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Optical absorption coefficients in two-dimensional semiconductors under strong magnetic field

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We calculate the photon absorption coefficient of hot two-dimensional electrons in the presence of a strong magnetic field. The electrons interact strongly with the optical phonons and the acoustic phonons in quantum wells. The dependence of the optical absorption on the magnetic field is obtained by using the quantum mechanical kinetic theory. It is found that the photon absorption spectrum displays a local magnetophonon resonance. The magnetophonon absorption resulting from inelastic scattering between Landau levels is more pronounced at higher temperature. The effect of subband nonparabolicity on the absorption coefficient is also discussed. © 2006 American Institute of Physics. [DOI: 10.1063/1.2206855]

I. INTRODUCTION

Many theoretical and experimental techniques have been developed to study the nonlinear transport and optical properties in low-dimensional semiconductor systems.^{1–6} The optical absorption in two-dimensional (2D) systems in a magnetic field is of particular interest as it exhibits various resonances due to the interplay between different length scales.^{7–10} There are several ways to tailor the optical properties of semiconductors. One of them is band-gap engineering, i.e., growing layered low-dimensional semiconductor structures (heterostructures) with prescribed electronic properties. In addition to that, one can apply time-dependent magnetic and electric fields to the systems for probing the dynamic properties. For 2D semiconductor structures under an external magnetic field, the energy levels are shifted and the in-plane electron motion is modified.^{11–13} In the presence of strong magnetic fields the electronic energy levels in a band are splitted into discrete Landau levels. Transitions between adjacent Landau levels are accompanied by the absorption or emission of a phonon.^{14–18} When the separation between the Landau levels matches the phonon energy the magnetophonon resonance (MPR) arises. In low-dimensional semiconductor structures the phonon emission and scattering by 2D electrons are dependent on the magnitude and direction of the magnetic field.^{19–21} The optical phonon confinement effects (i.e., confined and interface-phonon modes) which affect significantly the scattering rates in quantum wells have been extensively studied.^{22–26} The electron-optical-phonon interaction in quantum wells was studied using either dielectric continuum models^{27–29} or microscopic lattice dynamical models.^{30–32}

It is a quite common practice to assume that the semiconductor structure has a perfect parabolic band structure. However, in many cases this approximation is not well justified.^{33,34} The effect of the subband nonparabolicity on the optical absorption has not been fully studied. In this paper, the effect of the magnetic field on the electron-phonon interaction and the optical absorption is investigated. The magnetic field will significantly alter the electron-phonon coupling. We shall include the band nonparabolicity effect on the optical absorption mediated by emission of confined phonons. For our model systems of polar semiconductor quantum wells, the dominant scattering mechanisms are the electron interactions with longitudinal optical (LO) phonons and deformation-potential (DP) acoustic phonons.

II. ABSORPTION COEFFICIENT OF QUANTUM WELL IN THE PRESENCE OF MAGNETIC FIELD

Our model system is an $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}/\text{GaAs}$ single quantum well subjected to an in-plane magnetic field. The electron interacts with phonons and photons. The total Hamiltonian is given as

$$H = H_0 + H_{e\text{-ph}} + H_\gamma, \quad (1)$$

where H_0 is the Hamiltonian of a free electron in a quantum well under a constant magnetic field,

$$H_0 = \frac{1}{2}(P - eA)\frac{1}{m^*(z)}(P - eA) + U_0(z). \quad (2)$$

Here P is the electron momentum operator, A is the vector potential of the static magnetic field, $U_0(z)$ is the quantum well confining potential, and $m^*(z)$ is the position dependent electron effective mass. $H_{e\text{-ph}}$ includes electron interaction with both the LO phonons and DP acoustic phonons. H_γ is the electron-photon interaction. The electron-phonon interac-

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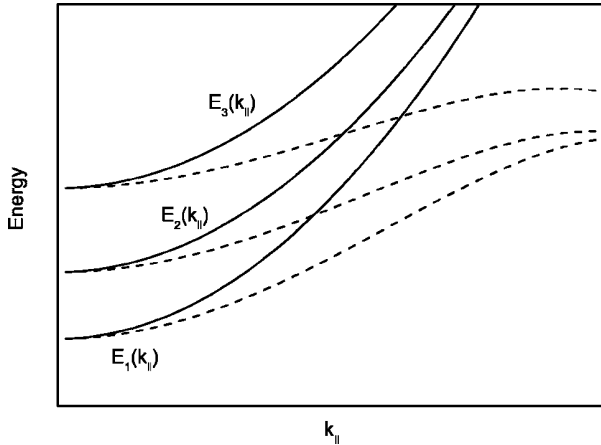


FIG. 1. A schematic diagram of phonon wave vectors with the absence of magnetic field which shows that for nonparabolic subbands the phonon wave vector is larger than for parabolic subband.

tion will be treated as a perturbation and we seek the linear response of the system to the photon field.

Our starting point is the zeroth order electron wave function determined by the eigenvalue problem of H_0 ,

$$H_0 \Psi_{k,j}(x, y, z) = E_{k,j} \Psi_{k,j}(x, y, z). \quad (3)$$

The zeroth order wave functions are the basis for calculating the transition probability due to H_{e-ph} and H_γ . The wave functions can be factorized into an in-plane part and a subband function, given as

$$\Psi_{k,j}(x, y, z) = e^{i(k_x x + k_y y)} \psi_j(z), \quad (4)$$

where k_x is the x component of wave vector, $\psi_j(z)$ is the envelope wave function for the j th subband, and $E_j(k_x, k_y) = \hbar^2 k_x^2 / 2m^* + \epsilon_j(k_y)$ is the corresponding energy spectrum. The envelope function is given as

$$\psi_j(z) = A \exp(-k_B z), \quad z < -L/2, \quad (5)$$

$$\psi_j(z) = B \sin(-k_W z) + C \cos(k_W z), \quad |z| < L/2, \quad (6)$$

$$\psi_j(z) = D \exp(k_B z), \quad z > L/2. \quad (7)$$

Here $k_W(k_B)$ is the electron wave vector in the well (barrier) region. The constants A , B , C , and D are determined by the normalization and boundary conditions.

The electron subband nonparabolicity³³ can be described by the energy dispersion relation and the appropriate energy effective masses. For semiconductor quantum wells, the nonparabolicity is mainly due to the coupling between k_{\parallel} and k_z through of nonparabolic parameter γ . In order to introduce the subband nonparabolicity, we expand the bulk conduction-band dispersion in terms of k^2 . The energy dispersion to the order of k^4 can be written $\epsilon - V = (\hbar^2 k^2 / 2m^*)(1 - \gamma k^2)$, where γ is the nonparabolicity parameter. $V=0$ in the well region and $V=V_0$ in the barrier region. Figure 1 shows the phonon dispersion in the absence of magnetic field. It can be seen that the phonon wave vector is larger for nonparabolic subbands than that for parabolic subband. The result indicates that the electron-phonon coupling is weaker in systems with nonparabolic subbands.

The transition probability can be derived by using the Fermi golden rule. As a consequence of the subband nonparabolicity, the energy conservation in the Fermi golden rule now becomes $\delta(\epsilon_i - \epsilon_f - \hbar\omega_q + \hbar\omega) = \delta(a \cos^2 \theta + b \cos \theta + c)$, where $\cos \theta$ is related to the in-plane momentum conservation for the emission of a phonon with momentum q_{\parallel} , and a , b , and c are defined as

$$a = \frac{\hbar^2}{2m^*} 4\gamma k_{\parallel}^2 q_{\parallel}^2, \quad (8)$$

$$b = \frac{\hbar^2}{2m^*} [2k_{\parallel} q_{\parallel} - 4\gamma(k_{\parallel}^2 + q_{\parallel}^2)k_{\parallel} q_{\parallel} - 4\gamma k_{\parallel}^2 k_{\parallel} q_{\parallel}], \quad (9)$$

$$c = \frac{\hbar^2}{2m^*} \{k_{\parallel}^2 [1 - 2\gamma(k_{\parallel}^2 + k_{zi}^2)] - (k_{\parallel}^2 + q_{\parallel}^2)(1 - 2\gamma k_{zi}^2) + \gamma(k_{\parallel}^2 + q_{\parallel}^2)^2 + Q^2\}, \quad (10)$$

and for phonon emission, $Q^2 = Q_{\pm}^2 = \pm(2m^*/\hbar)(E_i - E_f - \hbar\omega_q + \hbar\omega)$, where the upper (lower) sign is for intra-(inter-) band transitions. The energy conservation is now given as $\delta(\epsilon_i - \epsilon_f - \hbar\omega_q + \hbar\omega) = \delta[f(x)] = |b^2 - 4ac|^{-1/2} \delta(x - R^-)$, where R^- is the negative root of $f(x)=0$. Similarly, for phonon absorption, $Q^2 = Q_{\pm}^2 = \pm(2m^*/\hbar)(E_i - E_f + \hbar\omega_q - \hbar\omega)$.

Let $W_i^{\text{abs,em}}$ be the quantum-mechanical probabilities for an electron to make a transition from an initial state $|i\rangle$ to a final state $|f\rangle$ by absorbing a photon and simultaneously absorbing or emitting a phonon. It is given as

$$W_i^{\text{abs,em}} = \frac{2\pi}{\hbar} \sum_f | \langle f | M | i \rangle |^2 \times \delta(E_f - E_i \mp \hbar\omega \pm \hbar\omega_q), \quad (11)$$

where the operator M is given as

$$M = \sum_v \left\{ \frac{H_{k'v}^{\text{ph}} H_{vk}^i}{E_k - E_v + \hbar\omega} + \frac{H_{k'v}^i H_{vk}^{\text{ph}}}{E_k - E_v + \hbar\omega_i} \right\}, \quad (12)$$

and $\langle f | M | i \rangle$ are the transition-matrix elements of M between the initial and final states. The interaction matrix elements can be written as

$$H_{k+q,k}^s = \int_L \int_A \psi_{k+pq}^*(r, z) \langle n_{qj} | \hat{H}_s | n_{qj} \rangle \psi_k(r, z) d^2 r dz, \quad (13)$$

with $n_{qj} = [e^{\hbar\omega_q/k_B T} - 1]^{-1}$ the phonon occupation number. H_{vk}^{ph} is the matrix element of the electron interaction with the photon while H_{vk}^i is the matrix elements due to the i th scattering mechanism. Taking into account the nonparabolicity and using the Fermi golden rule, we obtain

$$W^{\text{abs,em}} = \frac{m^* \lambda^2 L}{\hbar^3} \sum_n |G_n|^2 (N_q + 1) \times \frac{1}{\alpha_{\mp}} \left\{ \frac{a_n L^2}{2\gamma} [(2\gamma k_z^2 - 1) + b_n] \right\}^{-1}, \quad (14)$$

where $|G_n|^2$ is the electron-phonon overlap integral, $\alpha_{\pm} = [(2\gamma k_z^2 - 1)^2 + 4\gamma Q_{\pm}^2]$, and $\lambda^2 = 4\pi e^2 \hbar \omega_{LO} (\epsilon_{\infty}^{-1} - \epsilon_s^{-1})$. a_n and b_n are the coefficients of the normalization of the phonon displacement, $a_n = 1$ for odd n and $= 3$ for even n , $b_n = (n$

$+1)^2\pi^2$ for odd n and $=n^2\pi^2$ for even n . It should be pointed out that the strong electron-phonon interaction causes electrons, making transitions by emitting and absorbing phonon. However, the coupling is not strong enough to alter the electronic wave functions. This is the justification for using the perturbation method given in Eqs. (7)–(10) to calculate the transition probability.

We now calculate the linear response of the electronic system to the photon field polarizing parallel to the layer. The electrons are described by a hot Fermi-Dirac distribution function and occupy only the ground state. The electron-phonon interaction is described by the dielectric continuum approach. The photon absorption due to 2D plasmon excitation is neglected because the coupling between the transverse photons and longitudinal plasmon is very weak. The absorption coefficient K can be written as⁷

$$K = \frac{\epsilon^{1/2}}{n_0 c} \sum_i (W_i^{\text{abs}} - W_i^{\text{em}}) f_i, \quad (15)$$

where ϵ is the dielectric constant of the material, n_0 is the number of photons in the radiation field, c is the speed of light, and f_i is the free-carrier distribution function. The index i is summed over all polar optical phonon modes involved in the absorption process. Substituting Eqs. (6)–(10) in (11), we obtain^{7,11,14,16}

$$K_i = \frac{\epsilon^{1/2}}{n_0 c} \sum_{N,N'} \sum_{p=\pm 1} \int_0^\infty dq \frac{q^2 \omega_q m^* \lambda^2 L}{4\pi^4 \hbar^3 n_e c \sqrt{\epsilon_q}} G_{N'N}(q) C_i(q) \times \left(n_q + \frac{1-p}{2} \right) (N_q + 1) \frac{1}{\alpha} \left\{ \frac{a_n L^2}{2\gamma} [(2\gamma k_z^2 - 1) + b_n] \right\}^{-1} \times \int_0^\infty dx \frac{f(x + \epsilon_{N'N})}{\sqrt{x}} [1 - f(x + \epsilon_{N'N} + p\hbar\omega - p\hbar\omega_{LO})], \quad (16)$$

where

$$G_{N'N}(q) = C_{N'N} \left[\frac{l_B^2}{2} \left(\frac{\omega_c^2}{\omega_c^2 + \omega^2} \right) q^2 \right], \quad (17)$$

$$C_{N,N+J}(y) = [N!(N+J)!] y^J e^{-y} [L_N^J(y)]^2, \quad (18)$$

with $L_N^J(y)$ being the associated Laguerre polynomial. For electron-LO-phonon interaction,

$$C_i(q) = 4\pi\beta\hbar^3 \omega_q / (2m^* q^2), \quad (19)$$

with $\beta=0.068$ is electron-LO-phonon coupling coefficient and $\omega_q = \omega_{LO}$. For electron deformation-potential acoustic phonon interaction,

$$C_i(q) = \hbar E_D^2 q / (2\rho v_l), \quad (20)$$

with E_D the deformation-potential constant, ρ the mass density of the material, and v_l the longitudinal sound velocity. $\epsilon_{N'N} = E_N + (E_{N'} - E_N + \epsilon_q - \hbar\omega_{LO})^2 / 4\epsilon_q$ in which $\epsilon_q = \hbar^2 q^2 / 2m^*$, $E_N = (N + \frac{1}{2})\hbar\sqrt{\omega_c^2 + \omega^2}$, $\omega_c = eB/m^*$ the cyclotron frequency, ω is photon frequency, and $l_B = (\hbar/m^*\sqrt{\omega_c^2 + \omega^2})^{1/2}$. The electronic density along the z direction is $n(z) = \sum_{j,k_y} |\psi_j(z)|^2 \int_0^\infty (dE/\sqrt{E}) f[E + \epsilon_j(k_y)]$, where $f(x)$

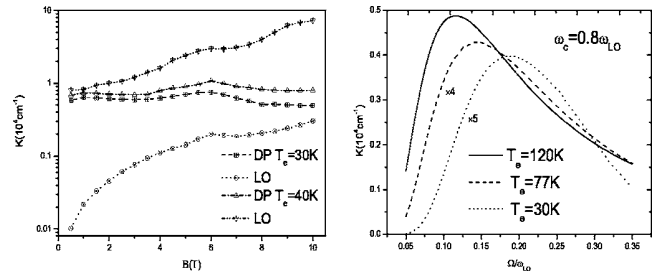


FIG. 2. In (a) the photon absorption coefficient is plotted as a function of B for a fixed value of Ω , and n_e with the interaction of electron and LO phonon (solid curve), and DP acoustic phonon (dotted curve) in the two different $T_e = 30$ and 40 K . In (b) the absorption coefficient is plotted as a function of ω/ω_{LO} at the electronic temperature $T_e = 30, 77$, and 120 K , at fixed cyclotron frequency $\omega_c = 0.8 \times \omega_{LO}$.

$= [\exp(x - E_F)/k_B T_e + 1]^{-1}$ is the Fermi-Dirac distribution function. The Fermi energy can be determined by the condition of the electron number conservation. The magnetic field induces a double-barrier-like potential $U_B(z, k_y)$ with potential minima at $z = \pm k_y l^2$. This double-well potential results in a density profile with two maxima. The ratio of the density maxima to the central minimum increases with the magnetic field.

III. NUMERICAL RESULTS AND DISCUSSIONS

We have calculated the absorption coefficients by using the formulas, Eq. (12). The material used in the calculations is an $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ single quantum well with 50 \AA well width and the barrier height is about 224 meV . The other parameters are $m^* = 0.0665 m_e$, $\hbar\omega_{LO} = 36.8 \text{ meV}$, $n_e = 1 \times 10^{16} \text{ m}^{-2}$, $\epsilon_s = 12.35$, and $\epsilon_\infty = 10.48$.

The electron temperature T_e appears in the velocity distribution functions of electrons. T_e differs from the lattice temperature because electrons are heated by photons. Due to the electron-lattice interaction, the hot electron relaxes by emitting or absorbing phonons. This process changes phonon number and is governed by T_e . Experimentally,³⁵ the energy-dependent distribution function is obtained from the photoluminescence spectra of a 2D electron system in the presence of a strong terahertz field. When the measured distribution is fitted to a theoretical model of velocity distribution (Maxwell distribution), an effective electron temperature can be determined, which is higher than the lattice temperature. The electron temperature can also be obtained from the excitation rate coefficient of spectral line density which is dependent on T_e .³⁶ In Fig. 2(a) we show the optical absorption coefficient of the quantum well under a strong magnetic field and in the absence of the subband nonparabolicity. The photon absorption coefficient is plotted as a function of B at a fixed value of ω . Two types of electron-phonon interaction are considered: the electron-LO-phonon interaction and the electron-DP acoustic phonon. When the electron temperature $T_e \leq 30 \text{ K}$ the electron-DP phonon interaction is more dominant. As T_e increase above 40 K the electron-LO-phonon interaction becomes more dominant. The absorption coefficient is oscillating weakly with the magnetic field. This magnetic oscillation is a manifestation of the interplay of various energy scales, such as the electronic energy (Landau levels),

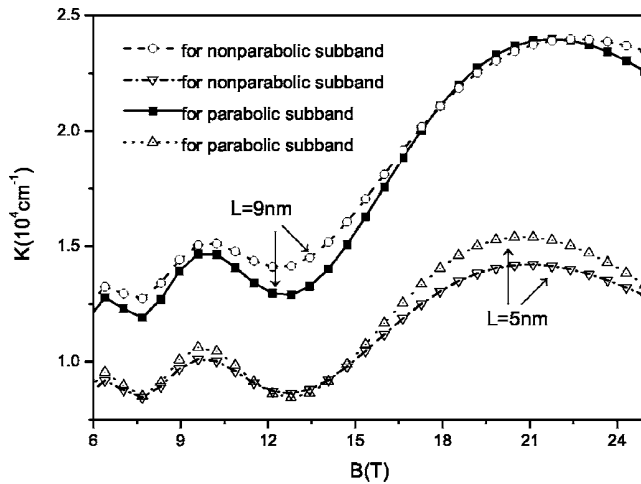


FIG. 3. Magnetic field dependent absorption coefficient of 2D electrons due to LO phonon scattering at $T_e = 77$ K and $n_e = 1.0 \times 10^{16} \text{ m}^{-3}$.

photon energy, and phonon energies. The coupling among electrons, photons, and phonons can lead to a phenomenon known as the magneto-photon-phonon resonance. The magnetic field intensities for the local maxima are given by $B = (\omega_q m^*) / (ne)$ ($n=1, 2, 3, \dots$ is an index difference between two Landau levels). In a two-dimensional electron gases (2DEG), electron-phonon resonance occurs when the energy difference between two electronic subbands equals the LO-phonon energy, i.e., when the condition $|\epsilon_n - \epsilon_m| = \hbar \omega_q$ is satisfied. This gives rise to a resonant electron-LO-phonon scattering between subbands n and m . As a consequence, the LO-phonon generation is enhanced. Here the lattice is not in equilibrium with the hot electrons of electron temperature T_e . The electron-phonon interaction and the net phonon generation are strongly dependent on the electron temperature and electron distribution. As a result, the phonon numbers increase with T_e .

In Fig. 2(b), the absorption coefficient is plotted as a function of ω/ω_{LO} at the electron temperatures of $T_e = 30, 77$, and 120 K, and at fixed cyclotron frequency $\omega_c = 0.8 \times \omega_{LO}$. The absorption shows a Lorentzian-type resonance. With the increase of the electron temperature T_e the total absorption increases and the absorption maximum moves to a lower frequency. The magnetophonon absorption resulting from the inelastic scattering between Landau levels with resonant absorption of phonons is more pronounced at higher T_e . The rapid increase of the optical absorption as a function of T_e is a direct consequence of optical process mediated by the electron-phonon interaction. The transition probability given in Eq. (7) requires that a phonon is emitted (absorbed) by the electron after absorbing (emitting) a photon. This leads to the net absorption proportional to both photon population and phonon population. Since the phonon population increases with T_e , the absorption also increases with T_e .

Figure 3 depicts the effect of the nonparabolicity on the resonant absorption due to LO-phonon scattering. The nonparabolic parameter γ is chosen to be $4.9 \times 10^{-19} \text{ cm}^{-1}$. The well widths are 9 and 5 nm, respectively. The solid line is for nonparabolic and the dotted line is for parabolic subbands. Local maxima can be seen in the absorption curves. In nar-

row wells the effect of the subband nonparabolicity is more important than that in larger wells. The nonparabolicity effects become more pronounced as energy increases.

IV. CONCLUSION

In conclusion, we provided quantitative analysis on the optical absorption in the GaAs/Al_{0.3}Ga_{0.7}As-based 2D semiconductor system in the presence of an in-plane magnetic field. The absorption is mainly due to the electron interaction with LO phonons and DP acoustic phonons. The calculated optical absorption coefficient indicates that an in-plane magnetic field significantly can affect the optical properties of the two-dimensional semiconductor system. The magnetophonon resonance occurs when $N\omega_c = \omega_q$. At fixed magnetic fields, the absorption coefficient increases with the electron temperature and the absorption peaks move to the lower energy region. The effect of magnetophonon resonance presented here should be observable in transport or optical experiments, e.g., measuring the dc resistivity or conductivity as a function of magnetic field at fixed photon frequency. The peaks can be observed when the resonant condition is satisfied.

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