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# Behavioural Modelling of High Temperature Superconducting Wires and Coils for Power Engineering Applications

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## **Keywords**

power, coils, wires, behavioural, temperature, high, modelling, engineering, applications, superconducting

## **Disciplines**

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# BEHAVIOURAL MODELLING OF HIGH TEMPERATURE SUPERCONDUCTING WIRES AND COILS FOR POWER ENGINEERING APPLICATIONS

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## Abstract

Many aspects of the behaviour of high temperature superconducting (HTS) wires are well understood for dc applications. However, models of HTS suitable for use in circuit analysis by power engineers designing HTS applications are not readily available. This paper describes the initial development of suitable models for HTS wire. The  $V-I$  characteristics for wires carrying dc transport current can be empirically determined using a power law  $V=kI^n$ . Experimentally measured  $V-I$  characteristics are fitted using the above power law. This enables the simulation of the behaviour of Bi-2223/Ag wire using the well-known circuit simulator PSpice. Other factors affecting the dc critical current density of the wire, such as bending diameter and some aspects of perpendicular and parallel applied magnetic fields have also been considered in the model. The performance of the model with dc current is compared with experimental results from a non-inductive pancake coil prepared using Bi-2223/Ag 27-core multifilament tape, as well as two short HTS tapes with different critical current ratings.

## 1. INTRODUCTION

The development of power engineering applications of high temperature superconductors (HTS) will benefit from a better knowledge of how HTS wire and tapes behave in power circuits. Typical HTS circuit components will include straight conductors, as well as a variety of HTS coils such as solenoids, pancakes, double pancakes, and toroids. It is necessary to predict the behaviour of such elements when they are incorporated into larger circuits. Many parameters, such as the coil dimensions, the number of turns of the wire, and the critical current of the wire must be considered in determining the behaviour of the coil. In addition, the type and geometry of a coil affects its performance.

This paper discusses the modelling techniques and the behaviour of HTS wires and coils carrying dc currents. Models of wire and of coil behaviour were developed based on  $V-I$  characteristics of Bi-2223/Ag wires. The wires were prepared using powder-in-tube processes. The degeneration of critical current due to the bending diameter has also been analysed and incorporated in the model based on the results of measurements of Bi-2223/Ag 27-core multifilament samples. The dc behaviour of an eight-metre length of tape wound in a non-inductive pancake coil has also been predicted using the model. Four samples from both ends of the wires were used in this work.

## 2. GENERAL PRINCIPLE OF THE MODELLING TECHNIQUES

The  $V-I$  characteristics of HTS wires can be expressed using a power law [1]  $V=kI^n$ , where  $V$  is the voltage rise across the voltage tap,  $I$  is the current flowing in the wire,  $n$  is a positive number indicating the index of transition or a measure of the sharpness of transition, and  $k$  is a constant of proportionality. Figure 1 shows a typical  $V-I$  characteristic of a HTS wire. Within the region where the current  $I$  is less than  $I_{max}$ , the voltage  $V$  increases non-linearly. This is due to the flux creep phenomenon in the superconductor. Afterwards, the voltage  $V$  increases linearly with the current  $I$  because of the homogeneous flux-flow of the wire. The slope corresponds to the full flux-flow resistivity [2]. In these modelling techniques, the working area is restricted to the non-linear region where the curve can be fitted by a power law:

$$V = kI^n \quad (1)$$

Figure 2 shows curves used for the basic description of the model. The  $V-I$  curve of the measurement result fits well with the model for the region  $I < I_{m1}$  as can be confirmed further in Figure 3 for a  $V-I$  measurement of a sample used in this work. The values of  $k$  and  $n$  for the wire used here are  $2 \times 10^{-11}$  and 9.4 respectively.

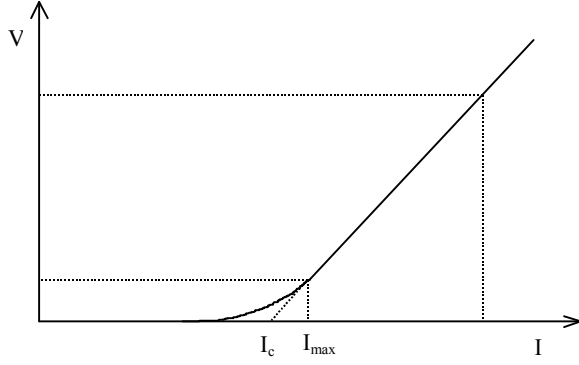


Figure 1. Typical  $V$ - $I$  characteristic of a HTS wire.

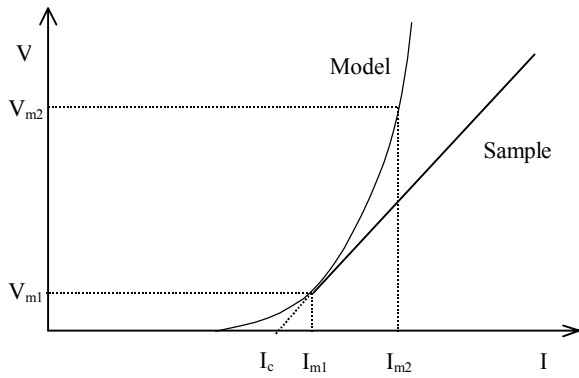


Figure 2.  $V$ - $I$  Curve used in the model.

Equation (1) can be rewritten as:

$$V = e^{\ln k + n \ln I} \quad (2)$$

In order to find the value of  $n$ , two points in the curve of Figure 2 will be considered, ie.  $(I_{m1}, V_{m1})$  and  $(I_{m2}, V_{m2})$ , where

$$V_{m1} = k I_{m1}^n \quad (3)$$

and

$$V_{m2} = k I_{m2}^n \quad (4)$$

By assuming that:

$$V_{m2} = \alpha V_{m1} \quad (5)$$

at

$$I_{m2} = I_{m1} + \Delta I_m \quad (6)$$

where the value of  $\alpha$  and  $\Delta I_m$  can be determined using the initial measurement of the sample, we obtain the value of  $n$  as follows:

$$n = \frac{\ln \alpha}{\ln \frac{I_{m2}}{I_{m1}}} \quad (7)$$

This can be rewritten as:

$$n = e^{\ln(\ln \alpha) - \ln(\ln(I_{m1} + \Delta I_m)) - \ln I_{m1}} \quad (8)$$

The  $\ln k$  factor in Equation (2) can be obtained by entering  $n$  in Equation (3), ie.  $V_{m1} = k I_{m1}^n$ , and therefore:

$$\ln k = \ln V_{m1} - n \ln I_{m1} \quad (9)$$

By choosing  $I_{m1} = I_c$ , so that  $V_{m1} = 1 \mu\text{Vcm}^{-1}$  is within the value of the field criterion it can be seen that

$$\ln k = -n \ln I_{m1} \quad (10)$$

and

$$n = e^{\ln(\ln \alpha) - \ln(\ln(I_c + \Delta I_m)) - \ln I_c} \quad (11)$$

Therefore the voltage  $V$  can be expressed as:

$$V = e^{n(\ln I - \ln I_{m1})} \quad (12)$$

Equation (12) can then be entered into PSpice to build a model diagram of the  $V$ - $I$  characteristic as can be seen in Figure 4.

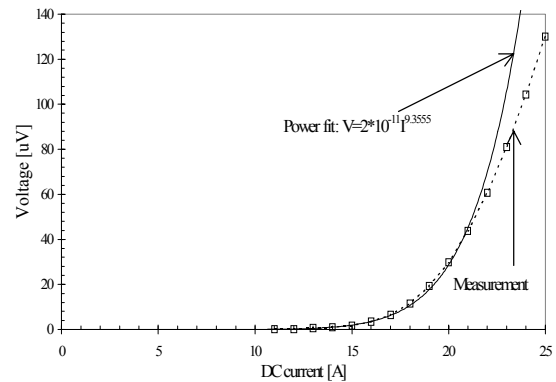


Figure 3. Fitted  $V$ - $I$  characteristic curve of Bi-2223/Ag 27-core wire with a critical current of 14 A.

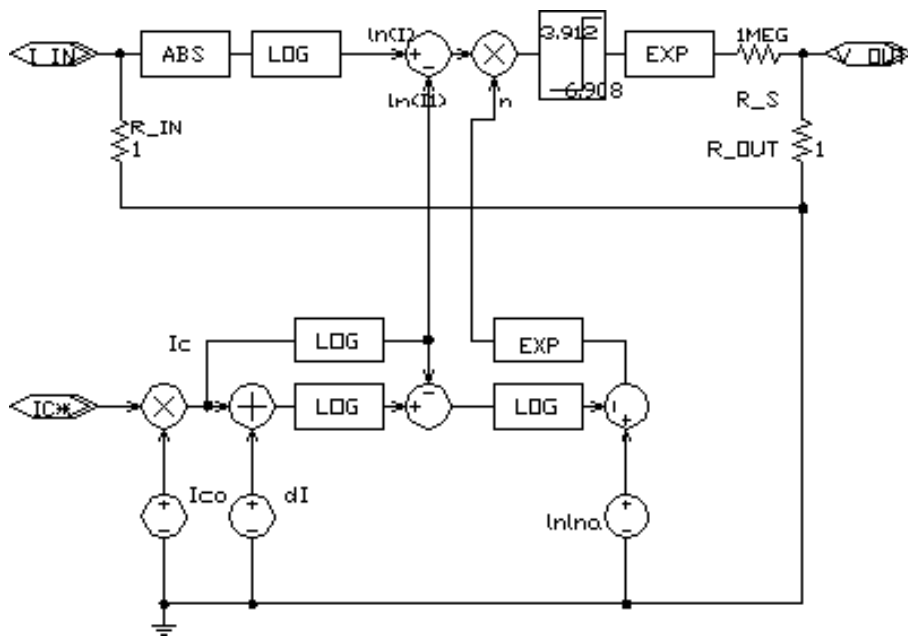


Figure 4. Model diagram written using PSpice circuit simulator. PSpice requires the resistors as dummy devices.

### 3. CRITICAL CURRENT DEGENERATION OF THE HTS WIRE

Critical current measurements of short samples from a single eight metre wire have been performed to quantify the degeneration of the wire, caused by mechanical stresses, which occur during the winding of coils. Initially, a  $V-I$  measurement of a straight sample was done to observe the original value of the critical current. Then, the sample was bent at a certain diameter and another  $V-I$  measurement was performed. This procedure was carried out several times for different bending diameters of the sample. Figure 5 gives  $V-I$  characteristics of the sample for different bending diameters varying from infinity to 0.2 cm.

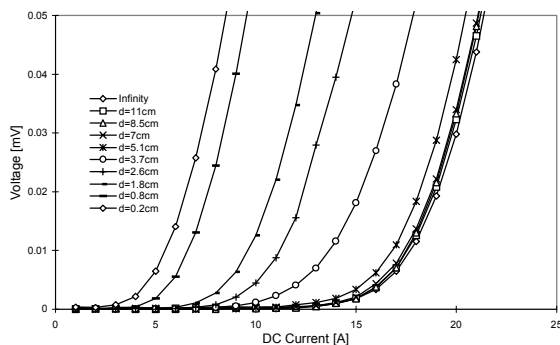


Figure 5.  $V-I$  characteristics of HTS sample under different bending diameter.

The same measurements were carried out for four different samples. The relationship between the

bending diameter and the normalised critical current is shown in Figure 6 for those samples. Sample 4 has the best bending properties, ie. it can be bent up to 5 cm of diameter with less than 10% critical current reduction. However, the critical current of sample 2 reduces drastically to about 70% of its original value for the same bending diameter. In addition, sample 1 may be damaged as it has a very low critical current. It can be concluded that this particular HTS wire has poor homogeneity in mechanical properties.

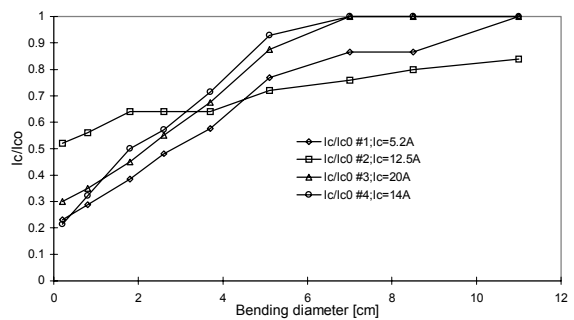


Figure 6. The relationship between the bending diameter and the normalised critical current.

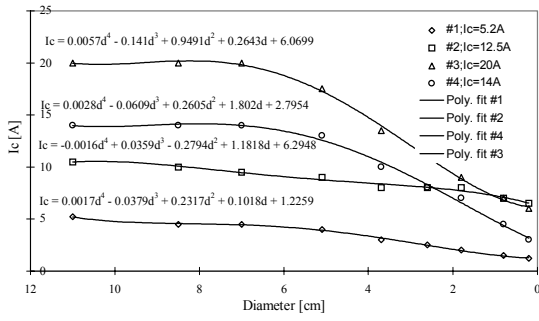


Figure 7. Effect of bending diameters on the critical currents.

In order to incorporate this property in the PSpice model it is necessary to find the curve of critical current as a function of bending diameter as shown in Figure 7. The polynomial equation describing this is then recorded in the model as shown in Figure 8 with bending diameter considered as the input parameter. In this figure the limiter is used to keep the input  $d$  within valid values and the voltage-controlled voltage source models the polynomial.

Figure 9 shows the results of the model for the same sample and bending diameter as in Figure 5. It can be seen that these two sets of curves closely coincide.

#### 4. MAGNETIC FIELD DEPENDENCE OF THE HTS TAPE

The magnetic field dependence of the sample has been investigated by measuring the critical current of the sample exposed to different strengths and orientations of the field. The measurement results for the fields both perpendicular as well as parallel to the wires are plotted in Figure 10.

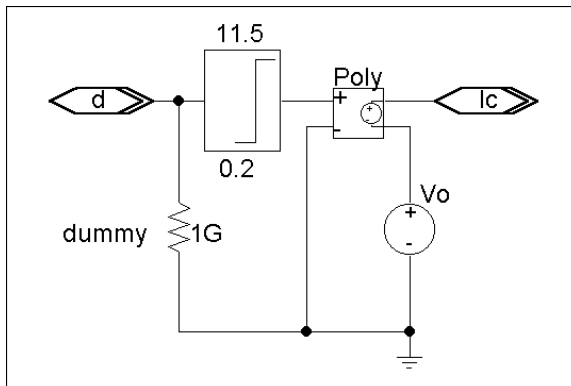


Figure 8. Model description to incorporate bending properties into the model of the coil.

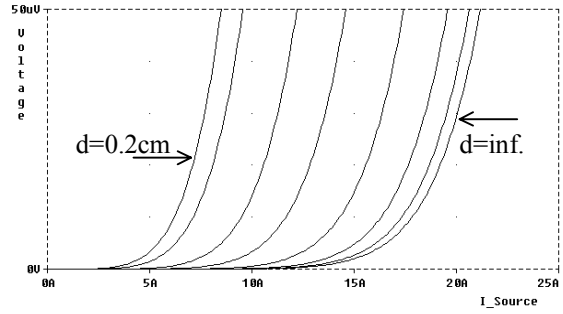


Figure 9. The  $V-I$  curves of the model for given bending diameters.

As expected the sample is significantly affected by the presence of a perpendicular field. The normalised critical current decreases to about 50% at about 20 mT. The presence of field parallel to the sample has much less effect on the critical current. For this wire the critical current drops to 50% at about 200 mT and then reduces slightly at higher fields. This phenomenon is caused by the anisotropic properties of the superconducting part of the wire. However, the effect of a relatively low field, say 5 mT or less, is negligible.

For modelling purposes these trends are approximated using curve-fitting methods. The effect on the critical current due to field both parallel as well as perpendicular to the sample can be fitted using Kim's model [8]:

$$\frac{I_c}{I_{co}} = \frac{1}{1 + \frac{H}{H_0}}, \quad (13)$$

where  $I_c/I_{co}$  is the normalised critical current and  $H$  is the magnetic field. The value of  $H_0$  for parallel field is 200 mT and that for perpendicular field is 20 mT. The curve fitting using this model can be seen in Figure 10.

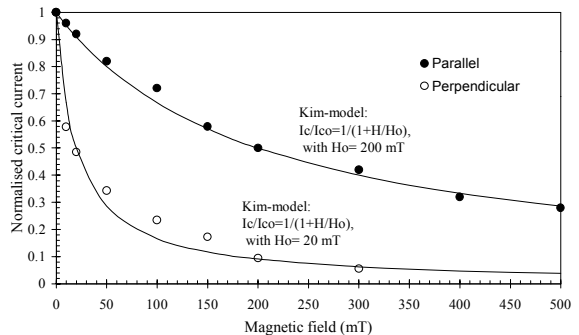


Figure 10. Magnetic field dependence of the sample.

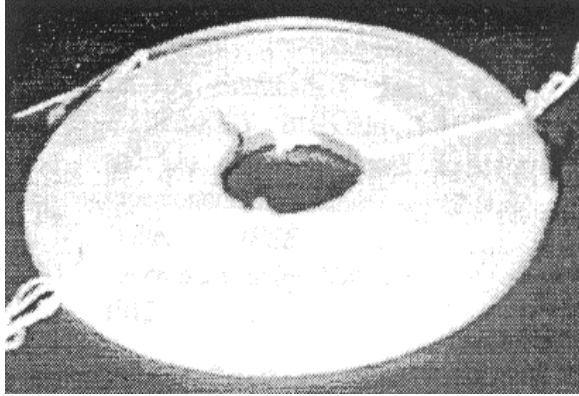


Figure 11. Photograph of the pancake coil prepared by Bi-2223/Ag wire.

However, in this paper the effect of the magnetic field was not considered in the model for the particular coil that is described in the following section, as it was a non-inductive coil, in which the effect of the field was insignificant.

## 5. MODEL OF NON-INDUCTIVE PANCAKE COIL

The critical current of Bi-2223/Ag wire demonstrates a strong dependence on the applied magnetic field, and has a brittle mechanical characteristic [3-5]. To develop this material for applications, several prototype coils have been made using this wire [6-7]. One of these coils, a single layer non-inductive pancake coil, is considered here. The wire, which is used to make this coil, is produced by the powder-in-tube techniques. It consists of 27-core multifilaments and the cross-section is  $3.4 \text{ mm} \times 0.28 \text{ mm}$  with total length of about 8 m. The area of the superconducting part as a percentage of total cross-section is about 27%. The critical current of short samples was measured to be  $16 \pm 4 \text{ A}$ .

The coil has inner diameter of 5 cm and outer diameter of 11.5 cm and has  $15 \times 2$  turns. The wire was insulated with Teflon tapes and insulation paper. Figure 11 shows a photograph of the coil. The experimentally measured  $V-I$  characteristic of the coil is depicted in Figure 12. The critical current of that coil is 7.2 A at a criterion of  $1 \mu\text{V}/\text{cm}$ .

When two currents are passed into the two winding in opposite directions, the generated magnetic fields tend to cancel each other, as shown in Figures 13 and 14. These figures were generated using a finite element simulations [9] and confirm that there are no significant fields within the coil when carrying currents.

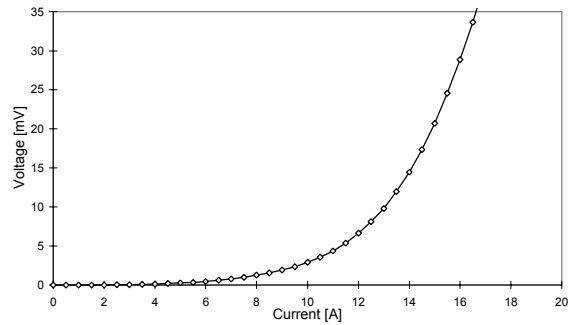


Figure 12. The  $V-I$  Curve of the pancake coil. The critical current is 7.2 A.



Figure 13. A magnetic field map of a non-inductive coil.

Figure 13 shows the field map of the coil, and the field distribution in the radial direction on the coil surface compared with a normal coil (inductive) can be seen in Figure 14. Thus the net field is close to zero and can be neglected in the model. As a consequence, the final Pspice model of the coil can be represented as a 'black-box', which only contains the information about the wire and that of the coil, such as the diameter of the coil, and length of the wire used to make the coil.

The predicted  $V-I$  characteristic of the model for the non-inductive pancake coil is shown in Figure 15. This model incorporates the bending property of the worst sample (sample 2) and that of the best sample (sample 4). The first result coincides reasonably well with the measurement results as shown in Figure 12. The critical current in this curve is 7.8 A and was experimentally measured to be 7.2 A. This discrepancy is probably because of the inhomogeneity of the current distribution along the wire which in some parts is in a superconducting state and some other parts in a normal state.

## 6. CONCLUSIONS AND DISCUSSIONS

For many power-engineering applications it is necessary to form HTS wires into coils. A circuit model has been developed to predict the behaviour of a Bi-2223/Ag 27-core multifilament wires as well as a single layer non-inductive pancake coil. Both mechanical and magnetic field properties of the superconducting wire have been considered. The

models were implemented in the widely used circuit simulator Pspice as a 'black box' which needs data inputs such as type of wire, length of wire, type of coil, coil dimension, and a current input. It is then able to provide the voltage across the wire/coil as the output.

Experimental results showed that the model provided accurate predictions of performance for HTS wire, and a reasonably accurate prediction of the performance of a non-inductive pancake coil. It is suggested that variations in the critical current distribution along the wire account for much of the difference between modelled and actual performance of the coil. A behaviour model has been developed and tested to confirm the behaviour of these devices.

Future work will include the modelling of inductive coils, which form elementary devices in power engineering applications. The effect of magnetic fields and other properties will be incorporated in the model to predict the behaviour and the losses of these coils in AC applications.

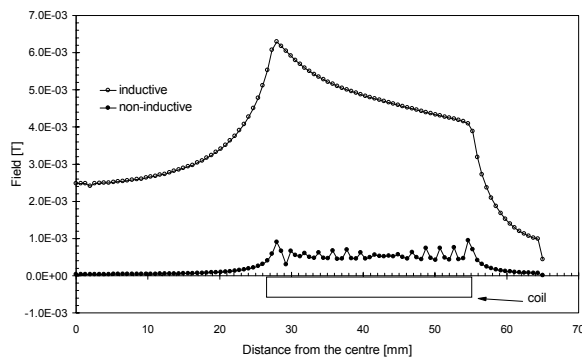


Figure 14. Field distribution of a non-inductive coil compared with a normal coil.

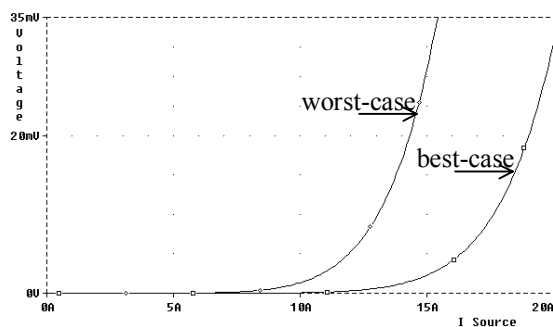


Figure 15. The  $V$ - $I$  curves of the model for the pancake coil.

## 7. ACKNOWLEDGMENTS

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