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Analytical solutions for modeling soft soil consolidation by vertical drains

Rohan Walker
University of Wollongong

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ANALYTICAL SOLUTIONS FOR MODELING SOFT SOIL CONSOLIDATION BY VERTICAL DRAINS

A thesis submitted in fulfillment of the
requirements for the award of the degree

Doctor of Philosophy

from

University of Wollongong, NSW Australia

by

Rohan Walker, B. Eng (Hons. I)

School of Civil, Mining and Environmental Engineering

2006

CERTIFICATION

I, Rohan Walker, declare that this thesis, submitted in fulfillment of the requirements for the award of Doctor of Philosophy, in the Department of Civil Engineering, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. The document has not been submitted for qualifications at any other academic institution.

Rohan Walker

2nd June 2006

ABSTRACT

Vertical drains increase the rate of soil consolidation by providing a short horizontal drainage path for pore water flow, and are used worldwide in many soft soil improvement projects. This thesis develops three new contributions to the solution of consolidation problems: (i) a more realistic representation of the smear zone where soil properties vary gradually with radial distance from the vertical drain; (ii) a nonlinear radial consolidation model incorporating void ratio dependant soil properties and non-Darcian flow; and (iii) a solution to multi-layered consolidation problems with vertical and horizontal drainage using the spectral method. Each model is verified against existing analytical solutions and laboratory experiments conducted at the University of Wollongong, NSW Australia. The nonlinear radial consolidation model and the spectral method are verified against two trial embankments involving surcharge and vacuum loading at the Second Bangkok International Airport, Thailand. The versatility of the spectral method model is further demonstrated by analysing ground subsidence associated with ground water pumping in the Saga Plain, Japan.

New expressions for the smear zone μ parameter, based on a linear and parabolic variation of soil properties in the radial direction, give a more realistic representation of the extent of smear and suggest that smear zones may overlap. Overlapping linear smear zones provide some explanation for the phenomena of a minimum drain spacing, below which no increase in the rate of consolidation is achieved. It appears this minimum influence radius is 0.6 times the size of the linear smear zone. The new smear zone parameters may be used with consolidation models (ii) and (iii), as mentioned above.

The analytical solution to nonlinear radial consolidation is valid for both Darcian and non-Darcian flow and can capture the behaviour of both overconsolidated and normally consolidated soils. For nonlinear material properties, consolidation may be faster or slower when compared to the cases with constant material properties. The difference depends on the compressibility/permeability ratios (C_v/C_k and C_r/C_k), the preconsolidation pressure and the stress increase. If $C_v/C_k < 1$ or $C_r/C_k < 1$ then the coefficient of consolidation increases as excess pore pressures dissipate and consolidation is faster.

The multi-layered consolidation model includes both vertical and radial drainage where permeability, compressibility and vertical drain parameters vary linearly with depth. The ability to include surcharge and vacuum loads that vary with depth and time allows for a large variety of consolidation problems to be analysed. The powerful model can also predict consolidation behaviour before and after vertical drains are installed and has potential for nonlinear consolidation analysis.

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PUBLISHED WORK

The following publications are related to this PhD thesis:

Walker, R. and Indraratna, B. (2006). “Vertical drain consolidation with parabolic distribution of permeability in smear zone”. *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, in press July, 2006.

In preparation:

Walker, R. and Indraratna, B. “Vertical drain consolidation with non-Darcian flow and void ratio dependent compressibility and permeability”.

Walker, R. and Indraratna, B. “Consolidation of stratified soil with vertical and horizontal drainage under surcharge and vacuum loading”.

Walker, R. and Indraratna, B. “Vertical drain consolidation with overlapping smear zones”.

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LIST OF SYMBOLS

a	PVD width
\mathbf{A}	vector of time dependant coefficients
A	smear zone permeability parameter; function; time dependant coefficient; cyclic load amplitude
A_η	smear zone compressibility parameter
b	PVD thickness
B	smear zone permeability parameter
B_η	smear zone compressibility parameter
C	smear zone permeability parameter; integration constant
\mathbf{c}	vector of constants
\tilde{c}_h	horizontal coefficient of consolidation for non-Darcian flow
\tilde{c}_{h0}	initial horizontal coefficient of consolidation for non-Darcian flow
C_c	compression index
c_h	horizontal coefficient of consolidation
c_{h0}	initial value of horizontal consolidation coefficient
c_{h1}, c_{h2}	horizontal consolidation coefficients for double layered ground
c_{he}	effective consolidation coefficient
c_{hf}	final value of horizontal consolidation coefficient
C_k	permeability change index
C_r	recompression index
CS	function
c_v	vertical coefficient of consolidation
\bar{c}_v	reference value of vertical consolidation coefficient
c_{v1}, c_{v2}	vertical consolidation coefficients for double layered ground
D_{85}	diameter of clay particles corresponding to 85% passing
d_e	diameter of influence area
dQ_1, dQ_2	infinitesimal volume flows
dT_h	reference horizontal time factor divided by time
dT_v	reference vertical time factor divided by time
d_w	drain diameter
e	void ratio
\mathbf{E}	matrix of eigenvalue exponential terms
e_0	initial void ratio
e_{00}	initial void ratio at soil surface
e_{0n}	void ratio at the depth below which soil is normally consolidated
e_f	final void ratio
e_r	error

F	function
f	source/sink term; function; cyclic load natural frequency
F_c	discharge capacity reduction factor due to bending
F_{fc}	discharge capacity reduction factor due to filtration and clogging
F_t	discharge capacity reduction factor due to lateral pressure
g	function
G_s	specific weight of soil solids
H	height of soil; drainage length
H_0	initial height of soil
H_1, H_2	layer depths for double layered ground
h_c	height of clay layer
h_s	height of sand layer
i	hydraulic gradient; integer
i_l	critical hydraulic gradient for non-Darcian flow
j	integer
J_0, Y_0	bessel functions
\bar{k}	reference value of permeability
\tilde{k}	non-Darcian coefficient of consolidation
\tilde{k}_h	undisturbed non-Darcian horizontal permeability
\tilde{k}_s	smear zone non-Darcian horizontal permeability
k_0	permeability at soil/drain interface; initial coefficient of permeability
k_1	ratio of vacuum pressure at drain bottom to drain top
k_{clay}	clay permeability
k_{filter}	PVD filter permeability
k_h	undisturbed horizontal coefficient of permeability
\bar{k}_h	reference value of horizontal permeability
k_{h1}, k_{h2}	horizontal permeability for double layered ground
k_s	smear zone permeability
k_{sand}	sand permeability
k_{soil}	soil permeability
$k_{\text{undisturbed}}$	coefficient of permeability for undisturbed soil
k_v	vertical coefficient of permeability
\bar{k}_v	reference value of vertical permeability
k_{v1}, k_{v2}	vertical permeability for double layered ground
k_{vB}	vertical permeability at bottom of soil layer
K_{ve}	equivalent vertical coefficient of permeability
k_{vT}	vertical permeability at top of soil layer
k_w	drain permeability
l	depth of vertical drain; integer

L	linear operator
$\#l$	number of soil layers
m	integer; overconsolidated shear strength index
M	summation term e.g. $M = (2m + 1)\pi/2$ in Terzaghi theory
M^-	function
M^+	function
m_v	coefficient of volume compressibility
\bar{m}_v	reference value of volume compressibility
m_{v0}	initial value of volume compressibility; volume compressibility at soil/drain interface
m_{va}	average value of volume compressibility
m_{vB}	volume compressibility at bottom of soil layer
m_{vS}	smear zone volume compressibility
m_{vT}	volume compressibility at top of soil layer
m_{vX}	volume compressibility where interacting smear zones begin to overlap
n	ratio of influence radius to drain radius; non-Darcian flow index
N	ratio of influence radius to drain radius; integer
n'	ratio of influence radius to equivalent mandrel radius based on the mandrel perimeter
O_{95}	95% of filter openings are smaller than this opening
OCR	overconsolidation ratio
P	cyclic load wave period
p_0	vacuum pressure at soil surface
P_{av}	averaging factor to account for changing coefficient of consolidation
PTIB	pervious top impervious bottom boundary condition
PTPB	pervious top pervious bottom boundary condition
q_w	drain discharge capacity
$q_{w(\text{required})}$	required discharge capacity
$q_{w(\text{specified})}$	discharge capacity specified to manufacture
r	radial coordinate
r_e	radius of influence area
r_m	equivalent radius of mandrel
r_s	radius of smear zone
s	integration variable
\bar{s}	mean square distance of the flownet draining a circular area to a rectangular drain.
s_{constant}	smear zone size calculated with constant permeability smear zone
$s_{\text{equivalent}}$	smear zone size calculated with linear or parabolic permeability producing equivalent consolidation to smear zone calculated with constant permeability
SN	function
S_p	drain spacing interval

S_u	undrained shear strength
s_X	ratio of smear zone interaction radius to drain radius
t	time
\tilde{T}	modified time factor
\tilde{T}_{Darcy}	Darcian time factor
\tilde{T}_m	modified time factor at application of m^{th} instantaneous load
\tilde{T}_p	modified time factor at preconsolidation pressure
t_{90}	time required to reach 90% consolidation
T_c	construction time factor
t_f	end time for integration
T_h	time factor for horizontal consolidation
T_{h0}	horizontal time factor calculated from initial parameters
t_{Ω}	drain installation time; piecewise nonlinear time marker
t_s	starting time for integration
T_v	time factor for vertical consolidation
U	degree of consolidation
u	pore water pressure
\bar{u}	average excess pore pressure
$\bar{\bar{u}}$	depth averaged pore pressure
\bar{u}_0	initial excess pore pressure
$\Delta\bar{u}$	change in pore pressure
\bar{u}_{δ}	fundamental pore pressure solution
U_h	average degree of consolidation in the horizontal direction
U_{hs}	degree of consolidation calculated with settlement data
u_m^-	pore pressure immediately before application of m^{th} instantaneous load
u_m^+	pore pressure immediately after application of m^{th} instantaneous load
u_s	smear zone pore pressure
U_z	average degree of consolidation in the vertical direction
\mathbf{v}	matrix of eigenvectors
w	pore pressure in the drain; vacuum pressure
W	normalised pore pressure
\mathbf{w}	matrix dependant on vacuum loading terms
W_p	normalised pore pressure at preconsolidation pressure
\mathbf{x}	vector of unknown coefficients
x	integration variable
y	transformed integration variable
z	depth coordinate
Z	normalised depth
z_n	depth below which all soil is normally consolidated

Greek symbols

α	non-Darcian radial consolidation parameter; function parameter; soil property parameter
λ	non-Darcian radial consolidation parameter; slope of Cam-clay consolidation line; spectral method eigenvalue
β	non-Darcian radial consolidation parameter; slope of Asaoka plot; function variable
χ	vector of constants
$\#\chi$	number of series term used in previous time step
Δ	change operator
δ	dirac delta function
ε	strain
$\partial\varepsilon/\partial t$	volumetric strain rate
η	lumped vertical drain parameter; ratio of undisturbed volume compressibility to drain/soil interface volume compressibility
$\bar{\eta}$	reference value of lumped drain parameter
η_X	ratio of interacting smear zone volume compressibility to drain/soil interface compressibility
Γ	matrix dependant on compressibility and geometry parameters
γ_w	unit weight of water
κ	ratio of undisturbed permeability to permeability at the drain/soil interface; slope of Cam-clay swelling line
κ_X	ratio of interacting smear zone permeability to drain/soil interface permeability
Λ	function
μ	smear zone parameter for Darcian flow
μ^*	composite smear zone parameter
μ_{m_v}	smear zone compressibility parameter
μ_w	well resistance parameter
μ_X	overlapping smear zone parameter
Ω	matrix dependant on non-zero initial condition
ω	cyclic load angular frequency
ϕ	basis function
Φ	vector of basis functions
$\bar{\phi}$	integrated basis function
$\bar{\Phi}$	vector of integrated basis functions
φ	cyclic load phase
Ψ	matrix dependant on permeability, and geometry parameters
ρ	settlement
ρ_c	consolidation or primary settlement
ρ_i	immediate or distortion settlement
ρ_∞	final settlement
ρ_l	settlement caused by lateral displacement
ρ_s	secondary compression
ρ_t	total settlement

σ	total stress
σ	matrix dependant on surcharge loading terms
$\bar{\sigma}$	average total stress
$\bar{\sigma}'$	average effective stress
σ'	effective stress
σ'_0	initial effective stress
σ'_{00}	initial effective stress at soil surface
σ'_{0n}	this effective stress marks the depth below which soil is normally consolidated
σ'_{0z}	initial effective stress at depth z
σ'_{\max}	evolving maximum effective stress
σ'_p	preconsolidation stress
σ'_{v0}	vertical effective stress
$\Delta\sigma$	change in total stress
τ	time
θ	function parameter
Θ	matrix dependant on cyclic loading terms
v	velocity of flow
v_r	velocity of flow in radial direction
v_v	velocity of flow in the vertical direction
Ξ	function
ζ	depth
