test Bed.. 55
3.2.2 Dynamic Modelling of the Friction Test Bed 60
3.3 MODELLING OF THE IMPULSE CONTROLLER... 67
3.3.1 Loop Topology.. 67
3.3.2 Hybrid PID + Impulse Control for Improved Stability 68
3.4 CONCLUSIONS 70

CHAPTER 4: IMPULSE CONTROLLER DESIGN.. 72
4.1 INTRODUCTION.. 72
4.2 MATHEMATICAL ANALYSIS 74
4.3 MINIMUM PULSE WIDTH.. 78
4.4 VARIABLE PULSE WIDTH IMPULSE CONTROL 80
4.4.1 Controller Design 80
4.4.2 Regulated Pulse Height 82
4.4.3 Simulation of the Variable Pulse Width Controller......................... 83
4.5 VARIABLE PULSE HEIGHT IMPULSE CONTROL 87
4.5.1 Controller Design 87
4.5.2 Simulation of the Variable Pulse Height Controller 88
4.6 PULSE SHAPING FOR IMPROVED PRECISION... 92
4.6.1 Impulse Shape 1 92
4.6.2 Impulse Shape 2... 94
4.6.3 Impulse Shape 3... 95
4.6.4 Impulse Shape 4... 96
4.6.5 Comparison of Pulse Shapes 97
4.7 LIMIT CYCLE OFFSET FOR IMPROVED POSITIONING.................................. 100
4.7.1 Motivation.. 100
4.7.2 Limit Cycle Offset 102
4.7.3 Controller Design 103
4.7.4 Simulation of the Limit Cycle Offset Function 104
4.8 SUMMARY AND CONCLUSIONS.. 106

CHAPTER 5: PERFORMANCE ANALYSIS AND IMROVEMENT OF THE IMPULSE CONTROLLER USING THE FRICTION TEST BED 112
5.1 INTRODUCTION.. 112
5.2 IMPROVING THE IMPULSE CONTROLLER FOR A REAL SYSTEM 113
5.2.1 Velocity Reversal Compensation 114
5.2.2 Direction Dependent Friction Values 115

5.3 EXPERIMENTAL EVALUATION OF PULSE SHAPES 1 TO 4 117
5.3.1 Position Pointing Using Pulse Shapes 1 to 4 117
5.3.2 Low Speed Position Tracking Using Pulse Shapes 1 to 4 120
5.3.3 Impulse Height Blending for Pulse Shapes 3 and 4 125
5.3.4 Vibration Analysis 126
5.4 HIGH SPEED POSITION TRACKING 127
5.5 LIMIT CYCLE OFFSET 129
5.6 IMPULSE CONTROL VERSUS TANGENTIAL DITHER 131
5.7 CONCLUSIONS 134

CHAPTER 6: PERFORMANCE ANALYSIS USING THE HIRATA SCARA ROBOT 138
6.1 INTRODUCTION.. 138
6.2 EXPERIMENTAL EVALUATION OF PULSE SHAPES 1 TO 4 139
6.2.1 Position Pointing Using Pulse Shapes 1 to 4 139
6.2.2 Low Speed Position Tracking Using Pulse Shapes 1 to 4 140
6.3 LIMIT CYCLE OFFSET 143
6.4 CIRCULAR TRACE EXPERIMENTS USING A AND B AXES 145
6.5 HIGH SPEED POSITION TRACKING 150
6.6 CONCLUSIONS 152

CHAPTER 7: CONCLUSIONS AND FUTURE WORK 154
7.1 CONCLUSIONS 154
7.2 FUTURE WORK... 160

REFERENCES.. 163

APPENDIX A - LIST OF SYMBOLS.. 172

APPENDIX B - EXPERIMENTAL EQUIPMENT DATA 175
B.1 FRICTION TEST BED 175
B.1.1 Friction Test Bed Direct Drive Motor Specifications 175
B.1.2 Friction Test Bed Direct Drive Digital Amplifier Specifications ... 176
B.1.3 Coulomb Friction Estimation – second method 177
B.2 HIRATA ROBOT 178
B.2.1 Position Data and Calibration 178
B.2.2 Physical Layout of the Robot 180
B.2.3 Movement Specifications 180
B.2.4 Inverse Kinematics.. 181
B.2.5 Motor and Drive Ratings 182
B.2.6 Electrical Circuit Response and Control Scheme 183
B.2.7 Mechanical System Parameters.. 185
B.3 ACCELEROMETER.. 186
B.4 SIMPLIFICATION OF EQUATION 4.13 188

APPENDIX C - EXPERIMENTAL FRICTION TEST BED DESIGN DRAWINGS.. 193
C.1 FRICTION TEST BED – DRAWING. NO. 2003-001.. 194
C.2 FRICTION TEST BED – DRAWING. NO. 2003-002.. 195
C.3 FRICTION TEST BED – DRAWING. NO. 2003-003.. 196
C.4 FRICTION TEST BED – DRAWING. NO. 2003-004.. 197
C.5 FRICTION TEST BED – DRAWING. NO. 2003-005.. 198
C.6 FRICTION TEST BED – DRAWING. NO. 2003-006.. 199
C.7 FRICTION TEST BED – DRAWING. NO. 2003-006.. 200
C.8 FRICTION TEST BED – DRAWING. NO. 2003-006.. 201

LIST OF FIGURES

Figure 1.1 A simple set-up for stick-slip motion [1]. 2

Figure 1.2 A simulation of stick-slip motion [1]. 3

Figure 1.3 Memoryless friction models. The friction force is given by a memoryless function except possibly at zero velocity. Figure a) shows Coulomb friction and Figure b) Coulomb plus viscous friction. Stiction plus Coulomb friction are shown in Figure c), and Figure d) shows the Stribeck friction [1].. 6

Figure 1.4 Full fluid lubrication, regime IV of the Stribeck curve [5]. 7

Figure 1.5 The generalized Stribeck curve, showing friction as a dynamic function of velocity for low velocities [5]... 8

Figure 1.6 Asperity contact behaving like springs [15]...................................... 9

Figure 1.7 Spring force profile during stick-slip motion at two velocities; spring force decreases when velocity increases [15]..................................... 11

Figure 1.8 Static Friction (breakaway force) as a function of dwell time, schematic; with stick slip cycle shown. Dwell time is the time in static friction, shown as T_2 in Figure 1.7 [15]. 11

Figure 1.9 The friction interface between two surface is thought of as a contact between bristles. For simplicity the bristles on the lower part are shown as being rigid [1]. 14

Figure 1.10 Bristle model; Figure a) shows the deflection of a single bristle. Figure b) shows the resulting static friction model for a single instance in time... 14

Figure 1.11 Friction compensation using: 1) Problem avoidance, 2) Non-model based control, and 3) Model based control... 18

Figure 1.12 Direction and effect of Dither [5]... 21

Figure 1.13 Model Based Friction Compensation [5]. 23

Figure 1.14 Experimentally determined displacement as a function of pulse width and pulse height [13]. 25

Figure 2.1 Three dimensional drawing of the friction test bed............................. 37

Figure 2.2 Exploded view of the friction mechanism.. 38

Figure 2.3 Communication flow diagram for the friction test bed... 40

Figure 2.4 Combined electrical and mechanical system block diagram............... 42

Figure 2.5 Electrical circuit block diagram... 43

Figure 2.6 Reduced block diagram. 43

Figure 2.7 Pre-sliding displacement and breakaway friction for: (a) counter clock wise rotation; and (b) clock wise rotation................................. 45

Figure 2.8 Position dependent static friction for: (a) counter clockwise rotation; (b) clockwise rotation; and (c) magnified counter clockwise rotation. 47

Figure 2.9 a) Velocity response to step torque input for clockwise and counter clockwise motion, and b) resulting friction curve using Least Squares Method. 49

Figure 2.10 Mechanical system time constant. 50

Figure 2.11 Photograph of the Hirata SCARA robot AR-i350.............................. 51

Figure 3.1 Simplified Simulink® model of the friction test bed – open loop. 55

Figure 3.2 Simulation model and measured results for step torque inputs. 56

Figure 3.3 Modified model with a friction function which includes mean static friction, Stribeck effect, Coulomb and viscous friction for both clockwise and counter clockwise rotation. 57

Figure 3.4 Static friction model including stiction and Stribeck effect. a) A general friction model given by [1], and b) Simplified model for simulation purposes. 58

Figure 3.5 Tuning the PID controller of the friction test bed using 5 to 10% maximum overshoot. 59

Figure 3.6 Classic staircase stick-slip motion using PID control. 61

Figure 3.7 Block Simulink® model of the “new” Bristle dynamic model. 62

Figure 3.8 The sticking behaviour for the simplified standard model without damping ($\sigma_1 = 0$). 63

Figure 3.9 Olsson [1] simulation. 63

Figure 3.10 The sticking behaviour for the standard model with velocity dependent damping. The friction increases until the velocity is reached when it drops abruptly to zero... 64

Figure 3.11 Olsson [1] simulation. 65

Figure 3.12 Breakaway behaviour for the model with $\sigma_1 = \sqrt{10^5}$ 65

Figure 3.13 Olsson [1] simulation. 66

Figure 3.14 Olsson simulation of varying break-away force as a function of the rate of increase of the applied force: for the default parameters (+); v_s = 0.001 (*); and σ_0 = 10,000 [1]... 66

Figure 3.15 Diagram indicating the typical nested loop structure used in servo systems. 67

Figure 3.16 Friction test bed Simulink® simulation model with the impulse controller and no velocity loop.. 67

Figure 3.17 Block diagram of the friction test bed experimental system controller.. 68

Figure 3.18 Hybrid controller output.. 69

Figure 4.1 Simulated dynamics of a single torque impulse................................. 72

Figure 4.2 Expected pulse shapes 78

Figure 4.3 Experimentally measured displacement (friction test bed) for both positive and negative impulses using successive pulse widths 1.5 ms and 2 ms.. 79

Figure 4.4 Simplified variable width impulse controller simulation model......... 82

Figure 4.5 Simulation of a servomechanism position pointing task using; a) ‘PID only’, and b) ‘PID + impulse control’. The third plot of each set of graphs uses a very fine axis resolution for position........................ 84

Figure 4.6 Simulation of a servomechanism low speed position tracking task using; a) ‘PID only’, and b) ‘PID + impulse control’. 85

Figure 4.7 Simulation of a servomechanism low speed sinusoidal position tracking task using; a) ‘PID only’, and b) ‘PID + impulse control’..... 86

Figure 4.8 Graphical representation of the pulse height as a function of the error *e(k)*.. 88

Figure 4.9 Simplified variable height impulse controller simulation model........ 89

Figure 4.10 Simulation of a servomechanism position pointing task using; a) variable width PID + impulse, and b) variable height impulse + PID. 90

Figure 4.11 Simulation of a servomechanism low speed position tracking task using; a) variable width PID + impulse, and b) variable height PID + impulse. 90

Figure 4.12 Simulation of a servomechanism low speed position tracking task using; a) variable width PID + impulse, and b) variable height PID + impulse. 91

Figure 4.13 Simulated rectangular impulse $F \cdot t$ where fp = 125% of F_s : Denoted *Shape 1*. 93

Figure 4.14 Simulated rectangular impulse having a negative trailing pulse with amplitude 90% F_s: Denoted *Shape 2*. 94

Figure 4.15 Simulated stepped pulse with 2 ms startup force having an amplitude 125% F_s followed by secondary pulse 3 ms having an amplitude 130% F_C: Denoted *Shape 3*. 96

Figure 4.16 Simulated stepped pulse followed by a trailing negative pulse 90% F_s: Denoted *Shape 4*. 97

Figure 4.17 Experimental displacements for varying pulse widths for shapes 1, 2, 3, and 4. 98

Figure 4.18 Exploded view of Figure 4.17 for first two pulse widths showing the variation in minimum displacement (precision) for shapes 1, 2, 3, and 4. 98

Figure 4.19 Typical impulse showing net available torque (yellow area) after subtracting friction; a) standard rectangular pulse: Shape 1, and b) modified shape to offset friction: Shape 3. 99

Figure 4.20 Simulated displacements as a function of pulse width. 100

Figure 4.21 Simulation of the impulse controller limit cycling around the position reference set-point where the final torque output is a pulse with minimum width and mean peak to peak oscillation is d1. 101

Figure 4.22 Conceptual example of reducing the steady state error using 'Limit Cycle Offset' with the limit cycle shifted up by d2-d1 and the new error that is guaranteed to fall within the dead-zone.. 103

Figure 4.23 Simulation model of the modified impulse controller with 'Limit Cycle Offset'. 105

Figure 4.24 Simulation of the limit cycle offset function used with the PID + impulse controller and Shape 4. 105

Figure 5.1 Modified Simulink® model to include xPC analog out and DSP blocks (red).. 112

Figure 5.2 Integral "windup" observed at zero velocity and velocity reversal when using a) PID only control, and b) PID with integral reset. 114

Figure 5.3 Modified impulse controller Simulink® model with direction

dependent friction parameters, non linear pulse width gain, regulated pulse height and limit cycle offset function. 116

Figure 5.4 A sample step input and position response using pulse Shape 1. Mean final oscillating displacement μ_d = 1.440e-4 radians for a sample of 10 repeated experiments. 117

Figure 5.5 A sample step input and position response using pulse Shape 2. Mean final oscillating displacement μ_d =1.001e-4 radians for a sample of 10 repeated experiments. 118

Figure 5.6 A sample step input and position response using pulse Shape 3. Mean final oscillating displacement μ_d = 0.957e-4 radians for a sample of 10 repeated experiments. 119

Figure 5.7 A sample step input and position response using pulse Shape 4. Mean final oscillating displacement μ_d = 0.901e-4 radians for a sample of 10 repeated experiments. 119

Figure 5.8 Magnified linear position ramp response showing an experimental comparison between the impulse controller with pulses shape 1 to 4.. 120

Figure 5.9 Integral Absolute Error (IAE) for impulse shapes 1, 2 3 & 4 for a low speed position tracking task... 121

Figure 5.10 Experimental speed regulated sinusoidal position tracking using PID and PID + impulse controllers... 122

Figure 5.11 Magnified velocity reversal showing an experimental comparison of precision between the impulse controllers with Shapes 1, 2, 3 & 4..... 122

Figure 5.12 Separating Shape 1 and Shape 4 from Figure 5.13. 123

Figure 5.13 Integral Absolute Error (IAE) for impulse shapes 1, 2 3 & 4 for a sinusoidal position tracking task. 124

Figure 5.14 Modified pulse shape as a function of increasing error e(k). 125

Figure 5.15 Photograph of the friction test bed with the attached Kistler type 5134 accelerometer for the measure of system vibration.................... 126

Figure 5.16 Spectral analysis of system vibration using FFT for pulse shapes 1 to 4. 127

Figure 5.17 Tracking response for the friction test bed using PID and PID + impulse controllers for varying position ramps (0.02 rad/s to 0.35 rad/s)... 128

Figure 5.18 Mean value of the absolute error for each of the position tracking ramps shown in Figure 5.17 for the period 7 – 10 seconds................. 128

Figure 5.19 Steady state limit cycle for the PID + impulse controller using pulse shape 3. The mean peak to peak displacement μ_d is the non-elastic

	part of limit cycle..	130
Figure 5.20	Using the 'Limit Cycle Offset' function to reduce the final steady state error.	131
Figure 5.21	PID control with velocity reversal compensation and gains Kp=70, Ki=130 and Kd=1.2.	132
Figure 5.22	PID + tangential dither with amplitude A_o=4.2 Nm and frequency ω_o=250Hz.	133
Figure 5.23	PID + impulse control using pulse shape 4.	133
Figure 5.24	Spectral analysis of system vibration using FFT for PID, PID + impulse and PID + dither control.	134
Figure 6.1	A sample step input and position response using pulse shapes 1 to 4. Mean final oscillating displacement μ_d measured in radians for a sample of 10 repeated experiments..	140
Figure 6.2	Magnified linear position ramp 0.0015rad/s showing an experimental comparison between the impulse controller with Shape 1 and Shape 4.	141
Figure 6.3	Integral Absolute Error (IAE) for impulse shapes 1, 2 3 & 4 for a low speed position tracking task...	142
Figure 6.4	Sinusoidal trace and comparison of Shape 1 and Shape 4..	143
Figure 6.5	Integral Absolute Error (IAE) for impulse shapes 1, 2 3 & 4 for a sinusoidal position tracking task.	143
Figure 6.6	Steady state limit cycle for the PID + impulse controller using pulse shape 1. The mean peak to peak displacement μ_d is the non-elastic part of limit cycle..	144
Figure 6.7	Using the 'Limit Cycle Offset' function to reduce the final steady state error using pulse shape 3.	145
Figure 6.8	Reference control signals for the A and B axes (ω=31.4 mrad/s).	146
Figure 6.9	Circle trace with a 100 mm diameter using the PID only controller....	147
Figure 6.10	Circle trace with a 100 mm diameter using the PID + impulse controller with Shape 4...	147
Figure 6.11	Circle tracking errors for PID and PID + impulse controllers.............	148
Figure 6.12	Circle tracking errors for PID + impulse controllers using pulse shape 1 and shape 4..	149

Figure 6.13 Tracking response for the A axis using PID and PID +impulse controllers for varying position ramps (0.02 rad/s to 0.35 rad/s)........ 150

Figure 6.14 Mean value of the absolute error for each of the position tracking ramps shown in Figure 6.13 for the period 7 – 10 seconds................... 151

Figure B.1 (a) Continuous velocity when torque is reduced after breakaway, and (b) Coulomb friction prevents continuous velocity when torque is reduced after breakaway.. 178

Figure B.2 Hirata robot physical layout and work volume [1].............................. 179

Figure B.3 Hirata robot workspace showing the rotational angles for the A and B axes used to calculate the inverse kinematics................................. 182

Figure B.4 Electrical circuit block diagram for the Hirata robot........................... 184

Figure B.5 a) Velocity response to step torque input for A-axis Hirata robot, and b) resulting friction curve using Least Squares Method...................... 184

Figure B.6 Mechanical system time constant.. 185

Figure B.7 Simulink model for the experimental friction test bed......................... 190

Figure B.8 Subsystem friction force... 191

Figure B.9 Subsystem dZ/dt.. 191

Figure B.8 Subsystem g(v).. 192

Figure B.9 Subsystem Sigma 1.. 192

LIST OF TABLES

Table 1.1	Comparison of impulse controller strategy with known experimentation.	30
Table 2.1	Electrical and mechanical time constants for the friction test bed.	43
Table 2.2	Friction test bed mechanical system parameters.	50
Table 2.3	Electrical and mechanical time constants for the Hirata robot.	52
Table 2.4	Hirata robot mechanical system parameters.	52
Table 3.1	Default parameter values for the simplified standard model [1].	63
Table 3.2	Default parameters for the standard model [1].	64
Table 4.1	Simulation parameters for the "new' friction model.	93
Table 5.1	Measured mean peak to peak displacement of the steady state limit cycle for shapes 1 to 4	118
Table 6.1	PID gains for the Hirata robot's *A* and *B* axes.	139
Table 6.2	Measured mean peak to peak displacement of the steady state limit cycle for shapes 1 to 4.	139
Table B.1	Direct Drive motor specifications.	175
Table B.2	Direct drive digital amplifier specifications.	176
Table B.3	Hirata robot encoder resolution.	180
Table B.4	Hirata robot movement specifications.	181
Table B.5	Hirata robot motor and drive ratings.	183
Table B.6	Accelerometer Type Kistler 8694M1 specifications.	186
Table B.7	Accelerometer amplifier Type Kistler 5134 specifications..	187

LIST OF ABBREVIATIONS

DC – Direct Current

DSP – Digital Signal Processing

emf – Electro-Magnetic Force

FFT – Fast Fourier Transform

IAE – Integral of the Absolute Error

PC - Personal Computer

PCD – Pitch Circle Diameter

PD – Proportional Derivative

PDTV – Position Dependent Torque Variation

PID – Proportional Integral Derivative

PWMH – Pulse Width Modulated Sampled Data Hold

SCARA – Selective Compliant Assembly Robot Arm

Sgn – Sign function

Stiction – Static Friction

TCP – Tool Centre Point

ABSTRACT

At low velocities, friction is highly non linear and difficult to control. In practical mechanisms, friction may also be position dependent and highly variable. This can lead to tracking errors, limit cycles, and a phenomenon referred to as 'stick-slip', when a periodic cycle of alternating motion and rest, limits the mechanism's velocity and position accuracy.

Impulse control is a friction compensator that does not require an accurate friction model. It achieves precise motion of a servomechanism by applying small impacts which overcome static friction with a controlled breakaway. The size of the impact and its duration determine how much the mechanism moves. By controlling the pulse, the positional accuracy of the mechanism can be improved.

The work presented in this thesis results in new impulse controllers which: 1) improve the precision of a servomechanism without mechanical modification for the tasks of position pointing and low speed position tracking; 2) eliminate phenomena such as stick-slip, quadrant glitch, and limit cycling; 3) minimise system vibration and low speed position tracking ripple.

The new controllers are tested by simulations, and experimentally verified on two different mechanical systems. One of these is a test bed built specifically for friction control experiments, and the other is a SCARA robot manipulator.

ACKNOWLEDGEMENTS

This thesis was carried out as part of an Engineering Manufacturing funded project within the School of Electrical, Computer and Telecommunications Engineering at the University of Wollongong. I would like to thank these organisations for their support which made this thesis possible.

I would like to give a special thanks to my supervisor Professor Chris Cook and my co-supervisors Dr. Zheng Li and Dr. Gursel Alici for their assistance and guidance throughout my candidature. I would also like to thank Assoc. Prof. Friso de Boer who was initially my supervisor and was a great help and mentor in the initial stages of my thesis.

I would like to thank my fellow researchers, Dr. Marta Fernandes, Mr. Laurence Bate, Mr. Jeff Moscrop and Mr. Simon Webb as colleagues working together in the same area in which ideas could be discussed clearly, openly and freely. A special thank you is reserved for Dr. John Simpson, who not only helped me with all engineering aspects of my thesis, but as a friend always took the time to answer my questions and offer support.

I would like to thank all the general staff of the school who helped me reach my goal. A special thanks to Mr. Brian Webb for the machining of the Friction Test Bed.

Finally, a very warm and special thanks to my dear wife Leesa, and daughters Tyla and Skye, who never once complained about the inconvenience and self indulgent undertaking of my Ph.D. thesis over so many years. Their sacrifice was undoubtedly greater than mine.