Dynamic Stability Analysis for a Self-Mixing Interferometry

Yuanlong Fan

University of Wollongong

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Dynamic Stability Analysis for a Self Mixing Interferometry

A thesis submitted in fulfillment of the requirements for award of the degree

Doctor of Philosophy

from

UNIVERSITY OF WOLLONGONG

by

Yuanlong Fan

School of Electrical, Computer and Telecommunications Engineering

February 2016
Dedicated to my family
Declaration

This is to certify the work reported in this thesis was carried out by the author, unless specified otherwise, and that no part of it has been submitted in a thesis to any other university or similar institution.

Yuanlong Fan

February 2016
Abstract

In recent years, as an emerging and promising non-contact sensing technique, Self Mixing Interferometry (SMI) has attracted much attention of researchers. The SMI is based on the self-mixing effect that occurs when a small fraction of laser light emitted by the semiconductor laser (SL) is reflected by an external target and re-enters the SL cavity. Therefore, the core components of an SMI consist of an SL, a micro-lens and a target. The target forms an external cavity for the laser, and when it moves along the light beam, a modulation in the emitted laser power can be observed. This modulated power is referred to as an SMI signal which carries the information of the motion of the target as well as the SL itself. Compared to other traditional interferometric schemes, e.g., Michelson or Mach-Zender, SMI has the advantages of a compact set-up, high sensitivity and low cost.

Under the conditions of a stable operation, an SL biased by constant current usually emits laser light with a constant intensity. However, with the introduction of external optical feedback (EOF), the laser light can become unstable. The stability has been intensively studied for an SL with a stationary external target. However, the reported results mostly were obtained by either ignoring the nonlinear gain or using an approximated system determinant derived under certain assumptions. Furthermore, the existing stability analysis can not be directly applied on the SMI because the external target in an SMI moves.

In this thesis, firstly, in Chapter 2, by lifting all the assumptions made in the existing work related to the stability, an accurate system determinant is presented by considering the nonlinear gain. Based on the system determinant, a stability boundary is obtained and presented in a 3-parameter space described by the feedback strength $\kappa$, the external cavity length $L$ and the injection current $J$. The results reveal more insight into the influence of these parameters on the stability as compared to the existing work.

Secondly, the stability on an SL with a moving target, which is the case for most SMIs, is examined in Chapter 3. For an SMI system, the parameters $\phi_0$ (feedback phase),
$C$ (feedback level factor) and $J$ describe the operational conditions for an SMI. It is very important to describe the stability of an SMI by using these three parameters. By considering a time varying $\phi_0$, based on the results gained from the system determinant in Chapter 2, the dynamic stability of an SMI is presented in a plane of $(\phi_0, C)$, from which three important regions called stable, semi-stable and unstable are recognized to characterize the features of an SMI. We realize that the existing SMI model is only valid in a stable region, and the semi-stable region has potential applications for sensing and measurement, however it needs remodeling of the system by considering the bandwidth of the detection components.

Thirdly, in Chapter 4, the stability of an SMI is described in a plane of $(\phi_0, J)$, from which a critical injection current is found, above which the SMI is always stable. Furthermore, the importance of the nonlinear gain on the stability of an SMI is revealed in this Chapter. The results reveal that a stable SMI can be achieved by either an increase in the injection current or an increase in the external cavity length.

Lastly, based on the above stability analysis, an experimental approach was presented to determine the actual stability boundary for a practical SMI system, shown in Chapter 5. In that Chapter, the measurement problem for the linewidth enhancement factor (LEF), which is an important parameter of the SL itself, caused by an SMI operating at semi-stable region is presented. Then a new method for measuring the LEF is proposed.

All the results presented in this thesis are confirmed by both simulations and experiment. The results can be regarded as a set of general criteria for designing a stable SMI for sensing and instrumentation.
### Acronyms

<table>
<thead>
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<th>Description</th>
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<tr>
<td>SMI</td>
<td>self mixing interferometry</td>
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<tr>
<td>SL</td>
<td>semiconductor laser</td>
</tr>
<tr>
<td>LEF</td>
<td>linewidth enhancement factor</td>
</tr>
<tr>
<td>FP</td>
<td>Fabry-Perot</td>
</tr>
<tr>
<td>LK</td>
<td>Lang-Kobayashi</td>
</tr>
<tr>
<td>EOF</td>
<td>external optical feedback</td>
</tr>
<tr>
<td>DDE</td>
<td>delayed differential equations</td>
</tr>
<tr>
<td>LFF</td>
<td>low frequency fluctuation</td>
</tr>
<tr>
<td>MLM</td>
<td>minimum linewidth mode</td>
</tr>
<tr>
<td>RK</td>
<td>Runge-Kutta</td>
</tr>
<tr>
<td>PD</td>
<td>photodiode</td>
</tr>
<tr>
<td>TC</td>
<td>temperature controller</td>
</tr>
<tr>
<td>LC</td>
<td>laser controller</td>
</tr>
<tr>
<td>BS</td>
<td>beam splitter</td>
</tr>
<tr>
<td>OSA</td>
<td>optical spectrum analyzer</td>
</tr>
<tr>
<td>EPD</td>
<td>external photodiode</td>
</tr>
<tr>
<td>RO</td>
<td>relaxation oscillation</td>
</tr>
<tr>
<td>VCSEL</td>
<td>vertical cavity surface emitting lasers</td>
</tr>
<tr>
<td>DFB</td>
<td>distributed feedback</td>
</tr>
<tr>
<td>CSP</td>
<td>channeled substrate planar</td>
</tr>
<tr>
<td>SNR</td>
<td>signal noise ratio</td>
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I would like to extend thanks to the many people who generously contributed to the work presented in this thesis. Special mention goes to my enthusiastic supervisors, Associate Professor Yanguang Yu, Professor Jiangtao Xi and Dr Qinghua Guo. My PhD has been an amazing experience and I thank them wholeheartedly, not only for their tremendous academic support, but also for giving me so many wonderful opportunities. Without their enlightened instruction, impressive kindness and patience, I could not have completed my thesis. Their keen and vigorous academic observation enlightened me not only during this thesis but I know it will continue to do so in my future study. Secondly, I shall extend my thanks to the staff in the School of Electrical, Computer and Telecommunications Engineering (SECTE) who have helped me to develop a fundamental and essential academic competence. My sincere appreciation also goes to the teachers and students from the Signal Processing for Instrumentation and Communications Research (SPICR) Lab, who participated in this study with enthusiastic cooperation. Finally, but by no means least, thanks go to my wife, my parents in-law and my parents, for almost unbelievable support. They are the most important people in my world and I dedicate this thesis to them.
Publications

Journals:


International Conferences:


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Chapter 1  Introduction and Background

In the last decades, the various applications of the Self Mixing Interferometry (SMI) based sensing system have attracted much attention of researchers. An SMI can be used for measuring metrological quantities, such as velocity, absolute distance, displacement and vibration, etc. or even the parameters associated with the laser itself [1-19]. A schematic diagram of the SMI is shown in the following figure (Fig. 1-1).

![Figure 1-1: A schematic diagram of the SMI.](image)

The core part of an SMI consists of a single longitudinal mode Semiconductor Laser (SL) and a moving target which forms an external cavity of the SL. An SMI works when a small portion of light is back-scattered or reflected by an external target and re-enters into the laser internal active cavity. The re-entered light modulates both amplitude and frequency of the emitted SL power [20, 21]. This modulated power is called an SMI signal which can be used to detect metrological quantities associated
with the external target and the parameters associated with the SL itself. The advantages of using an SMI-based sensing system to perform measurement have been presented in [3]:

- No optical interferometer external to the source is required. This leads to a simple and compact set-up.
- No alignment is needed because the spatial mode that interacts with the cavity mode is filtered out spatially by the laser itself. This means that detection of the diffusive target's movement becomes possible.
- Sensitivity of the scheme is very high (sub-nm sensitivity).

Due to these advantages, the SMI has been intensively investigated both theoretically and experimentally for exploring all kinds of sensing applications. In these applications, it is required that an SMI system operates in a stable mode, in which case the SL biased by constant injection current usually leads to SMI signals with symmetric sinusoidal-like fringes or asymmetric sawtooth-like fringes, depending on the external optical feedback level. However, with the change of operational conditions, such as injection current and parameters associated with the external cavity including the optical feedback level and external cavity length, the SL can also exhibit unstable behavior. In this case, an SMI will degrade or even lose its sensing ability. Therefore it is very important to investigate the stability of an SMI system with respect to its operational conditions, which lead us to the main goal of this thesis, i.e., analyzing the stability of the SMI.

This chapter gives an introduction and background for this thesis. The rest of this chapter is organized into the following sections. Section 1.1 gives detailed derivation of the existing mathematical model for describing the SMI. A literature review for the SMI has been given in Section 1.2 in terms of the stability analysis. Based on the investigation of reviewed articles, the existing problems for investigating the stability of the SMI are presented in Section 1.3. Section 1.4 shows the structure of this thesis.
1.1 The Mathematical Model of the SMI

1.1.1 Derived from the Three Mirrors Model

The mathematical model of the SMI can be derived from the classical three mirrors model consisting of a Fabry-Perot (FP) type laser with facet reflection coefficients $r_1$ and $r_2$, and the target with the reflection coefficient of $r_3$ [22]. Therefore, a simplified arrangement based on the schematic diagram shown in Fig. 1-1 can be used for carrying out the derivation, which is shown in the following figure (Fig. 1-2):

![Simplified schematic diagram for the three mirrors model.](image)

Assuming $|r_2 r_3| \leq 1$, i.e., there is only one reflection within the external cavity, the effective reflection coefficient $r_{\text{eff}}$ at the laser front facet can be expressed as [22, 23]:

$$r_{\text{eff}} = r_2 + (1 - |r_2|^2) r_3 e^{-j\omega s \tau}$$  \hspace{1cm} (1.1)

where $\omega_s$ and $\tau$ are respectively the perturbed laser angular frequency and one roundtrip time of the light in the external cavity, and $\tau = 2L/c$, where $c$ is the speed of light.

On the other hand, $r_{\text{eff}}$ can also be represented in another form with respect to its amplitude $A_{\text{eff}}$ and phase $\phi_{\text{eff}}$ as [22]:
\[ r_{\text{eff}} = A_{\text{eff}} e^{-j\phi_{\text{eff}}} \]  
(1.2)

where
\[ A_{\text{eff}} = r_1 \left[ 1 + \kappa \cos(\omega \tau) \right] \quad \text{and} \quad \phi_{\text{eff}} = \kappa \sin(\omega \tau) \]  
(1.3)

where \( \kappa \) is the feedback strength and \( \kappa = (1-r_2^2) r_3^2 / r_2 \). As the roundtrip phase in the internal cavity must be equal to a multiple of \( 2\pi \), the phase condition of compound cavity of the three mirror model can be described using the following equation (Eq. 1.4) [22]:
\[ \Delta \phi_{L} = -\alpha (g_c - g_{th}) d + \tau_{in} (\omega_s - \omega_0) + \phi_{\text{eff}} \]  
(1.4)

where \( \Delta \phi_{L} \) corresponds to a change in the round trip phase compared to \( 2\pi q \), where \( q \) is an integer. In Eq. (1.4), \( \alpha \) is called the linewidth enhancement factor (LEF) which is an important fundamental descriptive parameter of the SL because it determines the characteristics of SLs, such as the spectral effects, the modulation response, the injection locking and the response to the external optical feedback [15, 24-29]. \( \tau_{in} \) is the light roundtrip time in the internal cavity and \( \omega_0 \) is the angular frequency of the solitary laser. \( g_c \) and \( g_{th} \) are respectively the threshold gain with and without external cavity.
\[ g_{th} = a_s + d^{-1} \ln \left( \frac{\eta r_2}{r} \right)^{-1} \]  
(1.5)

where \( a_s \) represents the loss account for any optical loss in the internal cavity.

Note that \( g_c \) also must satisfy the amplitude condition of the compound cavity, that is [22, 23]:
\[ r_1 A_{\text{eff}} e^{[g_c - a_s]d} = 1 \]  
(1.6)

Substituting Eqs. (1.3) and (1.5) into Eq. (1.6), we can obtain the threshold gain difference:
\[ g_c - g_{th} = -\frac{\kappa}{d} \cos(\omega \tau) \]  
(1.7)

Then substituting Eqs. (1.2) and (1.7) into Eq. (1.4) and considering \( \Delta \phi_L = 0 \), equation (1.4) changes to [22]:

\[ \omega \tau = \omega_0 \tau - \frac{\kappa}{\tau_m} \tau \sqrt{1 + \alpha^2} \sin[\omega \tau + \arctan(\alpha)] \]  
(1.8)

If we denote \( \phi_s = \omega \tau \), \( \phi_0 = \omega_0 \tau \) and \( C = \frac{\kappa}{\tau_m} \tau \sqrt{1 + \alpha^2} \), the core part of the SMI model can be finally derived as:

\[ \phi_s = \phi_0 - C \sin[\phi_s + \arctan(\alpha)] \]  
(1.9)

where \( \phi_s \) and \( \phi_0 \) are respectively the light phase corresponding to the perturbed and unperturbed laser angular frequency. Note that although the three mirrors model explains some interesting results, it lacks some details of the physical setting of the phenomenon, e.g., the material and associated effects for an SL [4].

### 1.1.2 Derived from the Lang and Kobayashi Model

Interestingly, equation (1.9) can also be derived from the well known Lang and Kobayashi (LK) equations which are based on Lamb’s equation and modified with the additional equation for the state concentration [4]. Compared to the three mirrors model, the LK equations describe the active material and carry a description of laser oscillator equations which yield a much more complete description of the dynamic behaviour of a single mode SL with external optical feedback (EOF). The well known LK equations [30] were first proposed in 1980 and the model consists of three simultaneous Delay Differential Equations (DDE) which are shown as below:

\[
\frac{dE(t)}{dt} = \frac{1}{2} \left\{ G[N(t), E^2(t)] - \frac{1}{\tau_p} \right\} E(t) + \frac{\kappa}{\tau_m} \cdot \cos[\omega_0 \tau + \phi(t) - \phi(t - \tau)] 
\]  
(1.10)
\[
\frac{d\phi(t)}{dt} = \frac{1}{2} \alpha \left\{ G[N(t), E^2(t)] - \frac{1}{\tau_p} \right\} \sin \left[ \omega_0 \tau + \phi(t) - \phi(t - \tau) \right]
\]
\[
\frac{dN(t)}{dt} = \frac{J}{eV} \frac{N(t)}{\tau_s} - G[N(t), E^2(t)]E^2(t)
\]
(1.11)

where \( G[N(t), E(t)] \) is the modal gain per unit of time and is expressed as [31, 32]:
\[
G[N(t), E(t)] = G_N \left[ N(t) - N_0 \right] \left[ 1 - \varepsilon \Gamma E^2(t) \right]
\]
(1.12)

and when ignoring the nonlinear effect, the modal gain can be simplified as:
\[
G[N(t), E(t)] = G_N \left[ N(t) - N_0 \right]
\]
(1.13)

Equations (1.10)-(1.13) describe the dynamic behavior of the three variables, namely the electric field amplitude \( E(t) \), the electric field phase \( \phi(t) \) and the carrier density \( N(t) \) where \( t \) is the time index. \( \phi(t) \) is given by \( \phi(t) = [\omega(t) - \omega_0]t \), and where \( \omega(t) \) is the instantaneous optical angular frequency for an SL with EOF.

The dynamics of the SL with an EOF system are governed by the injection current \( (J) \) to the SL and the parameters associated with the external cavity including \( \kappa \) and \( \tau \).

The other parameters in Eqs. (1.10)-(1.13) are related to the solitary SL itself, and are treated as constants for a certain SL. These parameters are defined in Table 1-1 [31].

Note that the values of the parameters provided in Table 1-1 are adopted from [31].

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Physical Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G_N )</td>
<td>modal gain coefficient</td>
<td>( 8.1 \times 10^{-9} \text{m}^2\text{s}^{-1} )</td>
</tr>
<tr>
<td>( N_0 )</td>
<td>carrier density at transparency</td>
<td>( 1.1 \times 10^{26} \text{m}^{-3} )</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>nonlinear gain compression coefficient</td>
<td>( 2.5 \times 10^{-3} \text{m}^2 )</td>
</tr>
<tr>
<td>( \Gamma )</td>
<td>confinement factor</td>
<td>0.3</td>
</tr>
<tr>
<td>( \tau_p )</td>
<td>photon life time</td>
<td>( 2.0 \times 10^{-8} \text{s} )</td>
</tr>
<tr>
<td>( \tau_s )</td>
<td>internal cavity round-trip time</td>
<td>( 8.0 \times 10^{-8} \text{s} )</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>line-width enhancement factor</td>
<td>6.0</td>
</tr>
<tr>
<td>( e )</td>
<td>elementary charge</td>
<td>( 1.6 \times 10^{-19} \text{C} )</td>
</tr>
<tr>
<td>( V )</td>
<td>volume of the active region</td>
<td>( 1.0 \times 10^{-4} \text{m}^3 )</td>
</tr>
<tr>
<td>( \tau_c )</td>
<td>carrier life time</td>
<td>( 2.0 \times 10^{-7} \text{s} )</td>
</tr>
</tbody>
</table>
The core part of the SMI model is derived from the stationary solutions of the above LK equations. Let $E_s$, $N_s$ and $\omega_s$ represent the stationary solutions of LK equations for electric field amplitude, carrier density and angular frequency respectively. When the system described by Eqs. (1.10)-(1.13) enters into a stationary state, we have $dE(t)/dt = 0$, $d\phi(t)/dt = \omega_s - \omega_0$ and $dN(t)/dt = 0$. Substituting $E(t) = E(t-\tau) = E_s$, $N(t) = N_s$, and $\phi(t) = (\omega_s - \omega_0)t$ into Eqs. (1.10)-(1.13) and ignoring the nonlinear gain, the well known stationary solutions can be obtained as below [1, 4, 21, 30, 31, 33]:

\begin{equation}
\omega_s \tau = \omega_0 \tau - \frac{K}{\tau_{in}} \sqrt{1 + \alpha^2} \sin(\omega_s \tau + \arctan \alpha) \tag{1.15}
\end{equation}

\begin{equation}
N_s = N_0 + \frac{1}{\tau_p G_N} - \frac{2\kappa \cos(\omega_s \tau)}{\tau_{in} G_N} \tag{1.16}
\end{equation}

\begin{equation}
E_s^2 = \frac{J/(eV) - N_s/\tau_s}{G_N (N_s - N_0)} \tag{1.17}
\end{equation}

From Eqs. (1.15)-(1.17) and by considering a moving target, the existing SMI model can be obtained as below by introducing:

\begin{equation}
\phi_0 = \omega_0 \tau, \quad \phi_s = \omega_s \tau \quad \text{and} \quad C = \frac{K}{\tau_{in}} \sqrt{1 + \alpha^2} \tag{1.18}
\end{equation}

Then Eq. (1.15) becomes:

\begin{equation}
\phi = \phi_0 - C \sin[\phi + \arctan \alpha] \tag{1.19}
\end{equation}

where $\phi_0$ is associated with the external cavity length $L$, i.e., $\phi_0 = 4\pi L/\lambda_0$, where $\lambda_0$ is the unperturbed laser wavelength.

Equation (1.19) is called the phase equation which is the core part of the existing SMI model, and is the same equation as derived by the three mirrors model (see Eq. (1.9)). By substituting Eq. (1.16) into Eq. (1.17), the normalized variation of the SL output
power (that is the so called SMI signal  \( g \)) can be obtained and described as [21]:

\[
g = \cos(\phi)
\]  

(1.20)

Equations (1.19) and (1.20) constitute the existing SMI model which has been widely accepted to describe the waveforms of SMI signals [2, 3, 6, 21, 23, 33-36]. As seen in Eqs. (1.19) and (1.20), there is a straightforward procedure to constitute \( g \), i.e., \( \phi_0 \rightarrow \phi_s \rightarrow g \), and also a straight backward procedure, i.e., \( g \rightarrow \phi_s \rightarrow \phi_0 \), to obtain \( \phi_0 \), thus retrieving the external cavity information. Therefore, the knowledge of the theory of generating an SMI signal as well as its waveform is essential to achieve good performance of the SMI.

1.1.3 SMI Waveform

In the SMI model, \( C \) (called feedback level factor) is an important parameter as it characterizes the waveform of an SMI signal. When \( C < 1 \) (corresponding to weak feedback regime), equation (1.19) presents a unique mapping from \( \phi_0 \) and \( \phi_s \). In this situation, the movement of the external target will result in an SMI signal waveform with a fringe structure similar to the traditional interference fringes, and each fringe period corresponds to a phase shift that is equivalent to a displacement of half a wavelength of the external target [35]. Figure 1-3 shows the relationship between \( \phi_0 \) and \( \phi_s \) as well as \( g \) and \( \phi_0 \) when \( C = 0.7 \) and \( \alpha = 6 \).

Supposing that the external target moves according to a sinusoidal law with \( L(t) = L_0 + \Delta L \cdot \sin(2\pi ft) \), where \( L_0 \), \( \Delta L \) and \( f \) are the initial external cavity length the vibration amplitude and frequency respectively which are chosen as \( L_0 = 0.35m \), \( \Delta L = 1.5\lambda_0 \) and \( f = 75Hz \), thus leading to a time varying \( \phi_0 \), i.e.,

\[
\Delta \phi_0(t) = 4\pi \Delta L(t)/\lambda_0 = 6\pi \cdot \sin(2\pi ft) \quad (rad)
\] 

which is shown in Fig. 1-4(a), and Fig. 1-4 (b) shows the corresponding SMI signal when \( C = 0.7 \) and \( \alpha = 6 \).
Figure 1-3: (a) Relationship between $\phi_0$ and $\phi_s$, (b) relationship between $g$ and $\phi_0$ when $C = 0.7$ and $\alpha = 6$.

Figure 1-4: (a) A time varying $\phi_0$, i.e., $\Delta \phi_0(t)$, caused by the target movement, (b) the corresponding SMI signal when $C = 0.7$ and $\alpha = 6$.

In the situation of $C > 1$ which corresponds to a moderate ($1 < C < 4.6$) or strong ($C > 4.6$) feedback regime, equation (1.19) yields multiple possible $\phi_s$ and the SMI signal shows asymmetric hysteresis and produces sawtooth-like fringes. To illustrate the scenario behind the waveform of SMI signal when $C > 1$, figure 1-5(a) shows the relationship between $\phi_0$ and $\phi_s$ as well as $g$ and $\phi_0$ when $C = 3$ and $\alpha = 6$.

The actual behaviour is described in [21, 27, 35, 37], indicating that $\phi_s$ and $g$ will vary along the route $A_i \rightarrow B \rightarrow B_i$ when $\phi_0$ increases, and it will however track the route of $B_i \rightarrow A \rightarrow A_i$ when $\phi_0$ decreases. Note that, the stationary solutions
for $\phi_s$ within the range of $[\phi_{s,A}, \phi_{s,B}]$ are always unstable according to [31, 38, 39], and thus will never be an oscillating mode of the SMI signal.

Figure 1-5: (a) Relationship between $\phi_0$ and $\phi_s$, (b) relationship between $g$ and $\phi_0$ when $C = 3$ and $\alpha = 6$.

Similar to the weak feedback regime case discussed above, we present $\Delta\phi_0(t)$, which is same with Fig. 1-4(a), as well as the corresponding SMI signal $g(t)$ in the following figure (Fig. 1-6).

Figure 1-6: (a) A time varying $\phi_0$, i.e., $\Delta\phi_0(t)$, caused by the target movement, (b) the corresponding SMI signal when $C = 3$ and $\alpha = 6$.

The mechanism of generating an SMI signal as well as predicting its behaviour with respect to $C$ has been well-established and presented in [18, 23, 33, 35, 36].
Clearly, the existing SMI model, i.e., Eqs. (1.19) and (1.20), is based on a premise that the system is stable, or in other words, it does not give the condition of stability of the SMI with respect to the SL operational parameters, i.e., $\kappa$, $\tau$ and $J$. The goal of this thesis is to achieve a complete and accurate stability analysis for the SMI. Before presenting our approach, a detailed literature review related to the stability analysis of an SMI is presented in the following section.

### 1.2 Literature Review

As an SMI is an SL with EOF by considering a time varying light phase, also called a feedback phase [40-44] $\phi_0$ or $\phi$, it is necessary to discuss the stability of an SL with EOF where the feedback phase is fixed, or say the target is stationary. Note that in a not very strict sense, it is usually convenient to consider the feedback phase, i.e., $\phi_0 = \omega_0 \tau$, as an independent operation parameter with $\tau$ [39, 43, 45-47], because very tiny changes of $\omega_0$ or $\tau$ can result in significant variation on the phase. In practice, the value of $\omega_0 \tau$ can be varied by minuscule changes of the external cavity length (on the order of one wavelength of the laser) or by altering $\omega_0$ through slightly adjusting the injection current or temperature [45].

Under the condition of a stable operation, an SL biased by constant current usually emits a laser with a constant intensity when the target is stationary. However, with the change of operational conditions, the laser output can also exhibit unstable behaviour, such as periodic oscillation, quasi-periodic oscillation, low frequency fluctuations (LFFs) or chaos [43, 44, 48-56]. As the transition from stable to unstable states is caused by the change in the injection current and parameters associated with EOF, it is important to figure out when the transition occurs with respect to the values of these parameters, or the range of these parameters within which the SL is stable or unstable. Such a range is referred to as the stability limit or the stability boundary [31], or the
Hopf bifurcation [57, 58].

The first discussion of the stability of the SL with EOF originates in 1980 [30], together with the proposal of the LK equations. By looking at the characteristics of the LK equation in a small region near its stationary solutions, a set of linear partial differential equations were obtained, from which the system determinant can be derived to evaluate the stability of the SL with EOF system. The system is considered stable when the system determinant does not have zeros in the right side of the S-plane. Under the assumption of weak feedback strength and short external cavity, i.e., $\kappa \tau / \tau_{in} << 1$, the system determinant was simplified to a quadratic equation from which an expression (Equation (29) in [30]) for identifying the unstable system was derived. However, the influence of the parameters associated with SL operation on the stability boundary was not discussed in [30].

Then based on the work in [30], in 1984, Tromborg, et al. [45] presented results of an analysis on the stability boundary of an SL with EOF. Three different boundaries were described in [45], namely boundaries A, B and C. Boundaries A and B are for the bistability condition described in [51], [59, 60]. Boundary C is defined as a dynamic stability boundary, also called a feedback-induced intensity pulsations boundary in [45]. In this thesis, we mainly discuss the boundary C, as it is the boundary identifying the transition from stable to unstable states. In [45], by introducing further assumptions, i.e., $\omega_{kr}^2 >> \kappa / (\tau_{in} \tau_{kr})$ and $\omega_{kr}^2 >> (\kappa / \tau_{in})^2$, the system determinant presented in [45] was simplified to a transcendental equation, leading to a set of coupling equations (Equations (45) and (47) in [45]) for describing the stability boundary. Here, $\omega_{kr}$ and $\tau_{kr}$ represent the relaxation resonance angular frequency and carrier density relaxation time (also called damping time [31]) respectively for the solitary laser. In [45], the stability boundary was presented as a relationship of the feedback phase in the range of $[-\pi, \pi]$ versus $\kappa$ for a fixed $\tau$ (see Figure 9 in [45]). In fact, such presentation of the stability boundary, that is, in a two dimensional
plane of the feedback phase and one of the operational parameters, is useful for investigating the stability of the SMI in the future, as the SMI is a special case of SL with EOF where the feedback phase is time varying in the range of $2\pi$ with the movement of the target (note that $\phi_0$ is periodic, and so is $\phi_j$ according to Eq. (1.19) [61]). In the figure 9 in [45], it can be seen that below a certain value of $\kappa$, the SL with EOF is always stable no matter what the values of the feedback phase are, therefore indicating a stable operation range with respect to $\kappa$ for the SMI.

Two years later, in 1986, Olesen, et al. [39] improved the work in [45] in two respects: 1) using the system determinant without the assumptions in [30] and [45], and 2) employing $\omega_R$ and $\tau_R$ of an SL with EOF rather than those of a solitary SL. In [39], the stability boundary was discussed in the same way as in [45] but for three different fixed values of $\tau$, showing that the stable operation range of the SMI increases with the increase of $\tau$.

In 2002, another work was reported in [46] using the normalized LK equations for the case of short external cavity ($\tau = 0.02\text{ns}$). Due to the use of the normalized LK equations, the numerical analysis on LK equations is significantly simplified [52], [46, 62, 63]. In [46], starting from the normalized LK equations, a new system determinant with the normalized coefficients was derived. The stability boundary was numerically calculated and presented as a relationship of the feedback phase versus $\kappa$ for a fixed $\tau$. The same conclusion with [45] can be drawn from the relationship presented in [46].

In 2009, another set of results were reported in [61] obtained by means of a MATLAB package DDE-BIFTOOL [64]. The stability boundary acquired in [61] was presented as a relationship of the feedback phase versus $\kappa$ and the feedback phase versus $J$ respectively for a fixed $\tau$. The normalized LK equations were also used in [61] for the numerical calculation of the stability boundary. The underlying algorithm of the MATLAB package is the linear multi-step method [65, 66] which can approximately calculate the locations of zeros of the system determinant.
In 2012, Donati and Fathi [67] numerically presented the possible region for achieving a stable SMI for both short and long external cavities by investigating the stability boundary of an SL with EOF in a plane of the feedback phase and $\kappa$. The results in [67] showed that the SMI can be stable when $\kappa$ is small.

More recently, in 2013, Lenstra [40] found that when the product of $\omega_{\kappa}$ and $\tau$ is equal to an integral multiple of $2\pi$, the SL with EOF behaves like a solitary laser which is always stable no matter what the feedback phase is. The stability boundary was presented as a relationship of the feedback phase in the external cavity and $J$ for a fixed $\kappa$.

Note that in the research conducted in 2009 [61] and 2013 [40], a band stable phenomenon with respect to $J$ was predicated theoretically. The phenomenon indicates that, in some certain regions of $J$, the stability is free of the influence of the feedback phase, thus leading to a stable SMI, while in other regions of $J$, the stability strongly depends on the phase. However, the studies were only for short cavities and weak feedback strength ($\kappa \tau / \tau_{\omega} << 1$).

An outstanding issue associated with the results reported in [30, 39, 40, 45, 46, 61, 67], is that, none of them considers the effect of nonlinear gain, which was ignored in the process of linearizing the LK equations in [30]. As a matter of fact, nonlinear gain is an important factor in describing the dynamic behaviors of an SL with EOF [32, 68, 69] and it was reported that the stability boundary of an SL with EOF can be enhanced by the nonlinear gain [68, 70]. Hence, in order to better describe the stability, nonlinear gain in the LK equations should be considered. Some preliminary work was reported on this problem [31, 71-73]. The work in [31] was based on the expression in [45], but insert a term in the expression of $\tau_{\kappa}$ to take into account the influence of nonlinear gain (see Equation (19) in [45]). The stability boundary obtained in [45] is a relationship of $\kappa$ versus $\tau$ for a fixed $J$. Furthermore, in [31], by analyzing the characteristic points on the stability boundary, a simple analytic
expression was derived for the stability boundary described by the critical feedback strength \( \kappa \). Obviously, [31] considers only partially the influence of nonlinear gain, and it is still based on the approximations used in [45]. In 1993, Ritter and Haug [71] presented another expression of the system determinant by taking into account the nonlinear gain in the electric field amplitude equation (Equation (2.1) in [71]) of LK equations. However, the influence of the gain in the electric field phase equation was still assumed to be linear. In working out the system determinant, a number of terms are neglected, including terms proportional to \( (\tau_R/\omega_R)^2 \), \( \kappa^2/(\tau_m\omega_R)^2 \) and \( (\tau_R\kappa)/(\tau_m\omega_R^2) \), and the system determinant was simplified to a transcendental equation. In this work, the stability boundary was described as a relationship of \( \kappa \) versus \( J \) for a fixed \( \tau \).

Later, in 1994, Tager and Petermann [72] presented similar work to [71], in which the gain in the electric field amplitude equation was considered as nonlinear, but the gain in the electric field phase equation was still linear. Two new coupling equations were derived from the system determinant under assumptions different to [71], that is,

\[
\omega_R^2 \gg \gamma_p/\tau_R \quad \text{and} \quad \omega_R^2 \gg \kappa/(\tau_m\tau_R),
\]

where \( \gamma_p \) is the contribution of nonlinear gain to \( \tau_R \) in a similar way to [31]. The two coupling equations are similar to those derived in [45], but they include the influence of compound cavity mode competition on the stability of an SL with short external cavities. The stability boundary was then presented as a relationship of \( \kappa \) versus \( \tau \) and \( \kappa \) versus the feedback phase in the external cavity for a fixed \( J \) in [72], indicating that (see Fig. 7 in [72]) there is no region where the SL with EOF is always stable for all the values of the feedback phase with a fixed \( \kappa \). That is to say there is no region guaranteeing a stable SMI.

In 1998, Masoller and Abraham [73] investigated the influence of \( \kappa \) on the stability limit when the SL is biased well above the threshold. However, the nonlinear gain was still partially considered in the same way as in [31].
1.3 Outstanding Issues

The results of analyzing the above existing literature are summarized in Table 1-2 from which the outstanding research issues can be drawn as follows:

1. The existing approaches either ignore the nonlinear gain, which is an important factor in describing the system, or make use of an approximated expression of the system determinant, thus leading to use of incomplete system descriptions and hence inaccurate results.

2. The influence of all parameters associated with an SL with EOF operation, i.e., $\kappa$, $\tau$ and $J$ on the stability of an SL with EOF has been absent in the literature, as well as the SMI, both theoretically and experimentally.

<table>
<thead>
<tr>
<th>References</th>
<th>Assumptions made</th>
<th>Boundary description</th>
<th>For fixed parameters of</th>
</tr>
</thead>
<tbody>
<tr>
<td>[30]</td>
<td>$\kappa \tau/\tau_m &lt;&lt; 1$</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>[45]</td>
<td>$\omega_k &gt; \kappa/(\tau_m \tau)$, $\omega_k &gt; (\kappa/\tau_m)^2$</td>
<td>$\phi_0$ and $\kappa$</td>
<td>$\tau$, $J$</td>
</tr>
<tr>
<td>[39]</td>
<td>NA</td>
<td>$\phi_0$ and $\kappa$</td>
<td>$\tau$, $J$</td>
</tr>
<tr>
<td>[46]</td>
<td>Normalized LK equations</td>
<td>$\phi_0$ and $\kappa$</td>
<td>$\tau$, $J$</td>
</tr>
<tr>
<td>[61]</td>
<td>Normalized LK equations and $\kappa \tau/\tau_m &lt;&lt; 1$</td>
<td>$\phi_0$ and $\kappa$, $\phi_0$ and $\tau$</td>
<td>$J$</td>
</tr>
<tr>
<td>[67]</td>
<td>NA</td>
<td>$\phi_0$ and $\kappa$</td>
<td>$\tau$, $J$</td>
</tr>
<tr>
<td>[40]</td>
<td>$\kappa \tau/\tau_m &lt;&lt; 1$</td>
<td>$\phi_0$ and $J$</td>
<td>$\tau$, $\kappa$</td>
</tr>
<tr>
<td>[31]</td>
<td>$\omega_k &gt; k/(\tau_m \tau)$, $\omega_k &gt; (k/\tau_m)^2$</td>
<td>$\kappa$ and $\tau$</td>
<td>$J$, $\phi_0$</td>
</tr>
<tr>
<td>[71]</td>
<td>$(\tau_m/\omega_k)^2$, $\kappa^2/(\tau_m \omega_k)^2$, $(\tau_m \kappa)/(\tau_m \omega_k)$</td>
<td>$\kappa$ and $J$</td>
<td>$\tau$, $\phi_0$</td>
</tr>
<tr>
<td>[72]</td>
<td>$\omega_k &gt; \gamma_x/\tau$, $\omega_k &gt; \kappa/(\tau_m \tau)$</td>
<td>$\kappa$ and $\tau$, $\phi_0$ and $\kappa$</td>
<td>$J$</td>
</tr>
<tr>
<td>[73]</td>
<td>NA</td>
<td>NA</td>
<td>$\tau$, $\phi_0$, $J$</td>
</tr>
</tbody>
</table>
\[ d\phi(t)/dt = \omega - \omega_0 \quad \text{and} \quad dN(t)/dt = 0, \]
thus leading to the position that actual behaviour of the system cannot be described by the existing SMI model.

4. The existing SMI based applications, such as measurement of displacement as well as the parameters of the SL itself, e.g., the LEF, all assume and require that the system is in a stable status. However, research in Chapter 3 will subsequently show that when the system enters into the unstable states, an unstable SMI can pretend to be a ‘stable’ SMI, which confuses researchers due to the limited bandwidth of the detection component, thus leading to the degraded system performance.

1.4 Thesis Organization

This thesis consists of six chapters:

Chapter 1 presents a brief introduction and background to this research project. Firstly, in Section 1.1, the background of the SMI is introduced then its theoretical model originated from both the three mirror and LK models is presented. Based on the SMI model, the characteristics of the SMI waveform as well as its mechanism related to the model are described. Secondly, in Section 1.2, the literature related to the stability analysis of an SMI is reviewed from the viewpoint of their theoretical derivations as well as the manifestation of their results. At the end of Chapter 1, the summarization of the outstanding problems and organization of this thesis are given.

Chapter 2 gives a complete and accurate stability analysis for an SL with EOF with a particular fixed feedback phase value. In Section 2.1, a new and accurate system determinant is derived by removing the assumptions or approximations made in the existing work to ensure a more accurate and complete theoretical analysis. Then by varying \( \kappa, \tau, \text{and} J \), the trajectory of the zeros of the system determinant is determined by an effective numerical computation developed by this researcher, from which the stability boundary described by these parameters can be obtained. In Section 1.3, the relationships of each pair of the three parameters will be investigated in detail and a list of new and interesting discoveries is presented.
Chapter 3 analyzes the stability of the SMI. The stability of an SMI is firstly presented in a two dimensional plane of the feedback phase $\phi_0$ and the feedback level factor $C$ in Section 3.1, based on which three regions for characterizing the behaviour of an SMI are proposed, and potential applications for the three regions are described. Furthermore, a critical optical feedback factor $C_{\text{critical}}$, under which the SMI is guaranteed to be stable, is approximately determined in an analytical form. Finally, in Section 3.2, detailed experimental procedures are designed and described to verify the results presented in Section 3.1.

Chapter 4, in section 4.1 presents, inspired by an existing work, the stability boundary of an SMI in a two dimensional plane of $\phi_0$ and $J$ using the system determinant derived in Chapter 2, in order to see how the injection current $J$ influences the stability of an SMI. Moreover, comparing it with the existing work, a critical injection current $J_{\text{critical}}$ is observed from the plane, above which the SMI is always stable. Followed by extensive simulations performed in Section 4.2, the relationships between $J_{\text{critical}}$ and $C$ as well as $J_{\text{critical}}$ and $\tau$ are revealed. In Section 4.3, experiments are performed to verify the results.

Chapter 5 shows a simple method for measuring the linewidth enhancement factor (LEF) using the stable SMI signals obtained based on the guideline designed in the previous chapters. By investigating the relationship between the feedback phase and the stable SMI signal, it is found that the LEF can be measured from the overlapping point of two stable SMI signals. Simulations and experiments verify the proposed method.

Chapter 6 summarizes the research activities in, and contributions made by this thesis, and at last suggests the possible future research topics. Note that, regarding the citations for my publications in this chapter. My journal papers are denoted by J1, J2, J3, and J4. My conference papers are denoted by C1, C2, C3 and C4.
Chapter 2 Determining the Stability Boundary for an SL with EOF

This chapter presents comprehensive studies on the stability of a semiconductor laser (SL) with external optical feedback (EOF), where the external target is stationary. In particular, based on numerical computation on the Lang and Kobayashi (LK) equations, the stability boundary under the condition of minimum linewidth i.e.,

\[ \phi_0 = \phi_s = 2p\pi - \arctan(\alpha), \]

where \( p \) is an integer, is investigated, revealing how it is influenced by three major parameters associated with SL operation conditions, including feedback strength (\( \kappa \)), external cavity length (\( L \)) and the injection current (\( J \)). In contrast to existing work in literature, the presented removed all the approximations and assumptions, hence resulting in relatively complete description of the stability boundary. The work presented leads to a number of important and interesting discoveries, e.g., (1) the possible stable area is broader than what is described by the existing work; (2) for long external cavities, the stability boundary can be described by a linear relationship between \( \kappa \) and \( L \); (3) on the stability boundary, the critical feedback strength (\( \kappa_c \)) and critical external cavity length (\( L_c \)) are discovered respectively being proportional and inversely proportional to \( J \) and \( \kappa \).
2.1 Determine the Stability Boundary

2.1.1 System Determinant

The stability of a system is usually analyzed based on the system determinant. For an SL with EOF, its system determinant is obtained based on analysis of the LK equations near the stationary solutions [30, 39, 43, 45, 46, 70-74] described by:

\[
\phi_s = \phi_0 - \frac{\kappa}{\tau_m} \tau \sqrt{1 + \alpha^2} \sin(\phi_s + \arctan \alpha) \tag{2.1}
\]

\[
N_s = N_0 + \frac{1/\tau_p - 2(\kappa/\tau_m) \cos(\phi_s)}{G_n(1 - \epsilon \Gamma E_s^2)} \tag{2.2}
\]

\[
E_s^2 = \frac{N_s/\tau_s - J/(eV)}{1/\tau_p + 2(\kappa/\tau_m) \cos(\phi_s)} \tag{2.3}
\]

where \( E_s, N_s \) and \( \omega_s \) represent the stationary solutions of LK equations without ignoring the nonlinear gain for electric field amplitude, carrier density and angular frequency respectively. Note that, by ignoring the nonlinear term associated with \( E(t) \) in Eq. (1.13), i.e., \( \epsilon = 0 \), the above stationary solutions are same as the ones used in [31, 39, 45, 46, 61], that is Eqs.(1.15)-(1.17) in Chapter one of this thesis. Assuming that \( E(t), \omega(t) \) and \( N(t) \) exhibit small deviations from above stationary solutions (denoted by \( \delta_E(t), \delta_\phi(t) \) and \( \delta_N(t) \) respectively), we can obtain the following linear differential equations from the LK Eqs. (1.10)-(1.12) in matrix form:

\[
\begin{bmatrix}
\frac{d\delta_E(t)}{dt} \\
\frac{d\delta_\phi(t)}{dt} \\
\frac{d\delta_N(t)}{dt}
\end{bmatrix}
= A \begin{bmatrix}
\delta_E(t) \\
\delta_\phi(t) \\
\delta_N(t)
\end{bmatrix}
+ B \begin{bmatrix}
\delta_E(t-\tau) \\
\delta_\phi(t-\tau) \\
0
\end{bmatrix} \tag{2.4}
\]

where
\[ A = \begin{pmatrix}
\frac{-\alpha N_s}{\tau_m} \cos(\phi_t) - \Delta T G_N(N_s - N_0)E_s^2 & \frac{-\alpha N_s}{\tau_m} \sin(\phi_t) E_s & \frac{1}{2} G_N(1 - \Delta T E_s^3) E_s \\
\frac{-\alpha N_s}{\tau_m} \sin(\phi_t) - \Delta T G_N(N_s - N_0)E_s & \frac{-\alpha N_s}{\tau_m} \cos(\phi_t) E_s & \frac{1}{2} \alpha G_N(1 - \Delta T E_s^3) \\
-2G_N(N_s - N_0)E_s(1 - 2\Delta T E_s^2) & 0 & - \frac{1}{\tau_s} - G_N(1 - \Delta T E_s^2) E_s^2
\end{pmatrix} \hspace{1cm} (2.5) \]

\[ B = \begin{pmatrix}
\frac{-\alpha N_s}{\tau_m} \cos(\phi_t) & \frac{-\alpha N_s}{\tau_m} \sin(\phi_t) E_s & 0 \\
-\frac{\alpha N_s}{\tau_m} \sin(\phi_t) & -\frac{\alpha N_s}{\tau_m} \cos(\phi_t) E_s & 0 \\
0 & 0 & 0
\end{pmatrix} \hspace{1cm} (2.6) \]

The system determinant (denoted by \( D(s) \)) of Eq.(2.4) is derived as:

\[ D(s) = \det(sI - A - e^{-\tau R} B) \]

\[ = s^2 + s^2 \left[ 2 \frac{\alpha N_s}{\tau_m} \cos(\phi_t) (1 - e^{-\tau R}) + \omega_{\tau_1} + \frac{1}{\tau_R} \right] \]

\[ + s \left[ \left( \frac{\alpha N_s}{\tau_m} \right)^2 (1 - e^{-\tau R})^2 + \frac{\alpha N_s}{\tau_m} \left( \frac{2}{\tau_R} + \omega_{\tau_1} \right) \cos(\phi_t) - \alpha \omega_{\tau_1} \sin(\phi_t) \right] (1 - e^{-\tau R}) + \frac{1}{\tau_R} \omega_{\tau_1} + \omega_{\tau_2}^2 \]

\[ + \frac{1}{\tau_R} \left( \frac{\alpha N_s}{\tau_m} \sin(\phi_t) \right)^2 (1 - e^{-\tau R})^2 + \frac{1}{\tau_R} \left( \frac{1}{\tau_R} \omega_{\tau_1} + \omega_{\tau_2}^2 \right) \cos(\phi_t) - \alpha \sin(\phi_t) \right] (1 - e^{-\tau R}) \hspace{1cm} (2.7) \]

where \( \tau_R \) is the damping time of the relaxation oscillation of an SL with EOF, \( \omega_{\tau_1} = \omega_{\tau_1}^{-1} + G_N E_s^2 (1 - \Delta T E_s^2) \), \( \omega_{\tau_1} = \omega_{\tau_3} \Delta T [G_N (1 - \Delta T E_s^2)] \), \( \omega_{\tau_2} = \omega_{\tau_3} (1 - 2\Delta T E_s^2) \)

and \( \omega_{\tau_3} = G_N G(N_s, E_s) E_s^2 \), where \( \omega_{\tau_3} \) is the relaxation resonance angular frequency of an SL with EOF. As \( E_s \) and \( N_s \) vary with \( J \), so do \( \tau_R \), \( \omega_{\tau_1} \) and \( \omega_{\tau_2} \).

If taking the assumptions as described in [30, 31, 45, 70-72] and ignoring the nonlinear gain, the system determinant \( D(s) \) in Eq. (2.7) becomes the same as the ones given in [30, 31, 45, 71, 72]. Therefore Eq. (2.7) provides a more comprehensive description on the stability property of an SL with EOF system.

### 2.1.2 Stability Boundary

With the system determinant in Eq. (2.7), we are able to work out the stability boundary based on the locations of zeros of Eq. (2.7) on the S-plane. The system is
stable if all the zeros are located on the left hand side of the S-plane. The zeros on the imaginary axis give the stability boundary of the SL with EOF.

The zeros of \( D(s) \) are defined as the roots of \( D(s) = 0 \), which are usually complex numbers, that is, \( s = \delta + j\Omega \). In order to work out all the zeros, we insert \( s = \delta + j\Omega \) in Eq. (2.7) and rearrange the right hand side by separating the real and imaginary parts and set them to be zero, yielding two equations as follows:

\[
3\Omega^2 \delta + (\Omega^2 - \delta^2) \left[ a_1 \left[ 1 - e^{-s\tau} \cos(\Omega \tau) \right] + a_2 \right] + \Omega e^{-s\tau} \sin(\Omega \tau) \left[ 2b_1 \left[ 1 - \cos(\Omega \tau) \right] + b_2 + 2\delta a_1 \right] - (\delta b_1 + c_1) \left[ 1 - 2e^{-s\tau} \cos(\Omega \tau) + e^{-2s\tau} \cos(2\Omega \tau) \right] - (\delta b_2 + c_2) \left[ 1 - e^{-s\tau} \cos(\Omega \tau) \right] - \delta b_3 - \delta^3 = 0
\]  

(2.8)

and

\[
\Omega^3 + (\Omega^2 - \delta^2) a_1 e^{-s\tau} \sin(\Omega \tau) - \Omega \left[ 3\delta^2 + 2\delta a_1 \left[ 1 - e^{-s\tau} \cos(\Omega \tau) \right] + 2\delta a_2 \right] - \Omega \left[ b_1 \left[ 1 - 2e^{-s\tau} \cos(\Omega \tau) + e^{-2s\tau} \cos(2\Omega \tau) \right] + b_2 \left[ 1 - e^{-s\tau} \cos(\Omega \tau) \right] + b_3 \right] - 2(\delta b_1 + c_1) e^{-s\tau} \sin(\Omega \tau) \left[ 1 - e^{-s\tau} \cos(\Omega \tau) \right] - (\delta b_2 + c_2) e^{-s\tau} \sin(\Omega \tau) = 0
\]

(2.9)

where

\[
a_1 = 2\frac{k}{\tau_m} \cos(\phi_j) \quad \text{and} \quad a_2 = \omega_{R1} + \frac{1}{\tau_R}
\]

(2.10)

\[
b_1 = \left( \frac{k}{\tau_m} \right)^2, \quad b_2 = \frac{k}{\tau_m} \left[ \frac{2}{\tau_R} + \omega_{R1} \cos(\phi_j) - \alpha \omega_{R1} \sin(\phi_j) \right] \quad \text{and} \quad b_3 = \frac{1}{\tau_R} \omega_{R1} + \omega_{R2}^2
\]

(2.11)

\[
c_1 = \frac{1}{\tau_R} \left( \frac{k}{\tau_m} \right)^2 \quad \text{and} \quad c_2 = \frac{k}{\tau_m} \left[ \frac{1}{\tau_R} \omega_{R1} + \omega_{R2}^2 \right] \left[ \cos(\phi_j) - \alpha \sin(\phi_j) \right]
\]

(2.12)

It can be seen from Eqs. (2.8) and (2.9) that the locations of the zeros depend on the values of the three parameters \( \kappa \), \( \tau \), \( J \) and the feedback phase \( \phi_j \). However, given the complexity of Eqs. (2.8) and (2.9), it is impossible to have analytical solutions for the zeros.

In order to have complete knowledge on the stability of the system, we can only utilize numerical computation to work out the zeros with respect to all the possible values of the parameters. In this Chapter, we mainly focus on the influence of \( \kappa \), \( \tau \) and \( J \), and will discuss the influence of the feedback phases in the following
chapters. To achieve numerically calculating the zeros, we considered that the parameters $\kappa$ and $J$ take 200 points equally spaced within the ranges $\kappa \in [0.000, 0.010]$ and $J \in [17.8 mA, 27.5 mA]$ respectively. Without loss of generality, we assume that wavelength of the laser is $780 nm$, and the external cavity length ranges from 0 to 0.3255$m$, implying that the external cavity length is 417308 wavelengths. Hence $\phi_s$ will fall into the range $\phi_s \in [0 rad \cdot s^{-1}, 4173 \cdot 200 \cdot 2 \pi rad \cdot s^{-1}]$. Within this range, we take 200 points equally spaced in such a way that $\phi_s = 4173 \cdot 2 \pi \cdot m - \arctan(\alpha)$, where $m=0, 1, 2, \ldots, 199$.

Such a choice of $\phi_s$ corresponds to the minimal linewidth mode (MLM) which was shown to be the most stable mode comparing to others [48, 72, 74] and the same choice was also made in [31, 43, 48, 71, 72]. Such choice also leads to $\omega_s = \omega_0 = 2.4166 \times 10^5 rad \cdot s^{-1}$ according to Eq. (1.15). Note that such a range corresponds to $\tau \in [0 ns, 2.17 ns]$. Now, we have a parameter sample space of $8 \times 10^6$ combinations of the three parameter values. Then the stability boundary is obtained by numerical computation following the procedure below:

- Step1: Choose a set of parameter values from the above parameter sample space.
- Step2: Determine $\delta$ by solving Eqs. (2.8) and (2.9). Note that $\delta$ can exhibit multiple values, and the right-most one (denoted as $\delta_0$) will be selected as it determines the stability of the system.
- Step3: Repeat Step 2 in an exhaustive manner with respect to all possible combinations of the parameters. This will yield all the zeros.
- Step4: For each of the zeros found in Step 3, check the value of $\delta$. If $\delta = 0$, record the corresponding values for $\kappa$, $\tau$ and $J$, yielding a point on the stability boundary in the three parameter space.
- Step 5: All the points found in Step 4 for $\delta = 0$ will build the complete stability boundary in the three parameter space $\kappa$, $\tau$ and $J$.

With the procedures above, we can work out the stability boundary now. Note that all other parameters take the values in Table 1-1.

In order to facilitate the comparison with existing results, parameters are scaled in the same way as in [31], where $\phi_s$ is normalized by multiplying a constant factor $\omega_p/(\omega_s \cdot 2\pi)$, where $\omega_p = 14.5 GHz$ corresponding to the relaxation oscillation frequency of the solitary laser when $J/J_{th} = 1.3$, where $J_{th}$ is the threshold current.

The results of numerical computation are depicted in Fig. 2-1, showing that the stability boundary is a surface with periodical fluctuations. The area above the surface is the unstable region.

![Figure 2-1: Stability boundary described in the 3-parameter space of ($\kappa$, $\tau$, $J$). The unstable region is above the surface.](image)

In order to demonstrate the physical meaning of the results shown in Fig. 2-1, we consider the relationship between $\kappa$ and $\tau$ with $J$ as a constant. Choosing $J/J_{th} = 1.3$, we have Fig. 2-2(a), where the system in the shaded areas is unstable. For comparison, the results for the same $J/J_{th}$ obtained in [31] is shown in Fig. 2-2(b) where the slashed areas are the unstable regions. By comparing the two figures, we
can see that the bottom line of the unstable area, that is, the critical feedback strength (denoted as $\kappa_c$) in Fig. 2-2(a) ($\kappa_c = 2.6\times10^{-3}$) is higher than that in Fig. 2-2(b) ($\kappa_c = 1.9\times10^{-3}$). Considering that the average range of $\kappa$ studied is in the order of $10^{-3}$ in existing work [31, 39, 45, 61, 70-72] for the stability analysis of an SL with EOF, such a difference is indeed considerable and should not be ignored.

![Figure 2-2: Stability boundary described by $\kappa$ and $\tau$ for a fixed $J/J_{th} = 1.3$. (a) Stability boundary given in this thesis on Fig. 2-1, (b) stability boundary given in [31].](image)

It should be noted that the result shown in Fig. 2-1 only holds for the case of minimum linewidth mode. In the situations where the condition is not met, the laser can still be unstable within the stability boundary. Therefore, the boundary shown in Fig. 2-1 presents the necessary but not sufficient condition for the stable operation of a laser.

To further verify our result shown in Fig. 2-2(a), we also studied the dynamics of $E(t)$ in the time domain using the 4-th order Runge-Kutta (RK) integration method on LK equations (1.10)-(1.12). We choose an unstable point from Fig. 2-2(b) marked by ‘♦’ with the parameters $\kappa = 0.0023$, $\omega_{ph}\tau/(2\pi) = 0.5$ and $J/J_{th} = 1.3$. This point is determined as stable shown in Fig. 2-2(a). Other parameters take the values in Table 1-1. The intensity of the SL output $I(t)$ is calculated as $I(t) = \overline{E^2}(t)/\overline{E}^2$ where $\overline{E}$ is the stationary solution of the electric filed amplitude for a solitary SL. The
waveform of $I(t)$ in Fig. 2-3 shows that a constant $I(t)$ is achieved after the transient dies away. Hence, the system is stable at this chosen point. This result coincides the conclusion reported in [32, 70] that nonlinear gain enhances the stability of the SL with EOF.

![Laser intensity waveform](image)

Figure 2-3: Laser intensity waveform of $I(t)$ for the chosen point, $\kappa = 0.0023$, $\omega_{\kappa} \tau/(2\pi) = 0.5$ and $J/J_{\text{th}} = 1.3$.

### 2.2 Influence of External Cavity, Injection Current and Feedback Strength

As the locations of zeros of the system determinant change with parameters $\tau$, $J$ and $\kappa$, it is important to investigate the influence of $\tau$, $J$ and $\kappa$ respectively on the stability boundary of an SL with EOF system. Because $\tau$ is directly dependent on the external cavity length $L$ (that is, $L = \tau \cdot c/2$, where $c$ is the speed of light), in the following we replace $\tau$ by its corresponding external cavity length $L$ which provides a more informative physical meaning related to an SL with EOF.

#### 2.2.1 Influence of External Cavity Length

Let us firstly look into the influence of $L$ on the stability boundary. To this end, we extract the relationship between $J$ and $\kappa$ from Fig. 2-1 by setting $\omega_{\kappa} \tau/(2\pi)$ at
six different values, including 1.0, 1.5, 2.0, 2.5, 3.0 and 3.5. So, equivalently, we have $L = 65\, mm, 97\, mm, 130\, mm, 162\, mm, 195\, mm, 231\, mm$. The extracted relationships are plotted in Fig. 2-4.

![Figure 2-4: Influence of $L$ on the stability described by $J$ and $\kappa$ when $J_{\text{th}} = 13.7 \, mA$.](image)

From Fig. 2-4, we are able to observe the following features:

1. The stability boundary shows a finger structure underneath the same asymptote for all the different values of $L$. As area above the asymptote is guaranteed to be stable, a safe choice can be made above the asymptote which gives a guideline for the selection of feedback level and injection current in order to have a stable system. For example, in order to have a stable system, large injection current is required for the case of high feedback strength.

2. Not all areas below the asymptote are unstable. However, with the increasing of $L$, the number of the fingers also increases, making the unstable area to increase as well and tend to fill the area under the asymptote (that is, the shaded area in Fig. 2-4). For the case of relative long external cavities (e.g., Fig. 2-4(f)), the stability boundary is close to and can be approximately described by the asymptote. However, for the case of a relative short cavity, some areas of considerable size under the asymptotes (e.g., Fig. 2-4(a)) are still stable.
2.2.2 Influence of Injection Current

We also examine the influence of injection current on the stability boundary. By setting $J$ to six different values, the relationships of $\kappa$ and $\tau$ can also be obtained from Fig. 1. In the relationships, parameter $\tau$ is replaced by $L$ using $L = \tau \cdot c/2$. So, we have the relationships between $\kappa$ and $L$ shown in Fig. 2-5 where the shaded still denotes the unstable region. From Fig. 2-5, we can also find the following features of the stability boundary in terms of $\kappa$ and $L$.

![Figure 2-5: Influence of $J$ on the stability described by $\kappa$ and $L$ when $J_{th} = 13.7mA$.](image)

1. An SL with very short external cavities (e.g., $L < 6.5mm$ in Fig. 2-5(b)) can always endure a very high feedback strength, which is consistent with the results reported in [31, 74, 75].

2. The stability boundary also demonstrates a finger structure. The width of all the fingers (denoted as $T$ in Fig. 2-5) is nearly the same with respect to $L$, but it decreases with the increase of $J$.

3. The minimum value of $\kappa$ appeared in the unstable region (see the horizontal line adhered to the stability boundary in Fig. 2-5), $\kappa$ increases with the increase of $J$. The value of $\kappa_c$ was also discussed in the previous work [31,
70, 72, 76]. Figure 2-6 shows the comparison of our results against the results obtained in [31, 70, 76] where the parameters are the same as shown in Table 1-1. It can be seen, the values of $\kappa_c$ we obtain numerically is higher than those presented by [31, 70, 76]. This means that the actual stable region is larger than those presented by existing literatures. Note that [72] did not discuss the influence of $J$ on $\kappa_c$.

2.2.3 Influence of Feedback Strength

Similarly, we can discuss the influence of $\kappa$ on the stability by observing the relationship of $J$ and $L$. The stability described by $J$ and $L$ is presented in Fig. 2-7 with six different values of $\kappa$. The shaded is still the unstable region.
Figure 2-7: Influence of $\kappa$ on the stability described by $J$ and $L$ when $J_{th} = 13.7\text{mA}$.

We conclude the stability boundary as follows:

1. A critical injection current ratio denoted by $J_c/J_{th}$ (shown as the horizontal line adhered to the stability boundary in Fig. 2-7) can be used to determine the stability boundary in the plane of $J$ and $L$. Obviously, a high feedback strength $\kappa$ requires a high value of $J_c$ to reach stable region. For the case with a relative high feedback strength (e.g., Fig. 2-7(f)), $J_c$ can be used as the stability boundary for designing a stable system. However, for the case with a relative low feedback strength, as more regions under the critical $J_c$ still showing stable character, we need to use Eqs. (2.8) and (2.9) to determine the stability.

2. The stability boundary exhibits ‘distorted’ finger structure. More and more fingers are observed with increasing $\kappa$.

3. An SL with a short external cavity is always stable. Let us denote the critical external cavity as $L_c$. From Fig. 2-7, we can see that $L_c$ depends on $\kappa$. Figure 2-8 gives the relationship of $\kappa$ and $L_c$ obtained from the numerical simulation. Approximately, $L_c$ is inversely proportional to the value of $\kappa$, that is,
\[ L_c = \frac{\pi c}{\omega_R} \left( \frac{8.0 \times 10^{-4}}{\kappa} + 0.1 \right) \]  \hspace{2cm} \text{(2.13)}

Figure 2-8: The relationship of \( \kappa \) and \( L_c \).

### 2.3 Summary

In this Chapter, a comprehensive analysis on the stability boundary of a single mode SL with a feedback phase condition \( \phi_b = \phi_s = 2 \pi p \arctan(\alpha) \) is presented, where \( p \) is an integer, corresponding to the minimum linewidth mode. The work starts from the Lang and Kobayashi (LK) equations, following by the derivation of a system determinant. An effective numerical computation is then employed to analyze the locations of zeroes of the system determinant, yielding the stability boundary with respect to three major parameters associated with SL operation condition, including feedback strength \( (\kappa) \), external cavity length \( (L) \) and the injection current \( (J) \) to the SL. The results presented in this paper provide helpful guidance for assessing the stability of an SL with EOF, which is important for designing SL systems in various applications of sensing and instrumentations. It should be stated again that the stability boundaries obtained are subject to the condition of minimum linewidth, and hence the boundaries are only able to distinguish “possible stable areas” and “unstable
areas”. In other words, lasers operating in the areas within the boundaries can be stable if the laser operates in the minimum width mode, and they will not be stable if operating in areas outside the limits.

The influence of the other parameter, i.e., the feedback phase will be discussed in the following Chapters which will provide a detailed stability analysis for the SMI.
Chapter 3  Stability Analysis for an SMI

As mentioned in Chapter one, when the system enters into the unstable state, the premise for deriving the stationary solutions of the Lang and Kobayashi (LK) equations will be no longer valid, thus leading to the actual behavior of the system can not be described by the existing SMI model. As the SMI can be considered as an SL with a time varying EOF, or in other words, with a time varying feedback phase $\phi_0$, in this chapter, the stability boundary of an SMI is obtained and presented in a two dimensional plane defined by $C$ and $\phi_0$ in order to have a clear and intuitive description of the stability of the SMI. By studying the features of the boundary, a critical $C$ (denoted as $C_{critical}$) is derived. If only an SMI is designed with a feedback level below $C_{critical}$, its sensing performance can be guaranteed and the behavior of the system can be described by the existing SMI model, otherwise by the LK model. An experimental method for determining the $C_{critical}$ is presented. The influence of the initial external cavity length $L$ and the injection current $J$ on the $C_{critical}$ are investigated from both simulations and experiments which show that stability can be enhanced by increasing either $L$ or $J$. Furthermore, three regions on the plane of $(C, \phi_0)$ are proposed to characterize the behavior of an SMI, including stable, semi-stable and unstable regions. We found that the existing SMI
model is only valid for the stable region, and the semi-stable region has potential applications on sensing and measurement but needs re-modeling the system by considering the bandwidth of the detection components.

### 3.1 The Stability Boundary Described by $C$ and $\phi_0$

The investigation of the stability for the SL with EOF system is usually based on the system determinant which is obtained based on the linearization of the LK equations near the stationary solutions [30, 31, 45, 46]. For a fixed set of operational parameters, the system is stable if all the zeros of the system determinant are located on the left hand side of the S-plane. In this situation, the corresponding stationary solutions are also stable. However, for the case of an SMI system, as $L$ varies with time, according to Eqs. (1.18) and (1.19), $\phi_i$ also varies with $\phi_0$, thus leading to the variations of all the stationary solutions. Therefore, in order to make sure the SMI system is stable, it requires that all the stationary solutions are stable during the variation of $L$. To better illustrate this situation, we recall the mechanism of the SMI described in Chapter 1 and present an example in Fig. 3-1 here which shows the relationship between $\phi_0$ and $\phi_i$ according to Eq. (1.19), where $C = 3$ and $\alpha = 3$.

The actual behavior of $\phi_i$ for a stable SMI signal has been investigated in [35]. It shows that $\phi_i$ will vary along the route of $A_1 \rightarrow B \rightarrow B_1 \rightarrow D \rightarrow D_1$ when $\phi_0$ increases, and it will however track the route of $D_1 \rightarrow C \rightarrow C_1 \rightarrow A \rightarrow A_1$ when $\phi_0$ decreases. Therefore, in this case, to generate a stable SMI signal demands that all the stationary solutions falling within the range of $[\phi_{i,A}, \phi_{i,B}]$, $[\phi_{i,A}, \phi_{i,D}]$ and $[\phi_{i,C}, \phi_{i,D}]$ are stable during the variation of $\phi_0$. Note that, the stationary solutions within the range of $[\phi_{i,A}, \phi_{i,B}]$ and $[\phi_{i,C}, \phi_{i,D}]$ are always unstable according to [31, 38, 39]. These unstable stationary solutions satisfy the following equation:
As mentioned above, the stability of a system is usually analyzed based on the system determinant. Based on the system determinant, [31, 45] showed that, to a good approximation, the stable condition of the system as well as of the stationary solutions satisfies the following condition:

\[-\alpha \kappa \sin(\phi_s) + \kappa \cos(\phi_s) \left[ 1 - 2 \left( \frac{\Omega}{\omega_R} \right)^2 \right] \leq \frac{\Omega}{2\tau_R} \frac{\tau_m}{\sin^2(\Omega\tau/2)} \]  

(3.2)

for all the values of \( \Omega \) (\( \Omega \) is defined as the imaginary part for a complex number in Laplace transform domain. The details can be found in [31, 45]), satisfying:

\[ \Omega^2 - \omega_R^2 = \frac{\Omega}{\tau_R} \cot(\frac{\Omega \tau}{2}) \]  

(3.3)

where

\[ \omega_R = \sqrt{\frac{G_N}{\tau_p}} \bar{E}_x, \quad \frac{1}{\tau_R} = \frac{1}{\tau_s} + \left( \tau_p + \frac{\alpha^2}{G_N} \right) \omega_R^2 \]  

(3.4)
In Eqs. (3.3) and (3.4), $E_s$ is the stationary electric field amplitude of the solitary laser which is determined by the injection current $J$ [45]. $\omega_r$ and $\tau_r$ are called the relaxation oscillation frequency and the damping time of the solitary laser [45].

When designing a stable SMI, it is important to know how to configure the system in terms of a proper feedback level and suitable movement range for the external target (or the feedback phase $\phi_0$). That is, we need to know the stable boundary for the parameter $C$ and $\phi_0$. Hence, we propose to describe the stability of an SMI system in the plane of $(C, \phi_0)$. To achieve this, let us replace $\kappa$ by $C$ (via $C = \frac{\kappa}{\tau_m} \sqrt{1 + \alpha^2}$) in the stable condition described by Eq. (3.2). Note that the amount of movement of target should be much smaller than the initial external cavity length. Then, equation (3.2) can be written as:

$$C \left\{ \cos(\phi_s) \left[ 1 - 2 \left( \frac{\Omega}{\omega_r} \right)^2 \right] - \alpha \sin(\phi_s) \right\} \leq \left( \frac{\Omega}{\omega_r} \right)^2 \frac{\tau \sqrt{1 + \alpha^2}}{2 \tau_r \sin^2(\Omega \tau/2)}$$

(3.5)

where the equal sign corresponds to the condition of stability boundary. What we want is to work out the relationship between $C$ and $\phi_0$ to describe the stability for the SMI. Let us consider the parameters appeared in Eq. (3.5). Generally, $\alpha$ is treated as a constant with the value from 3 to 6 [6, 21]. $\phi_s$ is the dependent variable of $\phi_0$, which can be determined by Eq. (1.19). Both $\omega_r$ and $\tau_r$ are dependent on the injection current $J$ according to Eq. (3.4) via $\tilde{E}_s$ [31, 45]. $\Omega$ is determined by $J$ and $\tau$ according to Eq. (3.3). So we can say, in Eqs. (3.5) and (3.3), $J$ and $\tau$ are two governing parameters that determine the stability boundary of an SMI system described by $C$ and $\phi_0$. Therefore, it is very important to investigate how the two parameters influence the stability boundary.
In order to work out the stability boundaries in the plane of \((C, \phi_0)\), a specific example is considered here to demonstrate how to determine the stable range of the SMI system using Eq. (3.5). We assume that the external target is located \(L_0 = 0.25m\) away from the SL which is injected with a fixed current of \(17.8mA\) (correspond to \(1.1J_{st}\)). The vibration amplitude of the target is \(\Delta L = 1.17\mu m\) (correspond to an amplitude of \(1.5\lambda_0\), where \(\lambda_0\) is the wavelength of the laser). With these assumption, the variation range of \(\tau\) will be in \([1.67 \times 10^{-6} - 7.80 \times 10^{-6}], (1.67 + 7.80 \times 10^{-6})\]ns, and \(\phi_0\) in \([(4.04 \times 10^6 - 18.85), (4.04 \times 10^6 + 18.85)]\)rad according to Eq. (1.18). The variation range of \(C\) is set to be \([0, 6]\). The computation for the stability of the SMI system is based on the above mentioned situation. In the range of \(\phi_0\) and \(C\), we take 400 equally spaced samples. The values for all the other parameters used during the computation are shown in Table 1-1. The procedures performed for determining the stability of the SMI system are shown as follows:

- Step1: Choose a set of parameter values from the above \(\phi_0\) and \(C\) space.
- Step2: Determine \(\phi_s\) by solving Eq. (1.19). Note that there can be more than one \(\phi_s\), and we discard the one satisfying Eq. (3.1).
- Step3: Determine \(\Omega\) by solving Eq. (3.3). Note that there can be more than one \(\Omega\), and we choose the one closest to \(\omega_k\) [31].
- Step4: Choose one of the \(\phi_s\) determined in Step2.
- Step5: Substitute \(C, \phi_s, \Omega\) and \(\tau = \phi_0/\omega_0\) into Eq. (3.5), test if Eq. (3.5) is satisfied.
- Step6: Go back to Step 4 until all the \(\phi_s\) are chosen.
Step 7: If all the $\phi_i$ determined in Step 2 satisfy Eq. (3.5) for $\phi_0$, $C$, and $\Omega$, record the corresponding values for $\phi_0$ and $C$, yielding a point on the stability boundary in the two dimensional plane of $\phi_0$ and $C$.

The above procedures will be repeated in an exhaustive manner with respect to all possible combinations of $\phi_0$ and $C$. Thus the final stability boundary can be constructed in a two dimensional plane of $\phi_0$ and $C$. Figure 3-2 shows the reconstructed result. Furthermore, the result of the stability boundaries can be obtained under different values for the parameter pairs of $J$ and $\tau$. As $\tau$ is associated to $L$ via $\tau = 2L/c$, instead of $\tau$, we present the influence of $L$ on the stability boundary in order to provide a more informative physical meaning related to the SMI. In Fig. 3-2(a), the boundary is computed with three different $J$ for a fixed $L = 0.25m$. In Fig. 3-2(b), the boundary is computed with three different $L$ for a fixed $J = 1.3J_{th}$, where $J_{th}$ is the threshold injection current. The area below each boundary is the stable region. In Fig. 3-2, we also indicate the different feedback regimes defined by the value of $C$, where weak feedback regime is for $C < 1$, moderate feedback regime for $1 < C < 4.6$, and strong feedback regime for $C > 4.6$ [35].

Figure 3-2: Influence of $J$ and $L$ on the stability boundary of an SMI. (a) for a fixed $L = 0.25m$ with different $J$, (b) for a fixed $J = 1.3J_{th}$ with different $L$. 

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From Fig. 3-2, the following features of the stability boundary can be found:

1. The stability boundary shows periodic fluctuation with a period of $2\pi$ equivalent to a half wavelength movement of the external cavity.
2. The system is always stable at a weak feedback regime and may enter unstable when the feedback level is moderate or high feedback regime.
3. To achieve a stable status at a moderate or high feedback regime, we can either increase the injection current or choose a long external cavity.

Figure 3-3 shows a boundary when $J = 1.1J_{th}$ and $L = 0.35m$. In Fig. 3-3, we define three different regions referred to as stable, semi-stable and unstable respectively according to the dynamic behavior of an SMI described as below. As the existing SMI described in Eqs. (1.19) and (1.20) is not able to describe the actual behavior of an SMI when the system enters the region above the stability boundary, we need to start from LK equations to investigate the output power of an SMI, i.e., $E^2(t)$. The calculation of $E^2(t)$ by the LK equations uses the 4-th order Runge-Kutta integration algorithm.

![Stability Boundary Diagram](image-url)

**Figure 3-3:** The stability boundary of an SMI when $J = 1.1J_{th}$ and $L = 0.35m$.

Now, let us study the features of the SL output power (below we will call it as an SMI signal) obtained by the LK model at the different region defined in Fig. 3-3. We
choose $C = 1.5, C = 2.5, C = 4.0$ and $C = 9.0$ which respectively correspond to the stable, semi-stable, semi-stable and unstable regions. Other parameters in the LK model take the values shown in Table 1-1. The SMI signal $g(t)$ is calculated as the normalized $E^2(t)$. Supposing that the external target moves at a sinusoidal law with same vibration trace of Fig. 1-4(a), for the purpose of comparison, Fig. 3-4 presents the SMI signals predicted respectively by the LK model shown from Figs. 3-4(b)-(e) and the existing SMI model from Figs. 3-4(g)-(j). In each row of Fig. 3-4, the two SMI signals are obtained under the same operation condition, i.e., the same $C$ value.

![Figure 3-4: SMI signals predicated by the LK model and the existing SMI model respectively. (a) and (f): movement trace of the external target, (b)-(e): SMI signals obtained by the LK model with $C = 1.5, C = 2.5, C = 4.0$ and $C = 9.0$ respectively. (g)-(j) SMI signals obtained by the existing SMI model with $C = 1.5, C = 2.5, C = 4.0$ and $C = 9.0$ respectively.]

According to the LK model, obviously, only Fig. 3-4(b) with $C = 1.5$ shows a stable SMI signal which can also be described by using the existing SMI model shown in Fig. 3-4(g). $C = 1.5$ indicates the SMI is stable in Fig. 3-3. In the region with
1.8 \( < C < 8.4 \), simulations using the LK model shows that the SMI signal contains a high frequency oscillation close to the relaxation oscillation frequency of the solitary laser. Figures 3-4(c) and 3-4(d) give the two SMI signals at the semi-stable region, which are more complicated than Fig. 3-4(b). Hence, the behaviors described by the LK model are different from the ones by the existing SMI model resulting from the stationary solutions of the LK model. It is very interesting to observe that, even for a complicated waveform shown in Figs. 3-4(c) and 3-4(d), the movement information of the target is still visible. This is why we call the region \( 1.8 < C < 8.4 \) as the semi-stable region. With the aid of signal processing technology, the system operating at the semi-stable region can also be used for sensing and measurement.

In order to achieve this, the SMI waveform needs to be investigated to reveal its relationship to the movement of the target. Also, due to the limit in the rising time of the photodiode (PD) packaged at the rear of the SL, it may not be able to detect the details of the high frequency SMI waveform in the semi-stable region, and the SMI signal observed will be a distorted version of the high frequency waveform. A complete theoretical model is required to describe the influence of the limited bandwidth of the PD on the high frequency SMI waveform with the aim to detect the movement of the target from the distorted SMI waveform. Obviously, extensive work is required and could be an interesting topic for future research.

When \( C > 8.4 \), it is hard to see the vibration information from the SMI waveform, implying that the SMI system may lose its sensing ability. In this situation, the spectrum of laser is dramatically broadened, which is beyond the scope of this paper. Fig. 3-4(e) shows the SMI signal with \( C = 9 \) indicating that the SMI system is not suitable for sensing applications.

Note that the SMI model is derived from the LK equations by letting \( \frac{dE(t)}{dt} = 0 \), \( \frac{d\phi(t)}{dt} = \omega_c - \omega_b \) and \( \frac{dN(t)}{dt} = 0 \). These conditions will no longer be valid when the system enters semi-stable or unstable region, e.g., the relaxation oscillation will become undamped [77, 78]. In summary, for the system working in the semi-stable
or unstable region, the existing SMI model cannot be used, but we can still use the fundamental LK model to describe the system behaviour.

3.2 Critical Feedback Level Factor

Furthermore, from the stability boundary shown in Figs. 3-2 and 3-3, We noticed that from the stability boundary a critical \( C \) (denoted by \( C_{\text{critical}} \)) can be defined under which the system is guaranteed to be stable. As \( C_{\text{critical}} \) corresponds to the bottom on the stability boundary, by performing differentiation with respect to \( \phi_0 \) on both sides of Eq. (3.5), we can obtain \( \phi_i \) for \( C = C_{\text{critical}} \) as follows:

\[
\phi_i = \arctan \left( \frac{\alpha}{2(\Omega_i/\omega_r)^2 - 1} \right) + p\pi \tag{3.6}
\]

where \( p \) denotes an integer. As \( C_{\text{critical}} \) lies on the stability boundary described by Eq. (3.5), inserting Eq. (3.6) into Eq. (3.5), thus we can obtain:

\[
C_{\text{critical}} = \left( \frac{\Omega}{\omega_r} \right)^2 \frac{L\sqrt{1+\alpha^2}}{\tau_k \sin^2(\Omega L/c)} \sqrt{\left[ 2(\Omega_i/\omega_r)^2 - 1 \right]^2 + \alpha^2} \tag{3.7}
\]

for all the values of \( \Omega \) satisfying Eq. (3.3). Equation (3.7) can be used to estimate \( C_{\text{critical}} \) when designing an SMI system if the values of the parameters listed in Table 1-1 are available.

From Eq. (3.7), we can see that there are three parameters influencing \( C_{\text{critical}} \), i.e., \( \alpha \), \( L \) (via \( \tau \) and \( \Omega \)) and \( J \) (via \( \omega_r \) and \( \tau_r \)). Here, we mainly focus on the influence of \( L \) and \( J \). Using Eq. (3.7), we are able to determine stability boundary for a SMI in a 3-parameter space with respect to \( C_{\text{critical}} \), \( L \) and \( J \). To determine the boundary, the following computation procedures have been performed:
Step 1: Set \( L \in [0.05, 0.40]m \) and \( J \in [17.8, 37.8]mA \), both with 200 equally spaced samples. All the other parameters are kept as the same as set previously. Note that, in this thesis, we only focus the range of \( L \) in long external cavity range. An external cavity is said to be short when \( L < c/(2\omega_k) \), as generally assumed in literatures [63, 67, 72, 74, 75].

Step 2: Choose a set of parameter values from \( L \) and \( J \).

Step 3: Solve Eq. (3.3) for \( \Omega \) and then calculate \( C_{critical} \) using Eq. (3.7). Note that there can be more than one \( \Omega \), and we choose the one closest to \( \omega_k \) [31].

Step 4: Record the value of \( C_{critical} \) and corresponding \( L \) and \( J \), yielding a point on the stability boundary in a three dimensional space.

Step 5: Repeat Step 2 until all the combinations of \( L \) and \( J \) are chosen.

Figure 3-5 shows the complete stability boundary constructed using the above computation procedures.

![Figure 3-5: Stability boundary described in the 3-parameter space of \((C_{critical}, J, L)\).](image)

The unstable region is above the surface.

From Fig. 3-5, we can see that for a fixed external cavity length \( L \), the boundary shows a rising finger-structure. The number of fingers increases with the increase of \( L \). The features of the boundary for a fixed injection current \( J \) are similar. Figure
3-6 shows an example of the stability boundary in the plane of $J$ and $C_{\text{critical}}$ extracted from Fig. 3-5 when $L = 0.05\,m, 0.10\,m, 0.15\,m, 0.20\,m, 0.25\,m, 0.30\,m$.

Figure 3-6: The stability boundary described by $J$ and $C_{\text{critical}}$ for long external cavities.

The shaded area in Fig. 3-6 is the unstable area. The area beneath the thin line shown in Fig. 3-6 is guaranteed to be stable. Therefore a safe choice can be made under the thin line. From Fig. 3-6, it is interesting to notice that with the increase of $L$, the unstable area tends to fill the area above the thin line. Meanwhile, the stability boundary also tends to be described by the thin line. For a relatively long external cavity (i.e., $L \gg c/(2\omega_p)$), $\Omega \approx \omega_R$ [31]. To a good approximation, we substitute $\Omega = \omega_R$ into Eqs. (3.3) and (3.7), and Eq. (3.7) reduces to the same form as derived in [31, 76]:

$$C_{\text{critical}} = \frac{L}{c\tau_R}$$

(3.8)

In order to have more insight into $C_{\text{critical}}$ for long external cavities, in Eq. (3.8), we replace $\tau_R$ using Eq. (3.4). By doing so, Eq. (3.8) becomes:

$$C_{\text{critical}} = \frac{J}{eV} \left[ \left( \frac{N_0 G_N \tau_p + 1}{G_N \tau_p \tau_s} \right) \left( G_N \tau_p + \delta \right) + \frac{1}{\tau_s} \right]$$

(3.9)
which is the equation for the thin lines shown in Fig. 3-6. Therefore, a safe choice for obtaining a stable SMI signal can be made using (3.9) instead of Eq. (3.7).

### 3.3 Determining Critical Feedback Level Factor

In this section, we present a experimental method to determine $C_{\text{critical}}$, and investigate how $L$ and $J$ influence $C_{\text{critical}}$. Figure 3-7 shows the experimental SMI setup for such investigation. A 0.8μm band single mode GaAlAs triple quantum well SL (HL8325G) from Hitachi was employed in the experiments. The temperature of the SL was stabilized to within 0.01°C by using a temperature controller (TC) (model TED200). The injection current to the SL was controlled by an SL controller (LC) (model LDC2000). The light emitted from the SL was focused by a lens and split into three beams by a beam splitter (BS). One beam was directed to the external target and then was reflected back to the SL internal cavity. The second beam was collected by an optical spectrum analyzer (OSA) (Advantest Q8347) for monitoring the optical spectrum of the SMI. The third beam was passed to a fast external photodiode (EPD) (PDA8GS which is provided by the Thorlabs) with an 9.5GHz bandwidth in order to detect the dynamics of the laser intensity. The detected laser intensity is then transferred into a real time oscilloscope (Tektronix DSA70804) with an 8GHz bandwidth and 12.5GS/s sampling rate which is high enough to capture the time series of the laser intensity. A piece of mirror was attached on the surface of a loudspeaker so that to achieve a strong optical feedback level. The loudspeaker was driven by a sinusoidal signal with 75Hz generated by a signal generator. The optical feedback level of the SMI system was adjusted by an attenuator inserted in between the BS and the loudspeaker. Note that the SMI signals can also be detected by the PD packaged at the rear of the SL and the acquired by our designed detection circuit.
Figure 3-7: Experimental setup for investigating the stability of the SMI.

When the SMI is in the stable region, the observed optical spectrum is clean showing the SL operating on only one single mode as shown in Fig. 3-8(a). When the system enters into semi-stable region, the relaxation oscillation (RO) of the laser becomes undamped. In this case, a subpeak corresponding to the RO frequency appears near to the main peak of the optical spectrum [77, 78].

Figure 3-8(b) shows the optical spectrum observed when the system in the semi-stable region. As our spectrum analyzer has a relative low resolution with 0.002nm, it is not able to separate clearly the subpeak from the mean peak. However, it can still tell us the appearance of the RO of the laser with frequency about 2-4GHz, therefore determining the stability of the system changes from stable to semi-stable. One may argue that these subpeaks may be related to the external cavity modes. However, we can confirm that the subpeaks in Fig. 3-8(b) correspond to the RO frequency by the following calculation (The principle for identifying the RO frequency is based on Fig.3(2) in [77]).

As an example, we consider the case with $C = 4.6$ to $7$, which corresponds to the strong feedback regime. In this case, there are 5 external cavity modes according to the phase equation but only 3 of them are possible oscillating modes (the other 2 are anti-modes [31, 73, 79]). The frequency interval between two adjacent modes is $\Delta f = c/2L$ [77].

For the case of Fig. 3-8(b), $L = 0.25m$, thus $\Delta f = 600MHz$. Therefore, the distance between the main peak and its adjacent oscillating external cavity mode is
$2\Delta f = 1.2\text{GHz}$. However, from Fig. 3-8(b), we can see that the right subpeak is around $\lambda_1 = 827.740\text{nm}$ which corresponds to the optical frequency $f_1 = c/\lambda_1 = 362432.64\text{GHz}$. The main peak is $\lambda_2 = 827.731\text{nm}$ which corresponds to the frequency $f_2 = 362436.58\text{GHz}$. Therefore, the frequency interval is $\Delta f = f_2 - f_1 = 3.94\text{GHz}$.

Obviously according to the above calculation, the location of the subspeaks on Fig. 3-8(b) does not correspond to the external cavity mode which distance to the main peak is $1.2\text{GHz}$.

![Two optical spectra obtained with $L = 0.25m$ and $J = 1.7J_\text{th}$ for (a) the stable region, (b) the semi-stable region.](image)

In the following experiments, we varied the feedback level from weak to strong with the aid of the attenuator, the single mode spectrum displayed on the spectrum analyzer will thus change. Once the subpeaks were first observed from the spectrum, the SMI should be at the point of the critically stable. Then, we apply a tiny change to the attenuator by reversely rotating it 2 degrees. A stable SMI signal very close to the critical level can thus be obtained and we used the signal to calculate the parameter $C$ by the method presented in [28]. The $C$ calculated is approximately represented for $C_{\text{critical}}$.

Based on above experimental method for estimating $C_{\text{critical}}$, the influence of $J$ and $L$ respectively on the $C_{\text{critical}}$ are also investigated. Figure 3-9 (a) shows the
$C_{critical}$ goes up with the increase of the injection current for a fixed $L = 0.25 \text{m}$. Figure 3-9 (b) shows the longer the external cavity the higher $C_{critical}$ for a fixed $J = 1.3 J_{th}$. Obviously, the experimental results show the same trend with the simulation analysis shown in Fig. 3-2, that is, $C_{critical}$ can be increased by either increase of $J$ or $L$, thus leading to stability enhancement of the SMI. We note that the experimental results obtained do not exactly agree with simulations. The reason is that the actual values of the internal parameters for the SL used in the experiment are different from the parameters shown in Table 1-1 for the simulation.

![Figure 3-9](image)

**Figure 3-9:** Experimental results with the aid of the OSA. (a) for a fixed $L = 0.25 \text{m}$, (b) for a fixed $J / J_{th} = 1.3$.

Furthermore, we observe the dynamics of the SMI in the three different regions, i.e., stable, semi-stable and unstable, using the oscilloscope in order to verify the results shown in the left column of Fig. 3-4. The external target is placed 0.35m away from the SL and the injection current is fixed as $J = 1.1 J_{th}$. By increasing the feedback level with the aid of the attenuator, we record the time series of the laser intensity in Fig. 3-10. From Fig. 3-10, we can see that the waveform of the SMI signal in semi-stable region is quite similar to the simulation result obtained using the LK equations, which contains high frequency component close to the relaxation oscillation frequency of the SL. Note that we are unable to observe the SMI signal in
the stable region due to low SNR of the system, and thus unable to determine the onset of semi-stable region using the oscilloscope.

Figure 3-10: Experimental observations of the dynamics of the SMI signals using the fast oscilloscope with an 8GHz bandwidth and 12.5GS/s sampling rate. (a) the target movement trace (b) the SMI signal in stable region (c)-(d) the SMI signals in semi-stable region (e) the SMI signal in unstable region.

Based on the results obtained in Fig. 3-10 (a)-(e), we also simultaneously recorded the SMI signals using the detection circuit which bandwidth is 3MHz. Figure 3-11 shows the SMI signals correspond to the cases shown in Fig. 3-10 respectively.
Figure 3-11: Experimental observations of the dynamics of the SMI signals using the detection circuit which bandwidth is 3MHz. (a)-(e) correspond to the cases shown in Fig. 3-11(a)-(e) respectively.

From Fig. 3-11 (b)-(e), it is interesting to notice that the signals observed by the data acquisition unit in the three different regions all look like the stable SMI signals described by the existing SMI model. However, it should be pointed out that the bandwidth of the PD packaged at the rear of the SL is not as high as the EPD, thus leading to the disappearance of the high frequency component of the SMI signals shown in Fig. 3-10(b) to (e) and giving the distorted SMI signals or say untrue SMI signals shown in Fig. 3-11(b)-(e) which will negatively affect the sensing and measurement performance of the SMI, e.g., SMI based $\alpha$ measurement. Therefore, as we mentioned before in Section 3.1.3, a complete theoretical model is required to describe the influence of the limited bandwidth of the PD as well as the detection
circuit on the high frequency SMI waveform with the aim to achieve accurate sensing and measurement results.

### 3.4 Summary

The stability of an SMI is investigated in this Chapter. It is found that, to achieve a stable SMI signal for sensing purpose under moderate or strong feedback level, we can either increase the initial external cavity length or the injection current to the laser. By monitoring the spectrum of the SMI, a critical optical feedback factor $C_{critical}$ can be determined approximately. Under the $C_{critical}$, an SMI is guaranteed to be stable and the existing SMI model can exactly describe the waveform of an SMI signal. Furthermore, we presented another two regions on the plane of $(C, \phi_0)$ called semi-stable and unstable with boundaries corresponding to the undamped relaxation oscillation and the chaos status respectively. We found that semi-stable region has potential applications on sensing and measurement but may require further signal processing technology. The results presented in this Chapter provide useful guidance for designing various SMI based sensing and instrumentations.
Chapter 4 Influence of the Injection Current on the Stability of an SMI

In this Chapter, the influence of another SMI parameter, i.e., the injection current, on the stability of an SMI is both numerically and experimentally explore. In fact, the idea of work presented in the chapter is originally inspired by the two recently published papers [61] and [40]. In [61], Green found that, with the variation of \( \phi_0 \) within the range of \( 2\pi \), the system is always stable for some values of the injection current \( J \) (or called band stable phenomenon) when \( L \) is short and \( \kappa \) is weak, but no explanation was given in [61]. Later in 2013, [40] predicted that the always stable region found in [61] is due to the interaction between the excitation of relaxation oscillation (RO) of the lasers and \( L \), and discovered that the region exists when the product of RO frequency \( (v_{ro}) \) and \( \tau \) equals an integer. However, we notice that both [61] and [40] ignore the effect of nonlinear gain which is very important for describing and modelling an SL, and can provide a good agreement between numerical results and experimental results. It is also well known that the nonlinear gain has the effect of stabilizing the dynamics of an SL with an external target.
The results presented in this chapter show that the stable region is significantly wider than the region predicted previously, and such phenomenon is caused by the nonlinear gain inherently existed in the lasers. Furthermore, the relationship between the critical injection strength \( J_c \), above which guarantees a stable SMI, and the other two important system parameters, i.e., \( C \) and \( L \), is investigated. The results presented in this chapter are also useful guidance for designing a feedback phase independent stable semiconductor lasers with optical feedback.

### 4.1 Stability Boundary Described by \( J \) and \( \phi_0 \)

In this chapter, beginning with a revisit of the result obtained in [40], we perform the simulation with the same parameters’ values adopted in [40] by using the complete system determinant derived in Chapter 2 of this thesis, where the nonlinear gain was included. Figure 4-1 (a) presents the result obtained in [40] which does not consider the nonlinear gain (see Fig. 1 in [40]). In Fig. 4-1(a), the gray region is the always stable operation region for an SMI system where \( v_{RO} \tau = 1, 2, ..., \) and the dark shaded region is the unstable region. The vertical axis \( P \) is the injection strength which is defined as \( P = (J - J_{th}) / J_{th} \) [40]. From Fig. 4-1(a), we can see that to achieve a stable SMI system, we need to carefully choose \( P \) to meet the condition of \( v_{RO} \tau = 1, 2, .... \)

Figure 4-1 (b) shows the stability region obtained using the complete system determinant derived in Chapter 2 by considering the nonlinear gain. Clearly, comparing with Fig. 4-1(a), the unstable region is significantly suppressed, and the critical pump strength \( P_c \) (dotted line in Fig. 4-1(b)), above which guarantees an stable SMI system, is slightly lower than the condition obtained in [40], i.e., \( v_{RO} \tau = 1 \).
Figure 4-1: Stability region of an SMI system (or band stable phenomenon) when $C = 0.84$ and $L = 4.5cm$. (a) stability region obtained in [40], (b) stability region obtained in this paper using the complete system determinant derived in Chapter 2 of this thesis.

To verify our result, i.e., Fig. 4-1(b), we intentionally choose three values of $P$ on Fig. 4-1(b), respectively are $P = 2$, $P = 0.8$, and $P = 0.1$, which correspond to the case that SMI is unstable, stable and unstable in Fig. 4-1(a). Using these three values of $P$ and keeping all the other parameters as the same as used in Fig. 4-1, we numerically solve the LK equations by letting the feedback phase varying as the same trace as shown in Fig. 1-4(a), and Fig. 4-2 shows the simulation results. From Fig. 4-2, we can see that when $P = 2$, the SMI is stable which proves that our result is correct.

With the result shown in Fig. 4-1(b), it is naturally to think about if and how $P_c$ is associated with other two system parameters, i.e., $C$ and $L$, because, for an SMI system, it is usually preferred to be operated under a relatively high feedback strength and long external cavity, i.e., $C > 1$ and $L > c/(2\nu_{RO})$ [67]. However, due to the complexity of the system determinant i.e., Eq. (2.7), it is difficult to derive the analytic expression of $P_c$. We have to numerically solve Eq. (2.7) to see how $P_c$ varies with $C$ and $L$, which will be presented in Section 4.2.
From the previous comparison shown in Fig. 4-1 as well as the result in Fig. 2-2, we can see that the nonlinear gain plays an important role in describing the dynamics of the SMI, and it can greatly suppress the unstable region of the SMI. Therefore, such important factor should not be ignored and should be paid attention. To illustrate the importance of the nonlinear gain, we here investigate how the nonlinear gain will influence the stability of the SL with EOF and thus revealing it importance. Let us firstly review some background knowledge of how to determine to stability of the system as discussed in Chapter 2. With the system determinant in Eq. (2.7), we are able to work out the locations of zeros of $D(s)$ on the S-plane. The zeros of $D(s)$ are defined as the roots of $D(s) = 0$, which are usually complex numbers and can be found using various techniques. Note that for a fixed set of parameters, Equation (2.7) has multiple roots. Among the roots, the right most root (denoted as $s_0$) can be used to determine the stability of the system. The system is stable if $s_0$ lies in the left side of the S-plane. The stability limit is reached when $s_0$ is on the imaginary axis.

Figure 4-3(a) shows the locus of $s_0$ by varying the nonlinear gain confinement factor $\Gamma$ when $\omega_h \tau/(2\pi) = 0.5$, $J/J_{th} = 1.3$, $\phi_0 = -\arctan(\alpha)$ and $\kappa = 0.005$. 
Figure 4-3: The locus of $s_0$ on the $S$-plane by varying $\Gamma$ when $\tau = 0.21 \text{s}$, $J/J_m = 1.3$ and $\kappa = 0.005$. (b)-(d) are the corresponding normalized laser intensity for $\Gamma = 1.50$, $\Gamma = 0.80$ and $\Gamma = 0.05$ respectively.

The variation range of $\Gamma$ is from 0.00 to 1.50 with 31 samples. All the other parameters are adopted from Table 1-1. From Fig. 4-3(a), we can see that from $\Gamma = 0.05$ to $\Gamma = 0.80$, the locus of $s_0$ moves horizontally from the right half side of the $S$-plane to the imaginary axis, which means the variation of $\Gamma$ in this range does not change the RO frequency. Keep increasing $\Gamma$ from 0.80 to 1.50, $s_0$ moves to the left half side of the $S$-plane and the RO becomes damped, therefore leading to a stable laser output. Figure 4-3(b)-(d) shows the corresponding normalized laser intensity output obtained by numerically solving LK equations for $\Gamma = 1.50$, $\Gamma = 0.80$ and $\Gamma = 0.05$ respectively, which verifies the correctness of the system determinant we derived. The method for solving LK equations is still the 4th order Runge-Kutta integration method. The intensity of the SL output $I(t)$ is calculated as the same shown in Chapter 2, i.e., $I(t) = E^2(t)/\bar{E}_i^2$. From Fig. 4-3, we can see that
the nonlinear gain does play an important role in determining the stability of the system.

With the locus of $s_0$, we are able to present the stability limit in a two dimensional plane of $\kappa$ and $\Gamma$ (shown as in Fig. 4-4) by observing the locus of $s_0$ for two different values of $\tau$, i.e., $\omega_0 \tau/(2\pi) = 0.3, 0.6$, when $J/J_\infty = 1.3$. The shaded area in Fig. 4-4 is the unstable region. The variation ranges for $\kappa$ and $\Gamma$ are chosen as $\kappa \in [0.000, 0.015]$, $\Gamma \in [0.0, 1.2]$. For each of the parameters, we take 200 samples by equally-spaced sampling. From Fig.4-4, we can see that, with the increase of $\Gamma$, the value of $\kappa$ for guaranteeing a stable system also increases. This phenomenon just coincides with the results show in the previous chapter as well as in [70]. When $\tau$ increases, the relationship of $\kappa$ and $\Gamma$ changes from linear to distorted while keeps the size of the unstable region almost unchanged.

![Figure 4-4](image)

Figure 4-4: The stability limit of $\kappa$ and $\Gamma$ for different values of $\tau$ when $J/J_\infty = 1.3$

(a) $\omega_0 \tau/(2\pi) = 0.3$, (b) $\omega_0 \tau/(2\pi) = 0.6$.

### 4.2 Critical Injection Current

In this section, we firstly investigate the band stable phenomena for the case with a fixed initial external cavity length. Figure 4-5 shows the stability region on the plane of $(P, \phi_0)$, which is obtained by using $D(s)$ derived in Chapter 2 with six different
values of $C$ but for a fixed $L = 0.15m$. Note that, as $\phi_0$ is directly related to the movement of the external target for an SMI sensing system, we are more interested in how $\phi_0$ influences the stability. Hence we choose to use $\phi_0$ to represent the x-axis in the plane rather than $\phi$ according to Eq. (1.19).

From Fig. 4-5, it can be seen that the band stable phenomena occurs when the system operates at moderate or high feedback levels (e.g., $C > 1$). Let’s call the band stable related region as the feedback-phase-dependent stable areas. Obviously, such region occupies more and more space in the plane of $(P, \phi_0)$ with the increase of $C$.

Moreover, comparing with Fig. 4-1(a), the unstable region is significantly suppressed due to the inclusion of the nonlinear gain in $D(s)$ presented in chapter 2.

Then, we study the cases with a fixed $C$ but varying $L$. Figure 4-6 shows the results for six different values of $L$ when $C$ is fixed to a moderate feedback level ($C = 2$). From Fig. 4-6, we can see that the band stable related region can be compressed to a small range of $P$ with the increase of $L$. 

![Figure 4-5: Influence of $C$ on the stability region of an SMI with $L = 0.15m$.](image)
Figures 4-5 and 4-6 also indicate that there is a relative large stable region located above the band stable related region when the system operates at a moderate feedback level and with a relatively long external cavity. Hence, it is significant to determine the critical injection current \( P \) so that an SMI can operate in a full stable region without the influence of the feedback phase \( \phi_0 \). Apparently, the operation parameters \( C \) and \( L \) will determine \( P_c \). We made intensive calculations by varying the feedback level from weak (\( C = 0.5 \)) to moderate (\( C = 4.5 \)) and the cavity length from \( L = 0.1m \) to \( L = 0.4m \), from which the relationships between \( P_c \) and \( C \), and \( P_c \) and \( L \) are obtained and shown in Fig. 4-7. From Fig. 4-7(a), we can see that \( P_c \) goes up with the increase of \( C \). This means, to achieve a stable SMI at a high feedback level, a large injection current is required. Fig. 4-7 (b) shows that \( P_c \) goes down with the increase of \( L \). This result indicates us that we can have a stable SMI system operate with a low injection current when the external cavity is long.
4.3 Determining Critical Injection Current

We make the following experiments to verify the simulation results shown in Fig. 4-7. The experiments are carried out with the setup shown in Fig. 3-7. The SL used in the experiment is still HL8325G and its temperature is maintained at $25 \pm 0.1^\circ C$ with $J_{th} = 40mA$ and maximum operating injection current of $J_{max} = 120mA$. The laser focused by a lens hits a mirror surface glued on a loudspeaker. The loudspeaker is driven by a sinusoidal signal with 220Hz and peak-peak voltage of 400mV. This harmonic vibrating target can cause the feedback phase varying from 0 to $20\pi$. The stability of the system is monitored by an optical spectrum analyzer (OSA). The detail approach for monitoring the stability can refer to Section 3.2 in this thesis. An attenuator is inserted in between the beam splitter (BS) and the loudspeaker in order to adjust the feedback level and thus $C$, and a translation stage is used to hold the loudspeaker and vary the external cavity length $L$.

The first group of experiments for investigating the relationship between $P_c$ and $C$ for a fixed $L = 0.15m$ follows the procedures below:

- Step1: Set the injection current to a high value $J = 100mA$, that is $P = 1.5$. 

![Figure 4-7: Simulation results of the (a) relationship between $P_c$ and $C$ when $L = 0.15m$, and (b) relationship between $P_c$ and $L$ when $C = 2$.](image)
• Step2: While keeping the SMI system stable, adjust the attenuator so that \( C \) reaches 4.5 (note that the value of \( C \) can be obtained from the waveform of a SMI signal using the method reported in [17, 35]).

• Step3: While keeping \( C \) unchanged based on the observed SMI signal waveform, gradually decrease the injection current until the system just enter the critical stable state, then record the injection current at this moment, \( P_c \).

• Step4: Adjust the attenuator to decrease the \( C \) by a step of about 0.2, and then repeat the steps from 1-3.

With the procedures above, we are able to obtain a number of pairs of \( P_c \) and \( C \). The results are plotted in Fig. 4-8(a). In a similar way we can get the relationship between \( P_c \) and \( L \) by fixing \( C = 2 \) and varying \( L \) from 0.1m to 0.4m. The results are plotted in Fig. 4-8(b). It can be seen that the experimental results show the similar relationships presented in Fig. 4-7.

![Figure 4-8: Experimental results of the (a) relationship between \( P_c \) and \( C \) when \( L = 0.15m \), and (b) relationship between \( P_c \) and \( L \) when \( C = 2 \).](image)

4.4 Summary

In this chapter, we investigated the influence of the injection current on the stability of an SMI with a moving external cavity. Numerical calculations on the system determinant of the LK equations are performed. We found that the relaxation
oscillation has a strong dependence on the feedback phase when the injection current is low and the external cavity is short, especially for the cases when an SMI is operated at moderate or high feedback cases. Both simulation and experiment show that there is a critical injection current $P_c$ above which the SMI can be feedback-phase-independent stable. This critical $P_c$ is determined by $C$ and $L$.

The results and the method presented in this letter are helpful for designing a stable SMI sensing system.
Chapter 5 Designing a Stable SMI for Measuring the Linewidth Enhancement Factor

It is well known that semiconductor lasers (SLs) play a key role in the emerging field of optoelectronics, such as optical sensors, optical communication and optical disc systems. For these applications the linewidth enhancement factor (LEF), also known as the alpha factor or $\alpha$-parameter, is a fundamental descriptive parameter of the SL that describes the characteristics of SLs, such as the spectral effects, the modulation response, the injection locking and the response to the external optical feedback [24, 26]. Therefore, the knowledge of the value of the LEF is of great importance for SL based applications.

It has been proved that LEF exhibits a strong dependence on the combination of the refractive index $\eta$, gain $G$ and the injected carrier density $N_j$, and is defined by the following equation [27, 80-82]:

$$\alpha = \frac{\chi^R}{\chi^I} = -2\frac{\omega}{c} \frac{\partial \eta}{\partial N_j}$$

(5.1)

where $\chi$ is the complex electric susceptibility, superscripts “$R$” and “$I$” denote the real and imaginary parts of $\chi$, and $\omega$, $c$ and $N_j$ are the angular optical frequency of the SL and the speed of light respectively.
Over the past three decades, various techniques were developed for measuring the alpha factor. These techniques can be mainly classified [80] as: 1) the linewidth measurement, 2) the current modulation, 3) the optical injection and 4) the optical feedback technique. Using these techniques, different types of SLs were tested for the alpha values, which are summarized as followings:

1) Using the linewidth measurement technique: The alpha factor values from 3.2 to 5.3 were obtained by [80, 83] for vertical cavity surface emitting lasers (VCSELs). For distributed feedback (DFB) lasers, the alpha factor was measured as 2.2 and 5.4 by [84] and [85] respectively. Higher values of the alpha factor were found by [24, 26] for Fabry-Perot (FP) lasers which vary from 4.6 to 8.2. Besides the above basic laser structures, the values of 3.9 and 4.4 were reported respectively by [86] and [37] for channeled substrate planar (CSP) lasers.

2) Using the current modulation technique: The alpha factor value of 2.5 for a VCSEL was reported by [87]. A wide range of the alpha factor values from 1.8~6.5 for DFB lasers was reported by [88-94]. For CSP, mesa strip and buried optical guide lasers, the values of 1.3~2.0 [95], 6.9±0.3 [96] and 4.6±1.0 [97] were obtained respectively.

3) Using the optical injection technique: For DFB lasers, the value of 5.5±0.6 for the alpha factor was obtained by [98]. In 2003, also for DFB lasers, [99] calculated the values of the alpha factor as 4.4±0.9 and 3.2±0.3. For F-P lasers, a very wide range of values from 1.0~14.0 was reported by [100]. The alpha factor value was measured as 2.65±0.2 for a CSP laser [101].

4) Using the optical feedback technique: For VCSELs, the alpha factor value was measured as 5.2±0.7 [80]. Different values between 2.2 and 4.9 were obtained for DFB lasers [27]. A wide range from 1.8~6.8 was obtained for FP lasers [27, 102].

The above techniques can also be classified into two categories based on the amount of injection current to the SL. For the first category, the injection current is below the
threshold and in this situation, the LEF is regarded as a material parameter and is measured according to the definition of the LEF in Eq. (5.1). In the second category, the injection current is above or close to the threshold and a mathematical model for measuring the LEF was developed from the rate equations of the SL. In this situation, the LEF is considered as a model parameter or effective parameter which is detached from its physical origin to a certain extent [29, 80]. Among the techniques in the second category, the optical feedback method, i.e., the SMI, is an emerging and promising technique which does not require high radio frequency or optical spectrum measurements, thus providing ease of implementation and simplicity in the system structure [4, 27].

Based on the SMI, various methods were proposed for measuring the LEF. In 2004, Yu et al. [27] proposed an approach which can obtain LEF by geometrically measuring the SMI signals’ waveform. However, this approach requires the SMI signal to have zero crossing points, which means the optical feedback level \( C \) falls within a small range, i.e., \( 1 < C < 3 \) which is difficult to achieve for some types of lasers. Additionally, the movement trace of the target must be away from and back to the SL at a constant speed, which is also difficult in practice. In subsequent years, several approaches [12, 15, 19, 103] for measuring the LEF were developed, and these approaches are mainly based on the numerical optimization for minimizing the cost functions in parameters. Similarly to [27], these methods are also restricted to certain feedback levels, e.g., approaches in [15, 19, 103] require a weak feedback level, i.e., \( C < 1 \), and the method in [12] requires a moderate feedback level, i.e., \( 1 < C < 4.6 \). Furthermore, these methods are quite time consuming due to the large data samples to be processed. Recently, two different approaches [18, 28] were developed for measuring the LEF over a large range of \( C \), but they still face the problem of requiring a large amount of computation time.

Note that the above methods [12, 15, 18, 19, 27, 28, 103] all assume that the SMI signals they used for measuring the LEF are “true”, or in other words stable. However, Chapter 3 (Section 3.3) has shown that when the SMI operates in the semi-stable or
even unstable (chaos) region, the SMI signals detected by the PD packaged at the rear of the SL still look like “stable” SMI signals. Such a phenomenon is caused by the limit in the rising time (also known as cut-off frequency, or bandwidth) of the PD packaged at the rear of the SL, as well as the circuit used for detection. The PD may not be able to detect the details of the high frequency (close to the relaxation oscillation frequency of the SL) SMI waveform in the semi-stable or unstable regions, and the SMI signal observed is actually a distorted version of the high frequency waveform. Therefore, if the above methods [12, 15, 18, 19, 27, 28, 103] are applied to the distorted SMI signals to measure the LEF, it may induce significant measurement error. Furthermore, the methods mentioned above are either restricted to a certain optical feedback level or are quite time consuming, which will hinder the use of SMI for embedded and industrial applications.

In this chapter, starting from investigating the influence of the bandwidth of the detection components (including PD and the detection circuit) on the SMI signals, we firstly study the measurement error of the LEF at the semi-stable region, to determine whether it has a significant impact on the results. Then based on our previous stability theory (explained in Chapters 3 and 4), a set of external parameters is determined for designing a stable SMI system to measure the LEF. Finally, using the stable SMI system we designed, a simple method for measuring the LEF is proposed and demonstrated in order to lift the above mentioned limitations of previously proposed methods. This simple method is based on the relationship between the light feedback phase and the output power from the well known LK equations. It was found that the LEF can quite simply be measured by the power value overlapped by two stable SLs’ output power, i.e., an SMI signal, under two different optical feedback strengths.

5.1 Detection Problems Caused by SMI Signals in the Semi-stable Region

Figure 5-1(b) shows an SMI signal obtained by the LK equations in the semi-stable region when $C = 2.0$ and $\alpha = 6.0$. Note that all the other parameters’ values are the
same as for the parameters used for Fig. 3-4(c). Also note that the external target moves according to sinusoidal law with the same vibration trace as for Fig. 3-4(a). That is, \( L(t) = L_0 + \Delta L \cdot \sin(2\pi ft) \), where \( L_0 \), \( \Delta L \) and \( f \) are the initial external cavity length the vibration amplitude and frequency respectively, for which the following values were chosen: \( L_0 = 0.35m \), \( \Delta L = 1.5\lambda_0 \) and \( f = 75Hz \).

From Fig. 5-1(b), it can be observed that the semi-stable SMI signal exhibits the form of high frequency oscillation with its amplitude modulated by a slow-varying signal. This high frequency is close to the relaxation oscillation frequency of the SL (usually 1.5-4\(GHz\)). Interestingly, the slow-varying envelopes are similar to the SMI signal characterized by the same fringe structure. It can be seen from Fig. 5-1(b), that there are nearly 6 fringes corresponding to the peak-peak displacement (3\(\lambda_0\)) of the target. That is, each fringe in the semi-stable SMI signal also corresponds to a target displacement of \(\lambda_0/2\), and hence the semi-stable SMI signal can also be used to measure the displacement with the same resolution as the normal SMI operating in the stable region. However, taking into account of a practical SMI system, the bandwidth (BW) of PD packaged at the rear of the SL for detection is usually under 1\(GHz\), which is in the range of about 200\(MHz\) to 800\(MHz\). Furthermore, the BW of the detection circuit is also

![Figure 5-1](image-url)
usually less than 1 GHz. For example, the BW of our detection circuit shown in Fig. 3-7 is 3 MHz. Therefore, obviously, the detection part of a practical SMI system is actually equivalent to a low pass filter on the real SMI signal, as shown in Fig. 5-1(b). This will dramatically influence the waveform of the SMI signal. Figures 5-2(b)-(f) show the filtered results by applying the detection circuit with different BWs. That is, a low pass filter is used with different cut-off frequencies, denoted as $f_{\text{cut-off}}$, to process the SMI signal in Fig. 5-1(b). The range of $f_{\text{cut-off}}$ is from 600 MHz to 3 MHz, which should cover most practical situations.

![Figure 5-2: Filtered SMI signals after the detection components with different $f_{\text{cut-off}}$](image)

Figure 5-2: Filtered SMI signals after the detection components with different $f_{\text{cut-off}}$ when $C = 2.0$ and $\alpha = 6.0$. (a) SMI signal in Fig. 5-1(b), (b)-(f) filtered SMI signals of the SMI signal in (a).

From the Fig. 5-2(b)-(f), we can see the following problems:

1. Sparkling noise appears in the filtered SMI signals (Fig. 5-2(b) and (c)) when $f_{\text{cut-off}}$ are 600 MHz and 400 MHz respectively.

2. Filtered SMI waveforms are different after applying a low pass filter with different $f_{\text{cut-off}}$.

Most $\alpha$ measurement approaches [12, 15, 18, 19, 27, 28, 103] are performed based on the waveform of the SMI signal. Thus, the distorted or filtered SMI signals due to different BWs of the detection components will work out different results of $\alpha$. Here, we adopt the approach in [27] as an example to see how the filtered SMI signals
introduce the measurement error. The approach in [27] can obtain \( \alpha \) by geometrically measuring the SMI signals’ waveform, and this approach requires the characteristic points of the SMI signal, i.e., zero crossing points, to perform the measurement. Figure 5-3 shows a typical experimental SMI signal, where A and B are zero crossing points.

![Figure 5-3: Experimental SMI signal obtained in [27] (see Fig. 2 in [27]) under moderate feedback. Time interval \( t_{13} \) and \( t_{24} \) are relevant for the determination of \( \alpha \).](image)

The time intervals \( t_{13} \) and \( t_{24} \) shown in Fig. 5-3 allow us to determine the phase difference, and thus obtain the value of \( \alpha \) based on the following two equations:

\[
\phi_{13} = \sqrt{C^2 - 1} + \frac{C}{\sqrt{1 + \alpha^2}} + \arccos\left(-\frac{1}{C}\right) - \arctan(\alpha) + \frac{\pi}{2} \quad (5.2)
\]

\[
\phi_{24} = \sqrt{C^2 - 1} - \frac{C}{\sqrt{1 + \alpha^2}} + \arccos\left(-\frac{1}{C}\right) + \arctan(\alpha) - \frac{\pi}{2} \quad (5.3)
\]

where \( \phi_{13} = 2\pi \frac{t_{13}}{T_1} \) and \( \phi_{24} = 2\pi \frac{t_{24}}{T_2} \). Obviously, the locations of the characteristic points are very important. However, due to different detection BWs, the waveform we observe will be different. Figure 5-4 shows an example where two filtered SMI signals (Fig. 5-2(c) and (f)) are plotted together in order to see their significant impact on the measurement results using the method in [27]. The solid line is the SMI signal when the \( f_{\text{cut-off}} \) is 3MHz.
Figure 5-4: (a) Two filtered SMI signals obtained in Fig. 5-2(d) and (f), (b)-(c) the zoomed SMI signals.

From the zoomed SMI signals, i.e., Fig. 5-4(b) and (c), we can see that the positions of the characteristic points, i.e., Z1-4 in Fig. 5-4(b) and (c), of the two signals are quite different, and will thus give different measurement results. For the SMI signal filtered with $f_{\text{cut-off}}$ of 400 MHz, using the approach in [27], we obtain $\alpha = 5.16$ whereas $\alpha = 5.67$ when $f_{\text{cut-off}}$ is 3 MHz, which deviates from the true value of the LEF, i.e., $\alpha = 6.0$. The results show that different $f_{\text{cut-off}}$ of the detection components, thus leading to different SMI waveforms, do largely impact on the measurement results.

In addition, it is noted that the sparkling noise has a strong influence on the normalization of an SMI signal. The normalization can be achieved via various methods [27, 36, 104] used for SMI sensing.

We also measured the LEF for other filtered SMI signals as shown in Fig. 5-2 using the method described in [27]. The estimated LEF are respectively 4.41, 6.89, 5.51 for Fig. 5-2(b) (d) and (e), which are also quite different from the true value, i.e., $\alpha = 6.0$. Therefore, it is very important to design a practical SMI system working in the stable region.

5.2 Designing a Method for Achieving a Stable SMI System

A practical stable SMI system can be designed based on the methods described in the
previous chapters. Here, a specific example is presented, which applies the method
described in Chapter 3, to illustrate how to design a stable SMI system and thus to
acquire stable SMI signals. A practical SMI system can be represented by the
experimental setup shown in Fig. 3-7, which we re-draw here as a simplified version
shown in Fig. 5-5. Figure 5-6 shows the physical SMI system corresponding to Fig.
5-5.

![Figure 5-5: Schematic diagram of the simplified SMI system of Fig. 3-7.](image)

There are three controllable external cavity parameters in the system: 1. the injection
current $J$, which can be controlled by the current controller, 2. the optical feedback
level $C$, which can be adjusted by the attenuator, and 3. the external cavity length $L$.

The stable SMI system can be designed based on the following steps:

- **Step 1**: Place the external target at a certain distance, e.g., $L = 0.25m$, from the
  SL.
Step 2: Set the injection current $J$ to the SL to a certain value, e.g., $J = 1.3J_{th}$, where $J_{th}$ is the threshold current.

Step 3: Vary the feedback level $C$ from weak to strong with the aid of the attenuator.

Step 4: Once the system enters into the semi-stable region, the relaxation oscillation (RO) of the laser becomes undamped. In this case, a subpeak corresponding to the RO frequency appears near the main peak of the optical spectrum, where an example is shown in Fig. 5-6.

Step 5: Once the subpeaks are first observed from the spectrum, the SMI should be at the point of being critically stable. Then, we apply a tiny change to the attenuator by rotating it in reverse by 2 degrees. A stable SMI signal very close to the critical level can thus be obtained by the detection circuit, and we used the signal to calculate the parameter $C$ by the method presented in [28]. The $C$ calculated is approximately represented for $C_{\text{critical}}$.

In a practical SMI system, the external target is placed at $L = 0.25m$ and the injection current is set $J = 1.3J_{th}$. The temperature of the SL is stabilized at $25 \pm 0.1^\circ C$. The loudspeaker is driven by a sinusoidal signal with a frequency of 200Hz and peak-peak voltage of 200mV. Based on the above steps, we can obtain $C_{\text{critical}} = 4.05$. That means, as long as $C$ is less than 4.05 when $L = 0.25m$ and...
$J = 1.3J_{th}$, the SMI system is always stable, and thus so are the SMI signals.

Figure 5-7 shows the two stable SMI signals with different $C$ values when $L = 0.25m$ and $J = 1.3J_{th}$. These two SMI signals can be thus used for measuring the LEF. Note that the value of $C$ can be obtained from the waveform of an SMI signal using the method reported in [17, 35].

![Figure 5-8: Two stable SMI signals obtained from the stable SMI system we designed.](image)

### 5.3 A New Method for the LEF Measurement

Based on the stable SMI signals acquired using the above stable SMI system we designed, we now develop a new method for the LEF measurement. The proposed method is simple and eliminates the feedback level restrictions used in previous methods.

As mentioned before, the optical feedback technique for the LEF measurement relies on a theoretical model, using Eqs. (1.19) and (1.20) based on the stationary solutions of the well known Lang and Kobayashi (LK) rate equations, i.e., Eqs. (1.10)-(1.12) which describes the dynamics of an SL with optical feedback. The model mainly consists of two equations and is re-written as follows:

$$\phi_s = \phi_0 - C \sin \left[ \phi_s + \arctan \left( \frac{g}{100} \right) \right]$$  \hspace{1cm} (5.4)

$$g = \cos(\phi_s)$$  \hspace{1cm} (5.5)
where $\phi_s$ and $\phi_0$ are the phases corresponding to the perturbed and unperturbed laser frequency respectively and $g$ is the SMI signal. $\phi_0$ is associated with the target movement trace $L$ and is given as $\phi_0 = 4\pi L/\lambda_0$, where $\lambda_0$ is the unperturbed laser wavelength. Clearly, there is a straightforward procedure to establish $g$ when $L$ varies, i.e., $\phi_0 \rightarrow \phi_s \rightarrow g$. During the establishment of $g$, another important parameter i.e., optical feedback level $C$, determines the shape of $g$. For a target subjected to a simple harmonic vibration, $g$ is symmetrical with sinusoidal-like fringes when $C < 1$. When $C > 1$, $g$ shows asymmetrical hysteresis sawtooth-like fringes. Figure 5-8 shows the relationship between $\phi_0$ and $\phi_s$ as well as $\phi_0$ and $g$ for three different values of $C$, whereas $\alpha$ is fixed to be 4.0. For an explanation of the mechanism behind the relationship between $g$ and $C$, refer to section 1.1.3 of this thesis.

![Figure 5-9](image)

Figure 5-9: The relationship between (a) $\phi_0$ and $\phi_s$ as well as (b) $\phi_0$ and $g$ for a fixed value of $\alpha = 4.0$.

From Eq. (5.4), it is interesting to notice that when $\phi_0 = m\pi - \arctan(\alpha)$, and where $m$ is an integer, $\phi_s$ is always equal to $\phi_0$ no matter what the value of $C$ is, i.e.,:
\[ \phi_s = \phi_0 = m\pi - \arctan(\alpha) \] (5.6)

A linear relationship between \( \phi_0 \) and \( \phi_s \) is plotted in Fig. 5-9(a) and is shown as the thin solid line with circles. From Fig. 5-9(a), it can be seen that the three curves intersect at three points corresponding to \( \phi_s = \phi_0 = m\pi - \arctan(\alpha) \). Note that the points of \( \phi_s = \phi_0 = k\pi - \arctan(\alpha) \), where \( k \) is an odd number, correspond to the unstable mode as it does not meet the condition of \( d\phi_s/d\phi_0 > 0 \) [35], and this mode does not involve the process of the constitution of \( g \). Therefore, for the points \( \phi_s = \phi_0 = 2m\pi - \arctan(\alpha) \), according to Eq. (5.5), we have:

\[ g = \cos[2m\pi - \arctan(\alpha)] = \frac{1}{\sqrt{1 + \alpha^2}} \] (5.7)

or

\[ \alpha = \sqrt{\frac{1}{g^2} - 1} \] (5.8)

Thus equation (5.8) provides us a simple approach to measure the LEF, and can be obtained by using the value of \( g \) at the intersection of two SMI signals under any two different optical feedback levels.

To verify our proposed approach, we firstly carry out the computer simulations to generate SMSs. Without loss of generality, the external target is assumed to be subjected to a simple harmonic vibration, i.e.,

\[ L(n) = L_0 + \Delta L(n) = L_0 + \Delta L \cos(2\pi \frac{f_0}{f_s} n) \], where \( L_0 = 0.3m \), \( \Delta L = 1.18\mu m \),

\( f_0 = 200Hz \) and \( f_s = 100kHz \), and are respectively the initial external cavity length, vibration amplitude, vibration frequency and sampling frequency. \( n \) is the discrete time series and thus \( \phi_0(n) \) can be obtained as \( \phi_0(n) = 4.8 \times 10^6 + 6\pi \cos(0.0126n) \)

Then the SMI signal can be obtained from Eqs. (5.4) and (5.5) [35] for a given set of values of \( C \) and \( \alpha \). Figure 5-9 shows the vibration trace as well as two SMI
signals for two different values of $C$ when $\alpha$ is fixed to 4.0. Note that the SMI signals are superposed by a noise signal with SNR=20dB.

Figure 5-10: Simulation results of SMI signals for two different $C$ values when $\alpha = 4.0$.

(a) the vibration trace (b) two SMI signals for $C = 0.5$ (solid line) and $C = 2.0$ (dotted line).

Fig. 5-10(b) shows that there are ten intersection points between two SMI signals, as indicated by the large black dots which correspond to the condition of $\phi_s = \phi_0 = 2m\pi - \arctan(\alpha)$. Then the value of $\alpha$ can be calculated using Eq. (5.8) for each intersection point of two SMI signals and finally the average of all the values of $\alpha$ as the estimated LEF is taken and is denoted as $\hat{\alpha}$. The accuracy is considered as the standard deviation of calculated data, denoted as $\sigma$. The calculated results of $\hat{\alpha}$ and $\sigma$ are respectively 3.81 and 6.5%.

Similarly to the above case, computer simulations have been performed for various preset values of $\alpha$ and the corresponding estimated values of $\hat{\alpha}$ are presented in Table 5-1. In Table 5-1, we also present the $\hat{\alpha}$ values calculated using the approach in [27]. Note that the approach in [27] is only valid when the feedback level is moderate, i.e., $1 < C < 3$ and the value we choose for the results in Table 5-1 is
\[ C = 2.5. \] Also from Table 5-1, we can see that even for the value of \( C \) falling in the range of \( 1 < C < 3 \), the approach in [27] is still not valid when \( \alpha \) is equal to or less than 1.0 because there is no zero crossing point of SMI signals, which is essential for utilizing the approach in [27]. From Table 5-1, it can be seen that our proposed method is more accurate and has wider practical utility.

<table>
<thead>
<tr>
<th>preset ( \alpha )</th>
<th>0.1</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
<th>4.0</th>
<th>6.0</th>
<th>8.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimated ( \hat{\alpha} / \sigma ) using the approach in [27]</td>
<td>0.19/7.8%</td>
<td>0.37/6.9%</td>
<td>0.78/7.1%</td>
<td>1.83/5.1%</td>
<td>3.81/6.5%</td>
<td>6.14/4.1%</td>
<td>7.91/3.8%</td>
</tr>
<tr>
<td>Estimated ( \hat{\alpha} / \sigma ) using this paper</td>
<td>N.A</td>
<td>N.A</td>
<td>N.A</td>
<td>1.78/4.5%</td>
<td>3.71/7.1%</td>
<td>5.81/5.6%</td>
<td>8.21/5.1%</td>
</tr>
</tbody>
</table>

Our proposed approach is also verified by the experiment using the two stable SMI signals (Fig. 5-8) obtained in Section 5.2 of this Chapter. Similarly to the measurement method described previously, we are able to obtain the measured \( \alpha \) as 3.19 and \( \sigma \) as 4.05%, whereas \( \alpha \) and \( \sigma \) are respectively calculated as 3.01 and 6.8% using the approach in [27].

### 5.4 Summary

In this chapter, a stable SMI system is designed in order to measure the LEF. Then, a new and simple method for measuring the LEF by investigating the relationship between the light phase and power is demonstrated. The proposed method is superior over the existing methods due to the following two aspects: (i) the proposed method eliminates the feedback level restrictions used in previous methods, thus allowing the experimental design to be simpler, (ii) the LEF can be simply determined from the intersection point of two different SMI signals’ waveforms, thus providing a fast and easy measurement technique for the LEF.
Chapter 6  Conclusion and Future Work

Self-Mixing Interferometry (SMI) is an emerging non-contact sensing technique for the measurement of various metrological quantities, such as absolute distance, angle, displacement, and velocity. An SMI system is composed of a semiconductor laser (SL) with a photodiode (PD) packaged at the rear of the SL, a lens, an external target and a data processing unit. When the external target moves, a small portion of light reflected re-enters the internal cavity of the SL, leading to the modulation in both the amplitude and frequency of the SL output power. The modulated power is detected by the PD as an SMI signal, which is fed to the data processing unit for extracting useful information related to both the external target and the SL itself [1-19]. However, when the system enters into the unstable state, the performance of the above mentioned applications will be severely degraded as the SMI signal is contaminated by a fast oscillation component. Meanwhile the existing SMI model will be no longer valid.

In this thesis, starting from the well known Lang and Kobayashi (LK) equations, and by examining the root locus of the system determinant with respect to all the system parameters, including the feedback strength, external cavity length and injection current, a comprehensive stability analysis for the SMI was presented. In addition, a simple method for measuring the linewidth enhancement factor (LEF) is proposed in this thesis based on the stable SMI signals.
6.1 Research Contributions

According to the extensive computer simulations and experimental results presented in this thesis, the following contributions can be stated:

1. A new and accurate system determinant has been derived without any assumption or approximation in the LK equations by considering the nonlinear gain as compared to existing work. The complete theoretical analysis presented in this thesis paves the way for future investigation on the physical aspects of semiconductor lasers, even for mathematical specialists focusing on the LK equations. [Publication, J2, C3]

2. The presented results in this thesis lead to a number of important and interesting discoveries, which provide accurate and useful guidance for assessing the stability of an SL with EOF as well as other various SLs with EOF based applications, such as SMI sensing, data recording and optical communication. For example, two critical parameters, i.e., critical feedback level factor and critical injection current, which can guarantee a stable SMI are found. [Publication, J2, J3 and J4]

3. Three different regions, i.e., stable, semi-stable and unstable, are identified for describing the dynamics of the SMI. It was found that the SMI signal contains a high frequency oscillation close to the relaxation oscillation frequency of the solitary laser, which is more complicated than the stable SMI signal observed in the stable region, and this can still possibly be used for sensing and measurement. [Publication, J3, C2]

4. A new and simple method for measuring the LEF based on the SMI is proposed by investigating the relationship between the light phase and power. The proposed method applies to a wider range of feedback level compared to the existing work, and quite simply determines the LEF from the intersection point of two different SMI signals, thus providing a fast and easy
6.2 Suggested Future Research Topics

Subsequent to the investigations described in this thesis, conducting research into the following topics would be helpful for enhancing the stability of the SMI:

1. The stability boundaries obtained in Chapter 2 are based on minimum linewidth conditions, i.e., the feedback phase \( \phi_b = \phi_s = -\arctan(\alpha) \), and hence they can only distinguish “the possible stable” and “the unstable” areas. However, when increasing the injection current, a semiconductor laser experiences a red shift in wavelength corresponding to \(~0.5\text{GHz/mA}~\) [105]. One can conclude that the above feedback phase is extremely sensitive to, and wildly increases as a function of the injection current, encompassing several intervals of \( 2\pi \). In practice, control over the precise feedback phase is therefore impossible and stability should always be checked for all possible phase values. As a task for future work, studies are required to investigate how the stability limits presented in Chapter 2 vary with respect to changes of feedback phase over the entire range of \( 2\pi \), based on which a set of limits guaranteeing the stability of laser operation can be obtained.

2. It is shown in Chapter 3 and 5 that due to the bandwidth limit of the detection components, it may not be possible to detect the details of the high frequency SMI waveform in the semi-stable region, and the SMI signal observed will be a distorted version of the high frequency waveform which will strongly reduce the accuracy of classical measurement systems based on an SMI, e.g., displacement, vibration, speed and absolute distance measurements. Therefore, a complete theoretical model is required to describe the influence of the limited bandwidth of the detection components on the high frequency SMI waveform with the aid of the advanced signal processing technique, which apparently requires extensive work and could be an interesting topic for
future research.

3. Finally, it should be pointed out that the LK model itself is also derived under certain approximations, which may lead to discrepancies between the experimental results and numerical results. The work presented in this thesis is based on the LK model and its effectiveness is surely dependent on the accuracy of the LK model. Therefore, the limitations of the LK model due to the approximations in the process of its derivation should be investigated in the future.
References


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