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Dynamic Hall resistivity of electronic systems in the presence of Rashba coupling at zero field

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We report a phenomenon in electronic systems. It is shown that, at both zero and finite frequencies, Hall resistivity (ρ_{xy}) is finite in an electronic system with sufficient Rashba coupling and in the absence of an external magnetic field. It is found that the off-diagonal component of the resistivity is determined by the reactive part (real part) of the inverse dielectric functions. This is in contrast to any other electrical transport, including magnetotransport where all resistivity components (both diagonal and off diagonal) are determined by the absorptive part (imaginary part) of the inverse dielectric functions. The longitudinal resistivity (ρ_{xx}) decreases as the Rashba coupling increases. The transverse charge current reported here should be clearly distinguished from the intrinsic transverse spin current reported previously [Phys. Rev. Lett., **92**, 126603 (2004)].

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Spin-dependent transport phenomena are of great importance to both condensed matter physics and device applications.[1,2] It has been realized that spin-electronic (or spintronic) systems and devices can be realized on the basis of diluted magnetic semiconductors and narrow-gap semiconductor nanostructures. Basic design proposals for spintronic devices, such as field-effect switches,[3,4] spin transistors,[3,5] spin filters,[6] spin waveguide,[7] etc, use the fact that electron waves with opposite spin acquire different phase factors during their propagation in the presence of Rashba spin-orbit coupling.[8] The experimental results[9,10] have indicated that, in InAs- and InGaAs-based two-dimensional electron gas (2DEG) systems, the spontaneous spin splitting is mainly induced by the Rashba effect [with an SU(2) symmetry], which can be enhanced further by increasing the gate voltage applied.

A physical phenomenon called spin Hall effect (SHE) in the Rashba split systems has been reported recently.[11,12] For high-mobility systems with sufficient Rashba coupling, a spin Hall current j^{sH} was found. This is an intrinsic effect since an external magnetic field is not required to observe this transverse spin current. We will refer this effect as intrinsic spin Hall effect (ISHE).

One immediate question arises: is there a zero-field charge Hall current in the same system? If such a charge Hall current exists, what is the dynamic behavior of the charge Hall effect (CHE)? In this paper we shall show that such CHE does indeed exist in the Rashba split system. From the first principle, we formulate the problem of frequency-dependent electrical transport under zero magnetic field. We were able to show that the finite spin-orbit coupling can lead to a finite current in the direction perpendicular to the driving electric field. Furthermore the Hall resistivity, ρ_{xy}, is determined by the reactive (real) part of the inverse dielectric function. All other resistivity components under zero or finite magnetic field in any electronic system are directly determined by the absorptive (imaginary) part of inverse dielectric function.[13] Therefore an electronic system with finite Rashba coupling represents a physical system whose resistivity component is determined by $\mathrm{Re}[1/\epsilon(q,\omega)]$. This would allow us to use the transport measurement to probe the reactive part of the dynamical dielectric function.

The CHE reported here is entirely different from the SHE as we deal the real charge transport. Though charge and spin transport are not related, the CHE and SHE are equally important in studying properties of spintronics systems. Moreover, our result applies to both zero and finite frequencies. Works on electron transport in spintronic systems have studied the cases where the applied field that drives the electric current is static. For systems with finite spin splitting, transitions between different spin states due to photon scattering can play a significant role in the electrical transport.

We consider a two-dimensional (2D) electronic system in the x-y plane in narrow-gap semiconductor nanostructures (e.g., InGaAs/InAlAs quantum wells). The Hamiltonian of a free electron is given as

$$H_0 = \frac{1}{2m^*}(p_x^2 + p_y^2) + \frac{\lambda}{\hbar}(\sigma_y p_x - \sigma_x p_y), \tag{1}$$

where m^* is the electron effective mass. The wave function can be written in the form of $\psi(x,y) = u_{\mathbf{k}}(x,y)\xi$, where $u_{\mathbf{k}}(x,y) = \exp(ik_x x + ik_y y)$. The eigenvalue is

$$E_\alpha^{(0)} = \frac{\hbar^2 k^2}{2m^*} + \alpha\lambda k, \tag{2}$$

where $k = \sqrt{k_x^2 + k_y^2}$ and $\alpha = \pm 1$, and eigenfunctions ξ_α are

$$\xi_{k,\alpha} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -\alpha(k_y - ik_x)/k \end{pmatrix}. \tag{3}$$

We now use the wave function given above to construct electron quantum field operators, $\hat{\mathbf{\Psi}}(x,y) = \Sigma_{k,\alpha}\hat{a}_{\mathbf{k}\alpha}u_{\mathbf{k}}(x,y)\xi_{\mathbf{k}\alpha}$, and $\hat{\mathbf{\Psi}}^\dagger(x,y) = \Sigma_{k,\alpha}\hat{a}^\dagger_{\mathbf{k}\alpha}u^*_{\mathbf{k}}(x,y)\xi^*_{\mathbf{k}\alpha}$, where $\hat{a}_{\mathbf{k}\alpha}(\hat{a}^\dagger_{\mathbf{k}\alpha})$ is the creation (annihilation) operator for an electron with wave number $\mathbf{k}$ and in Rashba state α. These field operators satisfy the equal time anticommutation rules.

For a many-electron system in the presence of electron-impurity scattering, the total Hamiltonian can be written in the form of $H = H_0 + H_{e-e} + H_{im}$, where H_0 is the Hamiltonian for free electrons with the spin-orbit interactions, H_{e-e} is the Coulomb interaction between electrons, and H_{im} is the interaction between the electrons and singly charged random impurities. In the second quantized notation, the Hamiltonians are written as[14]

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$$H_0 = \sum_{\mathbf{k}\alpha} E^{(0)}_{\mathbf{k},\alpha} \hat{a}^\dagger_{\mathbf{k}\alpha} \hat{a}_{\mathbf{k}\alpha}, \tag{4}$$

$$H_{e-e} = \frac{1}{4} \int d\mathbf{q} V(q) \sum_{\mathbf{k},\mathbf{k}'} \sum_{\alpha\alpha'\alpha''\alpha'''} \hat{a}^\dagger_{\mathbf{k}-\mathbf{q},\alpha} \hat{a}^\dagger_{\mathbf{k}'+\mathbf{q},\alpha'} \hat{a}_{\mathbf{k}'\alpha''} \hat{a}_{\mathbf{k}\alpha'''} \times [U_{\alpha\alpha'\alpha''\alpha'''}(\mathbf{k},\mathbf{k}',\mathbf{q}) + iW_{\alpha\alpha'\alpha''\alpha'''}(\mathbf{k},\mathbf{k}',\mathbf{q})], \tag{5}$$

and

$$H_{im} = \frac{e^2}{2} \int d\mathbf{q} V(q) \sum_i e^{i\mathbf{q}\cdot\mathbf{R}_i} \sum_{\mathbf{k},\alpha,\alpha'} \hat{a}^\dagger_{\mathbf{k},\alpha} \hat{a}_{\mathbf{k}+\mathbf{q},\alpha'} V_{\alpha\alpha'}(\mathbf{k},\mathbf{k}',\mathbf{q}). \tag{6}$$

Here we have used the following notations $V(q)=2\pi e^2/\kappa q$, where κ is the dielectric constant of the host semiconductor, and

$$U_{\alpha\alpha'\alpha''\alpha'''}(\mathbf{k},\mathbf{k}',\mathbf{q}) = 1 + \alpha\alpha''' \frac{k^2 - \mathbf{k}\cdot\mathbf{q}}{k|\mathbf{k}-\mathbf{q}|} + \alpha'\alpha'' \frac{k'^2 + \mathbf{k}'\cdot\mathbf{q}}{k'|\mathbf{k}'+\mathbf{q}|} + \alpha\alpha'\alpha''\alpha''' \frac{S}{kk'|\mathbf{k}-\mathbf{q}||\mathbf{k}'+\mathbf{q}|},$$

with $S=[\mathbf{k}\cdot(\mathbf{k}-\mathbf{q})][\mathbf{k}'\cdot(\mathbf{k}'+\mathbf{q})]-[\mathbf{k}\times(\mathbf{k}-\mathbf{q})]\cdot[\mathbf{k}'\times(\mathbf{k}'+\mathbf{q})]$,

$$W_{\alpha\alpha'\alpha''\alpha'''}(\mathbf{k},\mathbf{k}',\mathbf{q}) = -\frac{\hat{z}\cdot[\mathbf{k}\times(\mathbf{k}-\mathbf{q})]}{k|\mathbf{k}-\mathbf{q}|} \times \left(\alpha\alpha''' + \alpha\alpha'\alpha''\alpha''' \frac{\mathbf{k}'\cdot(\mathbf{k}'+\mathbf{q})}{k'|\mathbf{k}'+\mathbf{q}|} \right) - \frac{\hat{z}\cdot[\mathbf{k}'\times(\mathbf{k}'+\mathbf{q})]}{k'|\mathbf{k}'+\mathbf{q}|} \times \left(\alpha'\alpha'' + \alpha\alpha'\alpha''\alpha''' \frac{\mathbf{k}\cdot(\mathbf{k}-\mathbf{q})}{k|\mathbf{k}-\mathbf{q}|} \right),$$

and

$$V_{\alpha\alpha'}(\mathbf{k},\mathbf{k}',\mathbf{q}) = \left(1 + \alpha\alpha' \frac{\mathbf{k}\cdot(\mathbf{k}+\mathbf{q}) + i\mathbf{z}\cdot[\mathbf{k}\times(\mathbf{k}+\mathbf{q})]}{k|\mathbf{k}+\mathbf{q}|} \right).$$

For the frequency-dependent charge transport, the electron momentum relaxation is through H_{im}. The electron-electron interaction, H_{e-e}, does not directly contribute to the current. In principle, an electron can absorb a photon without interacting with another electron. However, many-body effects such as photon absorption due to particle-hole and plasmon excitations are only possible with electron-electron interaction.

We construct the single-electron density matrix element between states $\langle\alpha,k,t|$ and $|\beta,k+p,t\rangle$, which is defined as $F_{\alpha\beta}(k+p,k,t)=\langle\hat{a}^\dagger_{k,\beta}(t)\hat{a}_{k+p,\alpha}(t)\rangle$. Alternatively we can define the density matrix in terms of the electron spin $F_{\sigma\sigma'}(\mathbf{k}+\mathbf{p},\mathbf{k},t)=\langle\hat{a}^\dagger_{\mathbf{k},\sigma'}(t)\hat{a}_{\mathbf{k}+\mathbf{p},\sigma}(t)\rangle$. Here σ and σ' can be up (u) or down (d) and the density matrix is a four-component vector given as

$$F(\mathbf{k}+\mathbf{p},\mathbf{k},t) = \begin{pmatrix} F_{uu}(\mathbf{k}+\mathbf{p},\mathbf{k},t) \\ F_{ud}(\mathbf{k}+\mathbf{p},\mathbf{k},t) \\ F_{du}(\mathbf{k}+\mathbf{p},\mathbf{k},t) \\ F_{dd}(\mathbf{k}+\mathbf{p},\mathbf{k},t) \end{pmatrix}. \tag{7}$$

Under an external electrical field $\mathbf{E}(t)=\mathbf{E}e^{-i\omega t}$ along the 2D plane that drives the electrical current, the equation of the motion for the single-electron density matrix is found in a matrix form as

$$\left(\hat{K}(\mathbf{k},\mathbf{p},\partial/\partial t) + \frac{e\hbar^2}{m^* c}\mathbf{p}\cdot\mathbf{A}(t) \right) F(\mathbf{k}+\mathbf{p},\mathbf{k},t) = \sum_{\sigma,\mathbf{q},\mathbf{k}'} \left[F_{\sigma\sigma}(\mathbf{k}'+\mathbf{q},\mathbf{k}',t) + e^2 \sum_i \exp(-i\mathbf{q}\cdot\mathbf{R}_i) \right] \times 2V(q)[F(\mathbf{k}+\mathbf{p},\mathbf{k}+\mathbf{q},t) - F(\mathbf{k}+\mathbf{p}-\mathbf{q},\mathbf{k},t)], \tag{8}$$

where $\mathbf{A}=\mathbf{E}e^{-i\omega t}/(i\omega)$ and $\hat{K}(\mathbf{k},\mathbf{p},\partial/\partial t)$ is a 4×4 square matrix whose matrix elements are: $\hat{K}_{ii}=[\hbar^2(\mathbf{k}+\mathbf{p})^2]/2m^* - \hbar^2k^2/2m^* - i\hbar(\partial/\partial t)$, $\hat{K}_{12}=\hat{K}_{34}=\lambda(k_y-ik_x)$, $\hat{K}_{13}=\hat{K}_{24}=-\lambda(k_y+p_y+ik_x+ip_x)$, $\hat{K}_{14}=\hat{K}_{23}=0$, and for $i\neq j$ $\hat{K}_{ij}=\hat{K}^*_{ji}$. The derivation of Eq. (8) is essentially the same as that of,[15] except now we have a multicomponent system.

From the definition of the current operator, $\hat{\mathbf{J}}=c\delta H/\delta\mathbf{A}$, we obtain the electrical current operators. The current components contain two parts: $J_x(t)=J_{1,x}(\omega)+J_{0,x}(\omega)$, and $J_y(t)=J_{1,y}(\omega)+J_{0,y}(\omega)$, where $J_{0,x}(\omega)=(e^2/m^*c)A_x\Sigma_{\mathbf{k}}n_\sigma(\mathbf{k})=(ine^2/m^*\omega)E_x=\sigma_0E_x$, and $J_{0,y}(\omega)=(ine^2/m^*\omega)E_y=\sigma_0E_y$. Here n is the two-dimensional electron concentration. The current $\mathbf{J}_0$ is pure imaginary and thus does not contribute to the resistivity. The current $\mathbf{J}_1$ is complex and its average values are given in term of electron density matrices,

$$J_{1,x(y)}(\omega) = \frac{e\hbar}{m^*} \sum_{\mathbf{k}\sigma} k_{x(y)} F_{\sigma\sigma}(k,\mathbf{k},\omega) \pm i\frac{e\lambda}{\hbar} \sum_{\mathbf{k}} [F_{ud}(\mathbf{k},\mathbf{k},\omega) \mp F_{du(ud)}(\mathbf{k},\mathbf{k},\omega)], \tag{9}$$

where the upper (lower) sign is for $J_x(J_y)$. To calculate the current given in Eq. (9), we use Eq. (8) to obtain the density matrix. By solving Eq. (8) up to the second order in electron-impurity scattering and the linear order in electron-electrical field coupling while the electron-electron interaction is treated in the self-consistent field approximation,[15] we obtain

$$F(\mathbf{k}+\mathbf{p},\mathbf{k},\omega) = \frac{e\hbar}{m^* c\omega}\mathbf{p}\cdot\mathbf{A}_0 V(p) \left[\frac{\Xi(\mathbf{k},\mathbf{p},0)}{\epsilon(p,0)} - \frac{\Xi(\mathbf{k},\mathbf{p},\omega)}{\epsilon(p,\omega)} \right] \times [F^{(0)}(\mathbf{k}+\mathbf{p}) - F^{(0)}(\mathbf{k})] \sum_i \exp(-i\mathbf{p}\cdot\mathbf{R}_i), \tag{10}$$

where $\Xi(\mathbf{k},\mathbf{p},\omega)=\hat{K}^{-1}(\mathbf{k},\mathbf{p},\omega)$, $\hat{K}(\mathbf{k},\mathbf{p},\omega)$ is the Fourier transformation of $\hat{K}(\mathbf{k},\mathbf{p},\partial/\partial t)$. Here the zeroth order density matrix is written as $F^{(0)}_{\sigma\sigma'}(\mathbf{k}+\mathbf{p},\mathbf{k})=f_{\mathbf{k},\sigma}\delta_{\mathbf{p},0}\delta_{\sigma\sigma'}$, and $f_{\mathbf{k},\sigma}=[e^{(E^{(0)}_{\mathbf{k},\sigma}-\mu)/k_BT}+1]^{-1}$ is the Fermi-Dirac distribution, where μ is the chemical potential. In this paper, we shall use the zero-temperature distribution function. The Fermi energy of the system is given as $E_F=\mu(T=0)=\hbar^2\pi n/m^* - m^*\lambda/\hbar^2=E_F(0)-m^*\lambda/\hbar^2$. The electrical current is now found as,

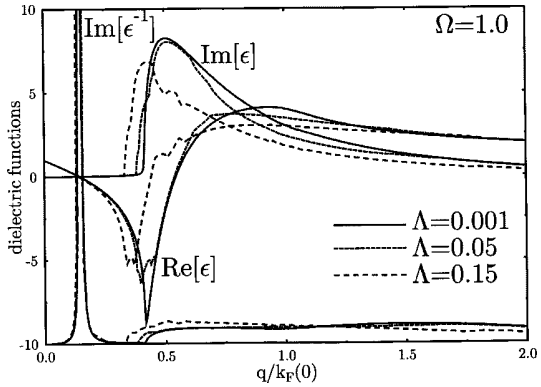

FIG. 1. Wave vector dependence of dielectric functions. The curve for $\mathrm{Im}[1/\epsilon]$ has been multiplied by 7 and shifted by −10.

$$\mathbf{J}(\omega)=\sigma_0\mathbf{E}-i\frac{ne^2}{2m^{*2}\omega^3}\int d\mathbf{q}V(q)\mathbf{q}\cdot\mathbf{E}\left[\frac{1}{\epsilon(q,0)}-\frac{1}{\epsilon(q,\omega)}\right]-\frac{ne^2\lambda^2}{m^*\omega^4}\int d\mathbf{q}\frac{[V(q)]^2}{\epsilon(q,0)}\mathbf{q}\cdot\mathbf{E}\left[\frac{\mathbf{D}_1(\mathbf{q},\omega)}{\epsilon(q,0)}-\frac{\mathbf{D}_2(\mathbf{q},\omega)}{\epsilon(q,\omega)}\right], \tag{11}$$

$$\mathbf{D}_1(\mathbf{q})=\sum_{\mathbf{k}}\frac{2\hat{z}\times\mathbf{k}}{\hbar^2\omega^2-4\lambda^2k^2}\sum_{\alpha}(f_{\mathbf{k},\alpha}-f_{\mathbf{k}+\mathbf{q},\alpha})\times\frac{-g^2+\lambda^2\zeta_1^2-\frac{2\lambda^2}{\hbar\omega}g\mathbf{k}\cdot\mathbf{q}+\frac{2\lambda^4}{\hbar\omega}(2k^2+\mathbf{k}\cdot\mathbf{q})}{g^3-2\lambda^2g\zeta_2^2+\lambda^4g},$$

$$\mathbf{D}_2(\mathbf{q},\omega)=\sum_{\mathbf{k}}\frac{\hat{z}\times\mathbf{k}\sum_{\alpha}(f_{\mathbf{k},\alpha}-f_{\mathbf{k}+\mathbf{q},\alpha})}{\hbar^2\omega^2-4\lambda^2k^2}\frac{2(A\lambda^2C_1+BC_2)}{A^2-B^2},$$

where $g=\hbar^2(q^2+2\mathbf{q}\cdot\mathbf{k})/2m^*$, $\eta^2=g^2+(\hbar\omega)^2$, $\zeta_1^2=4k^2+q^2+4\mathbf{k}\cdot\mathbf{q}$, $\zeta_2^2=2k^2+q^2+2\mathbf{k}\cdot\mathbf{q}$ $A=\eta^4+4(\hbar\omega)^2g^2+[4(m^*)^2\lambda^4/\hbar^2]g^2-2\lambda^2\eta^2\zeta_2^2$, $B=4\hbar\omega g(\eta^2-\lambda^2\zeta_2^2)$, $C_1=-g[g^2+3(\hbar\omega)^2]+\lambda^2g(4k^2+q^2)-(2\lambda^2/\hbar\omega)(\eta^2-2\hbar\omega g)\mathbf{k}\cdot\mathbf{q}+(4m^*\lambda^4/\hbar^2\omega)(2k^2+\mathbf{k}\cdot\mathbf{q})g$, and $C_2=\hbar\omega[3g^2+(\hbar\omega)^2]-\lambda^2\hbar\omega\zeta_1^2+4\lambda^2g\mathbf{k}\cdot\mathbf{q}$. Here we have defined an electronic polarizability function $Q(q,\omega)$ and the dielectric function $\epsilon(q,\omega)=1-2V(q)Q(q,\omega)$,

$$Q(q,\omega)=\sum_{\mathbf{k}}\frac{\Delta(\Delta^2-\lambda^2\zeta_1^2)\sum_{\alpha}[f_{\mathbf{k}+\mathbf{q},\alpha}-f_{\mathbf{k},\alpha}]}{[\Delta^2-(2m^*\lambda^2/\hbar^2)g]^2-(2\lambda k\Delta)^2},$$

where $\Delta=g-\hbar\omega$. The typical behavior of the dielectric function is shown in Fig. 1. In all numerical calculation, the plasma parameter, $r_s=m^*e^2/\kappa\hbar^2k_F(0)$, is taken to be 1 and the electron lifetime broadening is $0.001E_F(0)$ [where $k_F(0)$ and $E_F(0)$ are, respectively, the Fermi wave vector and Fermi energy of the system at $\lambda=0$]. In the absence of the spin splitting, there is a single value of momentum transfer corresponding to the transition energy for a photon absorption. Due to Rashba splitting, there are four different momentum transfers for a given frequency, two intra- and two interlevel transitions. This leads to the fine structures in both the real and imaginary part of ϵ. These fine structures are only resolved if λ is sufficiently large. The quantity that directly affects the transport is the spectral weight $\mathrm{Im}[1/\epsilon]$, which contains both a particle-hole contribution at large q and a plasma contribution at small q. The change of the single-particle energy due to Rashba coupling is $\alpha\lambda k$. For absorption of a given photon, the required momentum transfer of the electron q is less for a state with large λ. This results in a shift of the particle-hole contribution in the spectral weight towards the low q. The q shift of the plasma contribution is much less compared to that of the particle-hole contribution.

The typical value of the dimensionless Rashba parameter $\Lambda=\lambda k_F(0)/E_F(0)$ in narrow-gap semiconductors is around 0.01–0.2. Therefore it is justified to neglect terms of order $0(\Lambda^4)$. In this case the first term in Eq. (11) only contributes to the longitudinal conductivity (σ_{xx} and σ_{yy}) and the second term only contributes to the Hall conductivity (σ_{yx}). σ_{xx} only depends on λ through the dielectric function. σ_{xy} is directly proportional to λ^2. We now define the resistivity of the system, $\hat{\rho}(\omega)=\mathrm{Re}[\hat{\sigma}^{-1}(\omega)]$. Making use of the fact that σ_{xy} is smaller than σ_{xx} by at least Λ^2, we obtain

$$\rho_{xx}(\omega)=\frac{2\hbar}{e^2}\frac{r_s}{\Omega}\int d\mathbf{u}\frac{u_x^2}{u}\,\mathrm{Im}\left[\frac{1}{\epsilon(u,\Omega)}\right], \tag{12}$$

and

$$\rho_{xy}(\omega)=\frac{\hbar}{e^2}\frac{\Lambda^2r_s}{2\pi\Omega^2}\int\frac{d\mathbf{u}}{u}\frac{u_y^2}{\varepsilon(u,0)}\left[\frac{1}{\varepsilon(u,0)}-\mathrm{Re}\left(\frac{1}{\varepsilon(u,\Omega)}\right)\right]. \tag{13}$$

Here $u=q/k_F(0)$, $\Omega=\hbar\omega/E_F(0)$, and ε is the dielectric function at $\lambda=0$. The longitudinal resistivity for the system with finite Λ is given in the same form as those of other many-electron systems, i.e., $\rho_{xx}=\Sigma_q f(q)\mathrm{Im}(1/\epsilon)$, where $f(q)$ is a form factor depending on the details of electron-disorder scattering and $\mathrm{Im}(1/\epsilon)$ can be defined as the scattering cross-section $S(q)$. Our result shows that at finite λ, ρ_{xx} retains the form of $\Sigma_q f(q)\mathrm{Im}(1/\epsilon)$ but with a new dielectric function containing details of the spin splitting. This form of ρ_{xx} can also be understood as a weighted sum of the scattering cross-section $S(q)$. The numerical results indicate that while $S(q)$ is strongly λ dependent, its weighted sum $\Sigma_q f(q)S(q)$ changes very little with Λ. This means that the main effect of the Rashba coupling is to redistribute the scattering probability at different q but not to significantly change the total cross

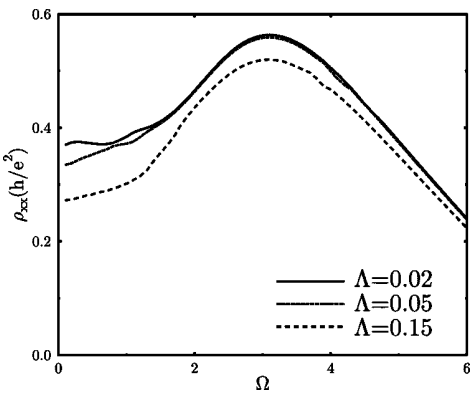

FIG. 2. Plot of the longitudinal resistivity vs frequency.

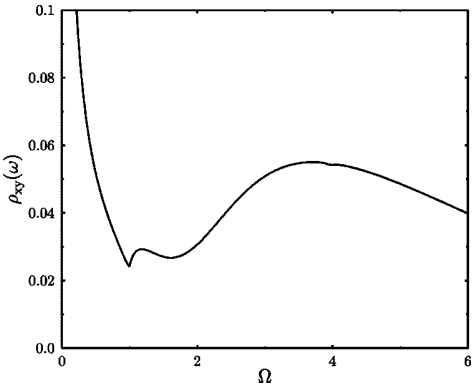

FIG. 3. Plot of the Hall resistivity (in unit of $\Lambda^2\hbar/e^2$) vs frequency.

section summed over all q. As a result, ρ_{xx} depends on λ rather weakly. Figure 2 shows the frequency dependence of ρ_{xx} for weak and strong Rashba coupling. The effect of Rashba coupling is more important at low frequency. When the frequency is low, particle-hole contributions dominate. Due to the q shift of the particle-hole contribution in the $S(q)$ discussed early, the resistivity is reduced. As frequency increases, the $S(q)$ is mainly due to the plasmon mode that is much less affected by the Rashba coupling. Therefore the reduction in ρ_{xx} at higher frequency is less. The reduction of ρ_{xx} as λ increases has a similar origin of the pass from weak localization to weak antilocalization.[16,17] The overall frequency dependence of ρ_{xx} reflects the interplay of the electron-impurity scattering probability and the available phase (energy) space for electronic transition. The scattering rate decreases with the Ω while the phase space increases with the Ω at low Ω and reaches the maximum value at some intermediate Ω. At the high-frequency regime, the phase space is constant and the decreasing ρ_{xx} is due the decreasing scattering probability. At low frequency, the increase of ρ_{xx} is basically due to the fact that phase space increases at a faster rate than the decreasing rate of the scattering probability. The screened potential has a weak oscillation due to $\mathrm{Re}[\epsilon]$ at a low-frequency regime. This leads to the weak oscillatory structure at a low-frequency regime.

While $\varepsilon(q,\omega)$ and ρ_{xx} showed some interesting features due to the Rashba coupling, the central result and the new finding of the current work is the prediction and derivation of zero-field dynamic Hall resistivity $\rho_{xy}(\omega)$. While a nonzero ρ_{xy} is expected for an isotropic electronic system, our result of ρ_{xy} exhibits several unexpected features. The magnitude of ρ_{xy} is proportional to Λ^2. Unlike other known resistivity components in any other many-electron systems, the Hall resistivity derived here is given in the form of $\Sigma_q f(q)\mathrm{Re}(1/\varepsilon)$. That is, the resistivity is determined by the reactive part of the response function. This is due to the reason that in the Rashba coupling, there is a $\pi/2$ phase difference between p_x and p_y. The real and imaginary parts of the dielectric functions have very different frequency and wave number dependence. Figure 3 shows the frequency dependence of the Hall resistivity. At low frequency, the Hall resistivity increases rapidly with the decreasing frequency. This is similar to the behavior of frequency-dependent electron effective mass,[18] as both are determined by the real part of the dielectric function. Apart from an additional screening $[1/\varepsilon(q)], \rho_{xy} \sim \mathrm{Im}(\sigma_{xx})$. As the quantity $[1/\varepsilon(q,0] - \mathrm{Re}[1/\varepsilon(q,\omega)])/\omega^2$ is finite when $\omega \to 0$, the dc limit of ρ_{xy} is well defined in our formalism. Since ρ_{xy} is determined by the reactive part of the dielectric function, there is no restriction due to the phase space. The reactive part $\mathrm{Re}[1/\varepsilon]$ increase with Ω slowly for $\Omega < 1$ and much more rapidly for $\Omega > 1$. On the other hand, function $\Sigma_q f(q)\mathrm{Re}(1/\varepsilon)$ will be amplified by Ω^{-2} for $\Omega < 1$ and be suppressed by it for $\Omega > 1$. This leads to a sudden turnaround (cusplike) behavior of ρ_{xy} at $\Omega = 1$. At high frequencies, $\Sigma_q f(q)\mathrm{Re}(1/\varepsilon)$ is approximately a linear function of Ω; as a result, ρ_{xy} decreases with Ω. Experimentally it is rather difficult to measure the dynamic effective mass while it is relatively easier to measure the Hall resistivity.

In conclusion, we report on a zero-field charge Hall effect in electronic systems with Rashba coupling. The Hall resistivity is determined by the real part of the dielectric function. It is also found that the Rashba coupling reduces the electromagnetic absorption in electronic systems. This reduction is most important at low frequencies. Both these features can be measured in a transport experiment.

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