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## Asterism decoding for layered space-time systems using 8PSK

### Abstract

The area of layered space-time (LST) systems has received enormous attention recently as they can provide a roughly linear increase in data rate by using multiple transmit and receive antennas. The optimal detection strategy for a LST receiver is to perform a maximum-likelihood (ML) search over all possible transmitted symbol combinations has an exponential complexity when the constellation size of number of transmit antennas increases. While sub-optimal decoders, such as VBLAST, provide linear decoding only where the number of receive antennas is at least equal to the number of transmit antennas. The decoding scheme proposed in this paper, called Asterism decoding, looks for a more efficient way of finding the ML solution by first considering the case of multiple transmit antennas and a single receive antenna. The decoder is then extended to achieve ML like performance for any number of receive antennas. It further shows that Asterism decoding has at least an approximate order of magnitude reduction in computational complexity when compared to ML decoding. Asterism decoding is the first lower complexity decoder that achieves ML-like performance for systems where the number of receive antennas is less than the number of transmit antennas without the additional use of error coding.

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# Asterism decoding for Layered Space-Time systems using 8PSK

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**Abstract**—The area of Layered Space-Time (LST) systems has received enormous attention recently as they can provide a roughly linear increase in data rate by using Multiple Transmit and Receive antennas. The optimal detection strategy for a LST receiver is to perform a Maximum-Likelihood (ML) search over all possible transmitted symbol combinations has an exponential complexity when the constellation size of number of transmit antennas increase. While sub-optimal decoders, such as VBLAST, provide linear decoding only where the number of receive antennas is at least equal to the number of transmit antennas.

The decoding scheme proposed in this paper, called Asterism decoding, looks for a more efficient way of finding the ML solution by first considering the case of multiple transmit antennas and a single receive antenna. The decoder is then extended to achieve ML like performance for any number of receive antennas. It further shows that Asterism decoding has at least an approximate order of magnitude reduction in computational complexity when compared to ML decoding. Asterism decoding is the first lower complexity decoder that achieves ML-like performance for systems where the number of receive antennas is less than the number of transmit antennas without the additional use of error coding.

## I. INTRODUCTION

In recent years the employment of multi-element antenna arrays at both transmit and receive sites has received much interest because it is capable of enormous theoretical capacity over wireless communications systems, in particular the area of Layered Space-Time (LST) systems [1]. However, the design of practical signaling and signal processing schemes capable of supporting data rates close to the capacity limit remains a major challenge.

A number of sub-optimal decoding schemes such as Zero Forcing (ZF), the Bell Labs Layered Space-Time (BLAST)[2] [3] and the QR Decomposition [4] perform best when the number of receive antennas is greater than the number of transmit antennas, while performance is less-optimal when antenna numbers are equal and cannot be used for systems where there are less receive antennas than transmit. Whereas the optimal detection strategy for a LST receiver is to perform a Maximum-Likelihood (ML) decoding has a computational complexity exponential with the number of transmit antennas and constellation size.

It was shown in [5], that using such a ML decoder with less receive than transmit antennas could still provide sufficient increase in data rate, hence removing the need for additional receive antennas on a receiving device, such as a mobile

terminal. Reducing the size and cost of mobile terminals by reducing the number receive antennas leads to the need for a lower complexity decoder that achieves ML like performance for systems where the number of receive antennas is less than the number of transmit antennas.

The decoding scheme proposed in this paper, called Asterism decoding, looks for a more efficient way of finding the ML solution by first considering the case of multiple transmit antennas and a single receive antenna. The decoder is then extended to achieve ML like performance for any number of receive antennas rather than previously described decoders that first try to have ML-like performance when the number of receive antennas is greater than (or equal) the number of transmit antennas.

The paper is ordered as follows. Section 2 gives a brief system description of Layered Space-Time systems and Maximum Likelihood decoding. Section 3 introduces an Asterism decoding for any number of transmit and receive antennas. While Section 4 give a simple comparison the complexity of Asterism decoding and ML decoding.

## II. LAYERED SPACE-TIME SYSTEMS

The Layered Space-Time Processing approach was first introduced by Lucent's Bell Labs, with their BLAST family of Space Time Code structures [2]. An uncoded Vertical Bell Laboratories Layered Space-Time (VBLAST) scheme, where the input bit stream is de-multiplexed into  $n_t$  substreams, is considered in this paper. Let  $n_t$  be the number of transmit and  $n_r$  be the number of receive antennas, and  $s = (s_1, s_2, \dots, s_{n_t})^T$  denote the vector of symbols of constellation size  $C$ , transmitted in one symbol period. The received vector  $R = (R_1, R_2, \dots, R_{n_r})^T$  is

$$R = Hs + n \quad (1)$$

where  $n = (n_1, n_2, \dots, n_{n_r})^T$  is the noise vector of additive white Gaussian noise of variance  $\sigma^2$  equal to  $\frac{1}{2}$  per dimension. The  $n_r \times n_t$  channel matrix

$$H = \begin{pmatrix} h_{1,1} & \cdot & \cdot & h_{1,n_t} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ h_{n_r,1} & \cdot & \cdot & h_{n_r,n_t} \end{pmatrix} \quad (2)$$

contains independent identical distribution (i.i.d.) complex fading gains  $h_{i,j}$  from the  $j^{th}$  transmit antenna to the  $i^{th}$

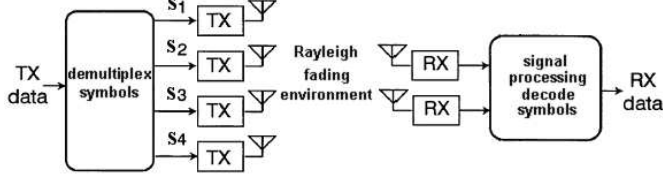


Fig. 1. Block diagram of a Layered Space-Time system where  $n_t = 4$  transmit and  $n_r = 2$  receive antennas.

receive antenna. We assume Rayleigh flat fading where the magnitude of the elements of  $H$  have a Rayleigh distribution. A block diagram of Layered Space-Time system is shown in Fig. 1.

Maximum Likelihood decoding is achieved by minimising

$$\|Hs - R\|^2 \quad (3)$$

for all elements of  $s$ , which are symbols of constellation of size  $C$ . This would produce a search of length  $C^{n_t}$ , which for a system using 4 transmit antennas and 8PSK gives 4096 possibilities. This leads to the development of ML performance decoding methods with a reduced computational complexity.

### III. ASTERISM DECODING

For the initial description we consider here the noiseless system where  $n_t = 4$  and  $n_r = 1$  using 8PSK modulation without coding and with Rayleigh fading. The formula for any received vector  $Y$  is:

$$Y = h_1 s_1 + h_2 s_2 + h_3 s_3 + h_4 s_4 \quad (4)$$

The results of (3) for all values of  $C$  substituted into  $s$  are plotted in Fig. 2. The ML solution is point that has the smallest complex distance calculation from the receive vector  $R$ . It can be seen that the complex constellation, of Fig. 2, can be divided into  $C$  (in this case 8) smaller groups or Asterisms [6], as shown in Figure 2b. Each of these Asterisms can in turn be divided into  $C$  smaller Asterisms, and so on for  $n_t$ . Finding the ML solution without having to test every point by grouping the complex constellation into Asterisms is the main concept behind the Asterism decoding.

For ease of explanation, we make the assumption that the magnitude of  $H$  in (3) is decreasing i.e.  $|h_1|$  is the largest and  $|h_4|$  is the smallest. Therefore, circles which cover the Asterisms are of radius  $|h_2| + |h_3| + |h_4|$  and are centered at  $h_1 \times s_i$ . Every possible combination is covered by these 8 Asterism circles. The size and the amount of overlap of these circles is determined by the number of transmit antennas, the magnitude of the elements of  $H$  and the size of the constellation.

Quite simply, if the received vector  $R$  is inside the one or more circles it is possibly the ML solution. The algorithm

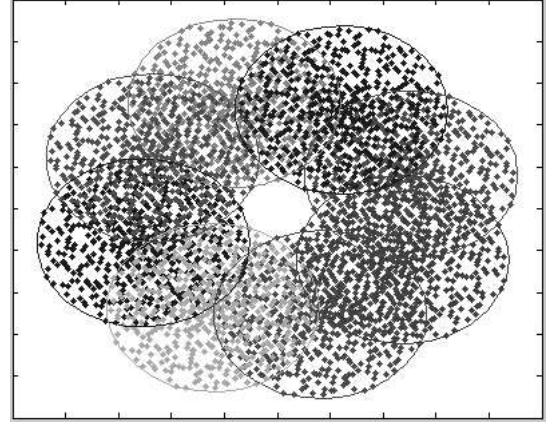


Fig. 2. Complex constellation  $Y$  grouped into 8 Asterisms.

then subtracts this possible solution from  $R$  and determines whether modified  $R$  is in one of the new Asterism circles centered at  $h_2 \times s_i$  and of radius  $|h_3| + |h_4|$ . This recursive process continues until all  $n_t$  symbols are found. If there are more than one combination found, the combination with the lowest complex distance measurement is chosen to be the ML solution.

While using Asterism decoding to multiple transmit and a single receiver antenna system produces the ML performance, the performance of this type of system is poor. To overcome this the use of multiple antennas at the receive is now considered. The received vector where  $n_t = 4$  and  $n_r = 2$  becomes:

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} & h_{1,3} & h_{1,4} \\ h_{2,1} & h_{2,2} & h_{2,3} & h_{2,4} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} \quad (5)$$

To take advantage of the information provided by the additional antenna a number of traditional receive diversity combining schemes, such as equal gain and Maximum Ratio combining, were tested and found to have sub-ML performance. This loss in performance was overcome by applying a modified Maximum Ratio combining at each stage of decoding and was found to have near ML performance. For the first detected symbol the modified Received vectors becomes:

$$R_{m_{1st}} = h_{11}^* R_1 + h_{21}^* R_2 \quad (6)$$

After the first stage of decoding the modified received vector becomes:

$$R_{m_{2nd}} = h_{12}^* R_1 + h_{22}^* R_2 \quad (7)$$

this process is continued to detect all  $n_t$  symbols are found. Because the order is which the symbols are detected, the detection order needs to be tested to ensure that the modified Maximum ratio combining does not change the detection ordering and that the symbol with the largest channel magnitude is detected first while the symbol with the smallest

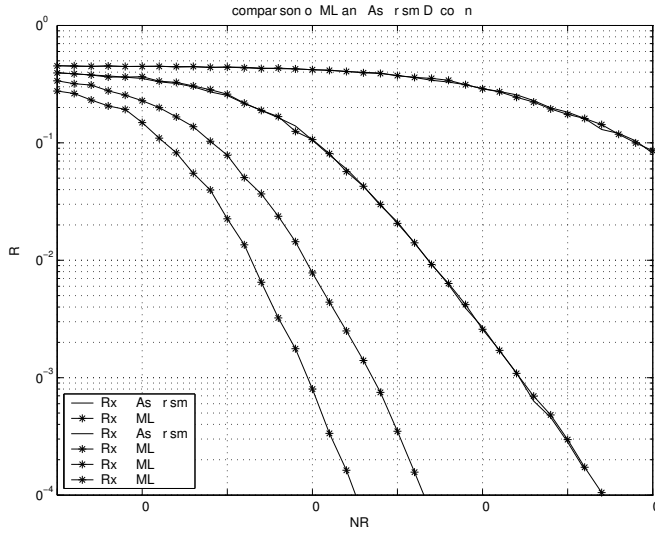


Fig. 3. Performance comparison of Asterism and ML decoding for  $n_t = 4$ , 8PSK and  $n_r = 1, 2, 3$  and 4 antennas.

channel magnitude is detected last. A performance comparison between Asterism and ML decoding with  $n_t = 4$  and  $n_r = 1, 2, 3$  and 4 is shown in Fig. 3. It can be seen that the greatest improvement is when the number of receive antenna is increased from 1 to 2.

#### IV. FIRST LOOK AT COMPLEXITY

Obviously the worse case scenario is when all channels have the same magnitude and computation will be equal to  $C^{n_t}$  (4096 for described system, or  $512 \times C$ ) complex distance calculations. Whereas the best case is when each channel magnitude is equal to or greater than the sum of the remaining channel magnitudes, equal to  $n_t \times C$  complex distance measurements.

Using Monte Carlo simulations for the system of  $n_t = 4$ , 8PSK and by counting the number of  $C$  complex distance operations to decode the system, it was found that computational complexity is reduced to an approximate mean of 30 with a standard deviation of 25. This suggests an approximate order of magnitude reduction in the complexity when compared to ML decoding. Figure 4 shows a distribution of the number of complex distance calculation.

#### V. CONCLUSION

This paper has described a new method of decoding uncoded Layered Space-Time systems. The decoding scheme proposed in this paper, called Asterism decoding, looks for a more efficient way of finding the ML solution by first considering the case of multiple transmit antennas and a single receive antenna. The decoder was then extended to achieve ML like performance for any number of receive antennas. It was then shown to have an approximate order of magnitude reduction in computational complexity when compared to ML decoding.

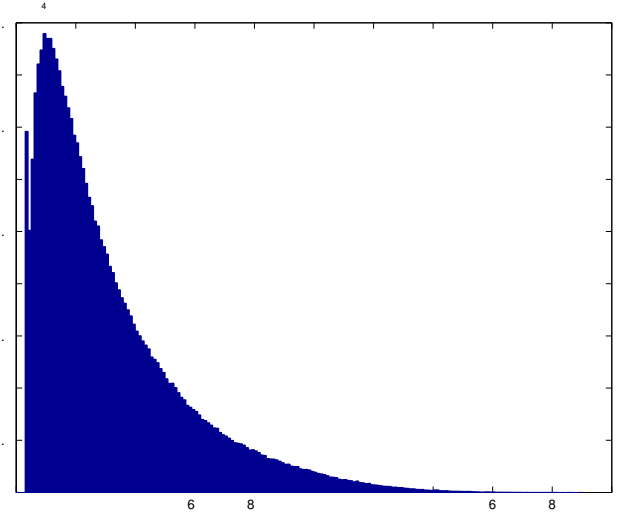


Fig. 4. Distribution of Complex distance calculations for Asterism decoding for  $n_t = 4$  antennas and QPSK.

#### REFERENCES

- [1] E. Telatar, "Capacity of Multi-antenna Gaussian Channels", AT & T Bell Labs, Murray Hill, NJ, Tech. Rep., 1995
- [2] G.J. Foschini, "Layered Space-Time Architecture for Wireless Communications in a Fading Environment when using Multiple Antennas", Bell Labs Technical Journal, Autumn 1996.
- [3] P.W. Wolniansky, G.J. Foschini, G.D. Golden, and R.A. Valenzuela, "V-BLAST: An architecture for realizing very high data rates over the rich-scattering wireless channel", Bell Labs., Lucent Technol., Crawford Hill Lab., Holmdel, NJ, Tech Rep., 1999.
- [4] D. Shiu and J.M. Kahn, "Layered Space-Time Codes for Wireless Communications using Multiple Transmit Antennas", in IEEE Proceedings of International Conference on Communication (ICC'99), British Columbia, June 1999.
- [5] D. Garrett, L. Davis, G. Woodward, "19.2 Mbit/s 4 /spl times/ 4 BLAST/MIMO detector with soft ML outputs", Electronics Letters, Volume: 39 Issue: 2, 23 Jan. 2003, Page(s): 233 -5.
- [6] K. Kronberg, Munich Astro Archive Constellation Pages, "http://www.maa.mhn.de/Maps/Const/asterism.html", [Last accessed Sept. 15 2003]