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Keywords

adaptive control, autoregressive moving average processes, matrix algebra, parameter estimation, process control, stochastic systems, time-varying systems, tracking

Disciplines

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PARAMETER TRACKING FOR STOCHASTIC TIME-VARYING SYSTEMS

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Abstract - This paper describes a method for tracking fast time-varying parameters of dynamic systems under stochastic disturbances. Auxiliary information is used in addition to the plant input and output for better tracking accuracy.

1. INTRODUCTION

Unknown and time-varying processes are common in industrial applications and one of the primary motivations for adaptive control is for dealing with such dynamics. In parameter adaptive control approaches the key difficulty is to track time-varying parameters accurately without delay.

For fast time-varying systems Xie and Evans suggested to approximate the time-varying parameters locally using time polynomials and developed a scheme based on linear approximation for adaptive control of deterministic systems [1]. Their scheme was extended by Li using high order algebraical polynomial approximation [2] based on the fact that a smooth enough parameter can be approximated very precisely by a low order Taylor polynomial in a small neighborhood around the expansion point. The smaller the neighborhood, the better the approximation precision in general. In order to achieve high modeling precision for the current parameters for adaptive control the expansion point has to run forward constantly such that it is close to the current time. On the other hand, the data, which appeared far away from the current time, have to be forgotten because of their correspondence to relatively large modeling errors. However, this may result in insufficiency of data for persistent excitation and the estimates are sometimes sensitive to noise.

To reduce the sensitivity Li introduced an additional measurement called auxiliary variable for parameter adaptive control of stochastic time-varying systems where the polynomial approximation is with respect to the auxiliary variable rather than time [3]. When an appropriate auxiliary variable is available the running speed of the expansion point can be very slow compared to those based on the time polynomial scheme and, consequently, more data are available for counteracting the noise. Moreover, if the auxiliary variable is uniformly bounded the dynamic model for the parameter variation is globally stable. In this paper we will show that the advantages of using auxiliary variable can be further extended by using a combination of local modeling and globe smoothing scheme.

2. PARAMETER MODELING

Consider the time-varying ARMAX model,

$$y(t)+a_1(r)y(t-1)+a_2(r)y(t-2)+\ldots+a_n(r)y(t-n)$$
$$=a_{n+1}(r)u(t-1)+a_{n+2}(r)u(t-2)+$$
$$\ldots+a_m(r)u(t-m+n)+w(t) \quad (1)$$

where $u(t)$ and $y(t)$ are the plant input and output, and $a_i(r)$, $i=1, 2, \ldots, m$, are unknown plant parameters. In the ARMAX model $w(t)$ is a zero mean independent Gaussian noise having uniformly bounded variance and $r=r(t)$ is a function of time which represents the third measurable variable in addition to the plant input and output. Because r is a variable the plant parameters are time-varying. In particular, if $r=t$ the plant (1) becomes the standard time-varying ARMAX model. The variable r is usually called an auxiliary variable in gain scheduling.

The objective is to track the time-varying plant parameters accurately. The following assumptions are made about the system.

A1) The plant orders m and n are known
A2) The auxiliary variable r wanders within a finite interval $(a, b]$
A3) The plant parameters are unknown and smooth enough with respect to the auxiliary variable over the closed interval $[a, b]$

There is no limitation on the variation speed of r with respect to time. Each of the m parameters can be modeled using an algebraical polynomial as follows

$$d(r)=d_0+d_1\frac{(r-s)}{1!}+d_2\frac{(r-s)^2}{2!}+\ldots+d_N\frac{(r-s)^N}{N!} \quad (2)$$

where r and s are values of the auxiliary variable and d_i, $i=0, 1, \ldots, N$, are constants to be determined. Letting $\Delta=r-s$ we define the following output and state transition matrix

$$\bar{C} = [0\ 0 \ldots\ 0\ 1]$$

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$$\bar{A}(r-s)=\begin{bmatrix} 0 & 0 & \dots & 0 & 1 \\ 0 & \dots & 0 & 1 & \frac{\Delta}{1!} \\ & & \dots & & \\ 0 & 1 & \frac{\Delta}{1!} & \dots & \frac{\Delta}{(N-1)!} \\ 1 & \frac{\Delta}{1!} & \frac{\Delta}{2!} & \dots & \frac{\Delta}{N!} \end{bmatrix} \quad (3)$$

The polynomial can then be expressed using a state space model

$$d(r)=\bar{C}\bar{X}(r)$$

$$\bar{X}(r)=\bar{A}(r-s)\bar{X}(s) \quad (4)$$

Considering m polynomials for modeling the m time-varying parameters we define the overall state transition matrix and the overall output matrix,

$$A(r-s)=\bar{A}(r-s)\otimes I$$

$$C=\bar{C}\otimes I \quad (5)$$

where $\otimes$ denotes the Kronecker multiplication and I is an $m\times m$ dimensional identity matrix. Thus, the m polynomials can be modeled as the output of the following state space model.

$$X(r)=A(r-s)X(s)$$

$$D(r)=CX(r) \quad (6)$$

Rewrite the ARMAX model in the compact form,

$$y(t)=\varphi^{\tau}(t)\theta(r)+w(t) \quad (7)$$

where

$$\varphi(t)=[-y(t-1)\ -y(t-2)\ \dots\ -y(t-n)\ u(t-1)\ u(t-2)\ \dots\ u(t-m+n)]^{\tau}$$

$$\theta(r)=[a_1(r)\ a_2(r)\ \dots\ a_n(r)\ a_{n+1}(r)\ a_{n+2}(r)\ \dots\ a_m(r)]^{\tau} \quad (8)$$

are the data and parameter vectors. The parameter vector can therefore be approximated by the output of the state-space model (6).

The assumption A3) can be relaxed to that the plant parameters are unknown continuous functions of the auxiliary variable on $[a, b]$ according to the famous Weiestrass Theorem that every continuous function on $[0, 1]$ can be uniformly approximated by algebraical polynomials. However, a much higher polynomial degree may be required to achieve sufficient approximation accuracy when a plant parameter is not smooth enough.

When A3) holds the polynomial modeling can be explained using the Taylor polynomial where s is the expansion point and d_i is the ith derivative of a plant parameter at s. When the auxiliary variable wandering range is wide, the polynomials with high degrees have to be applied in general to ensure sufficient global modeling accuracy so that both remote and local data with respect to the expansion point can be utilised to counteract noise. However, the higher the polynomial degree, the more the coefficients to be estimated and, consequently, the more sensitive the estimation algorithm. An alternative is to use low order polynomials to model the parameters over a small neighborhood around the current auxiliary variable.

3. LOCAL PARAMETER ESTIMATION

In order to model the plant parameters accurately using low order polynomials the local parameter estimation is based on a bank of local parameter models, each describes the variations in the plant parameters over a section of the auxiliary variable wandering range and handles the corresponding data. We divide the range into μ subintervals, $\delta_i=(a_{i-1}, a_i]$, $i=1, 2, \dots, \mu$, where $a_0=a$, $a_\mu=b$. We then apply a polynomial state space model (6) to each subinterval for modeling the plant parameters within that subinterval. The available data are therefore divided among the subintervals. We use the notation n_i to indicate the number of auxiliary variable measurements, which fall into δ_i up to and including time t. When $r(t) \in \delta_i$ we let $r_i(n_i)=r(t)$, $y_i(n_i)=y(t)$, $\varphi_i(n_i)=\varphi(t)$ and $v_i(n_i)=v(t)$ to associate them with the ith subinterval and for this sampling period the algorithms for the rest of the subintervals will remain unchanged.

Since the plant parameters are smooth enough with respect to r we know from Taylor Remainder that the local parameter dynamic model can achieve the approximation of arbitrarily high precision even by using low degree polynomials when the length of the subinterval is sufficiently small. Ignoring the modeling error and substituting the state space model (6) into the plant model (7) we have μ state space equations, each defined over one of the μ subintervals,

$$X_i(r_i(n_i))=A(r_i(n_i)-r_i(n_i-1))X_i(r_i(n_i-1))$$

$$y_i(n_i)=h_i(n_i)X_i(r_i(n_i))+v_i(n_i),\quad i=1, 2, \dots, \mu \quad (9)$$

where

$$h_i^{\tau}(n_i)=[0\ 0\ 0\ \dots\ 0\ \varphi_i^{\tau}(n_i)],\ i=1, 2, \dots, \mu \quad (10)$$

are time-varying observation matrices. The state space model (9) has a unit sampling period with respect to the local time index n_i, however, its sampling period with respect to the real

time t is time-varying. Letting $\hat{X}_i(r/n_i)$ and $\hat{\theta}_i(r/n_i)$ denote the estimates of $X(r)$ and $\theta(r)$ based on the measurements up to and including n_i the local parameter tracking algorithm for the *ith* subinterval is given by the following parameter tracker.

$$P_i(r_i(n_i)/n_i\text{-}2)=A(r_i(n_i)\text{-}r_i(n_i\text{-}1))P_i(r_i(n_i\text{-}1)/n_i\text{-}2)A^{\tau}(r_i(n_i) \text{-}r_i(n_i\text{-}1))+Q_i(n_i\text{-}1) \quad (11)$$

$$\hat{X}_i(r_i(n_i)/n_i\text{-}1)=A(r_i(n_i)\text{-}r_i(n_i\text{-}1))\hat{X}_i(r_i(n_i\text{-}1)/n_i\text{-}1) \quad (12)$$

where $r_i(n_i\text{-}1)$ is the expansion point at $n_i\text{-}1$ and $r_i(n_i)$ is the expansion point at n_i. The gain for correction is

$$K_i(n_i)=P_i(r_i(n_i)/n_i\text{-}2)h_i(n_i)[R_i(n_i) +h_i^{\tau}(n_i)P_i(r_i(n_i)/n_i\text{-}2)h_i(n_i)]^{-1} \quad (13)$$

The state estimate are the covariance matrix are updated by

$$\hat{X}_i(r_i(n_i)/n_i)=\hat{X}_i(r_i(n_i)/n_i\text{-}1) +K_i(n_i)[y_i(n_i)\text{-}\varphi_i^{\tau}(n_i)\hat{\theta}_i(r_i(n_i)/n_i\text{-}1)]$$

$$P_i(r_i(n_i)/n_i\text{-}1)=P_i(r_i(n_i)/n_i\text{-}2)\text{-}P_i(r_i(n_i)/n_i\text{-}2)h_i(n_i)[R_i(n_i) +h_i^{\tau}(n_i)P_i(r_i(n_i)/n_i\text{-}2)h_i(n_i)]^{-1}h_i^{\tau}(n_i)P_i(r_i(n_i)/n_i\text{-}2) \quad (14)$$

where $R_i(n_i)$ is a positive real number uniformly bounded away from both zero and infinite and $Q_i(n_i\text{-}1)$ is a positive semi-defined matrix uniformly bounded away form infinite. Parameter estimate can then be computed using

$$\hat{X}_i(r/n_i)=A(r\text{-}r_i(n_i))\hat{X}_i(r_i(n_i)/n_i)$$

$$\hat{\theta}_i(r/n_i)=C\hat{X}_i(r/n_i) \quad (15)$$

For the current parameter estimate we let $r=r_i(n_i)$. The initial condition is given by

$$\hat{X}_i(r_i(0)/0)=A(r_i(0)\text{-}r_0)\hat{X}(r_0/0)$$

$$P_i(r_i(0)/\text{-}1)=A(r_i(0)\text{-}r_0)P(r_0/\text{-}1)A(r_i(0)\text{-}r_0)^{\tau} \quad (16)$$

where $P(r_0/\text{-}1)\geq cI$, for some constant $c>0$ and an identity matrix I of appropriate dimension. At the starting time $\hat{X}(r_0/0)$ and $P(r_0/\text{-}1)$ denote initial estimates of $X(r_0)$ and an estimate of its estimation error covariance matrix. In the case of no prior knowledge we set, as in the least-squares case, $\hat{X}_i(r_i(0)/0)=0$ and $P_i(r_i(0)/\text{-}1)=KI$ with $K>>1$.

The above local trackers estimate the plant parameters locally and independently based on the localised data. The estimates are usually discontinuous at joint points of the subintervals, i.e. $r=a_i$, $i=1, 2, \ldots, \mu\text{-}1$. Moreover if the data share of a subinterval is small, the accuracy of local parameter estimates over this subinterval will be poor. In the next section we will show how the local parameter estimates can be smoothed globally to improve estimation accuracy.

4. GLOBAL PARAMETER SMOOTHING

The local parameter estimates can be joined together to form global parameter estimates for more accurate estimation, particularly around the join points of the subintervals, which are usually discontinuous due to the locality of the tracking algorithms, and for the subintervals which do not have enough data shares. We fit the local parameter estimates using higher order polynomials over several subintervals. The global parameter estimates based on all the data over two or several contiguous subintervals are defined as an m dimensional polynomial vector

$$\hat{\theta}(r/t)=\hat{F}R(r) \quad (17)$$

where

$$R(r)=[1 \; r \; r^2 \; \ldots \; r^K]^{\tau} \quad (18)$$

is an $K+1$ dimensional vector and K is the order of the global polynomial, the coefficients of the polynomial is given by an $m\times(K+1)$ dimensional matrix,

$$\hat{F}=[\hat{f}_1 \; \hat{f}_2 \; \ldots \; \hat{f}_m]^{\tau} \quad (19)$$

given by

$$\hat{F}=\underset{F}{argmin}\{\sum_{i=h}^{k}\int_{a_{i-1}}^{a_i} FR(r) -\hat{\theta}_i(r/n_i)]^{\tau}W_i^{-1}(r/n_i)[FR(r)-\hat{\theta}_i(r/n_i)]dr\} \quad (20)$$

where the weighting matrices

$$W_i(r/n_i)=CA(r\text{-}r(n_i))P_i(r(n_i)/n_i)A^{\tau}(r\text{-}r(n_i))C^{\tau} \quad i=h, h+1, \ldots, k \quad (21)$$

are the covariance matrices for the local parameter estimates. The cost functional in the right hand side of (20) can be rewritten as

$$I(F)=\sum_{i=h}^{k}\int_{a_{i-1}}^{a_i}[rs(F)(I\otimes R(r))-\hat{\theta}_i^{\tau}(r/n_i)]W_i^{-1}(r/n_i)$$
$$\times[(I\otimes R^{\tau}(r))cs(F^{\tau})-\hat{\theta}_i(r/n_i)]dr \quad (22)$$

where

$$rs(F)=cs(F^{\tau})^{\tau}=[f_1^{\tau}\ \ f_2^{\tau}\ \ \cdots\ f_m^{\tau}] \quad (23)$$

is an $m(K+1)$ dimensional row vector. It follows that

$$\frac{\partial I(F)}{\partial cs(F^{\tau})}=2\sum_{i=h}^{k}\int_{a_{i-1}}^{a_i}(I\otimes R(r))W^{-1}(r/n_i)[(I\otimes R^{\tau}(r))cs(F^{\tau})$$
$$-\hat{\theta}_i(r/n_i)]dr$$

$$\frac{\partial^2 I(F)}{\partial cs(F^{\tau})\partial cs(F^{\tau})^{\tau}}=2\sum_{i=h}^{k}\int_{a_{i-1}}^{a_i}(I\otimes R(r))W^{-1}(r/n_i)$$
$$\times(I\otimes R^{\tau}(r))dr \quad (24)$$

Noting

$$I\otimes R(r))W_i^{-1}(r/n_i)(I\otimes R^{\tau}(r)\geq 0$$
$$I\otimes R(r))W_i^{-1}(r/n_i)(I\otimes R^{\tau}(r)\neq 0 \quad (25)$$

hold almost everywhere within the interval $[a, b]$ and $b-a>0$ we have

$$\frac{\partial^2 I(F)}{\partial cs(F^{\tau})\partial cs(F^{\tau})^{\tau}}>0 \quad (26)$$

Thus, from the first equation in (24) we have

$$cs(\hat{F}^{\tau})=[\sum_{i=h}^{k}\int_{a_{i-1}}^{a_i}(I\otimes R(r))W^{-1}(r/n_i)[(I\otimes R^{\tau}(r))dr]^{-1}$$
$$\times[\sum_{i=h}^{k}\int_{a_{i-1}}^{a_i}(I\otimes R(r))W_i^{-1}(r/n_i)\hat{\theta}_i(r/n_i)dr] \quad (27)$$

The above solution requires calculation of the inverse of an $m(K+1)\times m(K+1)$ matrix, which is computationally expensive. Noting that the function of the weighting matrices is to accentuate accurate local parameter estimates in the smoothing, we replace $W_i(r/n_i)$ using

$$\lambda_i I=||W_i(\frac{a_{i-1}+a_i}{2}/n_i)||I \quad (28)$$

for all i to save the computational effort. It follows that

$$\frac{\partial I(F)}{\partial cs(F^{\tau})}=2\sum_{i=h}^{k}\int_{a_{i-1}}^{a_i}[(I\otimes R(r)R^{\tau}(r))cs(F^{\tau})$$
$$-(I\otimes R(r)\hat{\theta}_i(r/n_i)]dr$$
$$=2\sum_{i=h}^{k}\int_{a_{i-1}}^{a_i}[\begin{bmatrix}R(r)R^{\tau}(r)f_1\\R(r)R^{\tau}(r)f_2\\\cdots\\R(r)R^{\tau}(r)f_m\end{bmatrix}-\begin{bmatrix}R(r)\hat{\theta}_{i1}(r/n_i)\\R(r)\hat{\theta}_{i2}(r/n_i)\\\cdots\\R(r)\hat{\theta}_{im}(r/n_i)\end{bmatrix}]dr \quad (29)$$

Rearranging elements in the above equation we have

$$\frac{\partial I(F)}{\partial cs(F^{\tau})}=2\sum_{i=h}^{k}\int_{a_{i-1}}^{a_i}\{R(r)R^{\tau}(r)[f_1, f_2, \cdots f_m]$$
$$-R(r)[\hat{\theta}_{i1}(r/n_i)\ \hat{\theta}_{i2}(r/n_i)\ \cdots\ \hat{\theta}_{im}(r/n_i)]\}dr \quad (30)$$

Thus

$$\hat{F}^{\tau}=G^{-1}B \quad (31)$$

where

$$B=\sum_{i=h}^{k}\lambda_i\int_{a_{i-1}}^{a_i}R(r)\hat{\theta}_i^{\tau}(r/n_i)\,dr$$
$$G=\sum_{i=h}^{k}\lambda_i\int_{a_{i-1}}^{a_i}R(r)R^{\tau}(r)dr \quad (32)$$

Given the piecewise local parameter state estimates

$$\hat{X}_i(0/n_i)=[\hat{x}_{i1}^{\tau}(0/n_i)\ \ \hat{x}_{i2}^{\tau}(0/n_i)\ \ \cdots\ \ \hat{x}_{i(N+1)}^{\tau}(0/n_i)]^{\tau} \quad (33)$$

where $\hat{x}_{ij}^{\tau}(0/n_i)$, $i=h, h+1, \ldots, k$; $j=1, 2, \ldots, N+1$, are m dimensional substate vectors. From (32) we know

$$B=[b_1\ b_2\ \cdots\ b(K+1)]^{\tau}$$

$$G=[g_{pq}](K+1)(K+1) \tag{34}$$

are given by

$$b_p=\sum_{i=h}^{k}\lambda_i\sum_{j=1}^{N+1}\frac{a_i^{j+p-1}-a_{i-1}^{j+p-1}}{(j+p-1)(j-1)!}\hat{x}_{ij}(0/n_i)$$

$$g_{pq}=\frac{1}{p+q-1}\sum_{i=h}^{k}\lambda_i(a_i^{p+q-1}-a_{i-1}^{p+q-1}) \tag{35}$$

Comparing with (27), solution (31) requires to inverse an $(K+1)^2$ matrix and the integration is avoided.

The introduction of the weighting matrices (21) into the L_2 approximation cost functional (22) is in account for the difference in estimation accuracy between the local estimates. Roughly speaking, the weightings reflect the accuracy well when small norm covariance modification matrices $Q_i(n_i-1)$ are applied continuously or periodically in the estimation algorithm (12). However, if large norm covariance modification matrices are applied occasionally, the weightings may not be able to reflect the accuracy correctly. When there is no big difference in estimation accuracy among the local estimates, we set $\lambda_i=1$ for all i in order to save computational effort.

The choice of the global polynomial degree K is crucial to estimation accuracy for the smoothing. When all the local parameter estimates are well estimated within each subintervals and the purpose of the smoothing is to improve estimation accuracy at the discontinuous joint points between the subintervals the degree can be chosen so that $K\geq(k-h+1)(N+1)-1$. When there is big difference in estimation accuracy among the local parameter estimates equal or lower polynomial degree could be applied. The degree can be chosen by estimating numbers of turning points of the plant parameters using the local parameter estimates. If the estimated number of turning points for a parameter is H, the polynomial degree should be $K\geq H+1$.

For simplifying presentation we assume that all the global polynomials have the same degree. This assumption is not necessary and can be easily removed allowing different parameters to be approximated using global polynomials of different degrees.

5. SIMULATION

The plant for simulation is given by

$$y(t)+a_1(r(t))y(t-1)=a_2(r(t))u(t)+w(t) \tag{36}$$

where $u(t)$ is a square wave with zero mean, amplitude 10 and period 5 and $w(t)\sim N(0, 1)$ is an independent stationary Gaussian noise with zero mean and unit variance. The plant parameters are given by

$$a_1(r(t))=0.6\cos(0.8\pi r(t)+0.7\pi)$$

$$a_2(r(t))=0.3+0.2r(t)+0.1r(t)^2 \tag{37}$$

where the auxiliary variable has the form

$$r(t)=1+\cos(2\pi\frac{t+1}{100}) \tag{38}$$

For this simulation we choose the weighting factors as $\lambda_i=1$ for all i, the local polynomial degree $N=2$ while the global polynomial degree $K=5$. Apparently, the polynomial parameter model cannot describe the cosine parameter accurately. To accommodate the error we set for any i

$$Q_i(k)=\begin{cases}0.001I & k=10\ 20\ 30\ \ldots\\ 0I & else\end{cases} \tag{39}$$

The auxiliary variable wandering range is divided into two subintervals, $[0, 1]$ and $(1, 2]$. The parameters and their estimates *versus* the time variable are presented in Fig. 1 and Fig. 2 showing that large deviations between the parameters and their estimates vanish within 50 steps. However, small deviations persists. The large deviations are due to initial uncertainty while the small ones are due to the approximation error and the noise. The parameters and their estimates *versus* the auxiliary variable at $t=400$ are shown in Fig. 3 and Fig. 4. The discontinuous points at $r=1$ persist. The improvement on estimation accuracy by the global estimate around the discontinuous points is visible.

8. CONCLUSION

A parameter tracking scheme has been developed for time-varying systems subject to stochastic disturbances. In order to achieve accurate modeling and maximum data utilisation it consists of two steps: local tracking and global smoothing. Simulation results show the method is able to track fast time-varying parameters of dynamic systems accurately under stochastic disturbance.

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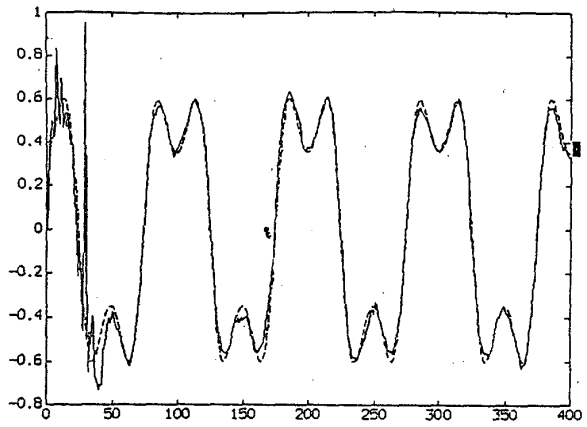

Fig. 1 The first parameter $a_1(r)$ (dashed line) and its estimate (solid line) *versus* the time variable.

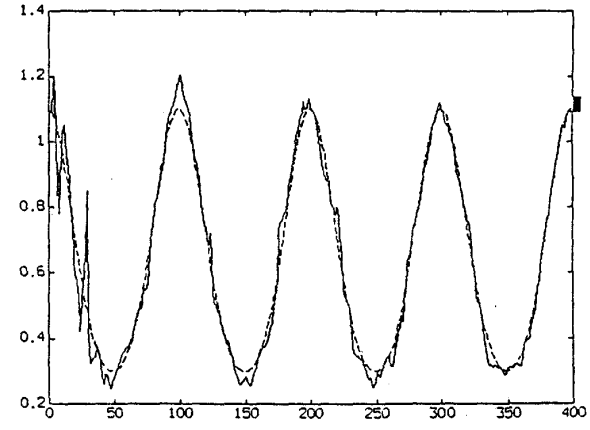

Fig. 2 The second parameter $a_2(r)$ (dashed line) and its estimate (solid line) *versus* the time variable.

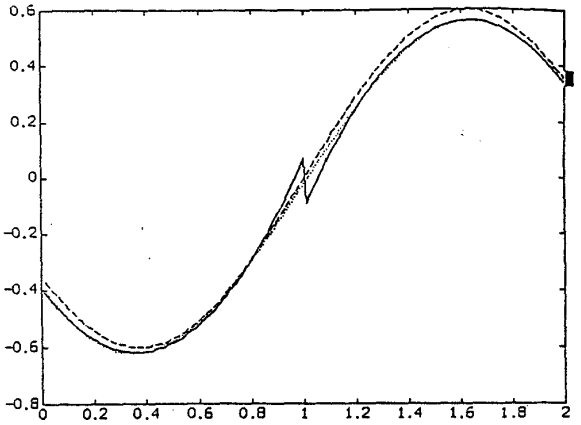

Fig. 3 The first parameter $a_1(r)$ (dashed line), its local (solid line) and global (dotted line) estimate *versus* the auxiliary variable at $t=400$.

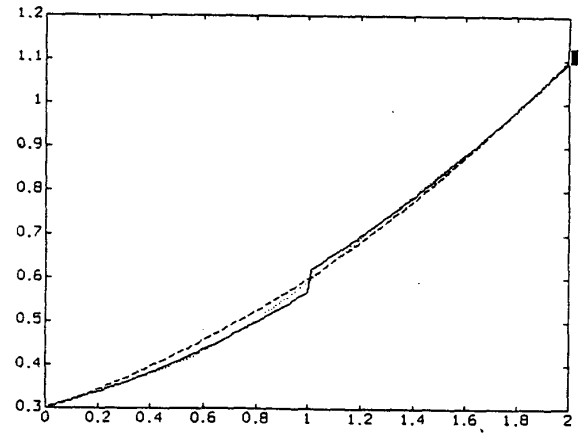

Fig. 4 The second parameter $a_2(r)$ (dashed line), its local (solid line) and global (dotted line) estimate *versus* the auxiliary variable at $t=400$.