2014

Railway rolling noise separation and wheel and rail roughness estimation from pass by measurement: field tests and theoretical modelling

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Recommended Citation
Li, Wenxu, Railway rolling noise separation and wheel and rail roughness estimation from pass by measurement: field tests and theoretical modelling, Doctor of Philosophy thesis, Department of Mechanical, Materials and Mechatronic Engineering, University of Wollongong, 2014.
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Department of
Mechanical Materials and Mechatronic Engineering

Railway Rolling noise separation and wheel and rail roughness estimation from pass by measurement—field tests and theoretical modelling

Wenxu Li

This thesis is presented as part of the requirements for the award of the Degree of Doctor of Philosophy
University of Wollongong

March 2014
ABSTRACT

A desirable capability for those seeking to understand, and manage, railway rolling noise is the ability to separate the contributions from the wheel and track from revenue service train pass-by events. Relevant aspects of these contributions are the contributions to either: the excitation mechanism, driven by contacting surface roughness; or, to the noise radiated by the various elements of the vehicle and track. Efficient methods for both rolling noise separation, and wheel and rail roughness estimation, have been proposed, including a method known as VTN for noise separation and Janssens’s Method for roughness estimation. Each relies on simplified models. The validity and acceptability for use have not been adequately tested. Field tests were conducted in order to test the validity of these methods. The results show that, using VTN as currently implemented, estimation errors relative to measured values, for overall noise level, of 2.3 dB(A). In contrast, predictions of the noise spectra result in errors of up to 20 dB at a certain frequencies. For roughness estimation, Janssens’s method returns roughness values with errors of 10dB for some wavelengths for the total wheel and rail effective roughness.

In an attempt to improve the accuracy of these methods, more accurate parameter estimation models have been developed. These are based on the novel use of: the Semi-Analytical Finite Element (SAFE) method for the rail vibration model; and the Wave-number Boundary Element (WBE) method for the rail radiation model.

The SAFE rail model has been developed to accommodate multiple layers of rail support. Rail vibration and radiation behaviours have been investigated utilising these rail vibration and radiation models.

The results show that 1) practical models for determining the key parameters for rolling noise and roughness prediction, incorporating more accurate assumptions for rail and rail support behaviour can be developed; 2) below 1 kHz, the assumed rail support has a significant effect on both the calculated rail vibration response and dispersion relations as well as the calculated rail radiation power; 3) the results also show that the assumed rail support has a limited effect on the rail radiation ratio, under a vertical excitation, and on the calculated rail radiation directivity, under either vertical or lateral excitation; 4) the Timoshenko beam-based rail model adequately predicts vertical rail vibration response for the rail foot centre but
inadequately predictions of the rail lateral response, should accurate predictions be required for particular frequencies, as is the case for the separation of wheel and rail rolling noise from a total noise measurement; and, 5) the analytical line source model is inadequate for the prediction of the rail radiation directivity factor above 1 kHz, and the rail radiation ratio given lateral excitation. The parameters obtained by using the SAFE and WBE rail models developed have been applied to both the VTN rolling noise separation method and Janssens’s roughness estimation method. Field tests undertaken show that 1) for those cases analysed, the application of the new models results in accurate prediction of rail radiation i.e. elimination of the over-estimation typical of the existing models, and 2) calculated wheel and rail total effective roughness correlation can be improved, with the total effective roughness from direct wheel and rail roughness measurements. In addition to providing more accurate parameters for the VTN and Janssens’s methods, the models developed in this work can also be used to improve the accuracy of rolling noise prediction software as well as to investigate the design of rail dampers and low profile noise barriers. Validation of these results using more extensive field trials is required in order to confirm these results. In addition, although the rail models developed allow the inclusion of multiple layers of support with different characteristics, they only accommodate the assumption of a continuous support. A more accurate model would allow discrete supports so representing the actual support provided by the sleepers. The effect of this limitation is restricted to specific frequencies.
ACKNOWLEDGEMENTS

The work described in this thesis has been carried out under a scholarship provided by the CRC for Rail Innovations, supported through the Australian Government’s Cooperative Research Centres Program.

First and foremost, I want to express my sincere gratitude to my supervisor, A/Prof. Richard Dwight, for his enthusiastic encouragement and great supervision on this research work, most of all, for his generosity to share his precious time with me whenever I need help. Without him, I could not have completed this work.

I am also indebted to my coordinate supervisor Prof. Kiet Tieu for his guidance.

Many thanks are also given to my assistant supervisor Dr. Jiandong Jiang for his valuable and constructive discussions on this research work. His kindness of sharing his knowledge and great experience on the area made my research work a lot easier.

I would also like to extend my thanks to the university workshop staff. Without their kind help, my experiments will take much longer time.

I am also indebted to the people from RailCorp, who kindly invited me to join their field activities many times and shared their experiment data generously, all of which contribute to the work presented here. Their help on the organisation of my access to the track and the conduct of field tests which forms a crucial part of my research work is also highly acknowledged.

To my parents words are not enough to show my gratitude for their relentless support and love which drove me through all of the hardships I have ever confronted.

Last but not least, I would like to thank my colleagues at University of Wollongong. With them, my study and life has become a happy experience which I will cherish forever.
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1 INTRODUCTION

1.1 Background
Railways are usually seen as a means of transportation more environmental friendly than aircraft and road traffic. They cause less annoyance than either aircraft or road traffic at a given high noise level (Fields and Walker, 1982). Conflicting results also indicate railway noise has more physiological impact than the others (Griefahn et al., 2006). Either way, the fact remains that railway noise has become a drawback, preventing the promotion of railways. Legislation to curb railway noise emission has been enforced in many countries around the world. More are expected to join the club.

It is apparent that complex systems are involved in the operation of railways. The sources of the noise can be many. Depending on sites and conditions, the same train may generate different levels of noise, and the dominant noise source may vary. Those normally of significance can be classified as follows:

1) Rolling noise: wheel and rail vibration excited by the undulations at the contacting surface. Its bandwidth is rather broad, ranging from 50 to 5000 Hz. It typically dominates over the other sources for train speeds from 50 to 300 km/h, particularly on tangent tracks.

2) Impact noise: due to discontinuities in the running surfaces of the rail and wheel. Features creating such discontinuities include: design features such as rail joints; points, crossings and defects, such as wheel flats; or dipped welds and squats. In each case, the excitation mechanism is similar to rolling noise: a vertical discontinuity in the running surface causes vibration of the wheel and rail. The most apparent difference between impact and rolling noise is the discrete nature of the event, and therefore the short time duration of the noise per wheel pass-by.

3) Traction noise: generated by power units of any kind, including diesel or electrical power sources. This covers a range of possible mechanisms associated with the function of converting the supply energy to mechanical work. Relevant situations include: diesel locomotives at idle; electric or diesel powered systems under traction; and high-speed trains at low speeds (below ~60 km/h) or at idling (Dittrich and Zhang, 2006).
4) Friction braking noise: generated by the interaction between the friction material and the rotating element. Some classification systems consider this a sub-set of traction noise.

5) Curving noise: caused by friction induced self-excitation of the wheel and rail in a lateral direction on low radius curves.

6) Aerodynamic noise: caused by the disturbance of the airflow over the train, which becomes a significant noise source for train speeds >300km/h.

7) Other noise sources include; wagon “bunching”, coupler noise, bridge noise, warning signals, communications system noise, stabling and yard noise, maintenance noise; internal noise such as air-conditioning noise, and gangway noise.

Among them, rolling noise is the most important in most circumstances, as its occurrence is ubiquitous and dominates over a wide range of train speeds. Clearly, without reducing the noise from the trains or the tracks, a significant restriction of train operation may be required in order to reduce noise to a permissive level. Noise barriers may be built but their life cycle cost is normally a significant impost on the rail system.

The mechanism of rolling noise generation is well understood to consist of two almost equally important components — wheel and track. Track noise contribution can be further separated into rail and sleeper noise radiation. To mitigate rolling noise, those three components have to be targeted.

Various noise control measures have been developed with the assistance of analysis tools, notably including Track Wheel Interaction Noise Software (Thompson and Jones, 2000). Trials of these measures have also be conducted during full scale operations (Thompson and Gautier, 2006). However, a control measure may be effective on wheel but not on the track, and vice versa. To evaluate how effective a control measure is, a single microphone is apparently inadequate. A better evaluation method to determine wheel and rail contribution separately is certainly required under this background. In order to direct and manage mitigation methods, it is desirable to be able to determine wheel and rail roughness.

Various methods have been developed to identify wheel and rail contributions to rolling noise, such as Vibro-acoustic Track Noise or VTN (Verheijen and Paviotti, 2003). The accuracy of these methods is still unknown in most cases, as there is a
lack of (or lack of access to) published trial results of these methods. Evaluation of these methods is therefore required.

Since the development of these methods, more advanced rail vibration and radiation models have been developed. However these advanced models have not been used for rolling noise calculation. After proper improvements, these models may be used to provide more accurate parameters, which can be used to upgrade the current noise calculation methods to achieve better accuracy.

1.2 Aims of the thesis
This work aims to establish a practical measurement approach that is able to separate rolling noise into wheel and rail noise components and estimate wheel and rail roughness.

1.3 Structure of the thesis
This thesis is separated into 7 chapters:

Chapter 1 is an introduction to the background of the work conducted.

In Chapter 2, current knowledge on rolling noise mechanism is reviewed, including the measurement methods available. Methods on more advanced rail modelling are introduced and their pros and cons compared.

In Chapter 3, the promising measurement methods identified in Chapter 2 are trialled through field tests. The pros and cons of each method are identified.

In Chapter 4, a rail vibration model, based on the Semi Analytical Finite Element method, is proposed, which accommodates the representation of rail, the support structure, as multiple layers, in addition to rail cross section shape. The parameters required in the model are determined from field tests. Dispersion relations and rail vibration responses generated by the application of these models are presented.

In Chapter 5, a rail radiation model is established based on the boundary element method. By coupling with the Semi Analytical Finite Element rail model presented in Chapter 4, important parameters characterizing rail radiation are calculated. These include rail radiation ratios and radiation directivity factors in the rail cross section plane, due to both vertical and lateral point force excitations.

In Chapter 6, the models developed in Chapter 4 and in Chapter 5 are used to obtain parameters that can be utilised in the noise/roughness calculation methods presented.
in Chapter 3. The data collected from the field are used to trial the methods with the upgraded parameters.

Chapter 7 summarizes the work of the thesis and sets out the next steps for this research.
2 LITERATURE REVIEW

The design of practical rolling noise separation methods is based on understanding of the rolling noise generation mechanism, which includes roughness excitation, wheel and rail vibration, sound radiation and propagation. Therefore theoretical work on each of these components will be reviewed.

Estimation of wheel/rail roughness requires the effects of the finite size of a wheel and rail in contact to be taken into account. This is especially true for roughness measured directly by using displacement/acceleration sensors with a very small tip. Available roughness processing methods will therefore be introduced.

Current knowledge on the modelling work of both wheel and rail is reviewed. Emphasis is given to the rail because: 1) modelling work related to wheel has been fully developed while development of rail model is still in progress and 2) rolling noise separation methods are mostly track based, due to ease of access and ability to conduct tests on the track. The noise contribution of the wheel is often inferred afterwards.

After comparing the pros or cons of the available rail models, both analytical and numerical, the model that most accurately represents rail vibration/radiation is identified. Directions of further improvements on these identified models are then established.

Secondly, available empirical methods on rolling noise separation and roughness estimation are reviewed. The most promising method for trial is identified and reviewed in more details.

In the conclusion, improvements on the identified empirical methods are proposed.

2.1 Rolling noise generation mechanism

Since the first series of papers on it published by Remington (1976a, 1976b), the rolling noise generation mechanism has been shown to be due to the vibrations of the wheel and rail excited by their surface undulations. This is known also as roughness among railway communities. Those vibrations propagate into the surrounding medium (air) as sound. A well-known diagram showing this process is given in Fig. 2.1.

The frequency range of acoustic interest for rolling noise is between 50 and 5000 Hz. This is a frequency range humans are most sensitive to and find hard to endure.
The first question arising from inspecting Fig. 2.1 is: which component is more important in radiating noise; wheel, track or sleepers? This question is relevant to the design of a noise control measure, as any proposed measure may not work if applied to a secondary component. It is now clear that both wheel and rail contribute significantly to the total rolling noise (Thompson, 1988). Rail radiated noise is often dominant from approximately 0.5 to 1 kHz. Above 1 kHz the wheel radiated noise dominates. Below approximately 0.5 kHz the sleepers radiate more effectively than rail, a result of their relatively large radiation area and similar vibration level relative to the rail. At which frequency rail radiated noise starts to dominate rolling noise is determined by the decoupling vibration frequency of the rail from the rest of the track structure. The most important affecting parameter is the stiffness of rail pads, normally installed between the rail and the sleepers. A higher pad stiffness indicates stronger coupling between rail and sleeper vibration, and higher sleeper noise radiation (Vincent et al., 1996). The dividing frequency between the rail and wheel component dominance is determined by other factors such as: wheel and rail sizes and shapes, train speed, and any damping measures employed on either rail or wheel.
2.2 Theoretical modelling and validation

2.2.1 Introduction

Considerable work has been published setting out various approaches to the modelling of rolling noise. The seminal work of Remington (1976a, 1976b) was extended by Thompson, in particular (1993a, 1993b, 1993c, 1993d, 1993e) which led to the development of ‘Track Wheel Interaction Noise Software’ (‘TWINS’) analysis tool, well known among the railway noise community. TWINS methodology is shown in the diagram in Fig. 2.2.

Fig. 2.2. TWINS methodology (Thompson et al., 2009, Section 2.5), continuous arrow: flow direction for roughness estimation from measured rail vibration; dashed arrow: flow direction for noise separation from measured rail vibration.
This work aims to establish a practical model for use in separating rolling noise components, particularly at a rail track site which has the track related parameters, e.g. vibration, measured. Rail (and sleeper) radiation models are of great interest to get separate wheel/rail noise components with measured rail vibration (optional sleeper vibration). To get roughness, the order of the flowchart shown in Fig. 2.2 has to be reversed, i.e. going upwards from rail vibration to wheel/rail roughness. As a result, knowledge on wheel and rail vibration and interaction model, and the roughness processing model, notably the contact filter, is required. A review on each of these points is given below.

2.2.2 Roughness processing methods

The roughness effective in exciting wheel and rail vibration has amplitude ranging from one to tens of microns, depending on the wavelengths. The relationship between the roughness wavelength \( \lambda \) and the vibration/sound frequency \( f \) is:

\[
f = \frac{V}{\lambda}
\]

where \( V \) is train speed.

Obviously, according to Eq. (2.1), the higher the train speed, the higher the vibration/acoustic frequency induced by roughness of a given wavelength. The range of roughness for a noise event can also be estimated from Eq. (2.1) at a given train speed.

Direct measurement of roughness, either on wheel or rail surface in practice, is done through an acceleration or displacement sensor, like a LVDT, which scans the measured surface. The problem arising from use of this type of sensor is that the sensor usually has a pined tip, which has a radius of several millimetres. In contrast, wheel and rail contact is between 10 and 15 mm in length and width, which is much larger than the tip of the sensor. This results in a too detailed profile being collected by the sensor, which however does not excite wheel and rail to vibrate. This attenuation effect, due to the finite size of wheel and rail contact area, is known as a ‘contact filter’. Remington (1976b) was the first to investigate this effect and gave an analytical formulation to estimate this effect, based on Hertz contact theory. Later, Remington and Webb (1996) developed a more sophisticated numerical model, which represents the contact between wheel and rail as a series of non-linear springs at a series of points distributed over the contact area. The contact force at each point
is assumed to be proportional to the square root of the local spring deflection. This model is known as ‘Three dimensional Distributed Point Reacting Spring’, or 3D DPRS model. As indicated by the name, the model requires information across the contact area, i.e. roughness measured at multiple positions across the contact surface. To improve the performance of the model when only measurement at a single line is available, Ford and Thompson (2006) developed a two-dimensional DPRS model, which they referred as the ‘mattress model’. In their model, wheel and rail are also connected by a series of distributed point springs but along a line rather than distributed over an area. Both of the two numerical models require adjustment of the wheel or rail radius to ensure a correct contact patch size.

Performance of the three filters is compared in Fig. 2.3. The data used for 3D and 2D DPRS was extracted from reference (Ford and Thompson, 2006) which was obtained by averaging six measurements. The parameters used were: wheel radius of 460 mm in rolling direction, rail radius of 460 mm in the transverse direction, infinite wheel radius in its transverse direction and infinite rail radius in its longitudinal direction at the typical contact area for rolling noise. The wheel load was 50 kN. The same parameters are used in Remington’s analytical model. The radius of the contact area is an equivalent one because practical wheel and rail contact is elliptical (Kalker, 1991), while Remington’s analytical model assumes a circular contact. The equivalent radius ensures that the circular contact area is equal to the elliptical contact area. The calculation process is given in Appendix A. An important parameter in the analytical model is $\alpha$ which indicates the correlation between roughnesses across the contact area. A higher $\alpha$ value implies poorer correlation, or the roughness is less effective in exciting wheel and rail to vibrate compared to well correlated roughness, e.g., roughness of a sine wave shape on the rail surface in the longitudinal direction. The results obtained by using analytical models presented in Fig. 2.3 include 4 $\alpha$ values, i.e. $\alpha = 0.1, 1, 3$ and 5.

Fig. 2.3 shows that analytical model agrees well with the 3D DPRS below 3 kHz when $\alpha = 1$. Even at higher frequencies, up to 4 kHz, the difference is within 5 dB. This frequency corresponds to roughness of wavelength 6.9 mm, smaller than the dimensions of a contact patch. When $\alpha = 0.1$, the analytical filter effect is higher than either the 3D or 2D DPRS models below 3 kHz. When $\alpha = 2$ or 5, the analytical filter is below the two numerical models in the current case. The 2D DPRS model is
generally below 3D DPRS, except between 1.5 and 2 kHz. This example confirms the finding observed by Thompson (2003a) that when choosing $\alpha$ a value between 1 and 3, the analytical solution can achieve good accuracy.

![Graph showing contact filter effects](image)

Fig. 2.3. Effects of different types of the contact filters: 3D DPRS, 2D DPRS, $\alpha=0.1$, analytical model, $\alpha=1$, analytical model, $\alpha=3$, analytical model, $\alpha=5$, analytical model. (adapted from Ref. (Ford and Thompson, 2006)).

Measured wheel and rail roughness can be used to calculate the total effective roughness $L_{r,tot}$, which is the roughness effective in exciting wheel and rail to vibrate. It can be calculated by,

$$L_{r,tot} = L_{r,wheel} \odot L_{r,rail} + CF$$  \hspace{1cm} (2.2)

where $L_{r,wheel}$ is wheel roughness spectra, $L_{r,rail}$ is rail roughness spectra, $CF$ denotes contact filter and $\odot$ denotes energy sum.

2.2.3 Wheel and contact modelling

Fig. 2.2 also shows that the wheel, rail vibration is a result of their interaction at the contact area. For each component involved, which includes wheel, rail and the contact, its role is measured by either impedance (Remington, 1976b), receptance.
or any other equivalent quantities that can qualify the ability of the response of the component under a point force excitation. Impedance is the ratio between the excitation force and the structure vibration velocity it induces. Receptance is the ratio of structure displacement and its excitation force. A higher impedance (or lower receptance) indicates a smaller vibration level for a given excitation. Determination of receptance for each component has been under intensive study.

Wheel vibration response nowadays has become a standard procedure, which can be obtained by either finite element (FE) analysis or experimentation for a stationary wheel. Wheel rotation effect may also be included in the model thereafter, see Thompson (1993e). Contact is often simplified as a damped spring model. The creepage (relative movement between two contacting bodies at the contact position) can also be included, see Thompson (1993d). To study contact stiffness at six degrees of freedom, he applied Kalker’s contact theory (Thompson, 1993d). All of this work is also available in Thompson’s book (2009).

2.2.4 Track modelling

Rail vibration model development in contrast has been a long lasting process. This is mainly because of the complexity of the rail cross section and the complexity of the rail support, such as the nonlinearity of the rail pad, ballast stiffness (Fenander, 1997, Thompson et al., 1998) or the whole rail foundation (Wu and Thompson, 1999c).

A comprehensive review of the track model of earlier development was given by Knothe and Grassie (1993). Thompson and Vincent (1995) compared the performance of the three alternative track models incorporated in TWINS. Two of the models are Timoshenko beam based models. The first model is a continuously supported spring-mass-spring rail model, developed by Grassie et al. (1982), which is shown in figure (a) of Fig. 2.4. The second model is a rail model with periodic support and is shown in figure (b) of Fig. 2.4. The third model is a numerical model with continuous support developed by Thompson (1993c). In their work, they also found a way to calculate cross coupling between vertical and lateral rail vibration responses by assuming it is proportional to the geometric average of the vertical and lateral response for beam based rail models. In a companion paper, Vincent and
Thompson (1995) presented the validation of these models through lab tests. Other single beam based rail models include a periodically supported rail model developed by Heckl (2002) who investigated coupling between various waves travelling along the rail and used Bloch’s theorem to include the effects of periodicity of the supports. In an earlier work (1995), Heckl developed a model to examine the effects of random support on rail response.

Fig. 2.4. Track beam models with: (a) a continuously support, (b) a discontinuously support.

In general, these single beam based models have inherent defects. They do not allow for rail cross-section deformation. Rail starts to deform around 1.5 kHz, such as “foot flapping” induced by vertical bending (Thompson, 1997). This renders the beam models ineffective in predicting rail vibration behaviours at higher frequencies.

Analytical track models using multiple beams have also been reported. Wu and Thompson (1999b) utilized 2 separate Timoshenko beams, representing the rail head (and web) and foot, respectively, to study vertical rail vibration behaviour. The beams are connected with a layer of continuously distributed springs, which allows the relative motion between the two beams. By limiting the research on the model to the first order vibration mode, they were able to study foot deformation, which is not possible using the single beam model. In this work, Wu and Thompson also investigated the effects of periodic supports on the rail vibration response, based on a method used by Heckl (1995). In this method the supports are represented by a series of point reaction forces applied on the infinite beam. By applying Green’s function and the superposition principle, they predicted rail vibration response. In another work, Wu and Thompson (1999a) used a multiple beam model, representing rail head, web and rail foot, respectively, to investigate rail lateral vibration at high
frequencies. However, multiple beams based track models still cannot take into account rail deformation, especially rail web deformation.

This work listed above is all based on an unloaded track and a uniform track property, e.g. constant rail pad and ballast stiffness along the track. However, as mentioned previously (at the beginning of Section 2.2.4), rail pad and ballast stiffness can have a non-linear behaviour with the increase of the load. Rail dynamics can be affected by the load because of the aforementioned resilient track components. This effect has been investigated by Wu and Thompson (1999c, 2000).

In their earlier work (Wu and Thompson, 1999c), they examined the local stiffening effects induced by a single wheel on the track and the impact of that on the rail vibration response. In their later work, they included multiple wheels on the track in their model, without the consideration of the wheel preload (Wu and Thompson, 2001). The latter effect was included in another work (Wu and Thompson, 2000). The general conclusions from their study were that the wheel load effect is only significant at a local region around the wheel, about several sleepers at each side of the wheel. Its impact on rail vibration response is limited. But wave reflections due to the presence of multiple wheel can cause rail vibration response to fluctuate around the rail response obtained without this effect. This effect is significant between 500 and 900 Hz and then reduced with increases of the frequency. Their findings implied that for rail vibration prediction, the effects of wheel loads and the nonlinearity of the support can be ignored in most cases, as long as the study is not limited to rail response close the wheel/rail contact region. The effect of multiple wheel presence on the track can also be neglected due to the limited frequency range of its influence.

So far, the rail dynamics reviewed is all based on a stationary harmonic point excitation. The moving harmonic load excitation, which more closely represents the reality of moving wheels, has been studied by Thompson (2009, Section 3.2.6) who considered this type of excitation working on an Euler-Bernoulli beam. Through this simple model Thompson demonstrated that the load motion induces the Doppler effect to the waves propagating along the rail, i.e. the waves are shortened ahead of the load position in the moving direction and lengthened behind the load position. Sheng et al. (2005b) developed a rail model with periodic supports excited by a moving point load, based on the Semi Analytical Finite Element method. They
further demonstrated that the method could also be applied to a conventional Timoshenko beam model, which they used to calculate both vertical and lateral rail responses. They observed the Doppler effects from the splitting of a vibration peak at the pinned-pinned frequency. In this work, they demonstrated the validity of the continuously supported rail model on the prediction of rail dynamics in the vertical direction in most of the frequencies investigated. They also demonstrated lateral rail responses predicted using the continuously supported rail model are lower than those predicted using the perodically supported rail model below 200 Hz. Despite these reported differences, the periodically supported rail model introduces very limited change on rolling noise prediction presented in decibel level for one-third octave bands, compared with the continuously supported rail model as reported by Thompson et al. (1996a).

The effects of multiple moving wheels on rail response have also been investigated by Sheng et al. (2007). By assuming track and surface roughness are periodic and the system is linear, Sheng et al. (2007) found that steady state wheel/rail interaction forces and displacements at the wheel/rail contact points are periodic functions of time. To enable the study of the moving axle excitations in a roughness based input model, such as TWINS, they proposed a method to transfer the moving axle excitations to equivalent roughness, which can be used as input into TWINS where the moving axle effects are excluded. From their trial, they found that equivalent roughness induced by moving axle excitations does not depend on speed while the contact force increases with the increase of the speed of moving axles. However, they did not investigate the effects of moving axles on the generation of rolling noise.

In addition to the analytical track models based on simple beams, numerical methods have also been employed to study rail dynamic behaviours. These models are based on FE or boundary element (BE) methods and allow for rail cross-section deformation. Two widely used models for waveguides, i.e. a structure of a constant cross-section and extending in another direction, are introduced first. One is the ‘Wave and Finite Element (WFE) Method’ and the other the ‘Semi-Analytical Finite Element (SAFE) Method’. The two methods may be referred with the same name in the literature, for example, Nilsson et al. (2009) called their method ‘Waveguide Finite Element method’ which is actually the SAFE method. Duhamel et al. (2006) named their method ‘Waveguide Finite Element method’ which is WFE method. The
two methods are different not only in their applications but also their underlying principle, though both of them aim to investigate the wave characteristics of waveguides. A diagram is given in Fig. 2.5 to show the different waveguides they are often applied to. WFE method is based on periodic structure theory and specialized in the study of structures like type (b) in Fig. 2.5. SAFE method is based on an analytical expression for the wave propagation in the waveguide elongation direction and therefore limited to the structure type (a) of Fig. 2.5.

![Diagram of waveguides](image)

Fig. 2.5. Waveguides: (a) continuous or circular, (b) periodic.

Two important parameters to characterize the vibration behaviour of a waveguide are its dispersion relations and its forced response. The dispersion relation relates the wave-number of a wave travelling along the waveguide with its frequency and determines the speed of wave energy propagating along a waveguide, i.e. the group velocity. Group velocity is the differentiation of the frequency with respect to wave-number or the inverse of the slope of a dispersion curve. Phase velocity can also be determined using the dispersion relations, which is the ratio of the frequency and the wave-number at a point (Fahy, 2007, Chapter 1). In addition, the real and imaginary parts of a wave-number reveal the propagating and decaying part of a travelling wave. The sign of the imaginary part of a wave-number signifies the wave’s travelling direction. These values can also be used to determine the travelling distance of various waves along the rail, e.g. by Ryue et al. (2009). The forced response of a waveguide, especially under a harmonic force excitation, is used to calculate the response of the waveguide to a general excitation.
<table>
<thead>
<tr>
<th>Methods(names)</th>
<th>Advantages</th>
<th>Disadvantages</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finite Element method (FEM)</td>
<td>It can rely solely on FE package; no development efforts are required on the establishments of governing equations.</td>
<td>A whole section of the structure has to be discretized; The length of the structure used for the modelling work is crucial for the range of wave-number investigated, which results in only a limited number of waves which can be modelled; no near field waves can be predicted</td>
<td>Thompson (1993c), Wu and Thompson (1999b, 1999a), Ryue et al. (2008)</td>
</tr>
<tr>
<td>Wave and finite element method (WFEM)</td>
<td>It harnesses the full power of conventional FEA; It is capable of including the periodic nature of the structure</td>
<td>The degree of freedom involved in the governing equations doubles that of the SAFE method, because of the requirement of meshing both ending cross-sections of the structure and therefore more computation; numerical difficulties are often encountered, depending on the length of the model used.</td>
<td>Thompson (1993c), Gry (1996), Gry and Gontier (1997), Gómez et al. (2006)</td>
</tr>
<tr>
<td>Semi Analytical Finite element method (SAFEM); Waveguide Finite Element Method (WFEM); 2.5 D Finite Element Method (2.5D FEM); Spectral finite element method.</td>
<td>It only requires discretization of one cross-section and assumes analytical expression in the other direction, the mesh of only the cross-section enables it to couple with 2.5D Boundary Element Method</td>
<td>The analytical expression in the longitudinal direction makes it difficult to include the discrete nature of a structure; the coefficient matrices of the governing equations have to be decided on a case-by-case basis and for each type of element, a complete analysis is required.</td>
<td>Gavrić (1995), Hayashi et al. (2003), Bartoli et al. (2006), Ryue et al. (2008), Nilsson et al. (2009), Ryue et al. (2009)</td>
</tr>
<tr>
<td>2.5D Boundary Element Method (2.5D BEM)</td>
<td>The method requires only the boundary of the structure, which results in a problem with matrices of much smaller size.</td>
<td>It has all of the inherent problems of Boundary Element methods, such as the singularity problem, which requires special regulation techniques. This adds difficulty to the application of the method.</td>
<td>Mazzotti et al.(2013)</td>
</tr>
</tbody>
</table>
The numerical methods that have been developed to model rail are reviewed and summarized in Table 1 where the pros and cons of each method are discussed. Though those methods have more general applications, only the publications dedicated to rail modelling, or used rail modelling as an application example, are selected and listed in this table, dating back to the early 1990s.

Thompson (1993c) built a rail model based on the so called ‘periodic structure theory’, which allows him to explore dynamic behaviour of rail of infinite length with the consideration of the complex rail cross-section. This method is the ‘Wave and Finite Element Method’ due to the underlying methodology used, see Table 1. In this model, he included the effects of rail pad, sleeper and ballast support on rail, as an equivalent continuous spring-mass-spring layer. By validating the predicted rail vibration responses with those from measurements, he confirmed the validity of his rail model. He identified serious rail cross-section deformation at frequencies above 1.5 kHz and the inefficiency of the Euler-Bernoulli beam model in the prediction of rail vibration response in the lateral direction. Through the study of rail support effect on rail response, he found it is limited at low frequencies below 1 kHz.

Gry and Gontier (1997) derived formulations for rail vibration using WFE method. They used a transfer matrix to relate the ‘state’ of the ending cross-sections of a track element, the components of which are shown in Fig. 2.6. The ‘state’ is a set of variables that define the status of a cross section. Available choices include displacements, forces or other parameters which are functions of displacements or forces, or both. From eigenvalue analysis of the transfer matrix, wave characteristics along the rail can be determined. Apparently the determination of the transfer matrix is vital to the accuracy of this method. They proposed both analytical and FE methods to determine the transfer matrix, including reduced basis to build the transfer matrix and an effective way to address the large eigenvalue spectrum of the eigenvalue problem, inherent in WFE method, which was widely adopted by: e.g., Mace et al. (2005) and Duhamel et al. (2006). However, they failed to find a solution to include waves of high frequencies which limited their model to frequencies below 1.2 kHz. This limitation was eliminated by Mace et al. (2005) who calculated rail response from waves decaying in amplitude over distance and waves with constant amplitude but decreasing time average power transmission along the propagation direction.
Further development on WFE method by Waki et al. (2009) considered forced response of the waveguides under point load excitations. Renno and Mace (2010) generalized the excitation to a convected harmonic force.

An inherent problem associated with WFE method is that the eigenvalue problem arising from solving the governing equation of this method is often ill-conditioned. Though reported by different authors, such as Gry and Gontier (1997), Mace et al. (2005) and Waki et al. (2009), this problem has not been completely addressed.

SAFEM was first proposed by Aalami (1973) to study waves propagating in bars with constant cross-section of any shape. More application examples using SAFEM include rail vibration (Gry, 1996), ground vibrations induced by travelling trains (Sheng et al., 2005a, Sheng et al., 2006), rib-stiffened plates (Orrenius and Finnveden, 1996), pipes (Finnveden, 1997), wind tunnels (Finnveden, 2004), curved structures (Finnveden and Fraggstedt, 2008) and ducts (Nilsson and Finnveden, 2008), and car tyres (Nilsson, 2004). The review here focuses only on application of SAFE method on rail track modelling.

Earlier SAFE rail models were based on a free and undamped rail and no forced rail response was calculated. Knothe et al. (1994) classified the numerical models into three categories: model 1: FE method in both rail cross-section and the longitudinal direction; model 2 and model 3: finite strip element method (FE method in the cross section and analytical wave solution in the longitudinal direction). Model 2 is two dimensional and model 3 is three dimensional. In fact, model 2 and 3 can be classified as SAFE method because of the similar underlying methodology used. They demonstrated the superiority of model 3 over model 1 in terms of computation costs.
They calculated dispersion relations for a free undamped rail up to 15 kHz. Gavrić (1995) assumed a free and undamped rail and calculated rail dispersion relations up to 6 kHz. He also presented the evolution of the rail cross section deformation of various waves with the increase of frequency.

Bartoli et al. (2006) introduced damping to the SAFE model by incorporating complex stiffness matrices. They investigated dispersion relations for various waveguides, including rail, and provided a formulation for the calculation of phase velocity, group velocity (for undamped waveguides) and energy velocity (for damped waveguides). Hayashi et al. (2003) also gave methods to calculate phase and group velocity but they used a free undamped rail.

Nilsson et al. (2009) and Ryue et al. (2009) included the effects of rail support into their SAFE rail model by representing the support as a layer of rail pad underneath the rail. They used an exaggerated rail pad size to clarify rail displacement and reduced pad density to avoid the waves travelling in the pad. Nilsson et al. (2009) also calculated forced rail responses.

The eigenvalue problem arising from solving the governing equation associated with SAFE method is better conditioned than WFE method. The numerical difficulties of the WFE method are due to the facts (Waki et al., 2009, Zhong and Williams, 1995, Duhamel et al., 2006) that: 1) the associated eigenvalue problem is sensitive to the length of the element used in the finite element model and 2) the eigenvalues are in pairs where one is reciprocal to the other. The absolutely analytical nature of SAFE method in the longitudinal direction also means that there is no low wave-number limit that can be calculated, when compared with WFE method. Its disadvantage is that it is difficult to include the discrete supports into the rail model. But this effect is very limited on vertical rail dynamics, except close to pinned-pinned frequency, as reported by Thompson (1993c).

BE method, as the name indicated, has the elements generated on the boundary of a structure. This method therefore reduces the problem by one dimension, in comparison with conventional FE method. Its application is primarily on problems involving infinite or semi-infinite domains, such as radiation or scattering problems, where the conventional FE method is unsuitable. This is because of the huge number of elements required for a FE model to cover the whole domain and also the complexity in applying proper boundary conditions. Coupled FE method and BE
method can also be used to study waves propagating from a source. In the railway community, examples include the work done by Sheng et al. (2003, 2005a) in studying ground vibration. Nilsson et al. (2009) used it to investigate rail radiation by coupling with a rail model based on SAFE method. Application of only BE method on the study of the dispersion of rail vibration has been conducted by Mazzotti et al. (2013).

In conclusion, the SAFE method is suitable for the modelling of rail despite the difficulty in including discrete support. It is expected to have better performance than conventional beam models by allowing for rail cross-section deformation and being better conditioned than WFE method. By using this model, it is understood that without the inclusion of discrete supports, lateral rail dynamics may be affected (Sheng et al., 2005b), especially below 200 Hz. Vertical rail dynamics however is less affected and the influence will be limited to the region near the pinned-pinned frequency. Improvement of the current rail model based on SAFE method however is required to include multiple layer of support, which is critical for the rail dynamics, especially below 1 kHz. Other effects, like moving axles, multiple wheels present on the rail and the non-linear nature of the rail supports, can be ignored due to their limited influence on rail dynamic behaviour, and therefore sound radiation, though preferably they should be included.

2.2.5 Radiation modelling

As shown in Fig. 2.2, vibration of wheels, rails or sleepers has to be transmitted into the air as sound before reaching an observer. A proper radiation model for each component is thus required to accurately predict their noise radiation. The critical parameters to characterize radiation characteristics from a structure are the radiation ratio and directivity factor. Radiation ratio relates the sound power generated by a radiating structure to the velocity averaged over the surface area of the structure. It is calculated as the ratio of the sound power radiated by the structure to that of a baffled plate, which has a same surface area, and a uniform surface velocity that equals the surface velocity of the structure averaged over time and space. The directivity factor is required to calculate SPL from the calculated sound power.

Radiation ratio for a wheel can be obtained either analytically or numerically. Earlier work is more on analytical formulation. Remington (1976) modelled the wheel as a
rigid, unbaffled disk to estimate its radiation ratio and assumed a uniform directivity for both radial and lateral wheel radiations. Thompson (1988) calculated wheel radiation by assuming it as a baffled piston. One drawback of these models is that they do not allow for the complex shape of the wheel and various wheel vibration modes. Later these effects were included by using BE based wheel models to determine wheel radiation characteristics. An example is Fingberg (1990) who used the BE method to predict radiation from a wheel in its various normal modes and Thompson and Jones (2002) who used BE method to investigate wheel radiation characteristics on a mode to mode basis. Thompson and Jones (2002) also explored the effects of wheel geometry on wheel radiation. Their work led to a few general analytical formulas, which are more convenient to use than the numerical BE analysis which has to be on a case to case basis.

Wheel directivity is often determined experimentally. Previous work includes Remington (1976b), Thompson and Dittrich (1991) and Zhang & Jonasson (2006). Thompson and Dittrich (1991) found that for a wheel with a straight web, its radial modes radiate more effectively in the radial direction than in the lateral direction and its axial modes radiate more effectively in the axial direction than in the radial direction. However, it should be noted that the coupling between radial and axial wheel vibration is weak for wheels with a straight web. They also measured the radiation characteristics of a curved wheel and found that both the radial and axial modes do not show any significant directivity because of the strong coupling of the vibration in the two directions. In the conclusion, Thompson and Jones (2002) suggested that radial and axial wheel motion should be treated separately for a wheel of straight web: wheel radial motion is better approximated with a monopole source and axial motion with a dipole source.

Radiation from the rail is more complex because waves travelling along it can be either decaying (causing rail to resemble a point source) or propagating (causing rail to behave like a line source) and the rail has serious cross-section deformation at high frequencies. In their work, Bender and Remington (1974) assumed that a rail radiates like a line source and neglected the effects of near field waves. Uniform radiation directivity was used. Thompson et al. (2003) built a rail model based on the BE method, calculated rail radiation ratio and radiation directivity in the longitudinal direction. They demonstrated that rail can be approximated by a two-dimensional
radiation model above 250 Hz. Below this frequency it is necessary to allow for three-dimensional radiation characteristics. Comparison of the predictions on radiation ratios from the line source model and the BE model is shown in Fig. 2.7. In this figure, the cylinder is called equivalent because its size is determined by tuning the rail radiation ratios obtained from an analytical line source model to those obtained from a numerical BE rail model.

Fig. 2.7 shows that the rail radiation ratios can be approximated by those of simple cylinder models, at low frequencies. At higher frequencies, rail radiation ratios calculated from the BE rail model fluctuate around those calculated from equivalent cylinder models. This is a result of the constructive or destructive interference of different parts of the rail due to its complex cross section. For example, the troughs occurring at 1 kHz and higher order harmonics at 2 kHz are due to destructive interference of rail head and rail foot, because the sound wavelengths in the air (around 0.34m and 0.17m) at these frequencies almost double or equal that of the rail height (around 0.17m).

![Graph showing rail (UIC60 rail) radiation ratios and equivalent cylinders](image)

**Fig. 2.7.** Rail (UIC60 rail) radiation ratios, vertical direction (BEM calculation), lateral direction (BEM calculation), equivalent cylinders, vertical direction, lateral directions (Thompson et al., 2009).
Thompson et al. (2003) also concluded that no uniform value can be used to represent the rail radiation directivity in the longitudinal direction. By representing it as the inclination of the intensity vector with respect to the direction normal to the rail, they found that rail longitudinal directivity is about 26.5° at low frequencies and tends to become 0° with an increase of the frequency. This finding provides a physical explanation to the observed overestimation of wheel noise contribution from microphone arrays when deployed along the rail, such as reported by Kitagawa and Thompson (2009). Thompson et al. (2003) used a free rail model in predicting rail radiation ratios. They did not consider rail directivity in the vertical plane, which has great implications to a single microphone based measurement.

In conclusion, wheel radiation ratios and directivities are ready to use, however work is still required for rail. Though rail radiation ratios have been calculated by Thompson et al. (2003), they were obtained for a free rail. By affecting the rail dynamic behaviours, the support is supposed to affect rail radiation characteristics. But this effect has not been reported. Rail radiation directivities in the vertical plane have significant implications to a single microphone based measurement or measurements using a microphone array surrounding the track but are still unknown despite the fact that rail radiation directivity in the longitudinal direction has been reported by Thompson et al. (2003). A model capable of investigating the two aspects is required. As mentioned previously in this section, BE method has been successfully used to determine radiation ratios of a rail with single layer of support. It is expected that BE method can also be used to calculate radiation ratios and radiation directivities of a rail with multiple layer of support by coupling with an improved SAFE rail model that can include multiple layers of support.

2.2.6 Validation and further development of the TWINS model

TWINS model has been validated intensively since its development. Earlier validation work is done by Thompson et al. (1996a). Extended validation has been made by Jones and Thompson (2003). It has been widely used for designing noise control measures (Vincent et al., 1996, Thompson and Gautier, 2006) and providing a benchmark to assess accuracy of other alternative noise separation methods, e.g. the Vibro-acoustic Track Noise (VTN) method (Verheijen and Paviotti, 2003). Further application includes the usage of TWINS to explain the apparent
underestimation of microphone arrays in measuring rail noise (Kitagawa and Thompson, 2006).

Recent work on TWINS development includes Railway Rolling Noise Prediction Software (‘RRNPS’) (Jiang et al., 2011), which is a Matlab version of TWINS with a Graphic User Interface. Jiang et al. validated this model under what they termed European (2011) and Australian conditions (Jiang and Meehan, 2012). In addition, they included a roughness growth prediction model into RRNPS (Jiang et al., 2012). They also investigated the effects of humidity and friction at the wheel rail contact area on the generation of rolling noise by a test rig (Jiang and Meehan, 2013).

![Fig. 2.8. Difference between predicted and measured sound pressure level, averaged over 34 wheel/track combinations, —— mean, --- one standard deviation range (Jones and Thompson, 2003).](image)

TWINS is found to be able to provide an accuracy within 2dB, in terms of total rolling noise level (Jones and Thompson, 2003). Differences in each frequency band may be larger, see Fig. 2.8 which is replicated from reference (Jones and Thompson, 2003). The results presented were reported to use only the radiation models of TWINS by inputting measured wheel and rail vibration. Noise predicted using roughness as input is slightly worse than the results shown due to the uncertainties of
the roughness inputs, e.g., due to variation of contact area and roughness variation across the contact area.

It should be noted that though the extensive validation has been made on the total SPL (assembled from every component) as shown in Fig. 2.8 and also on wheel and rail vibration, see reference (Thompson et al., 1996a), no validation has been made on the separate vehicle and track noise contribution or at particular frequency bands.

2.3 Empirical methods

2.3.1 A general review

As reviewed in subsection 2.2.6, TWINS has achieved success in the prediction of the rolling noise and in guiding the design of noise control measures. The introduction of low noise solutions gives rise to the need for improved measurement methods, which are accurate enough to quantify the effect of each individual noise control measure.

It is known that wheel, rail and sleeper can all significantly contribute to rolling noise and their contribution may vary from site to site and train to train. As a result, either track or vehicle can dominate the noise. It is then difficult to assess the performance of a control measure, which may be effective on reducing wheel noise but not track noise and vice versa. Measurement based on a single microphone is apparently inadequate for this purpose. A method able to separate vehicle noise and track noise is indispensable under this context.

Much of the pioneer work for this purpose has been done in Europe since 1990s under a series of projects, such as Methodologies and Actions for Rail Noise and Vibration Control (METARAIL) from 1997 to 1999 which aimed to develop techniques for railway noise measurement for type testing (Wirnsberger et al., 1999). Another example was the Strategies and Tools to Assess and Implement noise Reducing measures for Railway Systems (STAIRRS) from 2000 to 2003 in which methods of separating vehicle and track noise contributions were further improved and cost benefit analysis was conducted on available noise control measures (Hemsworth, 2002). Yet another example was the Harmonoise project from 2001 to 2004 which aimed to develop harmonised, accurate methods for the assessment of the environmental noise from railways and roads. The latest project is Improved Methods for the Assessment of the Generic Impact of Noise in the Environment.
(IMAGINE) which was commenced in 2003 and aimed to extend Harmonoise methods to include aircraft and industrial noise sources (Beuving and Hemsworth, 2006). A few effective rolling noise separation methods were developed during these projects and those are listed in Table 2.

Table 2 is adopted from reference (Verheijen, 2004) but is enhanced by the addition of methods Verheijen did not cover. Contrary to the options presented by Verheijen, the methods to separate rolling noise are reviewed instead of methods that can be used to build transfer functions. It should be noted that all of the data provided in Table 2 are in general relative terms: e.g. ‗fair‘, ‗good‘, ‗excellent‘ due to the unavailability of the actual accuracy. Each method will be reviewed in the following text.

In Table 2, VTN method was developed by Verheijen and Paviotti (2003) in the STAIRRS project. This method uses a few measured vibration signals from rail and sleeper to estimate track radiated noise by assuming simple radiation models for each of the components. Vehicle noise contribution can then be estimated by subtracting track noise contribution from the microphone measurement. Obviously the accuracy of vehicle contribution depends on the track noise calculation. No event better than ±2dB(A) for vehicle noise calculation was reported by Verheijen and Paviotti. But Verheijen and Paviotti reported that VTN is able to provide an accuracy up to 1.5 dB(A) for track noise contribution if a specific measurement procedure is followed during the measurement. The procedure is mainly about instrumentation set up and the data format required for data analysis. Verheijen and Paviotti validated the method by comparing it with other methods, like TWINS, Janssens’s method (Janssens et al., 2006a) and Multiply Input Single Output (‗MISO‘) (Letourneaux et al., 2002). The latter two methods will also be reviewed later this section. The results reported by Verheijen and Paviotti are shown in Fig. 2.9 where VTN prediction is compared with TWINS prediction. However, considering VTN is built upon TWINS, this validation may not be reliable.

MISO in Table 2 is developed by Letourneaux et al (2002) under STAIRRS project. This method attempts to estimate track noise contribution from rail vibration measurement while infer vehicle contribution by subtraction. The method however assesses track contribution by building a track transfer function between track vibration and track noise measured midway between the bogies which is:
\[ H_{tr}(f) = C^{-1}_{aa}(f)C_{ap}(f) \] (2.3)

where \( H_{tr}(f) \) is track transfer function, \( C^{-1}_{aa}(f) \) is the inverse of the cross spectra matrix of the measured vibration signals, \( C_{ap}(f) \) is the cross spectra matrix of the vibration and near rail sound measurements. The subscript \( a \) and \( p \) denote acceleration and pressure, respectively. Track contribution is obtained by multiplying vibration integrated over the whole train passing period and the obtained transfer function \( H_{tr}(f) \). As indicated by Eq. (2.3), accuracy of the method depends on calculated cross spectra, which are sensitive to relative phase of the two input signals. This makes the method vulnerable to the background noise as reported by Letourneaux et al (2002). In addition they did not provide a rigorous validation of the method in their work. The method uses a similar set up to VTN.

![Graph](image)

**Fig. 2.9.** Comparison between VTN and TWINS predictions, upper figure: passenger train; lower figure, freight train. (Verheijen and Paviotti, 2003).

Dittrich and Janssens (2000) and Janssens et al. (2006a) developed another method which will be referred to as Janssen's Method. It is noted that this method has been coded into a software package named PBA, standing for Pass By Analysis. This
method is based on track vibration and wayside noise measurement during train pass-bys to derive track radiated noise. This method estimates track noise contribution from established transfer functions. The same principle applies to vehicle contribution. A transfer function is the noise radiation of a component per unit roughness. Therefore this method has to be able to address two problems: 1) the separation of noise contribution from vehicle and track and, 2) the estimation of roughness at the wheel and track contact surface. For the first problem, they proposed three methods. The first method is the ‘Equivalent Force method’ (Dittrich and Janssens, 2000), which uses an impact hammer to create artificial excitations along the track at a series of positions and measures the generated rail vibration (or alternatively force) and sound at a number of positions. The response functions between the sound and the vibration can be used to estimate track noise contribution under real train operations, by measuring track vibration at the same positions. The advantage of this method is that it is easy to use and is capable of constructing a traffic noise map over the measurement surface. The impact of vehicle presence on the track is however not considered, which can affect the track dynamics. The second and the third methods proposed by Dittrich and Janssens are the silent track and the silent vehicle method. Each of the methods requires the silent component radiates significantly less than the other, e.g. 10 dB less. Contribution of the silent component can be neglected compared with the other in this case. Once the transfer function for a component is obtained, the other can simply be obtained through subtraction. They also proposed several methods to achieve a silent component, such as adding damping and maintaining a smooth surface. Janssens et al. (2006b) showed that a vehicle with smaller wheels can be used as a silent vehicle. In their work, they also addressed the second problem by developing a method which can transfer the measured rail vibration to the wheel and rail total effective roughness, as an alternative of direct roughness measurement. Track decay rates were also determined during the process and a corresponding calculation method was provided. To validate their methods, Janssens et al. compared the track decay rates derived with impact hammer tests and those obtained by using their method. They also compared the transfer functions obtained from pass bys and static measurements. In addition, they compared traffic SPL from wayside microphone measurement and from the prediction, using transfer functions and estimated roughness. Good agreements were
reported. However, the silent track or vehicle may not be easily accessed. The separate track or vehicle noise spectra were also not validated though the transfer functions of a same vehicle, determined at different sites, were compared. In addition, they did not validate the calculated total effective roughness with those from direct measurement, due to lack of wheel roughness. It should also be noted that the method uses a similar set up with VTN and MISO methods.

Another possible method also listed in Table 2 is the use of a microphone array. Microphone arrays can offer a possibility of identifying both location and amplitude of a noise source, as long as an appropriate number and shape arrangement of the microphones are selected. This technique has long been used to locate aerodynamic noise sources from high speed trains, such on Germany ICE (Brühl and Röder, 2000), French TGV (van der Toorn et al., 1996). Pallas et al. (2011) used microphone arrays to characterize tram noise sources. Use of a microphone array to identify rolling noise sources has also been performed, e.g., by Barsikow et al. (1987) who used both vertical and lateral line array to study rolling noise and by Kitagawa and Thompson (2006) who used a horizontal line array to study rolling noise. Except for the sophistication of the array support structure and the high expense, there are two other major limitations restricting the application of microphone arrays on rolling noise separation.

![Fig. 2.10. Noise map created by a T-shape microphone array from a passing train of speed 80 km/h (Dittrich and Janssens, 2000). The x axis is distance along the train, the y axis is the height above ground and the shading represents the measured sound pressure level in decibels.](image-url)
Table 2. Review of source separation methods developed in European studies (Verheijen, 2004), ‘Accuracy’: deviation of the results from the true value; ‘Robustness’: consistency of the results between different sites and different trains; ‘Economy’: cost of using these method; ‘Ease of use’: likelihood to obtain useful results without much experience; ‘Limitations’: difficulties or uncertainties when using these methods.

<table>
<thead>
<tr>
<th>Options</th>
<th>Accuracy of noise prediction/estimation</th>
<th>Robustness</th>
<th>Economy</th>
<th>Ease of use</th>
<th>Possible Limitations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equivalent forces method</td>
<td>good, depends on track function accuracy</td>
<td>Require validation</td>
<td>good</td>
<td>good</td>
<td>Cannot allow for the effects due to train presence</td>
</tr>
<tr>
<td>Vibro-acoustic Track Noise</td>
<td>Good, dependent on track noise</td>
<td>reasonable, tested in STAIRRS validation campaign</td>
<td>mainly measuring and analysis costs, trains can be measured in normal operation</td>
<td>track acceleration measurement needs some care</td>
<td>over-estimation of track radiation</td>
</tr>
<tr>
<td>Multiple Inputs Single Output</td>
<td>good, but dependent on vehicle noise</td>
<td>fair, dependent on track noise</td>
<td>reasonable, tested in STAIRRS validation campaign</td>
<td>similar as above option</td>
<td>success depends on degree of coherence between track acceleration and nearby noise</td>
</tr>
<tr>
<td>Janssens’s method + silent vehicle method</td>
<td>good, fair, dependent on track noise</td>
<td>reasonable, tested in STAIRRS validation campaign</td>
<td>a silent vehicle measurement campaign needs to be arranged; thereafter costs similar as for the first option</td>
<td>depends on organizational effort for silent vehicle; track acceleration measurement needs some care</td>
<td>Difficult to obtain a silent vehicle/track</td>
</tr>
<tr>
<td>Microphone arrays</td>
<td>underestimates rail contribution</td>
<td>Overestimates wheel contribution</td>
<td>good</td>
<td>expensive</td>
<td>no</td>
</tr>
<tr>
<td>Frid’s method</td>
<td>unknown</td>
<td>unknown</td>
<td>reasonable</td>
<td>good</td>
<td>Not fully validated</td>
</tr>
</tbody>
</table>
One limitation of using a microphone array on rolling noise separation is its spatial resolution, which does not allow differentiation of wheel and rail contribution (Dittrich and Janssens, 2000). This point can be seen through the example shown in Fig. 2.10 where radiation of wheel and rail is identified to be mixed together. Another limitation of the microphone array is that the rail is identified as a series of incoherent point sources (Kitagawa and Thompson, 2006). This results in the underestimation of rail noise radiation at high frequencies, where free waves cut on as reported by Barsikow et al. (1987). The low decay rates at these frequencies are the cause of the discrepancy because waves travel long distances along the rail and the vibration is coherent from point to point in these cases, which a microphone array cannot capture.

Frid (2000) developed an alternative method to separate wheel/track noise contribution by using two microphones, however in terms of sound power rather than SPL though SPL can also be calculated afterwards. In order to achieve this, he placed one microphone on the track bed and the other, a parabolic microphone at the wayside. Track radiation can be measured directly by the parabolic microphone due to its strong directivity, especially above 1 kHz. Only an instant of wheel contribution can be measured by the parabolic microphone when it is facing a wheel set. The decay rates of sound radiation away from a wheel were estimated using the SPLs measured by the parabolic microphone at a few other positions, which were then added to the measured wheel contribution. A calibration factor was used to calculate sound power from the measured SPL, which was determined by using the sound power calculated based on the track bed microphone and the SPL measured by the parabolic microphone when facing a wheel set. The method was also used to evaluate the performance of noise mitigation measures. However, valid validation of this method was not provided on either the track or wheel individual sound power in Frid’s work though comparison of a reconstructed total sound level with direct measurement shows a favourable agreement as reported by Frid.

2.3.2 Methods promising for trial

From the above review, it is concluded that for the purpose of noise separation, VTN is the most promising method for trial. This is due to its ease of use and relative ease of access to the measurement instruments (it requires only accelerometers and a data
acquisition system). The MISO method has a similar requirement to that of VTN in terms of instrumentation but its accuracy is liable to error induced by extraneous background noise. Irrespective of the attractiveness of the VTN method, it has been shown that a rigorous and reliable validation relative to its prime objective of separating wheel and rail noise contributions has not been set out in the literature. The only validation has been by comparing it with TWINS (2003). Validation of this method under a variety of track and vehicle types is required if implantation of the method in a wide scale is expected.

To estimate wheel and rail roughness, Janssens’s method is worthy of trial because no additional efforts are required on the measurement set up if carried out together with a trial on VTN. However the total effective roughness calculated by Janssens’s method was not validated.

2.3.3 VTN

VTN is identified as the promising method for rolling noise separation. The underlying methodology has to be understood which is reviewed in this section. VTN is built upon the track radiation algorithm integrated in TWINS (Thompson et al., 1996b). In this approach, track noise contribution, consisting of rail and sleeper components, is calculated based on measured accelerations of rail and sleeper. A simple radiation model is assumed for each component, accounting for the radiation area and radiation ratio of each component. In addition, the SPL at a receiver point from each component is estimated using a propagation model which includes the component’s specific directivity pattern and also the ground effects. Vehicle contribution is derived indirectly by subtracting the calculated track contribution from the measured total traffic noise,

\[ L_{p,veh} = 10 \log_{10} \left( 10^{L_{p,rot}/10} - 10^{L_{p,sl}/10} \right) \]  \hspace{1cm} (2.4)

where \( L_{p,veh}, L_{p,rot} \), and \( L_{p,sl} \) represent vehicle, total and track SPL (dB re 20 \( \mu Pa \)), respectively. The subtraction operation in Eq. (2.4) indicates that accurate \( L_{p,veh} \) is impossible unless it is higher than \( L_{p,sl} \).

Uncertainties of calculated rail or sleeper noise by using these simple radiation models have been reported by Verheijen and Paviotti (2003). This is particularly evident when either of the components exceeds the measured total traffic SPL. They
recommend assigning a value of 0.3 dB below the total SPL to the component exceeding the total SPL and a value 12 dB less than the total SPL to the remaining components. Clearly, in these cases, these values do not represent the real contribution of each component. This also casts doubt on the accuracy of predictions generally.

**Rail radiation model**

VTN assumes uniform rail vibration in both lateral and vertical directions. The phase difference of vibration along the rail is assumed to be zero. Based on the assumption, only one accelerometer is required in each direction. As a result, a rail can be seen as a line source in both the vertical and lateral directions. Lateral rail radiation is separated into two components, rail head and rail web, and is essentially seen as a line dipole source. In the vertical direction, the rail foot radiates significantly and will also cause a dipole field. However, this approximate dipole field is scattered by the ballasts and vehicle bodies such that it more resembles a monopole field (Verheijen and Paviotti, 2003). Thompson et al. (2009, Section 6.4.3) also made a similar conclusion.

The sound power, $W$, of a radiating structure is suggested to be (Thompson et al., 2009, Section 6.1)

$$W = \rho_0 c_0 \sigma S < \bar{v}^2 >$$

(2.5)

where $\rho_0$ is the air density, $c_0$ is sound speed in the air, $\sigma$ is the radiation ratio and $S$ is the surface area of the radiating structure, $< \bar{v}^2 >$ is the squared velocity averaged over time ($\bar{\cdot}$) and surface area ($< >$) of the structure.

The radiation area of a line source per unit length is given by

$$S = 2\pi R$$

(2.6)

where $R$ is the radius of the line source. According to Thompson (2009, Section 6.4.1), equivalent $R$ is around half of the rail height for lateral rail radiation and around a third of the rail width for vertical rail radiation. $R$ is less than a half of the rail width for vertical rail radiation because of the interference of the top and bottom surfaces of both rail head and rail foot. For UIC 60 rail, equivalent radiation radius $R$ is 86 mm for lateral rail radiation and 50 mm for vertical rail radiation.

$< \bar{v} >$ is given by
\[
\langle \nabla \rangle = \frac{\langle \tilde{a} \rangle}{2\pi f_c}
\]  
(2.7)

where \( f_c \) is the centre frequency of a frequency band, \( \langle \tilde{a} \rangle \) is acceleration spectra of the measured acceleration averaged over time and space.

The radiation ratios for line monopole and dipole are given by (Thompson et al., 2009, Section 6.2.2)

\[
\sigma = \frac{2}{\pi kR \left| H_1^{(2)}(kR) \right|^2}
\]  
(2.8)

and

\[
\sigma = \frac{2kR}{\pi \left| kRH_0^{(2)}(kR) - H_1^{(2)}(kR) \right|^2}
\]  
(2.9)

respectively.

In Eq. (2.8) and (2.9),

\[ k = 2\pi f_c / c_0 \]  
(2.10)

is the wave-number, \( H_n^m \) is Hankel function of \( n \)th kind of order \( m \) and \( R \) is the radius of the line source.

With the sound power determined, the SPL is given by

\[
p^2 = \frac{\rho_0 c_0 W}{2\pi d} \text{Dir}
\]  
(2.11)

where \( d \) is the distance between the receiver and the radiation source. \( \text{Dir} \) is the directivity pattern of the source. For a line monopole and a dipole source, \( \text{Dir} \) is given by

\[
\text{Dir} = 1
\]  
(2.12)

and

\[
\text{Dir} = 2 \cos^2 \theta
\]  
(2.13)

respectively.

In Eq. (2.13) \( \theta \) represents the angle between the receiver and dipole axis, which is given by,

\[
\theta = \tan^{-1} \left( \frac{h}{d} \right)
\]  
(2.14)
at a position, \( d \), away from the line source and \( h \) above the source.

The above procedure illustrates the calculation of radiation from one rail. To estimate track contribution, both rails are required and should be calculated independently.

**Sleeper radiation model**

Similar to the rail radiation model, VTN as applied by Verheijen and Paviotti (2003) assumes that each sleeper has uniform vibration on the top surface. This implies that each sleeper radiates like a baffled plate. Radiation from other surfaces of the sleeper is ignored: the bottom surface is highly scattered by the ballast; the side surface has a small area in the lateral direction and small vibration amplitude in the longitudinal direction.

Sleeper radiation power is calculated by Eq. (2.5). The radiation ratio of a sleeper is based on a baffled plate which is given by (Verheijen and Paviotti, 2003):

\[
\sigma = \frac{1}{1 + \left( \frac{f_{SC}}{f} \right)^2}
\]

(2.15)

where \( f_{SC} \) is sleeper critical frequency, which means below the frequency, sleeper does not radiate. \( f_{SC} \) is given by

\[
f = \frac{c_0}{\sqrt{2\pi S_{slp}}}
\]

(2.16)

where \( S_{slp} \) is sleeper radiation area given by

\[
S_{slp} = ab
\]

(2.17)

where \( a \) and \( b \) represent sleeper length and width, respectively.

Validity of the model is questionable based on the following two facts: 1) the train is moving and 2) sound wavelength changes with frequency. The first fact is equivalent to say that each sleeper is moving if the train is kept stationary, which does not make any difference to the receiver. The second fact implies that a number of sleepers may contribute to sound at a specific frequency, depending on the sound wavelength and sleeper configuration. For example, when half of a sound wavelength is longer than the sleeper spacing, several sleepers may vibrate/radiate coherently within a sound wavelength. The implications from the two facts are that each sleeper should be seen as a ‘moving point source’ and at low frequencies the relationship between sound
wavelength and the sleeper spacing and size has to be considered to get a correct
sleeper radiation power. This effect has been included in TWINS as reported by
Thompson and Janssens (1997) but not included in VTN.
The sleeper radiation area considering wavelength can be calculated as (Thompson
and Janssens, 1997)
\[
S_{\text{slp}} = ab \frac{\lambda}{2S_p}
\]  
(2.18)
where \( S_p \) is sleeper spacing.
For a moving point source model, each sleeper is a source and contributing to the
wayside noise. The sound power calculation in Eq. (2.5) has to include all of the
sleepers contributing to the wayside noise measurement. Sleepers covered in a
vehicle length are considered enough.
The SPL calculated using the moving monopole source model is given as (Thompson
et al., 2009, Section 6.6.1).
\[
P^2 = \frac{\rho_0 c_0 W}{2\pi LD} \tan^{-1}\left(\frac{L}{2D}\right)
\]  
(2.19)
where \( D \) is the distance between the line formed by the trajectory of the point source
and the receiver.

**Environmental effects**
All the calculations above are based on a free radiation field around the rail or
sleepers, i.e. there is no energy loss during sound propagation. This assumption will
normally either under or overestimate the SPL at an observation point, in practice.
Both ground and ballast will reflect or absorb sound during its propagation
depending on the surface conditions, as does the building or vegetation nearby. Wind
conditions can also affect the sound propagation. The effect varies from site to site.
VTN assumes a constant environmental effect for different sites, as long as they
comply with the test environment requirements given in reference (ISO, 2005-08-
15), i.e. the ground around the microphones is flat and the level variation is between
0 and 1m relative to the rail top and there is no obstacles that disturb the sound filed
and there is no reflective or absorptive ground cover.
A related issue about the free field assumption is the radiation from the far rail
relative to the observer. The sound radiation from far rail has to be reflected by the
vehicle subframe, running gear and ballast before reaching the trackside and eventually the receiver. This effect has been included in VTN by an absorption factor,

$$\alpha = \alpha_{\text{ballast}} + \alpha_{\text{vehicle}} - \alpha_{\text{ballast}} \cdot \alpha_{\text{vehicle}}$$  (2.20)

where \( \alpha, \alpha_{\text{ballast}} \) and \( \alpha_{\text{vehicle}} \) are the composite, ballast and vehicle absorption factor, respectively. Methods of determining the various factors have not been reported in the literature. Hemsworth (2002) proposed to use transfer functions between the measured rail vibration and the SPL measured by a microphone as an alternative method for determining the ground effect, if parameters in Eq. (2.20) are not available. No further detail was provided on how to achieve this. Possibly the ‘Equivalent force method’ (Dittrich and Janssens, 2000) can be used, which requires artificial excitations on the rail. By measuring the generated rail vibration and noise, the relationship between them can be established. However, a stationary vehicle may be needed on the track in this case to simulate the effects of train structure on the sound propagation from both rails. A summary of pros and cons of ‘Equivalent force method’ is also available in Table 2.

### 2.3.4 Janssens’s method

Janssens’s method is identified to be promising to trial for wheel and rail roughness estimation from pass by measurements. Its underlying methodology is shown in this section.

In this approach, wheel and rail combined roughness is estimated by using measured rail vibration. Contact filter has been included automatically. The obtained roughness is therefore the ‘total effective roughness’. Spectra of total effective roughness \( L_{\tau,\text{tot}}(f_c) \) can be calculated by,

$$L_{\tau,\text{tot}}(f_c) = L_{a,\text{rail}}(f_c) - A1(f_c) - A2(f_c) - A4(f_c) - 40\log_{10}(2\pi f_c)$$  (2.21)

where \( L_{a,\text{rail}}(f_c) \) are acceleration spectra of rail vertical vibration measured underneath rail foot which are averaged over distance and time. The last term of Eq. (2.22) converts acceleration spectra to displacement spectra. \( f_c \) is the centre frequency of an octave band. Other parameters convert the vibration measured at the bottom of the rail foot to the roughness in the contact area.
$A1(f_c)$ is the difference between vibration displacement at the measurement point on rail foot and the rail head at the contact point and given by,

$$A1(f_c) = 20\log10\left(\frac{\alpha_{foot}}{\alpha_{head}}\right) \quad (2.23)$$

where $\alpha_{head}$ and $\alpha_{foot}$ are rail head and foot receptance, respectively. Janssens et al. used zero for $A1(f_c)$ from 0 up to 4 kHz based on a finding of de Beer et al. (1998).

$A2(f_c)$ represents the difference between rail head vibration displacement and roughness at the wheel/rail contact area which is given by,

$$A2(f_c) = 20\log10\left(\frac{|\alpha_R|}{\alpha_W + \alpha_R + \alpha_C}\right) \quad (2.24)$$

where $\alpha_W$, $\alpha_R$ and $\alpha_C$ represent wheel, rail and contact receptance, respectively. A tabulated value for $A2(f_c)$ was given by Janssens et al. Use of the table has to be based on rail pad stiffness (Janssens et al., 2006a). These values are obtained based on calculation for a variety of wheels. The variation over different wheels, reported within ± 3dB by Janssens et al., was discounted by the fact that the uncertainties of the roughness measurements are more significant.

$A4(f_c)$ is the difference between the measured vibration and vibration across the contact area and given by

$$A4(f_c) = 10\log10\left\{\frac{8.686}{VD_{tr}T}\{1-e^{-VD_{tr}T/8.686}\}\right\} \quad (2.25)$$

where $V$ is train speed, $D_{tr}$ is track decay rate and $T$ is the wheel passage interval. It is obvious from Eq. (2.25) that $D_{tr}$ is the most influencing parameter on $A4(f_c)$ (product of $V$ and $T$ is a distance which is a constant, 1.8m, suggested by Janssens et al.). Track decay rate is a key parameter indicating the damping ability of the track. A high decay rate means a shorter length of track is vibrating and hence less energy and noise are generated from that rail. This parameter has been widely used to define the track radiation ability.
Track decay rates (TDRs)

Janssens et al. assume the vibration transmitted along the rail decays exponentially from the excitation point at a rate of $\beta$. When a train passes through the measurement site the wheels on it are assumed to represent independent and successive points of excitation, all with similar characteristics. TDRs are derived based on the ratio of the vibration energy near the excitation sources, in this case the wheels, to the whole vibration energy transmitted in the rail excited by these wheels.

The excitation vibration energy, denoted by $M_1$, is calculated by integrating the measured vibration over a finite and relatively short distance, $L_1$, of 1.8m centred at each wheel. All the rail vibration energy excited by the wheels, denoted as $M_2$, can be calculated by integrating the measured vibration over a longer distance, $L_2$. The typical length $L_2$, corresponds to one vehicle length or possibly one train length. The positions of the wheels at a particular point in time must be known. This will typically be determined by using a wheel trigger sensor. The vibration ratio is denoted as $R(f_c)$, which is a function of $\beta$ and can be expressed as,

$$R(f_c) = \frac{M_1}{M_2} = 1 - e^{-\beta L_1} \tag{2.26}$$

$D_{tr}(f_c)$ is expressed as,

$$D_{tr}(f_c) = 20 \log_{10}(e^{-\beta}) \tag{2.27}$$

By combining Eq. (2.26) and (2.27), $D_{tr}(f_c)$ can be expressed in terms of $R(f_c)$ rather than $\beta$ by,

$$D_{tr}(f_c) = \frac{20}{L_1} \log_{10}(1 - R(f_c)) \tag{2.28}$$

Normally the vibration measured at a particular point on the rail will be the result of excitation by a number of wheels adjacent to that point. The influence of adjacent wheels has to be accounted for. For high decay rates, the rail vibration attenuates over a sufficiently short distance so that the vibration excited by one wheel will have a negligible effect on the others. The effect is not negligible in the case of low decay rates. The effect is illustrated in Fig. 2.11 where two adjacent wheels are shown. The very low decay rates lead to the vibration induced being contributed by the 2 wheels.
adjacent to the measurement point. Noting that decay rate is a function of frequency, and denoting the measured vibration amplitude at the point when each of the adjacent wheels pass the measurement point as $B_1$, and $B_2$, and understanding that the decay rate does not vary with excitation event at a particular site, then a value for the decay rate ($\beta$) can be obtained by iteration expressed in equations:

$$B'_1 = B_1 - B_2 e^{-\beta x}$$  \hspace{1cm} (2.29)

$$B'_2 = B_2 - B_1 e^{-\beta x}$$  \hspace{1cm} (2.30)

where $x$ represents the distance between the adjacent wheels. Final values of $B'_1$ and $B'_2$ represent the rail vibrations excited by two independent wheels. Usually $\beta$ will converge within five iterations. Similarly, the effects of more than 2 wheels can also be taken into account.

It should note that Janssens et al. (2006b) suggest that when decay rate in a frequency band is less than 3 dB/m, the interaction of neighbouring wheels should be considered and removed from the results. This is a reasonable assumption, considering that rail vibration excited by a neighbouring wheel may already diminish before reaching another wheel if the decay rates are higher than 3 dB/m. For a typical bogie, the distance between axles of a bogie is around 1.7m, which means vibration from one wheel may have been reduced 5 dB/m or more and may be considered insignificant. Verification of this suggestion has not been reported.

![Diagram](image)

Fig. 2.11. Illustration of the influence on rail vibration due to the interactions of adjacent wheels as one wheel passes the measurement point.

Direct measurement of TDRs is also available. This method utilizes an impact hammer to excite the rail at a series of points on the rail head at various distances.
from a fixed measurement site. By measuring the rail vertical or lateral vibration responses and the corresponding impact excitations in the same direction, the decay rate for the rail in that direction can be derived separately. Typical application includes that used by Jones et al. (2006) and the standard impact hammer method (CEN, 2008b). By further decomposing the measured rail responses into individual travelling waves through a wave-number decomposition method developed by Thompson (1997), the decay rates of each wave can be evaluated. This measures track decay rate without train presence on track, which can affect the track dynamics. Janssens’s method, in comparison, measures track decay rate under train running conditions and therefore can more represent the reality.

2.4 Conclusion
From the review of the current theoretical and empirical models developed to separate rolling noise, we can conclude that
1. TWINS is fully validated in terms of total SPL but not on separate wheel and track noise contribution.
2. The empirical models discovered are all developed based on TWINS methodology which may or may not be validated. The validation process often involves comparison of the predicted noise level from a component with TWINS prediction and considers only the total noise level.
3. More effective methods for rail modelling have been developed after the appearance of TWINS, such as SAFE method, which represent improved predictive methods.
4. The SAFE method based Rail model is promising to calculate rail vibration characteristics. By coupling with the BE method, rail radiation characteristics can also be estimated. However, application of the method on rail modelling which allows for multiple track support is still lack at this stage.
5. VTN is promising for trial to separate rolling noise into its wheel and rail noise component. Jansssens’s method is promising to determine wheel and rail roughness indirectly.
6. VTN is based on a simplified track radiation model that assumes the rail radiates like a simple line source without consideration of rail cross section. Relaxation of this assumption by the use of a sophisticated radiation model, through taking account
complex rail cross section shape, may lead to more accurate radiation ratios and radiation directivities and therefore more useful separation results.

7. Parameters to estimate rail roughness in Janssens’s method are extracted from modelling of rail and wheel (including contact). These parameters may be improved by using more sophisticated wheel and rail models, especially those developed for the rail, such as the one based on SAFE method.
3 FIELD TRIALS OF ROLLING NOISE SEPARATION METHODS

3.1 Introduction and chapter outline
Chapter 2 shows that current rolling noise separation methods lack validation in terms of component contributions: either for estimating component roughnesses or radiated noise contributions. Examples of their application are also rarely found in the literature. This results in the current chapter having two objectives: 1) to trial these methods and 2) to evaluate their performance through field tests. Improvements on the methods will be made whenever applicable.

Firstly, the methodology of validation is introduced, followed by the test results of noise separation and roughness estimation, respectively. This will be followed by conclusions on the performance of the available methods, together with any suggested enhancements outlined.

3.2 Test plan
This section sets out the methodology to be used to compare VTN and Janssens’s method identified in Section 2.3 against field tests.

3.2.1 Verification of VTN calculation through field tests
Verification of VTN ideally involves comparing track noise contribution calculated by VTN with directly measured track noise contribution. This requires track noise to be isolated from the vehicle noise during measurement. A practical way to achieve this is by measuring track-side SPL in a moment when the track component is dominating rolling noise. The MISO method provides such a way by deploying a near rail microphone as reviewed in Section 2.3.1. This method is proposed to be used here. However, it is understood that wheel noise radiation, especially in radial direction, which is omnidirectional (Thompson and Jones, 2002), may contaminate the ‘track noise component’ measured by a single microphone. This effect is neglected here as done by Letourneaux et al. (2002).

3.2.2 Environmental effects on sound propagation
The surrounding environment, such as the ground, buildings and vegetation, can reduce or enhance sound during its propagation. Calculation of track noise by VTN has to take this effect into account to get correct SPL at an observation point.
By deploying two microphones at different distances from the track, the environmental effects can be measured by further assuming the traffic as a line source. Validity of this assumption depends on both traffic conditions, and the distance of the observation point from the track. A traffic dominated by rolling noise will be sufficient to see as a line source as long as measurement is made not too far from the track, such that the traffic is better viewed as a point source, for example more than 100 m away. Under this assumption, doubling a distance will lead to a reduction of SPL by 3 dB if a free space condition is met around the track. Above or below this level is considered to be from environmental influence. In other words, environmental effect \( \text{Diff} \) is given by,

\[
\text{Diff} = P_2 - P_3 - 3 \text{dB}
\]

where \( P_2 \) and \( P_3 \) are SPLs measured at for example 7.5m and 15m away from the track, respectively. Eq. (3.1) indicates that: if \( \text{Diff} = 0 \), there is no environmental effect during sound propagation; if \( \text{Diff} > 0 \), sound has been absorbed during propagation; if \( \text{Diff} < 0 \), the sound has been enhanced during propagation.

3.2.3 Validation of Janssens’s method

Validation of Janssens’s method consists of two parts: 1) validation of calculated wheel and rail total effective roughness and 2) validation of calculated TDRs. The two objectives can be achieved by: 1) comparing total effective roughness obtained by using Janssens’s method with those by direct wheel and rail roughness measurement; and 2) comparing TDRs calculated by using Janssens’s method with those measured from standard impact hammer tests (CEN, 2008b), respectively. A same train has to be used for both the total effective roughness measurement and the direct wheel roughness measurement to ensure that the validation for wheel and rail total effective roughness estimation by Janssens’s method is valid.

3.2.4 System design

**Measurement set-up**

Before conducting any test, quantities requiring measurement have to be determined. Those required for VTN include rail and sleeper vibration and two channels of SPLs used to determine the environmental effects, as discussed Section 3.2.2. For
evaluation of VTN performance, one additional microphone is required to measure track noise component as discussed in Section 3.2.1. Janssens’s method further requires wheel trigger signals and train speed. Wheel trigger signals can be measured by an optical sensor. In order to measure train speed, it is proposed to use two optical sensors deployed with a known span. The time interval a wheel passes over the two sensors can be used to derive the train speed.

To inspect the consistency of measurements at different sites, the field test can be carried out at two track sites which have apparently similar track forms and surrounding environment. Based on the above discussion, the measurement set up shown in Fig. 3.1 is proposed.

![Measurement set-up for the noise survey](image)

**Fig. 3.1.** Measurement set-up for the noise survey: 1, 2, 3, 4, 5 are accelerometers; 6, 7, 8 are microphones; 9, 10 are optical sensors.

**Site selection**

The test site has to be selected such that rolling noise is the dominant noise source on it. Flat sections with constant train speed and away from signals will reduce the incidence of traction and brake noise. Tangent track, or track with a shallow curve,
will eliminate the possibilities of squeal occurrences. Good track conditions can prevent localised noise, due to rail joints and crossing.

**Rail roughness measurement**

Rail roughness measurement can be conducted with a Corrugation Analysis Trolley (CAT) (Grassie, 2005). Depending on the width of the running band, several positions may be measured. For example, when the width of the running band is less than 30mm, measurements at three evenly distributed parallel lines are considered enough to cover the running band (CEN, 2008a).

**Track decay rate measurement**

Track decay rate measurement is through the impact hammer test following the standard procedure (CEN, 2008b).

**Wheel roughness measurement**

To measure wheel roughness, a device has been developed. This device measures wheel roughness through a LVDT, which scans wheel surface. To allow measurement to cover a wheel circumference, the measured wheel set is jacked up and rotated manually. Several paths on the wheel tread surface will be measured, starting from the nominal contact line (or treadline) which is 70mm from the flange back, to allow for the width of a running band. Detailed description of the device is given in Appendix C.

Diameters on which gross wheel defects were present, particularly wheel flats, were avoided.

### 3.3 Details of actual tests

The following attributes of the actual tests performed are relevant to the results obtained.

#### 3.3.1 Test sites

Two sites with geometry set out in Fig. 3.2 were selected for the test. On this part of railway line the track is tangent and supported with rail pad-sleeper-ballast structure. The sleepers are monobloc and made of concrete. The track forms and environment are apparently similar at both sites. The site conditions meet those proposed in Section. Both test sites have a mixed freight and passenger traffic. The instruments used at each site have been given in Appendix B.
3.3.2 Rail roughness measurement

Rail roughness was measured halfway through the test. From visual inspection on the rail head, the running band (brighter than other parts of the rail) was around 30mm wide, as shown in Fig. 3.3. Three positions on each rail were measured, 5 mm apart. The middle position coincides with the centreline of the running band.

Fig. 3.3. The running band on the rail head.
3.3.3 Wheel roughness measurement

During the day accessing the workshop, a four-car passenger train under maintenance was available for measurement. 5 out of 32 wheels on the train were measured for practical reasons.

3.4 Comparison of SPL measurement at the two sites

The two noise monitoring systems are compared in terms of the SPLs measured at 7.5m from the track for the same trains. Four examples are shown in Fig. 3.4. As shown, the measurement obtained from the two sites is close to each other which indicates the track conditions are similar.

![Graphs showing SPL measurement at two sites](image)

Fig. 3.4. Sound pressure level measured at: —— site 1 and –– –– site 2.

3.5 Noise separation results

During a seven day monitoring period around 500 trains were measured. Only the passenger trains were used for analysis.
3.5.1 Performance of proposed sleeper radiation model

The performance of the proposed sleeper radiation model in Section 2.3.3 is evaluated first. The result calculated from one train is given in Fig. 3.5. Also shown is the calculation from a single sleeper based plate radiation model (used in VTN) for comparison.

![Graph showing effects of various sleeper noise radiation models](image)

**Fig. 3.5.** Effects of various sleeper noise radiation models. ▼▼▼▼▼ total noise level, —— sleeper noise calculated by the proposed sleeper radiation model, ······· sleeper noise calculated by a single sleeper based plate radiation model, —— rail radiated noise.

Fig. 3.5 shows that the proposed sleeper model gives a higher SPL than the single sleeper based plate radiation model across the whole frequency range. Below 315 Hz calculated sleeper noise contribution is much closer to the total SPL. This finding confirms those reported elsewhere, e.g., in references (Thompson et al., 1996a, Thompson et al., 1999, Kitagawa and Thompson, 2006) that sleeper noise is the dominant rolling noise component at low frequencies below 500Hz, which is the decoupling vibration frequency of rail from the rest of the track structure. For example, the decoupling frequency is about 290 Hz for a UIC60 rail sitting on a very soft pad of 200 MN/m stiffness. The actual pad is likely to have a higher stiffness and therefore this decoupling frequency can be higher. Given this fact, sleeper
contribution shown in Fig. 3.5 is more reasonable. However, its accuracy is undetermined because no isolated sleeper noise component can be measured without being influenced by the rest of the rolling noise components in a pass-by measurement event.

In fact, the enhancement of the calculated sleeper noise component is attributed to the enhancement of sleeper radiation ratio, as shown in Fig. 3.6 in comparison of the radiation ratio obtained from a single sleeper based plate radiation model. By taking account of the relationship between sound wavelength and the sleeper configuration, more sleepers are included in the calculation of sleeper radiation ratio with the decrease of the frequency. This leads to an increase of equivalent sleeper radiation area with the decrease of the frequency, as also shown in Eq.(2.18). At high frequencies, each sleeper radiates independently and the proposed sleeper radiation model ceases to the single sleeper based plate radiation model.

![Fig. 3.6. Sleeper radiation ratio: --- a single plate based sleeper radiation model, --- proposed sleeper model.](image)

3.5.2 Results of environmental effects

The determined environmental effects on sound propagation from the traffic are shown in graph (a) and (b) of Fig. 3.7 for site 1 and 2, respectively. The methodology
used has been given in Section 3.2.2. The results shown are the statistical analysis results obtained by taking account of all of the trains running over the corresponding site. The standard deviations are also shown.

Fig. 3.7 shows that environmental effects vary around 0 dB at both sites. A trough occurs at around 250 Hz and a peak at around 600 Hz which indicates the sound has been enhanced and absorbed at the two frequencies, respectively. Excluding those around the two frequencies, environmental effects are limited to 2 dB in most of the remaining frequency range. In addition, Fig. 3.7 shows that site 2 has smaller environmental effects than site 1.

It was proposed that ground reflection may contribute to the trough and peak noticed in Fig. 3.7. A model to estimate the ground reflection effect is therefore built following Thompson et al. (2009, Section 6.6.4) and presented below.

The model is shown in Fig. 3.8 where the train is assumed to be a line source. The measurement point can be either P2 or P3, i.e. 7.5m away from the train and 1.2 m above the ground or 15m away from the train and 1.2 m above ground. It is noted
that the rail head height from the ground is ignored here for simplicity. Due to
ground reflection, there is an image line source corresponding to the train as shown
in Fig. 3.8. The sound pressure at the measurement point will be the combined
effects of the two line sources.

The sound pressure for a line source and its image source is given by

\[
p = p_0 \left( D(\theta_1)H_0^{(2)}(\kappa R_1) + D(\theta_2)H_0^{(2)}(\kappa R_2) \right) e^{i\omega t}
\]

(3.2)

where \( H_0^{(2)} \) is 2\textsuperscript{nd} Hankel function of zero order which is chosen to represent an
outgoing wave from the source. \( D \) is the directivity of the line source. For pulsating
line source, \( D = 1 \) and for oscillating line source, \( D = 2\cos^2 \theta \). Definition of \( \theta_1 \) and
\( \theta_2 \) are shown in Fig. 3.8 as well as \( R_1 \) and \( R_2 \). \( p_0 \) is a reference pressure.

![Diagram of train, image source, measurement point, and ground](image)

Fig. 3.8. Effects of ground reflection.

Using the geometrical relations shown in Fig. 3.8, we can get

\[
R_1 = \sqrt{d^2 + (h_2 - h_1)^2}
\]

(3.3)

\[
R_2 = \sqrt{d^2 + (h_1 + h_2)^2}
\]

(3.4)

The ratio of the sound pressure level with ground reflection effects included to the
sound pressure level obtained in a free field is

\[
\frac{p}{p_f} = \frac{D(\theta_1)H_0^{(2)}(\kappa R_1) + D(\theta_2)H_0^{(2)}(\kappa R_2)}{D(\theta_1)H_0^{(2)}(\kappa R_1)}
\]

(3.5)
To conform with the measurement conducted in the field, \( h_1 = 0.45m \), \( h_2 = 1.2m \) and \( d = 7.5m \) for P2 and \( d = 15m \) for P3. As the ground is not rigid, it is only partially reflective. After considering this effect (Thompson et al., 2009, Section 6.6.4), Eq. (3.5) can be further written as

\[
\frac{p}{p_f} = 1 + R(\phi) \frac{D(\theta_2)H_0^{(2)}(\kappa R_2)}{D(\theta_1)H_0^{(2)}(\kappa R_1)}
\]  

(3.6)

where \( R \) is the reflection coefficient and \( \phi \) is the angle between the incident wave and the normal to the ground for a plane wave. The detailed definition for \( R \) can be found in Ref. (Thompson et al., 2009, Section 6.6.4). For a line source, the incident wave to the ground may be seen as a plane wave. The flow resistivity of the ground at the measurement site presented is assumed to be 5e6, which is a typical value for roadside soil (Thompson et al., 2009, Section 6.6.4). Under these assumptions, the ground effects can be obtained using Eq.(3.5). The results are shown in Fig. 3.9.

![Graph](image)

Fig. 3.9. Effect of ground, pulsating line source. \( h_1 = 0.45m \), \( h_2 = 1.2m \) and \( d = 7.5m \).
As shown in Fig. 3.9, a trough and a peak occur at around 100 Hz and 1 kHz, respectively, which are different from the results shown in Fig. 3.7. Therefore, ground reflection may be discounted as a reason for the occurring of the trough and the peak noticed in Fig. 3.7.

### 3.5.3 Comparison VTN calculated results against direct track noise measurement

The validation results are presented in Fig. 3.10 for both sites where the difference between measured and calculated track SPLs is shown which is obtained by statistical analysis of the trains passing over each site.

![Figure 3.10](image_url)

**Fig. 3.10.** Measured – calculated track noise radiation. (a) site 1, (b) site 2. Shaded region: standard deviation. Continuous line: mean value.

Fig. 3.10 shows that measured track contribution is higher than those obtained from VTN calculation. In general, the difference between the measured and calculated values, and the standard deviation, decreases with the increase of frequency. The biggest difference can be as high as 10 dB, which occurs below 100 Hz. The reason for this difference is not fully understood. Three possible reasons may explain this: 1) the measurement is contaminated by wheel noise, especially wheel noise radiation
in radial direction, as discussed in Section 3.2.1. However this effect is expected to be more significant at high frequencies where rail contribution is less significant than the wheel; 2) the measurement is contaminated by the background noise from the highway nearby. The background noise however was not recorded during the monitoring period because the noise monitoring system was configured to only record the data when a train is passing over the test site; and 3) the acceleration measurement has a lot of uncertainties. During the test, the accelerometers were mounted on the rail by either magnet (test site 2) or instant glue (test site 1), see Fig. 3.11. A thin layer of soft rubber (around 2-3 mm thick) was inserted between magnet and rail. It was found that the magnet installed on rail side surface shifted its position over time. This apparently can affect the accuracy of the accelerometer measurement because the elastic rubber may act as a low pass filter in this case. At high frequencies, the accelerometer is isolated from the rail and its performance is significantly affected. Verification of the last theory is further discussed in Section 3.5.5. The glue mounting was supposed to have a better performance. The rail surface however was not properly cleaned during the test as later the rusted rail surface peeled off during the removal of the accelerometers. This occurred on the accelerometers used to measure lateral rail vibrations. The wind induced by a moving train is also expected to have some impacts but these impacts will be secondary and limited to very low frequency and the wind screen used for each microphone during the test will further reduce this effect.

Fig. 3.11. Field mounting of accelerometers at: (a) site a and (b) site b.
It is understood that other factors may contaminate the measured track noise. These factors may include:

- The track transfer function used by Letourneaux et al. (2002) relies on the near rail while the track noise at 7.5 m receives contributions from both rails.

- Near wind effects, when placing a microphone this close to a source (rail). The near field effect depends on frequency, as it is related to the dimensions of the source and its directional behaviour. Low frequencies are enhanced relative to high frequencies.

- The influence of reflections from the vehicle floor. At a distance of 2m from the track, the microphone ‘sees’ the floor of the wagon.

- Turbulent flow producing noise in the wind screen of the near rail microphone.

Further validation of VTN is made by comparing the calculated A-weighted track SPLs with those from measurements. The results obtained from site 1 and 2 are shown in graph (a) and (b) of Fig. 3.12, respectively. Note that only 20 trains measured in the first day are shown in graph (b) at site 2.

![Graph](image)

Fig. 3.12. Measured versus calculated A-weighted sound pressure level: (a) site 1 and (b) site 2.

In graph (a), the predicted SPL is 2.3 dB(A) below the measurement with a standard deviation of around 1.2 dB(A) at site 1. In graph (b), the predicted SPL is 1.0 dB(A) below the measurement with a standard deviation of 1.7 dB(A) at site 2. These
results indicate that VTN has a good performance in estimating overall track contribution.

A general conclusion can now be made based on Fig. 3.10 and Fig. 3.12 which states that: 1) VTN is able to calculate overall track contribution with an error less than 2.3 dB(A) in the current field trials and 2) in one individual frequency band, the difference between track contribution calculated by VTN and the value obtained from measurement can be as big as 10 dB.

3.5.4 More results on the trial of VTN

Results obtained from four trains passing over site 1 are shown in Fig. 3.13. Measured environmental effects in Section 3.5.2 are included in each of the track noise components in Fig. 3.13.

![Graphs showing sound pressure level vs. frequency for different configurations of SPL calculations.](image)

Fig. 3.13. VTN trials on four trains at site 1.
All the graphs in Fig. 3.13 show that sleeper radiation is significant below 300 Hz, exceeded by rail noise radiation in the middle frequencies and then both are outpaced by vehicle noise above 2 kHz. Overestimation of rail components can be found occurring at a frequency range between 1 and 2 kHz where predicted rail noise radiation exceeds the total measured noise level. Overestimation of sleeper component can also be noticed in graph (c) and (d) between 100 and 300 Hz. In these frequency bands, vehicle contribution will be negative but this is unrealistic in practice. Only explanation is that the vehicle contribution is insignificant at those frequencies. Therefore, whenever sleeper or rail noise is overestimated, vehicle component is not shown in Fig. 3.13.

3.5.5 Summary and discussion

During the test, the environmental effects on sound propagation were determined using two microphones deployed at different distances from the track. The results show that the effects are frequency dependent. The most prominent effects occur at frequency between 500 and 700 Hz, with a maximum value of 6 dB.

Comparison of VTN calculation against direct measurement of track noise is made by comparing its calculation on track contribution with the measurement from a near track microphone. Both the spectrum and the A-weighted SPLs of track contribution are compared. The results show that: 1) the measured track contribution is higher than the calculated values in most of the frequency range and the maximum difference is around 10 dB at frequencies below 100 Hz; 2) the spectrum difference reduces with the increase of the frequency; and 3) the calculated A-weighted SPLs are close to those obtained from measurements. Various reasons may contribute to the big difference between measured and calculated SPL spectra, ranging from background noise to uncertainties of the measurement, as discussed in Section 3.5.3.

Trials of the VTN show that rail noise contribution can be overestimated in some frequency bands. This may attribute to the simplification of the track noise prediction model. Another possible reason is the uncertainties of the acceleration measurements.

The first factor has to be investigated by using a more accurate rail radiation model, which is the work of Chapter 4 and 5. The second factor has been testified by a simple lab test rig.
Fig. 3.14. Test rig used to evaluate mounting effects on accelerometer performance.

The set-up of the test rig is shown in Fig. 3.14. It consists of a piece of rail (UIC 60) and a pendulum made of a metal ball hanging from the top bar of the frame by a string. The rail is also hanging to release it from the influence of any support. By dropping the ball at a same position each time, a constant excitation can be created. The rail response is recorded by an accelerometer on the other side of the rail head surface, opposite to the excitation position. The accelerometer is attached to the rail by either magnet or thread mounting. The total mass of the accelerometer and its accessories have a mass of less than 0.1kg which has a negligible impact on vibration characteristics of the piece of rail of a mass over 30.5kg.

The effect of magnet mounting on accelerometer measurement is investigated by comparing it with the measurement from thread mounting, which is the recommended mounting method, and supposed to have the best performance among all the mounting methods. In the field, this type of mounting has to be avoided because any damage to the rail surface is prohibited.
During the test, for each type of mounting methods, the rail was excited five times and an averaged result was calculated. By calculating the ratio of measurements under the two mounting methods, the accuracy of magnet mounting can be inferred. The measured acceleration spectra are shown in Fig. 3.15. The ratio of acceleration spectra obtained under the two mounting methods is shown in Fig. 3.16 where zero decibels indicate that the two methods are equivalent. Both figures show that magnet mounting is only close to the thread mounting measurement below 1 kHz. Above 1 kHz, rail acceleration measured under magnet mounting is higher than that measured under thread mounting. This finding demonstrates the inefficiency of magnet mounting method. No test on effects of inserting rubber spacer between rail and magnet is conducted but it is believed that this will further worsen the performance of accelerometer. This test implies that the mounting method has to be chosen properly in field test and magnet mounting should be avoided.

![Graph showing rail acceleration spectra under thread and magnet mounting.](image)

Fig. 3.15. Rail acceleration spectra measured under thread and magnet mounting.
It is noted that the mass of the accelerometer may cause errors in the measured rail vibration response and this impact may increase with frequency. A rail with two accelerometers installed adjacent to each other was also tested. However, it was found that the excitation was not constant. The repeatability of the tests was not good. Therefore comparison with the measured results from installation of a single accelerometer was not possible.

### 3.6 Results of validation of roughness estimation method

#### 3.6.1 Measured wheel roughness

Roughness conditions of two wheels are shown in Fig. 3.17 which were measured at the nominal contact position of each wheel, i.e. 70mm from the tread flange side. The two examples represent two extreme cases of wheel eccentricity of the measured data. The length of a period of the data shown in Fig. 3.17 corresponds to wheel circumference. Wheel eccentricity has been investigated by Johansson (2006). However it is not of a concern in this work, except the modulated signals on the periodic signal, or the surface roughness.
Repeatability
The repeatability of measured wheel roughness has to be investigated to ensure a reliable measurement. From the raw data shown in Fig. 3.17, strong periodicity of the roughness data can be seen which indicates the likelihood of good repeatability. In Fig. 3.18, roughness spectrum obtained by using data of different periods is further compared. Results obtained using data of more than one period is also presented.

Fig. 3.18 shows that roughness spectrum obtained from first to third period are close to each other except between the 10cm and 4 cm wavelength range. In this range, the difference is around 3 dB. This indicates a good chance of repeatability. It is noted that processing of wheel roughness followed the standard procedures given in CEN (2008a). As a result, sharp peaks and troughs that cannot be felt by a wheel when it contacts the rail were discarded. Hanning window was used to filter the data during the calculation of the spectrum shown in Fig. 3.18. In addition, the eccentricity evident in the raw data, as shown in Fig. 3.17, was removed as it is not relevant to the generation of the rolling noise for train speeds below 200 km/h.

Fig. 3.17. Raw data of measured wheel roughness.
However, the roughness spectrum obtained from data using one period and using more than one period has a big difference, especially in wavelength range between 20 cm and 6 cm. The maximum difference is around 10 dB. If data of more than two periods is used, the obtained roughness spectra changes insignificantly. The exact reason for this difference is unclear. It may be because the data length if data of only one period is used is not long enough. Nonetheless, the results imply that measurement is better to have more than two wheel rotation periods (not required to be integer times of wheel rotations) to ensure good consistency.

Fig. 3.18. Roughness calculated using the data of 1\textsuperscript{st} period, 2\textsuperscript{nd} period, 3\textsuperscript{rd} period, 1\textsuperscript{st} and 2\textsuperscript{nd} periods, 2\textsuperscript{nd} and 3\textsuperscript{rd} periods and all of data measured on a wheel.

More Results

The variation of wheel roughness over different positions is shown Fig. 3.19. Two examples are given.

Both graphs show that wheel roughness varies significantly across the lateral direction, especially at wavelengths between 4 cm and 1 cm, which correspond to rolling noise between 700 Hz and 2800 Hz at a rolling speed of 100 km/h. The maximum variation is about 5 dB in either case shown. For rolling noise prediction, 5 dB variation in roughness could lead to 5 dB variation in the predicted SPL,
because of the linear relationship between roughness and rolling noise (Thompson, 1996). The variation decreases with the decreases of wavelength.

Fig. 3.19. Wheel roughness of (a) wheel 1 and (b) wheel 2: —— 30mm from flange back, ------- 60mm from flange back, ——— 70mm from the flange back, ······· 80mm from flange back.

Roughness averaged over different positions of each wheel and over all of the wheels (in total 21 groups of data) is shown in Fig. 3.20. Fig. 3.20 shows that the standard
deviation decreases with the decrease of wavelengths. The maximum deviation is around 3 dB occurring at all wavelengths longer than 8 cm.

Fig. 3.20. Wheel roughness averaged over different positions. Shaded region: standard deviation region. Continuous line: mean value.

3.6.2 Measured rail roughness

Roughness measured on both rails at site 1 is shown in Fig. 3.21. Graphs (a) and (b) show the measured results from left and right rail, respectively. Fig. 3.21 shows that the variation of the roughness across the rail top surface is insignificant. All of the results exhibit a local maximum at a wavelength between 5 and 4 cm, attributable to rail grinding marks. Similar conclusion can be drawn for the roughness measured at site 2.
Fig. 3.21. Roughness spectra at three different positions on the rail head of (a) the left rail and (b) the right rail (left and right are with respect to train travelling direction). —— 25mm from inner gauge side, ----- 30mm from inner gauge side (centre of rail head), ——- 35mm from inner gauge side.
More Results

Rail roughness averaged over all of the measurement positions of a rail is shown in Fig. 3.22. Also shown are the ISO 3095 (ISO, 2005-08-15) and TSI (TSI, 2004-11-23) rail roughness limits. Fig. 3.22 shows that measured rail roughness is below the roughness limits in most of the wavelengths, which indicates a good rail surface condition at the measurement site. Measurements on other rails have similar results and therefore are not shown.

Fig. 3.22. Rail roughness spectra, shaded region: standard deviation, −−− mean value, –– ISO roughness limit, —— TSI limit.

3.6.3 Contact filter

The wheel and track parameters involved are chosen to be: wheel radius \( R_w = 470mm \), wheel radius in the transverse direction \( R_{wt} = \infty \), rail radius in the transverse direction \( R_t = 300mm \), rail radius in the longitudinal direction \( R_r = \infty \), Young’s modulus \( E = 210Gpa \), Poisson’s ratio \( v = 0.3 \), typical of a wheel and rail condition in the context of current measurement. The load is chosen to be 50 kN, a typical condition for a passenger vehicle. The analytical method used to determine the contact filter effects has been introduced in Section 2.2.2 or Appendix A.
The results are shown in Fig. 3.13. Each result is averaged over $\alpha = 1, 2, 3$. This is based on the finding that when $\alpha$ is a value between 1 and 3 gives good agreement with the numerical method (Thompson, 2003a) or Section 2.2.2.

Fig. 3.23. Contact filter obtained for a wheel with a diameter of 940mm under a load of 50kN.

3.6.4 Results of validation of TDRs measurement

**Impact hammer test results**

Vertical and lateral TDRs measured at both site 1 and 2 are shown in graphs (a) and (b) of Fig. 3.24, respectively. As shown, both vertical and lateral TDRs measured at the two sites are close to each other. Big difference can be found below 500 Hz and below 200 Hz for vertical and lateral TDRs, respectively. In either case, ballast plays an important role because at low frequencies, ballast stiffness together with the sleeper and rail mass determine the position of the first peak in the measured TDRs shown in Fig. 3.24. Ballast damping further determines the width of the peak. Therefore, Fig. 3.24 indicates that the track conditions may differ with each other, especially in terms of ballast stiffness and damping.
Fig. 3.24. TDRs measured at site 1 and site 2 in, (a) vertical direction; (b) lateral direction, –– site 1, ––– site 2.

Effects of wheel interaction on the calculation of TDRs
The effects of interaction of neighbouring wheels on the calculated TDRs are given in Fig. 3.25. The results were obtained from measurement at a site above a viaduct. Different with the suggestion given by Janssens et al. (2006a), the effects are removed regardless of the value of the TDR instead of 3 dB/m. Fig. 3.25 shows that the effects can still be as big as 1 dB/m even when the TDR is around 5 dB/m at a frequency like 200 Hz.

The significant effects of the interaction of neighbouring wheels on the calculation of TDRs, witnessed in Fig. 3.25, may result from various factors. By inspecting the data at each step, it was found that the main reason may be from the different excitation levels (or roughness) of different wheels. Though decaying fast along the rail, vibration generated by one wheel may still affect the neighbouring wheels, because it might excite rail vibration of a much bigger magnitude depending on its surface conditions relative to its neighbouring wheels. When reaching the neighbouring wheels, rail vibration excited by the wheel is still comparable to that excited by the neighbouring wheel. It is therefore insufficient to conclude the interaction of the

Fig. 3.25. Comparison of indicated track decay rates at a concrete viaduct (timber sleeper) track site: (−○−) without consideration of neighbouring wheels; (−▼−) with consideration of neighbouring wheels.
neighbouring wheels is negligible unless excitation generated by each wheel is similar.

Therefore, the best practice to calculate track decay rates is by considering the effects of neighbouring wheels regardless of the value of TDR at a frequency. This criterion has been followed throughout the rest of the work.

**Comparison of TDRs obtained from impact hammer test and pass by tests**

![Graphs showing TDRs](image)

Fig. 3.26. TDRs obtained from impact hammer test and pass by tests. (a) vertical TDRs at site 1, (b) lateral TDRs at site 1, (c) vertical TDRs at site 2, (d) lateral TDRs at site 2. —— mean value obtained from pass by tests, ----- standard deviation obtained from pass by tests, —— measurement from the impact hammer test.

A comparison of TDRs obtained from impact hammer test and pass by measurement is shown in Fig. 3.26. Graphs (a-d) show the vertical and the lateral TDRs of site 1 and site 2, respectively. The following observations can be made from the four graphs: 1) TDRs calculated from different trains are consistent with each other, with a deviation less than 2 dB/m and; 2) the difference of TDRs obtained by using the
two methods is limited to 50–600Hz for vertical TDRs and below 150 Hz for lateral TDRs.

In an earlier work, Janssens’s method was also trialled on tracks supported on a viaduct and on a bridge where TDRs from impact hammer tests were also available. The results are reported in graphs (a) and (b) of Fig. 3.27, respectively. Each result is averaged over five passing trains.

Both graphs of Fig. 3.27 show that TDRs obtained by using Janssens’s method are higher than those obtained from the impact hammer test over the majority of frequencies. In graph (a), this occurs in frequency between 200 and 2000 Hz, with a maximum of 7 dB/m difference at around 700 Hz. In graph (b), TDRs determined from Janssens’s method are constantly higher than the impact hammer test results for about 4 dB/m. One possible reason is the influence of train presence on the track dynamic characteristics. But this effect was yet investigated. During the measurement, intermittent flange contact on the rail gauge, or squeal events, were detected at either site. This is expected to cause additional vibration except from surface roughness. Because of the big discrepancy of results obtained by Janssens’s method from the results measured by an impact hammer test, Janssens’s method may not be suitable to such a type of track conditions.

In the study on tangent track shown in Fig. 3.26, the finding at the viaduct or bridge site was not reproduced. This further indicates that different track conditions have an important impact on the TDR measurement.

In general, results of this study suggest that the Janssens’s method performs well on the tangent track with good conditions, which agrees well with the impact hammer tests. A wide application throughout the rail industry is possible on this type of track condition.
Fig. 3.27. Vertical TDRs measured from the viaduct site (graph a) and the bridge site (graph b). (○○○) TDRs obtained from pass by tests; (▼▼▼) TDRs obtained from impact hammer tests.
3.6.5 Results of validation of roughness estimation

During the monitoring period, the train with wheel roughness measured passed over the test site two times. The obtained wheel and rail total effective roughness using Janssens’s method is compared with that obtained from direct wheel and rail roughness measurement. The results are shown in Fig. 3.28.

![Graph showing total effective roughness comparison](image)

Fig. 3.28. Total effective roughness: shaded region: standard deviation of the direct measurement, —— mean value of direct measurement, —— pass by measurement when the train first passed over the site, ----- pass by measurement when the train passed the site in a second time.

Fig. 3.28 shows that the two methods give results of significant difference at small wavelength below 3.15 cm. Part of the reason is that the analytical contact filter used does not have a good performance at low frequency (Thompson, 2003b). At longer wavelengths, the direct measurement method gives results higher than those predicted from Janssens’s method.

3.6.6 More results on the trial of Janssens’s method

Test results from pass by measurements at site 1 are presented in Fig. 3.29. Each result is averaged over measurements of one day. Good repeatability of the
measurements over different days can be seen from these figures. Results from site 2 are similar and not shown.

Fig. 3.29. TDRs calculated from pass by measurement at site 1, averaged on daily basis. (a) vertical TDRs, (b) lateral TDRs.
Total effective roughness calculated by using Janssens’s method is shown in Fig. 3.30 on a train to train basis. Fig. 3.30 shows that the inferred total effective roughness varies significantly from train to train. At wavelength of 12.5 cm, the variation is over 15 dB. Variation decreases with the decrease of the wavelength or increase of the frequency. This may indicate the large variation of the wheel roughness of different trains passing over the test site.

Fig. 3.30. Roughness obtained from pass by measurement during the first day, derived from individual trains.

3.7 Conclusion
An investigation on the performance of the current available noise/roughness separation methods is conducted through field tests. The test sites were arranged at a visually tangent track which is supported by rail pad-monobloc sleepers-ballasts. Two independent test sites on the track were monitored for one week. The conclusions drawn from this test campaign include:

1) VTN method overestimates the track (can be rail or sleeper) noise contribution at some frequencies. But in terms of overall track SPL, its deviations were found in this case to be within 2.3 dB (A) from field trials.
2) Developed wheel roughness measurement device proved to have good repeatability. Measurement over more than one turn is suggested in order to ensure the calculated roughness spectrum to have a good consistency.
3) Wheel roughness at different positions of a tread surface can vary over 5 dB. Variation of rail roughness across the surface can also be close to 5 dB.
4) Janssens’s method can give TDRs close to the measurement from the impact hammer test if the track has a good condition and it is a tangent, ballasted track. The performance of Janssens’s method differs with the impact hammer test significantly on the track supported by a viaduct or a bridge, due to the special track structures of the latter two cases.
5) Janssens’s method for roughness estimation using a pass-by approach was found not to be able to give total effective roughness that confirms well with those obtained from direct wheel and rail roughness measurement.
6) Validation of VTN using a microphone near to rail shows a big difference between track contribution calculated by VTN and those from measurement. The reason is unclear. It is possible that the measurement was contaminated by the background noise, including the radiation due to the wheel radial vibration.
7) In addition to the possible inaccuracy of the prediction methods, the magnet mounting of the accelerometers used during the test may also have an impact on the accuracy of the predicted results. These effects were studied through lab tests which indicate that the magnet mounting can indeed lead to measurement higher than the actual level of acceleration.

Irrespective of any measurement errors, the noise separation and total effective roughness estimation method contain significant assumptions that require testing. In the following chapters, a more complex rail vibration model and a more complex rail radiation model to that used in the current VTN method or Janssens’s method will be developed so that these assumptions can be relaxed.
4 RAIL VIBRATION MODEL BASED ON SAFE METHOD

4.1 Introduction and chapter outline

The previous chapter presented the results of roughness derived from pass by measurements and some discrepancies were identified with the results from direct wheel and rail roughness measurements. Parameters involved in the process are determined from rail and wheel modelling, as reported by Janssens et al. (2006a). This chapter aims to improve the accuracy of the parameters by building a more complex rail vibration model using the SAFE method. In addition, the SAFE rail model will be extended to include multiple layers of support.

The following content is organized into four sections. In section 4.2, the theory underlying the methodology of SAFE method is provided, including a method to consider rail support effects. In Section 4.3, the track parameters required for the rail modelling are determined, based on the field test. The discretized rail model is introduced in Section 4.4. The results, i.e. calculated rail dispersion relations and the rail forced responses are presented in the Section 4.5 and Section 4.6, respectively, for each of three assumed rail support characteristics, i.e. a free rail, a rail with a layer of single support and a rail with multiple layer of support. The work of this chapter is summarized in the last section.

4.2 Theory of SAFE method

![Fig. 4.1. Geometry of a waveguide. \( \Omega \) : cross-section domain, \( \Omega^e \) : element domain.](image)

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The geometry of a waveguide is shown in Fig. 4.1. Without loss of generality, the waveguide is aligned in the $z$ direction and its cross-section in the $x-y$ plane as shown in Fig. 4.1. In this example, the cross-section is meshed with quadrilateral elements. Other two-dimensional elements are also applicable. The domain of the cross-section is denoted by $\Omega$, and the element domain as $\Omega^e$. The displacement at any point in $\Omega$ has three components which are denoted as $u$, $v$ and $w$ in $x$, $y$ and $z$ direction, respectively.

Harmonic motion is assumed for the waveguide. More complex motion can be derived through Fourier transform. As a result, an implicit harmonic term $e^{i\omega t}$ is assumed throughout the chapter.

### 4.2.1 Derivation of governing equation

The governing motion equation for SAFE method can be built using various methods, such as virtual work (Hayashi et al., 2003) or the Hamilton’s principle (Nilsson et al., 2009, Bartoli et al., 2006). Hamilton’s principle was arbitrarily chosen. It utilises the concept that the motion of a mechanical system tends to minimize the time integral of the difference between potential energy and the sum of kinetic energy and the work done by external forces. The first variation of the Hamilton equation of the system will vanish at all material points (Petyt, 1989, Chapter 1)

$$\int_{t_1}^{t_2} (\delta(T-U) + \delta W) dt = 0$$

(4.1)

where $\delta$ denotes the first variation, $t_1$ and $t_2$ are arbitrary instants of time. $U$ and $T$ represent potential energy and kinetic energy of the system, respectively. $W$ is the work done by the external forces applied to the system. Each of the three terms is treated separately in the following text on the element basis following by assembly of these element matrices.

**Strain energy**

The strain (potential) energy is given by

$$\delta U^{(e)} = \int_{V^{(e)}} \delta \xi^{(e)^H} \sigma^{(e)} dV$$

(4.2)
where the upper script $H$ means a complex conjugate transpose, $(e)$ denotes element,

$$\xi^{(e)} = [\xi_x \quad \xi_y \quad \xi_z \quad \gamma_{xy} \quad \gamma_{yz} \quad \gamma_{xz}]^T$$

and

$$\sigma^{(e)} = [\sigma_x \quad \sigma_y \quad \sigma_z \quad \tau_{xy} \quad \tau_{yz} \quad \tau_{xz}]^T$$

are strain and stress vector, respectively. $V$ denotes volume.

The constitutive relationship can be written as

$$\sigma^{(e)} = D\xi^{(e)}$$

(4.3)

where $D$ is the stiffness matrix which is given as

$$D = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix}
1-\nu & \nu & 0 & 0 & 0 \\
\nu & 1-\nu & \nu & 0 & 0 \\
\nu & \nu & 1-\nu & 0 & 0 \\
0 & 0 & 0 & 0.5-\nu & 0 \\
0 & 0 & 0 & 0 & 0.5-\nu \\
0 & 0 & 0 & 0 & 0.5-\nu
\end{bmatrix}$$

(4.4)

For harmonic motion, structural damping can be added to account for the internal energy loss by making $D$ complex (Crandall, 1970) such that

$$D = D(1+i\eta)$$

(4.5)

where $\eta$ is the damping loss factor.

The strain-displacement relations are given by

$$\xi^{(e)} = [L_x \frac{\partial}{\partial x} + L_y \frac{\partial}{\partial y} + L_z \frac{\partial}{\partial z}]u^{(e)}$$

(4.6)

where

$$L_x = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}, \quad L_y = \begin{bmatrix}
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}, \quad L_z = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}$$

(4.7)

and $u^{(e)} = [u \quad v \quad w]^T$.

In this work eight-noded isoparametric quadrilateral elements are used to discretize the rail cross-section. The shape functions defined for each node, as derived by Petyt (1989, Section 4.6), can be given by:
\[ N_j(r,s) = \frac{1}{2}(1-r^2)(1+s_j) \]  \hspace{1cm} (4.8)

and

\[ N_j(r,s) = \frac{1}{2}(1-r_j)(1-s^2) \]  \hspace{1cm} (4.9)

for the nodes on the vertices of an element, indexed as \( j = 1, 2, 3, 4 \), and nodes at the middle of each edge on the element, indexed as \( j = 5, 6, 7, 8 \), respectively. \((r_j, s_j)\) are the coordinates of node \( j \) in the local coordinate system of an element.

Utilizing properties of isoparametric elements, \( u^{(e)} \) can be related to nodal displacements \( q^{(e)} \) via shape functions \( N(r,s) \) by

\[ u^{(e)} = N(r,s)q^{(e)} \]  \hspace{1cm} (4.10)

where \( N \) is a matrix form of the shape functions given as

\[
N(r,s) = \begin{bmatrix}
N_1 & 0 & 0 & N_2 & 0 & 0 & \ldots & N_8 & 0 & 0 \\
0 & N_1 & 0 & 0 & N_2 & 0 & \ldots & 0 & N_8 & 0 \\
0 & 0 & N_1 & 0 & 0 & N_2 & \ldots & 0 & 0 & N_8
\end{bmatrix}
\]  \hspace{1cm} (4.11)

where \( N_x \) and \( N_y \) denote the partial derivatives of \( N(r,s) \) with respect to \( x \) and \( y \), respectively. Each non zero element of \( N_x \) and \( N_y \) can be related to that of \( N_r \) and \( N_s \) as

\[
\begin{bmatrix}
\frac{\partial N_j}{\partial x} \\
\frac{\partial N_j}{\partial y}
\end{bmatrix} = J^{-1}
\begin{bmatrix}
\frac{\partial N_j}{\partial r} \\
\frac{\partial N_j}{\partial s}
\end{bmatrix}
\]  \hspace{1cm} (4.14)

where \( J \) is the Jacobian matrix and has a form of
where \( x^{(e)} = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7 \ x_8]^T \) and \( y^{(e)} = [y_1 \ y_2 \ y_3 \ y_4 \ y_5 \ y_6 \ y_7 \ y_8]^T \) are the \( x \) and \( y \) coordinates of element nodes, respectively.

Inserting Eq. (4.3), Eq. (4.12) and (4.10) into Eq. (4.2) and integrating by parts with respect to \( z \) yields

\[
\delta U^{(e)} = \int_z \delta q^{(e)H} (K_0^{(e)} + K_1^{(e)} \frac{\partial}{\partial z} + K_2^{(e)} \frac{\partial^2}{\partial z^2}) q^{(e)} dz
\]

where

\[
K_0^{(e)} = \int_{\Omega^{(e)}} (B_0^H DB_0) dA \\
K_1^{(e)} = \int_{\Omega^{(e)}} (B_0^H DB_1 - B_1^H DB_0) dA \\
K_2^{(e)} = -\int_{\Omega^{(e)}} (B_1^H DB_1) dA
\]

**Kinetic energy**

The first variation of kinetic energy is given by

\[
\delta T^{(e)} = \int_{V^{(e)}} \delta \ddot{u}^{(e)H} \rho^{(e)} \ddot{u}^{(e)} dV
\]

where \( \ddot{u}^{(e)} \) is the time derivative of \( u^{(e)} \) and \( \rho^{(e)} \) is the mass density of the element.

Using Eq. (4.10) and the harmonic motion assumption with time \( e^{j\omega t} \) yields

\[
\delta T^{(e)} = \omega^2 \int_z \delta q^{(e)H} m^{(e)} q^{(e)} dz
\]

where

\[
m^{(e)} = \int_{\Omega^{(e)}} N^{(e)H} \rho^{(e)} N^{(e)} dA
\]

is the element mass matrix.

**Work of external forces**

The first variation of the work done by the external force can be expressed as
where \( f^{(e)} \) is the external force. The internal work due to energy loss if damping exists can be accounted by a complex material matrix \( D \) as shown in Eq. (4.5).

By inserting Eq. (4.10) into (4.21) yields

\[
\delta W^{(e)} = \int_V \delta q^{(e)H} F^{(e)} dV
\]  

(4.22)

where

\[
F^{(e)} = \int_{\Omega^{(e)}} N^{(e)H} f^{(e)} dA
\]  

(4.23)

is the nodal force applied on one element.

Replacing \( \delta U \), \( \delta T \) and \( \delta W \) in Eq. (4.1) with Eq. (4.16), (4.19) and (4.22) yields

\[
\int_{t_1}^{t_2} \left\{ \int_z \delta q^{(e)H} \left( K_0^{(e)} + K_1^{(e)} \frac{\partial}{\partial z} + K_2^{(e)} \frac{\partial^2}{\partial z^2} - \omega^2 M^{(e)} \right) q^{(e)} dz - \int_z \delta q^{(e)H} F^{(e)} dz \right\} dt = 0
\]  

(4.24)

**Assembly**

Applying standard FE assembling procedures, e.g. Petyt (1989, Section 3.6) yields

\[
\int_{t_1}^{t_2} \left\{ \int_z \delta U^{H} \left( K_0^{(e)} + K_1^{(e)} \frac{\partial}{\partial z} + K_2^{(e)} \frac{\partial^2}{\partial z^2} - \omega^2 M \right) U dz - \int_z \delta U^{H} F dz \right\} dt = 0
\]  

(4.25)

where

\[
K_0 = \bigcup_{e=1}^{n_{el}} K_0^{(e)}, \quad K_1 = \bigcup_{e=1}^{n_{el}} K_1^{(e)}, \quad K_2 = \bigcup_{e=1}^{n_{el}} K_2^{(e)}, \quad M = \bigcup_{e=1}^{n_{el}} M^{(e)}
\]

\[
U = \bigcup_{e=1}^{n_{el}} q^{(e)}, \quad F = \bigcup_{e=1}^{n_{el}} f^{(e)}
\]  

(4.26)

, \( n_{el} \) is the total number of elements and \( e \) is the element index.

Because \( t_1 \) and \( t_2 \) are arbitrary time instants, the integrand inside the time integral must be zero. Similarly, because of the arbitrariness of \( \delta U \), the integrand of the inner integral over space must also be zero. Eliminating \( \delta U \) yields

\[
(K_0 + K_1 \frac{\partial}{\partial z} + K_2 \frac{\partial^2}{\partial z^2} - \omega^2 M)U(z) = F(z)
\]  

(4.27)

By defining a pair of Fourier transforms on displacements
\[
\hat{U}(\kappa) = \int_{-\infty}^{+\infty} U(z)e^{i\kappa z} \, dz \quad (4.28)
\]
\[
U(z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{U}(\kappa)e^{-i\kappa z} \, d\kappa \quad (4.29)
\]
and similarly for the force, Eq. (4.27) can be transformed into

\[
(K_0 + K_1(-i\kappa) + K_2(-i\kappa)^2 - \omega^2 M)\hat{U}(\kappa) = \hat{F}(\kappa) \quad (4.30)
\]

For free response, external force equals zero, Eq. (4.30) reduces to a homogenous form

\[
(K_0 + K_1(-i\kappa) + K_2(-i\kappa)^2 - \omega^2 M)\hat{U}(\kappa) = 0 \quad (4.31)
\]

Eq. (4.31) is a double quadratic eigenvalue problem, QEP, in terms of either wave-number \( \kappa \) or frequency \( \omega \). The associated solution can be made by solving this QEP either as a function of wave-number \( \kappa \) at a given frequency \( \omega \), or, as a function of frequency \( \omega \) at a given wave-number \( \kappa \). A comprehensive review of the solution methods has been given by Tisseur and Meerbergen (2001). In this text, the linearization method is used by converting Eq. (4.31) to a new form doubling its algebraic size as

\[
[A - \lambda B]Q = 0 \quad (4.32)
\]

where \( \lambda = -ik \),

\[
A = \begin{bmatrix}
0 & K_0 - \omega^2 M \\
K_0 - \omega^2 M & K_1
\end{bmatrix}, \quad
B = \begin{bmatrix}
K_0 - \omega^2 M & 0 \\
0 & -K_2
\end{bmatrix}, \quad
Q = \begin{bmatrix}
\hat{U} \\
\hat{\lambda}\hat{U}
\end{bmatrix} \quad (4.33)
\]

Eq. (4.32) leads to a first order general eigenvalue problem, GEP, which can be solved using standard solvers such as the Matlab function eig or eigs.

4.2.2 Forced responses

When a point excitation force is applied, Eq. (4.30) can be rearranged to get the waveguide receptance \( \hat{R}(\kappa) \) at frequency \( \omega \) and wave-number \( \kappa \) by

\[
\hat{R}(\kappa) = \hat{U}(\kappa)/\hat{f} = \left( K_0 + K_1(-i\kappa) + K_2(-i\kappa)^2 - \omega^2 M \right)^{-1} I \quad (4.34)
\]

where \( I \) is a vector of the same size to \( \hat{F}(\kappa) \) and related to \( \hat{F}(\kappa) \) by \( \hat{F}(\kappa) = \hat{f} I \). \( \hat{f} \) is scalar and can be obtained from Fourier transform of the non-zero component of \( F(z) \).
Assuming the excitation is at the cross-section of \( z = 0 \), the receptance of the waveguide in the spatial domain can be obtained by taking the inverse Fourier transform of Eq. (4.34) which yields

\[
R(z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left[ K_0 + K_1(-i\kappa) + K_2(-i\kappa)^2 - \omega^2 M \right]^{-1} e^{-i\kappa z} d\kappa \tag{4.35}
\]

To evaluate this integral, the contour integral can be used. For \( z \geq 0 \), the contour integral is over a semicircle at infinity in the lower half plane plus the real axis, i.e. \( \text{Im}(\kappa) \leq 0 \), which represents waves travelling in the positive direction. \( R(z) \) can be expressed with the residuals (Karasalo, 1994, Nilsson and Finnveden, 2007, Nilsson et al., 2009) as

\[
U(z) = 2\pi i \sum_n \text{Res}(\kappa_n) \text{ for } z \geq 0 \tag{4.36}
\]

where \( n \) is index for the poles and

\[
\text{Res}(\kappa_n) = \frac{1}{2\pi} \frac{y_n \hat{P}}{y_n P(\kappa_n) x_n} x_n e^{-i\kappa z} \tag{4.37}
\]

where \( P(\kappa) = K_0 + K_1(-i\kappa) + K_2(-i\kappa)^2 - \omega^2 M \), \( x_n \) and \( y_n \) are the right and left eigenvectors of \( P \) at a pole \( \kappa = \kappa_n \) and

\[
P'(\kappa_n) = (K_0 + K_1(-i\kappa) + K_2(-i\kappa)^2 - \omega^2 M) \bigg|_{\kappa = \kappa_n} = -iK_1 - 2i\kappa_n K_2 \tag{4.38}
\]

Hence the receptance is given by

\[
R(z) = i \sum_n \frac{y_n I}{y_n (-iK_1 - 2i\kappa_n K_2) x_n} x_n e^{-i\kappa_n z} \text{ for } z \geq 0 \tag{4.39}
\]

The waves travelling along the rail are symmetrical from an excitation point. Therefore similar results can be obtained for \( z \leq 0 \) by making the integral in Eq. (4.35) in the upper half plane.

For a point excitation force at a general position \( z_c \)

\[
F = F_0 \delta(z - z_c) \tag{4.40}
\]

where \( F_0 \) is the nodal force vector applied at the plane \( z = z_c \).

Its Fourier transform can be expressed as

\[
\hat{F}_0(\kappa) = \int_{-\infty}^{+\infty} F_0 \delta(z - z_c) e^{i\kappa z} dz = F_0 e^{i\kappa z_c} \tag{4.41}
\]
In this case, receptance can still be calculated by Eq. (4.39) except that an additional term \( e^{i\kappa z_c} \) is required on the left hand side of the equation. Nilsson et al. (2009) reported that an impulse force with a unit value across the wave-number domain tends to give rise to the unrealistic prediction of receptance with the increase of the wave-number. They proposed a solution of calculating the rail forced response utilizing a half cosine distributed unit force to represent the receptance expressed in Eq. (4.39). The formulation of this force is given by

\[
f(z) = \begin{cases} 
\frac{\beta}{2} \cos(\beta z) & |z| \leq \frac{\pi}{2\beta} \\
0 & |z| > \frac{\pi}{2\beta}
\end{cases}
\quad (4.42)
\]

where \( \frac{\pi}{\beta} \) is the length of the region of the distributed force. It can be easily proved that the total force in this region is unit by integrating the force over \( |z| \leq \frac{\pi}{2\beta} \). The force is symmetric around the excitation point. The forced response can be obtained: either as the convolution of the excitation force in Eq. (4.42) and the receptance function of Eq. (4.39) in the spatial domain; or, by calculating the product of the Fourier transform of the force and the receptance in the wave-number domain. Nilsson et al. (2009) gives the displacement at the excitation point in the spatial domain as

\[
U(0) = \sum_n A_n x_n c_n
\quad (4.43)
\]

where

\[
A_n = \frac{iy_n \hat{U}_0}{y_n (-iK_1 - 2i\kappa_n K_2)x_n}, c_n = \frac{i\kappa_n / \beta + e^{-i\pi\kappa_n / 2\beta}}{1 - \kappa_n^2 / \beta^2}
\quad (4.44)
\]

Rail receptance can be obtained as the ratio of \( U(0) \) and the resultant of \( f(z) \) which is unity for the current case. Mobility can be calculated by adding a factor \( i\omega \) to the receptance. The receptance and the excitation force in the wave-number domain have been given by Nilsson et al. (2009)

\[
\hat{R}_0(\kappa) = \sum_n A_n x_n \frac{2i\kappa_n}{\kappa^2 - \kappa_n^2}
\quad (4.45)
\]
\[
\hat{f}(\kappa) = \frac{\cos\left(\frac{\pi \kappa}{2 \beta}\right)}{1 - \left(\frac{\kappa}{\beta}\right)^2}
\]  

(4.46)

The forced response of the rail in the wave-number domain is the product of \( \hat{\mathbf{R}}_0(\kappa) \) and \( \hat{f}(\kappa) \). As shown in Eq. (4.45), \( \hat{\mathbf{R}}_0(\kappa) \) an even function with respect to \( \kappa \).

4.2.3 Inclusion of support

To include supports in the rail model, a layer of equivalent springs can be introduced which is distributed continuously along the rail and connected to the element nodes at the bottom of rail foot as shown in Fig. 4.2 alongside the model used by Nilsson et al. (2009) and Ryue et al. (2008). One result of using the spring support is fewer elements leading to faster computation speed over the discretized single layer support.

![SAFE rail models](image)

Fig. 4.2. SAFE rail models, (a) rail with one layer of support representing rail pad meshed by finite elements as assumed by Nilsson et al. (2009) and Ryue et al. (2008) and (b) rail with one layer of support represented by a layer of equivalent springs.

The application of this layer of springs can be extended to represent other support assumptions such as spring-mass, spring-mass-spring as used by Heckl (2002) to study wave propagation on a periodically supported Timoshenko beam. A summary of the support types is given in Table 3 which is replicated from Ref. (Heckl, 2002).

The forces exerting on the rail can therefore be estimated as

\[
f_s = \mathbf{S}\mathbf{U}(z)
\]

(4.47)
using Hooke’s law.

In Eq. (4.47), \( f_s \) is a force vector acting at every node on the rail cross-section, the subscript \( s \) denotes ‘support’, \( \mathbf{U} \) is the displacement at each node, \( \mathbf{S} \) is the stiffness matrix. It is important to note that the forces and displacements are calculated for every node on the rail cross-section. On the nodes other than those at the bottom of the rail foot the force and displacement are zero, as are stiffness elements in \( \mathbf{S} \).

<table>
<thead>
<tr>
<th>Support type</th>
<th>Stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass ( m )</td>
<td>( S(\omega) = -m\omega^2 )</td>
</tr>
<tr>
<td>Spring with stiffness ( s )</td>
<td>( S(\omega) = s )</td>
</tr>
<tr>
<td>Spring with stiffness ( s ) and mass ( m )</td>
<td>( S(\omega) = s - m\omega^2 )</td>
</tr>
<tr>
<td>Mass-spring system</td>
<td>( S(\omega) = \frac{s_1(s_2 - m\omega^2)}{s_1 + s_2 - m\omega^2} )</td>
</tr>
</tbody>
</table>

By calculating the support force at every node position, the support can be represented as an additional force applied to the system. By replacing the force in Eq. (4.25) as a combination of external force and the support force gives

\[
\mathbf{F} = \mathbf{F}_{\text{ext}} + f_s
\]

leading to

\[
(K_0 + K_1 \frac{\partial}{\partial z} + K_2 \frac{\partial^2}{\partial^2 z} - \omega^2 \mathbf{M}) \mathbf{U}(z) = \mathbf{F}(z) = \mathbf{F}_{\text{ext}}(z) + f_s(z)
\]

Replacing \( f_s \) with Eq. (4.47) and carrying out Fourier transform on both sides of Eq. (4.49) yields

\[
(K_2(-i\kappa)^2 + K_1(-i\kappa) + K_0 - \omega^2 \mathbf{M} - S(\omega)) \hat{\mathbf{U}}(\kappa) = \hat{\mathbf{F}}(\kappa)
\]

Eq. (4.50) has a similar form to Eq. (4.31) except that the eigenvalue problem of its homogeneous form can be solved as a QEP in terms of wave-number but not in terms of frequency. The methods reviewed in Section 4.2.2 to calculate forced rail
responses can be used directly to obtain the forced responses of supported rail represented by Eq. (4.50) without modification. It is important to note that the quadratic element is used in this work. The physical variables defined on each element vary non-linearly. This means the stiffness introduced by the supports should be varied non-linearly accordingly across each element. As a result the end nodes are given a value of 1/6 of the total stiffness and the middle node 1/3. This is determined as the ratio of the integral of the shape function defined at each node over the local coordinate to the sum of this type of integrals calculated for all of the nodes along an edge of an element.

4.3 Determination of track parameters

A field test was conducted to determine the track parameters such as the rail pad stiffness and damping and the ballast stiffness and damping required by the rail modelling. As a result, TDRs were measured at a test site which has a tangent track supported by rail pad, sleeper and ballast according to the standard procedure (CEN, 2008b), including both vertical and lateral TDRs. Since no direct measurements were available for rail pad and ballast properties of the track under study, these parameters have been chosen to obtain a good tuning for the measured TDRs at the test site using a Timoshenko beam-based rail model.

The supports were included in the Timoshenko beam-based model as two continuous layers of damped springs representing rail pad and ballasts, respectively between which is a layer of mass corresponding to the sleepers (Thompson et al., 2009, Section 3.3).

4.3.1 Measured and tuned TDRs

Fig. 4.3 shows predicted and measured vertical TDRs. As shown they correspond below 200 Hz and above 1 kHz. The measured peak at around 1.2 kHz was not predicted by the analytical model. When compared with measured TDRs, the Timoshenko beam-based rail model also overestimates the TDRs at frequencies between 200 and 800 Hz. This frequency range is known as the second blocked region (Thompson et al., 2009, Section 3.3). A blocked region is a frequency range where no wave propagates along the rail. This is caused by a high TDR resulting from the interaction of the rail with the resilient rail pad. The first blocked region occurs between zero frequency and the resonant frequency of the rail and sleeper.
mass on the ballast (Thompson et al., 2009, Section 3.3). This first high TDR region was not evident in the measurement results shown in Fig. 4.3. The observed peak of the measured TDRs at 1.2 kHz corresponds to the rail pinned-pinned frequency resulting from the discrete nature of the actual rail support structure: the sleeper supports. Vibration at this frequency has a wavelength equal to two times the sleeper span. The Timoshenko beam-based track model assumes a continuous support and therefore cannot predict the pinned-pinned frequency.

![Graph showing vertical track decay rates](image-url)

Fig. 4.3. Vertical track decay rates, --- measurement, --- prediction.

Predicted and measured lateral TDRs are shown in Fig. 4.4. These correspond for all frequencies apart from those below 80 Hz and between 400 Hz and 1 kHz. The region below 80 Hz corresponds to the first blocked region which is predicted but not measured. The second blocked region between 100 and 200 Hz is predicted accurately. The peak close to 1 kHz is due to the rail pinned-pinned vibration mode which cannot be predicted by a Timoshenko beam-based rail model that assumes a continuous support.
4.3.2 Track parameters determined

The tuned track parameters are listed in Table 4 along with typical values of material properties. In addition, β in Eq. (4.42) was chosen to be 10 mm which is the length and width of a typical wheel-to-rail contact patch.

There is no data available for track parameters in the longitudinal direction. The method used by Thompson (1993c) has been adopted: the same track parameters determined in lateral direction are also used for the longitudinal direction. For simplicity, the same sleeper mass is used in all three directions. Although it affects the rail dynamic behaviour, varying this assumption will not affect relative behaviour for the different rail support assumptions being studied and any actual effect is limited to the anti-resonance where sleeper mass on rail pad and ballast.

Fig. 4.4. Lateral track decay rates, --- measurement, —— prediction.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rail(UIC60)</strong></td>
<td></td>
</tr>
<tr>
<td>Vertical bending stiffness (MNm²)</td>
<td>6.42</td>
</tr>
<tr>
<td>Vertical Timoshenko shear coefficient</td>
<td>0.4</td>
</tr>
<tr>
<td>Vertical loss factor</td>
<td>0.03</td>
</tr>
<tr>
<td>Lateral bending stiffness (MNm²)</td>
<td>1.07</td>
</tr>
<tr>
<td>Lateral Timoshenko shear coefficient</td>
<td>0.4</td>
</tr>
<tr>
<td>Lateral loss factor</td>
<td>0.03</td>
</tr>
<tr>
<td>Mass per unit length (kg)</td>
<td>60</td>
</tr>
<tr>
<td>Young’s modulus (Gpa)</td>
<td>210</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>7850</td>
</tr>
<tr>
<td><strong>Rail pad</strong></td>
<td></td>
</tr>
<tr>
<td>Vertical stiffness (MN/m)</td>
<td>600</td>
</tr>
<tr>
<td>Vertical loss factor</td>
<td>0.5</td>
</tr>
<tr>
<td>Lateral stiffness (MN/m)</td>
<td>50</td>
</tr>
<tr>
<td>Lateral loss factor</td>
<td>0.5</td>
</tr>
<tr>
<td><strong>Ballast</strong></td>
<td></td>
</tr>
<tr>
<td>Vertical stiffness (MN/m)</td>
<td>10</td>
</tr>
<tr>
<td>Vertical loss factor</td>
<td>0.15</td>
</tr>
<tr>
<td>Vertical stiffness (MN/m)</td>
<td>40</td>
</tr>
<tr>
<td>Vertical loss factor</td>
<td>0.03</td>
</tr>
<tr>
<td><strong>Sleeper (Mono-bloc)</strong></td>
<td></td>
</tr>
<tr>
<td>Sleeper spacing (m)</td>
<td>0.6</td>
</tr>
<tr>
<td>Sleeper mass (kg)</td>
<td>300</td>
</tr>
</tbody>
</table>
4.4 Rail discretization

The cross-section of UIC60 rail has been used in building the SAFE rail model such that the model can comply with the rail type used in the field site. Other rail cross-section designs are expected to result in similar conclusions, because a rail design will not affect the investigation of the effects of different rail support on the rail dynamic behaviour.

It is well known that the discretization is crucial to the accuracy of a conventional FE analysis however its effect on SAFE analysis has not been established. To investigate this effect, two discretizations of different details were created for the UIC60 rail cross-section as shown in Fig. 4.5. Mesh type (a) in graph (a) has 14 elements and 69 nodes and mesh type (b) in graph (b) has 28 elements and 127 nodes. The excitation directions and positions used in each mesh type are also shown.

Fig. 4.5. Two FE meshes of UIC 60 rail cross-section, the arrows represent the direction and position of the excitation forces: (a) 14 elements and 69 nodes (b) 28 elements and 127 nodes.

4.5 Rail dispersion

4.5.1 Free rail

The real and the imaginary parts of the calculated dispersion relations for using the mesh shown in Fig. 4.5(a) are shown in Fig. 4.6 and Fig. 4.7, respectively. Also shown are the predictions using the rail model based on a free Timoshenko beam. Only wave-numbers with imaginary parts less than 35 dB/m are shown to ensure each dispersion curve is continuous. More waves are available but they have high
decay rates and diminish rapidly on the rail and will therefore make an insignificant contribution to either rail vibration or radiation.

Fig. 4.6 and Fig. 4.7 show that only 4 waves are predicted by the Timoshenko beam-based rail model. This number is far less than those predicted by the SAFE rail model even though only a few results from the latter model are shown. The number of waves predicted by a Timoshenko beam model is limited by the order of its governing equation which is 4. These waves correspond to the propagating bending waves and their near-field counterparts in the vertical and lateral directions. For a free undamped beam the torsional and longitudinal waves can be also predicted (Heckl, 2002). The number of waves calculated by the SAFE rail model in comparison is determined by the degree of freedom of the rail model.

Fig. 4.6. Dispersion relations of a free rail—real parts. The dotted lines represent results from the SAFE rail model, several of these are marked: a-e. The symbols represent 4 different wave predictions using a free Timoshenko beam-based rail model as follows: ◊, lateral bending wave; ○, vertical bending wave; □, lateral near-field wave; ∆, vertical near-field wave.

Other than the number of waves, predictions of the Timoshenko beam-based rail model and the SAFE rail model are close to each other, especially on the propagating
bending waves, in either the vertical or the lateral direction. Real parts of the
dispersion for lateral near-field waves are not predicted well by the Timoshenko
beam-based rail model close to 600 Hz and above 2 kHz. This finding indicates the
inefficiency of the Timoshenko beam-based rail model in predicting rail lateral
vibration responses.

![Dispersion relations of a free rail—imaginary parts](image)

Fig. 4.7. Dispersion relations of a free rail—imaginary parts. The dotted lines represent
results from the SAFE rail model, several of these are marked: a-e. The symbols represent 4
different wave predictions using a free Timoshenko beam-based rail model as follows: ◊,
lateral bending wave; ○, vertical bending wave; □, lateral near-field wave; Δ, vertical near-
field wave.

To further differentiate different waves travelling along the rail, real parts of the
dispersion curves are shown in Fig. 4.9 in a linear scale, where each propagating
wave at the marked position is shown in Fig. 4.8. Wave a results in the rail moving
laterally corresponding to a lateral bending wave; similarly, wave b is a torsional
bending wave; wave c is a vertical bending wave; wave d is the first order web
bending wave and wave e is a longitudinal wave. It is noted that the propagating
waves shown in Fig. 4.9 all cut on at zero frequency. The exception is wave d which
includes significant rail cross-section deformation. Relative to the free rail
assumption, the inclusion of rail supports implies that the prediction of waves a, b and c can be significantly affected by a rail support assumption.

Fig. 4.8. Deformed rail cross section for various waves analysed.

Fig. 4.9. Dispersion relations of a free rail in a linear scale—real parts. ●, positions where different waves are marked corresponding to deformation modes a to e in Fig. 4.8. a-e, wave labels.
4.5.2 Single layer supported rail

The single layer supported rail has a layer of spring support representing the rail pad. The real and imaginary parts of wave-number versus the frequency are shown in Fig. 4.10 and Fig. 4.11, respectively, calculated using the mesh type (b) of Fig. 4.5. Different propagating waves are marked and labelled in both figures. The predictions are compared with the equivalent Timoshenko beam-based rail model which is only able to predict waves travelling along the rail in the vertical direction up to around 2 kHz. Fig. 4.11 shows that above 2 kHz, the predicted rail vertical bending waves do not have the same imaginary parts. A large discrepancy with the SAFE rail model in the imaginary parts of dispersion for the propagating waves and the real parts for the near-field waves in the lateral direction can be observed above 700 Hz. This demonstrates the inadequacy of the Timoshenko beam-based rail model in predicting lateral rail vibration responses.

![Dispersion relations of a rail with a single layer of spring support—real parts. The dotted lines represent results from the SAFE rail model, several of these are marked: a-f. The symbols represent 4 different wave predictions using a Timoshenko beam-based rail model with a single layer of support as follows: ◊, lateral bending wave; ○, vertical bending wave; □, lateral near-field wave; Δ, vertical near-field wave. Wave shapes for waves f-h are shown in Fig. 4.8.](image_url)

Fig. 4.10.
Fig. 4.11. Dispersion relations of damped rail with a single layer of spring support—imaginary parts. The dotted lines represent results from the SAFE rail model, several of these are marked: a-f. The symbols represent 4 different wave predictions using a Timoshenko beam-based rail model with a single layer of support as follows: ◊, lateral bending wave; ○, vertical bending wave; □, lateral near field wave; ∆, vertical near field wave. Wave shapes for waves f-h are shown in Fig. 4.8.

The impact of the rail support can be observed for frequencies below 1 kHz by comparing Fig. 4.10 and Fig. 4.11 with the results show in Fig. 4.6 and Fig. 4.7, i.e. the bending waves and the near-field waves are indicated to have almost constant real and imaginary dispersions instead of increasing with frequency as was the case for the free rail assumption, as shown in Fig. 4.6 and Fig. 4.7. In addition, as indicated in Fig. 4.12, the cut on frequencies for waves a-c are not at zero Hz as was the case for the free rail assumption. However, the influence on the rail dispersion prediction of the inclusion of the rail support as a spring is only evident below 1 kHz.

Various waves can be identified from the corresponding deformed rail shape, or more conveniently by comparing with the waves that have been identified from a free rail model, as shown in Fig. 4.9. As a result, waves a-e can be identified same as waves a-e shown in Fig. 4.9. Deformed rail shapes for waves f-h in Fig. 4.12 at marked positions are shown in Fig. 4.8. From Fig. 4.8, it can be identified that wave f is the second web bending wave; wave h has serious rail foot out-of-plane...
movement; and wave g has rail head and rail foot vibrating out of phase in the longitudinal direction.

![Dispersion relations for a rail with a single layer of spring support in a linear scale](image)

Fig. 4.12. Dispersion relations of a rail with a single layer of spring support in a linear scale — real parts. •, positions where different waves are marked. a-h, wave labels.

4.5.3 Multiple layer supported rail

Dispersion relations determined for a rail model with spring-mass-spring support representing rail pads, sleepers and ballasts are shown in Fig. 4.13 and Fig. 4.14, calculated using the mesh type (b) of Fig. 4.5. Prediction from the equivalent Timoshenko beam-based rail model with the same spring-mass-spring support is also shown. Each wave is also marked and labelled. Identification of various waves is similar to those reported for rail of a single layer of support by comparing Fig. 4.15, where the real parts of dispersion are shown in a linear scale, with Fig. 4.12.
Fig. 4.13. Dispersion relations of a rail with spring-mass-spring support—real parts. The dotted lines represent results from the SAFE rail model, several of these are marked: a-f. The symbols represent 4 different wave predictions using a Timoshenko beam-based rail model with spring-mass-spring support as follows: ◊, lateral bending wave; ○, vertical bending wave; □, lateral near-field wave; Δ, vertical near-field wave.

Fig. 4.13 and Fig. 4.14 show that Timoshenko beam-based rail model is able to predict vertical bending waves travelling along the rail up to around 2 kHz. From this point and upwards, it deviates from the SAFE rail model in the prediction of vertical bending waves on the imaginary parts of the dispersion. The discrepancy noticed for the rail with only a spring support on the prediction of lateral waves travelling along the rail is once again demonstrated in Fig. 4.13 and Fig. 4.14. This further illustrates the inadequacy of the Timoshenko beam-based rail model on the prediction of lateral rail vibration response.

The effects of rail support can also be observed by comparing Fig. 4.13 and Fig. 4.14 with findings for the free rail and the rail with a single layer of support. Fig. 4.13 and Fig. 4.14 show a larger variation of the dispersion below 1 kHz. Further, Fig. 4.15, shows that waves a-c are not cutting on at zero Hz and the dispersion is more complex below 1 kHz compared with Fig. 4.9 and Fig. 4.12.
Fig. 4.14. Dispersion relations of a rail with spring-mass-spring support—imaginary parts. Keys are same to Fig. 4.13.

Fig. 4.15. Dispersion relations of a rail with spring-mass-spring supports-real parts in a linear scale—real parts. ●, positions different waves are marked. a-h, wave labels.
4.6 Rail forced responses

4.6.1 Free rail

Calculated vertical mobilities for a free rail are shown in Fig. 4.16 at the excitation point, at the centre of the rail foot and at the edge of rail foot. Calculations using both rail mesh types shown in Fig. 4.5 are presented.

Fig. 4.16 also shows that the centres of the rail head and rail foot respond significantly and increasingly differently above 1 kHz. This indicates severe rail cross-section deformation. It is noted that the Timoshenko beam-based rail model prediction is close to the rail foot centre response.

Calculated lateral rail mobilities are shown in Fig. 4.17, using mesh type (b) of Fig. 4.5. These include mobilities at the excitation point, at the rail head top centre and at the rail foot bottom centre. The prediction of the Timoshenko beam-based rail model is also shown for comparison.

Fig. 4.16. Vertical rail mobilities under a point force excitation at the centre of the rail head on the top calculated for a free rail. —— at the rail head centre, using the mesh type (b) of Fig. 4.5; —— at the rail foot centre using the mesh type (b) of Fig. 4.5; —— at the rail foot edge, using the mesh type (b) of Fig. 4.5; —– a the rail head centre, using the mesh type (b) of Fig. 4.5; —– at the rail foot centre, using the mesh type (b) of Fig. 4.5; —– at the rail foot edge, using the mesh type (b) of Fig. 4.5; — Timoshenko beam prediction.
Fig. 4.17 shows that rail head mobility at either the top centre or at the excitation point on the side surface is higher than the mobility at the rail foot centre. As also shown, there are two peaks on rail foot mobility at around 1.6 kHz and 4.3 kHz, respectively. These two peaks actually depend only on rail properties and are independent of the rail support assumption used. This will be tested in the application of the following rail models with varying support assumptions. The prediction of the Timoshenko beam-based rail model falls between the mobility predicted at the rail head centre and at the rail foot centre.

Fig. 4.17. Lateral rail mobilities under a point force excitation on the side of rail head calculated for a free rail: —— at the excitation point on rail head side surface; ··· at the rail head centre; – – at the rail foot centre; — Tiposhenko beam prediction.

4.6.2 Single layer supported rail

Calculated vertical rail responses at the excitation point, at the centre of rail foot and at the edge of rail foot are shown in Fig. 4.18, using the mesh type (b) of Fig. 4.5. Prediction by the equivalent Timoshenko beam-based rail model is also shown for comparison.
Fig. 4.18. Vertical rail mobilities under a point force excitation at the centre of the rail head on the top calculated for a rail with a single layer of support: —— at the rail head centre; •••••• at the rail foot centre; — at the rail foot edge; ○ Timoshenko beam prediction.

Fig. 4.18 shows that the predicted mobility using the Timoshenko beam-based rail model is close to the rail response at the bottom of the rail foot centre predicted by SAFE rail model, using the mesh type (b) of Fig. 4.5. Both of them deviate from the rail response at the rail head centre significantly at high frequencies. The rail foot edge has the highest response at high frequencies, especially near the peak at around 5.4 kHz. This peak is caused by the cut on of wave g as shown in Fig. 4.11 where at 5.4 kHz the imaginary part of its wave-number drops sharply. This frequency depends only on the properties of the rail and is independent of the assumed form of the support. The other peak at around 730 Hz is the bouncing frequency of the rail mass on the rail pad, which can be estimated using parameters given in Table 4. Since this frequency, rail vibration is decoupled from the rest of the track structure and starts to vibrate like a free rail.
Fig. 4.19. Lateral rail mobilities under a point force excitation on the side of rail head calculated for a rail with a single layer of support: —— at the excitation point on rail head side surface; ---- at the rail head centre; —— at the rail foot centre; —— Timoshenko beam prediction.

Lateral rail responses calculated at the excitation point, at the rail head centre and at the rail foot centre are shown in Fig. 4.19, using the mesh type (b) of Fig. 4.5. Fig. 4.19 shows that the mobility predicted by the Timoshenko beam-based rail model lies between the mobilities of the rail foot and rail head predicted using the SAFE rail model, confirming the results presented in Fig. 4.17 of a rail without support. The peak 3 and 4 at frequencies at around 1.6 and 4.3 kHz, respectively, reported in Fig. 4.17 are observed again, confirming the conclusion that they are independent of the rail support. They are actually the cut on frequency of the first web bending wave and the second web bending wave. This argument is illustrated in Fig. 4.11, where imaginary (decaying) parts of wave-numbers corresponding to wave d and f fall dramatically at the those frequencies. Peak 1 occurring at a frequency around 150 Hz is due to the cut-on of rail lateral bending wave as shown in Fig. 4.11 where the imaginary part of wave a reduces quickly to less than 1 dB/m. Similarly, peak is due to the cut-on of the torsional wave.
4.6.3 Multiple layer supported rail

Vertical rail point responses calculated at both the rail head centre and at the bottom of foot centre are shown in Fig. 4.20, using the mesh type (b) of Fig. 4.5. Prediction from the equivalent Timoshenko beam-based rail model is also shown for comparison.

![Graph showing rail mobilities](image)

**Fig. 4.20.** Vertical rail mobilities under a point force excitation at the centre of the rail head on the top calculated for a rail with spring-mass-spring support: - - - at the rail head centre; - - - - at the rail foot centre; - - at the rail foot edge; - ○ - Timoshenko beam prediction.

Fig. 4.20 shows again that the rail head centre response exceeds the rail foot centre response as the increase of frequency and the prediction of the Timoshenko beam-based rail model is closely following the rail response at rail foot centre. Comparing with Fig. 4.18, there is one more peak at the frequency of around 35 Hz. This peak is the bouncing frequency of rail and sleeper mass on ballast or the first cut on frequency of the vertical bending wave. This can be easily verified by calculating it using the parameters listed in Table 4 or testified in Fig. 4.14 where the imaginary part of wave c has a sharp drop at around 35 Hz. The other two peaks occur at same frequencies as shown in Fig. 4.18. Similarly with the finding from Fig. 4.18, foot
Fig. 4.21. Lateral rail mobilities under a point force excitation on the side of rail head calculated for a rail with spring-mass-spring support: —— at the excitation point on rail head side surface; ∙∙∙∙ at the rail head centre; – – at the rail foot centre; –○– Timoshenko beam prediction.

Calculated lateral rail responses are shown in Fig. 4.21 using mesh type (b) of Fig. 4.5. The prediction of the Timoshenko beam-based rail model once again falls between rail head and rail foot mobility. Peak 5 and 6 at around 1.6 kHz and 4.3 kHz, respectively, are once again observed, confirming further the conclusion that they are independent of the assumed form of rail support. Comparing with Fig. 4.19, Fig. 4.21 shows rail response has one peak at around 35 Hz and another peak at around 75 Hz. As mentioned above, the first peak is the first cut-on frequency of vertical bending wave. The second peak is corresponding to the bouncing frequency of rail and sleeper mass laterally on ballast or the first cut-on frequency of lateral bending wave as shown in Fig. 4.14.
4.7 Effects of rail discretization
The effects of rail cross-section discretization are investigated in this section by comparing rail vibration responses and wave dispersion relations calculated using the two different rail meshes shown in Fig. 4.5.

4.7.1 Influence on calculated wave dispersion relations
One indication of the impact of mesh detail is evident in the real parts of dispersion relations shown in Fig. 4.22 for 0–4 kHz. No difference can be observed in this frequency range. The same conclusion results from a review of the imaginary parts of the dispersion relations.

![Dispersion relations for a free rail](image)

Fig. 4.22. Dispersion relations of a free rail—real parts: dotted line: mesh type of Fig. 4.5 (a); circles: mesh type of Fig. 4.5 (b).

4.7.2 Influence on calculated rail vibration responses
Calculated rail vibration responses at various points using the two rail models shown in Fig. 4.5 are given in Fig. 4.23, Fig. 4.24 and Fig. 4.25. Fig. 4.25 shows that mobilities calculated at rail head centre are affected by the rail models used. This difference, though seems to be significant, cannot be used to indicate the divergence
of the results. This may be caused by the force application at the excitation point and this effect is limited to the excitation point as evidenced by the calculated rail mobilities at the head edge shown in Fig. 4.23 from the two rail models. Further evidence is the calculated rail foot response as shown in Fig. 4.24 where almost no difference can be observed from the results calculated using the two rail models.

Fig. 4.23. Calculated rail mobilities: at the centre of the rail head on the top using – mesh (a) and --- mesh (b) of Fig. 4.5 and; at the edge of the rail head on the top using -- mesh (a) and ---- mesh (b) of Fig. 4.5.
Fig. 4.24. Calculated rail mobilities at the bottom centre of the rail foot using: – mesh (a) and --- mesh (b) of Fig. 4.5 and at the edge of the rail foot on the top using: — mesh (a) and ---- mesh (b) of Fig. 4.5.

Fig. 4.25. Calculated rail mobilities at the top centre of the rail head using: – mesh (a) and --- mesh (b) of Fig. 4.5.
4.8 Conclusion

In this chapter, it is verified that the existing SAFE rail model can be extended to the real track situation by representing rail supports as a layer of continuous support made by equivalent masses and springs. By measuring track parameters such as rail pad stiffness and ballast stiffness using the standard hammer test, track support dynamic stiffness can be determined and incorporated into the developed SAFE rail model to represent the real track situation. By inspecting the calculated rail dispersion relations and frequency responses under assumptions of a free rail, a rail with a single layer of support representing rail pads and a rail with a multiple layer of support representing rail pads, sleepers and ballast, it has been found that

1) the track support stiffness can be determined and incorporated into the proposed extension of the SAFE rail model to represent track conditions at a specific site by measuring track component parameters including stiffness and damping;

2) rail support has a significant impact on the prediction of rail dispersion relations and vibration response but this effect is limited to below 1 kHz; and

3) the developed SAFE rail model is demonstrated to be a feasible means for more accurate prediction of rail vibration characteristics, compared with the Timoshenko beam-based rail models.

The effects of the mesh details on the convergence of the proposed SAFE rail models are investigated using two different rail meshes. The results show that

1) the calculated dispersion relations between the two rail meshes have no obvious difference; and

2) the influence of the mesh details on the calculated rail responses is limited to the point where excitation force is applied. On other points in the same cross-section plane, the calculated rail responses have no difference between the two rail meshes. In the longitudinal direction, the difference is also limited, which is less than 1 dB at frequencies above 1.5 kHz. This indicates that the proposed SAFE rail model can be used to predict rail responses even with a limited number of elements.
5 RAIL RADIATION MODEL BASED ON WBE METHOD

5.1 Introduction and chapter outline

As identified in Chapter 2, rail radiation ratio that can consider the effects of rail support, and the corresponding rail directivity factor in the rail cross-section plane, lack knowledge. The two parameters however are critical to rail sound radiation, as shown in also Chapter 2. A rail radiation model which allows for the rail cross-section, and also the inclusion of support, is therefore required. A SAFE rail vibration model with inclusion of support has been established in the previous chapter. This chapter will extend the work to investigate rail radiation characteristics, using the BE method, which is coupled with the SAFE rail model. As a result, the rail radiation ratio under various supports will be examined and rail radiation directivity in the vertical plane will be determined.

The following content in this chapter is structured into four sections. Section 5.2 looks into the theory of WBE method. The singularity problems confronted by BE methods are also addressed in this section. In Section 5.3, the discretized rail model will be introduced. The calculated results, including rail radiation power, radiation ratios and radiation directivity, using the WBE rail model, are presented in Section 5.4, Section 5.5 and Section 5.6, respectively for three assumed rail support characteristics, i.e. a free rail, rail with a layer of single layer of support and rail with multiple layer of support. The last section summarizes the work of this chapter.

5.2 Theory of WBE method

5.2.1 Governing equation

The governing equation for a radiation problem is the wave equation, the inhomogeneous form of which in a lossless medium is given by

$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = f$$  \hspace{1cm} (5.1)

where $p$ is the sound pressure, $f$ is a source, $c$ is sound speed in the medium and $t$ denotes time. For the radiation problem, the domain is the space exterior to the radiating structure and denoted as $\Omega$. The boundary of the domain is denoted as $\Gamma$. For the rail radiation problem, the space around the rail is assumed to be free, i.e. $f = 0$ and the medium is air.
By assuming a harmonic wave solution, \( p(x, y, z, t) = p(x, y, z)e^{i\omega t} \) (the term \( e^{i\omega t} \) is used to comply with the term used in rail vibration modelling in Chapter 4 so that the two models can be coupled directly), Eq. (5.1) reduces to the classic Helmholtz equation

\[
\nabla^2 p(x, y, z) + k^2 p(x, y, z) = f(x, y, z) \tag{5.2}
\]

where \( k = \omega / c \) is the wave-number of the sound in the air at a frequency \( \omega \).

The problem defined by Eq. (5.2) in \( \Omega \) is often subject to constraints on the boundary of the domain. For a radiation problem, the velocity on \( \Gamma \) is known or the Neumann boundary conditions are given. For rail radiation, interaction between air and rail can be ignored because of the much lower acoustic impedance of air as compared with rail.

The solution integral to Eq. (5.1) can be written as (Katsikadelis, 2002, Chapter 3)

\[
Cp(P) = \int_{\Gamma} p^*(P, Q) q(P, Q) d\Gamma - \int_{\Gamma} p(P, Q) q^*(P, Q) d\Gamma \tag{5.3}
\]

where \( P \) is a source point and \( Q \) is a point on boundary \( \Gamma \) of the domain. Coefficient \( C \) is given by

\[
C = \begin{cases} 
0 & P \notin \Omega \\
1 & P \in \Omega \\
\frac{\theta_0}{2\pi} & P \in \Gamma 
\end{cases} \tag{5.4}
\]

where \( \frac{\theta_0}{2\pi} \) is used when the source point \( P \) is at a corner on boundary \( \Gamma \). \( \theta_0 \) is the angle formed by the two corner edges and outside \( \Gamma \).

The solution can be proved to satisfy the Somerfield boundary condition which states that the sound pressure diminishes at an infinite distance from the source.

In Eq. (5.3), \( p^* \) is the fundamental solution to Eq. (5.1) which is obtained under a point source excitation, also known as the ‘free space Green’s function’. For a two-dimensional radiation problem, one expression for \( p^* \) is obtained by using the method given by Morse and Ingard (1968, Chapter 7)

\[
p^*(r) = -\frac{i}{4} H_n^{(2)}(kr) \tag{5.5}
\]

where \( r \) is the distance between source point \( P \) and evaluation point \( Q \).
It can be proved that \( p^*(kr) \) guarantees that 1) the wave amplitude diminishes to zero at an infinite distance, 2) sound pressure or energy is infinite when the evaluation point is approaching the source point, i.e. \( r \to 0 \). \( q^* \) is the normal derivative of \( p^* \) and given by

\[
q^*(r) = \frac{\partial p^*}{\partial n} = \frac{ik}{4} H_1^{(2)}(kr) \frac{\partial r}{\partial n}
\]  

(5.6)

Here \( H_m^{(n)} \) is the \( n^{th} \) Hankel function of \( m^{th} \) order. \( r \) is the distance between P and Q. \( \mathbf{n} = (n_x, n_y) \) is the normal to the \( \Gamma \). \( \frac{\partial}{\partial n} \) is derivative along \( \mathbf{n} \) and given by

\[
\frac{\partial}{\partial n} = \frac{\partial}{\partial x} n_x + \frac{\partial}{\partial y} n_y.
\]

\( p \) is the sound pressure and \( q \) is its derivative along \( n \).

By discretizing \( \Gamma \) into \( E \) elements and \( N \) nodes, a total of \( N \) unknowns on the boundary are generated. Each element is assumed to have \( i_{\text{max}} \) nodes. The collocation method (Brebbia and Dominguez, 1992, Chapter2) can be applied to calculate the unknowns by placing the source point \( P \) at each of the node points successively. Eq. (5.3) can be written as if \( P \) is collocating with a node \( P_n \)

\[
C_p(P_n) = \sum_{e=1}^{E} \sum_{i=1}^{i_{\text{max}}} q_i^e \Delta G_{ni}^e - \sum_{e=1}^{E} \sum_{i=1}^{i_{\text{max}}} p_i^e \Delta H_{ni}^e
\]  

(5.7)

where

\[
\Delta G_{ni}^e = \int_{\Gamma^e} N_i(\xi) p^*(P_n) \, d\Gamma^e
\]  

(5.8)

\[
\Delta H_{ni}^e = \int_{\Gamma^e} N_i(\xi) q^*(P_n) \, d\Gamma^e
\]  

(5.9)

where \( i \) is the local node index for each element, \( n = 1, 2, 3, \ldots N \) is the global node index for the whole model and ordered in an anti-clockwise direction, \( N_i \) is the shape function defined for every node of an element and \( \xi \) is the local coordinate defined on each element. It should be noted that when \( i \) is not used as a subscript, it represents the imaginary unit.

By combining the equations corresponding to each node and moving all of the unknown sound pressures \( \{p\} \) to one side and the given boundary conditions \( \{q\} \) to the other side, the form of the system of equations can be written as

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\[ [A]_{N \times N} \{p\}_{N \times 1} = [B]_{N \times 3E} \{q\}_{3E \times 1} \] (5.10)

It is noted that a standard FE assembly method can be used to determine coefficient matrix \([A]\) but not \([B]\) because the sound pressure \(p\) is continuous across neighbouring elements. Discontinuity of the derivative of sound pressure \(p\), i.e. \(q\), can occur at a node which is at a corner on the boundary. In this case the normal is different before and after the node. As a result, \(A\) is of a size of \(N\) by \(N\) while \(B\) has a size of \(N\) by \(3E\).

By solving Eq. (5.10), the sound pressures on the boundary can be obtained. The sound pressure at any point inside the domain \(\Omega\) can then be calculated using Eq. (5.7) and by noting that \(C=1\).

Particle velocity along a direction \(n_p\) is related to sound pressure at point \(P\) by

\[ v = -\frac{1}{i\omega \rho_0} \frac{\partial p}{\partial n_{OM}} \] (5.11)

where \(\rho_0\) is air density. \(\frac{\partial}{\partial n_p} = \frac{\partial}{\partial x_p} n_{x_p} + \frac{\partial}{\partial y_p} n_{y_p}\). Eq. (5.11) can, therefore, be written as

\[ v = -\frac{1}{i\omega \rho_0} \left( \frac{\partial p}{\partial x_M} n_{x_M} + \frac{\partial p}{\partial y_M} n_{y_M} \right) \] (5.12)

The derivation of \(p\) with respect to \(x\) is given below

\[ \frac{\partial p}{\partial x} = \sum_{e=1}^{E_{\text{max}}} \sum_{i=1}^{i_{\text{max}}} q_i^e \Delta R_{x}^e - \sum_{e=1}^{E_{\text{max}}} \sum_{i=1}^{i_{\text{max}}} p_i^e \Delta S_{x}^e \] (5.13)

where

\[ \Delta R_{x}^e = \int_{-1}^{1} N_i(\xi) \frac{\partial p^*}{\partial x_p}(kr) J(\xi) \, d\xi \] (5.14)

\[ \Delta S_{x}^e = \int_{-1}^{1} N_i(\xi) \frac{\partial q^*}{\partial x_p}(kr) J(\xi) \, d\xi \] (5.15)

where \(p^*(kr)\) is given in Eq. (5.5) and \(q^*(kr)\) in Eq. (5.6). \(J(\xi)\) is the Jacobian. The derivative of \(p^*\) and \(q^*\) with respect to \(x\) are given by

\[ \frac{\partial p^*}{\partial x_p} = \frac{ik}{4} H_1^{(2)}(kr) r_{x_p} \] (5.16)
\[
\frac{\partial q^e}{\partial x_p} = -\frac{ik^2}{4} \left[ H_0^{(2)}(kr) - \frac{1}{kr} H_1^{(2)}(kr) \right] \left( r_{xp} n_x Q + r_{xp} r_{yp} n_y Q \right) - \\
\frac{ik}{4} H_1^{(2)}(kr) \left[ \frac{r_{yp}^2}{r} n_y Q - \frac{r_{xp} r_{yp}}{r} n_y Q \right]
\]

(5.17)

, respectively, where \( r_{xp} \) and \( r_{yp} \) are the derivatives of \( r \) with respect to \( x_p \) and \( y_p \), respectively. More details on the derivation of Eq. (5.16) and Eq. (5.17) are given in Appendix D. The derivation of \( p \) with respect to \( y \) can be obtained similarly by replacing \( x \) with \( y \) in Eq. (5.14)-(5.17).

### 5.2.2 Singularity problem

\( \Delta G_{ni}^e \) and \( \Delta H_{ni}^e \) have a Hankel function and its derivative as kernels which tend to be infinite when source point \( P \) is on the integration element, i.e. \( r \rightarrow 0 \). This problem is known as a ‘weakly singular integral’ because integrals of the Hankel function and its derivative, i.e. \( \Delta G_{ni}^e \) and \( \Delta H_{ni}^e \), converge in the classic Riemann sense since their kernels have an order of \( O(\ln(r)) \) when \( r \rightarrow 0 \). If standard Gaussian quadrature is to be used, then the number of Gaussian points has to be increased significantly near the singularity to ensure acceptable accuracy and this renders the Gaussian quadrature method impractical. When \( P \) is distant from the integration element, \( \Delta G_{ni}^e \) and \( \Delta H_{ni}^e \) have regular integrands and conventional Gaussian Legendre quadrature can be applied. The importance of calculating singular integrals at the same level of accuracy as regular integrals has been shown by Treeby and Pan (2009). Therefore this must be done and practical methods have to be used.

A nearly weakly singular condition may also arise when the distance between source point \( P \) and the element where integration is to be calculated is small in relation to the size of the element. The kernels in this case also become close to infinity and a number of methods have been developed to counteract this. These include non-linear transformation (Telles and Oliveira, 1994, Elliott and Johnston, 2008, Johnston et al., 2013) and analytical formulation (Niu et al., 2007). This singularity will only occur when source point \( P \) is one of the nodes of an element near a corner of an irregular two-dimensional BE model when the collocation method is applied. For a meshed rail profile (shown in Fig. 5.1) there is only a small number of vertices compared to
the number of boundary elements, the outer edge of the eight-noded quadrilateral elements, distributed on the planar or near-planar rail profile edges. The inaccuracy induced by the small number of nearly weakly singular kernels will not be sufficient to give rise to significant error in the solution of Eq. (5.10). Treeby and Pan (2009) have supported the argument in their work by a scattering cylinder and an irregular scatter example. The nearly weakly singular problem is, therefore, not considered in the current model presented.

![SAFE rail vibration model](image)

**Fig. 5.1.** SAFE rail vibration model, arrows represent the positions and the directions of applied forces.

Various methods have been developed to overcome the weakly singular problem. These include non-linear coordinate transformation (Doblaré and Gracia, 1997, Yun, 2006), division of the integral at the singular point (Singh and Tanaka, 2001) and analytical solutions (Singh and Tanaka, 2000). Among them, the non-linear coordinate transformation method allows the use of the Gaussian quadrature method without modification, minimizing the changes to the algorithms of the BE method. It is chosen for use in this work.

The non-linear coordinate transformation method maps the coordinate associated with the singularity onto itself using a polynomial. After transformation, the Jacobian
becomes null at the singular point. As a result, the order of the original singularity of the weakly singular integrals is weakened and standard Gaussian quadrature can then be applied. Compared to other non-linear coordinate transformation methods, the one proposed by Sato et al. (1988) has been found to be one of the best performing ones (Singh and Tanaka, 2001) in terms of the smoothing property near the singular point, numerical convergence and accuracy. This method is, therefore, adopted in the current work. According to the findings of Treeby and Pan (2009) from trials of Sato’s method on a Hankel kernel-based integral, the order of Sato’s transformation polynomial is chosen to be 5 and the number of Gaussian points 20 to ensure a good performance of Sato’s method for solving weakly singular problems.

Weakly singular problems arise when the source point P is on an integration element. For a quadratic element, 3 situations have to be differentiated:

1) when P is on a node of a different index with \( i \) shown in Eq. (5.8) and (5.9), kernels \( p^* \) and \( q^* \) tend to be infinite but the shape functions also approach zero as distance \( r \) decreases. The integral of their products are finite as a result. Standard Gaussian quadrature can be applied to calculate \( \Delta G_{ni}^e \) and \( \Delta H_{ni}^e \).

2) when P is one of the ending nodes and \( i \) denotes the same node. In this case, kernels \( p^* \) and \( q^* \) tend to be infinite but the shape functions tend to be unit. Their product becomes infinite and so does the integral of their product. \( \Delta G_{ni}^e \) and \( \Delta H_{ni}^e \) are weakly singular as a result. Standard Gaussian quadrature becomes unsuitable. Sato’s method can be used in this case. For the ending nodes of an element, Sato’s transformation (Singh and Tanaka, 2001)

\[
\xi(\tau) = \xi_s - \frac{\xi_s}{2^4} (1-\xi_s \tau)^5, |\xi_s| = 1
\]  

(5.18)

maps local coordinates \( \xi \) defined on \((-1,1)\) of an element onto a new coordinate \( \tau \) of \((-1,1)\). The Jacobian of the transformation is

\[
J(\tau) = \frac{5\xi_s^2}{2^4}(1-\xi_s \tau)^4
\]  

(5.19)

From Eq. (5.19) \( J(\tau) \) equals zero at both of the ending points of the element. Substituting Eq. (5.18) and (5.19) into Eq. (5.8) and (5.9), \( \Delta G_{ni}^e \) and \( \Delta H_{ni}^e \) can be transformed into the integrals in terms of \( \tau \).
\[
\Delta G^e_{ni} = \frac{i}{4} \int_{-1}^{1} N_i(\xi(\tau)) H_0^{(2)}(kr(\xi(\tau))) J(\xi(\tau)) \frac{d\xi}{d\tau} \, d\tau 
\tag{5.20}
\]
\[
\Delta H^e_{ni} = \frac{i k}{4} \int_{-1}^{1} N_i(\xi) H_1^{(2)} (r_{x,n} + r_{y,n}) J(\xi(\tau)) \frac{d\xi}{d\tau} \, d\tau 
\tag{5.21}
\]

3) when P is node 2 and \( i = 2 \)

In this case, the singular point is belonging to the integration element. Similar with case 2), weak singularity occurs. Sato’s method can be used but transformation is required as the Sato’s method is only applied at the end points of an element. The interval is therefore partitioned at P (Treeby and Pan, 2009). Sato’s method can then be applied on each part after mapping the coordinates of each part onto (-1, 1).

The equations below

\[
\xi = \frac{1}{2} \left[ \gamma(\xi_0 + 1) - 1 + \xi_0 \right] 
\tag{5.22}
\]

\[
\xi = \frac{1}{2} \left[ \gamma(1 - \xi_0) + 1 + \xi_0 \right] 
\tag{5.23}
\]

map the first and second part of the integration element defined on \( \xi \) of (-1,\( \xi_0 \)) and (\( \xi_0 ,1 \)) onto \( \gamma \) of (-1,1), respectively. \( \xi_0 \) is local coordinate of the singular point. For the quadratic element used here, \( \xi_0 \) is zero. Note that singularity occurs at \( \gamma = 1 \) for the first part and -1 for the second part of the element.

Sato’s non-linear transformation (Eq. (5.18) and (5.19)) can now be applied to each part by remapping \( \gamma \) onto a new variable, \( \tau \) of (-1,1). Those formulations are

\[
\gamma(\tau) = 1 - \frac{1}{2^4} (1 - \tau)^5 
\tag{5.24}
\]

for the first part and

\[
\gamma(\tau) = -1 + \frac{1}{2^4} (1 + \tau)^5 
\tag{5.25}
\]

for the second part.

\( \Delta G^e_{ni} \) and \( \Delta H^e_{ni} \) can then be obtained by combining integrals of the two parts.

5.2.3 Wave-number domain BE method

Applying a Fourier transform to Eq. (5.2) with respect to \( z \) and neglecting the excitation source \( f \) for a free space condition yield
\[ \nabla_2 D^2 \bar{p} + \alpha^2 \bar{p} = 0 \]  \hspace{1cm} (5.26)

where

\[ \bar{p} = \int_{-\infty}^{+\infty} p e^{i\kappa z} d\kappa \]  \hspace{1cm} (5.27)

\[ p = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \bar{p} e^{-i\kappa z} dz \]  \hspace{1cm} (5.28)

and

\[ \alpha = \sqrt{k^2 - \kappa^2} \]  \hspace{1cm} (5.29)

where \( \kappa \) is wave-number of a wave travelling along the structure. Similarly, a pair of the transform can be defined for particle velocity given in Eq. (5.11).

It is noted that for a sound radiation problem, \( \alpha \geq 0 \) because no sound can be generated in the air from a structure if \( k \) is less than \( \kappa \), or the structural wavelength are shorter than the sound wavelength in the air at a frequency (Norton and Karczub, 2003, Section 3.3). As a result, Eq. (5.26) is a typical two-dimensional Helmholtz equation. Then the solution method introduced in the previous sections can be used to solve Eq. (5.26). The get the boundary conditions or the boundary velocities, WBE rail model can be coupled with the SAFE rail vibration model developed in Chapter 4 by sharing the boundary node velocities.

5.2.4 Radiation ratio

The formula to the calculate radiation ratio based on a WBEM has been proposed by Nilsson et al. (2009)

\[ \sigma = \frac{P}{P_0} = \frac{1}{2} \frac{Re \left( \int_{-k}^{+k} \bar{p}^H \bar{v} d\Gamma dk \right)}{\int_{-k}^{+k} \int_{\Gamma} \left| \bar{v} \right|^2 d\Gamma dk} \]  \hspace{1cm} (5.30)

where \( P \) is the radiation power of the rail, \( P_0 \) is the radiation power of the baffled plate with a same surface area to the rail and a surface velocity equal to the rail velocity averaged over the rail surface area. The superscript \( H \) denotes complex conjugate.

The internal integral of either the numerator or denominator around the contour of the rail can be calculated on an element basis. For example, the internal integral of the numerator of Eq. (5.30) can be discretized into
The calculation of the external integrals of both the numerator and the denominator requires careful consideration. One example of the displacement versus wave-number is shown in Fig. 5.2 obtained from one node at a particular frequency. This figure shows that the nodal displacement varies significantly for wave-numbers between -20 and 20. The velocity, which is proportional to displacement, will also vary significantly in this region. This indicates that external integrals have to be distributed in small steps in order to get a relatively accurate result in this wave-number range. From trials on series of nodal and integration points, it was found that the step size should be no greater than 0.01 when the wave-number is between -30 and 30, and 0.05 outside this region. Using these settings, the variation of the results is limited to the fourth significant figure. This setting was adopted in this work. The trapezoidal integral method is used throughout.

One important property of the integral is that the integrands are even with respect to wave-number \( \kappa \) as shown in Chapter 4. Velocity is proportional to the displacement.
and therefore is also even, with respect to \( \kappa \). Sound pressure is also even with respect to wave-number \( \kappa \) as it depends on \( \alpha \), which is even around \( \kappa \) as shown in Eq. (5.29). This property can significantly improve the calculation speed of the integrals in Eq. (5.30) as only half the number of the wave-numbers in the range is required.

5.2.5 Radiation directivity factor

The radiation directivity factor is defined as the ratio of the sound intensity, \( I_\theta \), at a distance \( r \) and direction \( \theta \) from the source to the sound intensity, \( I_0 \), produced by a uniform directional source of the same power (Norton and Karczub, 2003, Section 4.5). It can be written as

\[
D(r, \theta) = \frac{I_\theta}{I_0}
\]

(5.32)

Rail sound intensity is given by

\[
I_\theta = \frac{1}{2} \text{Re} \left( \int_{-\infty}^{+\infty} \tilde{p}^H \tilde{v} \, dk \right)
\]

at an evaluation point \((r, \theta)\). Here \( r \) is defined as the distance from centre of rail foot bottom to the evaluation point and \( \theta \) is the angle between the evaluation point and the plane of rail foot as shown in Fig. 5.3.

![Fig. 5.3. Position of a point \((r, \theta)\) with respect to rail.](image)

Transforming the integral from the space domain to the wave-number domain by applying Parseval’s theorem on Eq. (5.33) gives

\[
I_\theta = \frac{1}{2} \text{Re} \left( \int_{-k}^{+k} \tilde{p}^H \tilde{v} \, dk \right)
\]

(5.34)
\( I_0 \) can be calculated by

\[
I_0 = \frac{P_0}{2\pi r} = \frac{1}{2} \int_{-k}^{+k} \int_{\Gamma} |\tilde{p}|^2 d\Gamma dk
\]

(5.35)

5.3 Discretized WBE rail model

A WBE rail model has been built so that it confirms the discretization of the SAFE rail vibration model around the boundary and can be coupled with the SAFE rail vibration model. The isoparametric quadratic line element is used to comply with the eight-noded quadratic element used in the SAFE rail model. As a result, the current BE model has a total of 40 elements and 80 nodes. Under this setting, nodal displacements obtained from the SAFE rail model can be used in the BE rail model directly as boundary conditions. The order of the nodes is in the anti-clockwise direction for a rail radiation problem.

To cover the frequencies of interest for rolling noise, the model must be valid for a frequency \( \geqslant 5 \) kHz. In the current BE model, the maximum distance between the neighbouring nodes is 12.1 mm. This distance ensures the model is valid for frequencies up to 7 kHz with four nodes per wavelength.

5.4 Rail radiation power

The radiation power calculated for each of the 3 support assumptions is presented in Fig. 5.4. The rail is excited by a vertical point force at the centre of the top of the rail head, as shown in Fig. 5.1. A significant difference (>5 dB) in the results between the free rail case and both of the supported cases is evident for frequencies below 1 kHz. This difference generally increases with decreasing frequency for the single layer support assumption. For the multiple layer support assumption, the difference varies from 0 to 35 dB with an average of around 20 dB. This tends to indicate that for a UIC60 rail profile, the most significant influence (>5 dB) of the rail support on the rail radiated sound power occurs for frequencies below 1 kHz. Under different track parameters, particularly rail pad stiffness, the 1 kHz frequency may vary.

Fig. 5.4 also shows, as expected, that a free rail has the highest radiation power. Radiation power predicted by the rail model with multiple layers of support falls between the results predicted by the free rail model and the rail model of a single layer of support for vibration frequencies below 250 Hz. Above 250 Hz, the results
obtained from the two supported rail models progressively merge and are essentially equal above 1 kHz. This is consistent with the intuitive behaviour where, for high frequencies, the support ‘decouples’ from the rail.

Fig. 5.4. Rail radiation powers calculated under a vertical point excitation on the top of the rail head: —— free rail model; —— rail model with a single layer of spring support; —– rail model with spring-mass-spring support.

The stiffness change due to the inclusion of supports is expected to be the cause of differences in radiation power between the different cases. A free rail will have the lowest stiffness. Neglect of the relatively low stiffness afforded by the ballast results in the overestimation of the support stiffness below the frequency where rail and sleeper vibration decouples from the ballast, i.e. the ballast stiffness significantly influences the overall support stiffness in this frequency range and is best accommodated by the use of the multi-layer support assumption.

The correlation between lower support stiffness and higher rail radiated noise can be tested by comparing calculated rail mobilities using the SAFE rail vibration model. Fig. 5.5 shows the resulting rail mobilities for the 3 assumed support conditions at the driving point: the top of the rail head centre under a vertical point excitation. In general, the trend of the mobilities shown in Fig. 5.5 is close to the trend of radiation
power shown in Fig. 5.4. The differences occur at 1) frequencies above 500 Hz where the supported rail conditions have higher or similar mobilities but lower radiation powers than the free rail condition and 2) there appears to be a transitional frequency at 250 Hz where the single layer-supported rail has a larger radiation power than the multiple layer-supported rail which is higher than the corresponding transitional frequency of the rail mobilities at around 180 Hz.

The two observed differences may result from the different travelling distances of the various waves along the rail or the different decay rates of these waves under the three assumed support conditions. The calculated rail mobility at the driving point does not take into account these effects while the calculated radiation power does.

![Graph](image)

Fig. 5.5. Vertical rail mobilities at the centre of the rail head under a vertical point excitation force at this point: — free rail model; —— rail model with a single layer of support; ——— rail model with multiply layers of support.

The decay rates of the waves travelling along the rail determine the significant length of a radiating rail. The lower the decay rate, the longer the effective distance a wave can travel along the rail and the higher the contribution the wave can make to the rail radiation if it has a wave-number which is smaller than the wave-number of the sound in the air. In other words, it can produce sound in the air. By introducing a
support, additional damping is also introduced to the rail. As a result, the wave
behaviours are changed. This assumption can be tested using the calculated rail
vibration by the SAFE rail model. Such results are presented in Fig. 5.6 where the
decay rates of the propagating waves along the rail are shown for the three rail
support assumptions up to 4 kHz. Only the vertical and lateral bending waves are
shown. Other waves, such as the near-field waves, may also contribute to the noise
radiated by the rail but these are considered to be insignificant due to either: 1) their
relatively lower efficiency in disturbing air or generating sound than the propagating
bending waves or 2) their relatively shorter travelling distance due to higher decay
rates.

![Decay rates for vertical and lateral bending waves travelling along the rail.](image)

Fig. 5.6. Decay rates for vertical and lateral bending waves travelling along the rail. From
thin to very thick, the lines represent the rail model with multiple layers of support, the rail
with a single layer of support and the free rail model, respectively. Continuous lines
represent lateral bending waves and dashed lines, the vertical bending waves.

Fig. 5.6 shows that, of the 3 support assumptions modelled, a free rail has the lowest
vertical and lateral decay rates and therefore has the highest radiation power for a
similar or even smaller response. The multiple support condition yields lower decay
rates than the single support layer condition below 130 Hz for lateral bending waves.
and below 370 Hz for vertical bending waves. For a similar rail response, the multiple support layer support assumption is therefore expected to indicate a higher radiation than the single layer support assumption below a frequency between 130 and 370 Hz.

Fig. 5.4 also shows that calculated rail radiation power fluctuates at frequencies above 1 kHz. This is due to the interaction of rail head and foot which vibrate in or out of phase at these frequencies because of rail cross-section deformation. For example, there is a peak around 2 kHz, corresponding to a sound wavelength of 170 mm. This is roughly the height of the rail opening up the possibility of coincident peaks.

![Graph showing rail radiation power vs. frequency](image)

**Fig. 5.7.** Rail radiation powers calculated under a lateral point force excitation on the rail head side surface: ——— free rail model; ——— rail model with a single layer of spring support; ——— rail model with spring-mass-spring support.

Rail radiation power calculated under a lateral point excitation force on the rail head side surface is shown in Fig. 5.7 for the three support assumptions. Fig. 5.7 further demonstrates the effects of rail support on the prediction of rail radiation power, though the effects are less significant than those observed in Fig. 5.4. The influence of support also increases with the decrease of frequency and, as can be seen in Fig.
the difference between a free rail and the rail with multiple layers of support is smaller than that of a free rail with a rail of a single layer of support below 130 Hz. Similar to Fig. 5.4, the free rail generally has the highest radiation power above 180 Hz. The rail in the multiple layer support case has the highest radiation power from 20 to 80 Hz. The single layer support assumption yields the lowest radiation power below 130 Hz. Above 130 Hz, it has higher radiation power than the multiple layer support assumption until around 1 kHz, where they converge. This tends to indicate that rail support only affects rail noise radiation prediction at frequencies below 1 kHz. Other rails with different support conditions may produce different results from those presented here, but the general trend will apply, i.e. effects of different rail support conditions are limited to low frequencies because at high frequencies, the rail is decoupled from the support.

The influence of varying support stiffnesses between the support assumptions on the results presented in Fig. 5.7 can be tested by considering rail lateral mobilities. These are presented in Fig. 5.8 which shows that multiple layer support assumption results in a higher rail mobility than the single support assumption, and therefore a higher rail radiation power, below 120 Hz. Though the free rail assumption does not result in the highest mobility at every frequency above 180 Hz, it has the highest radiation power. Similar to the finding for the rail radiation power under vertical excitation, as discussed previously and illustrated in Fig. 5.6, rail radiation power calculated under lateral excitation is affected by the decay rates of various waves. The observed lower decay rates of the travelling waves under the free rail assumption may result in a higher radiation power than under the supported rail assumptions above 180 Hz as shown in Fig. 5.7 even though it has a lower vibration response at the driving point as indicated in Fig. 5.8.
Fig. 5.8. Rail mobilities at the centre of rail head under a lateral point force excitation on the rail head side surface: —— free rail model; ···· rail model with a single layer of support; –– rail model with multiply layers of support.

5.5 Rail radiation ratios

Fig. 5.9 shows the radiation ratios calculated for the three support assumptions under a vertical point excitation on the top of rail head. Comparing Fig. 5.4 to Fig. 5.9 shows that the effect of rail supports on rail radiation ratios is more limited. There is almost no difference between the three support assumptions above 400 Hz and below 30 Hz. The single layer support assumption results in the highest radiation ratio which increases almost monotonically at a rate of 30 dB per decade of frequency up to around 1 kHz. This corresponds to an infinite oscillating cylinder (Thompson et al., 2003). Both the free rail and multiple layer support assumptions exhibit a trough at around 60 Hz and 150 Hz, respectively, where the increasing speed of the radiation ratio changes.

Analytical radiation ratio assuming the rail as an oscillating cylinder is presented in Fig. 5.9. The equivalent cylinder radius of 50mm was obtained by tuning the model to the results obtained from the WBE rail model. This value equals one third of the rail foot width. The equivalent radius is less than half the rail width due to the
interaction of the sound radiated from the rail head and the foot surfaces during propagation of the sound (Thompson et al., 2009, Section 6.4).

Fig. 5.9. Rail radiation ratios calculated under a vertical point force excitation on the top of the rail head: —— free rail model; ——— rail model with a single layer of support; ——— rail model with multiple layers of support; ······ oscillating line source model with an equivalent radius of 50 mm.

In general, the analytical solution is close to the WBE prediction as illustrated in Fig. 5.9, but there are some important exceptions. The analytical radiation ratio only corresponds closely to the prediction using the single support assumption. The troughs noticed for the WBE results obtained from free rail and multiple layer support assumptions are not predicted by the analytical model. The fluctuations from WBE models in radiation ratio above 1 kHz are also not predicted by the analytical model due to the rigid body assumption.

WBE results for rail radiation ratios under lateral point excitation on the rail head side surface are shown in Fig. 5.10. As shown, the influence of the support assumption on rail radiation ratios is more significant compared with the results shown in Fig. 5.9. Significant differences can be observed between the free and the supported rail assumptions at frequencies below 700 Hz. The supported rail models, however, start to converge at around 250 Hz. The rail of a single layer support has
the highest radiation ratio with an increase speed of around 30 dB per decade until around 650 Hz. The free rail model has an increase of around 30 dB per decade below 150 Hz and 45 dB per decade upwards until around 650 Hz. This is different with the result reported by Thompson et al. (2003) where the lateral rail radiation ratio increases at a constant speed of 30 dB per decade up to around 600 Hz. However it should note that the two-dimensional free rail used by Thompson et al. (2003) is assumed to be rigid while the free WBE rail model considers the waves propagating along the rail. The decay rates of the lateral and vertical bending waves as shown in Fig. 5.6 increase with frequency, which indicates that rail radiation can be less efficient or the radiation ratio increases faster as the increase of the frequency up to 650 Hz as shown in Fig. 5.10. Radiation ratios for the multiple layer support assumption fall between predictions from the free rail and the single layer support assumptions. Interestingly, there is a peak at around 80 Hz, due to the bounce of rail and sleeper mass on the lateral ballast stiffness. This is shown by the second peak of mobility calculated for the multiple layer support assumption, as shown in Fig. 5.8, and the lower decay rates (<1 dB/m) of both vertical and lateral bending waves shown in Fig. 5.6 near 80 Hz.

Results obtained from an equivalent line source model are also presented in Fig. 5.10 for comparison with the results calculated by the WBE rail models. The equivalent radius of the line source tuned by using the WBE results was found to be 68 mm which is around \( \frac{2}{5} \) of the rail height. This finding is close to those reported by Thompson et al. (2009, Section 6.4) that the lateral radius of the equivalent line source model is equal to half of the rail height.

Similar to the findings for the vertical rail radiation ratio, the analytical line source model has a better match with the single support assumption than the free rail and the multiple layer support assumptions in predicting lateral rail radiation ratios. This is illustrated in Fig. 5.10. The radiation ratio calculated by the analytical line source model is around 5 dB higher than the multiple layer supported rail between 60 and 200 Hz and then is around 3 dB lower between 300 and 800 Hz, which also means that the predicted rail noise radiation has the same over- or under-estimation at these frequencies. However, the fluctuations occurring at frequencies above 700 Hz are not captured by this analytical model due to its simplified assumption of rigid body movement.
Fig. 5.10. Rail radiation ratios calculated under a point force excitation on the rail head side surface: ----- free rail model; --- rail model with a single layer of support; -- rail model with multiple layers of support; --- oscillating line source model with an equivalent radius of 68 mm.

5.6 Rail radiation directivity

5.6.1 Rail directivity predicted by WBE rail model

Rail radiation directivity factors were calculated at 72 different angles around the rail with a span of 5 degrees between two consecutive points. Those points are distributed on a circle of 1.5 m radius surrounding the rail. The results obtained for the free rail assumption under vertical point excitation and under lateral point excitation are shown in Fig. 5.11 and Fig. 5.12, respectively. The calculated directivity factors below 1 kHz depend only on position and are independent of the frequency. Above 1 kHz, directivity factors fluctuate significantly. The fluctuation is caused by rail profile deformation or different parts of rail vibrating in or out of phase. The directivity factor curve at the bottom of Fig. 5.12 corresponds to a position along the symmetrical axis of the rail profile.
Rail radiation directivity patterns calculated at 50 Hz, 500 Hz, 1 kHz and 3 kHz under vertical and lateral point excitation are shown Fig. 5.13 and Fig. 5.14, respectively. Both figures show that at low frequencies, rail radiation resembles a dipole source. At higher frequencies, rail radiation directivity patterns become more complex because of the increasingly complex rail profile deformation. Fig. 5.13 and Fig. 5.14 imply the validity of an oscillating line source model for both vertical and lateral rail sound radiation but the validity is limited to below 1 kHz.

The points along the symmetric rail axis always has a minimum at every frequency shown in Fig. 5.14 which confirms the conclusion made earlier that the directivity factor curve at the bottom of Fig. 5.12 corresponds to a position along the symmetric axis of the rail.
Fig. 5.12. Rail directivity factors obtained under a point force excitation on the rail head side surface using a free rail model at four positions: —— 0˚; — — 30˚; — — — 60˚; — — — 90˚, and 1.5 m away from the bottom centre of the rail foot.

Fig. 5.13. The free-rail vertical radiation pattern at four frequencies: (a) 50 Hz; (b) 500 Hz; (c) 1 kHz; (d) 3 kHz.
The directivity factors calculated for the two supported rail assumptions are shown in Appendix E. Conclusions similar to those of the free rail radiation directivity can therefore be drawn for the supported rails, i.e. radiation directivity factors are independent at frequencies below 1 kHz. Large fluctuations occur above 1 kHz due to the significance of rail profile deformation.

Directivity patterns of supported rails which are close to those obtained from a free rail are not shown. The rail resembles a line dipole source at low frequencies and the directivity pattern becomes more and more complex as the frequency increases.

![Figure 5.14](image)

Fig. 5.14. The free-rail lateral radiation pattern at four frequencies: (a) 50 Hz; (b) 500 Hz; (c) 1 kHz; (d) 3 kHz.

5.6.2 Comparison with analytical line source models

To further validate the current rail radiation model, the directivity factors obtained can be compared with those obtained from the analytical line source models. The directivity factor obtained from an oscillating line source model is \(2\cos^2 \theta\) where the definition of \(\theta\) is shown in Fig. 5.15. The directivity factor derived analytically depends only on position. For a particular position, it is constant.
Fig. 5.15. Definition of $\theta$ in an oscillating line source model.

Comparisons of directivity factors obtained from the analytical line source model and from the three WBE rail models under vertical rail point excitation and lateral rail point excitation are shown in Fig. 5.16 and Fig. 5.17, respectively. Four angles are shown for each direction. The definition of the angles relative to the rail has been given in Fig. 5.3.

Fig. 5.16. Vertical rail radiation directivity factors obtained from analytical model and WBE model: --- free rail; ----- rail of a single layer of support; --------- rail of multiple layers of support; ----- oscillating line source model.
Fig. 5.16 shows that vertical rail sound radiation directivity corresponds well with the prediction of the analytical line source model below 600 Hz for most of the angles under investigation. Significant differences occur for sound frequencies above 600 Hz due to the rail profile deformation. At the angle ($\theta = 10^\circ$) close to the axis which is normal to the dipole axis, (i.e. along the rail foot for vertical rail sound radiation), analytical and WBE rail models differ from each other for about 1.5 dB (re 1) but the magnitude of the directivity factor at this angle becomes insignificant when compared to the other three angles (more than 10 dB less) which means generated sound level at $\theta = 10^\circ$ is significantly lower than at the other three angles. Differences among the three WBE rail models is insignificant above 50 Hz for all of the angles shown, which indicates the insignificance of rail support on the calculation of rail directivity factors. In general, the analytical line source model cannot adequately model the vertical rail directivity factor above 600 Hz because of the rigid
body assumption. The influence of rail support is insignificant on the calculation of vertical rail directivity factors.

Fig. 5.17 shows that the numerical and analytical models give close predictions below 1 kHz. The difference between them increases with the increase of $\theta$ but remains insignificant (<2 dB). Fig. 5.17 also demonstrates the inadequacy of the analytical model in predicting rail directivity factors above 1 kHz. The WBE results exhibit large fluctuations with sound frequency due to the rail profile deformation which cannot be predicted by the analytical model. The three WBE rail models provide similar results above 100 Hz. In general, analytical line source models do not adequately represent the lateral rail directivity factor above 1 kHz.

5.7 Conclusion

A rail model based on the WBE method has been developed to investigate rail radiation characteristics. Assumptions made are that: 1) the interaction between air and rail is negligible and 2) rail cross-section is uniform in the longitudinal direction. It has been found that a rail radiation model based on the WBE method provides the possibility of including the effects of complex rail profile shape deformation and the rail support characteristics by coupling with a SAFE rail vibration model. This method is based on the wave superposition principle. This WBE rail model obtains the boundary conditions, i.e. rail surface velocity, from the outputs of the SAFE rail vibration model. By the use of this model, the effects of rail support on rail radiation characteristics have been investigated by comparing 3 rail models: a free rail model, a rail model with a single layer of support and a rail model with multiple layers of support. Both rail directivity factors and directivity patterns have been determined for each of these three assumptions. The efficiency of the analytical line source model in representing the rail radiation ratio and the directivity has also been analysed. The findings can be summarized as follows:

1) The calculated rail radiation power is significantly affected by assumptions about the rail support below 1 kHz. The influence of the support is more significant on rail radiation power calculated under vertical point excitation than calculated under lateral point excitation. This influence reduces with increasing vibration frequency.

2) Rail radiation ratio is less affected by the assumed rail support properties, especially when calculated under vertical point excitation. Under lateral point excitation, the effects of the rail support have been found to be significant below 700
Hz. Analytical radiation ratios calculated using an oscillating cylinder line source model are close to those calculated by the WBE rail models under vertical point excitation. Under lateral point excitation, prediction of radiation ratio by the analytical model is inadequate and can lead to an overestimation of rail noise by up to 4 dB below 200 Hz and an underestimation of rail noise by up to 3 dB between 300 and 800 Hz.

3) Rail radiation directivity patterns resemble oscillating line sources below 1 kHz but have more complex shapes above 1 kHz due to increasingly complex rail profile deformation under either vertical or lateral point excitation.

4) The analytical line source model which depends only on the observation position is not as suitable for calculating vertical rail directivity factors for sound frequencies above 600 Hz, as WBE models which accommodate more accurate assumptions for rail profile and rail support properties because the latter takes into account both position and frequency. The influence of rail support has been shown to be insignificant on the calculation of vertical rail directivity factors.

5) Similarly, the analytical line source model does not adequately represent lateral rail directivity factors for sound frequencies above 1 kHz. The influence of rail support on the calculation of lateral rail directivity factors above 100 Hz is insignificant.

This work implies that rail radiation ratio and radiation directivity factors have to be determined on a case to case basis by using the rail model including the support effects, especially below 1 kHz. Above 1 kHz, the two parameters determined under a free rail model are acceptable.
6 TRACK NOISE RADIATION AND TOTAL EFFECTIVE ROUGHNESS ESTIMATION WITH UPGRADED PARAMETERS

6.1 Introduction and chapter outline

In Chapter 3, track noise contribution and wheel/rail combined roughness are determined using measured rail and sleeper vibration. For track noise calculation VTN is used, which represents rail as a line source in both vertical and lateral directions. This apparently ignores the complex shape of rail cross-section and its deformation at higher frequencies. Janssens’s method is used to determine wheel/rail combined roughness. The parameters used are those given by Janssens et al. (2006a) but the accuracy of these parameters has been not fully investigated.

This chapter provides an upgrade of the parameters involved in the rolling noise separation and roughness estimation method with more accurate values. For noise separation, those parameters include rail radiation ratio and rail radiation directivity. For the roughness estimation, the parameters relevant are A1, A2 and A4. Definitions of A1, A2 and A4 have been given in Chapter 2 and those definitions comply with those used by Janssens et al. (2006a).

Under this context, rail vibration and radiation models based on the SAFE method and the WBE method have been developed. Preliminary results from trials of the two methods have been presented in Chapter 4 and 5, respectively. These models can be used to determine parameters characterising rail vibration and radiation. By allowing for complex rail cross-section shape and the inclusion the rail supports, these models are expected to give more accurate parameters than those used in the VTN and Janssens’s methods. The required track properties for the two models, such as rail pad and ballast stiffness, are determined from a field test by tuning the TDR predicted by the Timoshenko beam-based rail model to the measured values, see Chapter 4.

This chapter will apply the developed SAFE rail vibration model and WBE rail radiation model to determine the parameters required to predict rail noise radiation and wheel/rail combined roughness. The performance of the noise and roughness estimation methods with upgraded parameters will be evaluated.

The radiation ratio and radiation directivity factor used for the rail noise calculation are presented in Section 6.2 and Section 6.3 successively. The parameters used for wheel and rail combined roughness estimation are presented in Section 6.4 and
Section 6.5. The noise separation results and roughness calculation results, with the use of the upgraded parameters, in the corresponding methods are presented in Section 6.6 and Section 6.7, respectively.

### 6.2 Radiation ratios

![Graph of radiation ratios](image)

Fig. 6.1. One third octave spectra of rail radiation ratios: --- determined under lateral point excitation on the rail head side surface, --- determined under vertical point excitation on the top of rail head.

Fig. 6.1 shows the rail radiation ratios obtained by using the WBE based rail model developed in Chapter 5, under vertical and lateral point excitations. The effects of the support assumed can be seen for frequencies below 400 Hz noting that the radiation ratio is not increasing linearly with the increase in frequency as would be the case for a line source model. Variation of the radiation ratio above 1 kHz is also observed due to the effect of rail cross-section deformation, i.e. different parts of the rail cross-section vibrate in or out of phase which causes their contribution to the sound measured at an observation point, relative to each other, to either be constructive or destructive, as has been discussed in Section 5.5.
6.3 Directivity factor

The obtained rail directivity factors are shown in Fig. 6.2, obtained by using the supported WBE rail model for both of the rails, at a position of 7.5 m away from the track centreline and 1.2 m above rail head in order to conform to the measurement setup of a microphone used during field tests. The results are obtained for each rail under both vertical and lateral point excitations at the positions shown in Fig. 5.1.

![Diagram showing rail radiation directivity factors](image)

**Fig. 6.2.** Rail radiation directivity factors. The terms ‘near’ and ‘far’ are used to denote the relative distance of a rail with respect to the microphone position during measurement.

In contrast with the analytical line source model the directivity factors shown in Fig. 6.2 vary with frequency in addition to angle, especially above 800 Hz. This is also a result of rail cross-section deformation which cannot be predicted by a line source model based on a rigid beam, as has been pointed out in Section 5.6.2.

6.4 Vibration difference between rail head and rail foot $A_l(f_c)$

The calculated $A_l(f_c)$ using the SAFE rail vibration model is shown in Fig. 6.3, based on Eq.(2.23). Rail head and foot centre vibration is almost the same from 0 up to 1 kHz except for the large trough around 400 Hz which is thought to be caused by minor rail cross-section deformation induced by near-field waves travelling along the
rail. Above 1 kHz the rail head has a much higher vibration displacement than the rail foot. This contradicts the results reported by Janssens et al. (2006a) that vibration at the rail head is the same to the vibration at the rail foot up to around 4 kHz. It also indicates that by setting $A_l(f_c) = 0$ dB up to 4 kHz as assumed by Janssens et al. (2006a) may cause an overestimation of the estimated roughness spectra of up to 15 dB as shown in Fig. 6.3 at the frequency of 4 kHz.

![Graph showing vibration difference $A_l(f_c)$ between rail head and rail foot](image)

**Fig. 6.3.** Vibration difference $A_l(f_c)$ between rail head and rail foot calculated under a vertical point excitation at the centre of rail head.

### 6.5 The extent of rail vibration excited by wheel/rail surface roughness $A_2(f_c)$

#### 6.5.1 Wheel FE model

Rail vibration depends on the interaction of rail with the wheel. The vibration characteristics of the wheel therefore have to be known. The wheel shape widely operated through the field study site has a straight web and a diameter in the range of 940 to 860mm. The dimensions and the discretization details adopted for the wheel are shown in Fig. 6.4.
To determine the wheel vibration response under a point excitation at the nominal contact point, i.e. 70 mm from the flange, the free vibration modes have been determined up to 10 kHz. The mode superposition method (Petyt, 1989, Section 9.3) can be used to calculate the wheel response at any point. A single wheel which has the displacements constrained at the inner edge of the hub to account for the effects of wheel axle has been used for modelling. Modelling of the full wheel-set is not necessary because the flexible wheel modes are not influenced by the axle connection as pointed out by Thompson et al. (2009, Section 4.3). However, to get the forced wheel frequency response, the inclusion of wheel-set rigid modes (i.e. modal frequency is zero or very close to zero) is required (Thompson et al., 2009, Section 4.3). As a result, a separate wheel-set model was built to calculate wheel-set rigid body modes.

One important parameter affecting wheel response is the damping assumed for each mode which can significantly reduce wheel frequency response near the resonant frequency. Determining wheel modal damping can be achieved by conducting a modal test. However, Thompson et al. (2009, Section 4.2) pointed out that exact damping ratios are not critical in determining wheel vibration responses. Instead,
they gave a list of alternative nominal damping values. These values are assigned to each mode according to its nodal diameter number which is denoted as n. A nodal diameter is composed of a series of static points, for each mode, along the wheel radial direction. The values given by Thompson et al. are: 1) $10^{-3}$ for modes with $n=0$, 2) $10^{-2}$ for modes with $n=1$ and 3) $10^{-4}$ for modes with $n\geq2$. The damping ratio is set to 1 for rigid wheel-set modes.

The stationary wheel has been assumed for the FE modelling, i.e. wheel rotation effects are neglected. Considering wheel rotation will lead to a split of the wheel response at the resonance frequencies when viewed in a non-rotating frame such as a static point on the rail. This effect however is hard to notice if the audible sound is measured (Thompson, 1993e).

Wheel and rail contact is assumed to be a spring (Thompson et al., 1996b). The stiffness of the contact is determined by wheel and rail dimensions and their physical properties in addition to the load condition. The procedures to determine contact stiffness have been given by Thompson et al. (2009, Section 5.3).

### 6.5.2 Calculated $A_2(f_c)$

The calculated wheel, rail and contact mobilities are shown in Fig. 6.5. Wheel mobility is calculated at the nominal contact point of 70 mm from the flange. Rail mobility is calculated at the rail head centre under a vertical excitation force at the same point. The contact mobility was obtained using the following vehicle and track conditions: wheel rolling radius $R_w = 470\, \text{mm}$, wheel lateral radius $R_{wt} = \infty$, rail lateral radius $R_{rt} = 300\, \text{mm}$, rail longitudinal radius $R_r = \infty$, steel Young Modulus $E = 210\, \text{Gpa}$ and the Poisson’s ratio $\nu = 0.3$. The load was assumed to be 50 kN, a typical load condition for a passenger vehicle.
Fig. 6.5. Vertical mobility of contact spring ——, wheel ——- and rail ———.

Fig. 6.6. Narrow Band Spectra of A2.
The narrow band spectra of $A_2(f_c)$ are shown in Fig. 6.6, calculated using Eq. (2.24). As shown in Fig. 6.6, wheel mobility dominates $A_2(f_c)$ to around 20 Hz, from which and upwards till 900 Hz, rail mobility is dominating. Above 900 Hz, the contact mobility is dominating except at the wheel resonant frequencies where wheel mobility is the most important component. As a result, rail vibration is heavily influenced by the wheel and contact.

### 6.6 Validation of VTN with upgraded parameters

#### 6.6.1 Direct validation

![Diagram](image)

**Fig. 6.7.** Measured - calculated SPL. —— mean value calculated by VTN without upgraded rail radiation parameters with the light grey region indicating the standard deviation; ——— mean value calculated by VTN using upgraded rail radiation parameters with the dark grey region denoting the standard deviation.

The difference between the track noise, measured 2 m away from the track centreline, and the calculated noise level, using VTN with upgraded parameters including rail radiation ratio and rail radiation directivity factors, is shown in Fig. 6.7. It does not indicate any reduction in apparent error when using the upgraded parameters with a possible increase in the error above 3 kHz. A comparison of A-
weighted SPLs, indicated by the different methods with the measured values from the field testing, as presented in Fig. 6.8, indicates that the mean value using the upgraded parameters is 3.2 dB(A) which is higher than the 2.3 dB(A), obtained by VTN without using the upgraded parameters.

This is due to the predicted rail contribution being reduced as indicated in Fig. 6.7. Although the difference between the measured track contribution and the calculated track contribution using VTN seems to be increased after using the upgraded rail radiation parameters, the measured track contribution may have been over indicated because of the influence of the background noise as discussed in Section 3.5.3. The near field effect due to deployment of a microphone very close to the track (2 m) may also contribute to the large variation between measured and calculated SPA shown in Fig. 6.8. Therefore, this direct comparison of the field measurement results with those estimated using the VTN method, with upgraded parameters, is inconclusive.

![Graph](image)

Fig. 6.8. A-weighted SPL: measured versus calculated. (a) VTN without using upgraded rail radiation parameters; —— mean, 2.3 dB(A); —— standard deviation, 1.2 dB(A), replicated from Fig. 3.12. (b) VTN using upgraded rail radiation parameters; —— mean, 3.2 dB(A); —— standard deviation, 1 dB(A).

It is noted that in Fig. 3.12 and Fig. 6.8, there are some points well above the general trend line. A review conducted by listening to recording of the sound tends to indicate that this was not caused by unusual conditions of the wheel, such as the presence of wheel flats. A possible reason is the stronger near-field effect caused by the relative higher speed for these particular trains, although this cannot be verified.
6.6.2 Considering the results of noise separation for specific train pass-bys

The calculated rail and sleeper noise contributions from four train pass-bys are shown in Fig. 6.9 with and without the upgraded rail radiation ratios and radiation directivity factors. The measured environment effect has been included in the calculated rail and sleeper contributions. The estimated vehicle contribution is also shown. When compared with the VTN method without upgraded parameters, the overestimation of rail contribution, for VTN with upgraded parameters, at frequencies in the vicinity of 1 kHz is eliminated. This comparison indicates a significant improvement in the VTN estimates when using the upgraded parameters as the overestimation of the rail contribution has been eliminated, at least for all of the available observations. Further trials are required to verify this result. This finding is significant as noise components in this frequency band influence the A-weighted noise level disproportionately. This argument may be tested by Fig. 6.8 where the calculated overall noise level is very close to the measurement.

![Graphs showing sound pressure level vs frequency](image)

Fig. 6.9. Rolling noise separation examples calculated using upgraded rail radiation parameters.
6.7 Roughness estimation results

The calculated wheel and rail total effective roughness from pass by measurements with and without the upgraded parameters are shown in Fig. 6.10. Also shown is the total effective roughness obtained from direct measurement for comparison. As shown, calculated roughness with upgraded parameters have more data fallen into the region covered by the direct measurement than the calculated roughness without the upgraded parameters. However both of the spectra are higher than the roughness spectra from direct measurement at wavelength below 3.15 cm.

![Fig. 6.10. Wheel/rail total effective roughness. — mean value from direct wheel and rail roughness measurement. Grey region: standard deviation from direct wheel and rail roughness measurement. —— first pass-by measurement without using upgraded A1, A2; -- -- second pass-by measurement without using upgraded A1, A2; —○— first pass-by measurement using upgraded A1, A2; —∇— second pass-by measurement using upgraded A1, A2.]

6.8 Conclusion

Rolling noise can be separated into vehicle and track radiated noise contributions utilizing measured rail and sleeper vibration. Using the vibration data, wheel and rail combined total effective roughness can also be calculated.
The parameters used for the rolling noise separation, in particular radiation ratios and radiation directivity factors as determined by using a WBE rail radiation model provide improved results for the allocation of total radiated noise to individual components. The rail vibration parameters used for wheel and rail total effective roughness calculation as determined from a SAFE rail vibration model also improves the results of the calculated total effective roughness.

Specifically it has been found that:

1) The overestimation of rail contribution, associated with the application of VTN using established line source models, is avoided by using the upgraded radiation ratios and radiation directivity factors derived by using a WBE modelling approach which allows for the more accurate rail cross-section and for three-dimensional radiation characteristics of the rail.

2) The estimated wheel and rail total effective roughness is also closer to those from direct measurement with the upgraded parameters calculated using a SAFE rail vibration model which includes more accurate assumptions for rail cross-section deformation characteristics and for rail support arrangements, compared to the results calculated using the value in a look-up table provided by Janssens et al.(2006a).
7 CONCLUSIONS

7.1 Outcomes
This thesis aimed to address two problems related to railway rolling noise measurement and management: the separation of wheel and track contribution to the rolling noise; and, the quantification of wheel and rail surface roughness.

A review of current methods finds that the rolling noise separation and wheel/rail roughness estimation are proposed, being based on the coincident measurement of overall noise and rail and sleeper vibration. While VTN was identified as a feasible method for noise separation, and, Janssens’s method for roughness estimation, neither has been found to have been adequately validated.

Field trials of these methods, using the established modelling approaches that have been employed, indicate that:

1) the existing VTN method is able to estimate the overall rolling noise level with an accuracy of 2.3 dB when compared with direct measurement using a microphone. The variation over different frequency spectra however can be as large as 20 dB, particularly at low frequencies. These variations are contributed by measurement errors as well as inadequacies in the models used;

2) inaccuracies in the VTN method, as currently applied, are demonstrated by overestimates obtained for radiated rail and sleeper noise, which manifest as estimated rail or sleeper noise levels exceeding the total measured noise levels for some frequencies; and,

3) roughness separation trials indicate that Janssens’s method is not effective in estimating wheel/rail total effective roughness, with estimation errors of 10 dB for some wavelengths.

The modelling issues that contribute to the inadequacies in these approaches include:
1) VTN assumes rail behaves like a line source in both vertical and lateral directions when it radiates. This apparently ignores the complexity of the rail cross-section shape and the deformation of the rail cross-section at high frequencies, because line source model assumes uniform vibration along the rail; and 2) Janssens’s method uses parameters whose accuracy is unclear.

A rail vibration model, developed based on the so-called SAFE method, assuming vibration propagating along the rail longitudinal direction as waves and the
conventional FE method allows determination of the rail vibration characteristics in the cross-section plane. Improved from previous work, such as Nilsson et al. (2009) and Bartoli et al. (2006), this model can include various types of support into the rail model. This represents a more assumption while remaining computationally feasible. By utilising the SAFE method for three rail support assumptions: a free rail; a rail of a single layer of support; and, a rail with multiple layers of support and tuning TDRs predicted by the Timoshenko beam-based rail model to the measured values, such as rail pad and ballast stiffness, wave propagation characteristics along the rail and forced rail responses can be determined. In addition, the rail support has been shown to have a significant impact on the prediction of rail dispersion and vibration response for frequencies below 1 kHz. The Timoshenko beam-based rail model has also been shown to be inadequate in predicting rail vibration response, especially the lateral rail vibration.

Application of WBE has been demonstrated to adequately represent rail sound radiation. Based on this model, the two most important sound radiation parameters, i.e. rail radiation ratio and radiation directivities can be determined for all of the three rail support conditions investigated. The results show that the influence of the assumed support model:

1) is significant in determining the rail radiation power;
2) is more significant on the rail radiation ratio calculated under a lateral excitation than the ratio calculated under a vertical excitation; and,
3) has an insignificant impact on the calculation of rail radiation directivities.

In addition, the analytical line source model is inadequate in predicting rail radiation ratio under lateral excitation and in predicting rail directivity.

The rail vibration and radiation model developed can be used to calculate the parameters required for both the VTN and Janssens’s methods. Wheel and contact frequency responses can be calculated to obtain the ratio of rail receptance to the total of wheel, rail and contact receptances. Trials of the VTN and Janssens’s methods with the parameters calculated using the SAFE and WBE rail models indicate that:

1) the overestimation of rail contribution is avoided by using the upgraded radiation ratios and radiation directivity factors;
2) the estimated wheel and rail total effective roughness is closer to those from direct measurement when compared to the results calculated without using these upgraded parameters.

7.2 Contributions and their possible uses

The following are contributions of this work:

1) A SAFE model has been developed and applied for modelling of a rail with the possibility of representing multiple layers of rail support. This results in more accurate representation of the practical track condition than has so far been attempted.

2) A WBE radiation model has been developed and applied to calculate the rail directivity in the rail cross sectional plane which accommodates deformation of the cross-section and three-dimensional vibration behaviour. This has significant implications for single-microphone based measurement of railway rolling noise components. This model also allows the investigation of rail support designs in order to optimise noise performance.

3) As demonstrated, these models can be used within the VTN method and roughness estimation to more accurately determine the contributions to rolling noise by the wheel and the track. This can result in more effective deployment of mitigation methods: such as rail grinding and wheel refurbishment, directed at controlling rolling noise on a rail network.

4) The models developed can be adopted by noise prediction software to improve their accuracy in predicting railway rolling noise for specific frequencies and for determination of specific component contributions to noise radiation.

5) These models can be used to more accurately investigate the design of rail dampers and low profile noise barriers.

7.3 Limitations of this work

The field trials undertaken were limited in several respects due to the difficulties in both arranging and controlling such trial which necessarily were required to be conducted on mainline track with revenue service trains. To complicate this, the line made available was an electrified line with restrictions imposed on the method of mounting sensors, particularly accelerometers in this case, onto the rails. Uncertainties in the acceleration measurement resulting from accelerometer
mounting arrangements have been discovered through subsequent laboratory trials of various accelerometer mounting arrangements. These uncertainties will have had implications for the attempts at validating the various methods examined in this work. Subsequent trials with more dependable mounting arrangements were not possible. Irrespective of this, evidence of the benefits of utilising the more accurate models proposed in this work was found. Certainly however further field work is required in order to validate the findings with more certainty.

It is also noted that direct measurement of track noise is not possible. For the purpose of comparison/validation, an approximate method (based on MISO) has been used to investigate the track noise spectra produced by the upgraded method. Differences between the upgraded method and the results of the approximate method do not necessarily mean that the upgraded method is wrong or inaccurate.

7.4 Further Work

The following work is proposed as the next steps in this research:

1) The SAFE rail vibration model developed can only account for a continuous rail support. The rail is actually supported on discrete rail pads attached to sleepers or other structures on a conventional track. The effects are expected to be included in the current model. The corresponding rail radiation model for the discretely supported rail model should also be investigated.

2) More extensive and reliable field testing should be used to validate the new VTN method.

3) To improve the accuracy of noise separation for low frequencies (below 500-1000Hz, depending on the rail pad stiffness), a more accurate sleeper radiation model should be sought for calculating sleeper noise contribution.
REFERENCES


Sheng, X., Jones, C. J. C. & Thompson, D. J. 2005b. Responses of infinite periodic structures to moving or stationary harmonic loads. *Journal of Sound and Vibration* 282, 125-149.


APPENDIX A CONTACT FILTER

According to classical Hertz contact theory, the semi-axis $a$ and $b$ of wheel/rail contact area are given by (Thompson, 2003a).

$$a = \sigma_1 \left( \frac{3P_{ce}}{2E} \right)^{1/3}, \quad b = \sigma_2 \left( \frac{3P_{ce}}{2E} \right)^{1/3} \quad (A.1)$$

where $P$ is the normal load, $E^* = E / (1 - \nu^2)$ is the plane strain elastic modulus with $E$ the Young’s modulus and $\nu$ the Poisson’s ratio, and $r_e$ is given by, e.g., Thompson (2003a),

$$r_e = \frac{1}{2} \left( \frac{1}{R_w} + \frac{1}{R_{wt}} + \frac{1}{R_r} + \frac{1}{R_{rt}} \right) \quad (A.2)$$

where $R_w$ and $R_{wt}$ are wheel circumferential and tangential radius and $R_r$ and $R_{rt}$ are the rail longitudinal and lateral radius.

$\sigma_1$ and $\sigma_2$ are also dependent on $\cos \theta$ which is given by (Thompson, 2003a),

$$\cos \theta = -\frac{r_e}{2} \left( \frac{1}{R_w} - \frac{1}{R_{wt}} + \frac{1}{R_r} - \frac{1}{R_{rt}} \right) \quad (A.3)$$

Values of $\sigma_1$ and $\sigma_2$ have been tabulated, e.g., by Thompson (2009, Section 5.3).

Remington (1976b) developed an analytical formula for Hertz contact. Assuming a circular contact area of radius $b$, the contact filter is given by

$$|H_f(k)|^2 = \frac{4}{\alpha (kb)^2} \frac{1}{\tan^{-1} \alpha} \int_0^\infty J_1^2 (kb \sec \Psi) d\Psi \quad (A.4)$$

Where $k$ is wave-number of propagating waves in the rail, $\alpha$ is a constant determining the degree of correlation between parallel roughness profiles at a given wave-number, $J_1$ is the Bessel function of first order, $\Psi$ is a variable.

The equivalent radius $b$ is given by

$$b = \sqrt{\sigma_1 \sigma_2 \left( \frac{3P_{ce}}{2E^*} \right)^{1/3}} \quad (A.5)$$
## APPENDIX B FIELD TEST INSTRUMENTS

Table B.1 Equipments used in site 1 and site 2.

<table>
<thead>
<tr>
<th>DAQ Channel</th>
<th>Site 1</th>
<th></th>
<th></th>
<th>Site 2</th>
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<th></th>
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<td>Desc.</td>
<td>SN</td>
<td>Pos.</td>
<td>Desc.</td>
<td>SN</td>
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<td>136956</td>
<td>Down rail, vertical under foot.</td>
<td>Dytran model 3055B01 uniaxial accelerometer</td>
<td>TBA</td>
</tr>
<tr>
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<td>PCB uniaxial accelerometer</td>
<td>136955</td>
<td>Down rail, lateral head field side</td>
<td>Dytran model 3055B01 uniaxial accelerometer</td>
<td>TBA</td>
</tr>
<tr>
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<td>Up rail, vertical under foot.</td>
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<td>9448</td>
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<tr>
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<td>PCB model 354C03 triaxial accelerometer</td>
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<td>PCB model 378B02 ½” microphone</td>
<td>LW7832</td>
<td>Up Cess, 2m from track centre, 0m above top of rail</td>
<td>B&amp;K type 4189 ½” microphone</td>
<td>2386084</td>
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<td>PCB model 378B02 ½” microphone</td>
<td>LW7833</td>
<td>Up Cess, 7.5m from track centre, 1.2m above top of rail</td>
<td>B&amp;K type 4189 ½” microphone</td>
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<td>LW108024</td>
<td>Up Cess, 7.5m from</td>
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<td>microphone</td>
<td>track centre, 1.2m above top of rail</td>
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<td>5m from microphones</td>
<td>SICK model WL-34-V540 wheel sensor</td>
<td>1126</td>
</tr>
</tbody>
</table>

The DAQ system used at site 1 is National Instruments CompactRio 9024 and at site 2 is EDAQ system.

In addition to the above equipment, the following are used:

- 6x weather protection kits for microphones
- 6x tripods
- 4x reflector sets for wheel sensors
- 10x perspex discs for threaded mounts
- 10x threaded mounts for accelerometers
- 10x cables from each accelerometer to DAQ
- 2x battery system for DAQ to power continuously 8 days
APPENDIX C WHEEL ROUGHNESS MEASUREMENT DEVICE

System requirements
The device was developed to:
1) be able to measure wheel roughness at one position on wheel tread surface each time
2) have a resolution of better than 1µm as roughness responsible for rolling noise generation has amplitudes in the range from tens of microns at long wavelengths to less than a micron at short wavelengths.
3) allow for lateral movement to enable measurements across the rung band on a wheel
4) be able to have a sample distance no longer than 0.5mm as typical wavelengths of roughness relevant to rolling noise are between about 5 and 500mm.
5) be low cost, portable and self-contained.

Design methodology
Figure C.1 shows the design methodology. A LVDT sensor is used to contact and measure wheel surface roughness while the wheel is rotated by hand. An encoder is used to measure the displacement the wheel has been rotated over. Instead of installing the encoder on wheel axle, a small wheel was installed on the encoder shaft to contact wheel surface and rotate together with the wheel. For each revolution, the encoder will output a certain amount of pulses, which can be related to the displacement the wheel rotating over. Then the outputs from both the encoder and the LVDT sensor are sent to the DAQ where they can be saved for further data analysis.

The real set up in commissioning the device is shown in Figure C.2. As shown, the groove on the base plate enables the lateral movement of the LVDT. The thread near the LVDT sensor enables adjustment of vertical movement of the LVDT. The stud fixing the device where LVDT sensor is mounted enables the rotation of the LVDT. This vertical and rotation movement of LVDT enable a close contact between LVDT sensor and the wheel surface. The lateral movement enables the measurement of parallel lines on the wheel surface.

In addition, the mechanical manufacturing is in house work and materials are all very low cost.
Figure C.1 Diagram of wheel roughness measurement device.

Figure C.2 Designed wheel roughness measurement device.
APPENDIX D DERIVATION FOR WBE METHOD

Assuming \( r \) is distance between two points \( P(x_P, y_P) \) and \( Q(x_Q, y_Q) \), i.e.

\[
    r = \sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2}
\]

(Q is a point on the boundary \( \Gamma \) of the domain \( \Omega \). P is the source point to be evaluated.

Derivative of kernel \( p^* \) with respect to \( x \) is given by

\[
    \frac{\partial p^*}{\partial x_P} = \frac{ik}{4} H^{(2)}_1(kr)r_x
\]

by using the relation (Olver et al., 2010, 10.6.3)

\[
    H^{(2)\nu}(z) = -H^{(2)}_1(z)
\]

Derivative of kernel \( q^* \) with respect to \( x_P \) is given by

\[
    \frac{\partial q^*}{\partial x_P} = \frac{ik}{4} H^{(2)}_1(kr) \left[ \frac{1}{kr} H^{(2)}_0(kr) \left( \frac{r_{x_Q} n_{x_Q} + r_{y_Q} n_{y_Q}}{r} \right) - \frac{r_{x_P} r_{y_P}}{r} \right]
\]

Use of the relation (Olver et al., 2010, 10.6.2)

\[
    H^{(2)\nu}(z) = H^{(2)}_0(z) - \frac{1}{z} H^{(2)}_1(z)
\]

is made.

Derivation of the derivatives of kernels \( p^* \) and \( q^* \) with respect to \( y \) is similar.
Fig. E1. Rail directivity factors calculated under a vertical point excitation on the top of rail head using a rail model of a single layer of support at four positions: —— 0°; — — 30°; — — — 60°; ——— 90°, and 1.5 m away from the bottom centre of the rail foot.
Fig. E2. Rail directivity factors calculated under a lateral point excitation on the rail head side surface using a rail model of a single layer of support at four positions: ——— 0˚; ----- 30˚; ----- 60˚; ——— 90˚, and 1.5 m away from the bottom centre of the rail foot.
Fig. E3. Rail directivity factors calculated under a vertical point excitation on the top of rail head using a rail model of multiple layers of support at four positions: —— 0°; — 30°; —— 60°; —— 90°, and 1.5 m away from the bottom centre of the rail foot.
Fig. E4. Rail directivity factors calculated under a lateral point excitation on the rail head surface using a rail model of multiple layers of support at four positions: —— 0°; —— 30°; —— 60°; —— 90°, and 1.5 m away from the bottom centre of the rail foot.