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Abstract

We use a new algorithm to find new sets of sequences with entries from {0, ±a, ±b, ±c, ±d}, on the commuting variables a, b, c, d, with zero autocorrelation function. Then we use these sequences to construct a series of new three and four variable orthogonal designs in order 36. We show that the necessary conditions plus (.s1, s2, s3, s4) not equal to 12816 18 816 221313 26721 36 816 4889 12825 191313 23 424 289 9 381015 8899 14425 22 916 are sufficient for the existence of an OD(36; s1, s2 s3, s4) constructed using four circulant matrices in the Goethals-Seidel array. Of the 154 theoretically possible cases 133 are known. We also show that the necessary conditions plus (s1, s2 s3) ≠ (2,8,25), (6,7,21), (8, 9, 17) or (9, 13, 13) are sufficient for the existence of an OD(36; s1, s2, s3) constructed using four circulant matrices in the Goethals-Seidel array. Of the 433 theoretically possible cases 429 are known. Further, we show that the necessary conditions are sufficient for the existence of an OD(36; s1, s2 36 — s1 — s2) in each of the 54 theoretically possible cases. Further, of the 27 theoretically possible OD(36; s1, s2, s3, 36 — s1 — s2 — s3), 23 are known to exist, and four, (1,2,8,25), (1, 9,13,13), (2,6,7,21) and (3, 8,10,15), cannot be constructed using four circulant matrices. By suitably replacing the variables by ±1 these lead to more than 200 potentially inequivalent Hadamard matrices of order 36. By considering the 12 OD(36; 1, s1, 35—s1) and suitably replacing the variables by ±1 we obtain 48 potentially inequivalent skew-Hadamard matrices of order 36. A summary with all known results in order 36 is presented in the Tables.

Keywords

Orthogonal design, autocorrelation, circulant matrix, algorithm, Construction, AMS Subject Classification Primary 62K05, 62K15, Secondary 05B15

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Necessary and Sufficient Conditions for Three and Four Variable Orthogonal Designs in Order 36

S. Georgiou[*], C. Koukouvinos[†], M. Mitrouli[‡] and Jennifer Seberry[§]

Dedicated to Professor Sumiyasu Yamamoto

Abstract

We use a new algorithm to find new sets of sequences with entries from $\{0, \pm a, \pm b, \pm c, \pm d\}$, on the commuting variables $a,\ b,\ c,\ d$, with zero autocorrelation function.

Then we use these sequences to construct a series of new three and four variable orthogonal designs in order 36. We show that the necessary conditions plus (s_1, s_2, s_3, s_4) not equal to

1 2 8 16	1 8 8 16	2 2 13 13	2 6 7 21	3 6 8 16	4 8 8 9
1 2 8 25	1 9 13 13	2 3 4 24	2 8 9 9	3 8 10 15	8 8 9 9
1 4 4 25	2 2 9 16				

are sufficient for the existence of an $OD(36; s_1, s_2, s_3, s_4)$ constructed using four circulant matrices in the Goethals-Seidel array. Of the 154 theoretically possible cases 133 are known.

We also show that the necessary conditions plus $(s_1, s_2, s_3) \neq (2, 8, 25)$, $(6, 7, 21)$, $(8, 9, 17)$ or $(9, 13, 13)$ are sufficient for the existence of an $OD(36; s_1, s_2, s_3)$ constructed using four circulant matrices in the Goethals-Seidel array. Of the 433 theoretically possible cases 429 are known.

Further, we show that the necessary conditions are sufficient for the existence of an $OD(36; s_1, s_2, 36 - s_1 - s_2)$ in each of the 54 theoretically possible cases. Further, of the 27 theoretically possible $OD(36; s_1, s_2, s_3, 36 - s_1 - s_2 - s_3)$, 23 are known to exist, and four, $(1, 2, 8, 25)$, $(1, 9, 13, 13)$, $(2, 6, 7, 21)$ and $(3, 8, 10, 15)$, cannot be constructed using four circulant matrices.

By suitably replacing the variables by ± 1 these lead to more than 200 potentially inequivalent Hadamard matrices of order 36. By considering the 12 $OD(36; 1, s_1, 35 - s_1)$ and suitably replacing the variables by ± 1 we obtain 48 potentially inequivalent skew-Hadamard matrices of order 36.

A summary with all known results in order 36 is presented in the Tables.

AMS Subject Classification: Primary 62K05, 62K15, Secondary 05B15
Key words and phrases: Orthogonal design, autocorrelation, circulant matrix, algorithm, construction.

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1 Introduction

An *orthogonal design* of order n and type $(s_1, s_2, \ldots, s_u)$ $(s_i > 0)$, denoted $OD(n; s_1, s_2, \ldots, s_u)$, on the commuting variables $x_1, x_2, \ldots, x_u$, is an $n \times n$ matrix A with entries from $\{0, \pm x_1, \pm x_2, \ldots, \pm x_u\}$ such that

$$AA^T = (\sum_{i=1}^{u} s_i x_i^2) I_n$$

Alternatively, the rows of A are formally orthogonal and each row has precisely s_i entries of the type $\pm x_i$. In [1], where this was first defined, it was mentioned that

$$A^T A = (\sum_{i=1}^{u} s_i x_i^2) I_n$$

and so our alternative description of A applies equally well to the columns of A. It was also shown in [1] that $u \leq \rho(n)$, where $\rho(n)$ (Radon's function) is defined by $\rho(n) = 8c + 2^d$, when $n = 2^a b$, b odd, $a = 4c + d$, $0 \leq d < 4$.

A weighing matrix $W = W(n, k)$ is a square matrix with entries $0, \pm 1$ having k non-zero entries per row and column and inner product of distinct rows zero. Hence W satisfies $WW^T = kI_n$, and W is equivalent to an orthogonal design $OD(n; k)$. The number k is called the *weight* of W. If $k = n$, that is, all the entries of W are ± 1 and $WW^T = nI_n$, then W is called an Hadamard matrix of order n. In this case $n = 1, 2$ or $n \equiv 0 (mod\ 4)$.

Given the sequence $A = \{a_1, a_2, \ldots, a_n\}$ of length n the *non-periodic autocorrelation function* $N_A(s)$ is defined as

$$N_A(s) = \sum_{i=1}^{n-s} a_i a_{i+s}, \quad s = 0, 1, \ldots, n-1, \tag{1}$$

If $A(z) = a_1 + a_2 z + \ldots + a_n z^{n-1}$ is the associated polynomial of the sequence A, then

$$A(z)A(z^{-1}) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i a_j z^{i-j} = N_A(0) + \sum_{s=1}^{n-1} N_A(s)(z^s + z^{-s}), z \neq 0. \tag{2}$$

Given A as above of length n the *periodic autocorrelation function* $P_A(s)$ is defined, reducing $i + s$ modulo n, as

$$P_A(s) = \sum_{i=1}^{n} a_i a_{i+s}, \quad s = 0, 1, \ldots, n-1. \tag{3}$$

A $4n \times 4n$ matrix obtained by using four circulant matrices, A, B, C, D of order n, which satisfy

$$AA^T + BB^T + CC^T + DD^T = fI_n,$$

where R be the back diagonal matrix, in the following array, called the *Goethals- Seidel array,*

$$GS = \begin{pmatrix} A & BR & CR & DR \\ -BR & A & D^T R & -C^T R \\ -CR & -D^T R & A & B^T R \\ -DR & C^T R & -B^T R & A \end{pmatrix}$$

is said to be of *Goethals-Seidel type.*

The following theorem which uses four circulant matrices in the Goethals-Seidel array is very useful in our construction for orthogonal designs.

Theorem 1 *[2, Theorem 4.49] Suppose there exist four circulant matrices A, B, C, D of order n satisfying*

$$AA^T + BB^T + CC^T + DD^T = fI_n.$$

then these may be used in the Goethals-Seidel array to obtain a $W(4n, f)$ when A, B, C, D are $(0, 1, -1)$ matrices, and an orthogonal design, $OD(4n; s_1, s_2, \ldots, s_u)$, on $x_1, x_2, \ldots, x_u$, when A, B, C, D have entries from $\{0, \pm x_1, \ldots, \pm x_u\}$ and $f = \sum_{j=1}^{u}(s_j x_j^2)$. □

Corollary 1 *If there are four sequences A, B, C, D, of length n, with entries from $\{0, \pm x_1, \pm x_2, \pm x_3, \pm x_4\}$, with zero periodic or non-periodic autocorrelation function, then these sequences can be used as the first rows of circulant matrices which can be used in the Goethals-Seidel array to form an $OD(4n; s_1, s_2, s_3, s_4)$. If there are sequences of length n with zero non-periodic autocorrelation function, then there are sequences with zero non-periodic autocorrelation and the same list of weights as the sequences of length n, but of length $n + m$ for all $m \geq 0$.* □

This method for constructing orthogonal designs was used in [3, 4, 5, 6, 7]. Furthermore, in [6] a complete study for two variable orthogonal designs in order 36 was given, and some new four variable orthogonal designs of the same order were constructed. More results on three and four variable orthogonal designs in order 36 were recently presented in [3, 4].

2 Number of Possible $n-$tuples

We note the following lemma which has not previously been published but which is useful in determining the size of programs to search for orthogonal designs. The result is obtained by simple counting.

Lemma 1 *Let $n = 4m$ be the order of an orthogonal design. Then the number of cases (k-tuples, $k = 2, 3, 4$) which must be studied to determine whether all orthogonal designs exist is*

(i) $\frac{1}{4}n^2$ *when* $2-$*tuples are considered;*

(ii) N, *when 3-tuples are considered, where*

(a) $N = \frac{n}{72}(2n^2 + 3n - 6)$ *if* $\frac{1}{4}n \equiv 0(mod\ 3)$;

(b) $N = \frac{(n+2)}{72}(2n^2 - n - 4)$ *if* $\frac{1}{4}n \equiv 1(mod\ 3)$;

(c) $N = \frac{(n-2)}{72}(2n^2 + 7n + 8)$ *if* $\frac{1}{4}n \equiv 2(mod\ 3)$.

(iii) N, *when* $4-$*tuples are considered, where*

(a)́ $N = \frac{1}{576}(n^4 + 6n^3 - 2n^2 - 24n)$ *if* $\frac{1}{4}n \equiv 0(mod\ 3)$;

(b)́ $N = \frac{1}{576}(n^4 + 6n^3 - 2n^2 - 24n + 64)$ *if* $\frac{1}{4}n \equiv 1(mod\ 3)$;

(c)́ $N = \frac{1}{576}(n^4 + 6n^3 - 2n^2 - 24n + 64)$ *if* $\frac{1}{4}n \equiv 2(mod\ 3)$.

3 New Algorithm

The algorithm previously used to find OD's via four sequences of length $n \leq 10$ was prohibitively slow for length 11. Hence we tried a new algorithm, which depended on the previous algorithm, to find first a $W(4n, k)$ (or $OD(4n;\ s_1, s_2)$ or $OD(4n;\ s_1, s_2, s_3)$) made with four sequences of length n with $PAF = 0$ or $NPAF = 0$. In the new algorithm MATLAB was used to set up a series of equations to be solved for each individual k (or (s_1, s_2) or (s_1, s_2, s_3)) and then all solutions to these equations were found.

Example 1 We illustrate the algorithm by the following example. We start with the following four sequences of length 9 and type $(5, 9)$ with $NPAF = 0$.

$$\begin{array}{ccccccccc}
b & 0 & -b & 0 & 0 & 0 & 0 & 0 & 0 \\
b & a & -b & 0 & 0 & 0 & 0 & 0 & 0 \\
b & a & 0 & a & -b & 0 & 0 & 0 & 0 \\
b & a & b & -a & b & 0 & 0 & 0 & 0
\end{array}$$

We now fill each of the positions which are presented by 0 by one of the 22 variables $x_1, x_2, \ldots, x_{22}$. Thus we have the sequences

$$\begin{array}{ccccccccc}
b & x_1 & -b & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\
b & a & -b & x_8 & x_9 & x_{10} & x_{11} & x_{12} & x_{13} \\
b & a & x_{14} & a & -b & x_{15} & x_{16} & x_{17} & x_{18} \\
b & a & b & -a & b & x_{19} & x_{20} & x_{21} & x_{22}
\end{array}$$

We now use MATLAB to expand the first rows to make four circulant 9×9 matrices with row inner product zero. This corresponds to forming four sequences with $PAF = 0$. Thus we set up a series of equations, that when solved, yield, among others, the following solutions:

$$\begin{array}{ccccccc|cccccc|}
x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} & x_{11} & x_{12} & x_{13} \\
0 & -1 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 0 & -1 & 1 & 0 \\
0 & -1 & -1 & -1 & 0 & -1 & 1 & 0 & -1 & 0 & 1 & -1 & -1 \\
0 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & 0 & 0 & 1 & -1
\end{array}$$

$$\begin{array}{ccccc|cccc}
x_{14} & x_{15} & x_{16} & x_{17} & x_{18} & x_{19} & x_{20} & x_{21} & x_{22} \\
0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & -1 & 0 & -1 & 0 & 1 \\
0 & 1 & 0 & 0 & -1 & 1 & -1 & -1 & 1
\end{array}$$

The first solution leads to the following new three variable orthogonal design in order 36.

$$\begin{array}{c}
\underline{(5,9,9)} \\
\begin{array}{ccccccccc}
b & 0 & -b & -c & 0 & 0 & 0 & 0 & -c \\
b & a & -b & -c & -c & 0 & -c & c & 0 \\
b & a & 0 & a & -b & c & c & -c & 0 \\
b & a & b & -a & b & 0 & 0 & 0 & 0
\end{array}
\end{array}$$

The second solution gives the following $(5, 9, 14)$ orthogonal design in order 36.

$$\begin{array}{c}
\underline{(5,9,14)} \\
\begin{array}{ccccccccc}
b & 0 & -b & -c & -c & -c & 0 & -c & c \\
b & a & -b & 0 & -c & 0 & c & -c & -c \\
b & a & 0 & a & -b & c & 0 & c & -c \\
b & a & b & -a & b & 0 & -c & 0 & c
\end{array}
\end{array}$$

The third solution gives the following $(5,9,16)$ orthogonal design in order 36.

$$\begin{array}{ccccccccc} & & & & \underline{(5,9,16)} & & & & \\ b & 0 & -b & -c & -c & -c & -c & -c & c \\ b & a & -b & c & -c & 0 & 0 & c & -c \\ b & a & 0 & a & -b & c & 0 & 0 & -c \\ b & a & b & -a & b & c & -c & -c & c \end{array}$$

4 New Orthogonal Designs

In this section we give new sequences with entries from $\{0, \pm a, \pm b, \pm c, \pm d\}$, on the commuting variables a, b, c, d, and zero autocorrelation function. These sequences are used to construct some new orthogonal designs.

Theorem 2 *There are $OD(36; s_1, s_2, s_3, 36 - s_1 - s_2 - s_3)$ constructed using four circulant matrices in the Goethals-Seidel array for the following 4−tuples*

1, 1, 2, 32	1, 3, 8, 24	2, 6, 12, 16	3, 3, 15, 15	5, 5, 13, 13
1, 1, 9, 25	1, 5, 5, 25	2, 8, 8, 18	3, 6, 9, 18	6, 6, 12, 12
1, 1, 17, 17	1, 8, 9, 18	2, 8, 13, 13	4, 8, 8, 16	8, 8, 10, 10
1, 2, 6, 27	2, 2, 16, 16	3, 3, 3, 27	5, 5, 8, 18	9, 9, 9, 9
1, 2, 11, 22	2, 3, 6, 25	3, 3, 6, 24		

Proof. We use the sequences found by Koukouvinos [3] and given in the Appendices A and B, which have zero non-periodic and periodic autocorrelation function respectively, as the first rows of the corresponding circulant matrices in the Goethals-Seidel array to obtain the required orthogonal designs. □

Theorem 3 *There are 4-$NPAF(s_1, s_2, s_3, s_4)$ of length 9 for the following (new) 4-tuples*

(1, 2, 3, 24) (1, 4, 5, 20) (3, 3, 6, 24)

Proof. We use the sequences given in the Appendix A, which have zero non-periodic autocorrelation function. □

Remark 1 The existence of 4-$NPAF(1,2,3,24)$ gives the existence of 4-$NPAF(3,27)$ which improves the result, given in [6] for length 9. □

Remark 2 An exhaustive search shows that there are no 4-$NPAF(3,3,27)$, 4-$NPAF(3,3,3,27)$, 4-$NPAF(6,7,21)$, or 4-$NPAF(2,6,7,21)$ sequences of length 9. □

Using the results of Koukouvinos [3] and further searches we have

Remark 3 An exhaustive search shows that if the orthogonal designs $OD(36; 1,2,8,16)$, $OD(36; 1,2,8,25)$, $OD(36; 1,4,4,25)$, $OD(36; 1,8,8,16)$, $OD(36; 2,2,9,16)$, $OD(36; 2,2,13,13)$, $OD(36; 2,3,4,24)$, $OD(36; 2,8,9,9)$, $OD(36; 6,7,21)$, $OD(36; 2,6,7,21)$, $OD(36; 3,6,8,16)$, $OD(36; 3,8,10,15)$, $OD(36; 4,8,8,9)$, $OD(36; 8,8,9,9)$, $OD(36; 8,9,17)$, $OD(36; 9,13,13)$, and $OD(36; 1,9,13,13)$, exist, then they cannot be constructed using four circulant matrices in the Goethals-Seidel array. □

Remark 4 We have found the following new 4-$NPAF(s_1, s_2, s_3, s_4)$ sequences of length 8.

$$
\begin{array}{llllllllll}
(2,2,8,18) & d & c & 0 & c & -d & d & b & -d \\
 & d & c & 0 & c & -d & -d & -b & d \\
 & d & c & d & -c & d & d & a & -d \\
 & d & c & d & -c & d & -d & -a & d \\
\\
(2,8,10,10) & c & a & -c & b & d & 0 & -d & b \\
 & c & a & -c & -b & -d & 0 & d & -b \\
 & c & d & c & b & d & -c & d & -b \\
 & c & d & c & -b & -d & c & -d & b \\
\\
(4,6,8,12) & a & d & 0 & -d & a & b & d & -c \\
 & a & d & 0 & -d & a & -b & -d & c \\
 & a & d & c & d & -a & b & -d & c \\
 & a & d & c & d & -a & -b & d & -c
\end{array}
$$

which improves the results, given in [6] for length 9. □

Koukouvinos [3] has found many $OD(36; s_1, s_2, s_3)$ but in this paper we give the remainder that were not yet known. Many of the results described in the next theorem are new.

Theorem 4 *There are $OD(36; s_1, s_2, s_3)$ constructed using four circulant matrices in the Goethals-Seidel array for 3-tuples indicated as in Table 1.*

Proof. We use the sequences given in the Appendices A and B, which have zero non-periodic and periodic autocorrelation function respectively, as the first rows of the corresponding circulant matrices in the Goethals-Seidel array to obtain the required orthogonal designs. □

Theorem 5 *There are 4-$NPAF(s_1, s_2, s_3)$, of length 9, for the following (new) 3-tuples*

$(1,2,24)$	$(1,5,24)$	$(3,3,30)$	$(3,9,24)$	$(5,7,18)$	$(6,6,24)$
$(1,2,27)$	$(2,3,24)$	$(3,6,17)$	$(4,5,20)$	$(6,6,12)$	$(6,12,16)$
$(1,3,24)$	$(2,3,25)$	$(3,6,24)$	$(4,5,25)$	$(6,12,12)$	$(8,8,13)$
$(1,3,26)$	$(3,3,24)$	$(3,6,27)$	$(5,5,18)$		

Proof. We use the sequences $(1,2,3,24)$, $(3,3,6,24)$, $(3,6,17)$, $(1,4,5,20)$, $(4,5,25)$, $(5,5,18)$, $(5,7,18)$, $(6,6,12,12)$, $(6,12,16)$, and $(8,8,13)$, given in Appendix A, which have zero non-periodic autocorrelation function. (cf. the results of [4]). □

Theorem 6 *The necessary conditions are sufficient for the existence of the following* 23 *$OD(36; s_1, s_2, s_3, 36 - s_1 - s_2 - s_3)$.*

$(1,1,2,32)$	$(1,3,8,24)$	$(2,6,12,16)$	$(3,3,15,15)$	$(5,5,13,13)$
$(1,1,9,25)$	$(1,5,5,25)$	$(2,8,8,18)$	$(3,6,9,18)$	$(6,6,12,12)$
$(1,1,17,17)$	$(1,8,9,18)$	$(2,8,13,13)$	$(4,8,8,16)$	$(8,8,10,10)$
$(1,2,6,27)$	$(2,2,16,16)$	$(3,3,3,27)$	$(5,5,8,18)$	$(9,9,9,9)$
$(1,2,11,22)$	$(2,3,6,25)$	$(3,3,6,24)$		

A further four, $(1,2,8,25)$, $(1,9,13,13)$, $(2,6,7,21)$ *and* $(3,8,10,15)$, *cannot be constructed using four circulant matrices.*

Proof. We use the sequences $(s_1, s_2, s_3, 36 - s_1 - s_2 - s_3)$ given in the Appendices A and B, which have zero non-periodic and periodic autocorrelation function respectively, as the first rows of the corresponding circulant matrices in the Goethals-Seidel array to obtain the required orthogonal designs. □

Theorem 7 *The necessary conditions are sufficient for the existence of an* $OD(36; s_1, s_2, 36 - s_1 - s_2)$ *constructed using four circulant matrices in each of the following* 54 *cases:*

$(1,1,34)$	$(1,13,22)$	$(2,11,23)$	$(3,10,23)$	$(6,6,24)$	$(8,10,18)$
$(1,2,33)$	$(1,14,21)$	$(2,12,22)$	$(3,11,22)$	$(6,7,23)$	$(8,12,16)$
$(1,3,32)$	$(1,17,18)$	$(2,13,21)$	$(3,15,18)$	$(6,9,21)$	$(8,13,15)$
$(1,5,30)$	$(2,2,32)$	$(2,16,18)$	$(4,8,24)$	$(6,12,18)$	$(9,9,18)$
$(1,6,29)$	$(2,3,31)$	$(2,17,17)$	$(4,16,16)$	$(6,14,16)$	$(9,13,14)$
$(1,8,27)$	$(2,6,28)$	$(3,3,30)$	$(5,5,26)$	$(6,15,15)$	$(10,10,16)$
$(1,9,26)$	$(2,7,27)$	$(3,6,27)$	$(5,6,25)$	$(7,8,21)$	$(10,11,15)$
$(1,10,25)$	$(2,8,26)$	$(3,8,25)$	$(5,8,23)$	$(8,8,20)$	$(10,13,13)$
$(1,11,24)$	$(2,9,25)$	$(3,9,24)$	$(5,13,18)$	$(8,9,19)$	$(12,12,12)$.

Proof. We use the sequences $(s_1, s_2, 36 - s_1 - s_2)$ and $(s_1, s_2, s_3, 36 - s_1 - s_2 - s_3)$ given in the Appendices A and B, which have zero non-periodic and periodic autocorrelation function respectively, as the first rows of the corresponding circulant matrices in the Goethals-Seidel array to obtain the required orthogonal designs.

The sequences for $(1,1,34)$, $(1,6,29)$, $(2,2,32)$, $(2,7,27)$ and $(2,17,17)$ were given in [6]. □

Lemma 2 *Replacing the variables of the known* $OD(36; s_1, s_2, s_3, 36 - s_1 - s_2 - s_3)$, $OD(36; s_1, s_2, 36 - s_1 - s_2)$ *and* $OD(36; s_1, 36 - s_1)$ *by* ± 1 *leads to more than* 200 *potentially inequivalent Hadamard matrices of order* 36.

Theorem 8 *There are* $OD(36; s_1, 36 - s_1)$, *constructed using* 4*-NPAF sequences to obtain four circulant matrices for use in the Goethals-Seidel array, for the following* 2*−tuples*

$1,35$	$3,33$	$5,31$	$7,29$	$9,27$	$11,25$	$13,23$	$15,21$	$17,19$
$2,34$	$4,32$	$6,30$	$8,28$	$10,26$	$12,24$	$14,22$	$16,20$	$18,18$.

No cases are undecided.

Lemma 3 *Replacing the variables of the known* $OD(36; 1, s_1, 35 - s_1)$ *by* ± 1 *gives at least* 48 *potentially inequivalent skew-Hadamard matrices of order* 36.

Remark 5 An exhaustive search shows that there are no 4-$NPAF(s_1, s_2)$ sequences of length 9 for $(s_1, s_2) = (3,31),\ (5,30),\ (6,29),\ (8,27),$ *or* $(13,22)$. □

5 Summary

In this section we summarize all known results for three and four variable designs in order 36. The existence of designs constructed using four circulant matrices in the Goethals-Seidel array is completely resolved.

Remark 6 There are 1347 possible 3-tuples. Table 1 lists the 433 3-tuples which may correspond to designs in order 36: 914 cases correspond to 3-tuples eliminated by number theory. The design is known to be able to be constructed using four circulant matrices in the Goethals-Seidel array for 429 cases. For 4 cases, if designs exist for the corresponding 3-tuple, they cannot be constructed using circulant matrices (Y). P indicates that 4-PAF sequences with length 9 exist; n indicates 4-$NPAF$ sequences with length n exist. □

s_1,s_2,s_3	n	s_1,s_2,s_3	n	s_1,s_2,s_3	n	s_1,s_2,s_3	n	s_1,s_2,s_3	n
(1,1,1)	1	(1,1,2)	1	(1,1,4)	2	(1,1,5)	3	(1,1,8)	3
(1,1,9)	7	(1,1,10)	3	(1,1,13)	5	(1,1,16)	7	(1,1,17)	7
(1,1,18)	6	(1,1,20)	6	(1,1,25)	9	(1,1,26)	P	(1,1,29)	P
(1,1,32)	9	(1,1,34)	P	(1,2,2)	2	(1,2,3)	2	(1,2,4)	2
(1,2,6)	3	(1,2,8)	3	(1,2,9)	3	(1,2,11)	5	(1,2,12)	5
(1,2,16)	7	(1,2,17)	5	(1,2,18)	6	(1,2,19)	6	(1,2,22)	7
(1,2,24)	9	(1,2,25)	9	(1,2,27)	9	(1,2,32)	P	(1,2,33)	9
(1,3,6)	3	(1,3,8)	3	(1,3,14)	6	(1,3,18)	6	(1,3,24)	7
(1,3,26)	9	(1,3,32)	9	(1,4,4)	5	(1,4,5)	5	(1,4,8)	5
(1,4,9)	5	(1,4,10)	5	(1,4,13)	7	(1,4,16)	7	(1,4,17)	7
(1,4,18)	7	(1,4,20)	7	(1,4,25)	9	(1,4,26)	P	(1,4,29)	9
(1,5,5)	3	(1,5,6)	3	(1,5,9)	5	(1,5,14)	5	(1,5,16)	7
(1,5,20)	9	(1,5,21)	P	(1,5,24)	9	(1,5,25)	P	(1,5,30)	P
(1,6,8)	5	(1,6,11)	5	(1,6,12)	7	(1,6,14)	7	(1,6,18)	7
(1,6,20)	9	(1,6,21)	7	(1,6,27)	P	(1,6,29)	P	(1,8,8)	7
(1,8,9)	5	(1,8,11)	5	(1,8,12)	7	(1,8,16)	7	(1,8,17)	9
(1,8,18)	9	(1,8,19)	P	(1,8,22)	P	(1,8,24)	P	(1,8,25)	P
(1,8,27)	P	(1,9,9)	7	(1,9,10)	5	(1,9,13)	9	(1,9,16)	7
(1,9,17)	P	(1,9,18)	7	(1,9,20)	9	(1,9,25)	P	(1,9,26)	9
(1,10,10)	7	(1,10,11)	7	(1,10,14)	P	(1,10,16)	P	(1,10,19)	9
(1,10,25)	P	(1,11,18)	P	(1,11,22)	P	(1,11,24)	P	(1,12,14)	P
(1,12,18)	P	(1,13,13)	9	(1,13,14)	P	(1,13,16)	P	(1,13,17)	P
(1,13,22)	P	(1,14,19)	P	(1,14,21)	P	(1,16,16)	9	(1,16,17)	9
(1,17,17)	P	(1,17,18)	9	(2,2,2)	2	(2,2,4)	2	(2,2,5)	3
(2,2,8)	3	(2,2,9)	5	(2,2,10)	5	(2,2,13)	5	(2,2,16)	7
(2,2,17)	7	(2,2,18)	6	(2,2,20)	7	(2,2,25)	P	(2,2,26)	9
(2,2,29)	P	(2,2,32)	9	(2,3,4)	3	(2,3,6)	3	(2,3,7)	3
(2,3,9)	5	(2,3,10)	7	(2,3,15)	7	(2,3,16)	7	(2,3,22)	P
(2,3,24)	9	(2,3,25)	9	(2,3,28)	9	(2,3,31)	P	(2,4,4)	3
(2,4,6)	3	(2,4,8)	5	(2,4,9)	5	(2,4,11)	5	(2,4,12)	7
(2,4,16)	7	(2,4,17)	7	(2,4,18)	7	(2,4,19)	7	(2,4,22)	9
(2,4,24)	9	(2,4,25)	P	(2,4,27)	P	(2,5,5)	3	(2,5,7)	5
(2,5,8)	5	(2,5,13)	6	(2,5,18)	7	(2,5,20)	P	(2,5,22)	P
(2,5,23)	P	(2,6,7)	5	(2,6,9)	5	(2,6,12)	6	(2,6,13)	7
(2,6,16)	7	(2,6,19)	P	(2,6,21)	P	(2,6,25)	P	(2,6,27)	P
(2,6,28)	P	(2,7,10)	7	(2,7,12)	7	(2,7,13)	7	(2,7,19)	P
(2,7,20)	P	(2,7,21)	P	(2,7,24)	P	(2,7,27)	P	(2,8,8)	5
(2,8,9)	5	(2,8,10)	5	(2,8,13)	7	(2,8,16)	7	(2,8,17)	P
(2,8,18)	7	(2,8,20)	9	(2,8,25)	Y	(2,8,26)	9	(2,9,9)	5
(2,9,11)	6	(2,9,12)	7	(2,9,16)	P	(2,9,17)	P	(2,9,18)	P
(2,9,19)	P	(2,9,22)	P	(2,9,25)	P	(2,10,10)	6	(2,10,12)	6
(2,10,15)	9	(2,10,18)	9	(2,11,16)	P	(2,11,22)	P	(2,11,23)	P
(2,12,15)	9	(2,12,16)	9	(2,12,22)	P	(2,13,13)	P	(2,13,15)	9
(2,13,18)	P	(2,13,21)	P	(2,16,16)	9	(2,16,18)	9	(2,17,17)	P

Table 1: The existence of $OD(36; s_1, s_2, s_3)$.

s_1, s_2, s_3	n	s_1, s_2, s_3	n	s_1, s_2, s_3	n	s_1, s_2, s_3	n	s_1, s_2, s_3	n
(3,3,3)	3	(3,3,6)	3	(3,3,12)	7	(3,3,15)	7	(3,3,24)	9
(3,3,27)	P	(3,3,30)	9	(3,4,6)	5	(3,4,8)	5	(3,4,14)	7
(3,4,18)	7	(3,4,24)	P	(3,4,26)	P	(3,6,6)	5	(3,6,8)	5
(3,6,9)	5	(3,6,11)	5	(3,6,12)	7	(3,6,16)	7	(3,6,17)	9
(3,6,18)	7	(3,6,19)	7	(3,6,22)	P	(3,6,24)	9	(3,6,25)	P
(3,6,27)	9	(3,7,8)	6	(3,7,11)	7	(3,7,15)	P	(3,7,18)	7
(3,7,23)	P	(3,8,9)	7	(3,8,10)	7	(3,8,15)	7	(3,8,16)	P
(3,8,22)	P	(3,8,24)	P	(3,8,25)	P	(3,9,14)	7	(3,9,18)	P
(3,9,24)	9	(3,10,15)	9	(3,10,17)	9	(3,10,18)	P	(3,10,23)	P
(3,11,19)	P	(3,11,22)	P	(3,12,12)	P	(3,12,15)	9	(3,14,16)	P
(3,15,15)	P	(3,15,18)	P	(4,4,4)	3	(4,4,5)	5	(4,4,8)	7
(4,4,9)	5	(4,4,10)	5	(4,4,13)	7	(4,4,16)	7	(4,4,17)	7
(4,4,18)	9	(4,4,20)	7	(4,4,25)	P	(4,4,26)	9	(4,5,5)	5
(4,5,6)	5	(4,5,9)	5	(4,5,14)	7	(4,5,16)	7	(4,5,20)	9
(4,5,21)	9	(4,5,24)	P	(4,5,25)	9	(4,6,8)	5	(4,6,11)	7
(4,6,12)	7	(4,6,14)	7	(4,6,18)	7	(4,6,20)	8	(4,6,21)	P
(4,8,8)	7	(4,8,9)	7	(4,8,11)	7	(4,8,12)	7	(4,8,16)	7
(4,8,17)	9	(4,8,18)	8	(4,8,19)	P	(4,8,22)	9	(4,8,24)	P
(4,9,9)	6	(4,9,10)	7	(4,9,13)	9	(4,9,16)	P	(4,9,17)	P
(4,9,18)	P	(4,9,20)	P	(4,10,10)	7	(4,10,11)	P	(4,10,14)	7
(4,10,16)	9	(4,10,19)	P	(4,11,18)	P	(4,12,14)	8	(4,12,18)	9
(4,13,13)	P	(4,13,14)	P	(4,13,16)	P	(4,13,17)	P	(4,16,16)	9
(5,5,5)	7	(5,5,8)	7	(5,5,9)	5	(5,5,10)	5	(5,5,13)	7
(5,5,16)	7	(5,5,17)	P	(5,5,18)	9	(5,5,20)	9	(5,5,25)	P
(5,5,26)	P	(5,6,9)	7	(5,6,16)	P	(5,6,25)	P	(5,7,8)	7
(5,7,18)	9	(5,7,22)	P	(5,8,8)	7	(5,8,13)	7	(5,8,18)	P
(5,8,20)	P	(5,8,23)	P	(5,9,9)	P	(5,9,14)	P	(5,9,16)	P
(5,9,20)	P	(5,10,10)	7	(5,10,15)	9	(5,13,13)	P	(5,13,18)	P
(6,6,6)	7	(6,6,12)	7	(6,6,15)	P	(6,6,24)	9	(6,7,8)	P
(6,7,18)	P	(6,7,21)	Y	(6,7,23)	P	(6,8,9)	7	(6,8,12)	7
(6,8,13)	P	(6,8,16)	8	(6,8,19)	P	(6,9,11)	P	(6,9,12)	P
(6,9,14)	P	(6,9,18)	P	(6,9,21)	P	(6,11,12)	P	(6,11,16)	P
(6,12,12)	8	(6,12,16)	9	(6,12,18)	9	(6,14,16)	P	(6,15,15)	P
(7,7,7)	7	(7,7,14)	7	(7,8,10)	P	(7,8,12)	P	(7,8,13)	7
(7,8,19)	P	(7,8,21)	P	(7,11,12)	P	(7,12,15)	P	(8,8,8)	7
(8,8,9)	7	(8,8,10)	7	(8,8,13)	9	(8,8,16)	9	(8,8,17)	P
(8,8,18)	9	(8,8,20)	P	(8,9,9)	7	(8,9,11)	P	(8,9,12)	P
(8,9,16)	P	(8,9,17)	Y	(8,9,18)	P	(8,9,19)	P	(8,10,10)	7
(8,10,12)	8	(8,10,15)	P	(8,10,18)	9	(8,12,16)	P	(8,13,13)	P
(8,13,15)	P	(9,9,9)	9	(9,9,10)	P	(9,9,13)	P	(9,9,16)	P
(9,9,18)	9	(9,10,10)	P	(9,10,11)	P	(9,10,14)	P	(9,13,13)	Y
(9,13,14)	P	(10,10,10)	9	(10,10,13)	P	(10,10,16)	P	(10,11,15)	9
(10,13,13)	P	(11,11,11)	P	(12,12,12)	9				

Table 1(Cont): The existence of $OD(36; s_1, s_2, s_3)$.

Remark 7 There are 3396 possible 4-tuples. 3242 4-tuples do not satisy the necessary number theoretic conditions for the existence of 4−variable designs. So for the 154 cases which satisfy the number theoretic necessary conditions for the existence of 4−variable designs, 7 are excluded, in order 36 but not necessarily in higher orders by the Geramita-Verner theorem : $(1,5,9,20)$, $(1,9,9,16)$, $(2,5,8,20)$, $(2,8,9,16)$, $(5,5,5,20)$, $(5,5,9,16)$ and $(6,8,9,12)$; 133 are known to exist; the following 14 cannot exist by a construction using four circulant matrices.

1 2 8 16	1 8 8 16	2 2 13 13	2 6 7 21	3 6 8 16	4 8 8 9
1 2 8 25	1 9 13 13	2 3 4 24	2 8 9 9	3 8 10 15	8 8 9 9
1 4 4 25	2 2 9 16				

There are no unresolved cases. □

Remark 8 Table 2 shows that there are no unresolved cases with regard to the existence of $OD(36; s_1, s_2, s_3, s_4)$ constructed using four circulant matrices in the Goethals-Seidel array. □

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(Y) indicates that, if the designs exist, they cannot be constructed using circulant matrices.
P indicates that there are 4-PAF sequences of length 9.
* indicates that there are 4-PAF sequences for every odd length ≥ 7.
n indicates that there are 4-$NPAF$ sequences giving the design for every length $\geq n$.

4 -tuple	wt	ref	n	4 -tuple	wt	ref	n	4 -tuple	wt	ref	n
1 1 1 1	4	[GS]	1	1 4 8 18	31	AppB	P	2 8 8 18	36	AppB	P
1 1 1 4	7	[GS]	3	1 4 9 9	23	AppB	P	2 8 9 9	28		Y
1 1 1 9	12	[GS]	5	1 4 9 16	30	AppB	P	2 8 10 10	30	Rem4	8
1 1 1 16	19	[GS]	*, P	1 4 10 10	25	AppB	10, P	2 8 13 13	36	AppB	14, P
1 1 1 25	28	[6]	P	1 4 13 13	31	AppB	14, P	3 3 3 3	12	[GS]	3
1 1 2 2	6	[GS]	2	1 5 5 9	20	[2]	5	3 3 3 12	21	[5]	6
1 1 2 8	12	[GS]	3	1 5 5 16	27	AppB	P	3 3 3 27	36	AppB	P, 16
1 1 2 18	22	[2]	6	1 5 5 25	36	[3]	P	3 3 6 6	18	[GS]	5
1 1 2 32	36	[6]	P	1 6 8 12	27	AppB	P	3 3 6 24	36	AppA	9
1 1 4 4	10	[GS]	3	1 8 8 9	26	AppB	P	3 3 12 12	30	AppB	P, 12
1 1 4 9	15	[2]	5	1 8 8 16	33		Y	3 3 15 15	36	AppB	P
1 1 4 16	22	[GS]	7	1 8 9 18	36	AppB	P	3 4 6 8	21	AppB	P
1 1 4 25	31	[6]	P	1 9 9 9	28	AppB	P	3 4 6 18	31	AppB	P
1 1 5 5	12	[GS]	3	1 9 10 10	30	AppB	P	3 6 6 12	27	AppB	P, 12
1 1 5 20	27	[6]	P	1 9 13 13	36		Y	3 6 8 9	26	AppB	P
1 1 8 8	18	[GS]	5	2 2 2 2	8	[GS]	2	3 6 8 16	33		Y
1 1 8 18	28	[3]	P	2 2 2 8	14	[GS]	5	3 6 9 18	36	AppB	12, P
1 1 9 9	20	[GS]	5	2 2 2 18	24	[2]	7	3 8 10 15	36		Y
1 1 9 16	27	AppB	P	2 2 4 4	12	[GS]	5	4 4 4 4	16	[GS]	4
1 1 9 25	36	[6]	P	2 2 4 9	17	[2]	5	4 4 5 5	18	[GS]	5
1 1 13 13	28	[GS]	*, 14, P	2 2 4 16	24	[GS]	6	4 4 4 9	21	AppB	P
1 1 10 10	22	[2]	6	2 2 4 25	33	[6]	P	4 4 4 16	28	[GS]	7
1 1 16 16	34	[6]	9	2 2 5 5	14	[GS]	6	4 4 5 20	33		P, 18
1 1 17 17	36	[6]	P	2 2 5 20	29	AppB	18, P	4 4 8 8	24	[GS]	6
1 2 2 4	9	[GS]	3	2 2 8 8	20	[GS]	5	4 4 8 18	34	[6]	9
1 2 2 9	14	[GS]	5	2 2 8 18	30	Rem4	8	4 4 9 9	26	AppB	P
1 2 2 16	21	[2]	7	2 2 9 9	22	[GS]	6	4 4 9 16	33	AppB	P
1 2 2 25	30	[6]	P	2 2 9 16	29		Y	4 4 10 10	28	[GS]	7
1 2 3 6	12	[GS]	3	2 2 10 10	24	[GS]	6	4 4 13 13	34	AppB	P, 14
1 2 3 24	30	AppA	9	2 2 13 13	30		14, Y	4 5 5 9	23	AppA	9
1 2 4 8	15	[GS]	5	2 2 16 16	36	[6]	9	4 5 5 16	30	AppB	P
1 2 4 18	25	[2]	7	2 3 4 6	15	[GS]	5	4 6 8 12	30	[3]	8
1 2 6 12	21	[GS]	7	2 3 4 24	33		Y	4 8 8 9	29		Y
1 2 6 27	36	[6]	P	2 3 6 9	20	[2]	5	4 8 8 16	36	AppB	P, 10
1 2 8 9	20	[2]	5	2 3 6 16	27	AppB	P	4 9 9 9	31	AppB	P
1 2 8 16	27		Y	2 3 6 25	36	[3]	P	4 9 10 10	33	AppB	P
1 2 8 25	36		Y	2 3 10 15	30	AppA	9	5 5 5 5	20	[GS]	5
1 2 9 18	30	AppB	P	2 4 4 8	18	[GS]	5	5 5 8 8	26	[GS]	7
1 2 11 22	36	[3]	P	2 4 4 18	28	[6]	9	5 5 8 18	36	AppB	P
1 3 6 8	18	[2]	6	2 4 6 12	24	[GS]	6	5 5 9 9	28	AppB	P
1 3 6 18	28	[2]	7	2 4 8 9	23	[2]	7	5 5 10 10	30	[6]	9
1 3 8 24	36	[6]	P	2 4 8 16	30	[6]	9	5 5 13 13	36	AppB	P, 14
1 4 4 4	13	[GS]	5	2 4 9 18	33	AppB	P	6 6 6 6	24	[GS]	6
1 4 4 9	18	[3]	9	2 5 5 8	20	[GS]	5	6 6 12 12	36	[3]	9
1 4 4 16	25	[2]	7	2 5 5 18	30	AppB	P	7 7 7 7	28	[GS]	7
1 4 4 25	34		Y	2 6 7 21	36		Y	8 8 8 8	32	[GS]	8
1 4 5 5	15	[GS]	5	2 6 9 12	29	AppB	P	8 8 9 9	34		Y
1 4 5 20	30	AppA	9	2 6 12 16	36	AppB	10, P	8 8 10 10	36	AppB	10, P
1 4 8 8	21	[GS]	6	2 8 8 8	26	[GS]	7	9 9 9 9	36	[6]	9

Table 2: The existence of $OD(36; s_1, s_2, s_3, s_4)$.

Appendix A: Order 36 (Sequences with zero non-periodic autocorrelation function)

Design	A_1									A_2, A_3									A_4								
$(1,1,25)$	b	$-b$	0	$-a$	0	b	$-b$	0	0	b	b	$-b$	c	b	$-b$	$-b$	0	0									
										b	b	0	$-b$	$-b$	$-b$	b	$-b$	b	b	b	0	b	0	b	b	b	$-b$
$(1,2,3,24)$	c	c	c	$-b$	0	$-a$	$-c$	c	$-c$	c	c	$-c$	$-b$	0	a	$-c$	c	c									
										c	c	c	0	b	0	c	$-c$	c	c	c	$-c$	0	d	0	c	$-c$	$-c$
$(1,2,25)$	b	b	$-b$	0	$-a$	0	b	$-b$	$-b$	b	b	b	0	b	0	b	$-b$	b									
										b	$-b$	$-b$	0	b	0	$-c$	b	$-b$	b	b	c	0	$-b$	0	$-b$	$-b$	b
$(1,2,33)$	a	$-b$	b	b	b	b	b	$-b$	b	a	b	$-b$	$-b$	$-b$	$-b$	$-b$	b	$-b$									
										b	b	$-b$	$-b$	$-b$	b	b	$-b$	b	b	$-b$	b	b	c	$-b$	$-b$	b	$-b$
$(1,3,32)$	b	b	b	b	a	$-b$	$-b$	b	b	b	b	$-b$	b	$-a$	b	$-b$	b	b									
										b	b	$-b$	b	a	b	$-b$	$-b$	$-b$	b	b	$-b$	b	c	$-b$	b	$-b$	$-b$
$(1,4,5,20)$	b	0	b	$-a$	$-d$	a	$-b$	0	$-b$	b	$-b$	$-b$	$-a$	0	$-a$	$-b$	b	b									
										b	0	b	$-c$	$-c$	c	b	0	b	b	$-b$	$-b$	$-c$	0	$-c$	b	$-b$	$-b$
$(1,6,20)$	b	0	b	0	$-a$	0	$-b$	0	$-b$	b	$-b$	$-b$	c	$-c$	$-c$	$-b$	b	b									
										b	0	b	0	c	0	b	0	b	b	$-b$	$-b$	c	0	c	b	$-b$	$-b$
$(1,8,17)$	b	0	b	$-a$	$-b$	a	b	0	b	b	0	b	$-a$	c	a	$-b$	0	$-b$									
										b	0	0	a	b	a	b	0	$-b$	b	0	0	$-a$	b	$-a$	$-b$	0	b
$(1,8,18)$	a	0	b	c	$-b$	0	$-a$	0	0	a	b	0	b	$-b$	b	b	b	$-a$									
										a	b	b	0	$-b$	$-b$	a	$-b$	0	a	b	$-b$	b	b	$-b$	0	0	a
$(1,9,13)$	b	$-a$	0	0	0	a	b	0	0	b	$-a$	b	0	b	0	$-b$	a	b									
										b	a	c	$-a$	$-b$	0	0	0	0	b	a	0	a	b	0	$-b$	a	$-b$
$(1,9,26)$	a	$-b$	a	b	$-b$	b	$-b$	$-b$	$-b$	b	$-b$	b	b	$-b$	$-a$	a	a	a									
										b	a	b	$-a$	a	$-b$	b	b	b	b	b	b	$-b$	c	b	$-b$	$-b$	$-b$

Appendix A(cont): Order 36 (Sequences with zero non-periodic autocorrelation function)

Design	A_1									A_2, A_3									A_4								
$(1,10,19)$	b	a	0	a	$-c$	$-a$	0	$-a$	$-b$	b	0	b	0	b	0	$-b$	a	b									
										a	$-b$	a	$-b$	a	b	a	0	$-a$	a	a	a	$-a$	$-a$	a	a	$-a$	a
$(1,13,13)$	b	$-a$	$-b$	a	b	a	b	a	b	a	$-b$	$-b$	0	a	0	b	$-b$	a									
										a	$-b$	0	0	$-c$	0	0	b	$-a$	b	$-a$	0	a	0	a	0	$-a$	$-b$
$(1,17,18)$	b	a	$-b$	b	$-b$	$-b$	a	a	a	b	$-a$	b	$-a$	a	a	b	a	b									
										b	a	$-b$	$-b$	b	b	$-a$	a	$-a$	b	a	b	$-a$	c	a	$-b$	$-a$	$-b$
$(2,3,10,15)$	b	d	$-b$	0	$-b$	c	0	0	a	b	d	$-b$	0	b	$-c$	0	0	$-a$									
										a	a	b	$-b$	$-c$	$-b$	$-b$	a	$-a$	a	a	b	$-b$	b	b	b	$-a$	a
$(2,12,15)$	a	$-b$	0	b	a	b	$-b$	$-c$	b	a	$-b$	0	b	a	b	b	c	$-b$									
										a	$-b$	$-a$	0	a	$-b$	a	0	0	a	$-b$	a	$-b$	$-a$	$-b$	$-a$	0	0
$(2,29)$	b	0	b	0	a	$-b$	$-b$	b	$-b$	b	0	$-b$	0	a	b	b	$-b$	$-b$									
										b	$-b$	b	$-b$	b	b	b	b	b	b	$-b$	$-b$	b	0	$-b$	b	b	b
$(3,3,6,24)$	d	$-d$	$-d$	a	$-c$	b	$-d$	$-d$	d	d	$-d$	d	a	c	$-b$	$-d$	$-d$	$-d$									
										d	d	d	$-c$	a	c	d	$-d$	d	d	$-d$	$-d$	$-c$	$-b$	$-c$	d	d	$-d$
$(3,6,17)$	c	b	c	0	a	b	$-c$	0	$-c$	a	$-b$	c	$-b$	c	b	0	0	0									
										b	c	0	0	0	$-a$	0	c	0	c	c	c	$-c$	$-c$	c	c	$-c$	c
$(3,27)$	b	0	a	$-b$	$-b$	b	$-b$	b	b	b	b	$-b$	0	a	b	$-b$	$-b$	$-b$									
										b	b	$-b$	b	$-b$	0	$-a$	0	$-b$	b	b	b	0	b	0	$-b$	b	b
$(3,27)$	b	0	b	0	a	b	$-b$	b	b	b	0	0	0	$-a$	$-b$	b	b	b									
										b	b	0	b	$-a$	b	$-b$	$-b$	$-b$	b	b	$-b$	$-b$	b	b	$-b$	b	$-b$
$(3,27)$	a	$-b$	b	b	b	0	b	0	0	a	$-b$	$-b$	b	$-b$	0	$-b$	0	0									
										b	$-b$	b	b	a	b	$-b$	$-b$	$-b$	b	b	b	$-b$	$-b$	b	b	$-b$	b

Appendix A(cont): Order 36 (Sequences with zero non-periodic autocorrelation function)

Design	A_1	A_2, A_3	A_4
$(4,5,5,9)$	$d\ \ 0\ \ -c\ \ 0\ \ 0\ \ 0\ \ d\ \ 0\ \ c$	$a\ \ 0\ \ 0\ \ c\ \ a\ \ -c\ \ -b\ \ 0\ \ 0$	
		$d\ \ -b\ \ 0\ \ -a\ \ c\ \ -b\ \ -d\ \ 0\ \ -c$	$b\ \ -c\ \ -a\ \ -c\ \ -b\ \ 0\ \ a\ \ -c\ \ 0$
$(4,5,25)$	$c\ \ b\ \ 0\ \ a\ \ -c\ \ a\ \ c\ \ -b\ \ -c$	$c\ \ b\ \ 0\ \ a\ \ -c\ \ -a\ \ -c\ \ b\ \ c$	
		$c\ \ c\ \ c\ \ -c\ \ b\ \ -c\ \ -c\ \ c\ \ -c$	$c\ \ -c\ \ c\ \ c\ \ c\ \ -c\ \ c\ \ c\ \ c$
$(4,8,17)$	$b\ \ -c\ \ b\ \ 0\ \ -b\ \ 0\ \ b\ \ c\ \ b$	$b\ \ c\ \ b\ \ 0\ \ 0\ \ 0\ \ -b\ \ c\ \ -b$	
		$b\ \ a\ \ 0\ \ a\ \ b\ \ -a\ \ -b\ \ a\ \ b$	$b\ \ a\ \ -b\ \ a\ \ -b\ \ a\ \ 0\ \ -a\ \ -b$
$(4,9,13)$	$b\ \ 0\ \ 0\ \ -c\ \ b\ \ 0\ \ a\ \ c\ \ -a$	$b\ \ -b\ \ b\ \ b\ \ b\ \ -a\ \ 0\ \ a\ \ 0$	
		$b\ \ 0\ \ 0\ \ -c\ \ -a\ \ 0\ \ -b\ \ -c\ \ a$	$a\ \ -b\ \ a\ \ b\ \ a\ \ b\ \ 0\ \ -b\ \ 0$
$(5,5,18)$	$a\ \ b\ \ -a\ \ 0\ \ -b\ \ -b\ \ -b\ \ 0\ \ -b$	$c\ \ b\ \ -c\ \ 0\ \ b\ \ -b\ \ b\ \ 0\ \ b$	
		$b\ \ -b\ \ -b\ \ b\ \ 0\ \ c\ \ 0\ \ c\ \ a$	$c\ \ -a\ \ 0\ \ -a\ \ 0\ \ b\ \ b\ \ -b\ \ -b$
$(5,7,18)$	$b\ \ -b\ \ -a\ \ c\ \ a\ \ -b\ \ 0\ \ -b\ \ -b$	$b\ \ b\ \ b\ \ c\ \ c\ \ -c\ \ 0\ \ -b\ \ b$	
		$b\ \ -b\ \ -a\ \ c\ \ -b\ \ c\ \ 0\ \ b\ \ 0$	$b\ \ a\ \ -b\ \ a\ \ c\ \ b\ \ 0\ \ -b\ \ 0$
$(5,22)$	$b\ \ b\ \ 0\ \ -a\ \ 0\ \ -a\ \ 0\ \ b\ \ -b$	$b\ \ 0\ \ b\ \ -a\ \ a\ \ a\ \ b\ \ 0\ \ b$	
		$b\ \ 0\ \ b\ \ b\ \ 0\ \ -b\ \ -b\ \ b\ \ -b$	$b\ \ -b\ \ 0\ \ b\ \ -b\ \ -b\ \ -b\ \ 0\ \ b$
$(6,6,12,12)$	$c\ \ d\ \ a\ \ c\ \ d\ \ b\ \ -c\ \ -d\ \ a$	$c\ \ -d\ \ b\ \ c\ \ -d\ \ -a\ \ -c\ \ d\ \ b$	
		$c\ \ d\ \ a\ \ -c\ \ d\ \ -a\ \ c\ \ d\ \ -a$	$c\ \ -d\ \ b\ \ c\ \ d\ \ -b\ \ c\ \ -d\ \ -b$
$(6,12,16)$	$a\ \ c\ \ b\ \ -c\ \ b\ \ b\ \ c\ \ -a\ \ c$	$a\ \ c\ \ b\ \ -c\ \ b\ \ -b\ \ -c\ \ a\ \ -c$	
		$a\ \ c\ \ 0\ \ c\ \ -b\ \ b\ \ c\ \ -b\ \ -c$	$a\ \ c\ \ 0\ \ c\ \ -b\ \ -b\ \ -c\ \ b\ \ c$
$(7,20)$	$a\ \ 0\ \ 0\ \ -b\ \ 0\ \ 0\ \ a\ \ -b\ \ 0$	$a\ \ b\ \ b\ \ a\ \ b\ \ 0\ \ -a\ \ b\ \ 0$	
		$b\ \ b\ \ 0\ \ a\ \ -b\ \ 0\ \ -b\ \ -b\ \ b$	$b\ \ b\ \ -b\ \ -b\ \ a\ \ b\ \ -b\ \ b\ \ -b$
$(7,27)$	$a\ \ b\ \ -b\ \ -b\ \ -b\ \ b\ \ -b\ \ -b\ \ a$	$a\ \ -b\ \ b\ \ b\ \ a\ \ b\ \ b\ \ b\ \ -a$	
		$b\ \ a\ \ -b\ \ 0\ \ -b\ \ b\ \ b\ \ b\ \ -b$	$b\ \ -b\ \ b\ \ -b\ \ -b\ \ 0\ \ -b\ \ -a\ \ b$

Appendix A(cont): Order 36 (Sequences with zero non-periodic autocorrelation function)

Design	A_1	A_2, A_3	A_4
$(7, 29)$	a b $-b$ $-b$ $-b$ b $-b$ $-b$ a	a b b $-b$ a b b b $-a$	
		b b a b $-b$ $-b$ b $-b$ b	b $-b$ b b b $-b$ a $-b$ $-b$
$(8, 8, 13)$	a a $-a$ a b b b $-b$ 0	a $-a$ $-a$ $-a$ b $-b$ b b 0	
		c 0 $-c$ c $-c$ 0 0 c c	c c $-c$ c c c 0 0 $-c$
$(8, 10, 18)$	b $-a$ c $-b$ c $-b$ $-c$ a b	c a $-b$ $-b$ $-c$ $-b$ $-b$ $-a$ $-c$	
		c a b $-b$ $-b$ b $-b$ a c	b a $-c$ $-b$ b b $-c$ a $-b$
$(10, 11, 15)$	b $-c$ $-b$ b $-c$ $-c$ $-c$ $-a$ c	b $-c$ $-b$ $-b$ c $-c$ c a $-c$	
		a a b $-b$ $-c$ $-b$ $-b$ a $-a$	a a b $-b$ b b b $-a$ a

Appendix B: Order 36 (Sequences with zero periodic autocorrelation function)

Design	A_1,	A_2, A_3		A_4
$(1,1,1,16)$	$a\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$	$b\ 0\ -d\ -d\ d\ -d\ d\ d\ 0$	$d\ d\ -c\ -d\ -d\ 0\ d\ -d\ 0$	$d\ 0\ d\ 0\ d\ 0\ d\ 0\ 0$
$(1,1,9,16)$	$a\ -d\ -a\ -d\ -d\ 0\ -d\ 0\ 0$	$a\ b\ -a\ -d\ 0\ -d\ d\ 0\ d$	$a\ 0\ c\ 0\ -a\ d\ d\ -d\ -d$	$a\ -d\ a\ d\ a\ -d\ d\ 0\ 0$
$(1,1,13,13)$	$a\ -a\ -b\ -b\ -b\ -a\ -a\ 0\ 0$	$b\ -b\ a\ a\ a\ -b\ -b\ 0\ 0$	$a\ -b\ b\ d\ -b\ b\ -a\ 0\ 0$	$a\ -a\ -b\ 0\ 0\ b\ a\ -a\ -c$
$(1,2,9,18)$	$c\ a\ -c\ -d\ d\ 0\ 0\ -d\ d$	$c\ c\ c\ -d\ -d\ 0\ 0\ d\ d$	$c\ -c\ b\ 0\ -d\ -d\ -d\ d\ -d$	$c\ -c\ -b\ 0\ -d\ -d\ d\ -d\ -d$
$(1,2,11,22)$	$a\ -d\ -d\ -d\ d\ -d\ d\ d\ d$	$b\ c\ d\ -d\ -c\ d\ d\ c\ d$	$b\ -c\ -d\ c\ -d\ c\ -c\ -c\ -d$	$c\ -d\ c\ d\ -d\ -d\ c\ d\ -d$
$(1,4,8,18)$	$b\ a\ 0\ -b\ -d\ b\ 0\ -a\ -b$	$a\ b\ -a\ c\ -b\ -b\ 0\ b\ c$	$a\ 0\ a\ -c\ b\ b\ 0\ b\ c$	$a\ b\ -b\ a\ -b\ -b\ -b\ b\ -b$
$(1,4,9,9)$	$a\ 0\ -a\ 0\ -d\ -d\ 0\ -d\ 0$	$a\ b\ -a\ 0\ d\ 0\ 0\ -d\ 0$	$a\ c\ 0\ c\ -a\ -d\ 0\ d\ 0$	$a\ c\ a\ -c\ a\ -d\ d\ 0\ 0$
$(1,4,9,16)$	$b\ 0\ -b\ -c\ -c\ -c\ -c\ -c\ c$	$b\ d\ -b\ c\ -c\ 0\ 0\ c\ -c$	$b\ a\ 0\ a\ -b\ c\ 0\ 0\ -c$	$b\ a\ b\ -a\ b\ c\ -c\ -c\ c$
$(1,4,10,10)$	$b\ 0\ a\ -d\ -a\ 0\ -b\ 0\ 0$	$a\ -b\ -b\ 0\ 0\ -a\ -a\ -b\ 0$	$a\ -c\ -b\ a\ c\ a\ b\ -b\ 0$	$a\ -a\ c\ -b\ 0\ c\ 0\ b\ 0$
$(1,4,13,13)$	$a\ a\ -c\ b\ b\ -b\ b\ b\ c$	$a\ a\ -a\ a\ a\ 0\ -b\ -b\ 0$	$a\ -b\ 0\ -b\ b\ 0\ b\ -a\ d$	$a\ -a\ -c\ -a\ -b\ 0\ b\ a\ -c$
$(1,5,5,16)$	$a\ 0\ -b\ -b\ d\ b\ b\ 0\ -a$	$a\ 0\ b\ -b\ b\ -b\ 0\ c\ c$	$a\ a\ -c\ b\ -b\ -b\ b\ 0\ 0$	$b\ b\ 0\ b\ b\ c\ -c\ 0\ 0$

Appendix B(cont): Order 36 (Sequences with zero periodic autocorrelation function)

Design	A_1,									A_2, A_3									A_4								
$(1,5,5,25)$	a	$-d$	$-d$	$-d$	d	$-d$	d	d	d	b	b	$-d$	d	c	$-d$	d	$-d$	d									
										b	$-d$	d	d	$-c$	$-c$	$-d$	$-d$	d	d	d	d	c	$-c$	d	d	b	$-b$
$(1,6,8,12)$	b	c	a	$-c$	$-b$	0	0	0	0	d	$-c$	b	$-c$	$-b$	$-d$	d	0	d									
										b	0	$-d$	$-d$	b	$-c$	d	0	$-d$	b	d	b	$-d$	d	c	d	0	0
$(1,8,8,9)$	a	$-d$	0	0	$-c$	c	0	0	d	b	$-b$	$-c$	$-c$	$-d$	$-b$	$-b$	0	d									
										b	$-b$	d	0	$-b$	$-b$	c	c	$-d$	c	$-c$	d	0	d	0	0	0	d
$(1,8,9,18)$	b	c	a	$-b$	$-d$	b	$-a$	$-c$	$-b$	a	c	$-a$	b	c	c	$-b$	c	$-c$									
										a	b	a	b	b	c	$-b$	b	$-c$	a	b	$-b$	b	a	$-b$	$-b$	$-b$	$-b$
$(1,9,9,9)$	a	$-a$	b	$-b$	c	0	c	c	0	a	$-c$	d	c	$-a$	0	b	$-b$	0									
										a	a	0	$-b$	a	b	c	$-c$	0	a	0	$-c$	$-a$	$-b$	$-b$	$-b$	0	c
$(1,9,10,10)$	a	a	a	$-b$	b	0	0	$-c$	c	a	$-a$	$-c$	b	0	$-b$	$-c$	$-b$	$-c$									
										a	$-a$	$-b$	$-b$	0	$-c$	$-b$	c	c	d	$-a$	c	$-b$	0	0	b	$-c$	a
$(1,14,19)$	a	$-c$	$-b$	c	$-b$	b	$-c$	b	c	b	b	$-c$	c	c	$-c$	c	b	0									
										b	$-c$	c	b	c	$-c$	$-c$	$-c$	$-c$	c	b	b	$-b$	$-b$	b	c	c	0
$(1,14,21)$	a	$-c$	$-c$	$-c$	c	$-c$	c	c	c	b	c	b	$-c$	b	$-c$	c	$-c$	$-c$									
										b	$-b$	$-b$	$-b$	$-b$	b	b	$-b$	$-c$	c	c	$-c$	c	c	b	c	b	$-b$
$(2,2,5,20)$	a	$-a$	a	$-c$	a	a	a	$-c$	c	a	a	$-a$	$-c$	$-c$	$-a$	$-a$	$-a$	0									
										a	$-a$	$-a$	b	a	$-d$	0	0	0	a	$-a$	a	$-b$	$-a$	$-d$	0	0	0
$(2,3,6,16)$	b	a	$-d$	c	d	0	0	0	0	d	a	$-b$	$-c$	$-d$	d	$-d$	0	0									
										d	c	0	c	$-b$	0	$-d$	$-d$	d	d	$-c$	$-d$	c	$-d$	$-d$	$-d$	0	$-d$
$(2,3,6,25)$	a	$-d$	b	d	$-d$	d	$-d$	c	d	a	$-d$	$-b$	d	$-d$	d	$-d$	$-c$	d									
										b	$-d$	$-d$	$-d$	$-c$	$-c$	d	d	d	d	d	d	$-d$	d	d	d	c	$-c$

Appendix B(cont): Order 36 (Sequences with zero periodic autocorrelation function)

Design	A_1	A_2, A_3	A_4
$(2,4,9,18)$	$a\ \ -a\ \ b\ \ -b\ \ d\ \ -b\ \ -b\ \ -b\ \ 0$	$a\ \ -a\ \ -b\ \ -b\ \ -d\ \ -b\ \ b\ \ -b\ \ 0$	
		$a\ \ -b\ \ c\ \ b\ \ b\ \ c\ \ -b\ \ -a\ \ 0$	$b\ \ -a\ \ -a\ \ -a\ \ -b\ \ c\ \ b\ \ -b\ \ -c$
$(2,5,5,18)$	$a\ \ d\ \ -a\ \ -b\ \ -b\ \ -b\ \ b\ \ -b\ \ 0$	$b\ \ -b\ \ b\ \ b\ \ b\ \ c\ \ d\ \ -c\ \ 0$	
		$a\ \ b\ \ -b\ \ c\ \ b\ \ c\ \ -b\ \ 0\ \ 0$	$b\ \ -a\ \ b\ \ -a\ \ -b\ \ -b\ \ c\ \ 0\ \ 0$
$(2,6,9,12)$	$a\ \ -a\ \ b\ \ 0\ \ 0\ \ b\ \ d\ \ c\ \ 0$	$a\ \ -b\ \ -a\ \ b\ \ -c\ \ d\ \ -b\ \ -b\ \ 0$	
		$a\ \ -b\ \ b\ \ c\ \ c\ \ -b\ \ -a\ \ -b\ \ 0$	$a\ \ b\ \ a\ \ a\ \ -b\ \ -c\ \ c\ \ 0\ \ 0$
$(2,6,12,16)$	$a\ \ a\ \ -c\ \ a\ \ -b\ \ b\ \ -b\ \ -b\ \ c$	$a\ \ -b\ \ -d\ \ b\ \ -b\ \ b\ \ b\ \ b\ \ c$	
		$a\ \ b\ \ -b\ \ -b\ \ -b\ \ -c\ \ -a\ \ -a\ \ -d$	$a\ \ -a\ \ -b\ \ a\ \ -a\ \ c\ \ -a\ \ -b\ \ c$
$(2,7,19)$	$a\ \ 0\ \ -b\ \ b\ \ -c\ \ c\ \ c\ \ b\ \ 0$	$a\ \ 0\ \ 0\ \ -b\ \ -c\ \ c\ \ -c\ \ 0\ \ 0$	
		$b\ \ b\ \ -c\ \ -c\ \ c\ \ -c\ \ c\ \ c\ \ c$	$b\ \ -c\ \ -c\ \ -c\ \ 0\ \ c\ \ -c\ \ -c\ \ 0$
$(2,7,21)$	$a\ \ 0\ \ c\ \ 0\ \ -b\ \ b\ \ b\ \ c\ \ 0$	$c\ \ -c\ \ -a\ \ c\ \ c\ \ b\ \ c\ \ -c\ \ 0$	
		$b\ \ -c\ \ b\ \ c\ \ -c\ \ -c\ \ -c\ \ c\ \ -c$	$b\ \ -c\ \ -c\ \ c\ \ 0\ \ c\ \ c\ \ c\ \ 0$
$(2,7,24)$	$a\ \ c\ \ -c\ \ c\ \ -b\ \ b\ \ b\ \ c\ \ 0$	$a\ \ -c\ \ -c\ \ -c\ \ c\ \ -c\ \ -b\ \ c\ \ 0$	
		$c\ \ c\ \ b\ \ -c\ \ b\ \ c\ \ -c\ \ -c\ \ 0$	$b\ \ c\ \ -c\ \ c\ \ -c\ \ -c\ \ -c\ \ -c\ \ -c$
$(2,8,8,18)$	$b\ \ b\ \ c\ \ -b\ \ b\ \ c\ \ -b\ \ -b\ \ d$	$a\ \ b\ \ -c\ \ b\ \ -b\ \ -c\ \ -b\ \ -a\ \ d$	
		$a\ \ b\ \ -b\ \ a\ \ b\ \ c\ \ b\ \ b\ \ -c$	$a\ \ b\ \ -c\ \ -a\ \ b\ \ -a\ \ b\ \ -a\ \ c$
$(2,8,13,13)$	$a\ \ a\ \ -d\ \ b\ \ b\ \ -b\ \ b\ \ b\ \ d$	$a\ \ a\ \ -a\ \ a\ \ a\ \ -d\ \ -b\ \ -b\ \ d$	
		$a\ \ -b\ \ d\ \ -b\ \ b\ \ d\ \ b\ \ -a\ \ c$	$a\ \ -a\ \ d\ \ -a\ \ -b\ \ -c\ \ b\ \ a\ \ d$
$(2,8,17)$	$b\ \ a\ \ -b\ \ b\ \ 0\ \ b\ \ 0\ \ 0\ \ 0$	$b\ \ a\ \ -b\ \ -b\ \ 0\ \ -b\ \ 0\ \ 0\ \ 0$	
		$c\ \ -c\ \ c\ \ c\ \ c\ \ c\ \ -c\ \ c\ \ 0$	$c\ \ -c\ \ -c\ \ c\ \ c\ \ c\ \ c\ \ -c\ \ -c$
$(2,9,17)$	$a\ \ -b\ \ b\ \ -c\ \ 0\ \ c\ \ c\ \ c\ \ 0$	$a\ \ b\ \ -b\ \ -b\ \ -c\ \ 0\ \ b\ \ -c\ \ 0$	
		$b\ \ c\ \ b\ \ b\ \ -c\ \ 0\ \ c\ \ -c\ \ 0$	$c\ \ -c\ \ -c\ \ -c\ \ 0\ \ -c\ \ c\ \ -c\ \ 0$

Appendix B(cont): Order 36 (Sequences with zero periodic autocorrelation function)

Design	A_1,									A_2, A_3									A_4								
$(2,11,16)$	a	b	$-c$	0	$-c$	c	0	c	0	a	$-b$	$-b$	0	b	$-b$	0	b	0									
										b	c	b	$-c$	$-c$	c	$-c$	b	c	b	$-c$	$-c$	$-c$	$-b$	c	$-c$	$-c$	0
$(3,3,12,12)$	a	a	b	$-a$	a	$-d$	d	$-d$	$-d$	a	a	c	a	$-a$	$-d$	d	d	d									
										c	$-a$	a	0	$-b$	$-d$	0	$-d$	0	c	$-a$	$-a$	0	b	$-d$	0	d	0
$(3,3,3,27)$	a	b	$-c$	$-d$	$-d$	$-d$	d	d	d	a	$-b$	d	$-d$	d	d	d	$-d$	d									
										a	$-d$	c	$-d$	$-d$	d	d	$-d$	$-d$	b	c	d	$-d$	d	d	d	$-d$	d
$(3,3,15,15)$	a	b	c	c	$-c$	$-c$	$-c$	b	$-d$	a	$-b$	c	$-c$	c	c	c	$-c$	c									
										a	$-c$	$-b$	$-b$	$-b$	b	b	$-c$	d	b	$-b$	b	b	b	$-b$	b	c	d
$(3,4,6,8)$	a	c	0	c	$-d$	0	0	d	0	c	$-a$	$-d$	0	b	$-d$	0	$-b$	0									
										a	$-d$	0	$-c$	0	0	0	$-d$	0	b	d	c	b	$-c$	0	0	$-d$	0
$(3,4,6,18)$	a	$-c$	$-b$	$-d$	b	b	d	$-b$	$-c$	a	b	b	0	b	$-b$	0	b	c									
										a	$-b$	b	0	$-b$	$-b$	0	$-b$	c	b	d	b	$-b$	d	$-b$	c	0	$-c$
$(3,4,26)$	a	$-c$	c	$-b$	c	c	b	c	c	c	c	$-a$	$-c$	c	$-c$	c	c	0									
										c	c	$-a$	$-c$	$-c$	c	$-c$	c	0	c	$-c$	$-b$	c	c	$-b$	$-c$	$-c$	0
$(3,6,6,12)$	a	$-d$	b	$-d$	$-d$	$-c$	d	0	0	a	d	$-c$	$-d$	d	$-b$	d	0	0									
										a	0	$-b$	$-d$	c	b	d	c	0	b	d	$-c$	b	d	c	0	0	0
$(3,6,8,9)$	a	$-b$	$-d$	0	a	$-c$	0	b	0	a	$-b$	0	0	b	a	c	d	0									
										a	$-b$	0	0	$-b$	$-a$	$-c$	$-b$	c	a	$-c$	d	$-b$	$-a$	0	b	0	$-c$
$(3,6,9,18)$	a	$-a$	$-a$	$-a$	d	c	$-a$	b	$-c$	a	$-b$	$-a$	a	$-a$	$-a$	$-a$	$-a$	$-d$									
										a	c	c	$-a$	$-c$	c	$-d$	d	c	a	$-c$	$-a$	a	$-a$	$-d$	$-d$	c	b
$(3,6,22)$	a	b	b	$-c$	$-c$	c	c	$-c$	c	b	$-a$	c	$-c$	$-c$	$-c$	$-c$	0	0									
										c	$-a$	$-c$	b	c	c	c	0	0	b	$-b$	$-c$	$-c$	c	$-c$	c	$-c$	0

Appendix B(cont): Order 36 (Sequences with zero periodic autocorrelation function)

Design	A_1,									A_2, A_3									A_4								
$(3,7,15)$	a	0	0	b	$-c$	b	0	0	0	c	c	a	c	$-c$	$-b$	$-b$	b	0									
										a	$-c$	c	$-c$	$-b$	c	c	$-c$	$-c$	b	c	c	0	0	c	0	0	0
$(3,7,23)$	a	b	c	$-c$	$-c$	c	b	$-c$	0	b	a	$-b$	c	$-b$	$-c$	c	c	c									
										a	$-b$	$-c$	$-c$	c	$-c$	c	$-c$	0	c	$-b$	c	$-c$	$-c$	$-c$	$-c$	$-c$	0
$(3,8,22)$	a	b	b	$-c$	c	c	$-c$	$-c$	0	a	$-b$	$-c$	c	$-c$	$-c$	$-b$	c	0									
										a	$-c$	$-b$	b	$-c$	c	c	c	c	c	c	$-b$	c	$-c$	c	b	c	0
$(3,10,18)$	a	b	b	$-c$	$-c$	c	c	$-c$	c	c	$-c$	$-a$	b	$-c$	$-c$	$-c$	0	0									
										c	$-c$	a	$-c$	$-b$	$-c$	$-c$	0	0	b	$-b$	b	$-c$	$-b$	c	b	b	0
$(3,10,23)$	a	b	c	b	$-c$	c	$-c$	$-c$	$-c$	a	$-c$	c	$-b$	c	c	c	$-c$	c									
										b	$-a$	$-c$	$-c$	c	c	c	$-c$	c	c	c	b	b	b	$-b$	c	b	$-b$
$(3,11,19)$	a	b	$-c$	c	$-c$	$-c$	$-c$	c	$-c$	c	$-b$	b	$-a$	c	b	$-c$	$-c$	0									
										c	c	c	a	c	$-c$	$-c$	c	0	b	b	b	$-b$	$-b$	b	c	b	0
$(3,14,16)$	a	b	b	$-c$	b	c	c	$-c$	0	b	$-a$	b	c	c	$-c$	c	$-c$	$-c$									
										a	b	$-b$	$-b$	$-c$	$-b$	b	c	0	c	$-b$	c	b	b	$-b$	c	c	0
$(4,4,4,9)$	a	a	$-d$	$-c$	c	0	0	0	d	b	$-c$	b	c	0	0	0	0	0									
										b	0	$-b$	$-c$	0	$-c$	$-c$	0	0	a	$-a$	$-d$	0	$-c$	0	0	c	$-d$
$(4,4,5,20)$	b	$-b$	d	$-c$	$-c$	$-d$	b	$-b$	0	b	$-b$	$-d$	b	$-b$	$-d$	$-b$	b	0									
										b	b	a	$-c$	c	a	b	b	$-a$	b	b	a	$-b$	$-b$	0	$-b$	$-b$	a
$(4,4,9,16)$	c	d	c	$-b$	c	$-d$	c	0	b	c	d	c	$-b$	$-c$	d	$-c$	0	b									
										c	a	$-c$	0	$-c$	a	c	$-b$	b	c	a	$-c$	$-b$	c	$-a$	$-c$	$-b$	$-b$
$(4,4,13,13)$	a	a	$-c$	b	b	$-b$	b	b	c	a	a	$-a$	a	a	$-d$	$-b$	$-b$	d									
										a	$-b$	$-d$	$-b$	b	$-d$	b	$-a$	0	a	$-a$	c	$-a$	$-b$	0	b	a	c

Appendix B(cont): Order 36 (Sequences with zero periodic autocorrelation function)

Design		A_2, A_3	A_4
$(4,5,5,16)$	$c\ \ -c\ \ -b\ \ -a\ \ -b\ \ c\ \ -c\ \ 0\ \ 0$	$c\ \ c\ \ -a\ \ b\ \ -a\ \ -c\ \ -c\ \ d\ \ -d$	
		$c\ \ c\ \ -a\ \ 0\ \ a\ \ c\ \ c\ \ 0\ \ 0$	$c\ \ -c\ \ b\ \ 0\ \ -b\ \ -c\ \ c\ \ d\ \ d$
$(4,8,8,16)$	$a\ \ a\ \ b\ \ -b\ \ -c\ \ b\ \ b\ \ -d\ \ d$	$a\ \ d\ \ a\ \ b\ \ -b\ \ c\ \ -b\ \ -b\ \ -d$	
		$a\ \ -a\ \ b\ \ -b\ \ c\ \ b\ \ b\ \ -d\ \ -d$	$a\ \ -d\ \ -a\ \ b\ \ -b\ \ -c\ \ -b\ \ -b\ \ -d$
$(4,9,9,9)$	$a\ \ d\ \ -b\ \ -a\ \ b\ \ c\ \ 0\ \ c\ \ -d$	$a\ \ -b\ \ b\ \ 0\ \ 0\ \ -d\ \ -a\ \ -d\ \ -d$	
		$a\ \ -b\ \ c\ \ a\ \ -c\ \ -d\ \ a\ \ b\ \ d$	$a\ \ -b\ \ -b\ \ -a\ \ -b\ \ 0\ \ 0\ \ d\ \ -d$
$(4,9,10,10)$	$b\ \ d\ \ c\ \ -b\ \ d\ \ d\ \ -c\ \ d\ \ -d$	$a\ \ -b\ \ -b\ \ -a\ \ -b\ \ b\ \ -a\ \ -b\ \ 0$	
		$a\ \ -a\ \ -b\ \ a\ \ 0\ \ d\ \ a\ \ a\ \ -d$	$a\ \ -b\ \ -d\ \ -a\ \ c\ \ d\ \ 0\ \ b\ \ c$
$(5,5,8,18)$	$a\ \ -b\ \ -b\ \ -a\ \ b\ \ -b\ \ d\ \ -b\ \ d$	$b\ \ -b\ \ -b\ \ -c\ \ -b\ \ -d\ \ c\ \ -b\ \ -d$	
		$a\ \ b\ \ b\ \ c\ \ -b\ \ d\ \ c\ \ -b\ \ -d$	$b\ \ -a\ \ b\ \ -b\ \ -a\ \ -b\ \ -d\ \ c\ \ d$
$(5,5,9,9)$	$d\ \ -a\ \ 0\ \ 0\ \ c\ \ a\ \ b\ \ d\ \ -b$	$a\ \ -c\ \ -a\ \ b\ \ d\ \ 0\ \ 0\ \ -b\ \ -c$	
		$a\ \ -a\ \ a\ \ a\ \ a\ \ d\ \ 0\ \ -d\ \ 0$	$c\ \ 0\ \ b\ \ -b\ \ b\ \ b\ \ b\ \ -c\ \ 0$
$(5,5,13,13)$	$a\ \ b\ \ a\ \ -a\ \ c\ \ -d\ \ -a\ \ -b\ \ c$	$a\ \ a\ \ -b\ \ -c\ \ a\ \ -b\ \ -b\ \ b\ \ c$	
		$a\ \ -b\ \ c\ \ d\ \ -b\ \ b\ \ -a\ \ b\ \ d$	$a\ \ -a\ \ -a\ \ -b\ \ d\ \ -a\ \ -b\ \ -b\ \ -d$
$(5,7,22)$	$c\ \ -a\ \ -b\ \ a\ \ c\ \ -c\ \ 0\ \ -c\ \ -c$	$c\ \ -a\ \ -b\ \ -a\ \ -c\ \ -c\ \ c\ \ 0\ \ c$	
		$c\ \ b\ \ b\ \ -b\ \ c\ \ c\ \ c\ \ -c\ \ c$	$c\ \ -b\ \ a\ \ -b\ \ -c\ \ c\ \ c\ \ c\ \ -c$
$(5,9,14)$	$b\ \ 0\ \ -b\ \ -c\ \ -c\ \ -c\ \ 0\ \ -c\ \ c$	$b\ \ a\ \ -b\ \ 0\ \ -c\ \ 0\ \ c\ \ -c\ \ -c$	
		$b\ \ a\ \ 0\ \ a\ \ -b\ \ c\ \ 0\ \ c\ \ -c$	$b\ \ a\ \ b\ \ -a\ \ b\ \ 0\ \ -c\ \ 0\ \ c$
$(5,9,20)$	$b\ \ c\ \ a\ \ b\ \ -a\ \ -b\ \ c\ \ -b\ \ c$	$b\ \ c\ \ -b\ \ -c\ \ -b\ \ b\ \ 0\ \ b\ \ -b$	
		$b\ \ b\ \ b\ \ -c\ \ a\ \ b\ \ -b\ \ c\ \ b$	$c\ \ b\ \ b\ \ -a\ \ 0\ \ -a\ \ b\ \ -c\ \ -b$
$(6,7,23)$	$a\ \ a\ \ -b\ \ c\ \ -c\ \ c\ \ -c\ \ c\ \ c$	$a\ \ -a\ \ b\ \ c\ \ -b\ \ -a\ \ -b\ \ c\ \ c$	
		$a\ \ -c\ \ c\ \ b\ \ -c\ \ c\ \ c\ \ -c\ \ -c$	$b\ \ -c\ \ -c\ \ c\ \ b\ \ c\ \ c\ \ c\ \ c$

Appendix B(cont): Order 36 (Sequences with zero periodic autocorrelation function)

Design	A_1,									A_2, A_3									A_4								
$(6,8,19)$	b	$-c$	0	b	b	b	a	c	$-a$	b	b	$-b$	b	c	$-a$	$-b$	c	a									
										c	b	$-b$	$-b$	$-a$	$-b$	b	$-a$	0	b	$-b$	$-b$	$-c$	b	$-a$	b	$-a$	0
$(6,11,16)$	a	$-a$	$-b$	c	$-b$	$-b$	$-b$	$-c$	0	a	$-b$	b	b	$-b$	$-b$	b	0	c									
										a	$-a$	b	$-c$	$-a$	$-a$	$-b$	a	$-c$	a	a	$-b$	$-c$	$-b$	b	a	b	0
$(6,15,15)$	a	a	c	$-c$	$-b$	b	$-b$	$-c$	$-c$	b	a	$-a$	b	b	$-c$	a	c	c									
										a	$-c$	c	$-b$	c	c	c	$-c$	c	b	b	$-b$	$-b$	$-b$	$-b$	b	$-b$	c
$(7,8,10)$	a	a	$-a$	$-c$	c	0	c	c	0	a	0	a	$-c$	c	0	$-c$	$-c$	0									
										b	a	$-b$	b	0	b	0	0	c	b	a	$-b$	$-b$	0	$-b$	c	0	0
$(7,8,19)$	a	$-b$	$-b$	$-b$	c	$-a$	$-c$	$-b$	$-c$	b	$-a$	a	b	$-c$	$-b$	b	$-b$	$-c$									
										b	$-b$	a	a	b	b	$-b$	0	$-c$	a	$-b$	$-b$	$-b$	b	a	b	0	c
$(7,8,21)$	a	a	$-c$	c	c	c	$-c$	c	c	b	$-a$	a	b	b	$-b$	a	$-c$	$-c$									
										c	a	c	$-b$	$-c$	$-b$	$-c$	$-c$	$-c$	a	b	$-c$	$-c$	c	$-c$	c	$-b$	$-c$
$(7,11,12)$	a	c	a	$-a$	b	b	0	$-a$	0	a	b	$-c$	0	$-b$	0	b	b	$-c$									
										a	$-a$	c	0	b	c	b	$-a$	$-c$	a	a	a	b	c	$-b$	b	$-b$	0
$(7,12,15)$	a	b	a	0	b	$-b$	$-c$	c	c	b	a	b	a	$-b$	$-b$	$-c$	b	$-c$									
										a	b	$-a$	$-b$	b	$-a$	b	$-a$	$-c$	a	$-b$	0	$-a$	$-a$	a	$-b$	$-b$	$-c$
$(8,8,10,10)$	a	a	$-d$	d	$-b$	b	b	$-c$	$-c$	a	a	b	$-b$	b	d	c	$-d$	c									
										a	$-a$	a	$-b$	$-b$	$-d$	$-c$	$-d$	c	a	$-a$	$-a$	$-d$	$-d$	b	b	c	$-c$
$(8,8,17)$	b	$-a$	$-b$	b	$-a$	b	$-a$	0	a	b	a	$-b$	$-b$	$-a$	$-b$	$-a$	0	$-a$									
										c	$-c$	c	c	c	c	$-c$	c	0	c	$-c$	$-c$	c	c	c	c	$-c$	$-c$
$(8,9,11)$	a	$-b$	$-c$	b	a	0	c	0	c	a	$-b$	a	b	0	$-c$	0	c	$-c$									
										a	$-b$	$-c$	$-b$	$-a$	$-b$	c	0	0	a	$-b$	$-a$	$-c$	$-c$	b	0	0	$-c$

Appendix B(cont): Order 36 (Sequences with zero periodic autocorrelation function)

Design	A_1,	A_2, A_3	A_4
$(8, 10, 15)$	$c\ \ c\ \ -c\ \ b\ \ -b\ \ 0\ \ -b\ \ -b\ \ c$	$c\ \ -c\ \ c\ \ b\ \ -b\ \ c\ \ b\ \ b\ \ c$	
		$a\ \ c\ \ -a\ \ a\ \ 0\ \ a\ \ -c\ \ -c\ \ b$	$a\ \ 0\ \ -a\ \ -a\ \ -c\ \ -a\ \ b\ \ c\ \ -c$
$(9, 9, 13)$	$b\ \ -b\ \ c\ \ c\ \ -c\ \ -b\ \ -c\ \ -b\ \ 0$	$a\ \ -a\ \ -b\ \ b\ \ b\ \ b\ \ 0\ \ b\ \ 0$	
		$a\ \ b\ \ -c\ \ -b\ \ a\ \ -a\ \ -a\ \ -c\ \ -c$	$a\ \ a\ \ -b\ \ c\ \ -c\ \ b\ \ 0\ \ a\ \ 0$
$(9, 9, 16)$	$a\ \ b\ \ a\ \ -b\ \ a\ \ -c\ \ c\ \ -c\ \ c$	$b\ \ -b\ \ a\ \ 0\ \ -a\ \ c\ \ c\ \ -c\ \ -c$	
		$b\ \ -b\ \ -a\ \ 0\ \ a\ \ c\ \ c\ \ c\ \ c$	$a\ \ b\ \ b\ \ b\ \ -a\ \ -c\ \ c\ \ c\ \ -c$
$(9, 13, 14)$	$a\ \ a\ \ -b\ \ -c\ \ -b\ \ c\ \ c\ \ -c\ \ c$	$a\ \ b\ \ a\ \ -c\ \ b\ \ b\ \ -b\ \ -b\ \ c$	
		$a\ \ -a\ \ a\ \ a\ \ b\ \ -a\ \ b\ \ -c\ \ -c$	$b\ \ -b\ \ c\ \ -c\ \ -c\ \ -b\ \ -c\ \ -b\ \ -c$
$(11, 11, 11)$	$a\ \ -a\ \ -b\ \ -b\ \ c\ \ -c\ \ -b\ \ -c\ \ 0$	$a\ \ -c\ \ c\ \ -b\ \ -a\ \ -a\ \ c\ \ c\ \ c$	
		$a\ \ -a\ \ -b\ \ b\ \ b\ \ -a\ \ b\ \ -b\ \ 0$	$a\ \ a\ \ c\ \ 0\ \ -b\ \ a\ \ -c\ \ b\ \ c$