1-9-1999

When the necessary conditions are not sufficient: sequences with zero autocorrelation function

J. Horton
University of Wollongong, jeffh@uow.edu.au

Jennifer Seberry
University of Wollongong, jennie@uow.edu.au

When the necessary conditions are not sufficient: sequences with zero autocorrelation function

Abstract

Recently K. T. Arasu (personal communication) and Yoseph Strassler, in his PhD thesis, The Classification of Circulant Weighing Matrices of Weight 9, Bar-Ilan University, Ramat-Gan, 1997, have intensively studied circulant weighing matrices, or single sequences, with weight 9. They show many cases are non-existent. Here we give details of a search for two sequences with zero periodic autocorrelation and types (1,9), (1,16) and (4,9). We find some new cases but also many cases where the known necessary conditions are not sufficient. We instance a number of occasions when the known necessary conditions are not sufficient for the existence of weighing matrices and orthogonal de-signs constructed using sequences with zero autocorrelation function leading to intriguing new questions.

Keywords

Weighing matrices, orthogonal design, sequences, autocorrelation, construction, algorithm, AMS Subject Classification: Primary 05B20, Secondary 62K05, 62K10

Disciplines

Physical Sciences and Mathematics

Publication Details

This article was originally published as Horton, J and Seberry, J, When the necessary conditions are not sufficient: sequences with zero autocorrelation function, Bulletin ICA, 27, 1999, 51-61. ISSN 1183-1278.

When the necessary conditions are not sufficient: sequences with zero autocorrelation function

Jeffrey Horton and Jennifer Seberry
School of IT and Computer Science
University of Wollongong
NSW 2522
Australia

Abstract

Recently K. T. Arasu (personal communication) and Yoseph Strassler, in his PhD thesis, *The Classification of Circulant Weighing Matrices of Weight 9*, Bar-Ilan University, Ramat-Gan, 1997, have intensively studied circulant weighing matrices, or single sequences, with weight 9. They show many cases are non-existent. Here we give details of a search for two sequences with zero periodic autocorrelation and types (1,9), (1,16) and (4,9). We find some new cases but also many cases where the known necessary conditions are not sufficient.

We instance a number of occasions when the known necessary conditions are not sufficient for the existence of weighing matrices and orthogonal designs constructed using sequences with zero autocorrelation function leading to intriguing new questions.

Key words and phrases: Weighing matrices, orthogonal design, sequences, autocorrelation, construction, algorithm.
AMS Subject Classification: Primary 05B20, Secondary 62K05, 62K10

1 Introduction

A weighing matrix $W = W(n,k)$ is a square matrix with entries 0, ± 1 having k non-zero entries per row and column and inner product of distinct rows zero. Hence W satisfies $WW^T = kI_n$, and W is equivalent to an orthogonal design $OD(n;k)$ (see Geramita and Seberry [4] for more details). The number k is called the *weight* of W.

Weighing matrices were first studied by Hotelling [5] because of their use in weighing experiments.

Given a set of ℓ sequences, the sequences $A_j = \{a_{j1}, a_{j2}, ..., a_{jn}\}$, $j = 1, \ldots, \ell$, of length n the *periodic autocorrelation function, PAF,* $P_A(s)$, is defined, reducing $i+s$ modulo n, as

$$P_A(s) = \sum_{j=1}^{\ell} \sum_{i=1}^{n} a_{ji} a_{j,i+s}, \qquad s = 0, 1, ..., n-1. \tag{1}$$

Two sequences, of length n, will be said to be of type (s,t) if the sequences are composed of two variables, say a and b, and a and $-a$ occur a total of s times

and b and $-b$ occur a total of t times. Such sequences may be used as the first rows of two circulant matrices to obtain an orthogonal design, $OD(2n; s, t)$. The sequences are said to be of type $(0, \pm 1)$ and weight w if they have a total of w non-zero elements and may be used as the first rows of two circulant matrices to obtain a $W(2n, w)$.

We note, using the languages of sequences, the following conditions from [4]. In addition we note there is no square orthogonal matrix of size 18 which will have one element once in each row and column and the other 16 times.

Theorem 1 (Necessary Conditions) *The necessary conditions for the existence of a single sequence of length n, n odd, type $(0, \pm 1)$ and weight k are that k is a perfect square and $(n-k)^2 - (n-k) \geq n-1$.*

The following conditions are necessary for the existence of two sequences type $s, t)$ and length n with n odd:

i) s, t and $s+t$ must each be the sum of three squares,

ii) $s + t \leq 2n - 1$,

iii) there exists a 2×2 integer matrix P (called the sum-fill matrix), with all entries of modulus $\leq n$ which satisfies $PP^T = diag(s, t)$.

The necessary conditions for the existence of four sequences sequences of types (s_1, s_2), (s_1, s_2, s_3) or (s_1, s_2, s_3, s_4) are given in [4, 8].

In this note we concentrate on sequences of types (1,9), (1,16) and (4,9) which satisfy all known necessary conditions. We undertook a computer search which was unable to find sequences of type (4,9) except for lengths $19m$ and $21m$, $m \geq 1$. For types (1,9) and (1,16) we were able to find several new sequences. This leads to the intriguing question of what other necessary conditions must hold for these sequences to exist.

2 Preliminary Results

We make extensive use of the book of Geramita and Seberry [4]. For convenience, and since [4] is out of print, we restate some of the results in the language of sequences for reference. The next useful lemma is well known.

Lemma 1 *If there exists a sequence of length n, type $(0, \pm 1)$ and weight k then there exists a sequence of length pn, type $(0, \pm 1)$ and weight k for all $p \geq 1$.*

Theorem 2 (Circulant Kronecker Product) *If there exists a sequence of length n, type $(0, \pm 1)$ and weight k, and another sequence of length m, type $(0, \pm 1)$ and weight ℓ where $\gcd(n, m) = 1$ then there exists a sequence of length mn, type $(0, \pm 1)$ and weight $k\ell$.*

Theorem 3 *If there exist two sequences of length n and type (s, t) then there exist two sequences of length pn and type (s, t) for all integers $p \geq 1$.*

Theorem 4 *Suppose q is a prime power and $q^2 + q + 1$ is a prime. Then there exists a single sequence of length $q^2 + q + 1$, type $(0, \pm 1)$ and weight q^2. And there exist two sequences of length $q^2 + q + 1$, type $(0, \pm 1)$ and weight $(q+1)^2$.*

If, in addition, $q^2 + q + 1 \equiv 3 (mod\ 4)$ is prime, then there exist two sequences of length $q^2 + q + 1$ and type $(1, (q+1)^2)$.

Remark 1 We note that for $q = 2$ this means there are two sequences of length 7 and type (1,9); and for $q = 4$ there are two sequences of length 21 and type (1,16).

The first part of Theorem 4 allows us to construct a sequence of length 7, type $(0, \pm 1)$ and weight 4; a sequence of length 13, type $(0, \pm 1)$ and weight 9; and a sequence of length 21, type $(0, \pm 1)$ and weight 16. We use Lemma 1 to make a sequence of length $91m$, type $(0, \pm 1)$ and weight 4 and a sequence of length $91m$, type $(0, \pm 1)$ and weight 9. This ensures there are two sequences of length $91m$ and type (4,9) for all $m \geq 1$.

Two sequences of length 11 and type (1,16) are known from [4].

3 Weighing matrices of weight 9

Strassler [14] has completed the classification of single sequences of length n, type $(0, \pm 1)$ and weight 9.

When we consider two sequences of length n, type $(0, \pm 1)$ and weight 9 we find they are known for $n = 5, 7, 11, 13, 17, 19, 23$, and multiples of these numbers by p, $p \geq 1$.

In a number of cases the only weighing matrix known of weight 9 is described by writing out its elements as in the $W(15, 9)$, $W(17, 9)$ and $W(18, 9)$ (see Koukouvinos and Seberry [12] for details).

4 Sequences with Zero Autocorrelation

The sequences of lengths n and types (1,9) and (4,9) have proved even more elusive. The only known sequences for (4,9) are of length $91m$ (see Remark 1) and $19m$.

For order 14 the following matrix, quoted from [4, p331], has orthogonal rows and is of type (4,9) however no such other matrix is known for orders $m \equiv 2 (\mathrm{mod}\ 4)$, $m \leq 28$.

```
 0  x   y  y  y   x  x  x   y  y  y   y  y  y
 x  0   x  x  x  -y -y -y   y  y  y  -y -y -y

 y  x   0 -y -y   x -x -x   y  y -y   y  y -y
 y  x  -y  0 -y  -x  x -x  -y  y  y  -y  y  y
 y  x  -y -y  0  -x -x  x   y -y  y   y -y  y

 x -y   x -x -x   0  y  y  -y  y  y   y -y -y
 x -y  -x  x -x   y  0  y   y -y  y  -y  y -y
 x -y  -x -x  x   y  y  0   y  y -y   y  y -y

 y  y   y -y  y  -y  y  y   0 -x -x  -y  x -x
 y  y   y  y -y   y -y  y  -x  0 -x  -x -y  x
 y  y  -y  y  y   y  y -y  -x -x  0   x -x -y

 y -y   y -y  y   y -y -y  -y -x  x   0  x  x
 y -y   y  y -y  -y  y -y   x -y -x   x  0  x
 y -y  -y  y  y  -y -y  y  -x  x -y   x  x  0
```

This note details our search for sequences of these types.

4.1 Search Method

It is known that sequences $A = \{a_1, \ldots, a_n\}$ and $B = \{b_1, \ldots, b_n\}$ of length n and type (s,t), a variable a and $-a$ occurring s times, and variable b and $-b$ occurring t times, must satisfy the condition:

$$\left(\sum_{i=1}^{n} a_i\right)^2 + \left(\sum_{i=1}^{n} b_i\right)^2 = sa^2 + tb^2 \tag{2}$$

We use Equation 2 to determine in what quantities each of variables a, $-a$ and b, $-b$ can occur in each sequence: this helps limit the size of the space which must be searched for sequences with zero autocorrelation function.

Equation 2 is equivalent to:

$$\begin{aligned} sa^2 + tb^2 &= (N_{A,a}a + N_{A,b}b)^2 + (N_{B,a}a + N_{B,b}b)^2 \\ &= (N_{A,a}^2 + N_{B,a}^2)a^2 + (N_{A,b}^2 + N_{B,b}^2)b^2 + 2(N_{A,a}N_{A,b} + N_{B,a}N_{B,b})ab \end{aligned}$$

¿from which we obtain the following:

$$\begin{aligned} N_{A,a}N_{A,b} + N_{B,a}N_{B,b} &= 0 \qquad (3) \\ N_{A,a}^2 + N_{B,a}^2 &= s \qquad (4) \\ N_{A,b}^2 + N_{B,b}^2 &= t \qquad (5) \end{aligned}$$

Note that $N_{X,v} =$ (count of variable v) − (count of variable $-v$) in sequence X.

Consider the case for sequences of type (4,9); the case for (1,9) is similar. Then Equations 3, 4, 5 have solutions:

$$\begin{array}{llll} N_{A,a} = \pm 2 & N_{A,b} = 0 & N_{B,a} = 0 & N_{B,b} = \pm 3 \\ N_{A,a} = 0 & N_{A,b} = \pm 3 & N_{B,a} = \pm 2 & N_{B,b} = 0 \end{array}$$

We need consider only one of these two possible solutions; the other corresponds to exchanging sequences A and B.

Each solution has four cases, of which we need to consider only one. The other three cases correspond to changing the sign of each variable in one or both sequences, which does not affect the result of evaluating Equation 1.

This yields a table (Table 1) of counts of variables present in sequences A and B of type (4,9). A similar table (Table 2) can be generated for sequences of type (1,9).

We also observe that the result of evaluating Equation 1 is not affected by rotating a sequence; we are able to eliminate some rotations of a sequence from the search space by fixing the first element of each sequence. We choose the variable that is to occur in a sequence the fewest number of times as the first (fixed) variable in the sequence; this gives the greatest reduction in the search space.

Note also that the result of evaluating Equation 1 is the same for the reflection of a sequence; however, we have only been successful in eliminating a small proportion of these from the search space. No reflections have been eliminated from the search space sizes tabulated in Table 3.

A				B			
a	$-a$	b	$-b$	a	$-a$	b	$-b$
3	1	0	0	0	0	6	3
3	1	1	1	0	0	5	2
3	1	2	2	0	0	4	1
3	1	3	3	0	0	3	0
2	0	0	0	1	1	6	3
2	0	1	1	1	1	5	2
2	0	2	2	1	1	4	1
2	0	3	3	1	1	3	0

Table 1: Variable counts for sequences A and B of type (4,9).

A				B			
a	$-a$	b	$-b$	a	$-a$	b	$-b$
1	0	0	0	0	0	6	3
1	0	1	1	0	0	5	2
1	0	2	2	0	0	4	1
1	0	3	3	0	0	3	0

Table 2: Variable counts for sequences A and B of type (1,9).

We can speed up the process of evaluating Equation 1 for each pair of possible sequences if we count the number of possible sequences for A and for B given the variable counts for the particular s and t, evaluate Equation 1 for whichever has fewer possible sequences, and store these results in memory[1]. Equation 1 can then be fully evaluated for both sequences by evaluating it for each possibility of the sequence with the largest number of possibilities together with the results stored in memory.

Table 3 records the total size of the search space, amount of the space that was searched before finding a pair of sequences with zero autocorrelation function, and the time taken, for different (s, t) and various n. Results were obtained using three different computers.

4.2 Results

A summary of our results is given in Tables 4 and 5. Table 6 is derived from [12].

The results we now give in Table 4 marked $\surd$ are previously unpublished.

5 When necessary conditions are not sufficient

As we have seen in this paper, and from [8, 9, 11], the known necessary conditions are not sufficient for the existence of

- four sequences of lengths 5 and type $(3, 7, 8)$;

[1]Although in practice so much memory may be required for large n for some (s, t) that this step must be performed in parts. It should also be noted that considerable improvements in runtimes could be obtained by limiting the amount of memory used by a single part to a size that could be accommodated by cache memory.

Type	n	Tot. Sequences (billions)	Searched (billions)	Time (minutes)	Machine
(1,9)	27	3.78	All	14.5	Ultra-5
(1,9)	27	3.78	All	21.5	Ultra-2
(1,9)	29	7.16	All	27	Ultra-5
(1,9)	29	7.16	All	42.5	Ultra-2
(1,9)	31	12.94	All	55	Ultra-5
(1,9)	31	12.94	All	80	Ultra-2
(1,9)	31	12.94	All	50	PM 7300/200
(1,9)	33	22.44	All	99	Ultra-5
(1,9)	33	22.44	All	158	Ultra-2
(1,9)	33	22.44	All	116	PM 7300/200
(1,9)	37	60.93	All	275	Ultra-5
(1,9)	37	60.93	All	416	Ultra-2
(1,9)	41	148.07	All	767	Ultra-5
(1,9)	41	148.07	All	1102	Ultra-2
(1,9)	43	223.01	—	—	—
(1,16)	15	10.28	0.68	<1	PM 7300/200
(1,16)	17	155.83	0.12	<1	PM 7300/200
(1,16)	19	1538.97	2.39	5	PM 7300/200
(1,16)	23	63929.02	—	—	—
(4,9)	15	9.68	All	17	Ultra-5
(4,9)	15	9.68	All	23.5	Ultra-2
(4,9)	17	57.96	All	106	Ultra-5
(4,9)	17	57.96	All	151	Ultra-2
(4,9)	17	57.96	All	97	PM 7300/200
(4,9)	19	269.38	71.71	157	Ultra-5
(4,9)	19	269.38	71.73	245	Ultra-2
(4,9)	19	269.38	75.02	194	PM 7300/200
(4,9)	21	1035.27	834.60	1645	Ultra-5
(4,9)	21	1035.27	833.36	2222	Ultra-2
(4,9)	23	3431.72	—	—	—

Table 3: Time taken to search for sequences of types (1,9), (4,9) and (1,16) for various n, multiple computers.

Type	Length	Zero PAF	New?	Type	Length	Zero PAF	New?
(1,9)	7	Yes		(4,9)	19	Yes	$\surd$
(4,9)	7	No		(1,9)	21	Yes	
(1,9)	9	No		(1,16)	21	Yes	
(1,16)	9	No		(4,9)	21	Yes	$\surd$
(4,9)	9	No		(1,9)	23	Yes	$\surd$
(1,9)	10	Yes		(4,9)	23	No	$\surd$
(4,9)	10	No		(1,9)	25	Yes	$\surd$
(1,9)	11	No		(1,9)	27	No	$\surd$
(1,16)	11	Yes		(1,9)	29	No	$\surd$
(4,9)	11	No	$\surd$	(1,9)	31	No	$\surd$
(1,9)	13	Yes		(1,9)	33	No	$\surd$
(4,9)	13	No	$\surd$	(1,9)	35	Yes	
(1,16)	13	Yes	$\surd$	(1,9)	37	No	$\surd$
(1,9)	15	Yes	$\surd$	(1,9)	39	Yes	
(1,16)	15	Yes	$\surd$	(1,9)	41	No	$\surd$
(4,9)	15	No	$\surd$	(1,9)	43	No	$\surd$
(1,9)	17	No	$\surd$	(1,9)	45	Yes	
(1,16)	17	Yes	$\surd$	(1,9)	47	No	$\surd$
(4,9)	17	No	$\surd$	(1,9)	49	Yes	
(1,9)	19	Yes	$\surd$	(1,9)	51	No	$\surd$
(1,16)	19	Yes	$\surd$	(1,9)	53	No	$\surd$

Table 4: The results of an exhaustive search for the existence of two sequences of types (1,9), (1,16) and (4,9).

Length	Type	Sequences with zero autocorrelation function	Zero
7	(1,9)	b -b a b -b 0 0; b -b -b -b -b 0 0	PAF
7	(1,9)	a b b -b b -b -b; 0 b b 0 b 0 0	PAF
10	(1,9)	a 0 -b 0 0 0 0 0 b 0; b -b -b b b 0 b 0 b 0	PAF
11	(1,16)	a b-b 0 0 b 0 0-b 0 0 b-b; b b b b 0 b-b b b-b-b 0 0	PAF
13	(1,16)	a b-b 0 0 b 0 0-b 0 0 b-b; b b b b 0 b-b b b-b-b 0 0	PAF
15	(1,9)	0 0 0 0 b 0 0 a 0 0-b 0 0 0 0; b b b-b 0 0-b b 0 0 0 b 0 0 0	PAF
15	(1,16)	a b b 0 0 0 0 0 0 0 0 0 0 0 0-b-b; b b b b-b b-b b 0-b 0 b 0-b b 0 0	PAF
17	(1,16)	a b b 0 0 0 0 0 0 0 0 0 0 0 0-b-b; b b b b-b b-b b 0-b 0 b 0-b b 0 0	PAF
19	(1,9)	0 0 0 0 0 0 0 0 b a-b 0 0 0 0 0 0 0 0; b b b-b b 0 0 0-b 0 0 0 0 b 0 0 0 0 0	PAF
19	(1,16)	a b 0 0 b 0 0 0 0 0 0 0 0 0 0-b 0 0-b; b b b 0 0-b b b 0 b-b b-b-b 0 b 0 0 0	PAF
19	(4,9)	a a 0 0 b-b-b 0 0 0 0 0 0 0 b 0 0 0 0; a-a b 0 b 0 0 0 0 0 b 0 0 0 0-b 0 0 b	PAF
21	(1,9)	b 0 0 b 0 0 0 0 0 a 0 0 0 0 0-b 0 0-b 0 0; 0 b 0 0 0 0 0 b 0 0 0 0 0 b 0 0-b 0 0 b 0	PAF
21	(4,9)	a b b-b 0-b 0 a 0 0 0 0 0-b 0 0 0 0 b 0 0; a 0 0 0 0 0 0-a 0 0 0-b 0 0-b 0 0 0 0 0-b	PAF
21	(4,9)	a b 0 0 b 0 0 a 0 0-b 0 0-b 0 0 b 0 0-b 0; a 0 b 0 0 0 0-a b 0 0 b 0 0 0 0 0 0 0 0 0	PAF
23	(1,9)	b 0 0 0 0 0 0 0 0 0 0 a 0 0 0 0 0 0 0 0 0 0-b; b-b 0 b 0 0 b b b 0 0-b 0 0 0 0 0 0 0 0 0 0	PAF
25	(1,9)	0 b 0 0 0 0 0 0 0 0 0 0 a 0 0 0 0 0 0 0 0 0 0-b 0; b 0 b 0 0 b b-b 0 0 0 0 0 b 0 0 0-b 0 0 0 0 0 0 0	PAF

Table 5: Some new sequences.

Type	$2n$
(1,9)	12, 14, 16, (18), 20, (22), 24...
(1,16)	20...
(4,9)	14, 16, (18), 20, (22), 24, (26), 28...

Table 6: The asymptotic existence of $OD(2n;1,9)$, $OD(2n;1,16)$ and $OD(2n;4,9)$.

- four sequences of lengths 5 and types $(1,3,6,8)$, $(1,4,4,9)$, or $(2,2,5,5)$;
- two sequences of lengths 7, 9, 10, 11, 13, 15, or 17 and type $(4,9)$;
- four sequences of lengths 7 and type $(1,5,20)$;
- four sequences of lengths 7 and types $(1,4,9,9), (1,8,8,9), (2,8,9,9), (3,6,8,9)$, $(4,4,4,9)$ or $(4,4,9,9)$;
- two sequences of lengths 9, 11, 17, 27, 29, 31, 33 or 37 and type $(1,9)$;
- two sequences of length 9, type $(0, \pm 1)$ and weight 9;
- four sequences of lengths 9 and types $(1,2,8,25)$, $(1,4,4,25)$ or $(2,3,4,24)$;
- four sequences of lengths 11 and types (5,38), $(6,37)$, (8,35), $(10,33)$, $(12,31)$, $(13,30)$, $(14,29)$, (15,28), $(16,27)$, (19,24), (20,23), or $(21,22)$.
- four sequences of lengths 11 and types (1,5,38), $(1,6,37)$, (1,8,35), $(1,10,33)$, $(1,12,31)$, $(1,13,30)$, $(1,14,29)$, (1,15,28), $(1,16,27)$, $(1,19,24)$, (1,20,23) or (1,21,22).
- single sequence of length 15 or 17, type $(0, \pm 1)$ and weight 9;

We note there is an $(4,4,9,9)$ constructed of four circulant sequences of lengths 19 and 21.
Acknowledgement The authors wish to thank Dr Christos Koukouvinos for his careful reading and comments.

References

[1] K. T. Arasu, private email communication to J. Seberry, 22nd July 1996.

[2] K.T. Arasu and Jennifer Seberry, Circulant weighing designs, *Journal of Combinatorial Designs*, **4**, (1996), 439-447.

[3] Peter Eades and Richard Hain, On circulant weighing matrices, *Ars Comb.*, 2:265-284, 1976.

[4] A.V.Geramita, and J.Seberry, *Orthogonal designs: Quadratic forms and Hadamard matrices*, Marcel Dekker, New York-Basel, 1979.

[5] H. Hotelling, Some improvements in weighing and other experimental techniques, *Ann. Math. Stat.*, 16:294-300, 1944.

[6] C.Koukouvinos, Some new three and four variable orthogonal designs in order 36, *J. Statistical Planning and Inference,* (to appear).

[7] C.Koukouvinos, Some new orthogonal designs of order 36, *Utilitas Math.*, 51, (1997), 65-71.

[8] Christos Koukouvinos and Jennifer Seberry, New orthogonal designs and sequences with two and three variables in order 28, *Ars Comb.*, (to appear).

[9] C. Koukouvinos, N. Platis, and J. Seberry, Necessary and sufficient conditions for some two variable orthogonal designs in order 36, *Congressus Numerantium*, 114, (1996), 129-140..

[10] C.Koukouvinos, M.Mitrouli, J.Seberry, and P.Karabelas, On sufficient conditions for some orthogonal designs and sequences with zero autocorrelation function, *Australas. J. Combin.,* 13, (1996) 197-216.

[11] C.Koukouvinos, M.Mitrouli, and Jennifer Seberry, Necessary and sufficient conditions for some two variable orthogonal designs in order 44, *JCMCC*, (submitted).

[12] Christos Koukouvinos and Jennifer Seberry, New weighing matrices and orthogonal designs constructed using two sequences with zero autocorrelation function, (submitted).

[13] H. Ohmori, W(17,9) found by H. Ohmori and T. Miyamoto private email communication to J. Seberry, 16 May 1996.

[14] Yoseph Strassler, *The Classification of Circulant Weighing Matrices of Weight 9*, PhD Thesis, Bar-Ilan University, Ramat-Gan, 1997.

[15] Yoseph Strassler, private email communication to J. Seberry, 29 August, 1995.

[16] Jennifer Seberry and Mieko Yamada, Hadamard matrices, sequences and block designs, in *Contemporary Design Theory - a Collection of Surveys*, eds J. Dinitz and D.J. Stinson, John Wiley and Sons, New York, pp431-560, 1992.