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A refined general summation law for VU emission assessment in radial networks

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Keywords

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A Refined General Summation Law for VU Emission Assessment in Radial Networks

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Abstract

Voltage unbalance (VU) management of power systems requires the development of well researched engineering practices to maintain acceptable VU levels while utilising the total VU absorption capacity of the power system. In this regard, IEC/TR 61000-3-13:2008 prescribes a VU emission allocation methodology based on a stochastic approach which uses a general summation law in order to aggregate numerous sources of unbalance to take into account their random variations. On the other hand, recent deterministic studies on VU emission assessment at the post-connection stage of unbalanced installations present a complex VU factor based approach to determine constituent components of the post-connection VU emission at a point of evaluation. The primary objective of the work presented in this paper is to develop statistical approaches for compliance assessment using the outcomes of the deterministic methodologies on VU emission assessment, thus refining the existing general summation law. A revised general summation law is established, introducing weighting factors to evaluate the influences made by different sources of unbalance separately, in order to assess VU emission in radial networks.

Keywords: power quality, voltage unbalance, voltage unbalance emission assessment, general summation law, network asymmetry, load asymmetry

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1. Introduction

Voltage unbalance is defined as a condition in multi-phase electric power systems in which the magnitudes of the fundamental phase voltages are not equal and/or the associated phase angle separations defer from 120 degrees [1]. Thus, the VU factor is a complex number which carries phase angle information, although the common practice is to use the magnitude only [2],[3]. The highly variable characteristics of the power system makes VU levels vary randomly in time. Accordingly, the impact made by fluctuations of VU can be assessed using vector/phasor oriented deterministic approaches [4] and/or stochastic methods [5].

VU management in power systems essentially involves two major aspects; VU emission allocation at the pre-connection stage and compliance assessment at the post-connection stage of installations. The IEC Technical Report IEC/TR 61000-3-13:2008 [1] provides guiding principles to determine individual VU emission allocation limits for unbalanced installations in order to limit the total VU emission (injection from total individual customer installations and system inherent sources) to a level at or below the set planning levels when the system is fully utilised to its designed capacity. Although the resultant VU emission at a given instant can be defined as the vector summation of unbalanced voltage components caused by individual sources at the point of evaluation (POE) [1], reference values which govern the limits of VU (planning levels, compatibility levels and individual emission levels) cannot be expressed in terms of a phasor representation as phase angle can vary randomly. Instead, statistical values are used in the emission allocation methodology [1], adopting a general summation law in aggregating emission levels caused by multiple sources taking into account the effect of the phasor behaviour of individual VU sources. Some preliminary work on VU emission assessment in [6] follows the same stochastic approach to evaluate individual emission contributions made by load and line asymmetries at the post-connection stage of unbalanced installations. However, the methodologies described are not comprehensive for addressing the effects of simultaneously existing unbalance sources.

The recent deterministic studies on VU emission assessment work in [7] and [8] present a novel, complex VU factor based approach to determine the constituent components of post-connection VU emission level at a POE covering both radial and interconnected networks. These new methodologies provide a generalised classification on individual emission contributors at the POE which can be evaluated as decoupled quantities using pre- and post-connection voltage/current measurements and known network parameters. However, it is necessary to have continuous observability of three-phase voltages at all buses to accurately assess the voltage unbalance levels in the network. Since the monitors are installed at a reasonably small number of buses in the network due associated costs, the shortage of monitoring data presents a significant challenge in estimating the unbalance of the entire network [9].

Development of proper mechanisms to assess the post-connection VU emission levels employing statistical approaches can be used to enhance the existing IEC work on VU management as such methodologies will be in alignment with the VU emission allocation processes which also follow statistical principles [10]. Thus, development of statistical approaches based on the application of a general summation law to evaluate post-connection VU emission and its constituent components is the topic of interest in this paper. In this regard, already established deterministic methodologies on VU emission assessment (presented in [7] in relation to radial networks) are used to develop and validate such statistical approaches.

The work presented in this paper is organised as follows: Section 2 summarises the stochastic approach of VU emission allocation and assessment used in the IEC Technical Report [1] and key outcomes of deterministic methodologies on VU emission assessment in radial networks [7]. An analysis of VU emission assessment outcomes adopting the present statistical approach (application of the general summation law in relation to the determination of individual emission contributions) and hence a critical discussion on the validity of the existing general summation law for VU are given in Section 3. Establishment of the revised general summation law to assess the overall impact of randomly varing disturbances in radial networks are presented in Section 4. Finally, Section 5 presents conclusions drawn from the study.

2. VU Emission Allocation and Assessment

2.1. Stochastic Approach of VU Emission Allocation and Assessment Used in the IEC Technical Report [1]

As per the IEC Technical Report [1], the resultant VU emission at a POE is the vector summation of negative sequence[1] voltage components which arise as a result of the interaction of various sources of unbalance. The dynamic nature of the power system causes VU to vary randomly in time. Thus, representation of all randomly varing emission vectors in time as stochastic quantities using the general summation law [1] avoids the need for phase angle information:

$$u = \sqrt[\alpha]{\sum_{i=1}^{n} (u_i)^{\alpha}} \tag{1}$$

where;
u - magnitude of the resulting VUF for the considered aggregation of unbalance sources (probabilistic value);
u_i - magnitude of an individual VU emission level to be combined (95% or 99% probabilistic value); and
α - summation law exponent.
The exponent α mainly depends upon three factors:

- the chosen value of probability for the actual voltage unbalance level not to exceed the calculated value;
- the degree to which the combined individual unbalanced voltages vary randomly in magnitude and phase; and
- the number of random variations considered (either the number of summated sources or the variation in time).

The IEC/TR 61000-3-13:2008 gives an indicative value of $\alpha = 1.4$ in the absence of specific information considering a 95% non-exceeding probability level for VU emission coordination and based on the fact that the operation of most unbalanced installations are unlikely to produce simultaneous or in-phase emissions in practice. This indicative value is not based on measurement results, but has been proposed based on a uniform distribution of random vectors with a random phase variation of 360 degrees, and a magnitude range of 0.1 to 1 p.u. [1].

Use of the general summation law in the VU emission allocation process can be summarised as follows: total VU emission allowance for a particular system ($G_{\mathrm{uMV+LV}}$) is evaluated by incorporating planning levels (L_{uMV}) and VU transfer factors (T_{uUM}) as shown in (2). Total VU emission allowance is then apportioned using the $\mathrm{k_{uE}}$ factor[2] [11], [12] to account for network asymmetries to satisfy the relationship of;

(total VU emission allowance)$^{\alpha}$ = (VU allocation to unbalanced loads)$^{\alpha}$ +
(VU allocation to network asymmetries)$^{\alpha}$.

Then, the individual customer emission limits (E_{ui}) are derived based on an approach which considers the ratio between the agreed power (S_i for the installation i) and the total supply capacity of the system (S_t) as shown in (3).

$$G_{\mathrm{uMV+LV}} = \sqrt[\alpha]{L_{\mathrm{uMV}}^{\alpha} - (T_{\mathrm{uUM}} L_{\mathrm{uUS}})^{\alpha}} \tag{2}$$

$$E_{\mathrm{ui}} = \sqrt[\alpha]{\mathrm{k_{uE}}}\; G_{\mathrm{uMV+LV}} \sqrt[\alpha]{\frac{S_i}{S_{\mathrm{t}}}} \tag{3}$$

[1]IEC/TR 6100-3-13:2008 considers that the zero sequence unbalance can be controlled through system design and maintenance.

[2]$\mathrm{k_{uE}}$ factor represents a fraction of global emission allowance that can actually be allocated to unbalanced installations

Post-connection VU emission assessment guidelines given in [6] follow a similar statistical approach (use of summation law with pre- and post-connection VU measurements in the absence of phasor angle information) to determine the VU emission that arises at the POE due to the connection of load ($U_{2,i}$) as shown in (4). However, further stochastic analysis, i.e. a systematic approach for apportioning the total VU emission between load and line asymmetries or the identification of constituent components of total VU emission at the post-connection stage, is not available for managing VU in the IEC context.

$$|U_{2,i}| = (|U_{2,\mathrm{post-connection}}|^{\alpha} - |U_{2,\mathrm{pre-connection}}|^{\alpha})^{\frac{1}{\alpha}} \tag{4}$$

2.2. Post-connection VU Emission Assessment in Radial Power Systems : Key Outcomes of [7]

VU emission assessment techniques given in [7] present a generalised approach for identifying different sources of unbalance at the POE while separating the customer and network responsibility on VU emission. Accordingly, the resultant post-connection VU emission measurement at the POE was decomposed into three factors by identifying major VU emission contributors as follows:

- load asymmetry (load connected at the busbar under assessment, i.e. POE);
- line asymmetry (untransposed line connected between upstream source and the POE); and
- background unbalance caused by upstream unbalanced voltage source.

Hence, the constituent components of the resultant post-connection VU emission at the POE were established as asymmetrical load contribution, asymmetrical line contribution and upstream source contribution which are given in (5) in terms of the complex VU factors[3] for radial networks.

$$VUF_{POE} = VUF_{POE}^{\mathrm{load}} + VUF_{POE}^{\mathrm{line}} + VUF_{POE}^{\mathrm{source}} \tag{5}$$

VUF_{POE} is the resultant (total) VU factor at the POE; $VUF_{POE}^{\mathrm{load}}$ is the VU emission contribution made by load asymmetry; $VUF_{POE}^{\mathrm{line}}$ is the VU emission contribution made by line asymmetry and $VUF_{POE}^{\mathrm{source}}$ is the VU emission contribution made by the upstream source.

Mathematical formulations presented in [7] evaluate these three components as decoupled contributions in such a way that they reflect their own asymmetries through the decoupled formulation. These individual contributions are given in Table 1 for different load types considered. The reader should note that the mathematical symbols used to represent constituent components of resultant VU emission at the POE in this paper have been subjected to slight modifications compared to the original formulations presented in [7].

Referring to Table 1;

CUF - current unbalance factor, CUF_{Pload} - current unbalance factor of the passive load.
VUF_{source} - VUF (voltage unbalance factor) of the upstream busbar.
$Z_{xy,t}$ - mutual impedance of the transmission line in sequence domain. x and y can be replaced with 1 and 2 which stand for positive and negative sequence respectively.
$V_{\mathrm{reg\text{-}line}}$ - voltage regulation of the line defined as the ratio of positive sequence voltage drop in the line to positive sequence voltage at the POE.
$Z_{x,\mathrm{m}}$ - sequence impedance of the motor (x=1 or 2).

[3]VU factor (VUF) is defined as the ratio of negative sequence voltage to positive sequence voltage.

Table 1: VU emission assessment outcomes: Radial networks[7]

Row identifier	Individual contributor	Mathematical expression for the contribution made
Passive loads		
1	Load asymmetry VUF_{POE}^{load}	$(VUF_{POE} - CUF)\,\frac{V_{\text{reg-line}}}{(1+V_{\text{reg-line}})}$
2	Line asymmetry VUF_{POE}^{line}	$-\frac{Z_{21,t}}{Z_{11,t}}\frac{V_{\text{reg-line}}}{(1+V_{\text{reg-line}})}$
3	Source asymmetry $VUF_{POE}^{\text{source}}$	VUF_{source}
Induction motor loads		
4	Line asymmetry VUF_{POE}^{line}	$-\left(\frac{Z_{2,m}}{Z_{1,m}}\right)\left(\frac{Z_{21,t}}{Z_{22,t}+Z_{2,m}}\right)$
5	Source asymmetry $VUF_{POE}^{\text{source}}$	$\left(\frac{Z_{2,m}}{Z_{1,m}}\right)\left(\frac{Z_{1,m}+Z_{11,t}}{Z_{2,m}+Z_{22,t}}\right)VUF_{\text{source}}$
Mixed loads		
6	Load asymmetry VUF_{POE}^{load}	$(VUF_{POE} - CUF_{\text{Pload}})\left(\frac{Z_{1,m}Z_{2,m}\left(V_{\text{reg-line}}-\frac{Z_{22,t}}{Z_{1,m}}\right)}{Z_{1,m}Z_{2,m}(1+V_{\text{reg-line}})+Z_{22,t}(Z_{1,m}-Z_{2,m})}\right)$
7	Line asymmetry VUF_{POE}^{line}	$-\frac{Z_{21,t}}{Z_{11,t}}\left(\frac{Z_{1,m}Z_{2,m}V_{\text{reg-line}}}{Z_{1,m}Z_{2,m}(1+V_{\text{reg-line}})+Z_{22,t}(Z_{1,m}-Z_{2,m})}\right)$
8	Source asymmetry $VUF_{POE}^{\text{source}}$	$\left(\frac{Z_{1,m}Z_{2,m}(1+V_{\text{reg-line}})}{Z_{1,m}Z_{2,m}(1+V_{\text{reg-line}})+Z_{22,t}(Z_{1,m}-Z_{2,m})}\right)VUF_{\text{source}}$

3. Development of Statistical Approaches for VU Emission Assessment

3.1. Statistical Assessment of VU Emission in Radial Networks

As a first attempt, the complex VUF based formulation given in (5) ($VUF_{POE} = VUF^{\text{load}}_{POE} + VUF^{\text{line}}_{POE} + VUF^{\text{source}}_{POE}$) which represents the classification of constituent components of post-connection VU emission at the POE, is modified in order to incorporate the use of the general summation law as shown in (6). This enables the comparison of VU emission outcomes obtained using deterministic methodologies against statistical approaches.

$$VUF_{POE,\text{stat}} = \left(|VUF^{\text{load}}_{POE}|^{\alpha} + |VUF^{\text{line}}_{POE}|^{\alpha} + |VUF^{\text{source}}_{POE}|^{\alpha}\right)^{\frac{1}{\alpha}} \tag{6}$$

$VUF_{POE,\text{stat}}$ represents the statistical value of the resultant VU emission at the POE.

The application of a general summation law is justified in [1] under conditions where the VU randomly changes over time or where a large number of unbalanced installations are considered. The statistical analysis presented in this paper is based on the consideration of randomly varying unbalance which is cused by random load changes. The deterministic methodologies on VU emission assessment given in [7] utilise the snap-shot based voltage/current phasor measurements. A series of such measurements which are associated with random variations of unbalance caused by random load changes is used to evaluate individual emission contributions and the resultant VU factor at the POE. Such data is generated by running an unbalanced load flow program in MATLAB for consecutive load changes which follow normal and uniform distribution.

A simple radial system comprising a balanced upstream voltage source, untransposed transmission line and a randomly changing unbalanced load (constant power type) is considered for the investigations on statistical assessment of VU emission. The following aspects are to be noted in relation to the analysis procedure.

- Upstream voltage source in the radial network is considered to be balanced. Thus; $VUF^{\text{source}}_{POE} = 0$, resulting $VUF_{POE,\text{stat}} = \left(|VUF^{\text{load}}_{POE}|^{\alpha} + |VUF^{\text{line}}_{POE}|^{\alpha}\right)^{\frac{1}{\alpha}}$.
- Resultant VUF at the POE (VUF_{POE}) and its constituent components (VUF^{load}_{POE} and VUF^{line}_{POE}) were evaluated for each snap shot based measurement using the deterministic formulations given in Table 1 (using equations in 1$^{\text{st}}$ and 2$^{\text{nd}}$ rows).
- Magnitudes of all VU factor components (VUF_{POE}, VUF^{load}_{POE} and VUF^{line}_{POE}) are fitted to relevant probabilistic curves based on the load distribution (either normal distribution or uniform distribution).
- 95% probability levels of VUF^{load}_{POE} and VUF^{line}_{POE} which were obtained using cumulative probability distribution functions are used to evaluate the resultant VUF at the POE (application of (6)) and to compare with the 95% probability level of VUF_{POE} which gives the probabilistic value of the outcomes from the deterministic approach.

The following sections illustrate the above statistical analysis procedure in relation to different unbalanced load configurations and untransposed lines which are used in different case studies as listed below.

- Unbalanced loads:
  - Load 1: 10 MVA - 100 MVA constant power load; magnitude of the unbalanced load varies following a normal distribution as shown in Table 2 while power factors of three phases are fixed - 0.95, 0.85 and 0.9 in phases a, b and c respectively.

Table 2: Load 1: Normally distributed 10-100 MVA, constant power load

Magnitude	Phase a	Phase b	Phase c
mean (S) pu	0.7	0.85	0.8
standard deviation (σ) pu	0.1	0.1	0.1

Table 3: Load 2: Normally distributed 10-100 MVA, constant power load

Power factor	Phase a	Phase b	Phase c
mean (S)	0.95	0.9	0.85
standard deviation (σ)	0.01	0.05	0.05

- Load 2: 10 MVA - 100 MVA constant power load; magnitude of the unbalanced load is fixed while power factors of three phases vary following a normal distribution as given in Table 3
- Load 3: 10 MVA - 100 MVA constant power load; magnitude of the unbalanced load varies following a uniform distribution as given in Table 4 while power factors of three phases are fixed at 0.9 on phases a, b and c.

Table 4: Load 3: Uniformly distributed 10-100 MVA, constant power load

Magnitude	Phase a	Phase b	Phase c
minimum pu	0.8	0.7	0.7
maximum pu	0.85	0.85	0.85

- Untransposed lines:
  - Line I - 12.47 kV untransposed line:
    Calculated line impedance matrix ($[Z_{abc}]$ ohm/km)

$$\begin{pmatrix} 0.2494 + j0.8748 & 0.0592 + j0.4985 & 0.0592 + j0.4462 \\ 0.0592 + j0.4985 & 0.2494 + j0.8748 & 0.0592 + j0.4985 \\ 0.0592 + j0.4462 & 0.0592 + j0.4985 & 0.2494 + j0.8748 \end{pmatrix}$$

  - Line II - 66 kV untransposed line:
    Calculated line impedance matrix ($[Z_{abc}]$ ohm/km)

$$\begin{pmatrix} 0.0036 + j0.0168 & 0.0011 + j0.0089 & 0.0011 + j0.0088 \\ 0.0011 + j0.0089 & 0.0036 + j0.0168 & 0.0011 + j0.0092 \\ 0.0011 + j0.0088 & 0.0011 + j0.0092 & 0.0036 + j0.0168 \end{pmatrix}$$

3.1.1. Normally Distributed Loads Connected at the POE

Case I: 12.47 kV radial network with Line I (12.47 kV line) and Load 1

A 12.47 kV radial power system comprising a 3 km long line and a 10 MVA load (Load 1 configuration) is simulated using an unbalanced load flow program in MATLAB and a series of VU emission assessment data were generated (1000 samples) to represent randomly varying unbalance levels. The total VUF at the POE (VUF_{POE}) and individual contributions made by load and line asymmetries[4] (VUF_{POE}^{load} and VUF_{POE}^{line}) were evaluated using the proposed emission assessment formulation (6).

A phasor representation of all individual emission outcomes is shown in Fig. 1 as polar plots. The resultant VU emission vectors (VUF_{POE}) are seen to be scattered over the entire 360 degree polar plane, following the same behaviour exhibited by VU emission vectors that arise due to load asymmetries VUF_{POE}^{load}, although the VUF_{POE}^{line} phasors are restricted to a less scattered cluster (as expected since only a fixed line configuration is considered).

As shown in Fig. 2, the frequency distribution of the magnitudes of different VU emission components can be approximated to a normal distribution and hence the 95% probability level of different emission components can be evaluated using cumulative probability density functions (Fig. 3) as follows:

[4] $VUF_{POE}^{\text{source}} = 0$ as the upstream source is balanced

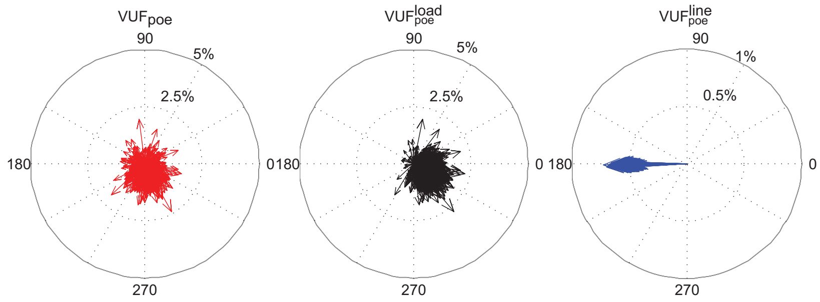

Figure 1: Phasor representation of VU emission phasors: Case I

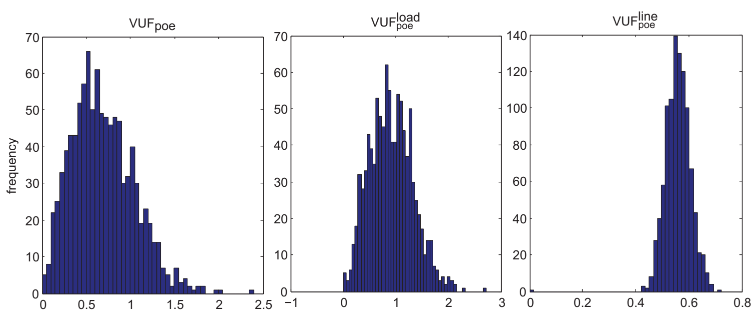

Figure 2: Frequency distribution of individual VU emission levels: Case I

- 95% value of $|VUF_{POE}| = 1.31$
- 95% value of $|VUF^{\text{load}}_{POE}| = 1.64$
- 95% value of $|VUF^{\text{line}}_{POE}| = 0.64$
- 95% value of $VUF_{POE,\text{stat}} = \left(|VUF^{\text{load}}_{POE}|^{\alpha} + |VUF^{\text{line}}_{POE}|^{\alpha}\right)^{\frac{1}{\alpha}} = 1.94$, with $\alpha = 1.4$

According to these values, the summation of individual emission contributions using the application of general summation law to obtain the resultant VUF ($VUF_{POE,\text{stat}}$) is observed to have some discrepancy over the 95% probability value of VUF_{POE} obtained employing the deterministic approach. Further analysis is carried out for the same radial network by varying the length of the asymmetrical line. Table 5 summarises the statistical outcomes of this VU emission assessment.

Table 5: Probabilistic outcomes of VU emission assessment: Case I

line length (km)	$\lvert VUF^{\text{load}}_{POE}\rvert$ 95% value	$\lvert VUF^{\text{line}}_{POE}\rvert$ 95% value	$\lvert VUF_{POE}\rvert$ 95% value	$VUF_{POE,\text{stat}}$ 95% value
1	0.41	0.20	0.34	0.52
2	0.94	0.42	0.74	1.15
3	1.64	0.64	1.31	1.94
4	2.55	0.88	2.04	2.95
5	3.82	1.13	3.14	4.34

The contributions made by both load and line asymmetries tend to increase with the line length, resulting in a higher VUF at the POE, as expected. However, there are significant discrepancies between the resultant VUF values at the POE obtained using the two approaches ($|VUF_{POE}|$ and $VUF_{POE,\text{stat}}$).

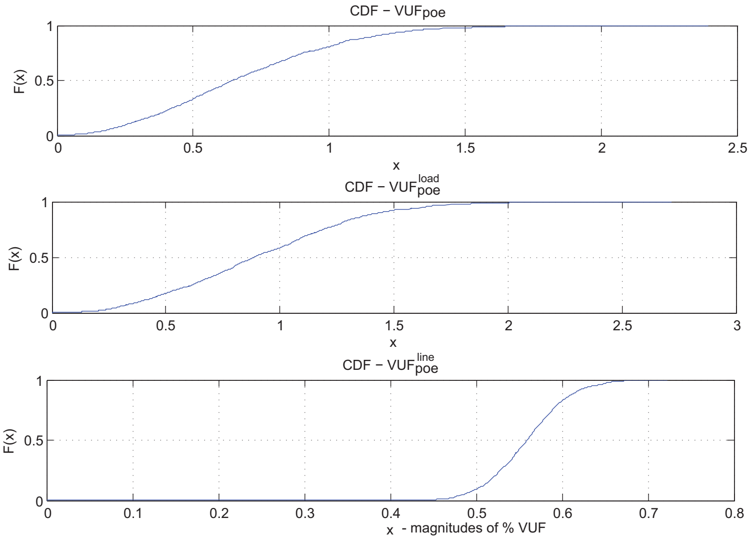

Figure 3: Cumulative probability distribution functions of VU emission levels: Case I

Case II: 66 kV radial network with Line II (66 kV line) and Load 1

A similar analysis was carried out for a radial system comprising a 66 kV line and a normally distributed 100 MVA unbalanced load (Load 1 configuration) for different line lengths. The resultant VU emission outcomes in terms of the phasor distributions are illustrated as polar plots in Fig 4. The probabilistic representation in terms of frequency distribution and cumulative probability density curves of VUF components in relation to the 15 km long line, are given in Figs. 5 and 6 respectively. Statistical outcomes (95% probability values of VUF_{POE}^{load}, VUF_{POE}^{line}, VUF_{POE} and $VUF_{POE,\text{stat}}$) are summarised in Table 6.

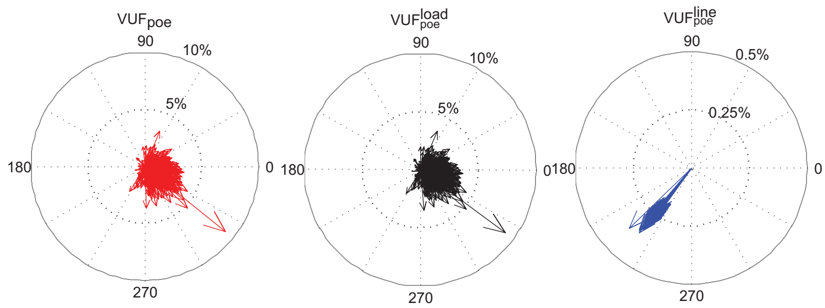

Figure 4: Phasor representation of VU emission phasors: Case II

In contrast to the 12.47 kV radial network, the resultant VUFs evaluated using a deterministic approach ($|VUF_{POE}|$) and the results from the application of the general summation law ($VUF_{POE,\text{stat}} = \left(|VUF_{POE}^{\text{load}}|^{\alpha} + |VUF_{POE}^{\text{line}}|^{\alpha}\right)^{\frac{1}{\alpha}}$) are in close agreement (maximum discrepancy is around 5.5%) for all cases, thus supporting the existing general summation law ($\alpha = 1.4$) in relation to the VU emission.

Case III: 12.47 kV radial network with Line I and Load 2

A 12.47 kV radial network containing the unbalanced load given by the Load 2 configuration is investigated to obtain the statistical outcomes of VU emission at the POE. Relevant 95% probabilistic emission levels are summarised in Table 7 for different line lengths. For the 3 km line, the polar plots in Fig. 7 show the distribution of VU emission phasors.

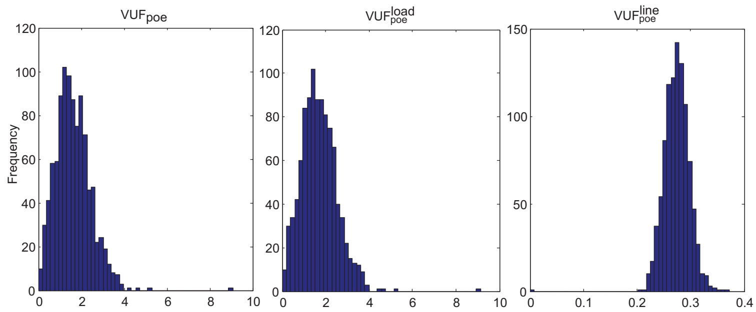

Figure 5: Frequency distribution of individual VU emission levels: Case II

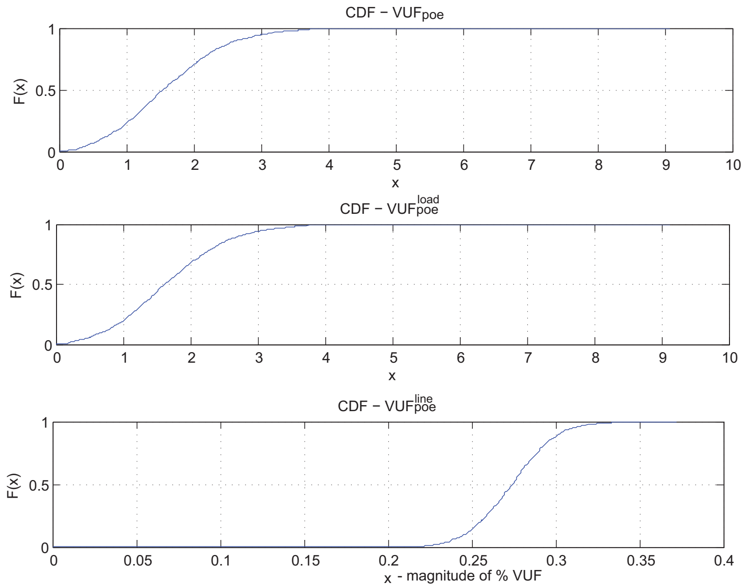

Figure 6: Cumulative probability distribution functions of VU emission levels: Case II

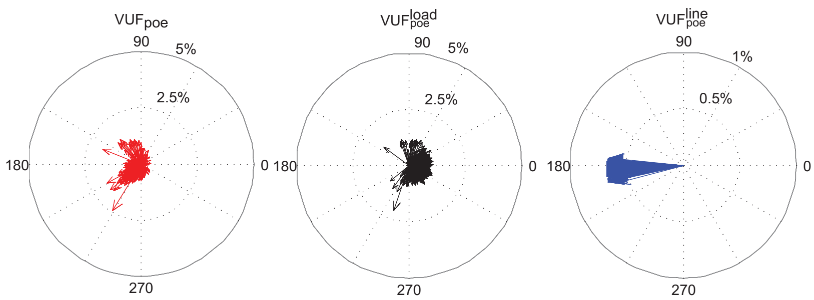

Figure 7: Phasor representation of VU emission phasors: Case III

Similar to Case I, an application of the general summation law is not conservative since there is a significant difference (50% - 60% discrepancy) between the total VUF values obtained from the two approaches ($|VUF_{POE}|$ and $VUF_{POE,\text{stat}}$).

Table 6: Probabilistic outcomes of VU emission assessment: Case II

line length km	$\lvert VUF_{POE}^{\text{load}} \rvert$ 95% value	$\lvert VUF_{POE}^{\text{line}} \rvert$ 95% value	$\lvert VUF_{POE} \rvert$ 95% value	$VUF_{POE,\text{stat}}$ 95% value
5	0.67	0.10	0.66	0.69
10	1.60	0.20	1.57	1.66
15	3.02	0.31	2.95	3.1
20	6.67	0.45	6.56	6.78

Table 7: Probabilistic outcomes of VU emission assessment: Case III

line length km	$\lvert VUF_{POE}^{\text{load}} \rvert$ 95% value	$\lvert VUF_{POE}^{\text{line}} \rvert$ 95% value	$\lvert VUF_{POE} \rvert$ 95% value	$VUF_{POE,\text{stat}}$ 95% value
1	0.35	0.24	0.29	0.48
2	0.68	0.47	0.58	0.95
3	0.98	0.71	0.9	1.40
4	1.27	0.94	1.18	1.82
5	1.56	1.18	1.52	2.25

3.1.2. Uniformly Distributed Loads Connected at the POE

Similar to the analysis related to the normally distributed loads connected at the POE, both 12.47 kV and 66 kV radial networks, containing uniformly distributed constant power type loads connected at the POE, are investigated in the following case studies. The corresponding statistical outcomes of VU emission levels are summarised in Tables 8 and 9. Polar plots which show the distribution of emission phasors for a selected line length are given in Figs. 8 and 9 for Case IV and Case V respectively.

<u>Case IV: 12.47 kV radial network with Line I and Load 3</u>

The statistical outcomes for VU emission levels are summarised in Table 8. The polar plot, which shows the distribution of emission phasors for a selected line length, is given in Fig. 8.

Table 8: Probabilistic outcomes of VU emission assessment: Case IV

line length km	$\lvert VUF_{POE}^{\text{load}} \rvert$ 95% value	$\lvert VUF_{POE}^{\text{line}} \rvert$ 95% value	$\lvert VUF_{POE} \rvert$ 95% value	$VUF_{POE,\text{stat}}$ 95% value
1	0.14	0.18	0.29	0.27
2	0.34	0.38	0.65	0.59
3	0.56	0.58	1.02	0.93
4	0.88	0.78	1.57	1.37
5	1.32	1.01	2.20	1.97

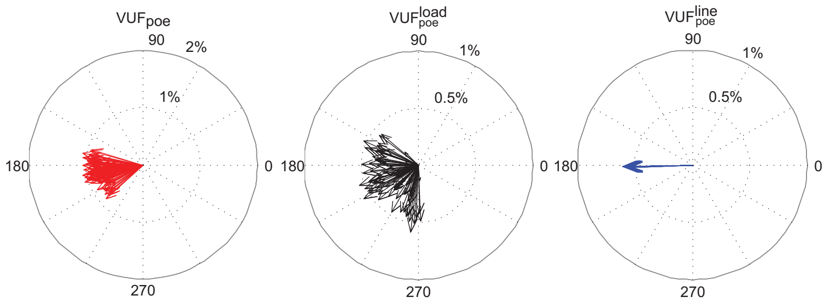

Figure 8: Phasor representation of all VU emission vectors: Case IV (3 km line)

Case V: 66 kV radial network with Line II and Load 3

The statistical outcomes for VU emission levels are summarised in Table 9. The polar plot, which shows the distribution of emission phasors for a selected line length, is given in Fig. 9.

Table 9: Probabilistic outcomes of VU emission assessment: Case V

line length km	$\|VUF_{POE}^{\text{load}}\|$ 95% value	$\|VUF_{POE}^{\text{line}}\|$ 95% value	$\|VUF_{POE}\|$ 95% value	$VUF_{POE,\text{stat}}$ 95% value
5	0.21	0.10	0.27	0.25
10	0.49	0.18	0.61	0.57
15	0.9	0.28	1.12	1.02
20	1.44	0.38	1.71	1.60
25	2.98	0.51	3.40	3.16

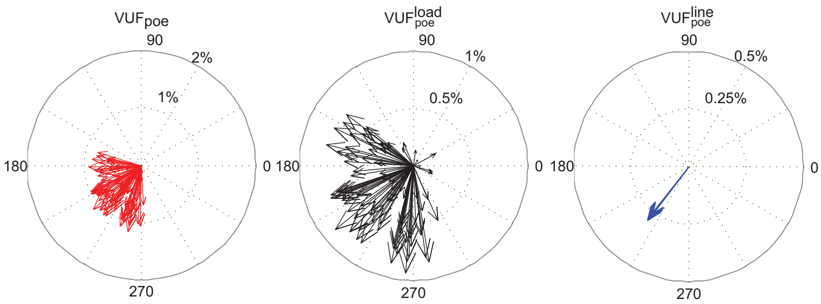

Figure 9: Phasor representation of VU emission phasors: Case V (15 km line)

In the case of uniformly distributed loads connected at the POE, the resultant VU factors evaluated using both approaches are seen to be similar in magnitude for both networks (Cases IV and V) thus supporting the validity of the existing general summation law.

3.2. Discussion

Statistical analysis of VU emission assessment based on deterministic approaches in relation to different radial network configurations as discussed in the preceding case studies has shown that the application of the general summation law for the aggregation of individual emission contributions is not in agreement in a consistent manner. Thus, the identification of the causes of the different outcomes in the presented case studies is required in order to revise the existing general summation law.

A complex VUF based formulation evaluates the resultant VUF at the POE as the phasor summation of individual emission contributions made by the unbalanced load and the line asymmetry in the presence of a balanced upstream voltage source. The resultant VUF at the POE will exhibit an increase or a decrease compared to the largest individual vector depending on the relative phasor orientations of individual emission vectors[5] [11]. However, this simple theory deviates when a large sample of randomly scattered emission contributions (phasors) are considered. In such a case, the statistically significant VU emission level resulting from randomly scattered emission phasors is determined by the most dominant emission vectors and their relative phasor orientations. This aspect has to be addressed when a revised general summation law is developed to account for randomly varying unbalance sources. Since, the application of the existing general summation law disagrees in certain cases as discussed in relation to the presented case studies (Cases I and III), it is vital to investigate the phasor representation of individual emission contributions in terms of polar plots.

[5] That is, if the phase angle between VUF_{POE}^{load} and VUF_{POE}^{line} is greater than 90 degrees, their vector summation (VUF_{POE}) leads to a value less than the value of the larger contributor and the inverse is true for the situations where the phase angle between VUF_{POE}^{load} and VUF_{POE}^{line} is smaller than 90 degrees.

3.3. Analysis of Polar Plots to Identify the Impact made by Phasor Distribution on Resultant VUF

As evident from the results presented in Section 3.1, in particular Cases II, IV and V, statistically processed VU emission outcomes based on the deterministic approach are seen to be in agreement with the emission outcomes obtained using the general summation law adopted in the IEC approach.

In Case II (66 kV network with a normally distributed load at the POE), the relevant polar plots (Fig. 4) show that the magnitudes of the VUF_{POE}^{line} phasors are comparatively small ($< 0.5\%$) compared to these of VUF_{POE}^{load} phasors (in the range of 2% - 4%) and are restricted to a narrow cluster in the polar plot. Further, VUF_{POE}^{load} phasors are scattered over the 360 degree plane while demonstrating their dominant emission contribution, resulting the phasor distribution of VUF_{POE} following the same pattern of VUF_{POE}^{load}.

In Case V (66 kV network with uniformly distributed load), the VUF_{POE}^{line} phasors are almost in phase, resulting in a very narrow cluster (Fig. 9), but the contributions made by VUF_{POE}^{line} phasors to VUF_{POE} are comparatively significant as the contributions made by VUF_{POE}^{load} phasors are in the range of 0.5% in magnitude approximately. In this case, VUF_{POE}^{load} phasors are mostly concentrated in a 150 degree (approximately) sector and the phasor distribution of the resultant VUF (VUF_{POE}) is further clustered in a smaller sector of 120 degrees due to the influence made by the phasors corresponding to the line contributions (VUF_{POE}^{line}) which are more or less uni-directional. Further, if two individual fictitious phasors are derived to represent all individual VUF_{POE}^{load} phasors and VUF_{POE}^{line} phasors, the phase angle between these two vectors will be less than 90 degrees, thus leading to an increase of the resultant VUF[6] in comparison to the individual contributions at the POE.

A similar observation can be made in relation to Case IV (12.47 kV network with uniformly distributed load) where the summation law is conservative for assessing resultant VU emission levels at the POE. Both emission contributors (VUF_{POE}^{load} and VUF_{POE}^{line}) are equally significant and clustered in small segments close to each other (Fig. 8) resulting an increased emission levels at the POE.

In Cases I and III (12.47 kV radial network with normally distributed loads) where there are differences between the VU emission outcomes at the POE obtained using deterministic and statistical approaches, the phasor distribution of VUF_{POE}^{line} is limited to narrow clusters (Fig. 1 and Fig. 7), similar to Cases II, IV and V. However, the overall impact made by VUF_{POE}^{line} is relatively more significant compared to Case II (compare the ratios $\left|\frac{VUF_{POE}^{\text{line}}}{VUF_{POE}^{\text{load}}}\right|$ for both cases) as the polar plot of VUF_{POE} phasors is not similar to the polar plot of VUF_{POE}^{load} as discussed in Case II. Although VUF_{POE}^{load} phasors are scattered in a larger sector, the interaction between the two contributors (VUF_{POE}^{load} and VUF_{POE}^{line}) seems to provide some degree of cancellation. That is, the phase angle between fictitious VU emission phasors which are developed to replace the group of VUF_{POE}^{load} phasors and the group of VUF_{POE}^{line} phasors is greater than 90 degrees. This arrangement of phasor distribution leads to a reduction in VUF_{POE} in comparison to the magnitude of the most influential emission contributor VUF_{POE}^{load}.

Based on this analysis, some observations can be made to identify the conditions to be satisfied in order to develop a general summation law for the aggregation of scattered unbalance levels which are generated by randomly changing loads. Accordingly, if two groups of randomly scattered VU sources are considered, the application of a general summation law is applicable where:

- the contribution made by one set of emission vectors is comparatively insignificant compared to the influential contributor (e.g. Case II - influence made by VUF_{POE}^{line} is very small) and/or
- all emission phasors are scattered in a small segment leading to an increase of the resultant emission level compared to the most influential contributors (i.e. when fictitious single emission phasors are derived to replace all individual emission phasors in a group, the phase angle between those two fictitious phasors should be smaller than 90 degrees) (e.g. Case IV and V).

These conditions can be mathematically reviewed by considering the deterministic formulation on VU emission assessment [7] as discussed in the Section 3.4.

[6] When considering the vector summation of VUF_{POE}^{load} and VUF_{POE}^{line}

3.4. Analysis of the Deterministic Formulation in Relation to the Statistical Outcomes

According to the deterministic methodology presented in [7], load contribution is given by (7) (1^{st} row of Table 1) and the contribution made by line asymmetry is given by (8) (2^{nd} row of Table 1).

$$VUF_{POE}^{\text{load}} = (VUF_{POE} - CUF)\frac{V_{\text{reg-line}}}{(1 + V_{\text{reg-line}})} \tag{7}$$

$$VUF_{POE}^{\text{line}} = -\frac{Z_{21,t}}{Z_{11,t}}\frac{V_{\text{reg-line}}}{(1 + V_{\text{reg-line}})} \tag{8}$$

VUF_{POE}^{load} can be modified as given in (9) noting that the magnitude of the CUF is much larger than that of the VUF in practice [11].

$$VUF_{POE}^{\text{load}} = (-CUF)\frac{V_{\text{reg-line}}}{(1 + V_{\text{reg-line}})} \tag{9}$$

Magnitudes of VUF_{POE}^{load} and VUF_{POE}^{line} can be evaluated using (7) and (8) respectively knowing the CUF and the allowable voltage regulation of the line as scalar quantities in addition to the known ratio of the negative-positive sequence coupling impedance to positive sequence impedance ($\frac{Z_{21,t}}{Z_{11,t}}$) of the transmission line. Thus, the most influential or the significant emission contributor can be identified and a ratio can be established to determine the least significant contribution to most significant emission contribution (i.e. the ratio $\frac{VUF_{POE}^{\text{line}}}{VUF_{POE}^{\text{load}}}$, assuming VUF_{POE}^{load} is more significant; the opposite is true for the case where VUF_{POE}^{line} is more significant).

According to the modified VUF_{POE}^{load} given in (9), the phase angle between load and line contributions is equal to the phase angle between the two vector quantities CUF and the ratio of $\frac{Z_{21,t}}{Z_{11,t}}$. Evaluation of the approximate phase angle between load and line contributions can be used to determine whether the resultant emission at the POE is leading to an increase or a decrease from the most significant individual contributor (i.e. VUF_{POE}^{load} or VUF_{POE}^{line}). That is, relative phasor orientation between VUF_{POE}^{load} and VUF_{POE}^{line} is important in the evaluation of the phasor summation.

4. A Refined General Summation Law for Aggregation of Randomly Varying Voltage Unbalance Levels

This section describes a revised general summation law which overcomes the shortcomings in the existing approach as discussed in Section 3. Here, emphasis is given to the observation that the randomly scattered VU emission phasors do not distribute over the entire 360 degree plane at all times, thus clustered emission phasors lead to a statistically significant dominant emission contribution influencing the resultant VU emission at the POE. Therefore, the existing summation law can be revised by introducing a weighing factor to signify the contribution made by individual VU sources on the resultant VU emission depending on the network specifications as shown in (10).

$$VUF_{POE,\text{stat}} = \sqrt[\alpha]{\sum K_i\,(VUF_i)^{\alpha}} \tag{10}$$

where $\alpha = 1.4$ and K_i is a coefficient which functions as a weighting factor on the influence made by each contributor depending on its magnitude and phasor orientation.

When the VU emission contributors (VUF_{POE}^{load} and VUF_{POE}^{line}) are considered in relation to the VU emission assessment in radial networks with passive loads connected at the POE, the coefficient K_i can be selected as given in Table 10 giving due consideration to following aspects:

- Summation of the scattered VU emission phasors is influenced by the relative phasor orientation of both contributors (VUF_{POE}^{load} and VUF_{POE}^{line}) which is approximately given by the phasor angle between CUF and $\frac{Z_{21,t}}{Z_{11,t}}$.

- Summation of scattered VU emission phasors is further influenced by the significance of both contributors which can be evaluated using the ratio of magnitudes of individual contributions as discussed in Section 3.4.

4.1. Selection of Weighting Coefficients K_i in the Modified General Summation Law

To establish a selection criteria for the weighing coefficients (K_i), the following notations are used in relation to the individual emission contributions made by load and line asymmetries of a given radial network configuration.

- A - Most significant contributor making an emission contribution of "a" (magnitude)
- B - Least significant contributor making an emission contribution of "b" (magnitude).

Thus, the ratio of least significant emission contribution to most emission contribution is given by $\frac{b}{a}$. K_i coefficients can be selected from Table 10, based on the phase angle between two emission contributors (given by the angle between CUF and $\frac{Z_{21,t}}{Z_{11,t}}$) and the ratio of $\frac{b}{a}$ which governs the influence of each emission contributor.

Table 10: Selection of the K_i coefficient for a radial network with passive loads

Emission contributor	K_i coefficient		
	Phase angle between CUF and $\frac{Z_{21,t}}{Z_{11,t}} < 90$ deg	Phase angle between CUF and $\frac{Z_{21,t}}{Z_{11,t}} > 90$ deg	
		Ratio ($\frac{b}{a}$)$\leq$0.5	Ratio($\frac{b}{a}$) > 0.5
Most significant contributor A	1	1	1
Least significant contributor B	1	-1	-0.3 to -0.6
	Existing summation law	Modified summation law	

The proposed formulation, including the selection of the coefficient K_i, was validated using extensive simulations although not derived through rigid fundamental theories.

Figure 10 gives an illustration of the process involved in the selection of the coefficients K_i required for the evaluation of the statistical value of the resultant VU emission level at the POE.

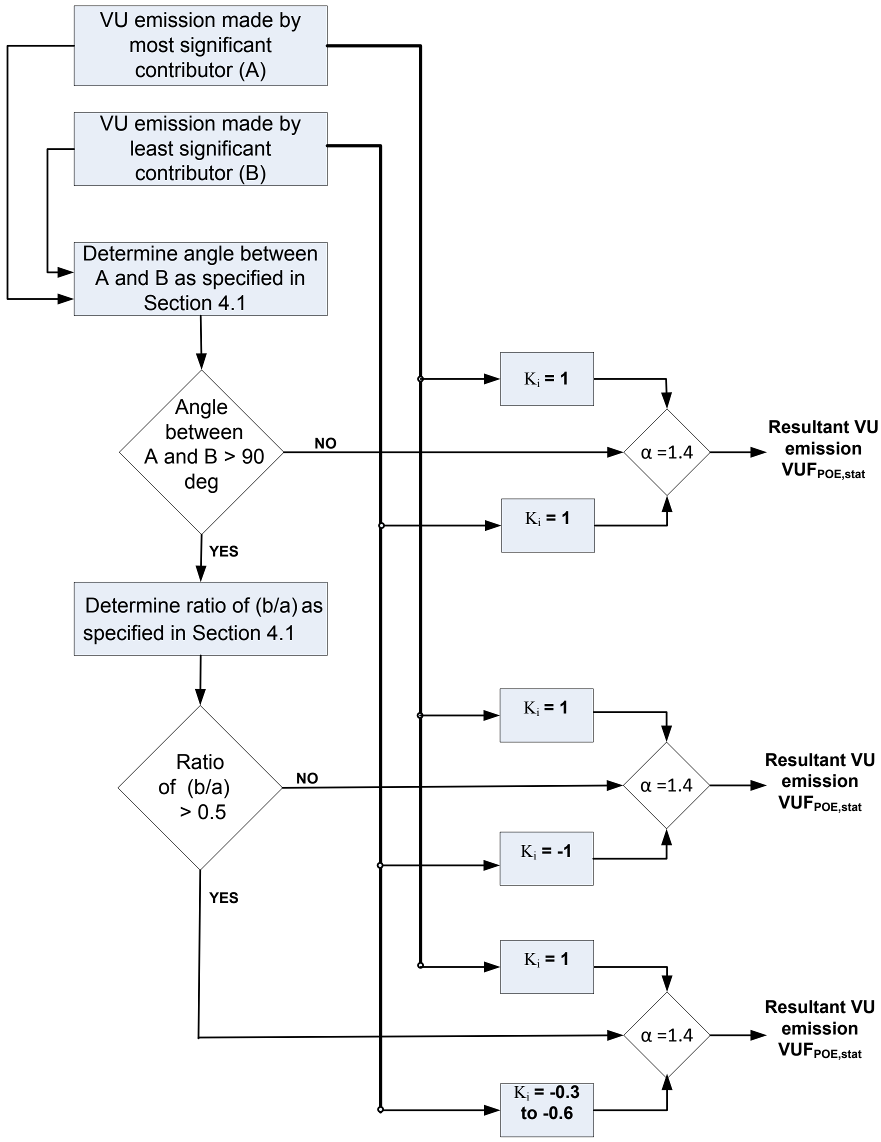

Figure 10: Overview of the coefficient K_i selection process and the evaluation procedure of the resultant VUF

4.2. Validation of the Modified General Summation Law in Relation to VU Emission Assessment

Statistical outcomes of the VU emission assessment in relation to the radial network configurations discussed in Section 3 which showed some discrepancy over the deterministic outcomes are re-evaluated employing the modified summation law in this section.

4.2.1. Case I Study (12.47 kV Network with Normally Distributed Load Given by Load 1 Configuration):

Application of the modified general summation law requires the selection of an appropriate K_i coefficient depending on the network configuration. Referring to the polar plot (Fig. 1), the phasor angle between load and line contributions can be noted to be greater than 90 degrees[7]. Polar plot analysis (Fig. 1) as well as the statistical outcomes of VU emission summarised in Table 5 shows that the most significant emission contribution is due to the load. Hence, the ratio of $|\frac{VUF_{POE}^{\text{line}}}{VUF_{POE}^{\text{load}}}|$ can be determined using the data entries (95% probabilistic levels of individual emission contributions) of Table 5. The corresponding ratio $|\frac{VUF_{POE}^{\text{line}}}{VUF_{POE}^{\text{load}}}|$ is less than 0.5 (this ratio varies from 0.3 to 0.5 when the line length increases). These findings suggest that the coefficient K_i should be selected as -1.

Table 11 shows the new emission assessment outcomes which are evaluated using the revised general summation law (for this case $VUF_{POE,\text{stat}} = \left(|VUF_{POE}^{\text{load}}|^{\alpha} - |VUF_{POE}^{\text{line}}|^{\alpha}\right)^{\frac{1}{\alpha}}$ as K_i = -1).

Table 11: Modified probabilistic outcomes of VU emission assessment: Case I

line length km	$\|VUF_{POE}\|$ 95% value	$VUF_{POE,\text{stat}}$ 95% value	
		Modified summation law	Existing summation law
1	0.34	0.30	0.52
2	0.74	0.70	1.15
3	1.31	1.32	1.94
4	2.04	2.12	2.95
5	3.14	3.35	4.34

4.2.2. Case III Study (12.47 kV Network with Normally Distributed Load Given by Load 2 Configuration):

A similar procedure has been adopted to re-evaluate the statistical value of the resultant VU factor at the POE ($VUF_{POE,\text{stat}}$). In this case, both emission contributions (VUF_{POE}^{line} and VUF_{POE}^{load}) significantly influence the resultant VU emission at the POE thus making the ratio $|\frac{VUF_{POE}^{\text{line}}}{VUF_{POE}^{\text{load}}}|$ greater than 0.5 (this ratio approximately varies from 0.68 to 0.75 for different line lengths considered). The phase angle between the CUF and $\frac{Z_{21,t}}{Z_{11,t}}$ is greater than 90 degrees. Therefore K_i can be selected as 0.4 considering the variation of the magnitude ratio ($\frac{b}{a}$). The corresponding VU emission assessment outcomes are summarised in Table 12.

Table 12: Modified probabilistic outcomes of VU emission assessment: Case III

line length km	$\|VUF_{POE}\|$ 95% value	$VUF_{POE,\text{stat}}$ 95% value	
		Modified summation law	Existing summation law
1	0.29	0.29	0.48
2	0.58	0.56	0.95
3	0.9	0.81	1.40
4	1.18	1.03	1.82
5	1.52	1.32	2.25

5. Conclusions

The work presented in this paper focused on the development of a revised statistical approach for VU emission assessment based on recently developed deterministic studies on compliance assessment. In this

[7]The phase angle between load and line asymmetries is approximately equal to the angle between the CUF and $\frac{Z_{21,t}}{Z_{11,t}}$. Referring to the 12.47 kV line impedance matrix, the ratio, $\frac{Z_{21,t}}{Z_{11,t}}$ can be calculated ($0.08\angle-34°$)

regard, application of the existing general summation law was reviewed by employing the outcome of the deterministic methodology which can evaluate constituent components of the post-connection VU emission in radial power systems. The concept of random variations of voltage unbalance was investigated by simulating randomly varying unbalanced loads subjected to a specified distribution. Considering a simple radial network with a passive load at the POE, deterministic outcomes were compared against the statistical outcomes of post-connection VU emission at the POE. These analyses led to the conclusion that the existing general summation law is not applicable in all cases, especially when the VU emission phasors are not scattered on the 360 degree plane (i.e. when emission phasors are clustered in small segments). In the IEC approach, there is an underlying assumption that emission vectors are entirely scattered on the 360 degree plane. Thus, the emphasis has been given to evaluate the influences made by individual contributions introducing different weighting factors which are assigned to each contributor. Accordingly, a modified general summation law was established specifying the selection criteria of influence coefficients for different network configurations. The new methodology was validated by considering radial networks with passive loads connected at the POE.

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