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Nonlinear response of lateral piles with compatible cap stiffness and p-multiplier

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Abstract
Response of lateral pile groups is modeled using the more accurate (than any other numerical modeling) p−y curves-based load transfer model. It is essentially underpinned by limiting force per unit length pm pu, modulus of subgrade reaction pmk, and p-multiplier pm (to cater for pile-pile interaction, pm ¼ 1 for single piles). With the model, new closed-form solutions are developed incorporating the cap-rotational stiffness kr. The solutions are presented in nondimensional charts for free-head (kr ¼ 0) through fixed-head (kr > 10 times the pile bending stiffness). The study reveals that the existing pm (bearing no link to the stiffness kr) is inconsistent with pm ¼ 0.25 for capped piles (at limiting state of elastic solutions). This casts doubt about the accuracy of available solutions, and a compatible stiffness kr and pm is required. The compatible normalized stiffness knr is equal to 0.275-0.333 (n ¼ 0.7) and 0.333-0.564 (n ¼ 1.7) for the associated pm at the design level of (ground-level) bending moment specified in the JGJ code. Use of the solutions is elaborated for a typical offshore pile group against measured response, which largely substantiates the deduced stiffness knr. The coupled kr and pm revealed are fundamental to design of the capped piles using any methods. The new solutions using the knr and pm values should be employed to conduct pertinent design.

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Title: Nonlinear response of lateral piles with compatible cap stiffness and $p$-multiplier

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Response of lateral pile groups is modelled using the more accurate $p$-$y$ curves based load transfer model (than any other numerical modelling). It is essentially underpinned by limiting force per unit length $p_{mpu}$, modulus of subgrade reaction $p_{mk}$, and $p$-multiplier $p_m$ (to cater for pile-pile interaction, $p_n = 1$ for single piles). With the model, new closed-form solutions are developed incorporating the cap-rotational stiffness $k_r$. The solutions are presented in non-dimensional charts for free-head ($k_r = 0$) through fixed-head ($k_r > 10$ times the pile bending stiffness). The study reveals that

- The existing $p_m$ (bearing no link to the stiffness $k_r$) is inconsistent with ‘$p_m = 0.25$’ for capped piles (at limiting state of elastic solutions). This casts doubt about the accuracy of available solutions; and a compatible stiffness $k_r$ and $p_m$ is required.
- The compatible normalised stiffness $k_{nr}$ is equal to $0.275 \sim 0.333$ ($n = 0.7$) and $0.333 \sim 0.564$ ($n = 1.7$) for the associated $p_m$ at the design level of (ground-level) bending moment specified in the JGJ code.

Use of the solutions is elaborated for a typical offshore pile group against measured response, which largely substantiates the deduced stiffness $k_{nr}$. The coupled $k_r$ and $p_m$ revealed are fundamental to design of the capped piles using any methods. The new solutions using the $k_{nr}$ and $p_m$ values should be employed to conduct pertinent design.

**Key words:** nonlinear response, rotating cap, limiting force profiles, lateral loading, piles
Introduction

Piles are customarily cast into a pile cap that restrains pile-head rotation (Mokwa and Duncan 2003; Ooi et al. 2004; Guo 2009), but allows horizontal translation during lateral loading. However at a working load level, cap-rotation generally occurs (Mokwa and Duncan 2003; Ooi et al. 2004), owing to relaxation of soil resistance underneath the pile-cap or around piles (Guo 2009), insufficient cap-restraint and possible cracking of the piles. The translation (displacement) and rotation [see Fig. 1(a)] may be associated with ~4 times different resistance as noted for rigid piles. This difference has to be properly considered to provide reliable design, especially, for wind-turbine foundations and subjected to earthquake loading.

Lateral pile-soil interaction may be mimicked using a series of independent springs-sliders distributed along the shaft and a membrane (to incorporate couple interaction among the springs) (e.g. Fig. 1(b)). The slider is characterised by the profile of limiting force per unit length ($p_u$ profile) to a (slip) depth $x_p$, and the spring has a modulus of subgrade reaction $k$ for the pile-soil system. In the context of the interaction model, elastic-plastic closed-form solutions (Guo 2012) were developed for free-head rotation but for soil resistance (FreH piles), and capped-head (no rotation, FixH piles), respectively. The FixH solutions are also used to predict group response using a reduced limiting resistance $p_{m}p_u$ and modulus $p_{m}k$ by incorporating the pile-pile interaction factor $p_{m}$ ($= p$-multipliers $\leq 1.0$) (Brown et al. 1998) [see Fig.1 (c) and (e)].

Fixed-head solutions using numerical finite element method, and finite difference approach (Ooi et al. 2004) overestimate measured maximum bending moment and underestimate measured deflection of capped piles (Duncan et al. 2005) at large; whereas free-head solutions offer incorrect depth of maximum bending moment and overestimate the head-deflection. The
concept of \( p \)-multiplier is less rigorous but offers more reliable and efficient prediction of overall pile response than finite element and finite difference methods (Guo 2009; Guo 2012). Nevertheless, the ~ 4 times difference in resistance due to cap-rotational stiffness \( k_r \) must be quantified to conform with the \( p_m \) to gain reliable solutions, which is not available. To meet more stringent design, new solutions underpinned by load transfer model are developed in this paper, for piles with a cap stiffness \( k_r \). With the solutions, the impact of the stiffness \( k_r \) is examined and presented in non-dimensional charts. The stiffness \( k_r \) and the \( p_m \) are obtained for typical cases.

**Overall Solutions for A Single Pile**

**Load Transfer Model**

Fig. 1(b) shows under a pile-head load \( H \), the pile-soil system is simulated using the uncoupled (with \( N_p = 0 \), plastic zone) and coupled (\( N_p \neq 0 \), elastic zone) load transfer models (Guo 2006). The pile-head has a rotational stiffness \( k_r \), and a ground-level bending moment \( M_o \) of \( k_i \omega_g \) (\( \omega_g = \) pile-head rotation angle in radian, at zero loading eccentricity of \( e_p = 0 \)). The pile-soil system retains the salient features of free-head (\( k_i = 0 \)) and fixed-head piles (\( k_i > 10E_pIp, E_pIp = \) flexural stiffness of a pile), as recaptured in Appendix I. The net limiting force per unit length, \( p_u \) [FL\(^{-1}\)] along the pile shaft [see Fig. 1(c)] is the sum of the passive soil resistance acting on the face of the pile in the direction of soil movement, and sliding resistance on the side of the pile, less any force due to active earth pressure on the rear face of the pile. The \( p_u \) is described by (Guo 2006)

\[
p_u = A_L(x + \alpha_o)^n
\]

\[
A_L = \tilde{s}_u N_g d^{1-n} \text{ (cohesive soil)}, \quad A_L = \gamma_s N_g d^{2-n} \text{ (cohesionless soil)}
\]

where \( d \) is pile diameter [L]; \( x \) is depth below ground level [L]; \( \alpha_o \) is an equivalent depth to include the \( p_o \) at ground-line level [L]; \( n \) is a power to the corrected depth of \( x + \alpha_o \); \( \tilde{s}_u \) is an
average undrained shear strength $s_u$ of the soil [FL$^{-2}$] over the max $x_p$ anticipated; $N_g$ is a gradient
correlated clay strength or sand density to the limiting $p_u$; $\gamma'$ is effective unit weight of the
overburden soil [FL$^{-3}$] (i.e. dry weight above water table, and buoyant weight below). The
parameters $\alpha_o$, $N_g$, and $n$ were deduced against measured response of 70 free-head piles and ~30
capped piles in single or layered soil (Guo 2013a). It is noted that $n = 0.5$–$0.7$ (for a uniform
strength profile, e.g. clay), and 1.3–1.7 (for a linearly increasing strength profile, e.g. sand). The
$p_u$ along with a subgrade modulus $k$ are sufficiently accurate to model pile response. The $p_u \leq$
9.14–11.94$sud$ clay (Randolph and Houlsby 1984)] from equation (1) is only effective to the max
$x_p$. It may be estimated using angle of internal friction and cohesion of a subsoil by Hansen’s
solution (Hansen 1961; Guo 2013a).

The uncoupled and coupled load transfer models (Hetenyi 1946; Guo and Lee 2001)
allow the governing equations for the pile (see Fig. 1) to be obtained as

$$ E_p I_p w''(x) = -p_u \quad \text{(Elastic zone, \(0 \leq x \leq x_p\)) (3) }$$

$$ E_p I_p w''(z) - N_p w(z) + kw(z) = 0 \quad \text{(Plastic zone, } x_p \leq x \leq L, \text{ or } 0 \leq z \leq L-x_p) \quad \text{(4) }$$

where $w(x)$ is the pile deflection at depth $x$; $w''(x)$ is 4th derivative of $w(x)$ with respect to $x$; $I_p$
and $E_p$ are moment of inertia and Young’s modulus of an equivalent solid cylindrical pile,
respectively. As with those for free-head and fixed-head piles (Guo 2006), response of the lateral
pile (see Fig. 1) is presented against the depth $x$ (measured from ground level) in the upper
plastic zone, and a depth $z (= x- x_p, \text{ measured from the slip depth } x_p)$ in the lower elastic zones,
respectively, to generate compact expressions. The values of $N_p$ and $k$ are calculated using the
average modulus $G_s$ of the soil over the effective length $L_e$ (see Appendix I).
Pile-head Rotational Stiffness \( k_r \)

A laterally loaded pile group generally exhibits the cap-rotation owing to more compression in piles located in front rows than back rows. The rotational stiffness of the pile-cap \( k_r \) may be taken as that of pile-head for a fully cast concrete cap with a sufficient rigidity. It may then be calculated using axial stiffness and capacity of a single pile, with due account of the pile group spacing (Mokwa and Duncan 2003). This treatment, however, is not valid to capped single piles or pile groups with other head constraints. The rotational stiffness \( k_r \) does affect the magnitude of the \( p_u \) (Guo 2009), which may be determined using \( N_g \) [see Eq. (2)]. A fictitious gradient \( N_g^{FreH*} \) is deduced by matching a measured load-displacement relationship (indicated by an asterisk * in the \( N_g \)) with free-head solutions; and a \( N_g^{FixH*} \) with fixed-head solutions. Elastic theory for a laterally loaded rigid pile (Scott 1981; Guo 2012) provides \( w_g^{FreH} = H^{FreH}/k^{FreH}L \), and \( w_g^{FixH} = 4H^{FixH}/k^{FreH}L \). Assuming that \( 4w_g^{FixH} = w_g^{FreH} \) and \( p_m^{FreH} = k^{FixH} \) leads to \( p_m^{FixH} = H^{FreH} \), which renders at limit state \( p_mN_g^{FixH*} = N_g^{FreH*} \). This, by no means, indicates a factor of \( p_m \) between fixed-head and free-head \( N_g \), as the 4 times different displacements would not be warranted between the piles. In fact the ratio becomes far greater than 4 once plastic deformation is induced (Guo 2013b). The form of \( N_g^{FixH*} = p_mN_g^{FreH} \) is adopted here to deduce \( N_g \) with a stiffness \( k_r \).

Ground-level bending moment \( M_o \) may be measured for a lateral load \( H \) at a loading eccentricity \( e_p \) above ground level. Owing to semi-fixity, the Chinese JGJ design code (JGJ 1994) recommends a design bending moment \( M_o \) be 0.4 times the moment (denoted as \( M_o^{FreH} \)) induced on a pile-head with translation only (FixH) pile; and the design pile-head deflection \( w_g \) be 1.25 times the \( w_g^{FixH} \) gained using the FixH solutions (JGJ 1994). The ratios will be used later to determine the stiffness \( k_r \).
Elastic-plastic Solutions

Equations (3) and (4) were resolved using the technology for free-head piles (Guo 2006) by enforcing the bending moment \(-M_0 = E_p I_p w^*(0) = k_\omega \omega_g + H_e p\), and the shear force \(-Q(0) = H\) at the pile-head level (\(x = 0\)). Note the \(\omega_g\) is of a negative value, and offers a counter moment against \(H_e p\) as expected. The elastic-plastic solutions developed are provided in Appendix I for response profiles and head response, respectively. They involve the reciprocal of a characteristic length \(\lambda = \sqrt[4]{k/(4E_p I_p)}\), the on-pile force per unit length \(p = p_u\) at \(x \leq x_p\) [see Fig. 1(c)], the normalised ground-level resistance \(\alpha_o(= \lambda \alpha_o)\), and the two coupled parameters \(\alpha_n\) and \(\beta_n\) in elastic zone (\(\alpha_n = \beta_n = 1\) for uncoupled springs with \(N_p = 0\)). The response profiles are dominated by the normalised depths \(\bar{x} (= \lambda x)\) and \(\bar{z} (= \bar{x} - \bar{x}_p, \bar{x}_p = \lambda x_p)\), respectively for plastic and elastic zones, and involve normalised pile-head load \(\bar{H} (= H \lambda^{n+1}/A_L)\), ground-level deflection \(\bar{w}_g\) (via \(\bar{w}_g = w_p \lambda^n/A_L)\) and rotation \(\bar{\omega}_g\) (via \(\bar{\omega}_g = \omega_p \lambda^{n-1}/A_L)\) for the normalised stiffness \(k_{nr} [= k, \lambda^3/k]\).

Simplified Expressions

The dimensionless expressions of \(\bar{H}, \bar{w}_g,\) and \(\bar{\omega}_g\) (see Appendix I) largely reflect the consequence of mobilization depth (via \(\bar{x}_p\)) of the limiting force per unit length \(p_u\) along a laterally loaded pile. They may be simplified to the following form, given negligible on-pile resistance at ground-line and the coupled interaction (thus \(\alpha_o \approx 0\) and \(N_p \approx 0\)).

\[
\bar{\omega}_g = \frac{\bar{x}_p^n}{2(\bar{x}_p + 1)^2 k_{nr} + 1 + \bar{x}_p + \bar{\omega}_p} \left\{ \frac{4\bar{x}_p(\bar{x}_p + \bar{\omega}_p + 1)}{(n+1)(n+2)(n+3)} + 2\frac{\bar{x}_p^2(1 - \bar{x}_p^2 - 2\bar{x}_p \bar{\omega}_p)}{(n+1)(n+2)} - 2\frac{\bar{x}_p^2(1 + \bar{x}_p) + (1 + 2\bar{x}_p)\bar{\omega}_p}{n+1} - (1 + \bar{x}_p)(1 + \bar{x}_p + 2\bar{\omega}_p) \right\} (5)
\]
\[
\overline{H} = \frac{0.5\overline{x}_p^2[(n+1)(n+2) + 2\overline{x}_p(2+n+\overline{x}_p)]}{(\overline{x}_p + 1 + \overline{\sigma}_p)(n+1)(n+2)} - \frac{k_{nr}\overline{\sigma}_g}{\overline{x}_p + 1 + \overline{\sigma}_p} \tag{6}
\]

\[
\overline{W}_g = -\omega_g\overline{x}_p \left( \frac{2k_{nr}\overline{x}_p(2\overline{x}_p + 3)}{3(\overline{x}_p + \overline{\sigma}_p + 1)} + 1 \right) + \frac{4\overline{x}_p^{4+n}}{(n+1)(n+2)(n+3)(n+4)} - \left[ \frac{2\overline{x}_p^2 + (2\overline{x}_p + n + 1)(n + 2)}{3(1 + \overline{x}_p + \overline{\sigma}_p)(n+1)(n+2)} \right] \overline{x}_p^{2+n} + \overline{x}_p^n \tag{7}
\]

where \( k_{nr} = k_r \lambda^3 / k = 4k_r/(E_p I_p) = \overline{k}_r/\overline{\sigma}_g \). The maximum bending moment is likely equal to the ground-line moment \( M_o \) (with \( M_o = \overline{H}\overline{\sigma}_p + k_{nr}\overline{\sigma}_g \)) for semi-FixH piles. The solutions possess similar features to FreH and FixH solutions (Guo and Lee 2001; Guo 2006) (see Appendix I), but for the \( N_g \) and \( n \) (thus \( A_L \)) values. The impact of the stiffness \( k_r \) is significant, but there is no simple way to estimate its value. The \( k_r \) may be calculated from the upper structure behaviour as suggested in some numerical program manuals, but this is often hard to be achieved. Assuming a design ratio \( M_o/M_o^{\text{FixH}} = 0.4 \) (JGJ 1994), the \( k_r \) is deduced here using Eqs. (5) and (6) for elastic case, which is simplified as Eq. (8) for \( \epsilon_p = 0 \).

\[
\frac{\overline{\sigma}_f k_{nr}^{\text{FixH}}}{\overline{\sigma}_g k_{nr}^{\text{FixH}}} = \frac{\omega_g k_{nr}^{\text{FixH}}}{\omega_g k_{nr}^{\text{FixH}}} p_m^{0.25(n-1)} = \frac{M_o}{M_o^{\text{FixH}}} p_m^{0.25(n-1)} \tag{8}
\]

The FixH condition is enforced using a \( k_{nr}^{\text{FixH}} \) (ie. \( k_{nr} \) for fully fixed-head) = 50,000, although ‘\( k_{nr} > 50 \)’ is sufficiently accurate. The \( k_{nr} \) values are determined for \( n = 0.7 \) and 1.7, which describe most piles well in clay and sand, respectively. Using Eq. (8) (taking \( \overline{x}_p = 0.0001 \)) and \( M_o/M_o^{\text{FixH}} = 0.4 \), the \( k_{nr} \) at \( n = 0.7 \) was deduced as 0.275 ~ 0.333 with \( k_{nr}/p_m = 0.275/0.2, 0.288/0.3, 0.298/0.4, 0.306/0.5, 0.313/0.6, 0.319/0.7, 0.324/0.8, 0.329/0.9, \) and 0.333/1.0, respectively; and \( k_{nr}(n = 1.7) = 0.333 \sim 0.564 \), or more specifically \( k_{nr}/p_m = 0.564/0.2, 0.488/0.3, 0.443/0.4, 0.412/0.5, 0.389/0.6, 0.371/0.7, 0.356/0.8, 0.344/0.9, \) and 0.333/1.0, respectively. These values
may be used in initial design in light of the JGJ code. Note a $k_{nr} = 0.5$ for $n = 0.7 \sim 1.7$ is deduced using Eq. (9) for elastic case, and $w_g/w_g^{FixedH} = 1.25$ (JGJ 1994), which is yet to be confirmed.

\[
\frac{w_g}{w_g^{FixedH}} = \frac{w_g}{w_g^{FixedH}} p_m^{0.25n} \tag{9}
\]

Validation and Parametric Analysis

The newly established closed-form (CF) solutions reduce to those for free-head piles at $k_{nt} = 0$ (Guo 2006) and for fixed-head piles at $k_{nr} > 50$ (Guo 2009), which compare well with finite element approach and experimental data. Thereby, the nonlinear response of capped piles was examined for a rotational stiffness $k_r = (0 \sim 10)E_pI_p\lambda$ at the typical $n = 0.7$ and 1.7, respectively. Given $n = 0.7$, normalised load ($H$)-displacement ($w_g$) curves at ground level are depicted in Fig. 2(a), and displacement ($w_g$) - bending moment ($M_g$) curves in Fig. 2(b), respectively. The same response concerning $n = 1.7$ is plotted in Fig. 3(a) and (b). The normalised displacement ($w_g$) versus rotational ($\omega_g$) response is presented in Fig. 4(a) and (b) for $n = 0.7$ and 1.7, respectively. The profiles of non-dimensional displacement, slope, bending moment and shear force, for example at $x_p = 1$ are illustrated in Fig. 5(a) through (d) for $n = 0.7$; and in Fig. 6(a) through (d) concerning $n = 1.7$. The impact of the rotational stiffness on the response is evident in Figs. 2 through 6. This is elaborated next in an example.

A Pile in 10-pile Group in Clay (Matlock et al. 1980)

Matlock et al (1980) performed lateral loading tests on a single pile, two circular groups with 5-pile and 10-pile, respectively in Harvey, Louisiana. The tests were conducted in a pit 2.4 m deep.
The site consists of a highly plastic gray-clay with an occasional thin layer of peat, sand or silt within a depth of 2.4 m - 18 m. The plasticity index was 77~100 (at a depth 0~1.2 m below the test pit) and 100~185 (1.2~2 m below), respectively, which allow the angle of the soil friction $\phi$ (drained case) to be estimated as 12 degree (BSI 1985), and a cohesion $c$ as 1 kPa (Guo 2013a).

The 10-pile group is analysed herein. The tubular steel piles were installed in a circle [see Fig. 7(a)] at a center-to-center spacing of 1.8 pile diameters. Each was 13.4 m in length, 168 mm in outside diameter, 7.1 mm in wall thickness, and had a bending stiffness of 2.326 MN-m$^2$. The piles were driven 11.6 m into a uniform soft clay that had a undrained shear strength $s_u$ of 20 kPa. Deflections of the group during the tests were enforced at two support levels (of 0.305 and 1.83 m) above ground-line to mimic FixH restraints. The measured $H_{av}$-$w$ and $w$-$M_o$ curves for a pile in the group are plotted in Fig. 7(c) and (d), respectively. The measured bending moment profiles are plotted in Fig. 7(e) for four typical values of ground-level deflection $w_g$.

The response of a capped pile in the group was predicted using free-head and fixed-head solutions (Guo 2012), $n = 0.85$, $p_m = 0.2$ (for either solution), $G_s = 13s_u$ (fixed-head solutions) or $33s_u$ (free-head solutions), and the respective, fictitious profile of limiting force per unit length $p_u$ [Fig. 7(b)]. Note the correlation of $G_s^{\text{FixH}} \approx 0.39G_s^{\text{FreH}}$ is resulted from semi-fixed head constraints and less interaction from the special (circle) layout of the group, otherwise $G_s^{\text{FixH}} = p_mG_s^{\text{FreH}}$. The curves of average load per pile ($H_{av}$) versus mudline deflection ($w_g$) predicted are plotted in Fig. 7(c), which agree remarkably well with the measured $H_{av}$-$w$ curve (thus $w \approx w_g$).

The predicted $M_o$ values using fixed-head solutions are much higher than the measured values [see Fig. 7(d) and 7(e)] (Guo 2009), even after deducting the moment $H_{av}e_p$ caused by loading eccentricity above ground level $e_p$. The new solutions are next used to examine impact of rotational stiffness $k_r$ on the modulus $G_{ss}$, the limiting force per unit length $p_u$, and the response of
a pile in the group. It is based on revised modulus $G_s$ and $p_u$ using a partial factor $\alpha_r$ (rather than $p_m$) determined iteratively. The $\alpha_r$ is initially taken as the ratio of $M_o/M_o^{\text{FixH}}$ (JGJ 1994).

First, assuming a factor $\alpha_r = 0.42$, the modulus $G_s$ was taken as $G_s^{\text{FixH}} + \alpha_r(G_s^{\text{FreH}} - G_s^{\text{FixH}})$. Given $G_s^{\text{FixH}} = 13s_u$, $G_s^{\text{FreH}} = 33s_u$, the $G_s$ is estimated as $25s_u [= 13s_u + 0.42 \times (33-13)s_u$ or $p_m = 0.76 = G_s/G_s^{\text{FreH}}$. This yields $k = 1.59 \times 10^3$ kPa (= 3.18 $G_s$, $s_u$ = 20 kPa), and $\lambda = 0.643/m$ (\cite{Guo_2009} $\lambda = [k/(4 \times 2.326)]^{0.25}$).

Second, the maximum $p_u$ profile was estimated using Hansen’s expression for free-head piles with $c = 1$ kPa, and $\phi = 12^o$, and is plotted in Fig. 7(b). As with the calculation of $G_s$, the $A_L$ is estimated as $2.26A_L^{\text{FixH}}$ using $A_L^{\text{FixH}} + \alpha_r(A_L^{\text{FreH}} - A_L^{\text{FixH}})$, and $A_L^{\text{FixH}} = 4A_L^{\text{FixH}}$, or $p_m = 0.565 = A_L/A_L^{\text{FreH}}$. Note this $p_m$ value should be the same as that for calculating the shear modulus, but for the special group layout (Guo 2009). The $A_L$ is calculated as 5.22 kN/m$^{1.85}$ (= $1 \times 20 \times 0.1681^{0.15} \times 0.2$) (Guo 2009), which was reduced to 5.142 kN/m$^{1.85}$ to compensate the impact from using $e_p = 0$ against a real $e_p (> 0)$. With $n = 0.85$, and $A_L = 5.142$ kN/m$^{1.85}$, the $p_u$ profile was obtained and is plotted in Fig. 7(b) as Guo LFP. It matches well with the Hansen LFP. Both Guo LFP and Hansen LFP offer weaker or slightly stronger (above or below a depth of 15$d$) resistance than the Bogard & Matlock’s $p_u$ does. This difference has limited impact on the pile response (with a maximum $x_p$ of 22.14$d$).

Third, with $A_L$, $n$, and $k$, the $\lambda$ is calculated, normalised load $\bar{H}$, moment (thus $\bar{\omega}_g$ with a known $\bar{e}_p$), and displacement $\bar{w}_g$ are obtained for measured load $H$, ground-level bending moment $M_o$ at a typical displacement $w_g$. The three measured values allow the three unknown $k_{nr}$, $A_L$ (thus $\alpha_r$) and $\bar{e}_p$ (via $k$) to be resolved iteratively using Eqs. (5), (6) and (7). The $\alpha_r$ value should be within an acceptable difference to the assumed value, otherwise, a new $\alpha_r$ is...
stipualated (resulting in new $G_s$, $A_L$, and $k$), the three steps are repeated. The calculation can be readily done using a professional mathematical program (e.g. Mathcad\textsuperscript{TM}).

This iterative calculation is illustrated using the design chart for $n = 0.7$ (note $n = 0.85$ for the current pile), which encompasses three steps: (1) a pair of measured load $H$ and displacement $w_g$ are normalised, respectively, using $A_L$, $n$, and $k$ gained for free-head piles (e.g. $H = 1.56$ at $w_g = 10$, see bold values in Table 1). (2) The pair $H$ and $w_g$ allow $k_r/(E_p I_p \lambda) = 1.1$ (or $k_{nr} = 0.275$) to be ascertained in Fig. 2(a), which gives a stiffness $k_r$ of 1.643 MNm ($= 1.1 E_p I_p \lambda$). (3) The $\alpha_r$ is estimated using $H$ (measured) = $H$ \text{FixH*} + $\alpha_r(H$ \text{FreH*} - $H$ \text{FixH*}). For instance, at $w_g = 10$, $H$ \text{FixH*} = 1.0 and $H$ \text{FreH*} = 2.2 [see Fig. 2(a)], and $H$ (measured) = 1.5, the $\alpha_r$ is obtained as 0.42 from $1.5 = 1.0 + \alpha_r(2.2-1)$. The deduced values are $k_{nr} = 0.275$ and $\alpha_r = 0.42$ for the measured $H$ and $w_g$. The calculation may be repeated for other measured pairs of $H$ and $w_g$, and similar $k_{nr}$ and $\alpha_r$ should be obtained (Guo 2013a).

With the $p_u$ ($n = 0.85$), the $G_s$ and the $k_{nr} (= 0.275)$ obtained, the pile response is readily predicted. This is provided next for $e_p = 0$ (zero loading eccentricity) for a ground-level deflection $w_g$ of 9.4, 23.5, 47.3 and 83.1 mm (at which bending moment profiles are provided).

With $w_g = 9.4$ mm, normalised slip depth $\bar{x}_p$ is estimated as 0.863 ($x_p = 1.342$ m). (1) The $\omega_g k \lambda^{n-1}/A_L$ is calculated using Eq. (5).

$$\omega_g k \lambda^{0.15}/A_L = \frac{-0.863^{0.85}}{1.863 \times 2 \times 0.275 + 1} \left(2 \times 0.863^2 \left[0.863 + 3 + 0.85\right] + 1 + 0.863\right) = -1.091 \quad (10)$$

Likewise, the values of $H \lambda^{1.85}/A_L$ = 0.686 and $w_g k \lambda^{0.85}/A_L = 1.997$ were obtained using Eqs. (6) and (7), respectively, along with $-M_o \lambda^{2.85}/A_L = 5.427$. (2) With $k = 1.59 \times 10^3$ kPa and $A_L = 5.142$ kN/m$^{1.85}$, the ground-level rotation angle $\omega_g = -3.3035 \times 10^{-3}$, and $H = 7.981$ kN were obtained at
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\( w_g = 9.4 \text{ mm}. \) (3) With expressions in Appendix I, \( F(1,0) = F(2,0) = 0, \) constants \( C_5 = 4.154 \times 10^{-3} \) and \( C_6 = -1.574 \times 10^{-3} \) were obtained respectively. The pile deflection at depth \( x \) (plastic zone) or \( z \) (elastic zone) is calculated respectively as

\[
\begin{align*}
w(x) &= \frac{5.142}{E_p I_p} \left[ -x^{4.85} \frac{x^3}{4.85 \times 3.85 \times 2.85 \times 1.85} + \frac{1.33017 x^3 - 2.7135 x^2}{5.142} - (3.3035x - 9.4) \times 10^{-3} \right] \text{ (m)} \quad (11a) \\
w(z) &= e^{-0.643(1-1.342)} \{4.154 \cos[0.643(x - 1.342)] - 1.574 \sin[0.643(x - 1.342)]\} \times 10^{-3} \text{ (m)} \quad (11b)
\end{align*}
\]

The profile of bending moment \( M(\bar{x}) \) or \( M(\bar{z}) \) is obtained as

\[
\begin{align*}
-M(x) &= -x^{2+n} A_k \frac{A_k}{n+2} + Hx + k_r \omega_g \\
&= -\frac{5.142 x^{2.85}}{1.85 \times 2.85} + 7.98x + 1.645 \times 10^3 \times (-3.3035 \times 10^{-3}) \\
-M(\bar{z}) &= 2E_p I_p \lambda^2 e^{-\bar{z}} \left[-C_6 \cos(\bar{z}) + C_5 \sin(\bar{z})\right] \\
&= 1.923 \times 10^3 e^{-0.643(x-x_p)} \left[-C_6 \cos[0.643(x-x_p)] + C_5 \sin[0.643(x-x_p)]\right] \\
\end{align*}
\]

The calculated bending moment profile is plotted in Fig. 7(e) as a solid line.

As with \( w_g = 9.4 \text{ mm}, \) the normalised slip depth \( \bar{x}_p \) was estimated as 1.3484, 1.7823, and 2.1764 for \( w_g = 23.5, 47.3 \) and 83.1 mm respectively. The \( H, \omega_g, C_5 \) and \( C_6 \) values (see Table 1) were obtained; and the bending moment profiles are plotted in Fig. 7(e). The predicted moment profiles agree well with the respective measured data but not the fixed–head predictions (via the program GASLGROUP). Note the profile of shear force \( Q(\bar{x}) \) or \( Q(\bar{z}) \) (not shown herein) can be predicted using the expressions in Appendix I.

Importantly, this capped pile has a moment ratio \( M_o/M_o^{\text{FixH}} \) of 0.371–0.442 (with 0.442, 0.396, 0.377 and 0.371 at \( w_g = 9.4, 23.5, 47.3, \) and 83.1 mm, respectively, see Table 1). The average (of 0.406) is rather close to 0.4 specified by the JGJ code, despite the special group layout, cap configuration, and nonlinear response. This consistency is further echoed between
the deduced values of \( k_{nr}/p_m = 0.275/0.565 \sim 0.76 \) (for \( M_o/M_o^{\text{FixH}} = 0.371\sim0.442 \)) and the theoretical values of \( 0.31/0.565 \) (for \( M_o/M_o^{\text{FixH}} = 0.4 \)). The reliability of the current solutions and the \( k_{nr} \) and \( p_m \) values is thus vindicated.

### Conclusions

Response of lateral pile groups is modelled using the more accurate \( p-y \) curves based load transfer model (than any other numerical modelling). It is essentially underpinned by limiting force per unit length \( p_m/p_k \), modulus of subgrade reaction \( p_m k \), and \( p \)-multiplier \( p_m \) (to cater for pile-pile interaction, \( p_m = 1 \) for single piles). With the model, new closed-form solutions are developed incorporating the cap-rotational stiffness \( k_r \). The solutions are presented in non-dimensional charts for free-head \( (k_r = 0) \) through fixed-head \( (k_r > 10E_pI_p) \). The study reveals that

- The existing \( p_m \) (bearing no link to the stiffness \( k_r \)) is inconsistent with ‘\( p_m = 0.25 \)’ for capped piles (at limiting state of elastic solutions). This casts doubt about the accuracy of available solutions; and a compatible stiffness \( k_r \) and \( p_m \) is required.

- The compatible normalised stiffness \( k_{nr} \) is equal to \( 0.275 \sim 0.333 \) (\( n = 0.7 \)) and \( 0.333\sim0.564 \) (\( n = 1.7 \)) for the associated \( p_m \) at the design level of (ground-level) bending moment specified in the JGJ code.

Use of the solutions is elaborated for a typical offshore pile group against measured response, which largely substantiates the deduced stiffness \( k_{nr} \). The coupled \( k_r \) and \( p_m \) revealed are fundamental to design of the capped piles using any methods. The new solutions using the \( k_r \) and \( p_m \) values should be employed to conduct pertinent design.

### NOTATION

The following symbols are used in the paper:
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A = coefficient for the LFP;  
$c = \text{cohesion;}$

$d = \text{diameter of an equivalent solid cylinder pile;}$

$E_p = \text{Young’s modulus of an equivalent solid cylinder pile;}$

$e, e_p = \text{eccentricity (} = e_p\text{), eccentricity of the location above the ground level for applying the}$

$H(H_g);$

$E_p I_p = \text{flexural stiffness of a lateral pile;}$

$FreeH = \text{free head, allowing translation and rotation at head-level;}$

$FixH = \text{fixed-head, allowing translation but not rotation at head level;}$

$G_s, G^* = \text{average soil shear modulus over the critical length, } L_c\text{, and } G^* = (1+0.75\nu_s)G_s;$

$I_p = \text{moment of inertia of an equivalent solid cylinder pile;}$

$k = \text{modulus of subgrade reaction;}$

$K_i(\gamma) = \text{modified Bessel function of second kind of } i^{th} \text{ order (Appendix I);}$

$L, L_c = \text{embedded pile length, and critical length beyond which any increase in the embedment}$

$\text{would not affect the pile response; }$

$LFP = \text{the profile of net limiting force per unit pile length;}$

$M_o, M(x) = \text{bending moment at the mudline level, the moment at the normalised depth } x;$

$\bar{M}_o = M_o \lambda^{2+n/AL}, \text{normalised } M_o;$

$n, = \text{power for the LFP;}$

$N_g = \text{gradient correlated soil strength to the } p_u, \text{ deduced from measured load-displacement of}$

$a \text{ capped pile using current solutions;}$

$N_g^{\text{FreeH}}, N_g^{\text{FixH}} = \text{gradient } N_g \text{ deduced from measured load-displacement using the free- and fixed-}$

$\text{head piles, respectively;}$

$N_p = \text{fictitious tension for the membrane tied together the springs around the pile shaft;}$

$H = \text{lateral load exerted on a single pile;}$

$\overline{H} = H \lambda^{1+n/AL}, \text{normalised pile-head load;}$

$H_{av}, H_g = \text{average load per pile in a group, and total load imposed on a group;}$

$k_{nr} = k \lambda^3/4k_i(E_p/\lambda) = \bar{k}_r/\bar{\omega}_g, \text{ and } \bar{k}_r = k \omega_g \lambda^{2+n}/AL;$

$k_r = \text{rotational stiffness of the pile cap;}$

$p_m = \text{p-multipliers used to reduce stiffness, and limiting force for individual piles in a group;}$

$p_u = \text{limiting force per unit length;}$

$p(\bar{x}), Q(\bar{x}) = \text{net force per unit length, and shear force at the normalised depth } \bar{x};$
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$\bar{s}_u = \text{undrained shear strength of soil. (Average } s_u \text{ over a maximum slip depth anticipated);}$

$t = \text{wall thickness of a pipe pile; }$

$x, x_p, \bar{x}, \bar{x}_p = \text{depth below ground level, slip depth from elastic to plastic state, } \bar{x} = \lambda x, \bar{x}_p = \lambda x_p;$

$w, w(x), w(z) = \text{lateral deflection of a pile, } w \text{ in the plastic, and } w \text{ in elastic zone, respectively; }$

$\bar{w}_g = w_g k \lambda / A_L \text{ normalised mudline deflection;}$

$w_p = \text{local limiting deflection beyond which the force mobilised on the depth is } p_u;$

$w_l, w_g = \text{pile deflection at loading level, and the deflection at mudline; }$

$w(\bar{x}), w'(\bar{x}) = \text{deflection and rotation at the normalised depth } \bar{x};$

$z, \bar{z} = \text{depth below the slip depth } x_p \text{ (i.e. } z = x - x_p), \text{ and } \bar{z} = \lambda z, \text{ respectively;}$

$\alpha_N, \beta_N = \text{normalised stiffness factors;}$

$\alpha_o(\overline{x}_o) = \text{an equivalent depth to account for ground level limiting force with } \overline{x}_o = \alpha_o \lambda;$

$\alpha_r = \text{a partial factor to gain the } A_L \text{ and } G_s \text{ using those for free-head and fixed-head piles; }$

$\gamma = \text{load transfer factor (Appendix I);}$

$\gamma' = \text{effective unit weight of the overburden soil; }$

$\phi = \text{angle of internal friction of soil;}$

$\omega = \text{rotation angle of pile at ground-level }$

$\overline{\omega}_g = \omega_g k \lambda / A_L, \text{ normalised mudline rotation;}$

$\lambda = \text{reciprocal of characteristic length;}$

$\nu = \text{Poisson's ratio of soil, taken as 0.25 for sand, otherwise 0.4;}$

Superscript ‘**’ denotes parameters deduced by matching the same pile response using free-head and fixed-head solutions; Subscript ‘g’ for ground level; ‘s’ soil.
References


APPENDIX I – Solutions for Semi-fixed head piles

This appendix provides load transfer approach and solutions for semi-fixed head piles.

Assumptions and features of load transfer approach

- The springs are characterized by an idealised elastic-plastic $p$-$y$ curve [see Fig. 1(e)], which constitute, respectively, the uncoupled ($N_p = 0$) and coupled load transfer models for the plastic and elastic zones that transfer at a ‘slip’ depth $x_p$.

- The $k$ and $N_p$ for elastic zone are gained theoretically as (Guo and Lee 2001):

$$ k = \frac{3\pi G_s}{2} \left[ 2\gamma \frac{K_i(\gamma)}{K_0(\gamma)} - \gamma^2 \left( \frac{K_i(\gamma)}{K_0(\gamma)} \right)^2 - 1 \right] $$

and $N_p = \pi r_o^2 G_s \left\{ \left[ \frac{K_i(\gamma)}{K_0(\gamma)} \right]^2 - 1 \right\}$

where $K_i(\gamma)$ is modified Bessel function of second kind of $i$-th order ($i = 0, 1$);

$\gamma = 0.65(E_p/G_s)^{0.5} (L/r_o)^{-0.04}$ (for long fixed-head piles with $L > L_c + \text{max.} x_p$); $G^* = (1+3\nu_s/4)G_s$; $r_o = 0.5d$, radius; $G_s$ is an average over a depth of $L_c + \text{max.} x_p$; and $L_c = 1.05d(E_p/G_s)^{0.25}$. The expression for $k_e = 0$ (Guo 2013a) may be used to gain the modulus ratio of $k/G_s$ for caps with rotational stiffness using an equivalent loading eccentricity $e$ ($= e_p + k_r \omega_g/H$, $\omega_g/H$ is calculated for an average load level $H$ and using $k$ for $e = 0$).

- At the slip depth, $x_p$, the pile deflection $w(x_p)$ equals the limiting $w_p (= p_u/k)$, and $p = p_u$ with $x < x_p$, see Fig. 1(d)). Below the $x_p$ ($x \geq x_p$), the deflection $w(x)$ is less than $w_p$; and the resistance $p (= k w(x) < p_u$ at the depth) is linearly proportional to the $k$. The $p_u$ profile (or LFP) is described by the parameters $N_g$, $\alpha_o$, and $n$ gained using average soil properties over a maximum slip depth $x_p$.

- In plastic zone, interaction among the springs is negligible (i.e. $N_p = 0$, Figure 1), as it cannot be transferred in the plastic zone. The resistance per unit length, $p$ is fully mobilised to $p_u$. The $N_p$ (in vertical direction), regardless of its value, does not involve in the governing equation for plastic zone.

- Pile-soil relative slip [e.g. value of $w(x) - w_p$] can only be initiated from mudline, and it can only move downwards.
Response profiles of a semi fixed-head pile

Response profiles in plastic zone \((\bar{x} \leq \bar{x}_p)\) are given by

\[
\omega \bar{x} + H = \frac{4A_L}{k\lambda} \left(-F(4,\bar{x}) + F(4,0) + F(3,0)\bar{x} + \left[k_w \bar{e}_y + \bar{H}e_p + F(2,0)\right] \frac{\bar{x}^2}{2} + [F(1,0) + \bar{H}] \frac{\bar{x}^3}{6}\right) + \frac{\omega_g}{\lambda} \bar{x} + w_g
\]

\[
w'(\bar{x}) = \frac{4A_L}{k\lambda^2} \left[-F(3,\bar{x}) + F(3,0) + \left[k_w \bar{e}_y + \bar{H}e_p + F(2,0)\right] \bar{x} + [F(1,0) + \bar{H}] \frac{\bar{x}^3}{2}\right] + \omega_g
\]

\[-M(\bar{x}) = E_p I_p w_n(x) = \frac{A_L}{\lambda^2 + n} \left(-F(2,\bar{x}) + F(2,0) + (F(1,0) + \bar{H})\bar{x} + k_w \bar{e}_y + \bar{H}e_p\right)\]

\[-Q(\bar{x}) = E_p I_p w_n(x) = \frac{A_L}{\lambda^2 + n} \left[-F(1,\bar{x}) + F(1,0) + \bar{H}\right]\]

where \(\bar{k}_r = \frac{k}{\omega_g} \frac{\lambda^2 + n}{A_L} = \frac{k_w}{\omega_g}; F(m, \bar{x}) = (\bar{x} + \overline{e}_p)^m / (n + m) \cdots \cdot (n + 2)(n + 1) (1 \leq m \leq 4); \overline{e}_o = \lambda \alpha_o; F(0, \bar{x}) = (\bar{x} + \overline{e}_p)^n; \text{ and } \overline{e}_p = \lambda \overline{e}_o. \text{ Note both } \omega_g \text{ and } \omega are used in the } w(\bar{x}) \text{ expression.}

Response profiles in elastic zone \((\bar{x} > \bar{x}_p, \text{ or } \bar{z} > 0, \bar{z} = \lambda \bar{z} = \lambda (x - x_p))\) are as follows:

\[
w(\bar{z}) = e^{-\alpha \bar{z}} \left[C_5 \cos(\beta \bar{z}) + C_6 \sin(\beta \bar{z})\right]
\]

\[
w'(\bar{z}) = \lambda e^{-\alpha \bar{z}} \left[-\alpha_C C_5 + \beta_C C_6\right] \cos(\beta \bar{z}) \left[-(\alpha_C - \alpha_5) C_5 - \alpha_5 C_6\right] \sin(\beta \bar{z})\]

\[-M(\bar{z}) = E_p I_p w_n'(\bar{z})\]

\[-Q(\bar{z}) = E_p I_p w_n(\bar{z}) = E_p I_p \lambda e^{-\alpha \bar{z}} \left\{\left(\alpha_5^2 - \beta_5^2\right) C_5 - 2 \alpha_5 \beta_5 C_6\right\} \cos(\beta \bar{z}) + \left[2 \alpha_5 \beta_5 C_5 + (\alpha_5^2 - \beta_5^2) C_6\right] \sin(\beta \bar{z})\]

where

\[
C_5 = \frac{4A_L}{k\lambda^2} \left\{(1 - 2 \alpha_5^2) [F(2, \bar{x}_p) - F(2,0) - k_w \overline{e}_y - \bar{H}e_p] - \alpha_5 F(1, \bar{x}_p) + \alpha_5 - (1 + 2 \alpha_5^2) \bar{x}_p [F(1,0) + \bar{H}]ight\}
\]

\[
C_6 = \frac{4A_L}{k\beta_5 \lambda^2} \left\{\alpha_5 (2 \alpha_5^2 - 3) [F(2, \bar{x}_p) + F(2,0) + (F(1,0) + \bar{H}) \bar{x}_p + k_w \overline{e}_y + \bar{H}e_p] + (\alpha^2_5 - 1)[F(1, \bar{x}_p) + F(1,0) + \bar{H}]ight\}
\]

\[
\alpha_5 = \sqrt{1 + N_p / \sqrt{4 E_p I_p k}} \quad \beta_5 = \sqrt{1 - N_p / \sqrt{4 E_p I_p k}}
\]

These expressions are independent of the head constraint, and are identical to those for free-head piles.

\[
\bar{H}, w_g \text{ and } \overline{M}_o \text{ of a semi fixed-head pile}
\]

(a) Normalised pile-head load, \(\bar{H}\)
The $H$ is deduced from the following relationship obtained for the depth, $x_p (z = 0)$:

$$w_p'' + \alpha_N w_p'' + 2 \lambda^2 w_p'' = 0$$

where $w_p''$, $w_p''$, and $w_p'''$ are values of $2^{nd}$, $3^{rd}$, and $4^{th}$ derivatives of $w(x)$ with respect to depth $z$. Given $x_p = 0$, the minimum head load to initiate slip is obtained.

(b) Normalised ground-line deflection, $\overline{w}_g$

$$\overline{w}_g = 4[F(4, \overline{x}_p) - F(4,0) - \overline{x}_p F(3, \overline{x}_p)] - 2(2 \alpha_N^2 - 1)[F(2, \overline{x}_p) - F(2,0) - k_w \overline{w}_g] - (2k_w \overline{x}_p + 1) \overline{x}_p \overline{w}_g$$

$$-2 \alpha_N F(1, \overline{x}_p) - 2F(2,0) \overline{x}_p^2 + 2[F(1,0) + H][\frac{1}{3} \overline{x}_p^3 + (2 \alpha_N^2 - 1) \overline{x}_p^2 + \alpha_N] + 2 \overline{w}_g \overline{H}[2 \alpha_N^2 - 1 - \overline{x}_p^2]$$

The $w_g$ is deduced from $w(x)$.

(c) Normalised rotation at ground level, $\overline{\omega}_g$

$$\overline{\omega}_g = \frac{4(\overline{x}_p + \alpha_N + \overline{\epsilon}_p)[F(3, \overline{x}_p) - F(3,0)] - 2(\overline{x}_p^2 - 2 \alpha_N^2 + 1 + 2 \overline{x}_p \overline{\epsilon}_p) F(2, \overline{x}_p)}{2k_w (\overline{x}_p^2 + 2 \alpha_N \overline{x}_p + 2 \alpha_N^2 - 1) + \overline{x}_p + \alpha_N + \overline{\epsilon}_p}$$

$$- \frac{2(\overline{x}_p^2 + 2 \alpha_N \overline{x}_p + 2 \alpha_N^2 - 1) F(2,0) + 2[\overline{x}_p (\alpha_N \overline{x}_p + 2 \alpha_N^2 - 1) + \overline{\epsilon}_p (2 \alpha_N \overline{x}_p + 2 \alpha_N^2 - 1)] F(1, \overline{x}_p)}{2k_w (\overline{x}_p^2 + 3 \alpha_N \overline{x}_p + 2 \alpha_N^2 - 1) + \overline{x}_p + \alpha_N + \overline{\epsilon}_p}$$

$$- \frac{[\overline{x}_p^2 + 2 \alpha_N \overline{x}_p + 1 + 2(\alpha_N + \overline{x}_p)] F(0, \overline{x}_p) - 2 \overline{\omega}_p (\overline{x}_p^2 + 2 \alpha_N \overline{x}_p + 2 \alpha_N^2 - 1) F(1,0)}{2k_w (\overline{x}_p^2 + 3 \alpha_N \overline{x}_p + 2 \alpha_N^2 - 1) + \overline{x}_p + \alpha_N + \overline{\epsilon}_p}$$

(d) Normalised ground-level bending moment

$$- \overline{M}_o = \overline{H} \overline{w}_p + k_w \overline{\omega}_g$$

Finally it should be mentioned that the constants $C_j$ are determined using the compatibility conditions of $Q(\overline{x}), M(\overline{x}), w'(\overline{x})$, and $w(\overline{x})$ at the normalised slip depth, $x_p$ [ $\overline{x} = x_p$ or $\overline{z} = 0$]. Elastic solutions validated for $N_e < 2(kE_p I_p)^{0.5}$ is ensured by $L > L_c + \max. x_p$.
Table 1: Key response and parameters for a pile in 10-pile group

<table>
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<tr>
<th>$w_g$ (mm)</th>
<th>$\bar{x}_p$</th>
<th>$\bar{H}/\bar{w}_g$</th>
<th>$H$ (kN)</th>
<th>$-\alpha_g$ ($\times 10^{-3}$)</th>
<th>$C_s/C_6$</th>
<th>$-M_o/M_{o}^{\text{FixH}}$</th>
<th>$M_o/M_{o}^{\text{FixH}}$</th>
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<tr>
<td>9.4</td>
<td>0.863</td>
<td>0.686/1.997</td>
<td>7.981</td>
<td>3.3035</td>
<td>4.154/1.574</td>
<td>5.43/12.29</td>
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<td>1.78227</td>
<td>1.559/10.05</td>
<td>18.616</td>
<td>12.0</td>
<td>7.964/7.462</td>
<td>19.43/51.5</td>
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<td>24.23</td>
<td>17.9</td>
<td>9.118/10.975</td>
<td>29.40/79.10</td>
<td>0.371</td>
</tr>
</tbody>
</table>

* Values of $M_o^{\text{FixH}}$ is cited from Guo (2009).*
Figure Captions

Fig. 1. Schematic model for a laterally loaded capped pile: (a) A single pile, (b) schematic model, (c) limiting force profile (LFP), (d) Pile deflection and $w_p$ profiles, and (e) $p$-$y$ curves for a single pile or a pile in a group.

Fig. 2. Nonlinear response of capped piles ($n = 0.7$): (a) load; (b) bending moment

Fig. 3. Nonlinear response of capped piles ($n = 1.7$): (a) load; (b) bending moment

Fig. 4. Nonlinear rotational slope of capped piles: (a) $n = 0.7$; (b) $n = 1.7$

Fig. 5. Non-dimensional profiles ($n = 0.7$): (a) displacement; (b) slope; (c) bending moment; and (d) shear force

Fig. 6. Non-dimensional profiles ($n = 1.7$): (a) displacement; (b) slope; (c) bending moment; and (d) shear force

Fig. 7. Predicted versus measured (Matlock et al. 1980) response of a pile in 10- pile group: (a) pile group layout; (b) $p_u$ profile; (c) $w_g \sim H_{av}$ curves; (d) $-M_o \sim w_g$ curves; (e) moment profiles
Lateral piles with compatible cap stiffness and $p$-multiplier

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Fig. 2. Nonlinear response of capped piles ($n = 0.7$): (a) load; (b) bending moment
Lateral piles with compatible cap stiffness and $\rho$-multiplier

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Fig. 4. Nonlinear rotational slope of capped piles: (a) $n = 0.7$; (b) $n = 1.7$.
Lateral piles with compatible cap stiffness and p-multiplier

Wei Dong Guo (2014)

Fig. 5. Non-dimensional profiles (n = 0.7): (a) displacement; (b) slope; (c) bending moment; and (d) shear force
Lateral piles with compatible cap stiffness and $p$-multiplier

Wei Dong Guo (2014)

Fig. 6. Non-dimensional profiles ($n = 1.7$): (a) displacement; (b) slope; (c) bending moment; and (d) shear force.
Lateral piles with compatible cap stiffness and \( p \)-multiplier

Wei Dong Guo (2014)

(a) Layout of the 10-pile groups

Fig. 7. Predicted versus measured (Matlock et al. 1980) response of a pile in 10-pile group: (a) pile group layout; (b) \( p_u \) profile; (c) \( w_g \sim H_{av} \) curves; (d) \( -M_o \sim w_g \) curves; (e) moment profiles