2005

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http://ro.uow.edu.au/engpapers/4166

Publication Details

WORK-PER-CYCLE ANALYSIS FOR ELECTROMECHANICAL ACTUATORS

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Work-per-cycle calculations have been demonstrated for two modes of actuation behaviour: (1) working against a dead weight (isotonic actuation) and (2) working against a restoring spring. Importantly, the influence of the electrical stimulus on the elastic modulus of the actuator material has been included in the analysis. The change in elastic modulus is shown to significantly influence the actuator strain and work-per-cycle. The maximum work-per-cycle for each case has been calculated. Comparison of different materials has been made on the basis of our theoretical prediction and other published models. Work-per-cycle is shown to be significantly affected by the mode of loading.

Actuator, work-per-cycle, isotonic, elastic modulus.
INTRODUCTION

The work performed per expansion/contraction cycle is an important performance criterion for electromechanical actuators. The maximum work-per-cycle (often expressed as per mass or per volume of the actuator material) defines how effectively the actuator can perform mechanical work. Various values of maximum work-per-cycle have been reported for different actuator materials [1-3], however, the means for calculating this important parameter also varies for the different studies. In this paper, a theoretical basis for calculating work-per-cycle is presented and different actuator materials are compared.

Importantly, the analysis includes the effect of changing elastic modulus of the actuator material due to the applied electrical stimulus. This effect has often been ignored in previous studies, however, it has recently been shown to make a major contribution to actuator strain [4-7] for polypyrrole actuators. Interestingly, in a review of various actuator materials, Hunter and Lafontaine report that a change in elastic modulus occurs during the actuation cycle for all actuator materials. Table 1 lists various actuator materials, along with typical actuator strains and material properties. The change in modulus between the activated and un-activated states can be as high as 500% (for skeletal muscle and polypyrrole) but is much smaller (~10%) for other materials such as piezoelectric ceramics.

In this paper, the effect of the changing modulus on actuator strain and work-per-cycle is explicitly demonstrated. Two modes of actuator operation are considered: actuators working against a dead-weight (isotonic) and working against a restoring spring. The former describes actuators used in positioning systems (eg. as in piezoelectric actuators used in the atomic force microscope) and is depicted in Figure 1a). The latter is more generally used since the spring assists the actuator to return to its rest condition. The latter also includes the common bi-layer (or tri-layer) actuator design where laminating the actuator against a passive (yet, flexible) substrate generates a bending displacement. In this design, the elastic bending of the substrate generates the restoring force and the magnitude of the force is proportional to the degree of deformation, as in simple springs. Figure 1b) shows actuators operating against restoring springs.
The general condition to be considered is an actuator material that is caused to contract when stimulated by an applied voltage and is subjected to a constant tensile force. The length changes occurring are shown in Figure 1a) for the case of a film actuator operating by contracting against an attached dead weight. Applying a force ($\Delta f = f$) produces an initial elastic deformation ($\Delta L_{AB}$) while the applied voltage stimulus produces a length change that (for simplicity) is separated into two parts: the length contraction due to the actuation process ($\Delta L_{BC}$) and the length extension due to a change in elastic modulus ($\Delta L_{CD}$). The magnitude of each of these length changes can be estimated assuming that the materials obey Hooke’s law (generally true at small strains):

$$\Delta L_{AB} = -\left(\frac{fL_0}{YA}\right)$$

(1)

$$\Delta L_{BC} = \Delta L_0$$

(2)

$$\Delta L_{CD} = -\frac{fL_0}{A} \left(\frac{1}{Y'} - \frac{1}{Y}\right)$$

(3)

In these equations $f$ is the (constant) applied force; $L_0$ is the initial length of the actuator; $Y$ is the elastic modulus when no voltage is applied; $Y'$ is the elastic modulus after the voltage has been applied; $A$ is the cross-sectional area of the actuator (assumed not to change during the actuation process); and $\Delta L_0$ is the “free stroke” of the actuator, or the length change that occurs when the actuator operates against no external force. Note that the actuation direction (contraction) is arbitrarily taken as positive, so that expansion is negative. Note also that when $Y' < Y$ the actuator lengthens, hence equation (3) carries a negative sign.

The approach taken in previous studies (e.g., Giurgiutiu et al.) has assumed that the length changes due to changing elastic modulus are negligible. The net change in length of the actuator when stimulated under a constant (isotonic) load ($f$) is then given by combining equations (1) and (2):

$$\Delta L_f = \Delta L_0 - \frac{fL_0}{YA}$$

(4a)
where $\varepsilon$ is now the actuator strain. The result leads to the familiar relationship, shown in Figure 2, where the actuator stroke decreases linearly from the free stroke at zero load to zero at a load known as the “blocking force”.

To include the effects of a changing elastic modulus, a third term is introduced by combining equations (1), (2) and (3):

$$\Delta L_f = \Delta L_0 - \frac{fL_0}{YA} - \frac{fL_0}{A} \left( \frac{1}{Y'} - \frac{1}{Y} \right)$$

Expanding this equation gives:

$$\Delta L_f = \Delta L_0 - \frac{fL_0}{Y'A}$$

or

$$\varepsilon_f = \varepsilon_0 - \frac{f}{Y'A}$$

Equation (6) demonstrates that the modulus after application of the drive voltage ($Y'$) determines the effect of the applied load. Figure 2a) includes the effects of changing modulus on the actuator strain at different loads. For larger changes in modulus, the strain decreases more rapidly leading to smaller blocking forces.

In other studies, [8] the work cycle has only included steps B to D in Figure 1a), since the application of the applied load produces just a one-off contribution to the actuator length. In this scenario, the net length change is given by:

$$\Delta L_f = \Delta L_0 - \frac{fL_0}{A} \left( \frac{1}{Y'} - \frac{1}{Y} \right)$$

and is illustrated in Figure 3a) for hypothetical actuator materials having a free stroke of 5% and different ratios of $Y:Y'$. Now the change in modulus has a more dramatic effect on the actuator stroke, with the blocking force decreasing rapidly when the ratio of $Y:Y'$ increases. In fact, rearranging (7) shows this relationship explicitly as a hyperbolic function, taking $\beta=Y/Y'$:

$$f_{\text{blocking}} = \frac{\Delta L_0 AY}{L_0(\beta - 1)}$$
The calculation of the work performed by the actuator depends upon the definition of the work cycle. In the analysis of Giurgiutiu et al. [1] the work cycle includes steps A to D as shown in Figure 1a). The actuator stroke is then given by equation (6a) and the work done is given by:

\[ W = f \Delta L_f \]  

(7)

or \[ W = f \Delta L_0 - \frac{(f)^2 L_0}{Y' A} \]  

(8)

Differentiating to find the maximum work gives:

\[ \frac{dW}{df'} = \Delta L_0 - \frac{2 \beta L_0}{Y' A} = 0 \]  

(9)

\[ W_{\text{max}} = \frac{1}{4} \frac{\Delta L_0^2 Y' A}{L_0} \quad \text{when} \quad f'_{\text{max}} = \frac{1}{2} \frac{\Delta L_0 Y' A}{L_0} \]  

(10)

If V is the sample volume and \( V = A L_0 \), the volumetric (\( W_v \)) and gravimetric (\( W_g \)) work-per-cycle can be expressed by:

\[ W_v = \frac{1}{4} \sigma_0^2 Y' \quad (\text{J/m}^3) \]  

(11)

\[ W_g = \frac{1}{4 \rho} \sigma_0^2 Y' \quad (\text{J/kg}) \]  

(12)

where \( \rho \) is the actuator material density.

The analysis is the same as that given by other workers [2], with the major exception that the modulus after the voltage is applied determines the work done. This effect has been neglected in previous studies, although the reduction in modulus has a major effect on the actuator work as demonstrated in Figure 2b). As the modulus decreases, the maximum work occurs at a smaller applied force and gives a significantly smaller work-per-cycle. The work capacities of various actuators calculated using equation (11) are included in Table 1.

The analysis by Baughman [8] also includes the work-per-cycle calculations where the work-per-cycle is expected to depend upon the applied load:
The analysis can also be extended to show that the maximum volumetric work and gravimetric work are given by:

\[ W_v = \frac{1}{4} \varepsilon_0^2 \left( \frac{Y \cdot Y'}{Y - Y'} \right) \] (J/m³) (14)

\[ W_g = \frac{1}{4 \rho} \varepsilon_0^2 \left( \frac{Y \cdot Y'}{Y - Y'} \right) \] (J/kg) (15)

Figure 3b) illustrates the effect of changing modulus on the work per cycle for the case where the initial isotonic loading step is ignored. It should be noted that when \( Y = Y' \), the value of \( f_{max} \) increases to infinity and therefore an infinite work per cycle can be obtained (Fig.3). In reality, the maximum load that can be applied is limited by yield or rupture. Baughman [8] has suggested that the practical maximum applied load is 50% of the breaking or yield force \( (f_b) \), so that \( W_{max} = \Delta L_0 \cdot f_b \).

**ACTUATORS OPERATED AGAINST A RESTORING SPRING**

In many applications using commercially available actuators (electrostrictive ceramics or shape-memory alloys) the actuator works against a restoring spring, as schematically illustrated in Figure 1b). A typical scenario involves the actuator at rest but under a pretension caused by the external spring (initial force of \( f' \)). Application of a voltage causes contraction of the actuator and extension of the spring (\( \Delta L_{act} \)), so that the restoring force increases with increasing actuator displacement (from \( f' \) to \( f'' \)). However, the voltage also causes a change in elastic modulus of the actuator so that the external spring produces an extension (\( \Delta L_{ext} \)) and the final spring force is reduced to \( f''' \). Since the application of the external force and the actuator stroke occur simultaneously, the effect of the external load on the actuator strain cannot be neglected.

The actuator stroke is first estimated for the case where the elastic modulus does not change:
\[ \Delta L_f = \Delta L_{AB} = \Delta L_0 - \left( \frac{f'' - f'}{k_i} \right) \]  

(16)

where the first term is the free stroke of the actuator and the second term accounts for the elastic stretching of the actuator caused by the increasing spring force. \( k_i \) is the internal stiffness of the actuator material and is equivalent to \( Y/A/L_0 \). The change in spring force is determined by how much the spring is deformed and in this case is given by:

\[ f'' - f' = k_e \Delta L_{AB} \]  

(17)

Combining equations (16) and (17) gives the actuator stroke as:

\[ \Delta L_f = \Delta L_0 \left( \frac{1}{1 + r} \right) \]  

(18)

where \( r \) is the ratio of the external spring stiffness \( (k_e) \) to the internal stiffness of the actuator material \( (k_i) \) and (18) is the same result as reported by Giurgiutiu and co-workers [2]. These same workers have used equation (18) to show that the actuator work is given by:

\[ W = \frac{r}{(1 + r)^2} \left( \frac{1}{2} k_i \Delta L_0^2 \right) \]  

(J)  

(19)

and is optimised when \( r=1 \) (so called “impedance matching”) so that the maximum work is:

\[ W_{\text{max}} = \frac{1}{4} \left( \frac{1}{2} k_i \Delta L_0^2 \right) \]  

(J)  

(20)

This expression gives a maximum volumetric and gravimetric work capacity of:

\[ W_v = \frac{1}{4} \left( \frac{1}{2} Y \varepsilon_0^2 \right) \]  

(J/m³)  

(21)

\[ W_v = \frac{1}{4 \rho} \left( \frac{1}{2} Y \varepsilon_0^2 \right) \]  

(J/kg)  

(22)

The same analysis used by Giurgiutiu et al. can be extended to include the changing internal stiffness (modulus) of the actuator material during the actuator stroke. Now the actuator strain is given by the free stroke \( (\Delta L_0) \), less the displacement caused by the increasing spring force \( (\Delta L_{AB}) \), and less the displacement caused by a change in modulus \( (\Delta L_{\eta_k}) \):

\[ \Delta L_f = \Delta L_0 - \frac{f'' - f'}{k_i} - \left( \frac{f''' - f''}{k_i} \right) \]  

(23)
where $k'_i$ is the internal stiffness of the polypyrrole after the electrical stimulus has been applied ($k'_i = Y' A/L_0$). Expanding this expression and assuming the spring to be initially at rest gives:

$$\Delta L'_i = \frac{\Delta L_0}{1 + r'}$$

(24)

where $r'=k_e/k'_i$. The work-per-cycle is given simply by:

$$W = \frac{r'}{(1 + r')^2} \left( \frac{1}{2} k'_i \Delta L_0^2 \right)$$

(J) (25)

and is optimised when $r'=1$ so that the maximum work is:

$$W_{\text{max}} = \frac{1}{4} \left( \frac{1}{2} k'_i \Delta L_0^2 \right)$$

(J) (26)

This expression gives a maximum volumetric and gravimetric work capacity of:

$$W_v = \frac{1}{4} \left( \frac{1}{2} Y' \epsilon_0^2 \right)$$

(J/m$^3$) (27)

$$W_g = \frac{1}{4 \rho} \left( \frac{1}{2} Y' \epsilon_0^2 \right)$$

(J/kg) (28)

The effect of changing modulus on actuators operating against a restoring spring is illustrated in Figure 4. The actuator stroke decreases as the external spring stiffness increases (Fig. 4a) but the decrease in stroke is more significant when the actuator modulus is also decreasing ($Y=2Y'$ and $Y=4Y'$). The effect on work-per-cycle is shown in Figure 4b). Again, the maximum work possible is significantly reduced when the actuator modulus is decreasing during the actuator contraction cycle. The peak in work also occurs when softer external springs are used (the peak occurs when $r'=1$). Table 1 includes the predicted maximum work-per-cycle for various actuator materials operating against a restoring spring.
COMPARISON OF DIFFERENT ACTUATOR MATERIALS

The expected performance of various actuator materials is given in Table 1 and Figure 5. The actuator strain and elastic modulus data included in Table 1 represents typical values for each material and has been obtained from Cheng et al. [9] for piezoceramics (PEC) and electrostrictive polymers (ESP); Pelrine et al. [10,11] for electrostatic elastomers (ESE) and from our own laboratories for polypyrrole (PPy) and carbon nanotube actuators (CNT). Figure 5 shows the work-per-cycle of the actuators operated isotonically (ignoring the pre-load stretch) and operated against a restoring spring. The maximum work capacities for each of these two conditions are also included in Table 1.

Under isotonic conditions, the maximum work capacity in most cases is limited by the breaking strength of the material (estimated at Y/100 for materials where literature values could not be found). As shown in Figure 5a), only polypyrrole shows a maximum in the work capacity at a stress level below the breaking stress. This material also showed a very low work capacity, reflecting the large effect caused by the huge changes in elastic modulus occurring during actuation. The maximum work output for conducting polymers is estimated at 66 kJ/m$^3$ which agrees reasonably well with literature reports for isotonic operation (83 kJ/m$^3$ [13]; 73 kJ/m$^3$ [4]).

The highest isotonic work capacities are produced by electrostatic elastomers, due to the very high strains possible. These materials are, however, limited by their low tensile strengths. It is expected that improvements in other materials will generate similarly high work capacities. It was recently reported, for example, that carbon nanotube fibers having a modulus of 40 GPa had been generated by a fiber spinning process [14]. Individual nanotube ropes have been shown to have an axial modulus of 640 GPa [15]. With further improvements in fiber spinning processes it is conceivable that carbon nanotube assemblies with moduli of at least 100 GPa will soon be realised. Given that these aligned CNT fibres produce similar strains to that reported for unaligned sheets (0.5% [16]) then the achievable work capacity for carbon nanotubes would increase dramatically to 2500 kJ/m$^3$ or 8333 J/kg, far exceeding the gravimetric work capacity of even the electrostatic elastomers.

More published data is available for actuators operated against restoring springs. As shown in Table 1, the work-per-cycle is much lower in most cases than in the isotonic operation since the increasing restoring force causes deformation within the actuator that partially cancels the actuation. The predicted result for piezoelectric ceramics (29 kJ/m$^3$) is somewhat different to literature values. In the work by Cheng et al. [9] the work capacity is over-estimated by simply taking the product of $Ye_0^2/2$ rather than using equation (27). The
work by Giurgiutiu [2] gives a highest work capacity of 12 kJ/m³ which is only half the predicted value since the actuators used in their study (from various commercial sources) only produced ~0.1% strain which is ½ that assumed in the calculated results given in Table 1.

The highest work capacity for actuators working against a restoring spring is achieved by electrostrictive polymers. Once again, however, improvements in carbon nanotubes would see the work capacity increase to 313 kJ/m³ and 1042 J/kg, exceeding all other actuator materials. Another key aspect for actuators operating against a restoring spring is the so-called “impedance matching” where the maximum work is achieved when the actuator is operated against an internal spring of equivalent stiffness. As can be seen from Figure 5b) the electrostatic elastomers only produce useful work when they are matched against soft springs. More practically useful are the piezoelectric ceramics that can deform stiff springs, such as metallic cantilever beams. Again, the improvement in performance of carbon nanotubes would enable actuation against similarly stiff components.

Finally, the analysis described in this paper can also be applied to the situation where the elastic deformation due to the external force works in phase with the actuation. For example, in the case of polypyrrole it is known that the elastic modulus decreases when the polymer is reduced (at negative potentials) [6,17] which tends to partially counter the contraction of the polymer that usually occurs at these potentials. However, it is also known that some counterions cause expansion of the polymer at negative potentials [18], and the elastic stretching of the polymer that occurs simultaneously would enhance this expansion. The two processes reinforce each other and would lead to the behaviour shown in Figures 2-4 for the case where $Y=0.5Y'$. For both isotonic and isometric actuation, the strain produced and work-per-cycle is higher for the case where modulus increases upon actuator contraction compared with the situation where modulus decreases when the actuator is made to contract. Clever design of actuator materials could take advantage of this strain magnification.

**CONCLUSIONS**

The work-per-cycle generated by electromechanical actuators has been re-analysed to account for the change in modulus that occurs upon electrical stimulation of many actuator materials. Both isometric and isotonic actuation have been considered and the isotonic case has both included and excluded the initial strain caused by loading the actuator material. All three analyses have been reported previously in the literature, but the influence of a changing modulus on the actuation strain and work-per-cycle is presented for all three cases for the first time. The analyses show that the strain and work-per-cycle are decreased when the elastic modulus of the actuator decreases at the same time that the actuator is made to contract by electrical stimulus. The work-per-cycle (and impedance matching) is then determined by the modulus after stimulation. A comparison of the typical performance
values of different actuator materials has shown that considerably different work-per-cycle is possible both by varying the type of material and by considering different loading cycles.

REFERENCES


FIGURE CAPTIONS

Figure 1  Length changes occurring during the loading and actuation processes for a) actuators working against a dead –weight (constant force or isotonic); and b) actuators working against a restoring spring.

Figure 2  Calculated actuator stroke (a) and volumetric work-per-cycle (b) for hypothetical actuators operated isotonically against different loads (steps A-D in Figure 1a). The applied load opposes the actuation direction. The effect of changing modulus during the actuation cycle is shown for the cases where $Y \neq Y'$. The free stroke ($\varepsilon_0$) is arbitrarily taken as 5%, the resting modulus ($Y$) is taken as 0.1 GPa and the cross-sectional area is taken as 1 mm².

Figure 3  Calculated actuator stroke (a) and volumetric work-per-cycle (b) for actuators operated isotonically against different loads but the initial deformation due to loading is ignored (steps B-D in Figure 1a). The applied load opposes the actuation direction. The effect of changing modulus during the actuation cycle is shown for the cases where $Y \neq Y'$. Other conditions are the same as quoted in Figure 2.

Figure 4  Calculated results showing a) the actuator stroke and b) the actuator work-per-cycle for actuators working against a restoring spring of different spring constants (the stiffness ratio is the ratio of the external spring stiffness to the internal stiffness of the actuator material). The effects of varying actuator modulus are also shown. Other conditions are the same as quoted in Figure 2.

Figure 5  Calculated work-per-cycle for various actuator materials using data from Table 1 and calculated for a) isotonic operation and b) against a restoring spring. The isotonic condition ignores the pre-load strain and includes the actuation strain only (steps B-D in Figure 1). The influence of applied isotonic stress and external spring stiffness on the work-per-cycle are shown in parts a) and b), respectively. Calculations use the conditions quoted in Figure 2.
### Table 1  Calculated work capacity for various actuator materials using different theoretical approaches.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\varepsilon_0$ (%&lt;br&gt;&lt;br&gt;Y (GPa)</th>
<th>Y/Y'</th>
<th>$\rho$ (g/cm$^3$)</th>
<th>Isotonic (A-D) W&lt;sub&gt;v&lt;/sub&gt; (kJ/m$^3$)</th>
<th>W&lt;sub&gt;g&lt;/sub&gt; (J/kg)</th>
<th>Isotonic (B-D) W&lt;sub&gt;v&lt;/sub&gt; (kJ/m$^3$)</th>
<th>W&lt;sub&gt;g&lt;/sub&gt; (J/kg)</th>
<th>Against Spring W&lt;sub&gt;v&lt;/sub&gt; (kJ/m$^3$)</th>
<th>W&lt;sub&gt;g&lt;/sub&gt; (J/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piezoelectric Ceramic</td>
<td>0.2</td>
<td>64</td>
<td>1.1</td>
<td>7.5</td>
<td>58</td>
<td>8</td>
<td>640</td>
<td>85</td>
<td>29</td>
</tr>
<tr>
<td>Electrostrictive Polymer</td>
<td>5</td>
<td>0.4</td>
<td>1.2</td>
<td>1.8</td>
<td>208</td>
<td>116</td>
<td>1250</td>
<td>694</td>
<td>104</td>
</tr>
<tr>
<td>Conducting Polymer 1</td>
<td>2</td>
<td>0.11</td>
<td>1.16</td>
<td>1.5</td>
<td>9</td>
<td>6</td>
<td>66</td>
<td>44</td>
<td>5</td>
</tr>
<tr>
<td>Conducting Polymer 2</td>
<td>5</td>
<td>0.1</td>
<td>5</td>
<td>1.5</td>
<td>13</td>
<td>8</td>
<td>16</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>Carbon Nanotube</td>
<td>0.5</td>
<td>5</td>
<td>1</td>
<td>0.3</td>
<td>31</td>
<td>104</td>
<td>125</td>
<td>417</td>
<td>16</td>
</tr>
<tr>
<td>Electrostatic Elastomer</td>
<td>63</td>
<td>0.001</td>
<td>1</td>
<td>1.4</td>
<td>99</td>
<td>71</td>
<td>7875</td>
<td>5625</td>
<td>50</td>
</tr>
</tbody>
</table>

The values in the table represent various properties of different actuator materials under different theoretical approaches, including isotonic (A-D) and isotonic (B-D) work capacities as well as work against a spring.
Figure 1
Figure 2
Figure 3
Figure 4
Figure 5
Figure 5 (continued)