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Application of an Advanced Bounding Surface Plasticity Model in Static and Seismic Analyses of Zipingpu Dam

Mojtaba E. Kan¹ and Hossein A. Taiebat²

Abstract:

The strong ground motion of Wenchuan earthquake which hit Zipingpu dam in China in 2008 has provided an excellent benchmark to study the behaviour of large modern rockfill dams subjected to seismic loading. The performance of the dam during construction and prior and after the earthquake loading has been recorded with good accuracy and provides a reliable database to examine the reliability of available constitutive models and numerical methods in predicting the static and dynamic behaviour of embankment dams. In this paper, an advanced bounding surface plasticity model has been used in a series of numerical analyses to study the static and dynamic behaviour of Zipingpu dam. The model can take into account particle breakage that may occur in monotonic and cyclic loading of rockfill materials. The material parameters required for the model are calibrated based on the results of available monotonic and cyclic triaxial tests. In the numerical procedure the staged construction of the dam and the subsequent impounding of the reservoir are simulated followed by dynamic loading. At each stage, the results of the numerical analysis are compared with in-situ monitoring records of the dam. The results of the numerical simulation and the displacements measured after the earthquake are also compared with those estimated by two simplified engineering procedures which are routinely used in practice. The effectiveness and applicability of the simplified procedures to such a large dam subjected to an earthquake with a long duration is also discussed.

Keywords: Bounding surface plasticity, Zipingpu dam, Particle breakage, Dynamic analysis

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Introduction

Zipingpu dam is one of the largest modern rockfill dams in the world which has experienced a severe earthquake and suffered substantial internal deformation and crest displacement. The ‘5.12’ Wenchuan Earthquake hit Zipingpu dam hard in May 2008 and rendered relatively large permanent displacements to the dam (Kong et al., 2010). Therefore simulation of the behaviour of this dam during earthquake is imperative to geotechnical engineers for better understanding of the seismic behaviour of compacted rockfill embankments, for assessing the safety of modern rockfill dams, and for evaluating the capabilities of numerical methods in simulating the seismic response of earth structures.

Different constitutive models have been used to simulate the behavior of geo-materials. Since 1970’s the nonlinear elastic Duncan-Chang model (Duncan and Chang, 1970) has been widely used to simulate construction of rockfill dams. The popularity of this model has most likely been due to the fact that the parameters of the model are relatively simple, easy to obtain and well established in the literature. However, the Duncan–Chang model does not take into account the dilatancy and plastic deformation of rockfill materials and therefore cannot be used for dynamic analysis of rockfill dams. More advanced elastoplastic constitutive models are required for dynamic analysis of rockfill dams, but only limited research can be found on numerical analyses of rockfill dams where advanced elastoplastic models have been utilised. Lollino et al. (2005) used the Lade model for sands to simulate the behaviour of rockfills. Zhang and Zhang (2009) used an elastoplastic model for rockfill to simulate construction of Zipingpu dam under static conditions. Zhou et al. (2011) used a creep model to capture the behaviour of rockfill during the operation period of a concrete faced rockfill dam, but Duncan–Chang model was used to simulate the construction process of the dam.
Few numerical studies have been conducted to fully understand the damage to Zipingpu dam due to Wenchuan earthquake. In a very first attempt, Kong et al. (2011) performed equivalent linear analyses to study the damage mechanism and the seismic performance of the dam. This model could not estimate the residual volumetric compression of the rockfill material properly. Later, Zou et al. (2013) used a modified generalized plasticity model in a three-dimensional finite element analysis to simulate the dynamic response of Zipingpu dam. Although the results of their simulations were generally in good agreement with the field data, their model failed to predict the effects of breakage of the rockfill particles on dynamic response of the dam.

In the current study, a robust numerical tool based on a bounding surface plasticity model has been employed to simulate the dynamic responses of Zipingpu rockfill dam. The effect of particle breakage on the behaviour of rockfill materials has been incorporated in the simulations using the concept of translating critical state line and isotropic compression line. Using this model, the residual deformation of the dam during the earthquake could be captured directly. For comparison purposes, the results of deformation analyses using current engineering procedures, i.e. the well-known Newmark sliding block method as modified by Makdisi and Seed, are also presented in this paper.

**Zipingpu Dam**

Zipingpu dam is a concrete faced rockfill dam which was affected severely by the strong motion of Wenchuan earthquake on 12 May 2008. The dam is located in Wenchuan County of Sichuan Province, 60 km northwest of Chendu city in southwest China (Chen and Han, 2009). The epicentre of the earthquake and the causative fault, located at the foot of the Tibetan Plateau, were 17 km and 7 km far from the dam site, respectively (Guan, 2009).

A cross section of the dam at its deepest point is shown in Figure 1. This section is located at station 0+251 and has detailed monitoring devices. The maximum height of the dam is 156 m,
with a 664 m long 12 m wide crest. The upstream slope of the dam is 1V:1.4H. Two downstream berms at EL 796.0 and 840.0 m with a width of 6 m provide an average downstream slope of 1V:1.5H. The total volume of the rockfill in the dam is $11.83 \times 10^6$ m$^3$ (Xu et al., 2012).

**Constitutive Model**

The capability of any numerical analysis to predict the behaviour of earth and rockfill materials is largely restricted to the availability of a reliable constitutive model that can adequately simulate the behaviour of the materials. Of particular importance is an adequate description of the plastic strains induced by particle breakage which is known to be a fundamental property of granular materials and rockfills. The need for a robust constitutive law becomes more evident for cases involving complex seismic loading conditions in boundary value problems. To this end, a newly developed bounding surface plasticity model for highly crushable materials and rockfills (Kan and Taiebat, 2014a) is used in this paper for the analysis of the dam. The main feature of the model, which differs from some existing models for rockfills, is its capability to incorporate the effect of particle crushing by translating the critical state and the limiting isotropic compression lines. The translation of the critical state line captures irrecoverable plastic strains induced by particle crushing more realistically compared to those models with fixed critical state line. Detailed description of the model can be found in Kan and Taiebat (2014a). A brief summary of the model is given here.

Following the general theory of plasticity, the total strain rate is decomposed into elastic and plastic parts so that:

$$\dot{\varepsilon} = \dot{\varepsilon}_e + \dot{\varepsilon}_p$$  \hspace{1cm} (1)
where a superimposed dot indicates incremental values, and the subscripts e and p denote the elastic and plastic components, respectively. The incremental elastic strains are linked to the incremental stresses through

\[ \dot{\sigma}' = D^e \dot{\varepsilon}' \tag{2} \]

where \( D^e \) is the elastic property matrix which can be defined as a function of the bulk modulus, \( K \), and the shear modulus, \( G \). The elastic-plastic stress-strain relationship is therefore defined as

\[ \sigma' = \left( D^e - \frac{D^e m n^T D^e}{h + n^T D^e m} \right) \dot{\varepsilon} \tag{3} \]

where \( n \) is the unit vector normal to the loading surface at the current stress state, \( \sigma' \), \( m \) is the unit direction of plastic flow at \( \sigma' \), and \( h \) is the hardening modulus.

A modified form of the critical state line (CSL) has been introduced in the model which has two curvatures and limits the minimum and maximum specific volumes corresponding to very large and very small stresses:

\[ \nu = \nu_l - \nu_l I_G + (\nu_h - \nu_l) \exp \left[ - \left( \frac{p'}{p_c} \right)^{\alpha_l} \right] \tag{4} \]

where \( \nu_h \) and \( \nu_l \) are the maximum and minimum specific volumes for the single sized particle grading \( (I_G = 0) \), and \( p_c \) and \( \alpha_l \) are parameters which control the size and the shape of the CSL, and \( \nu_c = \nu_l - I \). Here \( I_G \) is a grading index (Muir Wood, 2007) which represents the changes in particle grading and is defined as the ratio of the area under the current grain size distribution to the area under the fractal grading curve (McDowell et al., 1996). The grading index changes from 0 for single size particles to 1.0 for fractal grading. Figure 2 shows a schematic diagram of an evolving grading curve and the definition of grading index.
The CSL in the $q \sim p'$ plane is defined using a straight line passing through the origin with a slope of $M_{cs}$ which is related to the critical state friction angle, $\phi'_cs$:

$$M_{cs} = \frac{6 \sin \phi'_cs}{3\tilde{t} - \sin \phi'_cs}$$  \hspace{1cm} (5)

where $\tilde{t} = +1$ represents compression and $\tilde{t} = -1$ extension.

Similar to the critical state line a curved isotropic compression line is used in the model:

$$\nu = \nu_i - \nu_c I_G + \left( \nu_{max} - \nu_i + \nu_c I_G \right) \exp\left[ -\left( p' / p_{ic} \right)^{\alpha_i} \right]$$  \hspace{1cm} (6)

where $p_{ic}$ is a reference stress which controls the shape of the isotropic compression line and $\nu_{max}$ is an adjustable parameter as shown in Figure 3.

The limiting isotropic compression line (LICL) is also taken as a curve, located at a constant shift from the CSL along the isotropic compression line in $\nu - \ln p'$ plane with the following equation:

$$\nu = \nu_i - \nu_c I_G + \left( \nu_k - \nu_i \right) \exp\left[ -\left( \bar{p}' / R p_{cs} \right)^{\alpha_i} + \left( \bar{p}' / R p_{ic} \right)^{\alpha_i} - \left( \bar{p}' / p_{ic} \right)^{\alpha_i} \right]$$  \hspace{1cm} (7)

where $R$ is a model parameter and denotes the shift ratio in the $\nu - \ln p'$ plane.

The grading index $I_G$ describes the change in the mechanical properties of the rockfill due to particle crushing. Following Hardin (1985) a hyperbolic relationship is used for evolutions of grading index:

$$I_G = 1 - \exp\left[ -\left( \frac{p_{cr} - p_{cr0}}{p_{cr}} \right)^{\alpha_i} \right]$$  \hspace{1cm} (8)

where $p_{cr}$ describes the crushing stress at the current stress state and is defined as:
\[ p_{cr} = p' \left[ 1 + \frac{1}{2} \left( \frac{q}{M_{cr} p'} \right)^{\alpha_2} \right] \]  

(9)

\( p_{cr0} \) is the crushing stress for \( I_G = 0 \) and introduces the initiation of particle crushing, \( p_r \) is a reference stress and \( \alpha_2 \) is a model parameter.

Assuming isotropic elasticity and constant Poisson’s ratio, \( \nu \), the elastic bulk and shear moduli are defined as:

\[
K = \frac{\nu}{\nu - \nu_l + \nu I_G} p_{cr0}^{\alpha_1} p'^{\nu - \alpha_1} 
\]

(10)

\[
G = \frac{3(1 - 2\nu)}{2(1 + \nu)} K 
\]

(11)

The loading surface has a tear drop shape with the following equation:

\[
f(p', q, p'_c) = \left( \frac{q}{M_{cr} p'} \right)^N - \ln \left( \frac{p'_c}{p'} \right) \ln R = 0
\]

(12)

where \( p'_c \) is an isotropic hardening parameter that controls the size of the loading surface. The bounding surface has similar shape to the loading surface and can be defined by equation (12) if \( p'_c \) is replaced by \( \bar{p}'_c \) which is a function of the plastic volumetric strain and defines the size of the bounding surface. Constants \( N \) and \( R \) control the shape and curvature of the surfaces.

Figure 4 shows the shape of loading surfaces for first time loading as well as for unloading/reloading case. The similarity ratio of the loading and bounding surfaces is a function of the location of the centre of homology (Dafalias, 1986) based on a radial mapping rule. For first time loading this centre is at the origin and it moves to the last point of stress reversal for any subsequent unloading and reloading. To maintain similarity with the bounding
surface, loading surfaces undergo kinematic hardening during loading and unloading such that they remain homologues with the bounding surface at the centre of homology. The image point for cyclic loading is located by simple radial projection of the stress point on the bounding surfaces passing through each point of stress reversal (Kan et al., 2014).

The plastic potential \((g)\) is assumed to be unaffected by the grading index and defined as

\[
g(p', q, p_o) = \tilde{q} + \frac{AM_{cs}p'}{A-1}\left(\frac{p'}{p_o}\right)^{A-1} - 1 \quad A \neq 1
\]

\[
g(p', q, p_o) = \tilde{q} + M_{cs}p' \ln\left(\frac{p'}{p_o}\right) \quad A = 1
\]

where \(p_o\) controls the size of the plastic potential, \(A\) is a material parameter, and \(\tilde{q}\) is a scalar, the sign of which controls the direction of plastic flow in the deviatoric plane.

As the usual approach in bounding surface plasticity, the hardening modulus \(h\) is divided into two components of \(h_b\) and \(h_f\), the plastic moduli on the bounding surface and on the current stress state, respectively. \(h_b\) can be calculated based on the consistency condition at the bounding surface and isotropic hardening of the bounding surface with plastic volumetric compression:

\[
h_b = \frac{\nu}{\lambda^* - \kappa^* + \bar{p}'\nu c} \ln R \left| \frac{\partial g}{\partial \sigma} / \left| \frac{\partial g}{\partial \sigma} \right| \right|
\]

\[
\lambda^* = \kappa^* + \bar{p}'\nu c \frac{\partial I_c}{\partial p_c} \frac{\partial p_c}{\partial \sigma}
\]

where \(\lambda^*\) and \(\kappa^*\) denote the current slope of the LICL and the isotropic compression line in the \(\nu - \ln p'\), respectively. In this way, any softening effect due to particle crushing is assumed to be incorporated into plastic volumetric strain. \(h_f\) is defined such that it is zero on the bounding surface and infinity at the point of stress reversal:
\[ h_f = \frac{\nu p'}{\lambda' - \kappa' + \frac{\overline{\ddot{p}}'}{\overline{p}_c}} \left( \frac{\overline{p}_c}{\overline{p}_c'} - 1 \right) k_m (\eta_p - \nu) \]  

where \( k_m \) is a scaling parameter controlling the steepness of the \( q \sim \varepsilon_q \) curve and \( \eta_p \) is the slope of the peak strength line in the \( q \sim p' \) plane; it is a function of the state parameter, \( \xi \), and the slope of the critical state line and is given by:

\[ \eta_p = \nu (1 - k \xi) M_{cs} \]  

where \( k \) is a material parameter and \( k_m \) is a scaling parameter.

\( k_m \) for granular materials can be expressed as a function of the initial state parameter, \( \xi_0 \), and the initial confining pressure, \( p'_0 \):

\[ k_m = k_{m0} (1.0 - \beta_1 \exp(\xi_0)) (p'_0)^{\beta_2} \]  

Here \( k_{m0} , \beta_1 \) and \( \beta_2 \) are material parameters and \( \xi \) is the state parameter of Been & Jefferies (1985) which is defined as the vertical distance between the current state and the CSL in the \( \nu \sim \ln p' \) plane, as shown in Figure 3.

The constitutive model has been numerically implemented in the explicit finite difference program FLAC (Itasca Consulting Group Inc., 2008) and have been used successfully in simulation of fully coupled dynamic loading in boundary value problems (Kan, 2014).

**Calibration of Material Parameters**

The bounding surface plasticity model requires 11 model parameters for crushable rockfill materials, all of which can be obtained by conventional experimental tests. Note that based on the simulations of a variety of rockfill materials with rounded and angular particles, Kan and
Taiebat (2014a) found that values of 1.5, 2.0 and 3.0 were sufficiently accurate for $N, k$ and $R$, respectively. Also a default value of 1 kPa can be used for $p_{cv0}$ since granular materials exhibit particle crushing from very low stress levels.

The elastic parameters $p_{ic}$ and $\nu$ with the critical state constants $M_{cs}, \nu_i, \nu_h, p_{cs}$ and $\alpha_i$ can be determined from triaxial tests using conventional procedures; $p_{ic}$ is determined based on the shape of the elastic unloading-reloading line on $\nu - \ln p'$ plane, and $\nu$ is the Poisson’s ratio. $M_{cs}$ is the slope of the critical state line on the triaxial $q - p'$ plane. $\nu_i$ and $\nu_h$, which are the minimum and maximum specific volumes for single sized grading material ($I_0 = 0$), as well as $p_{cs}$ and $\alpha_i$, by which the critical state lines in $\nu - \ln p'$ plane are described, can be determined by the results of undrained triaxial compression tests. The constitutive parameters $\alpha_2$ and $p_r$ define the evolution of the grading index and can be determined by variation of the particle size distribution during isotropic or one-dimensional compression. $A$ can be calibrated by plotting the stress ratio $\eta$ against the measured total dilatancy in the standard drained triaxial compression tests, assuming that elastic strains are negligible compared to plastic strains. For detailed process of calibration of model parameters see Kan & Taiebat (2014a).

A well-known problem in application of any advanced constitutive model for very coarse granular materials and rockfills is that the material parameters cannot be obtained from laboratory tests on samples of the actual material since the maximum size of the particles is beyond the attainable size of the test specimen (e.g. Desai and Siriwardane, 1984). Large scale triaxial tests with specimen diameters in the ranges of 300 mm to 500 mm have been used for rockfill materials (e.g. Varadarajan et al., 2003). However the maximum size of rockfill materials used in the construction of embankment dams is commonly around 1000 mm and
cannot be directly used for triaxial testing. This problem has led to the introduction of modelled grading techniques to obtain representative samples of materials with smaller particles that are suitable for testing (e.g. Zeller and Wullimann, 1957, Lowe, 1964, Fumagalli, 1969, Frost, 1973). Among these techniques “parallel grading” (Lowe, 1964) is more common in practice (e.g. Varadarajan et al., 2003, Gupta, 2009). Xu et al. (2012) have also used this technique to obtain representative samples of the rockfill material of Zipingpu dam.

The problem with modelled grading techniques is that the tested material which has smaller particle size may exhibit different behaviour to that of the actual material. Saboya and Byrne (1993) and Hunter and Fell (2003) showed that smaller size rockfills have higher shear strength and stiffness. Varadarajan et al. (2003) concluded that an increase in the size of the rockfill particles results in higher volumetric strain at the same confining pressure. Varadarajan et al. (2006) introduced correlations between the mechanical parameters of the actual and the smaller representative samples of quarried and alluvial rockfills based on data obtained from six different dams. They concluded that for alluvial rockfills, the actual material has higher deviatoric stress response compared to the representative samples of the material, while for quarried rockfills the actual material has lower deviatoric stress response. For both types, contraction of the actual rockfills is higher than those of the tested samples.

For Zipingpu dam rockfill, Xu et al. (2012) assumed that the behaviour of samples with a lower density would represent that of the actual material and can be used for calibration of model parameters. While the void ratio of the actual material in the dam is 0.259, they used samples with void ratios in the range of 0.31 to 0.32 for triaxial testing. This assumption implies that as the size of particles in the actual rockfill increases, it exhibits a behaviour corresponding to the looser samples. Since the rockfills of Zipingpu dam were obtained from quarried limestone, this assumption is also compatible with findings of Varadarajan et al. (2006) for quarried rockfills and therefore the same assumption is used here in the current study.
The material parameters for the rockfill material of Zipingpu dam, calibrated from the results of triaxial tests reported by Xu et al. (2012), are presented in Table 1. The performance of the model is first verified by simulating the behaviour of samples under monotonic and cyclic loading conditions. Then the model is used to predict the behaviour of the dam under construction, impounding, and seismic loading. Also included in the numerical procedure here is the method proposed by Kan and Taiebat (2014b) to prevent the well-known phenomenon of overshooting in application of bounding surface plasticity models in solving boundary value problems.

**Simulations and Results**

**Triaxial tests**

Three monotonic and two cyclic triaxial tests are selected for simulation purposes. The monotonic tests are the conventional triaxial compression tests on dense samples of Zipingpu dam, as reported in Xu et al. (2012). The maximum particle size of the samples was 60 mm with a mean grain size of 9.5 mm which formed triaxial samples of 300 mm in diameter and 600 mm in height. For cyclic tests, two tests on dense and loose samples are simulated based on the experimental data presented by Liu et al. (2014).

The initial mean effective stress of the samples in the monotonic tests is in the range of 200 to 800 kPa with initial specific volume, $\nu$, from 1.317 to 1.31. The results of simulations of the monotonic triaxial tests are compared to the experimental data in Figure 5. An excellent agreement is observed in all three simulations. Apart from good predictions of deviatoric stresses, the initial dilative behaviour of the rockfill at lower confining pressures as well as the dilation throughout compression at higher confining pressures are captured appropriately.
The initial mean effective stresses for samples subjected to cyclic loading were 300 kPa and 500 kPa, with initial specific volumes of 1.42 and 1.313 for loose and dense samples, respectively. The results of simulations of the cyclic tests are compared with experimental data in Figure 6 for dense and in Figure 7 for loose samples. It can be seen that the simulation results for both tests are in good agreement with the laboratory data which shows the capability of the model in predicting the cyclic behaviour of rockfills in a wide range of strain from, 0 to 0.006 for the dense sample and 0 to 0.06 for the loose sample. This is a feature which is particularly important if the behaviour of a material in a boundary value problem is to be predicted under different loading conditions using a single set of material parameters. For example in the case of Zipingpu dam, the range of strain measured during the construction phase of the dam is relatively high, while during the seismic loading much lower shear strains were recorded.

**Construction and reservoir impoundment**

Construction of the Zipingpu dam commenced in 2001 and the reservoir impoundment and power generation started in 2005 (Chen and Han, 2009). The construction procedure consisted of 3 stages as shown in Figure 8.

The body of the dam consists of rockfill materials from quarried limestone with a maximum particle size of 800 mm and a dry density \((\rho_d)\) of 2160 kg/m\(^3\) (Xu et al., 2012). The upstream concrete slab rests on cushion and transition zones which both were sourced from the same limestone but with smaller grain sizes. The downstream side of the dam is partly constructed by a zone of rockfill, zone 3C in Figure 1, with slightly lower average dry density of 2150 kg/m\(^3\). In this study it is assumed that the behaviour of all the rockfills as well as that of the thin cushion and transition zones can be described by a single set of material parameters.

Since the main focus of the current research is on the behaviour of the rockfill materials, the upstream concrete slab of the dam and the plinth are not simulated separately. In other words, it
is assumed that the stiffness of the thin slab does not have any significant effect on the overall dynamic response of the dam (Uddin and Gazetas, 1995) and therefore it is simply simulated as a seepage barrier.

The numerical analyses of the dam are performed assuming plane strain conditions. A full 3D analysis which includes the geometry of the valley could potentially give more accurate results than a 2D analysis. However, given the crest length is more than 4 times the height of the dam, a 2D analysis with a relatively fine mesh is believed to provide sufficient accuracy for the purpose of seismic analysis. The geometry of the dam is discretised into 662 elements as shown in Figure 9. To increase the accuracy of the numerical solution, triangular elements are avoided in the finite difference mesh and the size of the elements are kept reasonably consistent throughout the dam section. Note that triangular elements may become badly distorted in the course of seismic loading when they are subjected to large strains, especially along the slope faces.

The construction procedure is modelled by simulating the three stages of construction of the dam (Figure 8) through 25 “lifts” where different layers of material in the finite different mesh (Figure 9) are activated sequentially by increasing the gravity of each layer from zero to 9.81 m/s². The newly activated material in each lift should represent closely the state of the material after compaction which is under very low vertical stress. Therefore, the initial specific volume of the material is set to the value expected after compaction; ie, 1.259 which corresponds to a relative density of 85%. Contours of settlements predicted at the end of each construction stage are shown in Figure 10. The magnitudes of the maximum settlement at the end of the three successive stages are 370, 650 and 850 mm. The predicted maximum settlement at the end of construction of the dam is slightly higher than the actual accumulated settlement which was 820 mm (Guan, 2009). In order to evaluate the results of the numerical analysis, settlement profiles at four different elevations are compared to the actual field data
In general it is evident that the settlement pattern at the end of the construction is captured very well by the numerical simulation; only at elevation El.850 the predicted settlements are slightly larger than those measured in the field.

The impoundment of the dam to its maximum normal level (elevation El. 877) is numerically reproduced by applying the reservoir water pressure directly on the upstream face of the dam in 10 stages. The stress and strain states obtained at the end of construction procedure are used as the initial states for the impoundment, assuming that no creep occurs. The displacements predicted during the construction phase are set to zero to reflect the incremental displacements occur due to the impoundment more clearly. Contours of displacements in the horizontal and vertical directions predicted due only to the impoundment are shown in Figure 12. This figure shows that the maximum horizontal and vertical displacements reach 330 mm and 210 mm, respectively, both are predicted to be in the upstream slope of the dam. The maximum resultant displacement in the upstream slope of the dam is 380 mm. No field data on the deformation of the dam after impoundment is reported in the literature. However, Xu et al. (2012) obtained a resultant displacement of 232 mm based on a numerical analysis using a generalised constitutive model where the upstream slab and the interface between the concrete slab and the rockfill were also modelled. This is slightly less than the resultant displacement predicted by the current analysis.

**Seismic loading**

The Wenchuan Earthquake had a magnitude of about 8 on the Richter scale \( M_s = 8.0 \). It hit Zipingpu dam severely and caused a maximum permanent settlement of 1 m and a horizontal displacement of 0.6 m to the dam crest (Chen and Han, 2009). The seismic observation network of the Zipingpu dam failed to record the acceleration time history on the bedrock during the earthquake. According to deductions from the peak acceleration measured by an accelerometer
installed in the dam crest, the peak ground acceleration (PGA) for the bedrock in the dam site has been estimated to be 0.5 to 0.6g (Guan, 2009). In the current study, following Kong et al. (2010) and Zou et al. (2013), the time history of the earthquake acceleration recorded in Mao Town is adopted and scaled to attain a PGA of 0.55g. The North-South component of the earthquake, which is in the transverse direction of the dam axis, is used as input motion for the numerical analysis and shown in Figure 13. This is an 80 seconds long record with high frequency content and an extremely low predominant period of 0.12 seconds. The input motion has been filtered in order to omit frequencies higher than 20 Hz. The baseline drift correction is then applied to the input motion using a low-frequency sinusoidal wave, in order to achieve zero residual displacement at the end of the earthquake.

**Seismic analysis of the dam**

The horizontal acceleration time history of the seismic excitation is applied to the base of the model. Although the interaction between the dam and the reservoir may cause complex hydrodynamic forces to the upstream concrete slab and affect the overall seismic response of the dam, it has not been included in the present study and only the static water pressure corresponding to the water level at the time of earthquake, El. 828.7, is applied on the upstream face of the dam. The bounding surface plasticity model can capture the material damping at finite strain. However, it predicts much smaller damping than that of the actual rockfills at infinitesimal strains. To damp the high frequency components of the motion, a small amount of Rayleigh damping of 1% is added into the dynamic simulation.

The results of the simulation of the dam under seismic loading are presented in Figure 14 to Figure 18. Contours of the horizontal and vertical displacements of the dam at the end of dynamic loading are shown in Figure 14(a) and (b). These displacements are due only to the seismic loading since the deformations predicted during the construction and impoundment
stages are set to zero before the dynamic analysis. These contours show that the maximum settlement of the dam after seismic loading is around 1.05 m while the maximum horizontal displacement is around 0.4 m. It appears that a potential sliding block is also formed at the downstream slope of the dam, as it can be seen in displacement vectors presented in Figure 14(c).

The time history of the crest acceleration predicted by the numerical analysis is shown in Figure 15. Kong et al. (2010) reported that the maximum crest acceleration has been recorded as 1.65 g. They have also reported a value of 0.8 g for filtered peak crest acceleration. These two field data are also shown in Figure 15 for comparison. It can be seen that the numerical simulation predicts a peak crest acceleration of about 1.44 g which is less than that recorded by the crest accelerometer, but is in between the actual and filtered field data with an amplification factor of 2.62 with respect to the peak ground acceleration.

The time history of crest displacements in the horizontal and vertical directions is given in Figure 16. The numerical predictions of Zou et al. (2013) for the seismic behaviour of the dam are also shown in this figure for comparison. It can be seen that the settlement predicted at the end of excitation is very close to the observed settlement in the field. The maximum and the permanent horizontal displacements predicted by the current study are 0.7 m and 0.4 m, as compared to 0.6 m observed in the field, while almost half of the actual field displacement was predicted by Zou et al. (2013).

In order to evaluate the predicted pattern of the internal settlements at the end of seismic excitation, the settlement profiles at four different elevations are compared with those recorded in the field (Kong et al., 2010) in Figure 17. At elevations El. 790 and El. 820, the maximum actual settlement is slightly lower than those predicted by the analysis, while at elevation El. 850, in a zone below the dam crest, the numerical analysis predicts lower settlements compared to the field data.
Contours of the change in particle grading index \( \Delta I_G = I_{G,f} - I_{G,0} \) predicted to occur in the course of seismic loading is shown in Figure 18, where \( I_{G,0} \) and \( I_{G,f} \) are the particle grading indices after the reservoir impoundment and at the end of seismic loading, respectively. This figure shows that a maximum of 28% extra particle breakage occurs due to the earthquake, mostly close to the crest and on the downstream side of the dam, while most of the lower parts of the dam experience less than 4% of extra particle breakage. Although the actual particle grading of the material after earthquake is not available for a quantitative comparison, these results are qualitatively consistent with observations of Chen and Han (2009).

**Simplified methods of seismic analysis**

In this section an attempt is made to compare the displacements predicted by the numerical analysis with those that could be obtained from simplified engineering techniques currently used in practice. Since good quality field data are also available, such a comparison gives insights into the pros and cons of application of the simplified engineering techniques to such a complex problem.

The method of dynamic slope stability analysis developed by Newmark (1965) and its variations and extensions are widely used in standard engineering practice (see Kan, 2014 for a detailed literature review). The Newmark analogy of a potential sliding mass of a slope as a sliding rigid block on a sloping plane utilises a combination of a critical yield acceleration, \( k_y \), at which the mass would reach its ultimate strength, and the acceleration time history of the slope. Only if the earthquake acceleration becomes larger than the yield acceleration permanent displacement could occur for the mass. The total accumulated displacement is calculated by double integration of the acceleration time history that exceeds the critical yield acceleration.

Makdisi and Seed (1978) modified and improved the original Newmark method. They considered the effect of deformability of embankment dams during earthquake loading and the
variation of the induced acceleration along the dam height and presented a set of practical charts for evaluation of crest settlements of dams. Although this contribution was based on analyses of some real and hypothetical dams with heights in the range of 30 to 60 metres, this method is still widely used in practice. Two different approaches have been derived from the Makdisi and Seed method. The first approach treats the slope as a deformable media and calculates the time history of the sliding block accordingly. This approach is called decoupled approach after Kramer and Smith (1997). The second approach is on direct application of the Makdisi and Seed charts.

**Decoupled approach**

In the decoupled approach, four sliding blocks on the downstream side of the dam are taken into account as shown in Figure 19(a). In order to calculate the yield acceleration for each sliding block, pseudo-static analyses are performed. The shear strength parameters of the rockfill are assumed to be consistent with the material parameters of the advanced constitutive model. With $M_{cs} = 1.757$, the critical state friction angle is calculated as $\phi'_c = 42.8^\circ$. Taking the relative density of the material into account and using the procedure outlined in FLAC manual (2008) for calculation of the parameters for UBCSAND model, the peak friction angle is evaluated to be $45^\circ$. A nominal small cohesion ($c' = 15$ kPa) is assumed for the material to prevent failure of very shallow sliding blocks in the analyses. The pseudo-static analyses are performed using the Spencer method in the commercially available software Slope/W of the GeoStudio package (Geo-Slope International Ltd., 2007). The horizontal yield acceleration which brings a block to the onset of failure is found by a trial and error approach and presented for each blocks in Figure 19(a).

To calculate the induced accelerations for each sliding blocks, a numerical analysis is performed on the dam utilizing a hyperelastic constitutive model for the rockfill material. For
this analysis the elastic bulk and shear moduli for the hyperelastic constitutive model should be estimated. For rockfill materials the elastic shear modulus can be evaluated from the equation proposed by Kokusho and Esashi (1981) for coarse gravels:

\[
G_{\text{max}} = \frac{13000(2.17 - e)^2}{1 + e}(p_0')^{0.55}
\]

(19)

where \( G_{\text{max}} \) is the small strain shear modulus, \( p_0' \) is the mean effective stress and \( e \) is the void ratio. Values of \( G_{\text{max}} \) are calculated for each element using the actual value of \( p_0' \) and \( e \) evaluated after the reservoir impoundment phase. The small strain bulk modulus is also calculated using the theory of elasticity assuming a Poisson’s ratio of 0.3 for rockfill materials. These small strain elastic parameters are subjected to degradation at higher shear strains when the material undergoes cyclic loading. The degradation function is assumed to follow the upper-bound curve proposed by Seed and Idriss (1970) for granular materials.

The average induced acceleration on a block at any time is calculated as the weighted average of the accelerations of all grid points inside the block obtained by the numerical analysis using the hyperelastic model. For example, the average induced acceleration calculated for block #4 is shown in Figure 19(b). Also shown in Figure 19(c) is the time history of the permanent displacement of this sliding block, which is the largest among the displacements of the four considered blocks. The permanent displacements of blocks #1, #2, #3 and #4 are calculated as 0.001 m, 0.003 m, 0.07 m and 0.14 m, respectively. Clearly these values are much less than that observed in the field or predicted by the advanced numerical analysis.

The natural period, \( T_n \), of the dam is calculated as 0.753 sec (see Table 2). The mean period, \( T_m \), of the Wenchuan earthquake is 0.21 sec. Therefore, the tuning ratio \( (T_n / T_m) \) of the system, as defined by Rathje and Bray (1999), will be around 3.6. This value is much higher than 1, the critical threshold beyond which decoupled method can be potentially non-conservative.
according to Rathje and Bray (2000). Rathje and Bray (1999) also concluded that for tuning ratios higher than 4, the decoupled method is most likely non-conservative. This is consistent with the non-conservative results of the decoupled approach in the current study.

**Makdisi and Seed direct approach**

In this approach, the dynamic properties of the dam including the maximum crest acceleration, $\ddot{u}_{\text{max}}$, and the natural period, $T$, should be evaluated using a proper procedures. Makdisi and Seed (1979) proposed a simplified method to evaluate these two parameters, based on shear beam theory with some engineering simplifications, and using a trial and error procedure on the mobilized shear strain and the dynamic response of the dam. This method is used here to calculate the first three natural periods of the dam ($T_1$, $T_2$, and $T_3$ ) and the maximum acceleration at the dam crest. These values are shown in Table 2. Using $k_y$ and $\ddot{u}_{\text{max}}$, the maximum induced acceleration on any potential sliding block, $k_{\text{max}}$, and the displacement of the sliding block, $U$, can be extracted from the charts provided by Makdisi and Seed (1978) for earthquakes of different magnitudes. Zipingpu dam has experienced an earthquake with a magnitude of about 8. Therefore, the upper bound, lower bound and average displacements of any sliding block can be obtained by interpolation between values given for earthquake magnitudes of 7 and 8.25. These displacements, which are in the in the horizontal direction (Makdisi and Seed, 1978), are also presented in Table 2. It can be seen that block #3 shows the highest displacement which ranges from 832 to 1388 mm. The actual horizontal displacement measured after the earthquake is 0.4m which is about half of the lower range estimated by the direct approach.

Further adjustment is required to evaluate the displacement in the direction of sliding along the slope of the dam, to be able to evaluate the magnitude of crest settlement. As mentioned before, the direct method estimates displacements in the horizontal direction. Makdisi and Seed (1978)
found that for inclined failure planes with slope angles of 15° to the horizontal, the displacements are 10%-18% larger than those based on horizontal plane assumption. For Zipingpu dam with an average downstream slope angle of 56° to the horizontal, the displacement along the slope is estimated by applying the calculated horizontal displacement to the centre of gravity of the sliding block #3 and rotating the block along its failure plane. This leads to displacements along the slope of about 15% larger than the calculated horizontal displacements. The vertical displacement or crest settlement is also calculated as 83% of the displacements along the slope or 95% of the horizontal displacements estimated by the direct method. Unlike the decoupled approach, the direct approach can better estimate the settlement of the dam crest. The average settlement estimated by this approach is 0.91 m which is close to the predicted displacement by the advanced numerical approach and the measured settlement of the dam. However, the upper bound value of the crest settlement estimated by the direct approach is 1.15 m which is larger than the observed settlement of the dam.

**Summary and conclusion**

As of 2015, Zipingpu dam is the highest concrete faced rockfill dam, with a height over 150 m, which has been subjected to a very strong earthquake. Although the dam is a recently constructed modern compacted rockfill, it suffered large deformations. This unique event has provided a rare opportunity to verify the seismic design methods and the numerical tools available to simulate the performance and the safety of large modern rockfill dams.

In the current study presented here, the bounding surface plasticity model proposed by Kan and Taiebat (2014a) was used to investigate the static and dynamic behaviour of Zipingpu dam. The main feature of this model is the translating critical state line and isotropic compression line through which particle breakage of rockfill materials can be properly captured. The predictive capabilities of the model were first illustrated through simulation of a series of monotonic and
cyclic triaxial tests performed on the rockfill of the dam. The numerical tool was then employed to simulate the behaviour of the dam under a variety of loading conditions, including staged construction, reservoir impoundment, and under the effects of the Wenchuan earthquake. The results of the simulations were discussed and compared with those measured in the field. It was shown that although the construction process was complex, due to non-symmetric internal zoning of the dam, the predicted settlements at different monitoring points and different stages of construction were consistent with the field measurements, indicating that the proposed numerical procedure can be comfortably used to analyse the deformation of rockfill dams under static loading. The impoundment of the dam was also numerically simulated before the effects of the Wenchuan earthquake was applied to the model. The pattern and the values of displacements of the dam due to earthquake loading were satisfactorily reproduced using the advanced numerical procedure. The numerical tool predicted the deformation pattern, the crest acceleration and displacements very closely and estimated the excess particle breakage of the rockfill materials during the course of earthquake loading. The predicted settlement at the end of earthquake loading is slightly less than the measured value. This difference, although acceptable in any numerical analysis of complex geotechnical problems, could be due to the adoption and scaling of the earthquake motion, due to the effects of interaction of reservoir water with the dam which was ignored in the current analysis, and due to the proposed constitutive model and numerical procedures as well.

The reliability of two commonly used engineering techniques in predicting the deformation of the dam was also discussed. These techniques are based on the modified Newmark method for deformation analysis of slopes, as proposed by Makdisi and Seed (1978). Both decoupled approach and direct approach were taken into account. It was shown that the decoupled approach gives a crest displacement much less than those observed in the field or predicted by the numerical tool. This can be attributed to the nature of the input motion which has a very high
frequency, rendering a tuning ratio much higher than 1.0; a critical threshold for decoupled approach proposed by Rathje and Bray (2000). Unlike the decoupled approach, the direct approach of Makdisi and Seed (1978) provided a better range for the final settlement of the dam.

The results of this study show that the advanced constitutive model proposed here can properly simulate the behaviour of rockfill materials under static, cyclic, and dynamic loadings conditions using a single set of material parameters. The performance of this constitutive model can further be improved in order to account for complex cyclic loading and seismic problems more efficiently while maintaining the simplicity of the model. The time dependent creep deformation of rockfill materials is another important feature which should be addressed appropriately in any future work.
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$v = v - 1.0$
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<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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Table 2: Calculation of crest deformation based on Makdisi and Seed (1978)

direct approach

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<th>3</th>
<th>4</th>
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</table>

| $U/k_{max}gT_i$  |        | Upper bound | 0.0048 | 0.0551 | 0.1030 | 0.0646 |
| $U$ (mm)        |        |            | 13.5   | 268    | 702    | 490    |

$M = 7.5$
(from charts)

| $U/k_{max}gT_i$  |        | Average    | 0.0051 | 0.0400 | 0.0766 | 0.0472 |
| $U$ (mm)        |        |            | 14     | 195    | 521    | 358    |

| $U/k_{max}gT_i$  |        | Lower bound | 0.0023 | 0.0295 | 0.0567 | 0.0348 |
| $U$ (mm)        |        |            | 6.5    | 144    | 387    | 264    |

$M = 8.25$
(from charts)

| $U/k_{max}gT_i$  |        | Upper bound | 0.0128 | 0.1442 | 0.2544 | 0.1673 |
| $U$ (mm)        |        |            | 36     | 702    | 1732   | 1271   |

| $U/k_{max}gT_i$  |        | Average    | 0.0076 | 0.0990 | 0.1825 | 0.1189 |
| $U$ (mm)        |        |            | 21     | 482    | 1242   | 903    |

| $U/k_{max}gT_i$  |        | Lower bound | 0.0037 | 0.0793 | 0.1550 | 0.0962 |
| $U$ (mm)        |        |            | 10     | 386    | 1055   | 731    |

$M = 8.0$
(interpolation)

| $U$ (mm)        |        | Upper bound | 284    | 557    | 1388   | 1011   |

| $U$ (mm)        |        | Average     | 19     | 386    | 1002   | 721    |

| $U$ (mm)        |        | Lower bound | 9      | 305    | 832    | 575    |
References


