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Abstract

In this paper, we proposed a new algorithm, referred to as inverse function analysis (IFA) method based on the derived mathematical model to reconstruct 3-D surfaces using fringe pattern profilometry (FPP) technique. Compared with traditional methods, our algorithm has neither the requirement for the structure of projected fringe patterns, nor the prior knowledge of the characteristics of projection systems. The correctness of inverse function analysis (IFA) method has been confirmed by simulation results. It can be seen that the measurement accuracy has been significantly improved by inverse function analysis (IFA) method, especially when the expected sinusoidal fringe patterns are distorted by unknown nonlinear factors.

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FRINGE PATTERN PROFILOMETRY BASED ON INVERSE FUNCTION ANALYSIS

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ABSTRACT

In this paper, we proposed a new algorithm, referred to as inverse function analysis (IFA) method based on the derived mathematical model to reconstruct 3-D surfaces using fringe pattern profilometry (FPP) technique. Compared with traditional methods, our algorithm has neither the requirement for the structure of projected fringe patterns, nor the prior knowledge of the characteristics of projection systems. The correctness of inverse function analysis (IFA) method has been confirmed by simulation results. It can be seen that the measurement accuracy has been significantly improved by inverse function analysis (IFA) method, especially when the expected sinusoidal fringe patterns are distorted by unknown nonlinear factors.

1. INTRODUCTION

Fringe pattern profilometry (FPP) is one of the most popular non-contact methods for measuring the three-dimensional surface of an object in recent years. With FPP, a Ronchi grating or sinusoidal grating is used to project fringe patterns onto a three-dimensional diffuse surface which results in deformed grating images. The deformed grating images are captured by a CCD camera and are processed to yield the shape of the object. In order to reconstruct the 3-D surface information from the pattern images, a number of fringe analysis methods have been developed, including Fourier Transform Profilometry (FTP)[1, 2], Phase Shifting Profilometry (PSP)[3, 4], Spatial Phase Detection (SPD)[5], Phase Locked Loop (PLL)[6] and other analysis methods[7, 8].

There are various methods to generate fringe patterns. In recent years, digital projectors have been widely used to obtain fringe patterns[9, 10]. The advantage of utilizing digital projectors for FPP is its simplicity and controllability. However, it is very difficult to obtain a pure sinusoidal fringe pattern by digital projectors due to the existence of geometrical distortion and colour distortion. Meanwhile we can note that for all the methods mentioned above, it has been assumed that fringe patterns are pure sinusoidal or can

be filtered to be sinusoidal by using digital filters to pick up the fundamental frequency component while eliminating the higher harmonics. However, in most practical cases, the filter can not be ideal and the results are still not pure sinusoidal. Therefore, errors will arise if the measurement is based on pure sinusoidal assumption. This problem motivates us to look for a new method to reconstruct the 3-D profile based on non-sinusoidal fringe patterns.

In this paper, we proposed a new approach to measuring object surface by projecting arbitrary fringe patterns onto the object. The proposed approach is based on the idea that the difference between original fringe pattern signal and deformed fringe pattern signal contains the 3-D information of the object, irrespective of whether they are sinusoidal or not. In this paper, firstly we introduce the principle of FPP system and a generalized model which mathematically describes the relationship between original fringe pattern signal and deformed fringe pattern signal. And then based on this model, a new approach for fringe pattern analysis, called inverse function analysis (IFA) method is proposed. Finally, Simulation results are provided to demonstrate the much improved measurement accuracy by IFA.

2. SYSTEM MODEL AND PRINCIPLE

A schematic diagram of a typical FPP system is shown in Figure 1. For the sake of simplicity we assume that the distance between the camera and the reference plane is long enough so that the reflected light beams captured by CCD camera from the reference plane and object are parallel. Meanwhile, because of the long distance, those parallel light beams reflected to the camera can be regarded as being vertical to the reference plane. The fringe pattern projected onto the reference plane is denoted by $s(x, y)$, which is the intensity of the signal at the location of (x, y) where x, y and a typical fringe pattern are depicted in Figure 1. Similarly the deformed fringe pattern is denoted by $d(x, y)$.

As shown in Figure 1, the projected fringe pattern on the reference plane is designed to exhibit a constant intensity along the y direction. Hence we can simply consider

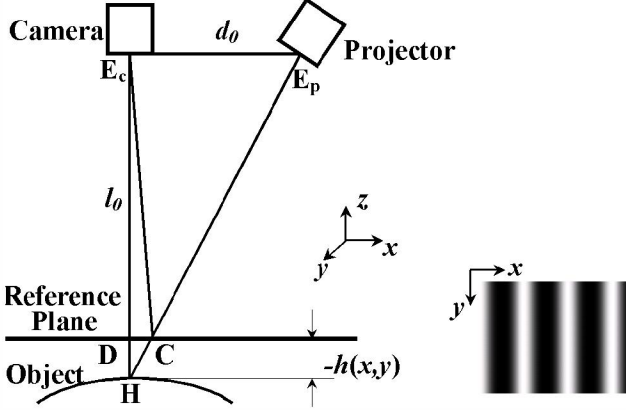


Fig. 1. Schematic diagram of fringe pattern profilometry (FPP) system

the signal intensity along the x direction for any given y coordinate, and the results can be extended to other values of y . Therefore, instead of $s(x, y)$ and $d(x, y)$, we simply use $s(x)$ and $d(x)$ to denote the projected signal on the reference plane and the deformed signal respectively. Similarly the height distribution function $h(x, y)$ of the object surface can also be represented as $h(x)$, a function with only one independent variable x , when we are considering the sections of object surfaces.

In order to establish the relationship between $s(x)$ and $d(x)$, we consider a beam of light corresponding to a pixel of the fringe pattern, denoted as E_pCH in Figure 1. It is seen that the light beam is projected at point C and reflected back to the camera if the reference plane exists. When the reference plane is removed, the same beam will be projected onto the point H on the surface of the object, which is reflected to the camera via the point D. This implies that the x coordinate of point H in the image of object surface equals to the x coordinate of point D in the image of reference plane, because point D and H are on the same reflected beam from point H to the camera. Assuming that the object surface and the reference plane have the same reflective characteristics, $s(x)$ at location C should exhibit the same intensity as $d(x)$ does at location D because they originate from the same point of the fringe pattern created by the projector. Hence we have

$$d(x_d) = s(x_c) \quad (1)$$

We use u to denote the distance from C to D, that is

$$u = x_d - x_c \quad (2)$$

where x_c and x_d are the coordinate locations of point C and point D, respectively. From Eqs.(1) and (2), we have

$$d(x_d) = s(x_d - u) \quad (3)$$

Obviously, u varies with the height of the point H on the object surface.

Meanwhile, because triangles E_pHE_c and CHD are similar, we have

$$\frac{x_c - x_d}{-h(x_h)} = \frac{d_0}{l_0 - h(x_h)} \quad (4)$$

where x_h is the x coordinate of point H, l_0 is the distance between the camera and the reference plane and d_0 is the distance between the camera and the projector.

As mentioned above, point H has the same x coordinate as point D does in captured images, which implies $x_h = x_d$. So Eq.(4) can be rewritten as

$$\frac{x_c - x_d}{-h(x_d)} = \frac{d_0}{l_0 - h(x_d)} \quad (5)$$

As defined in Eq.(2), Eq.(5) can be expressed as

$$\frac{-u}{-h(x_d)} = \frac{d_0}{l_0 - h(x_d)} \quad (6)$$

As the height distribution $h(x)$ is a function of x_d , u should also be a function of x_d . Then we have

$$u(x_d) = \frac{d_0 h(x_d)}{l_0 - h(x_d)} \quad (7)$$

An equivalent representation is

$$h(x_d) = \frac{l_0 u(x_d)}{d_0 + u(x_d)} \quad (8)$$

Therefore, Eq.(3) can be expressed as

$$d(x_d) = s(x_d - u(x_d)) \quad (9)$$

where $u(x_d)$ is given by Eq.(7).

To simplify the Eq.(9) and consider the model only mathematically, we let $x_d = x$, then we can derive a general mathematical model from Eqs.(8) and (9).

$$d(x) = s(x - u(x)) \quad (10)$$

$$h(x) = \frac{l_0 u(x)}{d_0 + u(x)} \quad (11)$$

Eq.(10) reveals that the deformed signal $d(x)$ is a shifted version of $s(x)$, and the shift function $u(x)$ varies with the height of the object.

Obviously, letting $s(x)$ be a cosinusoidal signal in Eq.(10), we can easily derive the conventional phase-modulation model. Let $s(x) = A \cos(2\pi f_0 x)$ where A is the amplitude of intensity. So the deformed signal is

$$\begin{aligned} d(x) &= s(x - u(x)) = A \cos(2\pi f_0 (x - u(x))) \\ &= A \cos(2\pi f_0 x - 2\pi f_0 u(x)) = A \cos(2\pi f_0 x + \phi(x)) \end{aligned} \quad (12)$$

where $\phi(x) = -2\pi f_0 u(x)$. i.e. $u(x) = -\frac{\phi(x)}{2\pi f_0}$. By substituting this equation into Eq.(11), we can derive a well-known equation:

$$h(x) = \frac{l_0 \phi(x)}{\phi(x) - 2\pi f_0 d_0} \quad (13)$$

As a classical formula, Eq.(13) appears in most of the articles on fringe pattern profilometry (for example[2]). This implies that the conventional model is a special case of our proposed model.

In practical cases, especially when we are using a digital projector to generate fringe patterns, the challenge we have to face to is the projected signal could not be pure sinusoidal signal. Eq.(10) and Eq.(11) reveal that the shift signal $u(x)$ contains all the 3-D information of object surface. Hence in theory we should be able to obtain the profile by projecting any signal. In next section, we will present an inverse function analysis (IFA) method to retrieve the shift function $u(x)$.

3. CALCULATION OF THE SHIFT

A straightforward method to calculate the shift distribution is to use inverse function. We assume the projected signal function $r = s(t)$ is a monotonic function or it is monotonic in intervals of t , in which $s(t)$ has a unique inverse function. Denoting the inverse function of $s(t)$ as $s^{-1}(v)$, we have

$$s^{-1}(s(t)) = t \quad (14)$$

Therefore, if we apply the inverse function $s^{-1}(v)$ to deformed signal $d(t)$, we will have

$$s^{-1}(d(t)) = s^{-1}\{s[t - u(t)]\} = t - u(t) \quad (15)$$

which means that we can obtain the shift function $u(t)$ by

$$u(t) = t - s^{-1}(d(t)) \quad (16)$$

It is obvious that from Eq.(16), the shift distribution can be calculated based on the fringe pattern projected on the reference plane and the deformed fringe pattern on the surface of the object. The key to calculating the shift distribution is to obtain the inverse function $s^{-1}(v)$. A possible way is to employ polynomial curve fitting, which will consequently introduce fitting errors. We use the mean square error to evaluate the curve fitting error, which is defined as:

$$e_f = E[(y_f(x) - y(x))^2] \quad (17)$$

where $E(w)$ is the operation of calculating the mean value of w , $y(x)$ are the data to be fitting and $y_f(x)$ are the values of the curve fitting results calculated by the approximate polynomial. The fitting error e_f will decrease with the increasing of the polynomial degree. Therefore, In order to determine the degree of polynomial used for curve fitting, we setup an upper bound of e_f in advance, and then we find out the minimum degree of polynomial which makes the curve fitting error e_f less than the upper bound we have setup. Hence the procedure of surface reconstruction is as follows:

Step 1. Set an upper bound of curve fitting error, e_m , and initialize k , the degree of polynomial used for curve fitting. The initial value of k equals 1.

Step 2. Based on the captured fringe pattern on the reference plane, $s(t)$, work out j_k , a polynomial of degree k to approximate the inverse function $s^{-1}(v)$ in least squares sense. More detailedly, at first, we take the straight line $t = r$ as a symmetry axis to obtain a symmetrical curve of $s(t)$ in each monotonic interval, which actually is the curve of the inverse function $s^{-1}(v)$. And then, we make curve fitting to the obtained symmetrical curve and obtain the curve fitting result j_k . This process is equivalent to directly fitting the inverse function $s^{-1}(v)$ by regarding the value of $s(t)$ as the variable and t as the value of the inverse function, rather than obtaining an approximate polynomial of the original function $s(t)$ before fitting the inverse function $s^{-1}(v)$.

Step 3. By Eq.(17), calculate the curve fitting error when using j_k to approximate $s^{-1}(v)$, if the error is less than e_m , continue to do Step 4, otherwise, set $k = k + 1$ and return to Step 2.

Step 4. Based on the curve fitting result $s^{-1}(v) \approx j_k$, and the values of deformed signal $d(t)$, we calculate the shift function $u(t)$ by Eq.(16).

4. SIMULATION RESULTS

In our simulation, we use a paraboloid as the object surface, and still use sinusoidal fringe patterns. We only consider the effect coming from the second order harmonic, so that the distortion of measurement system can be expressed as a polynomial of degree 2. We assume the expected signal is a cosinusoidal wave with an amplitude of A and a frequency of f_0 . Meanwhile, we assume the second order harmonic only has -20db of power, compared with fundamental component. Hence, making the second order harmonic be 0.1A, we have the assumed system distortion function as:

$$w(s) = \frac{1}{5A}s^2 + s + m \quad (18)$$

where m is DC component, which does not effect on the result of reconstructions. And s is the cosinusoidal signal we expect, which can be expressed as

$$s(x) = A \cos(2\pi f_0 x) \quad (19)$$

Normally, A is equal to 128, because of the maximum degrees of the intensity captured by the CCD camera. Then the coefficient of square item s^2 is 0.0016 approximately. In our simulation, A is also set to be 128.

Then, the signal we actually get is:

$$\begin{aligned} w(x) &= \frac{1}{5A}s^2(x) + s(x) + m \\ &= A \cos(2\pi f_0 x) + \frac{A}{10} \cos(2\pi \cdot (2f_0)x) + m \end{aligned} \quad (20)$$

After discarding DC component, the useful signal for our analysis can be expressed as

$$\tilde{w}(x) = A \cos(2\pi f_0 x) + \frac{A}{10} \cos(2\pi \cdot (2f_0)x) \quad (21)$$

In our simulation, we set the upper bound of the curve fitting error to be 0.1, so that by using inverse function analysis (IFA) method described in Section 3, the degree of polynomial is determined to be 39. Hence we use the polynomial of degree 39 to approximate the inverse function. The reconstruction results by PSP and IFA method are showed in Figure 2. The dashed line refers to the surface of object,

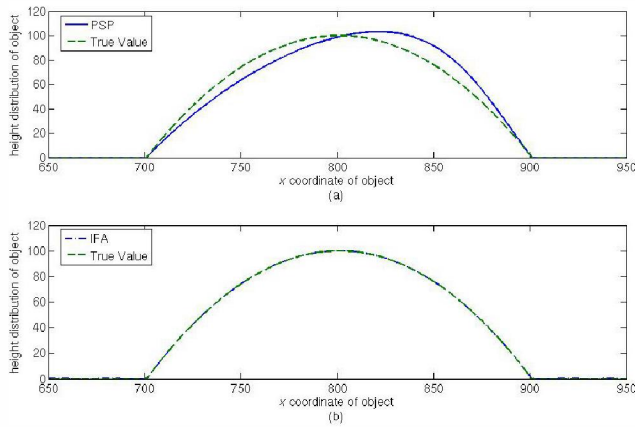


Fig. 2. Reconstruction result by PSP

which is the true value of the height distribution. The solid line is the measurement result by PSP method. This figure shows non-linear distortion will introduce noticeable errors when PSP method is used, even though the non-linear distortion is so slight that coefficient of square item is only 0.0016. However, by inverse function analysis method, we can obtain a much better reconstruction result shown in Figure 2(b). Same to Figure 2(a), dashed line is the true value. solid line is the reconstruction result by inverse function analysis (IFA) method. It can be seen that the reconstruction result by inverse function analysis (IFA) method is almost identical to the true values.

5. CONCLUSION

In this paper, we have presented a new method, called inverse function analysis (IFA) method, to analyze fringe patterns for profilometry, which is based on revealing the general relationships between projected signals and deformed signals. With IFA, the constraint of using sinusoidal signals has been completely removed. This implies, instead of sinusoidal fringe patterns, any kind of fringe patterns could be utilized for profilometry in theory. Meanwhile, as IFA has no requirement for the prior knowledge of the structure of

fringe patterns, it can be applied in theoretical analysis and practical operation. Our mathematical analysis and simulation results demonstrate that new method has higher accuracy when the projected signal is not exactly sinusoidal.

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