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New orthogonal designs and sequences with two and three variables in order 28

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We give new sets of sequences with entries from {0, ±a, ±b, ±c} on the commuting variables a, b, c and zero autocorrelation function. Then we use these sequences to construct some new orthogonal de-signs. We show the necessary conditions for the existence of an OD(28; s1, s2, s3) plus the condition that (s1, s2, s3) ≠ (1,5,20) are sufficient conditions for the existence of an OD(28; s1, s2, s3). We also show the necessary conditions for the existence of an OD(28; s1, s2, s3) constructed using four circulant matrices are sufficient conditions for the existence of 4 — NPAF(s1, s2, s3) sequences of of length n for all lengths n ≥ 7. We establish asymptotic existence results for OD(4N; s1, s2) for 2 ≤ s1 + s2 ≤ 28. We show the necessary conditions for the existence of an OD(28; s1, s2) with 25 ≤ s1 + s2 ≤ 28, constructed using four circulant matrices, plus the condition that (s1, s2) ≠ (1,26), (2, 25), (7, 19), (8, 19) or (13, 14), are sufficient conditions for the existence of 4 — NPAF(s1, s2) sequences of of length n for all lengths n ≥ 7.

Keywords

Autocorrelation, construction, sequence, orthogonal design, AMS Subject Classification: Primary 05B15, 05B20, Secondary 62K05.

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New orthogonal designs and sequences with two and three variables in order 28

C. Koukouvinos,* and Jennifer Seberry†

Abstract

We give new sets of sequences with entries from $\{0, \pm a, \pm b, \pm c\}$ on the commuting variables a, b, c and zero autocorrelation function. Then we use these sequences to construct some new orthogonal designs.

We show the necessary conditions for the existence of an $OD(28; s_1, s_2, s_3)$ plus the condition that $(s_1, s_2, s_3) \neq (1, 5, 20)$ are sufficient conditions for the existence of an $OD(28; s_1, s_2, s_3)$. We also show the necessary conditions for the existence of an $OD(28; s_1, s_2, s_3)$ constructed using four circulant matrices are sufficient conditions for the existence of $4 - NPAF(s_1, s_2, s_3)$ sequences of of length n for all lengths $n \geq 7$.

We establish asymptotic existence results for $OD(4N; s_1, s_2)$ for $2 \leq s_1 + s_2 \leq 28$. We show the necessary conditions for the existence of an $OD(28; s_1, s_2)$ with $25 \leq s_1 + s_2 \leq 28$, constructed using four circulant matrices, plus the condition that $(s_1, s_2) \neq (1, 26)$, $(2, 25)$, $(7, 19)$, $(8, 19)$ or $(13, 14)$, are sufficient conditions for the existence of $4 - NPAF(s_1, s_2)$ sequences of of length n for all lengths $n \geq 7$.

Key words and phrases: Autocorrelation, construction, sequence, orthogonal design.

AMS Subject Classification: Primary 05B15, 05B20, Secondary 62K05.

1 Introduction

Throughought this paper we will use the definition and notation of Koukouvinos, Mitrouli, Seberry and Karabelas [4].

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Theorem 1 (Geramita and Seberry and Lemma 3 [1, 2]) *Write $(s_i, s_j)_p$ for the Hilbert norm residue symbol. The following conditions are necessary for the existence of an $OD(4n; s_1, s_2, \cdots, s_\ell)$ in orders with n odd:*

i) for $\ell = 2$; $(-1, s_1 s_2)_p (s_1, s_2)_p = 1$ for all primes p;

ii) for $\ell = 3$; $(s_1, s_2)_p (s_1, s_3)_p (s_2, s_3)_p (-1, s_1 s_2 s_3)_p = 1$ for all primes p; if $s_1 + s_2 + s_3 = n - 1$ then $s_1 s_2 s_3$ is a square and $(s_1, s_2)_p (s_1, s_3)_p (s_2, s_3)_p = 1$ for all primes p;

The Goethals-Seidel construction can only be used if

iii) there exists an integer matrix P, with all entries of modulus $\leq n$ which satisfies $PP^T = diag(s_1, s_2, s_3)$.

In this paper there are no 2- or 3-tuples which satisfy (i) and (ii) which do not also satisfy (iii). However in others orders, such as 20, this does happen.

2 New orthogonal designs

Theorem 2 *There exist orthogonal designs $OD(4n; s_1, s_2, s_3)$ where (s_1, s_2, s_3) is one of the 3-tuples*

$(1,1,17)$	$(1,3,14)$	$(1,6,11)$	$(1,8,11)$	$(1,8,16)$
$(1,9,16)$	$(2,5,7)$	$(2,7,10)$	$(2,7,13)$	$(2,8,18)$
$(3,4,14)$	$(3,6,8)$	$(3,7,11)$	$(3,8,10)$	$(3,9,14)$
$(4,4,13)$	$(4,5,14)$	$(5,5,10)$	$(7,8,13)$	

for all $n \geq 7$, constructed using four circulant matrices in the Goethals-Seidel array.

Proof. We use the $4 - NPAF$ sequences given in the Appendices A and B, as the first rows of the corresponding circulant matrices in the Goethals-Seidel array to obtain the required orthogonal designs. Since these sequences have zero non-periodic autocorrelation function, the sequences are first padded with sufficient zeros added to the end to make their length $n \geq 7$. □

Theorem 3 *There exist orthogonal designs $OD(4n; s_1, s_2, s_3)$, constructed using four circulant matrices in the Goethals-Seidel array, where (s_1, s_2, s_3) is one of the 3-tuples*

$(1,1,17)$	$(1,3,14)$	$(1,6,11)$	$(1,8,11)$	$(1,8,16)$
$(1,8,17)$	$(1,9,16)$	$(2,4,22)$	$(2,5,7)$	$(2,7,10)$
$(2,7,13)$	$(2,8,18)$	$(2,9,11)$	$(2,9,17)$	$(3,4,14)$
$(3,6,8)$	$(3,6,16)$	$(3,6,17)$	$(3,7,11)$	$(3,7,15)$
$(3,8,10)$	$(3,8,15)$	$(3,9,14)$	$(3,10,15)$	$(4,4,13)$
$(4,5,14)$	$(4,10,11)$	$(5,5,10)$	$(5,5,18)$	$(5,9,14)$
$(5,10,10)$	$(6,7,8)$	$(6,9,11)$	$(7,8,10)$	$(7,8,13)$
$(8,8,9)$	$(8,9,11)$	$(9,9,10)$.		

Proof. We use the sequences given in Appendices B and C, which have zero periodic and non-periodic autocorrelation function, as the first rows of the corresponding circulant matrices in the Goethals-Seidel array to obtain the required orthogonal designs.

The designs for $(3,6,16)$, $(3,8,15)$, $(4,6,11)$, $(4,9,13)$, $(8,8,9)$, and $(8,9,9)$ are from [3]. □

Theorem 4 *If* $OD(28;1,5,20)$ *and* $OD(20;3,7,8)$ *exist they cannot be constructed using four circulant matrices in the Goethals-Seidel array.*

Proof. By an exhaustive search. □

Theorem 5 *There are no orthogonal designs* $OD(4n;s_1,s_2,s_3)$ *where* $(s_1,$
$s_2,s_3)$ *is one of the* 3*-tuples*

$(1,3,22)$	$(1,5,19)$	$(2,5,15)$	$(2,6,11)$	$(2,6,17)$
$(2,11,11)$	$(2,11,13)$	$(2,11,15)$	$(3,7,10)$	$(3,11,14)$
$(4,5,19)$	$(5,6,14)$	$(5,6,15)$	$(5,7,10)$	$(5,7,14)$
$(6,8,11)$	$(7,10,11)$,			

constructed using four circulant matrices in the Goethals-Seidel array.

Proof. There is no an integer sum-fill matrix P as described in [4, Lemma 3]. □

Theorem 6 *There are no* $4-NPAF(s_1,s_2,s_3)$ *and* $4-NPAF(s_1,s_2,s_3,s_4)$ *of length* 5 *for the following* 3− *and* 4−*tuples*

$(1,3,14)$ $(1,4,13)$ $(1,5,20)$ $(3,7,8)$ $(1,3,6,8)$ $(1,4,4,9)$

Proof. An exhaustive search showed the 3-tuples were not given $4-NPAF$ sequences.

The the existence of $4-NPAF(1,3,6,8)$ of length 5, after equating the last two variables would prove the existence of $4-NPAF(1,3,14)$ of length

5: hence the nonexistence of the $4-NPAF(1,3,14)$ of length 5 gives the nonexistence of $4-NPAF(1,3,6,8)$.

Also, the existence of $4-NPAF(1,4,4,9)$ of length 5, after equating the last two variables would prove the existence of $4-NPAF(1,4,13)$ of length 5: the nonexistence of the $4-NPAF(1,4,13)$ of length 5 gives the nonexistence of $4-NPAF(1,4,4,9)$. □

Theorem 7 *The necessary conditions plus the condition that* $(s_1,s_2,s_3)\neq(1,5,20)$ *are sufficient conditions for the existence of an* $OD(28;s_1,s_2,s_3)$.

Proof. See Table 1.

2.1 Two variable designs

Lemma 1 *The necessary conditions for the existence of an* $OD(28;s_1,s_2)$ *plus the condition that* $(s_1,s_2)\neq(1,26),(2,25),(7,19),(8,19),$ *or* $(13,14)$ *are sufficient for the existence of* $4-NPAF(s_1,s_2)$ *sequences.*

Proof. The theoretical necessary conditions are used to prove the following 2−tuples (s_1,s_2) do not give orthogonal designs $OD(28;s_1,s_2)$:

$(1,7)$	$(3,5)$	$(4,7)$	$(5,12)$	$(7,9)$	$(9,15)$	$(12,13)$
$(1,15)$	$(3,13)$	$(4,15)$	$(5,19)$	$(7,16)$	$(10,17)$	$(12,15)$
$(1,23)$	$(3,20)$	$(4,23)$	$(5,22)$	$(7,17)$	$(11,13)$	
$(2,14)$	$(3,21)$	$(5,11)$	$(6,10)$	$(8,14)$	$(11,16)$.	

The specific cases mentioned in the enunciation have been eliminated by exhaustive search. □

Theorem 8 *There are no* $4-NPAF(s_1,s_2,s_3)$ *of length* 7 *for the following* 3−*tuples*

$(1,1,25)$	$(1,1,26)$	$(1,2,25)$	$(1,8,17)$	$(1,8,18)$	$(1,8,19)$
$(1,9,13)$	$(1,10,14)$	$(1,13,13)$	$(1,13,14)$	$(2,4,22)$	$(2,7,19)$
$(2,9,17)$	$(2,13,13)$	$(3,6,17)$	$(3,7,15)$	$(3,10,15)$	$(4,4,18)$
$(4,9,13)$	$(4,10,11)$	$(5,5,18)$	$(5,9,9)$	$(5,9,14)$	$(6,7,8)$
$(6,9,11)$	$(7,8,10)$	$(8,9,11)$	$(9,9,9)$	$(9,9,10)$.	

Proof. By exhaustive search there are no $4-NPAF(s_1,s_2)$ for the following 2−tuples

$$(1,26) \qquad (2,25) \qquad (7,19) \qquad (8,19) \qquad (13,14).$$

Hence, there can be no $4-NPAF(s_1,s_2,s_3)$ of length 7 for the following 3−tuples

$(1,1,25)$	$(1,1,26)$	$(1,2,25)$	$(1,8,18)$	$(1,8,19)$	$(1,13,13)$
$(1,13,14)$	$(2,7,19)$.				

There are no $4-NPAF(s_1,s_2,s_3)$ of length 7 for the following $3-$tuples

$(1,8,17)$	$(1,9,13)$	$(1,10,14)$	$(2,4,22)$	$(2,9,17)$	$(2,13,13)$
$(3,6,17)$	$(3,7,15)$	$(3,10,15)$	$(4,4,18)$	$(4,9,13)$	$(4,10,11)$
$(5,5,18)$	$(5,9,9)$	$(5,9,14)$	$(6,7,8)$	$(6,9,11)$	$(7,8,10)$
$(8,9,11)$	$(9,9,9)$	$(9,9,10)$			

by exhaustive search. $\square$

Remark 1 The 236 3-tuples given in Table 1 are possible types of orthogonal designs in order 28. There are no cases unresolved. We use

n if they are made from 4-sequences with zero NPAF and length n;

P if they are made from 4-sequences with zero PAF;

F if they are made from the $OD(28;4,4,9,9)$;

X if they are not known because the integer sum-fill matrix does not exist;

Y if it does not exist by exhaustive search for length 7.

(1,1,1)	1	(1,5,16)	7	(2,4,8)	5	(3,4,8)	5	(4,8,12)	7
(1,1,2)	1	(1,5,19)	X	(2,4,9)	5	(3,4,14)*	P	(4,8,16)	7
(1,1,4)	2	(1,5,20)	Y	(2,4,11)	5	(3,4,18)	7	(4,9,9)	6
(1,1,5)	3	(1,6,8)	5	(2,4,12)	7	(3,6,6)	5	(4,9,10)	7
(1,1,8)	3	(1,6,11)	5	(2,4,16)	7	(3,6,8)	5	(4,9,13)†	7
(1,1,9)	7	(1,6,12)	7	(2,4,17)	7	(3,6,9)	5	(4,10,10)	7
(1,1,10)	3	(1,6,14)	7	(2,4,18)	7	(3,6,11)	5	(4,10,11)	P
(1,1,13)	5	(1,6,18)	7	(2,4,19)	7	(3,6,12)	7	(4,10,14)	7
(1,1,16)	7	(1,6,21)	7	(2,4,22)	P	(3,6,16)†	7	(5,5,5)	7
(1,1,17)*	P	(1,8,8)	7	(2,5,5)	3	(3,6,17)	P	(5,5,8)	7
(1,1,18)	6	(1,8,9)	5	(2,5,7)	5	(3,6,18)	7	(5,5,9)	5
(1,1,20)	6	(1,8,11)	5	(2,5,8)	5	(3,6,19)	7	(5,5,10)	5
(1,1,25)	P	(1,8,12)	7	(2,5,13)	6	(3,7,8)	6	(5,5,13)	P
(1,1,26)	P	(1,8,16)	7	(2,5,15)	X	(3,7,10)	X	(5,5,16)	7
(1,2,2)	2	(1,8,17)	P	(2,5,18)	7	(3,7,11)	7	(5,5,18)	P
(1,2,3)	2	(1,8,18)	P	(2,6,7)	5	(3,7,15)	P	(5,6,9)	7
(1,2,4)	2	(1,8,19)	P	(2,6,9)	5	(3,7,18)	7	(5,6,14)	X
(1,2,6)	3	(1,9,9)	7	(2,6,11)	X	(3,8,9)	7	(5,6,15)	X
(1,2,8)	3	(1,9,10)	5	(2,6,12)	6	(3,8,10)*	P	(5,7,8)	7
(1,2,9)	3	(1,9,13)*	P	(2,6,13)	7	(3,8,15)†	7	(5,7,10)	X
(1,2,11)	5	(1,9,16)	7	(2,6,16)	7	(3,9,14)	7	(5,7,14)	X
(1,2,12)	5	(1,9,18)	7	(2,6,17)	X	(3,10,15)	P	(5,8,8)	7
(1,2,16)	7	(1,10,10)	7	(2,7,10)	7	(3,11,14)	X	(5,8,13)	7
(1,2,17)	5	(1,10,11)	7	(2,7,12)	7	(4,4,4)	3	(5,9,9)*	P
(1,2,18)	6	(1,10,14)	P	(2,7,13)	7	(4,4,5)	5	(5,9,10)*	P
(1,2,19)	6	(1,13,13)	P	(2,7,19)	P	(4,4,8)	7	(5,9,14)	P
(1,2,22)	7	(1,13,14)	P	(2,8,8)	5	(4,4,9)	5	(5,10,10)	7
(1,2,25)	P	(2,2,2)	2	(2,8,9)	5	(4,4,10)	5	(6,6,6)	7
(1,3,6)	3	(2,2,4)	2	(2,8,10)	5	(4,4,13)	7	(6,6,12)	7
(1,3,8)	3	(2,2,5)	3	(2,8,13)	7	(4,4,16)	7	(6,7,8)	P
(1,3,14)	6	(2,2,8)	3	(2,8,16)	7	(4,4,17)	7	(6,8,9)	7
(1,3,18)	6	(2,2,9)	5	(2,8,18)	7	(4,4,18)	P	(6,8,11)	X
(1,3,22)	X	(2,2,10)	5	(2,9,9)	5	(4,4,20)	7	(6,8,12)	P
(1,3,24)	7	(2,2,13)	5	(2,9,11)	6	(4,5,5)	5	(6,9,11)	P
(1,4,4)	5	(2,2,16)	7	(2,9,12)	7	(4,5,6)	5	(7,7,7)	7
(1,4,5)	5	(2,2,17)	7	(2,9,17)	P	(4,5,9)	5	(7,7,14)	7
(1,4,8)	5	(2,2,18)	6	(2,10,10)	6	(4,5,14)*	P	(7,8,10)	P
(1,4,9)	5	(2,2,20)	7	(2,10,12)	6	(4,5,16)	7	(7,8,13)	7
(1,4,10)	5	(2,3,4)	3	(2,11,11)	X	(4,5,19)	X	(7,10,11)	X
(1,4,13)	7	(2,3,6)	3	(2,11,13)	X	(4,6,8)	5	(8,8,8)	7
(1,4,16)	7	(2,3,7)	3	(2,11,15)	X	(4,6,11)†	7	(8,8,9)†	7
(1,4,17)	7	(2,3,9)	5	(2,13,13)	P	(4,6,12)	7	(8,8,10)	7
(1,4,18)	7	(2,3,10)	7	(3,3,3)	3	(4,6,14)	7	(8,9,9)†	7
(1,4,20)	7	(2,3,15)	7	(3,3,6)	3	(4,6,18)	7	(8,9,11)	P
(1,5,5)	3	(2,3,16)	7	(3,3,12)	7	(4,8,8)	7	(8,10,10)	7
(1,5,6)	3	(2,4,4)	3	(3,3,15)	7	(4,8,9)	7	(9,9,9)	P
(1,5,9)	5	(2,4,6)	3	(3,4,6)	5	(4,8,11)	7	(9,9,10)	P
(1,5,14)	5								

Table 1: Known 3-variable designs in order 28: for the 3-tuples marked by
* the corresponding orthogonal design is known for $n \geq 6$ and orders ≥ 24.
† the corresponding orthogonal design is known for $n \geq 7$ from [3].

3 Asymptotic Results

Tables 2, 3 and 4 indicate the smallest known length, ℓ, such that $4-NPAF(4n; s_1, s_2)$ sequences, with $\sigma = s_1 + s_2 \le 28$. exist for every length $\ge \ell$.

Theorem 9 *The known asymptotic results for* $4-NPAF(s_1, s_2)$ *with* $\sigma = s_1 + s_2$
≤ 28 *are summarized in Tables* 2, 3, 4.

Proof. All the results quoted except those improved here, may be found in Geramita and Seberry [2, p168] and [4, 5]. □

Theorem 10 *Suppose* (s_1, s_2) *satisfies the necessary conditions to be the type of orthogonal design. Then an* $OD(4N; s_1, s_2)$ *exists for*

(i) $N \ge 2$ *for* $2 \le s_1 + s_2 \le 8$;

(ii) $N \ge 4$ *for* $9 \le s_1 + s_2 \le 16$;

(iii) $N \ge 5$ *for* $17 \le s_1 + s_2 \le 20$ *except possibly for the 2-tuples* $(3,16)$, $(6,13)$, $(7,11)$, $(7,12)$ *which are the types of orthogonal designs for orders* $4N$, $N \ge 6$;

(iv) $N \ge 6$ *for* $21 \le s_1 + s_2 \le 24$;

(v) $N \ge 7$ *for* $25 \le s_1 + s_2 \le 28$.

Proof. Every 2-tuple with $2 \le s_1 + s_2 \le 8$ is the type of an $OD(8; s_1, s_2)$ ¿from [2, p.368]. This combined with Table 2 gives the result.

Every 2-tuple with $9 \le s_1 + s_2 \le 16$ is the type of an $OD(16; s_1, s_2)$ ¿from [2, p.389]. This combined with Table 3 gives the result. Geramita and Seberry give the existence of $OD(24; s_1, s_2)$ for $(s_1, s_2) = (3,16)$, $(6,13)$, $(7,11)$ and $(7,12)$. Hence, using Table 2 these are the types of orthogonal designs in all orders $4N$, $N \ge 6$. The remainder of the 2-tuples for which $17 \le s_1 + s_2 \le 20$ are given in Table 3.

All 2-tuples $2 \le s_1 + s_2 \le 24$ are the types of an $OD(24; s_1, s_2)$ from [2, p.391]. This combined with Tables 3 and 4 gives the result for (iv).

Table 3 gives the result for the first part of (v) for all 2-tuples except $(1,26)$, $(2,25)$, $(7,19)$ $(8,19)$ and $(13,14)$. Table 1 shows an $OD(28; s_1, s_2)$ exists for each of these 2-tuples. [2, p.394 and p.395] shows each of these 5 2-tuples is the type of an orthogonal design in orders 32 and 40. Now [5] gives $OD(4m; s_1, s_2)$, for these (s_1, s_2) with $m \ge 9$ and so we have that these designs exist for all $N \ge 7$. □

s_1, s_2	σ	ℓ	s_1, s_2	σ	ℓ	s_1, s_2	σ	ℓ	s_1, s_2	σ	ℓ	s_1, s_2	σ	ℓ
$(1,1)$	2	1	$(1,2)$	3	1	$(1,4)$	5	2	$(1,6)$	7	3	$(1,8)$	9	3
			$(1,3)$	4	1	$(1,5)$	6	2	$(1,7)$	*no*		$(1,9)$	10	3
			$(2,2)$	4	1	$(2,3)$	5	2	$(2,5)$	7	3	$(2,7)$	9	3
						$(2,4)$	6	2	$(2,6)$	8	2	$(2,8)$	10	3
						$(3,3)$	6	2	$(3,4)$	7	3	$(3,6)$	9	3
									$(3,5)$	*no*		$(3,7)$	10	3
									$(4,4)$	8	2	$(4,5)$	9	3
												$(4,6)$	10	3
												$(5,5)$	10	3

Table 2: The indicated $4 - NPAF(s_1, s_2)$ sequences with $1 \leq s_1 + s_2 \leq 10$ exist for every length $N \geq \ell$.

s_1, s_2	σ	ℓ	s_1, s_2	σ	ℓ	s_1, s_2	σ	ℓ	s_1, s_2	σ	ℓ	s_1, s_2	σ	ℓ
$(1,10)$	11	3	$(1,12)$	13	4	$(1,14)$	15	5	$(1,16)$	17	5	$(1,18)$	19	5
$(1,11)$	12	3	$(1,13)$	14	5	$(2,14)$	*no*		$(1,17)$	18	5	$(1,19)$	20	5
$(2,9)$	11	5	$(2,11)$	13	5	$(2,13)$	15	5	$(2,15)$	17	5	$(2,17)$	19	5
$(2,10)$	12	3	$(2,12)$	14	5	$(2,14)$	*no*		$(2,16)$	18	5	$(2,18)$	20	5
$(3,8)$	11	3	$(3,10)$	13	5	$(3,12)$	15	5	$(3,14)$	17	5	$(3,16)$	19	7
$(3,9)$	12	3	$(3,11)$	14	5	$(3,13)$	*no*		$(3,15)$	18	5	$(3,17)$	20	5
$(4,7)$	*no*		$(4,9)$	13	5	$(4,11)$	15	5	$(4,13)$	17	5	$(4,15)$	*no*	
$(4,8)$	12	3	$(4,10)$	14	5	$(4,12)$	16	5	$(4,14)$	18	5	$(4,16)$	20	5
$(5,6)$	11	3	$(5,8)$	13	5	$(5,10)$	15	5	$(5,12)$	*no*		$(5,14)$	19	5
$(5,7)$	12	3	$(5,9)$	14	5	$(5,11)$	*no*		$(5,13)$	18	5	$(5,15)$	20	5
$(6,6)$	12	3	$(6,7)$	13	5	$(6,9)$	15	5	$(6,11)$	17	5	$(6,13)$	19	7
			$(6,8)$	14	5	$(6,10)$	*no*		$(6,12)$	18	5	$(6,14)$	20	5
			$(7,7)$	14	4	$(7,8)$	15	5	$(7,10)$	17	5	$(7,12)$	19	7
						$(7,9)$	*no*		$(7,11)$	18	7	$(7,13)$	20	5
						$(8,8)$	16	5	$(8,9)$	17	5	$(8,11)$	19	5
									$(8,10)$	18	5	$(8,12)$	20	5
									$(9,9)$	18	5	$(9,10)$	19	5
												$(9,11)$	20	5
												$(10,10)$	20	5

Table 3: The indicated $4 - NPAF(s_1, s_2)$ sequences with $11 \leq s_1 + s_2 \leq 20$ exist for every length $N \geq \ell$.

s_1, s_2	σ	ℓ	s_1, s_2	σ	ℓ	s_1, s_2	σ	ℓ	s_1, s_2	σ	ℓ
$(1,20)$	21	6	$(1,22)$	23	7	$(1,24)$	25	7	$(1,26)$	27	9
$(1,21)$	22	6	$(1,23)$	*no*		$(1,25)$	26	7	$(1,27)$	28	7
$(2,19)$	21	6	$(2,21)$	23	7	$(2,23)$	25	7	$(2,25)$	27	9
$(2,20)$	22	6	$(2,22)$	24	6	$(2,24)$	26	7	$(2,26)$	28	7
$(3,18)$	21	6	$(3,20)$	*no*		$(3,22)$	25	7	$(3,24)$	27	7
$(3,19)$	22	7	$(3,21)$	*no*		$(3,23)$	26	7	$(3,25)$	28	7
$(4,17)$	21	7	$(4,19)$	23	7	$(4,21)$	25	7	$(4,23)$	*no*	
$(4,18)$	22	6	$(4,20)$	24	6	$(4,22)$	26	7	$(4,24)$	28	7
$(5,16)$	21	7	$(5,18)$	23	7	$(5,20)$	25	7	$(5,22)$	*no*	
$(5,17)$	22	7	$(5,19)$	*no*		$(5,21)$	26	7	$(5,23)$	28	7
$(6,15)$	21	7	$(6,17)$	23	7	$(6,19)$	25	7	$(6,21)$	27	7
$(6,16)$	22	6	$(6,18)$	24	6	$(6,20)$	26	7	$(6,22)$	28	7
$(7,14)$	21	7	$(7,16)$	*no*		$(7,18)$	25	7	$(7,20)$	*no*	
$(7,15)$	22	7	$(7,17)$	*no*		$(7,19)$	26	9	$(7,21)$	28	7
$(8,13)$	21	7	$(8,15)$	23	7	$(8,17)$	25	7	$(8,19)$	27	9
$(8,14)$	*no*		$(8,16)$	24	6	$(8,18)$	26	7	$(8,20)$	28	7
$(9,12)$	21	7	$(9,14)$	23	7	$(9,16)$	25	7	$(9,18)$	27	7
$(9,13)$	22	6	$(9,15)$	*no*		$(9,17)$	26	7	$(9,19)$	28	7
$(10,11)$	21	6	$(10,13)$	23	7	$(10,15)$	25	7	$(10,17)$	*no*	
$(10,12)$	22	6	$(10,14)$	24	6	$(10,16)$	26	7	$(10,18)$	28	7
$(11,11)$	22	6	$(11,12)$	23	7	$(11,14)$	25	7	$(11,16)$	*no*	
			$(11,13$	*no*		$(11,15)$	26	7	$(11,17)$	28	7
			$(12,12)$	24	6	$(12,13)$	*no*		$(12,15)$	*no*	
						$(12,14)$	26	7	$(12,16)$	28	7
						$(13,13)$	26	7	$(13,14)$	27	9
									$(13,15)$	28	7
									$(14,14)$	28	7

Table 4: The indicated $4 - NPAF(s_1, s_2)$ sequences with $21 \leq s_1 + s_2 \leq 28$ exist for every length $N \geq \ell$.

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Appendix A: Order 20 (Sequences with zero nonperiodic autocorrelation function)

Design	A_1					A_2					A_3					A_4				
(1, 3, 14)	do not exist with length 5																			
(1, 4, 13)	do not exist with length 5																			
(1, 6, 11)	$-c$	b	a	$-b$	c	b	c	c	0	c	b	c	$-c$	0	$-c$	$-c$	b	c	b	$-c$
(1, 8, 11)	c	b	a	$-b$	$-c$	c	b	b	$-c$	c	b	$-c$	b	c	$-c$	c	c	c	$-b$	b
(2, 5, 7)	b	a	$-b$	0	c	$-c$	0	$-c$	a	c	c	0	b	0	c	$-c$	0	b	0	b
(3, 6, 8)	b	$-c$	b	c	a	$-b$	c	0	c	a	0	$-c$	a	$-c$	$-b$	$-b$	$-c$	b	c	0
(7, 10)	$-b$	b	b	0	a	a	0	$-b$	b	b	a	b	a	b	$-a$	a	$-b$	0	$-b$	a

Appendix B: Order 28 (Sequences with zero periodic autocorrelation function)

Design	A_1							A_2							A_3							A_4						
(1, 8, 17)	a	$-c$	$-c$	c	$-c$	c	c	b	b	0	$-c$	c	c	c	b	b	$-c$	$-c$	c	$-c$	0	$-b$	b	$-b$	c	b	c	c
(2, 4, 22)	$-c$	c	c	$-c$	$-c$	$-b$	a	$-c$	c	$-c$	c	a	b	c	c	$-c$	c	c	c	$-b$	c	$-c$	c	c	c	c	b	$-c$
(2, 9, 11)	a	b	0	$-c$	$-b$	0	0	a	$-b$	c	c	b	$-c$	0	b	b	$-c$	c	b	0	0	c	$-c$	c	c	c	b	$-b$
(2, 9, 17)	a	$-b$	c	$-b$	b	c	b	a	c	$-c$	c	$-c$	$-c$	$-c$	b	b	$-c$	c	b	$-c$	c	$-b$	$-c$	b	c	c	c	c
(3, 6, 17)	a	b	$-c$	c	c	0	c	a	$-c$	b	c	$-c$	0	$-c$	a	$-b$	$-b$	$-c$	$-c$	c	c	$-b$	b	c	$-c$	c	c	c
(3, 7, 11)	a	b	0	c	$-c$	c	0	a	b	0	c	0	$-c$	0	c	c	b	$-a$	b	$-c$	0	c	b	$-b$	$-b$	c	c	0
(3, 7, 15)	a	$-b$	b	b	c	c	0	a	b	$-c$	$-c$	c	c	$-c$	a	c	$-b$	$-c$	$-b$	$-c$	0	b	$-c$	c	$-c$	0	$-c$	$-c$
(3, 10, 15)	a	$-b$	b	b	c	b	c	a	b	$-b$	$-b$	c	$-c$	$-c$	a	$-c$	$-b$	$-c$	$-c$	c	c	b	$-c$	b	$-c$	c	$-c$	$-c$
(4, 9, 13)	a	a	$-b$	$-c$	c	b	0	b	b	$-c$	c	b	$-c$	c	c	c	c	c	$-c$	b	$-b$	a	$-a$	$-b$	$-c$	$-c$	b	0
(4, 10, 11)	a	a	$-b$	b	0	$-c$	c	$-a$	a	$-c$	b	c	c	$-b$	b	b	b	c	$-c$	0	$-c$	$-b$	b	c	c	0	b	c
(5, 5, 18)	a	b	$-c$	b	c	$-c$	c	a	$-c$	a	c	b	$-b$	$-b$	a	$-c$	$-a$	$-c$	$-c$	c	$-c$	$-c$	$-c$	c	c	c	c	c
(5, 9, 14)	a	$-b$	b	$-c$	c	c	c	a	b	$-c$	a	c	$-b$	$-c$	b	$-b$	$-b$	$-b$	$-c$	$-b$	c	$-a$	c	c	a	c	$-c$	c
(5, 10, 10)	$-a$	a	a	b	c	b	c	a	0	a	$-b$	$-c$	$-c$	c	b	0	$-b$	$-b$	c	c	$-c$	$-b$	b	b	$-c$	0	$-c$	b
(6, 7, 8)	$-c$	c	c	0	c	b	a	c	$-c$	$-c$	0	$-c$	b	a	b	a	0	$-a$	b	0	0	$-b$	a	$-b$	a	b	0	0
(6, 9, 11)	a	$-c$	c	c	c	0	c	a	$-b$	b	c	$-c$	0	$-c$	a	b	$-c$	a	$-b$	$-b$	b	$-a$	b	b	a	c	b	$-c$
(7, 8, 10)	$-a$	a	a	b	b	$-c$	0	$-a$	b	b	c	$-c$	c	0	a	$-b$	$-c$	$-c$	b	$-b$	b	a	$-c$	a	c	c	0	c
(8, 9, 11)	$-c$	b	$-b$	a	c	$-c$	a	b	a	a	c	a	$-a$	$-b$	c	$-a$	b	b	b	$-c$	a	$-c$	c	c	c	b	c	$-b$
(9, 9, 10)	a	a	a	$-b$	b	$-c$	c	$-b$	b	c	b	b	b	$-c$	a	b	$-a$	$-b$	c	$-c$	$-c$	$-a$	a	$-a$	c	a	c	c

Appendix C: Order 28 (Sequences with zero nonperiodic autocorrelation function)

Design	A_1							A_2							A_3							A_4						
(1, 1, 17)	$-c$	a	c	0	0	0	0	$-c$	b	c	0	0	0	0	c	c	c	$-c$	$-c$	c	$-c$	c	c	c	0	c	$-c$	c
(1, 3, 14)	$-c$	0	$-c$	a	c	0	c	c	0	0	0	b	$-c$	c	c	c	$-c$	0	0	c	b	$-c$	$-c$	0	0	$-c$	0	b
(1, 8, 16)	$-c$	$-b$	c	a	$-c$	b	c	b	b	0	c	$-c$	c	$-c$	b	$-c$	$-c$	0	b	c	c	$-c$	$-b$	b	$-c$	0	$-c$	$-c$
(1, 9, 16)	$-c$	$-b$	c	a	$-c$	b	c	c	b	$-c$	b	$-c$	b	c	$-c$	0	$-c$	b	$-c$	$-b$	$-c$	c	0	c	b	$-c$	$-b$	$-c$
(2, 7, 10)	$-c$	0	a	0	c	b	$-c$	b	a	$-b$	0	$-b$	c	0	$-c$	0	0	b	$-c$	0	0	$-c$	b	c	0	c	b	c
(2, 7, 13)	a	b	$-c$	0	$-c$	0	0	a	$-b$	c	0	c	0	0	c	$-c$	$-b$	b	b	c	c	c	c	b	$-c$	b	c	$-c$
(2, 8, 18)	b	c	c	a	$-c$	$-c$	b	$-b$	$-c$	c	a	$-c$	c	$-b$	b	c	$-c$	c	c	c	$-b$	b	c	c	c	$-c$	c	$-b$
(3, 4, 14)	c	c	a	$-c$	c	$-c$	0	c	b	0	a	0	$-b$	c	$-c$	c	a	$-c$	$-c$	$-c$	0	c	b	0	0	0	b	$-c$
(3, 6, 16)	c	b	c	a	$-c$	0	$-c$	c	b	$-c$	0	c	a	$-c$	c	b	$-c$	b	$-c$	$-a$	c	c	0	c	$-b$	c	b	c
(3, 7, 11)	c	0	a	b	0	c	$-c$	c	0	$-c$	0	a	b	0	$-c$	$-c$	$-b$	a	$-b$	c	0	$-c$	0	$-c$	$-b$	b	b	$-c$
(3, 8, 10)	b	c	c	0	a	$-c$	b	$-b$	$-c$	c	0	a	c	$-b$	b	$-c$	0	a	0	$-c$	$-b$	b	c	0	0	0	c	$-b$
(3, 8, 15)	b	c	c	0	a	$-c$	b	b	c	$-c$	0	$-a$	$-c$	b	b	$-c$	$-c$	a	c	$-c$	$-b$	b	c	c	c	$-c$	c	$-b$
(3, 9, 14)	c	$-b$	0	b	c	c	a	$-c$	b	$-c$	a	$-c$	$-b$	c	$-c$	b	0	$-b$	$-c$	c	a	$-c$	b	c	b	c	b	$-c$
(4, 4, 13)	a	$-c$	$-c$	0	c	c	a	b	$-c$	c	0	$-c$	c	b	a	c	0	0	0	c	$-a$	b	c	0	c	0	c	$-b$
(4, 5, 14)	a	0	$-b$	0	b	0	a	c	0	$-c$	b	c	0	c	a	$-c$	b	c	b	$-c$	$-a$	c	c	c	c	$-c$	c	$-c$
(4, 6, 11)	b	0	a	0	a	0	$-b$	c	c	c	b	$-c$	c	0	c	$-c$	c	b	$-c$	$-c$	0	b	0	$-a$	$-c$	a	0	b
(5, 5, 13)	b	c	c	a	$-c$	$-c$	b	a	$-c$	c	$-b$	$-c$	c	a	a	c	0	0	0	c	$-a$	b	c	0	c	0	c	$-b$
(5, 10, 10)	b	c	c	a	c	$-c$	b	$-b$	0	$-b$	a	$-c$	a	b	c	$-c$	$-c$	0	$-b$	b	b	b	0	$-c$	$-a$	b	a	$-c$
(7, 8, 13)	$-c$	b	$-a$	a	a	b	$-c$	$-a$	c	b	$-c$	c	b	c	a	$-c$	$-b$	c	c	b	c	$-c$	b	a	c	a	$-b$	c
(8, 8, 9)	b	$-c$	a	c	b	0	a	b	$-c$	b	c	$-a$	0	$-a$	b	$-c$	$-a$	$-c$	$-b$	$-c$	a	b	$-c$	$-b$	0	a	c	$-a$
(8, 9, 9)	a	c	c	0	c	$-c$	a	a	b	$-c$	0	$-c$	$-b$	a	a	b	$-b$	b	b	b	$-a$	a	c	b	$-c$	$-b$	c	$-a$
(4, 22)	b	0	a	a	$-b$	$-b$	b	$-b$	0	$-a$	a	b	b	b	b	b	$-b$	b	$-b$	b	b	$-b$	b	b	b	$-b$	b	b
(5, 23)	b	$-b$	$-a$	a	a	b	b	$-b$	$-b$	a	$-b$	a	b	b	b	b	$-b$	b	$-b$	b	b	$-b$	b	b	b	$-b$	b	b
(11, 14)	$-a$	a	a	$-b$	$-b$	b	b	b	b	a	0	b	$-a$	a	a	0	a	a	$-b$	b	$-b$	$-b$	b	a	b	b	0	$-a$
(11, 15)	a	b	$-b$	a	b	b	$-a$	a	b	b	0	$-a$	$-b$	a	a	b	$-b$	0	a	$-b$	a	a	b	b	b	$-b$	b	$-a$
(11, 17)	$-a$	a	a	$-b$	b	b	b	b	a	$-b$	$-b$	$-a$	a	$-b$	$-b$	b	a	a	b	a	$-b$	a	b	$-b$	b	$-a$	b	b