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Self-shape optimisation application: Optimisation of cold-formed steel columns

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\begin{abstract}
This paper presents the optimisation of cold-formed steel open columns using the recently developed self-shape optimisation method that aims to discover new profile shapes. The strength of the cold-formed steel sections is calculated using the Direct Strength Method, and the rules developed in the present work to automatically determine the local and distortional elastic buckling stresses from the Finite Strip and constrained Finite Strip Methods are discussed. The rules are verified against conventional and optimum sections yielded in this research, and found to accurately predict the elastic buckling stresses. The optimisation method is applied to singly-symmetric (mono-symmetric) cold-formed steel columns, and the operators behind the method for the special case of singly-symmetric open profiles are introduced in this paper. “Optimum” cross-sections for simply supported columns, 1.2 mm thick, free to warp and subjected to a compressive axial load of 75 kN are presented for column lengths ranging from 1000 to 2500 mm. Results show that the optimum cross-sections are found in a relatively low number of generations, and typically shape to non-conventional “bean”, “oval” or rounded “Σ” sections. The algorithm optimises for distortional and global buckling, therefore likely subjecting the cross-sections to buckling interaction. A manual attempt to redraw the “optimum” cross-sections to include limitations of current manufacturing processes is made. Future developments of the method for practical applications are also discussed.
\end{abstract}

\begin{keywords}
Self-shape optimisation \hfill Light gauge steel
Direct strength method \hfill Cold-formed steel columns
\end{keywords}
2. Optimisation problem

The present optimisation problem is illustrated in Fig. 1 and is concerned with minimising the cross-sectional area \( A \) of a column subjected to an axial compressive load \( N = 75 \text{ kN} \). The column is composed of 1.2 mm thick cold-formed steel open section, and is free to warp at the supports. The yield stress \( f_y \) is 450 MPa and the Young’s modulus \( E \) is 200 GPa and the shear modulus \( G \) is 77 GPa. Buckling lengths ranging from 1000 to 2500 mm are included in the present study.

The unconstrained optimisation problem suitable for GA consists of minimising the “fitness function” \( f \) as

Minimise \( f = \frac{A}{A_{\text{quash}}} + x \frac{N^*}{N_c} - 1 \) \hspace{1cm} (1)

where \( N_c \) represents the nominal axial capacity of the column (automatic determination of \( N_c \) for is discussed in Section 3), the parameter \( x \) is a penalty factor, and \( A_{\text{quash}} \) represents the lower bound cross-sectional area of the profile, defined as

\[ A_{\text{quash}} = \frac{N^*}{f_y} \] \hspace{1cm} (2)

3. Automatic determination of the nominal axial compression capacity \( N_c \) using the DSM

3.1. The direct strength method for columns

In order to estimate the nominal axial compression capacity \( N_c \) of the column in Eq. (1), the Australian design standard AS/NZS 4600 Cold-formed Steel Structures [10] is used in the present work. The standard allows the determination of the axial capacity using two distinct methods, referred to as the “Effective Width Method” (EWM) and the “Direct Strength Method” (DSM). The DSM, developed by Schafer and Pekoz [11], looks at the entire member rather than individual elements as in the EWM and has the advantages of offering the same design simplicity for complex and simple sections. Its recent development for the design of cold-formed steel sections [12,13] has simplified the design procedure when compared to earlier methods based on the EWM [10,14,15]. More importantly, it allows a more direct route to section optimisation as the three fundamental buckling modes (local, distortional, and global) are now represented by direct strength equations thus allowing the GA to operate with a more clearly defined set of constraints. This was not possible previously [16].

In the DSM, the global, local and distortional axial member capacities, \( N_{\text{ce}}, N_{\text{cl}}, \) and \( N_{\text{cd}} \), respectively, are determined, and the nominal member capacity in compression \( N_c \) is equal to the lowest of them

\[ N_c = \min(N_{\text{ce}}, N_{\text{cl}}, N_{\text{cd}}) \] \hspace{1cm} (3)

3.1.1. Flexural, torsional or flexural–torsional member capacity \( N_{\text{ce}} \) in the DSM

The global buckling mode does not involve change in the cross-sectional shape, but translation (flexure) and/or rotation (torsion) of the entire cross-section. For singly-symmetric open cross-section, the columns will either fail in “flexural” or “flexural–torsional” buckling. The DSM specified in AS/NZS 4600 [10] calculates the nominal capacity \( N_{\text{ce}} \) for global buckling based on the non-dimensional slenderness ratio \( \lambda_c = \sqrt{N_e/N_{\text{ce}}} \) as

\[ N_{\text{ce}} = \frac{A f_{\text{ce}}}{2} \] \hspace{1cm} (4)

For \( \lambda_c > 1.5 \), \( N_{\text{ce}} = \frac{0.677}{\lambda_c^2} N_y \) \hspace{1cm} (5)

where \( N_{\text{ce}} \) is the elastic global buckling load taken as

\[ N_{\text{ce}} = A f_{\text{ce}} \] \hspace{1cm} (6)

and \( N_y \) is the nominal yield capacity defined as

\[ N_y = A f_y \] \hspace{1cm} (7)

\( A \) is the gross cross-sectional area and \( f_{\text{ce}} \) is the elastic global buckling stress.

In this study, the elastic global buckling stress \( f_{\text{ce}} \) is determined using Timoshenko’s buckling theory as given in Clause 3.4.3 of AS/NZS 4600 [10]. For singly-symmetric open cross-sections, where \( x \) is the axis of symmetry, \( y \) is the axis perpendicular to the axis of symmetry and \( z \) is the member axis, \( f_{\text{ce}} \) is given as

\[ f_{\text{ce}} = \min(f_{\text{ce}x}, f_{\text{ce}y}) \] \hspace{1cm} (8)

where \( f_{\text{ce}x} \) is the elastic flexural–torsional buckling stress and \( f_{\text{ce}y} \) is the elastic flexural buckling stress about the axis perpendicular to the axis of symmetry.

3.1.2. Local member capacity \( N_{\text{cl}} \) in the DSM

Local buckling involves a change in the cross-sectional shape and includes only rotation, not translation, at the fold lines (e.g. the corners of a plain channel section) [13]. The DSM specified in AS/NZS 4600 [10] accounts for the interaction between local and global buckling, and calculates the nominal capacity \( N_{\text{cl}} \) for local buckling based on the non-dimensional slenderness ratio \( \lambda_l = \sqrt{N_{\text{cl}}/N_{\text{cl}}^0} \) as

\[ N_{\text{cl}} = 0.15 \left( \frac{N_c}{N_{\text{cl}}^0} \right)^{0.4} \left( \frac{N_{\text{cl}}}{N_{\text{cl}}^0} \right)^{0.4} \] \hspace{1cm} (9)

where \( N_{\text{cl}}^0 \) is the elastic local buckling load taken as

\[ N_{\text{cl}} = A f_{\text{cl}} \] \hspace{1cm} (10)

and \( f_{\text{cl}} \) is the elastic local buckling stress. The determination of \( f_{\text{cl}} \) is discussed in Section 3.2.3.

3.1.3. Distortional member capacity \( N_{\text{cd}} \) in the DSM

Distortional buckling involves distortion of the cross-section, which includes translation and rotation at one or more fold lines. The half-wavelength falls between local and global buckling [13]. The DSM specified in AS/NZS 4600 [10] calculates the nominal column capacity \( N_{\text{cd}} \) for distortional buckling based on the non-dimensional slenderness ratio \( \lambda_d = \sqrt{N_{\text{d}}/N_{\text{cd}}^0} \) as

\[ N_{\text{cd}} = 0.561 \] \hspace{1cm} (11)

where \( N_{\text{cd}}^0 \) is the elastic local buckling load taken as

\[ N_{\text{cd}} = A f_{\text{cd}} \] \hspace{1cm} (12)

and \( f_{\text{cd}} \) is the elastic distortional buckling stress. The determination of \( f_{\text{cd}} \) is discussed in Section 3.2.4.
3.1.4. Current research to enhance the DSM

Research is currently undertaken to account for interactions between buckling modes other than local and global buckling [17–25]. However, further investigation is needed to completely consider these interactions for practical design, and the DSM as adopted in Clause 7 of AS/NZS 4600 [10] is adopted in this study. However, the present optimisation procedure is anticipated to consider all buckling mode interactions when these phenomena are fully incorporated in the DSM. A discussion of buckling mode interactions for the “optimum” cross-sections found in this study is given in Section 6.1.

3.2. Elastic buckling stresses

3.2.1. General

As shown in Sections 3.1.1–3.1.3, the elastic global, local and distortional buckling stresses \( P_{oc}, P_{od} \) and \( P_{ob} \) respectively, are needed to calculate the global, local and distortional member capacities \( N_{oc}, N_{od} \) and \( N_{ob} \) respectively. The elastic global buckling stress \( P_{oc} \) can be estimated by either the Finite Strip Method (FSM) [3–5] or Timoshenko’s buckling theory, whereas the elastic local and distortional buckling stresses \( P_{od} \) and \( P_{ob} \) are typically estimated using the FSM.

A Finite Strip analysis provides a buckling curve, also referred to as the “signature curve”, of the buckling stresses against the half-wavelength with the associated buckling modes. Fig. 2 shows the buckling curve obtained from a Finite Strip analysis of a 90 mm deep, 50 mm wide and 1.2 mm thick lipped Cee section, referred to as C9012.

Ideally, a buckling curve, such as the one shown in Fig. 2, has two minima corresponding to the elastic local (first minimum) and distortional (second minimum) buckling stresses. However, Finite Strip analyses often result in one or no local minimum, and fail to directly identify the local and/or distortional buckling stresses. Indistinct buckling modes can be manually identified as discussed in Ref. [13]. Yet, the recent development of the constrained Finite Strip Method (cFSM) [6–9] opened new possibilities in optimisation of cold-formed steel members by providing automatic identification of indistinct buckling modes [26]. The cFSM enables calculations of “pure” buckling modes [7] and separates buckling modes into four subspaces referred to as “global”, “distortional”, “local” and “other”. The buckling curves for each “pure” mode can be calculated individually with the mode definitions adopted from the Generalised Beam Theory (GBT) [7]. The pure mode decompositions for distortional and local buckling using the cFSM are shown in Fig. 2 for the C9012 lipped Cee section.

Currently, no clear set of proven rules exists to automatically determine the local and distortional elastic buckling stresses for shape optimisation. For general optimisation purposes, Schafer [26] recommends the use of the cFSM to determine the critical half-wavelengths from the “pure” modes (i.e. determining the half-wavelengths corresponding to the minimum of the “pure” mode buckling curves) in conjunction with the use of FSM to determine the buckling stresses. Additionally, Li and Shafer [27] advises to perform constrained Finite Strip analyses on straight-line models, ignoring the corners. The latter recommendation is however not suitable for shape optimisation purposes that typically generate rounded cross-sections, as shown in Section 5 and Ref. [28]. Alternatively, if the signature curve from a Finite Strip analysis has unique minima, the need for performing a constrained Finite Strip analysis may be avoided [27].

For shape optimisation purposes, Leng et al. [28] only performed Finite Strip analyses and, if more than one local minimum exist on the buckling curve, chose the first local minimum of the buckling curve for \( P_{od} \) and the smallest of the remaining local minima, for \( P_{ob} \). If only one local minimum exists, then this minimum is chosen for \( P_{od} \) if it occurs at a half-wavelength less than a reference length. Otherwise, the local minimum is chosen for \( P_{ob} \). The reference half-wavelength is initially taken as the “perimeter length” and regularly updated through the optimisation process as the distortional critical half-wavelength when more than one local minimum exists. However, it is not clear if the method consistently determines the actual elastic buckling stresses, as if only one local minimum exists and is greater than the reference length, the algorithm is likely to overestimate the critical half-wavelength \( L_{oc} \) for local buckling. Conversely, if the local minimum occurs at a half-wavelength less than the reference, the critical half-wavelength \( L_{oc} \) for distortional buckling may be underestimated.

Fig. 2. Signature curve and mode decomposition for a C9012 lipped Cee-section.
3.2.2. The use of the cFSM for local buckling and shape optimisation

The calculation of the "pure" local buckling curve from the cFSM requires intermediate nodes, referred to as "sub-nodes", to be inserted between "main nodes". The main nodes are located at the intersection of two strips having a non-zero angle relative to each other [6]. Consecutive sub-nodes are therefore aligned and the plates are only able to buckle between main nodes.

Consequently, the cFSM for local buckling is well suited for cross-sections with straight lines and no rounded corners. For randomly drawn cross-sections where strips are likely to have non-zero angles relative to each other or for cross-sections with not perfectly flat sides, it is unclear which nodes have to be considered as sub-nodes. Moreover, it is likely that the transition from a sub-node to a main node is a gradual process, with sub-nodes partially preventing the plate to buckle between main nodes.

Currently, Finite Strip analysis programs, such as CUFSM [29] used in this study, checks if three or more consecutive nodes are aligned, within a given tolerance, to make the distinction between sub-nodes and main nodes, and is likely to consider too many nodes as main nodes in the current optimisation process, give low critical half-wavelengths and therefore overestimate the local elastic buckling stress \( f_{ol} \). This statement is illustrated in Ref. [30] using two C9012 lipped Cee sections with one having misaligned nodes in the web by half the profile thickness. Finite Strip analyses of the two cross-sections show little difference in the buckling curve and both cross-sections have the same critical half-wavelength \( L_{cr} \) for local buckling at 72 mm. However, a constrained Finite Strip analysis predicts the correct critical half-wavelength for local buckling at 72 mm for the "aligned" cross-section but results in a critical half-wavelength at 44 mm for the "misaligned" cross-section. The error in determining the half-wavelength results in the overestimation of the elastic local buckling stress by 50%. See Ref. [30] for more details. Determining the critical local half-wavelength using cFSM is therefore not recommended for arbitrarily drawn or rounded cross-sections that have node misalignments, and the recommendation in Refs. [26,27] described in the previous section cannot be used for local buckling and shape optimisation.

3.2.3. Proposed rule for determining the elastic local buckling stress \( f_{ol} \)

The critical half-wavelength \( L_{cr} \) for local buckling for a member in compression is typically less than or equal to the largest outside dimension \( d \) of the cross-section [13], and the elastic local buckling stress \( f_{ol} \) would typically correspond to the minimum of the buckling curve at a half-wavelength lower than \( d \). Therefore, following this observation, the elastic local buckling stress \( f_{ol} \) of a cross-section is determined from the smallest local minimum, if it exists, or from the smallest gradient of the buckling curve, in the half-wavelength interval \([r_0, d]\), where \( r_0 \) is the least radius of gyration of the column.

3.2.4. Proposed rule for determining the elastic distortional buckling stress \( f_{od} \)

Distortional buckling occurs at a half-wavelength significantly greater than local buckling, typically between three and nine times the largest outside dimension \( d \) of the cross-section [13]. Stub column tests do not generally pick up distortional buckling [31], and AS/NZS 4600 [10] recommends a maximum length for stub-column tests of twenty times the least radius of gyration \( r_0 \). Therefore, the literature shows that distortional buckling likely occurs at a half-wavelength between the lesser of \( 20r_0 \) and \( 3d \), and \( 9d \). However, verification of the present rules in Section 3.2.5 showed that a value of \( 10d \) is a better upper limit for distortional buckling, and is adopted herein.

Following these observations and the recommendations by Schafer [26] discussed in Section 3.2.1, the half-wavelength \( L_{crd} \) for distortional buckling is determined using the cFSM in the half-wavelength interval \([\min (20r_0, 3d), 10d] \), and the elastic buckling stress is then determined using the FSM. If more than one local minimum exist on the "pure" distortional buckling curve, the half-wavelength for distortional buckling is taken at the smallest local minimum.

It may be noted that local minimum may not always be found on the "pure" distortional buckling curve in the interval \([\min (20r_0, 3d), 10d] \), and the search interval needs to be extended to an upper limit of \( 13d \) [30].

3.2.5. Validation of the proposed rules

The proposed set of rules for determining the elastic local and distortional buckling stresses is validated in this section against a manual method, subjected to engineering judgement and best practice for handling indistinct buckling modes, as discussed in Ref. [13]. If indistinct local mode occurs, options to determine the critical local half-wavelength \( L_{cr} \) include: (i) refining the half-wavelengths, (ii) basing judgement on the definition of the buckling mode given in Section 3.1.2, keeping in mind that local buckling should occur at a half-wavelength less than the largest outside dimension of the member in compression \( d \), or (iii) if possible, pin internal fold lines to force local buckling. Similarly, if indistinct distortional mode occurs, options to determine the critical local half-wavelength \( L_{crd} \) include: (i) refining the half-wavelengths, (ii) basing judgement on the definition of the buckling mode given in Section 3.1.3, (iii) slightly varying the dimensions of the model to recognise a trend in distortional buckling minima or (iv) if possible, pin appropriate internal fold lines to force distortional buckling.

Forty eight conventional cross-sections and twelve "optimum" cross-sections, found in Section 5, are used to validate and cross-validate, respectively, the proposed set of rules. Specifically, the following cross-sections are considered:

- 16 lipped Cee-sections and 16 lipped Zed-sections commonly used in Australia and manufactured by BlueScope Steel Lysaght [32], as shown in Fig. 3. The nominal depth of the profiles ranges from 100 to 350 mm, and the nominal wall thickness from 1.0 to 3.0 mm.

Fig. 3. (a) Cee-section and (b) Zed-section.
Comparison between manual method and automated rules.

<table>
<thead>
<tr>
<th>Section type</th>
<th>No. of section analysed</th>
<th>Depth/thickness</th>
<th>Difference in elastic buckling stresses relative to the manual method [%]a</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>Cee</td>
<td>16</td>
<td>52.6</td>
<td>133.3</td>
</tr>
<tr>
<td>Zed</td>
<td>16</td>
<td>52.6</td>
<td>133.3</td>
</tr>
<tr>
<td>Rack with lip stiff.</td>
<td>9</td>
<td>22.9</td>
<td>60.0</td>
</tr>
<tr>
<td>Rack w/o lip stiff.</td>
<td>7</td>
<td>22.9</td>
<td>60.0</td>
</tr>
<tr>
<td>Optimum</td>
<td>12</td>
<td>66.6</td>
<td>100.0</td>
</tr>
<tr>
<td>All cross-sections</td>
<td>60</td>
<td>22.9</td>
<td>133.3</td>
</tr>
</tbody>
</table>

a A negative percentage value in the table means that the automated set of rules provides a lower elastic stress than the manual method.

- 16 typical storage rack uprights, including 9 profiles without lip stiffeners and 7 profiles with lip stiffeners, as shown in Fig. 4. The nominal depth of the profiles ranges from 55 to 90 mm for the profiles without lip stiffeners, and from 55 to 110 mm for the profiles with the lip stiffeners. The nominal wall thickness ranges from 1.2 to 2.4 mm. All 55 mm deep uprights have no web stiffener, as shown in Fig. 4 (c–d), while all remaining uprights have one web stiffener, as shown in Fig. 4 (a–b).
- 12 “optimum” cross-sections found in Section 5, corresponding to the three fittest cross-sections for each of the four column lengths investigated.

Table 1 shows the average difference in determining the local and distortional elastic buckling stresses from the manual method and automated set of rules. Detailed results can be found in Ref. [30]. Table 1 shows that the two methods give similar results, with an average difference of less than 1% for all cross-sections analysed and for the two modes of buckling. The maximum difference is equal to 8.6% and is encountered for a 55 mm deep and 2.4 mm thick storage rack upright without lip stiffeners. The standard deviation in predicting the elastic buckling stresses between the two methods is equal to 1.6% and 1.9% for the local and distortional buckling, respectively.

Manually determining the critical half-wavelength for local buckling proved challenging for the optimum sections, as the first minimum on the buckling curve may occur at a half-wavelength between the typical values for local and distortional buckling (of about two times the largest outside dimension of the cross-section), as illustrated in Fig. 5 for the three 2500 mm long columns. These minima are likely to be disregarded by an engineer for local buckling, as occurring at a half-wavelength greater than \( d \) [13], and \( f_o \) was therefore manually chosen at a lower half-wavelength herein based on the definition of the buckling mode given in Section 3.1.2.

For the “optimum” 1000 mm long columns, only one minimum typically occurs on the buckling curve in the half-wavelength range for distortional buckling, but was not selected by the two methods, as the associated buckled mode is a combination of local and distortional buckling, as illustrated in Fig. 6.

4. Self-shape optimisation principles for singly-symmetric cold-formed steel profiles

4.1. Initial population

As described in the companion paper [1], initial cross-sections are generated using self-avoiding random walks. As the cross-sections of interest are singly-symmetric, only half of each cross-section is modelled in the optimisation process.

A design space of 100 mm \( \times \) 100 mm is used in generating the half cross-sections. This design space may represent imposed constraints for the depth and width of the profile, to a maximum of 200 and 100 mm, respectively.

The cross-sectional areas of the initial population are deliberately generated to be uniformly distributed (see Ref. [2]) in five categories, for cross-sectional areas ranging from \( A_{\text{ref}} - 70 \text{ mm}^2 \) to \( A_{\text{ref}} + 70 \text{ mm}^2 \); where \( A_{\text{ref}} \) represents a reference value in the order of magnitude of the “optimum” cross-sectional area. \( A_{\text{ref}} \) is estimated for each column length by pre-running the algorithm with a large initial population of cross-sectional areas ranging from \( A_{\text{quasi}} \) to five times \( A_{\text{quasi}} \). Values of \( A_{\text{ef}} \) equal to \( 1.5 \times A_{\text{quasi}} \) (250 mm\(^2\)), \( 1.75 \times A_{\text{quasi}} \) (292 mm\(^2\)), \( 2 \times A_{\text{quasi}} \) (333 mm\(^2\)) and \( 2.25 \times A_{\text{quasi}} \) (375 mm\(^2\)) were found to produce a reasonable estimation of the “optimum” cross-sectional areas for the 1000, 1500, 2000 and 2500 mm long columns respectively.

As discussed in Refs. [1,2], the element size shall be small enough to ensure the accuracy of the cross-sectional area and allow complex cross-sectional shapes, including stiffeners, to be drawn. A nominal element size of 4 mm (i.e. 3.33 times the thickness) was found to be a reasonable compromise between accuracy and computational time in this study.
Cross-sections are therefore drawn on the $x_{\text{max}} = 100 \text{ mm}$ × $y_{\text{max}} = 100 \text{ mm}$ design space, and the reference point at $(x_{\text{max}}/4, 0)$ is chosen to be the origin of the cross-sections on the axis of symmetry. Similar steps to the ones presented in the companion paper [1] are used, with the exception that the cross-sections are drawn until they reach a predefined cross-sectional area corresponding to one of the five distributive categories. All steps are detailed in Ref. [30].

Initial cross-section examples are presented in Fig. 7, where only half of the profile is drawn. The horizontal axis $x=0$ is the axis of symmetry.

4.2. Cross-over operator

The cross-over operator is similar to the one presented in the companion paper [1], with two points $P_{\text{parent1}}$ and $P_{\text{parent2}}$ chosen at $\delta\%$ along the length of the first and second parents, respectively, with $\delta$ being a random number in the open interval [0,100], so the first and last points of the parents are not selected. The two points $P_1$ and $P_2$ are defined using a linear interpolation between $P_{\text{parent1}}$ and $P_{\text{parent2}}$, as illustrated in Fig. 8.

Two offsprings are created per operation with the first offspring built using the right-hand part of the first parent (i.e. the part including the extremity of the cross-section, point $P_{\text{end}}$ in Fig. 8) and the left-hand part of the second parent (i.e. the part including the point on the axis of symmetry, point $P_x$ in Fig. 8) as shown in Fig. 8 and detailed in Ref. [30]. Elements constituting the offsprings are merged or subdivided to keep all elements about 4 mm long, in the interval [3 mm, 6 mm]. A typical cross-over probability of 0.8 is used.

4.3. Mutation operator

Mutation allows new cross-sectional shapes to be introduced in the population by redrawing a part or several parts of a cross-section. The operator is similar to the one presented in the
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companion paper [1], with the exceptions that (i) point \( P_x \), on the axis of symmetry (see Fig. 8), is always kept at the reference point \( (\frac{X_{\text{max}}}{4}, 0) \) to avoid duplicate cross-sections and (ii) if the end point \( P_{\text{end}} \), at the extremity of the cross-section (see Fig. 8), is included in a mutated part, then a new end point \( P_{\text{new end}} \) is defined at a random distance \( d_r \) from the former point \( P_{\text{end}} \). The upper limit of \( d_r \) is inversely proportional to the distance between \( P_{\text{end}} \) and the mutated point. Point \( P_{\text{new end}} \) allows cross-sections to be created with end points different from the ones of the initial population.

The mutation operator is detailed in Ref. [30].

4.4. Augmented Lagrangian method

The augmented Lagrangian method for GA described in Adeli and Cheng [33] is used in this study. The optimisation problem given in Eq. (1) is then expressed as

\[
\text{Minimise } g = \frac{A}{A_{\text{squash}}} + \frac{1}{2} \gamma \left( \frac{N_c}{N_c^\star} - 1 + \mu \right)^2
\]  

(15)

Initial values for the Lagrangian coefficients of \( \gamma = 2 \) and \( \mu = 0 \) have been found to be appropriate values to ensure convergence of the algorithm [1], and are therefore used in the present work. A penalty increasing constant \( \beta = 1.05 \) is used to avoid premature convergence of the algorithm at a convergence rate \( \alpha = 1.5 \) [2,30]. Similar to the companion paper [1], to visualise the convergence of the algorithm, the fitness function \( f \) in Eq. (1), with a constant penalty factor \( \alpha \), is preferred through this paper.

5. Results

This section presents the “optimum” singly-symmetric open cross-sections obtained for the 1000, 1500, 2000 and 2500 mm long columns. For each column length, 10 runs were performed with an initial population of 500 individuals. A maximum of 80 generations were analysed per run.

Fig. 9 through Fig. 12 plot the three fittest cross-sections and the tenth fittest cross-section out of the 10 runs at the 80th generation for the 1000, 1500, 2000 and 2500 mm long columns, respectively. The fitness \( f \) of the cross-sections is evaluated using Eq. (1) with a penalty factor \( \alpha \) of 1.0. The entire design space is not plotted in Fig. 9 through Fig. 12 for clarity. All cross-sections found in this study are given in Ref. [30].

Table 2 summarises the optimum average cross-sectional areas \( A_{\text{optimum}} \) and axial compression capacity \( N_c \) after 10 runs.

Fig. 13 plots the average product between the fitness function \( f \) (with a penalty factor \( \alpha = 1.0 \)) and \( A_{\text{squash}}/A_{\text{optimum}} \) for each column length. The term \( A_{\text{squash}}/A_{\text{optimum}} \) allows comparison between the fitness functions of columns of different lengths. Fig. 13 shows

![Fig. 7. Example of initial cross-sections on a 100 mm x 100 mm design space of (a) 41 elements, (b) 49 elements and (c) 53 elements.](image)

![Fig. 8. Cross-over operator (a) defining cross-over points, (b) creation of the first offspring and (c) creation of the second offspring.](image)
that the algorithm converges to the “optimum” cross-sections in a relatively low number of generations, around 70 generations.

Fig. 14 shows the evolution of the fittest cross-section for the 1500 mm column shown in Fig. 10(a). The algorithm tends to smooth and close the profile through the optimisation process.

Table 3 gives the main properties of the fittest cross-sections shown Fig. 9(a), Fig. 10(a), Fig. 11(a) and Fig. 12(a) for the 1000, 1500, 2000 and 2500 mm long columns, respectively.

6. Discussion

6.1. General

The cross-sections mainly converge to three different shape types, namely a “bean” shape (as in Fig. 9(a) through Fig. 9(c), or Fig. 10(c)), an “oval” shape (as in Fig. 10(a) and Fig. 10(b), Fig. 11(a) through Fig. 11(c), or Fig. 12(a)), and a rounded “Σ” shape (as in Fig. 9(d), Fig. 10(d), or Fig. 12). The overall depth of the cross-sections is about 80, 95, 110 and 120 mm for the 1000, 1500, 2000 and 2500 mm long columns, respectively.

Typically, the “oval” and “bean” cross-sections are like closed profiles, whereas the “Σ” cross-sections tend to be open. Moreover, the “oval” and “bean” cross-sections usually behave...
Fig. 11. "Optimum" cross-sections for a column length of 2000 mm in increasing fitness order from (a) fittest cross-section ($A = 336.8$ mm$^2$, $N_c = 75.0$ kN), (b) second fittest cross-section ($A = 336.8$ mm$^2$, $N_c = 75.0$ kN), (c) third fittest cross-section ($A = 336.9$ mm$^2$, $N_c = 75.0$ kN) and (d) tenth fittest cross-section ($A = 340.5$ mm$^2$, $N_c = 74.8$ kN).

Fig. 12. "Optimum" cross-sections for a column length of 2500 mm in increasing fitness order from (a) fittest cross-section ($A = 385.8$ mm$^2$, $N_c = 74.9$ kN), (b) second fittest cross-section ($A = 386.3$ mm$^2$, $N_c = 75.0$ kN), (c) third fittest cross-section ($A = 386.8$ mm$^2$, $N_c = 75.0$ kN) and (d) tenth fittest cross-section ($A = 396.4$ mm$^2$, $N_c = 75.0$ kN).

Fig. 13. Evolution of the average fitness for 10 runs for each column length.
better than the “Σ” shape type cross-sections, with smaller cross-sectional areas.

On the other hand, the algorithm typically produces rounded cross-sectional shapes which have the advantages of (i) yielding high elastic local buckling stresses and (ii) maximising the second moments of area while minimising the cross-sectional area, as seen from the companion paper [1]. Therefore, local buckling is never the dominant failure mode and the local member capacity \(N_{cl}\) is always equal to the global member capacity \(N_{ce}\) in Eq. (9).

Global buckling is typically the critical buckling mode for all “optimum” cross-sections with \(N_c = N_{ce}\) for 38 runs out of the total 40 runs. However, the algorithm optimises for both distortional and global buckling modes and the distortional nominal capacity \(N_{cd}\) is on average equal to 76.05 kN for the 40 runs, with a coefficient of variation of 0.025, i.e. 1.4% higher than the targeted capacity of 75 kN. Table 4 gives the average distortional nominal capacities \(N_{cd}\) and elastic buckling loads \(N_{od}\) of the “optimum” columns for 10 runs. The close values between distortional and

<table>
<thead>
<tr>
<th>Column length (mm)</th>
<th>(N_{cd}) (kN)</th>
<th>(N_{cd}) CoV</th>
<th>(N_{od}) (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>75.12</td>
<td>0.008</td>
<td>89.0</td>
</tr>
<tr>
<td>1500</td>
<td>75.08</td>
<td>0.003</td>
<td>71.8</td>
</tr>
<tr>
<td>2000</td>
<td>75.53</td>
<td>0.006</td>
<td>61.6</td>
</tr>
<tr>
<td>2500</td>
<td>78.49</td>
<td>0.032</td>
<td>58.2</td>
</tr>
</tbody>
</table>

Fig. 14. Evolution of the optimum cross-section in Fig. 10(a) from (a) 1st generation (initial population), (b) 5th generation, (c) 10th generation, (d) 15th generation, (e) 20th generation, (f) 25th generation, (g) 40th generation, (h) 60th generation and (i) 80th generation (last generation).
Table 4

Table 5 gives the cross-sectional areas and axial capacities \( N_c \) of the "optimum" cross-sections redrawn with straight lines.

<table>
<thead>
<tr>
<th>Column length (mm)</th>
<th>Area (mm²)</th>
<th>Diff. with optimum (%)</th>
<th>( N_c ) (kN)</th>
<th>Diff. with optimum (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>244.7</td>
<td>1.32</td>
<td>74.2</td>
<td>0.80</td>
</tr>
<tr>
<td>1500</td>
<td>289.7</td>
<td>0.87</td>
<td>73.8</td>
<td>1.20</td>
</tr>
<tr>
<td>2000</td>
<td>340.3</td>
<td>1.01</td>
<td>74.4</td>
<td>0.80</td>
</tr>
<tr>
<td>2500</td>
<td>392.6</td>
<td>1.63</td>
<td>73.4</td>
<td>2.13</td>
</tr>
</tbody>
</table>

7. Future research

The self-shape optimisation principle will be extended in the future to incorporate the moment capacity \( M_c \) estimated from the DSM in the fitness function \( f \), in a similar manner to the axial capacity \( N_c \) (see Section 2).

The final aim of this study is to optimise cross-sections for practical industrial uses, and the limitations of cold-forming processes will be added to the algorithm. In addition to the manufacturing constraints, construction constraints specific to various types of cold-formed steel applications, such as purlins or girts, will be added to the algorithm.

As discussed in Section 6.1, the algorithm optimises for distortional and global buckling, and the buckling mode interaction will need to be taken into account in the optimisation process.

8. Conclusions

The extension of the self-shape optimisation method introduced in the companion paper to strength optimisation of singly-symmetric open cold-formed steel columns has been presented. The Direct Strength Method (DSM) as specified in AS/NZS 4600 Cold-formed Steel Structures was used to determine the axial member capacity \( N_c \) of the columns. Rules to automatically select the moment capacity \( M_c \) estimated from the DSM in the fitness function \( f \) are selected for redrawing. The redrawn cross-sections are shown in Fig. 15.

Table 5 shows that the first attempt to manually redraw the optimum cross-sections to allow roll-forming and brake-pressing processes, gives reasonable results. When compared to the raw "optimum" cross-sections, the cross-sectional areas increase by less than 2% while the axial capacities only decrease by 1% to 2%.

Fig. 15. "Optimum" cross-sections redrawn with straight lines for column lengths of (a) 1000 mm, (b) 1500 mm, (c) 2000 mm and (d) 2500 mm.
Columns with a wall thickness of 1.2 mm, lengths varying from 1000 to 2500 mm and subjected to an axial compressive load of 75 kN were optimised. The cross-sections converged to “bean”, “oval” or rounded “Z” shape types, in a relatively low number of generations, around 70 generations. The rounded shapes have the advantages of increasing the local buckling strength while maximising the global buckling strength. The algorithm mainly optimises the cross-sections for distortional and global buckling, which may lead to distortional/global buckling interaction, currently not considered in the DSM.

A manual attempt to redraw the raw “optimum” cross-sections with straight lines in order to include the current limitations of cold-forming processes was made. The performance of the redrawn cross-sections was found to be close to the raw “optimum” cross-sections.

Acknowledgement

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References


[26] Li Z, Schafer BW. Application of the finite strip method in cold-formed steel member design. Journal of Constructional Steel Research 2008;64:766–78.


