A terahertz system of units

R.A. Lewis

University of Wollongong, roger@uow.edu.au
A terahertz system of units

Abstract
It is proposed that a system of units based on the THz frequency and the properties of the THz photon has both aesthetic and technical advantages.

Keywords
terahertz, units, system

Disciplines
Engineering | Science and Technology Studies

Publication Details

This conference paper is available at Research Online: http://ro.uow.edu.au/eispapers/6718
Abstract—It is proposed that a system of units based on the THz frequency and the properties of the THz photon has both aesthetic and technical advantages.

I. INTRODUCTION AND BACKGROUND

The systems of units in wide use today tend to be based on quantities of importance in mechanics, as exemplified by length-mass-time employed in the MKS or CGS systems. Yet modern technology emphasizes electronic and optical rather than mechanical devices. The time may have come to shift the system of units from a mechanical to an optical basis.

Historically, many units have been defined through artefacts that are not widely accessible or reproducible. Over time these units have been replaced by definitions based on more fundamental phenomena, such as well-defined atomic transitions. This trend to employing universal phenomena may be taken further by utilizing the properties of a photon (specifically here, the THz photon) to define various quantities in a terahertz system of units.

II. FREQUENCY AND TIME

A. Frequency

The fundamental quantity in a terahertz system of units is frequency. Frequency is arguably the simplest physical quantity of all to measure, requiring only counting. Moreover, by counting for longer, the precision of the measurement can be increased arbitrarily. Thus frequency as a base quantity has technologically simple and attractive features.

Many physical systems exhibit characteristic frequencies readily expressed in THz. For example, the frequency of oscillation of an electron in the first Bohr orbit is 6580 THz. Within condensed matter physics, the transverse optical (TO) oscillation of an electron in the first Bohr orbit is 6580 THz. Many physical systems exhibit characteristic frequencies readily expressed in THz. For example, the frequency of light of frequency 1 THz.

B. Time

To deal with time in the THz system, the derived unit of the inverse THz is introduced:

$$1 \text{ iTHz} = (1 \text{ THz})^{-1} = 10^{-12} \text{ s} = 1 \text{ ps}$$

So 1 s = 10^{12} iTHz = TiTHz.

The unit of the iTHz is of the order of the half-life of the bottom quark. It lies between fundamental times based on the atom on the one hand and macroscopic times associated with the terrestrial day on the other hand, as indicated in Table II.

III. LENGTH AND SPATIAL FREQUENCY

A. Length

In the SI, once the second is given and the light speed fixed at 299 792 458 m·s⁻¹ the definition of the meter follows as “the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second” [2]. In terms of the base frequency unit of the THz, the meter could rather be defined as “The wavelength in a vacuum of light having a frequency of 0.000299792458 THz”.

A conceptually simpler idea than this is to regard the THz frequency photon as the basis for the unit of distance and then to define the THz length, THzL, as

$$\text{The terahertz length is the wavelength in vacuum of light of frequency 1 THz.}$$

By this definition, 1 THzL corresponds to ~0.3 mm, of the order of the thickness of a human hair, and a convenient length for engineering measurements and calculations. Some examples of the use of the THzL are given in Table III.

The terahertz is a frequency 108.7827757 times that of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom in its ground state at zero temperature.
Concerning lengths, visible radiation is often characterized by its wavelength, in units of nm (or Ångstrom) – partly because the numbers are conveniently typically of the order of some hundreds (or thousands). Using the frequency expressed in THz is just as convenient, as again the numbers are in the order of some hundreds. As well, stating light in terms of frequency, rather than wavelength, is more fundamental, as the frequency remains the same as the light transverses media of different refractive indices. Some visible optical phenomena are listed in Table IV according to their frequency in THz.

### Table III
**Examples of Lengths Conveniently Expressed in THzL**

<table>
<thead>
<tr>
<th>Conventional units</th>
<th>Length in THzL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 inch</td>
<td>85 THzL</td>
</tr>
<tr>
<td>1 meter</td>
<td>3336 THzL</td>
</tr>
</tbody>
</table>

The reciprocal of the THzL is the derived unit of **inverse THzL:**

\[ 1 \text{ iTHzL} = (1 \text{ THzL})^{-1} \]

and corresponds to the number of entities in a THzL. This may be directly related to the unit of wavenumber, \( \text{cm}^{-1} \), in common use by spectroscopists, \( \nu = 1/\lambda \). (This is to be distinguished from the wavenumber, \( k \), defined by \( k = 2\pi/\lambda \).) One THz corresponds to approximately 33.4 \( \text{cm}^{-1} \).

### Table IV
**Examples of Visible Phenomena Conveniently Expressed in THz**

<table>
<thead>
<tr>
<th>Phenomenon</th>
<th>nm</th>
<th>THz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed low frequency limit</td>
<td>400</td>
<td></td>
</tr>
<tr>
<td>Conventional long wavelength limit</td>
<td>428</td>
<td></td>
</tr>
<tr>
<td>He Ne laser</td>
<td>474</td>
<td></td>
</tr>
<tr>
<td>Na D-line</td>
<td>509</td>
<td></td>
</tr>
<tr>
<td>Conventional short wavelength limit</td>
<td>749</td>
<td></td>
</tr>
<tr>
<td>Proposed high frequency limit</td>
<td>750</td>
<td></td>
</tr>
</tbody>
</table>

IV. **Mass and Energy**

In the same spirit as used to define the THz length, the THz photon could be utilized in the definition of mass. While the photon itself is massless, the energy it carries according to the Planck formula \( hf \) can be equated with an equivalent mass through the Einstein formula \( mc^2 \) to define the terahertz mass, **THzM**. This turns out to be a very small mass, even relative to subatomic particles. Rather than use this in practice, the MKS kg or the CGS g could be retained, but defined via the THzM.

Alternatively, rather than use the three mechanical base units of time, length and mass, the three base units of frequency, length and energy might be employed, with the THz photon energy, **THzE** (~4 meV) as the base energy unit.

The unit of **THzE** is convenient for energies that are typical in condensed matter physics, for example, for expressing bandgaps of semiconductors. This is illustrated in Table V.

### Table V
**Examples of Bandgaps (300 K) Conveniently Expressed in THzE**

<table>
<thead>
<tr>
<th>Semiconductor</th>
<th>THzE</th>
<th>eV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>268</td>
<td>1.11</td>
</tr>
<tr>
<td>Ge</td>
<td>160</td>
<td>0.66</td>
</tr>
<tr>
<td>InSb</td>
<td>41</td>
<td>0.17</td>
</tr>
<tr>
<td>InAs</td>
<td>87</td>
<td>0.36</td>
</tr>
<tr>
<td>InP</td>
<td>307</td>
<td>1.27</td>
</tr>
<tr>
<td>GaAs</td>
<td>346</td>
<td>1.43</td>
</tr>
</tbody>
</table>

V. **Electrical Units**

In the SI definition, the base electrical unit is the current, which is defined in mechanical terms via force. Alternative approaches to defining the SI electrical units, for example, by involving the von Klitzing constant, are under investigation at present.

Rather than these approaches, the electrical units may also be considered from the perspective of frequency. Basing the electric units on frequency has the attractive feature that the unit of current may be defined in the same fundamental way that the quantity current is defined; namely, by the charge per unit time; or, equivalently, by the charge multiplied by unit frequency. The technology now exists to count individual fundamental charges in, for example, the single-electron transistor.

A second approach to electrical units based on frequency would be to exploit the metrologically well-established phenomenon of the ac Josephson effect through the (exact) value of the conventional Josephson constant \[ K_{J,90} = 0.4835979 \text{ THz/mV}. \]

In such an approach the THz Josephson frequency would be defined to correspond to a potential of about 2 mV. From potential could be derived current, then the other electrical quantities such as resistance, capacitance, inductance, and so on.

VI. **Conclusion**

A proposal is put forward to define base units of frequency, length, energy/mass, and electrical potential using THz photons. Alternative ways to deal with energy/mass and the electrical units have been set out. Three further base units could be added to these to then constitute “Le Système International du THz” (SIT).

REFERENCES