28-8-2005

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Abstract

This paper presents a new model that describes the behaviour of optical feedback self-mixing interferometry (OFSMI). Compared to the existing model, the proposed one does not suffer from the multiple value ambiguity problems. The proposed model is a dynamic one that gives a unique relationship between the external target moving law and the self-mixing signal (SMS) waveform (SMS) when other parameters are given. A computer simulation tool for OFSMI has been implemented to generate SMS waveforms and its effectiveness has been verified by experimental data.

AN IMPROVED DYNAMIC MODEL FOR OPTICAL FEEDBACK SELF-MIXING INTERFEROMETRY-BASED MEASUREMENT AND INSTRUMENTATION

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ABSTRACT

This paper presents a new model that describes the behaviour of optical feedback self-mixing interferometry (OFSMI). Compared to the existing model, the proposed one does not suffer from the multiple value ambiguity problems. The proposed model is a dynamic one that gives a unique relationship between the external target moving law and the self-mixing signal (SMS) waveform (SMS) when other parameters are given. A computer simulation tool for OFSMI has been implemented to generate SMS waveforms and its effectiveness has been verified by experimental data.

1 INTRODUCTION

The optical feedback self-mixing interferometric effect (OFSMIE) occurs when a small fraction of the light emitted by a semiconductor laser (SL) is backscattered or reflected by an external target and re-enters the laser active cavity, resulting in the modulation of both the amplitude and the frequency of the laser oscillating field. Since the modulation carries information about the external target as well as the SL, the observed emitted power, also called the self-mixing signal (SMS), can be used to measure the metrological quantities [1,2] as well as the parameters of the SL s itself [3,4].

The key issue that governs the use of OFSMIE on measuring various parameters is the theoretical model that describes the relationship between the light phase of the external cavity and the self-mixing signals. In the cases of weak optical feedback, the existing model can fully describe the OFSMI effect. In the situations of moderate optical feedback, the existing model suffers from multiple value ambiguity problems, which yields many possible modes for the laser oscillations and the model does not tell which mode will actually occur. However, in practice, this uncertainty does not exist as only one stable mode can be excited at one time due to modes competition in a SL. The above mentioned problems have dampened the use of the model for SMS processing and parameter estimation based on OFSMI.

In this paper we will develop a new model that is in accordance with the practical reality. When the displacement of external target and the values of other parameters associated with the OFSMI are given, the waveform of SMS can be obtained based on the new model. The proposed model can be used in the following ways. Firstly, it can be used to simulate the behavior of OFSME, which is significant as until now there is still not an effective tool to simulate the SMS waveform at the moderate feedback yet. Secondly, the model can be used for date-to-model fitting in order to achieve accurate estimation of the target movement as well as the parameters such as the linewidth enhancement factor (LEF) and the optical feedback level factor (OFLF).

The paper is organized as follows: Section 2 briefly reviews the theory of self-mixing optical feedback interferometric effect, where a set of well-known mathematical models is presented and described. Then in Section 3 the improved model is proposed, in which multiple value ambiguity problem is eliminated. Computer simulations on the proposed model are presented Section 4 in order to show the effectiveness of the new model. Finally Section 5 concludes the paper.

2 EXISTING THEORETICAL MODEL

The OFSMI effect has been studied extensively, resulting in a well-known mathematical model. There are two alternative and equivalent methods for the analysis of OFSMI effects: the Long and Kobayashi equations based approach[5] and the three-mirror cavity based approach[6]. Both approaches yield the same description about the behavior of a single-mode SL with optical feedback, given by the following equations:

$$\phi_F(\tau) = \phi_0(\tau) - C \cdot \sin[\phi_F(\tau) + k] \tag{1}$$

$$P(\phi_F(\tau)) = P_0\left[1 + mG(\phi_F(\tau))\right] \tag{2}$$

$$G(\phi_F(\tau)) = \cos(\phi_F(\tau)) \tag{3}$$

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where $k = \arctan(\alpha)$ and α is the LEF, $\phi_0(\tau) = \omega_0 \tau$ and $\phi_F(\tau) = \omega_F(\tau)\tau$, where ω_0 and $\omega_F(\tau)$ are the angular frequencies of the SL without and with feedback respectively; $\tau = 2L/c$, where L is the length of the external cavity and c the speed of light; C is the OFLF.

In Equation (2) $P(\phi_F(\tau))$ and P_0 are the power emitted by the SL with and without the external cavity respectively. It is seen that with the external cavity, the emitted power deviated from P_0 by a factor of $mG(\phi_F(\tau))$ where m is called modulation index (typical m=10^{-3}), and $G(\phi_F(\tau))$ is called the interferometric function which gives the effect of the external cavity length to the emitted power. With a self-mixing experimental set-up, the emitted power $P(\phi_F(\tau))$ can be observed with respect to different values of τ. By intentionally varying the length of external cavity, a trace of $P(\phi_F(\tau))$ with respect to τ (and hence time) can be obtained which is referred to as self-mixing signal (SMS).

Equations (1)-(3) describe the relationship between the observed self-mixing signals, parameters α and C, and the length of external cavity, which implies that the observed self-mixing signal contains information about the parameters and the external cavity length. Many applications have been implemented based on this principle, such as the LEF measurement [3,4] and other metrological quantity measurements [1,2, 5, 7].

Obviously Equations (1)-(3) play a crucial role in the above applications. It is desired that Equations (1)-(3) are sufficient to accurately describe behavior of the self-mixing effect. For this reason we revisit these equations as follows.

In order to get deeper understanding of the existing model, let us rewrite Equation (1) as follows:

$$y = x - C \cdot \sin[y + k] \tag{4}$$

where $x = \phi_0(\tau)$ and $y = \phi_F(\tau)$. Logically y (the phase with feedback) is a function of x (the phase without feedback). However, the mapping from x to y may not be unique. Hence we consider the following relationship:

$$x = y + C \cdot \sin[y + k] \tag{5}$$

Firstly we look at the gradient of x with respect to y as follows:

$$\frac{dx}{dy} = 1 + C \cdot \cos[y + k] \tag{6}$$

By letting the above gradients to zeros we can show that there are two possible maximum or minimum points for x within each of the interval $[2m\pi, 2(m+1)\pi]$ for y, where m=0, 1, 2, É .:

$$y_{m,1} = 2m\pi + \phi_0 - k \tag{7}$$

$$y_{m,2} = 2(m+1)\pi - \phi_0 - k \tag{8}$$

where $\phi_0 = \arccos\left(-\frac{1}{C}\right)$ and $0 < \phi_0 < \pi$. The values of x at these two points are:

$$x_{m,1} = 2m\pi + \phi_0 + \sqrt{C^2 - 1} - k \tag{9}$$

$$x_{m,2} = 2(m+1)\pi - \phi_0 - \sqrt{C^2 - 1} - k \tag{10}$$

In order to see the nature of these two points, we derive the second order gradients of x with respect to y at those points as follows:

$$\left.\frac{d^2x}{dy^2}\right|_{y=y_{m,1}} = -C \cdot \sin[y_{m,1} + k] = -\sqrt{C^2 - 1} < 0 \tag{11}$$

$$\left.\frac{d^2x}{dy^2}\right|_{y=y_{m,2}} = -C \cdot \sin[y_{m,2} + k] = \sqrt{C^2 - 1} > 0 \tag{12}$$

Hence x exhibits a maximum at $y_{m,1}$ and a minimum at $y_{m,2}$.

The above analysis also reveals the possible excited modes of the SL. When C<1, for any specific x there is a unique solution for y, that is, there is only a single mode in this situation. However, with the increase of C there may be multiple possible solutions for y for a specific value of x as can be seen from the following:

Three possible excited mode region: It is seen that there may be three possible solutions for y when $x_{0,1} > x_{0,2}$ and $x_{0,1} < x_{1,2}$. $x_{0,1} > x_{0,2}$ implies that C>1. From $x_{0,1} < x_{1,2}$ we have:

$$\phi_0 + \sqrt{C^2 - 1} - k < 4\pi - \phi_0 - \sqrt{C^2 - 1} - k \tag{13}$$

or

$$\phi_0 + \sqrt{C^2 - 1} < 2\pi \tag{14}$$

solving the inequity for C gives:

$$C < 4.6 \tag{15}$$

Hence we conclude that there are three modes for SL when 1<C<4.6, which is consistent with the results reported [5]. We call the situation a moderate feedback range.

Five possible excited mode region: Also there can be five possible solutions for y when the first maximum is greater than the second minimum (i.e. C>4.6) but smaller than the third minimum i.e. $x_{0,1} < x_{2,2}$ which means:

$$\phi_0 + \sqrt{C^2 - 1} - k < 6\pi - \phi_0 - \sqrt{C^2 - 1} - k \tag{16}$$

or

$$\phi_0 + \sqrt{C^2 - 1} < 3\pi \tag{17}$$

Solving (17) for C gives

$$C < 7.79 \tag{18}$$

Hence we can say that there are five possible modes when $4.6 < C < 7.79$.

Similarly if we increase C further, there are seven

possible modes when $7.79 < C < 10.95$. However, the OFSMIE effect may not exist in these cases.

3 THE NEW MODEL

Section 2 reveals that at the weak optical feedback where C<1, there is a unique mode for the resulting phase, and hence equations (1)-(3) are sufficient to describe the OFSMI. At the moderate or strong feedback, there are multiple possible excited modes. Hence it is necessary to work out a theoretical model that is consistent to what happens in practice.

In the cases that 1<C<4.6, there are the following different situations:

1. when $x_{m,1} < x < x_{m+1,2}$ there is an unique y which implies a single model for the SL;
2. when $x_{m,2} < x < x_{m,1}$, there are three solutions for Equation (1), that is, three possible modes;

There is no problem with the single mode region. However, in the three modes regions, the model in Equation (1) does not specify which model will occur in practice. In other words, Equation (1) is not able to fully describe the behaviour of the SL. Hence it is necessary to build an improved model that complete describes what happens in practice.

In addition to the existing model in Equations (1)-(3), extensive experimental work has been done during the past years and it is well-known that although there are three solutions for Equation (1), only one mode will be excited in any specific situation, and the excited mode can be determined according to the following rules:

1. when x increases, y will also tend to increase. In other words, $dy/dx \geq 0$;
2. when x varies continuously, y will also tend to vary continuously, unless there is nowhere to go to keep the continuity;

By combing the two rules above to Equation (1), we have the new model described by systematic block diagram in Figure 1.

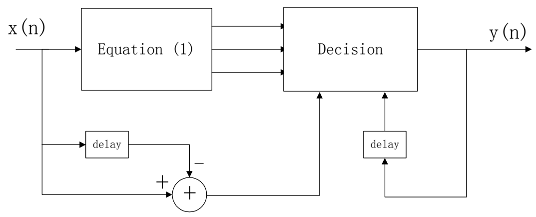


Figure 1. The systematic block diagram for determing the excited mode of the model

In Figure 1 $x(n)$ and $y(n)$ denote the values of $\phi_0(\tau)$ and $\phi_F(\tau)$ at the time instance n respectively. The decision block is implemented to select the correct $y(n)$ from the three possible solutions of Equation (1), which is described as follows:

Step 1: We use $y1$, $y2$ and $y3$ to save the three possible solutions for Equation (1). If Equation (1) has only one solution $y1$, set $y2$=INF and $y3$=INF; If it has two solutions $y1$ and $y2$, we will make $y1<y2$ and set the third one $y3$=INF; if it has three solutions, we will make $y1<y2<y3$. Here INF is a very large number which is not a possible solution of $y(n)$ for any given $x(n)$.

Step 2: Check the three solutions against the condition $yi \neq$ INF (for i=1,2,3) discard the ones if the condition is not met;

Step 3: Check the three solutions against the condition $\left. dy/dx \right|_{y=yi} \geq 0$ (for i=1,2,3) and discard the ones if the condition is not met. In the case of three solutions, the middle one will be discarded, and we will rearrange the two solutions and save them as $y1$ and $y2$;

Step 4: If only $y1$ is left after Steps 2 and 3, we have $y(n) = y1$, go the Step 1 for next input sample; If $y1$ and $y2$ are left after Steps 2&3, go to Step 5;

Step 5: if $x(n) - x(n-1) > 0$ (i.e. $x(n)$ increases) we choose $y(n) = yi$ if $yi - y(n-1) > 0$ and $|yi - y(n-1)| < |yj - y(n-1)|$ for $i, j = 1,2$ and $i \neq j$. If $x(n) - x(n-1) < 0$ (i.e. $x(n)$ decreases) we choose $y(n) = yi$ if $yi - y(n-1) < 0$ and $|yi - y(n-1)| < |yj - y(n-1)|$ for $i, j = 1,2$ and $i \neq j$.

The model in Figure 1 gives unique relationship between the moving law of the external target and the SMS waveform when α and C are given. Obviously the multiple value ambiguity problem has been eliminated.

4 SIMULATION RESULTS

A computer simulation tool for OFSMI has been developed based on the model in Figure 1. The input arguments for the tool are a, C as well as the variance of the external cavity length with time $x(n)$. With the software tool SMS waveforms incorporating various parameter values of a and C have been generated. In order to verify the effectiveness of the proposed model we compared the results with experimental SMS data recorded in our laboratory. The SMOFI experimental setup used for generating SMS is shown in Figure 2. The SL is biased with dc current; a lens is used to focus the light emitted by the SL on the target. A metal plate is used as the target, which is made to vibrate harmonically by positioning it close to a loudspeaker driven by a sinusoidal signal. The SMS is detected by the monitor photodiode (PD) and is amplified by a trans-impedance amplifier. The amplified signal is then acquired by personal computer via an A/D card with sampling frequency of 20KHz.

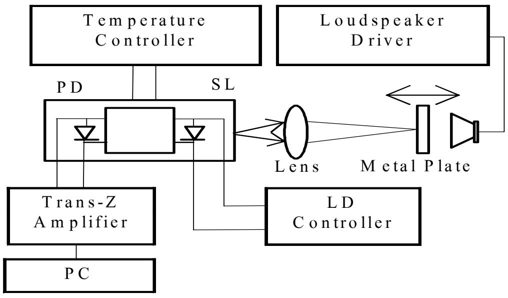
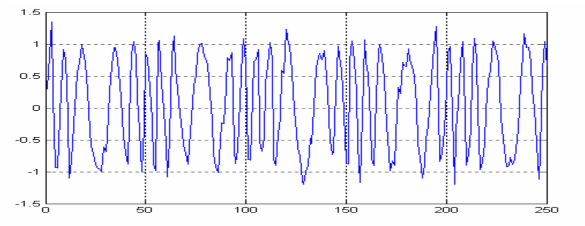
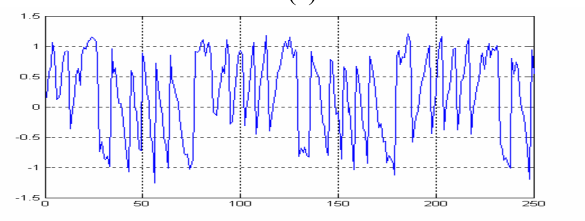


Figure 2. The experimental OFSMI system

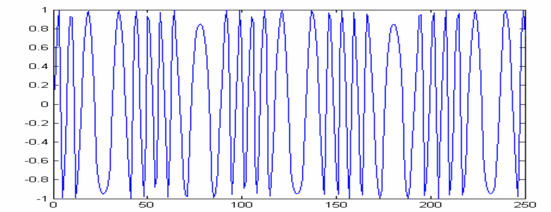


(a)

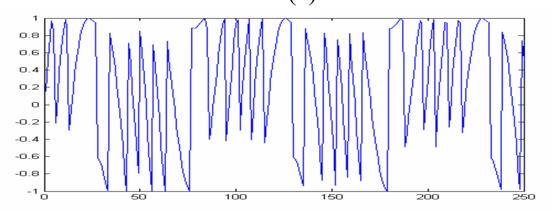


(b)

Figure 3 Recorded SMS Waveforms: (a) weak optical feedback level $a=3.71$, $C=0.5$; (b) moderate optical feedback level $a=3.71$, $C=2.4$



(a)



(b)

Figure 4 Simulated SMS Waveforms: (a) weak optical feedback level $a=3.71$, $C=0.5$; (b) moderate optical feedback level $a=3.71$, $C=2.4$

We recorded various SMS waveforms and estimated the values of a and C using the approaches developed in [7]. Figure 3 gives two SMS waveforms in the cases of weak and moderate feedback respectively. For comparison purpose, the simulated SMS waveforms with the same parameter values are depicted in Figure 4. It is seen that the simulated SMS waveforms are very close to the actual ones. Hence we can say that the proposed model is able to describe the OFSMI.

It should be mention that with the proposed model, various data fitting techniques can be developed for measuring the values of a and C as well as the displacement or movement of the external target.

5 CONCLUSION

This paper has presented a new systematic model for the OFSMI systems. Compared to the existing model, the proposed one is a dynamic one in which the multiple solution ambiguity problem is totally eliminated. The model gives a clear and unique relationship between the variance of external cavity length and the SMS waveforms when parameters a and C are given. The model can be used as a simulation tool to study the effect of these parameters to the SMS waveforms, and it can also be used to develop various parameter estimation and meterological quantities based on OFSMI systems.

6 REFERENCES

[1] G. Giuliani, M. Norgia, S. Donati, T. Bosch, "Laser diode self-mixing technique for sensing applications", *J. Opt. A*: *Pure Appl. Opt.*, vol. 4, no. 6, pp. S283-S294, 2002.

[2] L. Scalise, Y. Yu, G. Giuliani, G. Plantier and T. Bosch, □ Self-mixing laser diode velocimetry: application to vibration and velocity measurement,Ó IEEE *Trans. on Instrumentation and Measurement*, vol. 53, n. 1, pp. 223-232, 2004.

[3] G. Giuliani, M. Norgia, "Laser diode linewidth measurement by means of self-mixing interferometry", IEEE *Photon. Technol. Lett.*, vol 12, pp.1028-1030, Aug. 2000.

[4] Y. Yu, G. Giuliani and S. Donati, □Measurement of the linewidth enhancement factor of semiconductor lasers based on the optical feedback self-mixing effect,ÓIEEE *Photon. Technol. Lett*. Vol. 16, pp. 990-992, April 2004

[5] S. Donati, G. Giuliani, and S.Merlo, □Laser diode feedback interferometer for measurement of displacements without ambiguity,ÓIEEE *J. Quantum Electron*. Vol.QE-31, pp.113-119, Jan.1995

[6] G. Mourat, N. Servagent, and T. Bosch, □optical feedback effects on the spectral linewidth of semiconductor lasers using the self-mixing interference ,ÓIEEE *J. Quantum Electron*., vol. QE-34, pp. 1717-1721, Sep. 1998.

[7] Yanguang Yu, Jiangtao Xi, Enbang Li, Joe F. Chicharo and Thierry Bosch. Measuring Multiple Parameters in a Self-mixing Optical Feedback System, The IEEE--2004 Conference on Optoelectronic and Microelectronic Materials and Devices COMMAD2004 (in press)