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Performance Analysis of PowerWindow: a Linear Wind Generator

Seyed AmirHosein Jafari

University of Wollongong
UNIVERSITY OF WOLLONGONG

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School of Electrical, Computer and Telecommunications Engineering
Faculty of Engineering and Information Sciences

Performance Analysis of PowerWindow: a Linear Wind Generator

Seyed AmirHosein Jafari

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ABSTRACT

Linear wind generators (LWGs) are a new type of wind turbine developed recently. Unlike the conventional horizontal or vertical axis wind turbines, the power extraction mechanism of linear wind generator is based on the translational movement of blades along a path perpendicular to the incoming wind. This study focuses on a LWG design developed at the University of Wollongong, named PowerWindow, which represents a modular wind power generator, and investigates its characteristics and performance.

The thesis develops an analytical model for PowerWindow using the modified blade element momentum theory. It also develops a simulation of this device using the computational fluid dynamic method. It is envisaged that in practice PowerWindow modules can be installed and operated in two different positions, either suspended on a frame or landed on ground. The suspended configuration means that the module is placed in an elevated position for example, between two towers, and landed configuration means that the module is placed on a flat surface, for example, the roof of a tall building. Aerodynamic mechanisms of PowerWindow in both positions are analyzed and compared using the modified blade element momentum theory and the computational fluid dynamic model, and also validated by the experimental data obtained from the prototype tests in the wind tunnel.

This study shows that the PowerWindow turbine can operate with acceptable efficiency in very low blade to wind velocity ratio, which is not achievable by conventional wind turbines at the same value of tip speed ratio. It is shown that installation position (suspended or landed) greatly affects its performance. This study
also shows that the front blades can significantly impact on the performance of the rear blades, by increasing their angle of attack. Increasing the angle of attack also increases the possibility of stall. However it is also shown that stall condition can be postponed by increasing the solidity.
CERTIFICATION

I, Seyed AmirHosein Jafari, declare that this thesis, submitted in fulfillment of the requirements for the award of Master of Philosophy, in the School of Electrical, Computer and Telecommunications Engineering, Faculty of Engineering and Information Sciences, University of Wollongong, is wholly my own work unless otherwise acknowledged. The document has not previously been submitted for qualification at any academic institution.
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NOMENCLATURE

\( a \) \hspace{1cm} \text{Induction factor (dimensionless)}

\( A \) \hspace{1cm} \text{Air swept area (m}^2\text{)}

\( B \) \hspace{1cm} \text{Airfoil plan area (m}^2\text{)}

\( C_D \) \hspace{1cm} \text{Drag coefficient (dimensionless)}

\( C_L \) \hspace{1cm} \text{Lift coefficient (dimensionless)}

\( C_p \) \hspace{1cm} \text{Coefficient of performance (dimensionless)}

\( C_{sp} \) \hspace{1cm} \text{Surface Pressure Coefficient (dimensionless)}

\( F_D \) \hspace{1cm} \text{Drag force (N)}

\( F_L \) \hspace{1cm} \text{Lift force (N)}

\( \dot{m} \) \hspace{1cm} \text{Mass flow rate (kg m}^{-2}\text{s}^{-1}\text{)}

\( N \) \hspace{1cm} \text{Number of blades in one side of PowerWindow}

\( P \) \hspace{1cm} \text{Power (Watt)}

\( P \) \hspace{1cm} \text{Pressure (Pa)}

\( R_v \) \hspace{1cm} \text{Velocity ratio (dimensionless)}

\( V \) \hspace{1cm} \text{Air velocity (m} \text{s}^{-1}\text{)}

\( \alpha \) \hspace{1cm} \text{Angle of attack (degree)}

\( \beta \) \hspace{1cm} \text{Effective angle (degree)}

\( \theta_0 \) \hspace{1cm} \text{Design angle (dimensionless)}

\( \varepsilon \) \hspace{1cm} \text{Flow affected ratio (dimensionless)}

\( \sigma \) \hspace{1cm} \text{Solidity (dimensionless)}

\( \rho \) \hspace{1cm} \text{Air density (kg m}^{-3}\text{)}

\( \lambda \) \hspace{1cm} \text{Linear velocity ratio (dimensionless)}
CHAPTER 1. INTRODUCTION

Wind turbines use wind kinetic energy of wind to rotate a shaft and generate electrical power. There are different designs for wind turbines but based on the orientation of the axis of rotation there are generally categorized into two groups: horizontal axis wind turbines (HAWTs); and vertical axis wind turbines (VAWTs). Coefficient of performance of HAWTs is generally higher than VAWTs. The maximum coefficient of performance of HAWTs is reported 45% to 50% while it is reported below 40% for efficient VAWTs [1]. Despite all the developments in this area many researches are still being done to develop new devices which can compete with the conventional technologies in capturing wind energy.

The challenge for developing a new device is to have high efficiency as well as being cost effective in comparison to conventional designs. Linear wind generator (LWG) is a new generation of wind turbines which are proposed for this purpose. Power extraction mechanism of LWG is based on the transitional movement of its blades over two straight lines in opposite directions perpendicular to the incoming wind. One LWG feature which makes it superior to the VAWTs and HAWTs is its ability to operate with a very low operation velocity but acceptable efficiency compared to those turbines. However very few studies have been done on optimization of LWG and enhancement of its performance; hence this device has not still been broadly utilized. PowerWindow, developed at the University of Wollongong, Australia, is a particular LWG design with some new characteristics and advantages, which is introduced and analyzed in this study.
1.1 Aims of the Thesis

PowerWindow is a new wind turbine design and many optimizations can be applied to enhance its performance. The main aim of this study is to develop a suitable analytical model for its power generation mechanism and verify this model using experimental and/or simulation methods. This model can later be used to optimize the design or evaluate various characteristics of this particular type of LWG.

1.2 Contributions of the Thesis

Three methods are developed in this study for analysis of power generation mechanism and performance of PowerWindow:

(i) An analytical model using the blade element momentum theory.

(ii) A numerical simulation using the computational fluid dynamic method.

(iii) Comparison with experimental data obtained from a wind tunnel test of the current prototype of the device.

1.3 Structure of the Thesis

The thesis is structured as follows.

Chapter 1. Introduction, describes the background, aims and contributions of the thesis.

Chapter 2. The Current Status of Research on Wind Generator Mechanisms and Performance Analysis, categorizes different wind turbines including PowerWindow based on their aerodynamics characteristics, and gives a general overview of their performance and reliability.
Chapter 3. Modified Bladed Element Momentum Theory for PowerWindow, derives an analytical model for PowerWindow using a modified blade element momentum method.

Chapter 4. PowerWindow Computational Fluid Dynamic model, describes the transitional model, mesh structure and solution method used in the computational fluid dynamic software for PowerWindow simulation.

Chapter 5. Results and Discussion, presents and analyses the coefficient of performances obtained by the analytical and numerical models for PowerWindow, validates them with the experimental data from the prototype wind tunnel tests, and compares the performance of PowerWindow in the two operating positions described in chapter 2.

Chapter 6. Conclusions, briefly discusses the accuracy of the PowerWindow analytical and numerical models and summarizes the results obtained by these models in both operation positions.

1.4 Publication Arising from the Thesis

Based on the investigations of this thesis a journal article entitled “Power Generation Analysis of PowerWindow, a Linear Wind Generator, using Computational Fluid Dynamic Simulations” has been submitted to the Journal of Wind Engineering & Industrial Aerodynamics, and is still under review. Another journal article entitled “Analysis of Aerodynamic Performance of Landed and Suspended PowerWindow using Numerical and Analytical Models” is also in the final stages of preparation.
CHAPTER 2. THE CURRENT STATUS OF RESEARCH ON WIND GENERATOR MECHANISMS AND PERFORMANCE ANALYSIS

2.1 Aerodynamics and Power Generation Mechanisms of Wind Turbines

The mechanisms of wind generators can be categorized by their aerodynamics. Exposing a flat object to an incident wind creates aerodynamics forces known as drag and lift forces. As shown in Figure 2.1, drag force is the component created parallel to the flow direction, and lift force is the component created perpendicular to the flow direction. As the aerodynamic quality of the object is better, the lift force is higher and the drag force is lower. Different aerodynamics of wind generators is explained in this part and they are categorized based on their aerodynamic mechanism.

Many studies have been done to improve aerodynamic quality of wind turbine blades. A number of commercial airfoils with different applications are shown in appendix A. Bak and Fuglsang [2], studied on enhancement of aerodynamic performance the NACA 622-415 airfoil, and as they have reported HAWTs structures are extremely dynamic, and are mostly subjected to complex distributions of aerodynamic forces. Malcolm [3] reported heavy vertical wind shear might result in motion in the vertical plane and yaw system of the rotor.

Based on the design of the turbines’ rotors they can be lift-based or drag-based. Savonius wind turbine, (among VAWTs) is categorized as a drag-based wind turbine because its rotor rotates by the drag component of the aerodynamic force. Darrieus wind turbines (among VAWTs) and HAWTs are lift-based wind turbines. Their rotors basically rotate because of the lift component of the aerodynamic force. The
analytical model of some wind turbines such as Darrieus type is quite complex. Hence computational fluid dynamics is used for their power generation prediction. However, several analytical models are developed for HAWTs.

![Aerodynamic forces over a blade (Drag and Lift components).](image)

Figure 2-1. Aerodynamic forces over a blade (Drag and Lift components).

Many studies have been done on aerodynamic of wind turbine blades. Miller [4] developed aerodynamics analysis of HAWTs and emphasized the necessity of a comprehensive design theory. Schreck et al. [5] studied three-dimensional (3D), unsteady, vortex dominated flows creating dynamic stall on HAWT rotor blades in various wind speeds and yaw angles.

2.1.1 *Horizontal Axis Wind Turbine (HAWT)*

In the HAWTs the main rotor shaft is arranged on a horizontal axis. The first windmills were vertical axis wind turbines but, later HAWTs received more attention and HAWTs are currently more popular, because HAWTs primarily have higher coefficient of performance compared to other wind turbines. The maximum coefficient of performance of a modern HAWT has been reported up to 45% to 50% [1]. In the HAWTs the rotor shaft and the electrical generator are located on the top of a tower. The rotor is not able to capture the wind energy from all direction and
should be pointed to the wind direction. Hence a special mechanism is required to turn the rotor to that direction. Similar to the other lift-type wind turbines, HAWTs are very sensitive to variation in its blade surface roughness and profile design [6].

Large scales HAWTs have a yaw system which is a component basically that is adjust the orientation of the HAWT rotor towards the direction of the wind. In small scale HAWTs, the yaw system consists of a tail with a wing mounted on its end which creates a regulator moment which turns the wind turbine rotor towards the wind direction. This yaw system is also named ‘passive yaw system’. However the yaw system in the large size HAWTs is ‘active yaw mechanism’. In this yaw systems there is a wind sensor which can sense the wind direction, and there is a servo motor which creates a torque that is required to rotate the rotor and generator above the stationary tower.

Interests in the design and development of small scale wind turbines has world widely been increased during the last decades [7, 8]. The main idea behind that might be power generation from wind in the urban built environments. The idea is underpinned by the benefits from having power generated at the point of use. Despite this significant benefit, there are technological, economical and social hurdles which undermine wind turbine installations in the urban built environments [9] such as: (i) lack of suitable area for medium-large size wind turbines; (ii) noise pollution generated from (mainly medium-large size) wind turbines in high wind velocity conditions; and (iii) relatively low power output and unreliable performance due to unfavorable urban wind conditions such as low wind energy content (low wind velocity), continuously variable wind directions, high turbulence level and strong gust occurrences.
Billinton and Guung [10] showed that the site wind condition (speed and direction) extremely affect the reliability performance of a generating system. Therefore new large wind turbines use the active yaw mechanism to orient the wind turbine rotor to the wind direction. Minimizing the yaw angle maximizes the power output and minimizes the non-symmetrical loads. However, yawing cannot make a significant reduction in high wind speeds as it can in low-to-medium speeds, because the wind direction is less variable at high wind speeds compared to low-to-medium speeds.

Figure 2-2. (a) Number of three-bladed HAWTs used in a wind farm. (b) A counter rotating HAWT.

Commercial HAWT rotors used in wind farms for electric power generation normally have three blades as shown in Figure 2.2. (a), these rotor should be pointed to the wind direction by computer-controlled motors. These rotors usually have high tip speed and efficiency, with a low torque ripple, which creates a good reliability. There are, however, limitations on how closely one can place wind turbines next to each other. This is because the rotary model creates a rotation of air flow in its vicinity and wake due to tip vortices and rotational torque imparted on air flow. The
interference from these turbulence flows will reduce the efficiency of adjacent wind turbines at close distances.

An alternative design of HAWT is counter-rotating HAWT shown in Figure 2.2 (b). This turbine has two rotors rotating oppositely on one axis. This technique assists in enhancing the maximum coefficient of performance of the HAWT. The maximum coefficient of performance of a counter-rotating HAWT without any losses is reported 64% [11]. Lee et al. [12] investigated aerodynamic characteristics of a counter-rotating HAWT using three kinds of rotor configurations (single rotor with 2 and 4 blades, and counter-rotating rotor with 2 blades on each rotor) and compared them using a numerical method. Hwang et al. [13] showed that the interactions of the front and the rear rotor creates a complex flow field in counter-rotating HAWTs. Choosing the rotational speeds, radius ratios and pitch angles of both rotors as the design parameters, optimized a counter-rotating HAWT, and observed variations of the coefficient of performances and thrust coefficients.

2.1.2 Vertical Axis Wind Turbine (VAWT)

In the VAWTs the main rotor shaft is arranged on a vertical axis. The primary advantage of the VAWTs compared to HAWTs is that there is no yaw mechanisms required for these types, which significantly simplifies their design and configurations [6]. Hence the VAWTs are more applicable than the HAWTs in mountainous areas and urban areas with extremely gusty winds. Another advantage of the VAWTs over the HAWTs is that they are less noisy than the HAWTs, which is very important for urban areas. However the VAWTs also have some main constraints compared to the HAWTs. Their TSR is basically lower than the HAWTs.
They are unable to self-start and control the output power or rotational speed by adjusting blades pitch angle [14].

VAWTs are effectively applicable on high buildings in the cities where wind speed reaches 14 m/s or greater. Similarly [15]. There are many VAWT designs, but they all have been categorized three groups: (a) Savonius VAWT, (b) Curved-blade Darrieus VAWT, and (c) Straight-blade Darrieus or H-rotor VAWT which are shown in Figure 2.3 (a), (b) and (c). Similar to the HAWTs, Darrieus (Curved-blade and Straight-blade/ H-rotor) VAWTs are lift-type wind turbines which typically have the maximum coefficient of performance from 30% to 45%. Savonius VAWTs are the only drag-type wind turbines, which its maximum coefficient of performance does not exceed 25% according to most investigators [6].

Savonius VAWT was invented by S.J. Savonius (Finnish engineer) in 1929. This type of VAWT is suitable for lower wind speeds and power applications. The greatest advantage of a Savonius rotor compared to the Lift-type VAWTs is its self-start ability [16]. Savonius VAWTs also have other advantages such as having low cost, simple construction, insensitivity to the wind direction, low angular velocity and noise in operation [17].
George Jeans Mary Darrieus (French engineer) invented the Darrieus VAWT in 1931. Although Darrieus VAWTs have the highest coefficient of performance among VAWTs, they generally have problems such as low starting torque and weak configuration structure [15]. The Eole, with 96 m height and maximum power of 3.8 MW was the largest Darrieus VAWT ever, which was built in 1986 [1]. Darrieus VAWTs have number of blades (usually aerofoil-shaped) attached to a vertical shaft. VAWTs are basically lift-type wind turbines. The wind creates an aerodynamic momentum on the blades due to the lift force and rotates them around the shaft. Gupta and Biswas [18] studied application of twisted blades in Darrieus VAWT rotor at the trailing edge. Sharpe and Proven [19] presented the idea of Cross-flex wind turbine. This idea uses an innovative configuration in the primary concept of Darrieus VAWT. From the coefficient of performance aspect the Darrieus VAWT is more desirable than the Savonius VAWT [20].

Many researches have been done on power augmentation of both VAWT types. Zhang et al. [21] studies on the semi-rotary VAWT with two perpendicular blades,
and found that adding disks at the bottom and top of the rotor enhances the confident of performance to till 30%. They have also stated that increasing the number of the turbine blades from 4 to 6 can improve its coefficient of performance up to 6–7% [22]. Bedon et al. [23] constructing a configuration characterized by rotor performance improvement, enhanced the coefficient of performance of Darrieus VAWT above 30% using an algorithm named W.O.M.B.A.T\(^1\) [24].

Straight-blade or H-rotor VAWTs were developed in the United Kingdom through the research carried out during the 1970–1980s. It was shown that drag/stall effect created by a blade when leaving the wind flow constraints the speed which the contrary blade can propel the entire rotor. As a result the straight-blade/H-rotor Darrieus VAWT is self-regulating and in a short time after its cut-in wind speed can achieve its optimal rotational speed in all wind velocities[1]. Although the Darrieus VAWTs are known to have a lower\( C_p \) than the HAWTs, Mertens et al. [25] have shown that the coefficient of performance of an straight-blade/H-rotor Darrieus VAWT can be greater than HAWT if it is located on a rooftop. Another advantage of the straight-blade/H-rotor Darrieus VAWT over the HAWT is that unlike the HAWTs, their blades are not twisted and have much easier manufacturing process.

2.1.3 **Linear Wind Generator (LWG)**

Ponta et al. [26] studied on applications of large scale Darrieus VAWTs and presented a new design which was variable-geometry oval-trajectory (VGOT) Darrieus turbine. As Figure 2.4 shows the blades move on rail tracks located in an

---

\(^1\) W.O.M.B.A.T is a software package for quantitative genetic analyses of continuous traits, fitting a linear, mixed model; estimates of covariance components and the resulting genetic parameters are obtained by restricted maximum likelihood [20].

23
elevated position, instead of rotating around a single rotor shaft. The blades are mounted on wheels which are coupled with electrical power generators. This design uses multi-directional power absorption capability of VAWT but operates with a high coefficient of performance (nearby 57% in the optimum design configurations), and resolve the low starting torque problems [27, 28]. The results from these studies show that VGOT Darrieus turbine achieves greater coefficient of performance with higher number of blades ($N = 120 - 160$) in low TSR ($\sim 2$) while at higher TSR, greater coefficient of performance can be achieved by a comparatively fewer number of blades ($N = 60 - 80$).

![Figure 2-4. VGOT Darrieus turbine configuration [26].](image)

As can be seen in Figure 2.5 the VGOT Darrieus blades are attached to a wagon which can follow a non-circular trajectory. Increasing the ration of transit perpendicular area to the total incoming wind area may results in increasing wind the energy conversion and optimizing efficiency of the entire plant. The VGOT Darrieus blades generate higher power output when tracking along the perpendicular line to the incoming wind direction, but they consume power instead of generation when tracking along the line parallel to the incoming wind direction.
VGOT Darrieus configuration allows increasing the swept area by increasing height of the trajectory line and/or widening the blades. On the other hand, the inflow direction remains constant along these straight tracks, which also results in the system aerodynamic and structural stability, while in the traditional Darrieus VAWTs the blades are subjected to a variable inflow in both magnitude and direction over the blades.

PowerWindow developed at the University of Wollongong, Australia, is another design among the LWGs. The variable-geometry oval-trajectory (VGOT) Darrieus turbine (which is initially a VAWT) [26-28] aerodynamic mechanism might be most similar to the PW, which allows them to operate efficiently at very low linear velocity ratios. This ratio is analogous to tip speed ratio (TSR) in HAWT and VAWT, but is required to be at much lower value than typical tip speed ratio of HAWT. However a major deference existing between these two designs is that the PW has also benefits the counter-rotating HAWT [11-13] power generation.
mechanism which enables the front blades to enhance the power generation of the rear blades.

2.2 PowerWindow Mechanism

The approach to develop the PowerWindow is to abandon the rotary model (with its undesirable effects on creating turbulence in the vicinity and the wake of the turbine) and use a modular approach in building a large harvesting area. This design would have scalability with respect to technology, manufacturing and cost because the modules can be mass produced. A PowerWindow prototype is sketched in Figure 2.6. The current prototype dimensions are $2m \times 2m \times 0.4m$.

![Figure 2-6. Sketch of PowerWindow module](image)

There are two positions in which, PowerWindow can be located subjected to the wind: (i) somewhere elevated from the ground so that the wind can expand from
both side of the PowerWindow when facing, e.g. between two buildings which is named suspended position; (ii) and landed on the ground so that the wind can expand only from top side of the PowerWindow when facing it, e.g. on the top of a building which is named landed position. Figure 2.7 show PowerWindow inside the axial stream tube in (a) suspended position and (b) landed position.

A couple of rotating disks are mounted on this fixed frame and connected together with a shaft. The generator will be attached to this shaft using a gearbox. There are two belts running over each disk and guiding rollers at the top of the module. The blades are mounted on these belts. The wind will apply a lift force onto blades causing the belt to roll (similar to a garage door opening or closing). As the belt goes over the top roller guides and the bottom rotating disks, the blades will change side and orientation. The top and bottom sides of the PowerWindow are covered using an aerodynamically shaped (plastic) cover that guides the wind towards the center of the
module and enables the tips of the blades and their flipping over at the top and bottom to happen outside the wind.

As mentioned before, an important characteristic of the PowerWindow is that the blades do not rotate in the wind but are only experiencing a translational motion (up or down). As such, all the points on a blade move at the same speed and the blade does not impart a torque on the air flow. In addition PowerWindow edges are out of wind and covered so the tip vortices are likely to be small with little chance of interfering with neighboring modules. Consequently, the modules can be placed next to each other without significant loss of efficiency to build a large PowerWindow plant.
2.3 **Summary of Wind Turbines Mechanisms**

The key differences between the mechanisms of VAWT, HAWT and PowerWindow are briefly summarized in Table 2.1.

<table>
<thead>
<tr>
<th></th>
<th>Straight-blade Darrieus VAWT</th>
<th>Curved-blade Darrieus VAWT</th>
<th>HAWT</th>
<th>PowerWindow (LWG)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Blade Profile</strong></td>
<td>Simple</td>
<td>Complicated</td>
<td>Complicated</td>
<td>Simple</td>
</tr>
<tr>
<td><strong>Need for Yaw Mechanism</strong></td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Possibility of Pitch Mechanism</strong></td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No need</td>
</tr>
<tr>
<td><strong>Tower</strong></td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td><strong>Guy Wires</strong></td>
<td>Optional</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td><strong>Noise</strong></td>
<td>Low</td>
<td>Moderate</td>
<td>High</td>
<td>Very low</td>
</tr>
<tr>
<td><strong>Blade Area</strong></td>
<td>Moderate</td>
<td>Large</td>
<td>Small</td>
<td>small</td>
</tr>
<tr>
<td><strong>Generator position</strong></td>
<td>On ground</td>
<td>On ground</td>
<td>On tower</td>
<td>On frame</td>
</tr>
<tr>
<td><strong>Blade load</strong></td>
<td>Moderate</td>
<td>Low</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td><strong>Self-start</strong></td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Tower/frame interference</strong></td>
<td>small</td>
<td>small</td>
<td>High</td>
<td>small</td>
</tr>
<tr>
<td><strong>Foundation</strong></td>
<td>Moderate</td>
<td>Simple</td>
<td>Extensive</td>
<td>Simple</td>
</tr>
<tr>
<td><strong>Overall Structure</strong></td>
<td>Simple</td>
<td>Simple</td>
<td>Complicated</td>
<td>Moderate</td>
</tr>
</tbody>
</table>
2.4 Wind Turbines Performance and Reliability

There are many limitations and difficulties which influence wind generators reliability and performance of wind generators. The most important is Betz's limitation. Betz's law computes the maximum energy which can be captured from the wind energy in free stream, disregarding the wind turbine design. Albert Betz (a German physicist) published it in 1919. Betz's law applying mass and momentum conservation principles of the flow stream passing through an ideal disk including the rotor, named "actuator disk" extracts the maximum energy from the wind stream. Betz's law shows that, no wind power generator can capture more than 59.3% of the wind kinetic energy. Therefore this factor is known as the Betz's limit.

Considering an air stream-tube entering the actuator disc (Figure 2.8), the wind speed at the upstream of stream-tube is equal to \( u_1 \) and its cross-sectional area is equal to \( A_1 \). As the actuator disc captures greater kinetic energy from the wind its exit velocity would be decelerated. By assuming air as an incompressible flow in low speeds (\( u \leq 0.3 \text{ Ma} \)), the cross-sectional area of the stream-tube expands instead of compressing the flow in the the stream-tube and this decelerates its velocity. The cross-sectional area of the stream-tube increases to \( A_2 \) in this section. Wind static pressure also drops from \( P^+ \) to \( P^- \) when passing the actuator disk. As a result the downstream flow continue the expansion till static pressure of the flow reaches the atmospheric pressure, \( P_\infty \). This increases the cross-sectional area of the stream-tube far from the actuator disk from \( A_2 \) to \( A_3 \), where wind velocity is decelerated to \( u_3 \).
Figure 2-8. Stream-tube at the up-stream and down-stream of the actuator disc.

Assuming constant mass flow rate $\dot{m}$ through the stream-tube, using continuity equation, its value would be:

$$\dot{m} = \rho A_1 u_1 = \rho A_2 u_2 = \rho A_3 u_3 \quad (2-1)$$

Equation 2.2 gives the total wind power available in the wind flow when its speed is $u_1$ and passing through the cross-sectional area $A_2$ at the actuator disk section.

$$P_{max} = \frac{1}{2} \dot{m} u_1^2 = \frac{1}{2} (\rho A_2 u_2) u_1^2 = \frac{1}{2} \rho A_2 u_1^3 \quad (2-2)$$

This is the maximum wind power available in the wind flow.

Equation 2.3 gives the power extracted by the wind turbine

$$P_{ext} = \frac{1}{2} \dot{m} u_1^2 - \frac{1}{2} \dot{m} u_3^2 = \frac{1}{2} \rho A_2 u_2 (u_1^2 - u_3^2) \quad (2-3)$$

Also, using Bernoulli’s equation, it can be written as:

$$\frac{1}{2} \rho u_1^2 + p_\infty = \frac{1}{2} \rho u_2^2 + p^+ \quad (2-4)$$

$$\frac{1}{2} \rho u_2^2 + p^- = \frac{1}{2} \rho u_3^2 + p_\infty \quad (2-5)$$
From equations 2.4 and 2.5, it can be derived that:

\[ P^+ - P^- = \frac{1}{2} \rho u_1^2 - \frac{1}{2} \rho u_3^2 \]  

(2-6)

The total axial thrust exerted by the turbine over the wind flow equals to the change rate in the momentum of the flow.

\[(P^+ - P^-)A_2 = \dot{m}(u_1 - u_3) \]  

(2-7)

Or \((P^+ - P^-)A_2 = \rho A_2 u_2 (u_1 - u_3)\)  

(2-8)

From equations 1.7 and 1.8:

\[ \rho A_2 u_2 (u_1 - u_3) = A_2 (\frac{1}{2} \rho u_1^2 - \frac{1}{2} \rho u_3^2) \]  

(2-9)

This implies, \(u_2 = \frac{1}{2} (u_1 + u_3)\)

The coefficient of performance \((C_p)\) of a wind turbine is defined as the ratio of the extracted power over the total available power, shown in equation 2.10:

\[ C_p = \frac{p_{\text{ext}}}{p_{\text{max}}} = \frac{\frac{1}{2} \rho A_2 u_2 (u_1^2 - u_3^2)}{\frac{1}{2} \rho A_2 u_1^3} \]  

(2-10)

Therefore \(C_p\) can be written as shown in equation 2.11 and 2.12:

\[ C_p = \frac{\frac{1}{2} \rho A_2 (u_1 + u_3)(u_1^2 - u_3^2)}{\frac{1}{2} \rho A_2 u_1^3} \]  

(2-11)

\[ C_p = \frac{1}{2} (1 + \frac{u_3}{u_1})(1 - \frac{u_3^2}{u_1^2}) \]  

(2-12)

If \(y = \frac{u_3}{u_1}\)

\[ C_p = \frac{1}{2} (1 + y)(1 - y^2) \]  

(2-13)
The maximum coefficient of performance occurs when \( \frac{dC_P}{dy} = 0 \), so:

\[
\frac{dC_P}{dy} = \frac{1}{2}(1 + y)(1 - 3y) = 0
\] (2-14)

Since, \( y = \frac{u_3}{u_1} \neq -1 \) and \( y = \frac{1}{3} \). This gives maximum value of coefficient of performance: \( C_{p,max} = C_p \left( y = \frac{1}{3} \right) = \frac{16}{27} \).

The ideal turbine defined by Betz is a HAWT operating with infinite blades at infinite TSR and no energy losses. This turbine is very similar to the actual wind turbines, which typically operate at high TSRs. At high TSRs three blades are enough for interacting with the entire flow passing through the rotor area. A diffuser can be used for collecting further wind flow and conducting it through the turbine, which results in greater energy extraction. However, configuration of these shrouded turbines namely diffuser augmented wind turbines (DAWT) are more expensive because of the additional required structure.

Another factor which greatly influences the performance of the wind generators is their scale and scalability. Initially, the notion of a small wind turbine was defined based on its power generation capability which should be enough to supply an individual household electricity demand. However this is an approximation because the average household electricity demand is fixed around the world. There have also portable wind turbines been emerged for vehicles and small wind turbines for domestic applications. Nevertheless, developing larger wind turbines for capturing more wind power attracts significant interest.

Before 1990, wind turbines power generation capacity was typically less than 100 kW. This capacity increased to 500 kW by 1990, and increased to 750 to 1000 in
a few years. By 2000 and 2005, the turbine power generation capacity reached 2500 and 3500 kW respectively [29]. The largest turbine in world is currently Enercon E126 wind turbine (shown in Figure 2.9) with 7.5MW power rating introduced in 2007. This turbine with 127 metre rotor diameters with segmented steel-composite hybrid blades is mounted on a concrete tower with a 135 metre hub height. However the V164 with 8MW power rating and 164 metre rotor diameter will be the world's most powerful turbine in close future.

Figure 2.9. Enercon E126 wind turbine.

There always been some common problems facing the wind turbines. Since the wind turbines are under dynamic forces, they are always subjected to fatigue due to vibration. Hence it has been tried to develop models in order to analyse vibration
problem in these devices. Ramsamooj, et al. [30] presented a new analytical model for corrosion fatigue. Ye, Z.Q., et al. [31] studied the ‘structural dynamic characteristics of rotor blades to avoid sympathetic vibration problem’ using theoretical and experimental methods. The test revealed that flap-wise vibration is the main vibration of the rotor blade and this problem is emphasized for the larger wind turbines because as the rotor is larger, it is exposed to greater fatigue load.

2.5 Methods for Wind Turbines Performance Analysis

PowerWindow is still in development process. In order to enhance its performance a model is needed which can accurately predict its flow mechanism and loads exerted over it blades. Since very few studies have been done on LWG types, an investigation is needed to be done over the same studies on the other wind generator types. Loading calculation over the blades is the most basic of power generation analysis in the wind turbines. The methods for calculating the aerodynamic forces on the turbine blades are currently: (i). Blade Element Momentum (BEM) theory; (ii). Computational Fluid Dynamics (CFD) simulations; (iii) experimental prototyping in wind tunnel” [32]. These three methods have been used in this study to develop models which can analyse the aerodynamic mechanism of PowerWindow.

2.5.1 BEM Method

A mathematical method for fluid dynamics analysis of wind turbines and evaluating its performance is BEM method [33]. The BEM aerodynamic analysis concept is based on Glauert’s airscrew theory [34]. This method was previously getting used for the analysis of propellers, exclusively in the helicopter industry [35].
Later its application was extended to wind turbines and computing the performance of wind turbines [36]. BEM has recently broadly used for HAWTs analysis [37], [38], [39], [40] and VAWTs [23], using tabulated airfoil data. This method is also successfully applied for the tidal turbines [41], [42], [43].

BEM has been used for optimization of different types of wind turbines [44]. This method needs the blade two dimensional (2D) airfoil data, distributions of chord length and the twist angle along the blade length to find the optimum shape for blade. One major issue is that once HAWT blade optimization is done at one operating condition, the result is valid only for the relative TSR and angle of attacks. Hence that design will no longer be optimal for other TSR and angle of attacks [45].

Two assumptions are made in applying the BEM theory:

(i) There is no aerodynamic interaction between the elements; and

(ii) The forces on the blades are determined solely by the lift and drag characteristics of the airfoil shape of the blades [46].

The BEM model calculation is directly based on airfoil data and dependent on empirical corrections to two dimensional (2D) airfoil results to account for three-dimensional (3D) effects, such as tip losses, rotational flow, and dynamic stall [46]. Including the total loss (tip and hub loss) factor can improve BEM calculation accuracy. Prandtl [47] and Byand [48] developed BEM tip loss correction models. The modified BEM method has also been developed based on comparisons with actuator disc simulations [49].
The predictions obtained by the BEM method is reliable while it requires much less computational calculations compared to CFD simulation models [23]. However comparing CFD simulations and BEM method for small scale propellers, Carroll and Marcum [50] showed that the BEM method acceptably predicts the thrust with acceptable accuracy when the propeller operates with little separation and the blade has a high aspect ratio with little or no chord variation. However, in large regions of separated flow and blades of lower aspect ratio and chord variation, the accuracy of BEM diminishes. Moreover one key limitations of the BEM method compared to CFD simulation is that it cannot analyse the rotor impact on the surrounding flow [51]. Therefore a combination of BEM method and CFD simulations are being used in some recent studies [32], [51], [42], [52].

2.5.2 CFD Simulation

CFD is a numerical calculation to analyse to predict physical phenomenon such as flow and heat conduction in a flow [32]. The three-dimensional (3D) CFD simulations solving the Navier-Stokes equations are very physically realistic, but they need very long calculation times [53]. The CFD simulations can give explicit modelling of turbine blades and estimate the complex turbulent flows adjacent to its blades and wake regions created at the far downstream [54].

CFD simulation is very useful when a rotor is subjected to a complex flow conditions which needs three-dimensional investigation. High turbulence level and variable flow directions can create such a complex condition. Another condition is when a significant part of blades is operating in stall condition. In such a condition it is not possible to rely on pre-determined lift coefficients achieved by wind tunnel
There are also very accurate and realistic CFD simulation methods for representation turbine wakes, which need very long calculation times, so are not usually used for their performance evaluation [56].

Many researchers have performed wind turbine CFD simulations using different methods. Sezer-Uzol and Long [57] computed the NREL Phase VI turbine in various wind speeds and yaw angles using the finite volume flow solver PUMA2 with rotating unstructured tetrahedral grids. Their results well agreed with experimental tests, but since the nature of their code was inviscid, it had limited ability for prediction in massive flow separation conditions. A comprehensive aerodynamic study was later performed by Buning, et al. [58] who computed the NREL Phase VI turbine with the NASA compressible RANS flow solver Overflow-D, based on a finite differences approach and overset grid [59]. They compared the results with the experiments, and using the validated results discussed the aerodynamic mechanisms of the wind turbine such as shaft power, normal force and pressure coefficient.

Sørensen et al. [60] used a multi-block finite volume and incompressible RANS flow solver EllipSys3D to study the three-dimensional (3D) aerodynamic effects on a rotor-only configuration. Their computational results had good agreement with the experimental measurements. Mark and Dimitri [61] used the unstructured multi-grid RANS code NSU3D to predict the aerodynamics of an isolated wind turbine rotor. Bazilevs et al. [62] using a finite element and a (Non-Uniform Rational B-splines) NURB-based [63] method studied the NREL 5 MW baseline wind turbine rotor. NURB-based approach enables coupling the aerodynamic and structural analysis.
CFD simulations which have been widely used in recent studies about wind turbines [64], [53], [59], [65]. The major advantage of CFD simulations compared to experimental prototyping is that they not only take less time and cost, but also can give further insight over flow mechanism passing through the wind turbines [66]. CFD simulations have also some advantages compared to BEM model. As mentioned before, in large regions of separated flow and blades of lower aspect ratio and chord variation, the accuracy of BEM diminishes [51]. While CFD simulations have successfully been used for performance analysis of large-scale wind turbines [67], [68], and wake effects on turbines downstream [69]. CFD simulations are also able to predict VAWT performance more accurately than BEM model [70].

2.5.3 Experimental Prototyping

Experimental prototyping is the most expensive and time consuming but also the most reliable method for performance analysis of the wind devices. There are some approaches to measure power generation performance of the wind turbines in a wind tunnel. Hirahara et al. [71] and Koki et al. [72] used small scale wind turbine models and measured the mechanical torque over the blade by installing a torque converter on the shaft. Hirahara et al. [71] successfully derived performance curves of a HAWT by applying an electronic load which could change the rotational speed of the rotor. Hailiang et al. [73] presented improved control strategies for doubly fed induction generator (DFIG)-based generators. Experimental prototyping has been previously widely used for modelling and analysis of HAWTs [74] and VAWTs [75].
CHAPTER 3. MODIFIED BLADE ELEMENT MOMENTUM MODEL FOR POWERWINDOW

As mentioned before BEM method combines two theories of momentum theory and blade element theory. Applying the momentum theory, the maximum coefficient of performance of a wind turbine cannot exceed over 59% (Betz limit). However the maximum coefficient of performance of conventional single rotor HAWTs is about 40–50% which can be resulted by the energy losses such as viscous loss, three-dimensional loss, and transmission loss [11]. There is almost no three-dimensional loss in the PowerWindow, however its viscous loss is greater than VAWTs and HAWTs because unlike the other turbines, the PowerWindow blades have been allocated in a cascade configuration. When the fluid passes through the cascade, there will be a decrease in total pressure between the inlet to the cascade and at the section downstream of the cascade which is due to: (i) Frictional loss due to the formation of boundary layer on blades; and (ii) Losses due to mixing of blade wakes. Therefore BEM theory has been modified in this study by replacing lift and drag coefficient of the single blade by lift and drag coefficients of cascade.

3.1 Aerodynamics of Cascade

Operating condition, blade geometry and the airfoil lift and drag coefficients are essential parameters to be provided to generate the simulation code. As the blades might be under several different wind directions, the aerodynamic lift and drag curves are need in different angle of attacks(\(\alpha\)). Most of the databases developed for aeronautical use are limited to a range of angle of attack (\(\alpha\)) for Reynolds and Mach numbers which are greater than the values typically experienced by wind turbine
blades during operation [23]. For the airfoil used in PowerWindow, its lift ($C_L$) and drag ($C_D$) coefficients data were generated for $-14^\circ < \alpha < 36^\circ$ using Computational Fluid Dynamic (CFD) simulations. $C_L$ and $C_D$ can be defined in equations 3.1 and 3.2.

$$C_L = \frac{F_L}{\frac{1}{2} \rho B V_{rel}^2} \quad (3-1)$$

$$C_D = \frac{F_D}{\frac{1}{2} \rho B V_{rel}^2} \quad (3-2)$$

$F_L$ is the component of the force that is perpendicular to the oncoming flow direction (not chord line direction) and $F_D$ is the component of the surface force parallel to the flow direction (not chord line direction), $B$ is the airfoil plan area, $\rho$ is the air density and $V_{rel}$ is the relative velocity of wind to the airfoil. Hence by obtaining $F_L$ and $F_D$ from the CFD simulations and having $B, \rho$ and $V_{rel}$, the $C_L$ and $C_D$ of the airfoil can be calculated for each $\alpha$. The CFD simulations have been done using $K - \omega$, SST transitional model for isolated airfoil and cascade configuration with three different solidities. The inlet wind velocity was set to $8 \text{ m/s}$ which is the average velocity that the PowerWindow is designed to operate in. As a result Reynolds number and Mach number of the flow over the airfoil were $7.1 \times 10^4$ and $2.33 \times 10^{-2}$ respectively.

Figure 3.1 shows the PowerWindow airfoil cross-section. As mentioned before this airfoil is axisymmetric to the vertical axis since it is designed to extract power when flipped over and moves in the opposite direction in the rear side. Figure 3.2 shows a “goe 15k-il aerofoil” which is a commercial airfoil and partly similar to the PowerWindow aerofoil. The maximum thickness is $15\%$ of the chord at $50\%$ of its length. Therefore $C_L$ and $C_D$ data obtained by the CFD simulations for the isolated
PowerWindow airfoil is compared to goe 15k-il $C_L$ and $C_D$ from aerofoil database in Figure 3.3 (a) and (b) and 3.4 (a) and (b). $C_L$ and $C_D$ curves are not exactly the same but comparison shows that the CFD results are reasonable.

Figure 3-1. Cross section view of PowerWindow airfoil.

Figure 3-2. Cross section view of goe 15k-il aerofoil.

Figure 3-3. (a) $C_L$ and (b) $C_D$ of the PowerWindow isolated airfoil.
Figure 3.6 (a) and (b) show $C_L$ and $C_D$ of isolated airfoil and cascade configuration with three different solidities ($\sigma = 0.428, 0.857$ and $1.714$) against $\alpha$ obtained by the CFD simulations. $C_L$ and $C_D$ data is also reported in Table B.1 and B.2 in appendix B for $-14^\circ < \alpha < 36^\circ$. $\sigma$ shows a relation between number of the blades, their blade and the rotor swept area, which is not similarly defined for every wind turbine. For a PowerWindow, it is defined as written below.

$$\sigma = \frac{NB}{A} \quad (3-3)$$

$N$ shows the blade numbers and $A$ is the swept area of a PowerWindow.
Figure 3-5. (a) $C_L$ and (b) $C_D$ of the PowerWindow isolated airfoil and cascade for different $\sigma$. 
Figure 3-6. Velocity (on the left) and pressure (on the right) contours of (a) isolated airfoil and cascade configuration in (b). $\sigma = 0.428$ and (c) $\sigma = 0.857$ in $\alpha = 20^\circ$. 
As can be seen from Figure 3.5 (a), the maximum $C_L$ for cascades are lower than the maximum for the isolated airfoil. Moreover as the solidity of the cascade increases, the maximum $C_L$ decreases, and shifts to higher $\alpha$. The reason can be investigated in the velocity and pressure contours around the blades. Figures 3.6 (a), (b) and (c) show the velocity (on the left) and pressure on the right) contours of the isolated airfoil and cascade configuration when $\sigma = 0.428$ and $\sigma = 0.857$ in $\alpha = 20^\circ$ in terms of velocity ratio and surface pressure coefficient which are later discussed in the discussion and results.

The velocity contours show that the low velocity region on the suction side of the blade decreases as $\sigma$ increases. Comparing the relative pressure contours shows that decreasing in the area of the low velocity region results in decreasing the area of the sub atmospheric region on the blade suction side. The reason is that when a blade is located on top of another one, the flow within the blades gets trapped. This pushes the separation point toward the trailing edge. The flow over the blades becomes more uniform and minifies the area of the low pressure region on the top of the lower blade. However meanwhile the pressure increases on the pressure side but the overall effect is decreases in $C_L$. Hence increasing $\sigma$ shifts the separation point from the leading edge to the trailing edge. Hence as the $\sigma$ is higher, $C_L$ and stall occurs at higher $\alpha$.

### 3.2 Applying Modified BEM Theory

As mentioned before in BEM theory power generation of the device should be calculated using both momentum theory and blade element theory. Therefore both calculation methods have been derived.
3.2.1 The modified momentum theory

Figure 2.7 (a) and (b) show the flow expansion when entering PowerWindow in suspended and landed position. These figures show that two stages of blades block the wind entered the PowerWindow. This mechanism is very similar to the counter-rotating HAWTs. Lee, S., et al. [11] showed that in a counter-rotating HAWT the stream tube behind the front rotor and before entering the rear one can be assumed to be fully developed, unless on cases of very closely spaced rotor. Figure 3.7 shows flow model of a counter-rotating HAWT with a rear rotor operating inside the stream tube of the front rotor. As can be seen the flow enters the rear rotor is assumed to be entirely induced by the front rotor.

Distance between the front and rear blades in the PowerWindow prototype is almost equal to the blades cord line. Hence unlike the counter rotating HAWT, the axial stream behind the front rotor and before entering the rear one is assumed to be not developed and air flow cannot expand within two stages. As a result it is assumed.
that in Figures 2.7 (a) and (b) $P_4 = P_3$ (also shown in Figure 5.11) and $V_3 = V_4$. In
the first flow expansion air velocity reduces from $V_1$ to $V_2$ and air pressure increases
from $P_1$ to $P_2$. This expansion should magnify upstream pressure to create enough
pressure-gradient for air to pass through both front and rear blades, so upstream and
downstream velocities should decrease with a higher induction factor ($a$) compared
to HAWT to create a higher ultra-atmospheric pressure region at the inlet of the
PowerWindow and a lower sub-atmospheric pressure region at its outlet.

Induction factor is the ratio of reduction of the air velocity to that far away from the
wind and defined in equation 3.4.

$$a = \frac{v_1 - v_2}{v_1} \quad (3-4)$$

Therefore $V_2$ and $P_2$ can be calculated based on $V_1$ and $P_1$ using equation 3.5 and 3.6.

$$V_2 = V_1 (1 - a) \quad (3-5)$$

$$P_2 = P_1 + \frac{\rho(v_2^2 - v_1^2)}{2} \quad (3-6)$$

Air pressure drops from $P_2$ to $P_3$ when the flow passes the front region and from
$P_4$ to $P_5$ when passes the rear region, while the low distance between the section 2
and 3 and section 4 and 5 does not allow the flow to expand and change its velocity
in both stages.

$$P_4 = P_3 \quad (3-7)$$

$$V_5 = V_4 = V_3 = V_2 \quad (3-8)$$

So the power extracted from these regions will be:

$$P_{\text{wind, front}} = V_2 A (P_2 - P_3) = V_1 A (1 - a)(P_2 - P_3) \quad (3-9)$$

$$P_{\text{wind, rear}} = V_4 A (P_4 - P_5) = V_1 A (1 - a)(P_3 - P_5) \quad (3-10)$$

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A shows the swept area of the PowerWindow facing the wind. Adding the power extraction from front and rear regions gives:

\[ P_{\text{wind,total}} = V_1 A (1 - a) (P_2 - P_3) \]  
\[ (3-11) \]

Assuming equal air velocity reduction rates in upstream and downstream of the flow, it can be written:

\[ V_6 = V_1 (1 - 2a) \]  
\[ (3-12) \]

\[ P_6 = P_5 + \frac{\rho (v_2^2 - v_6^2)}{2} = P_5 + \frac{\rho (v_2^2 - v_6^2)}{2} \]  
\[ (3-13) \]

Therefore:

\[ P_{\text{wind,total}} = V_1 A (1 - a) \left( (P_1 + \frac{\rho (v_1^2 - v_2^2)}{2}) - (P_6 - \frac{\rho (v_6^2 - v_2^2)}{2}) \right) \]  
\[ (3-14) \]

Since that \( P_1 = P_6 = P_{\text{atm}} \), it can be written:

\[ P_{\text{wind,total}} = V_1 A (1 - a) \left( \frac{\rho (v_1^2 - v_2^2)}{2} + \frac{\rho (v_6^2 - v_2^2)}{2} \right) \]  
\[ (3-15) \]

Re-writing the velocities in equation 3.15 based on \( V_1 \) and \( a \), \( P_{\text{wind,total}} \) and considering that \( V_1 \) equals to \( V_{\text{wind}} \) (wind velocity far from the PowerWindow before entering its inlet region) can be stated as below:

\[ P_{\text{wind,total}} = \frac{1}{2} \rho A V_{\text{wind}}^3 4a (1 - a)^2 \]  
\[ (3-16) \]

### 3.2.2 The Modified Blade Element Theory

Blade element theory is mathematical approach, originally designed for estimation of the propeller’s behavior. This method divides a blade into several small elements and calculates the forces on each element. Integrating the forces along the entire blade and calculating the resulted moments gives the entire torque created over
propeller or rotor [76]. Calculation of the vertical and horizontal forces exerted on the PowerWindow blades is straight forward, since the PowerWindow blades are not twisted and have same shape along the span. The vertical and horizontal forces can be calculated using $F_L$ and $F_D$ from equations 3.1 and 3.2 in equations 3.17 and 3.18.

$$F_x = (F_L \sin \beta + F_D \cos \beta) \quad (3-17)$$

$$F_y = (F_L \cos \beta - F_D \sin \beta) \quad (3-18)$$

In figure 3.8, $\beta$ defines the angle between the drag force direction and the horizontal axis and/or between the lift force direction and the vertical axis and $\alpha$ is angle of attack of the wind when facing the blade in any position. Four blade locations of the PowerWindow are shown in Figure 3.8: for front blade, region 1; for rear blade, region 2; for top front blade, region 3; and for top rear blade, region 4 of the PowerWindow. In Figure 8, $\theta_0$ is the design pitch angle of the blades in the region 1 which is reversely repeated in the region 2. $\theta$ is the angle of the blades from horizontal axis when canter of the blade is located on the semicircular region, 3 and 4 on the top, which varies with $\varphi$ (the angle of from the horizontal axis). Equation 3.19 shows the relations between $\theta$ and $\varphi$ and $\theta_0$ in the regions 3 and 4.

$$\theta = \theta_0 \left(1 - \frac{\varphi}{90}\right) + \varphi \quad (3-19)$$

$\beta$ (effective angle) and $\alpha$ values depend on $\theta$ and $\varphi$ value in the regions 3 and 4. However $\beta$ and $\alpha$ values are constant in the regions 1 and 2 and can be calculated based on $\theta_0$. Based on the PowerWindow blade configuration and the gears diameters, minimum zero and maximum two blades can be located in the semicircular region (3 and 4). In the case that two blades are located in this region
one would be in region 3 and the other in the region 4. In the BEM analysis, calculation of the power generation in regions 3 and 4 is more complicated than regions 1 and 2, because as mentioned before the power extracted from these blades is dependent on their orientations ($\varphi$).

![Diagram of wind turbine regions and forces](image)

Figure 3-8. Path view followed by the PowerWindow blades in regions 1, 2, 3 and 4, and the velocity composition of the wind facing the front and rear blades which result in the aerodynamic forces acting on the blades.
\( \lambda \) defines the PowerWindow blades absolute velocity (which is also equal to their vertical velocity) to the wind velocity ratio (equation 3.20).

\[
\lambda = \frac{V_{blade}}{V_{wind}} = \frac{V_{y,absolute}}{V_{wind}}
\] (3-20)

Considering the PowerWindow blades move vertically in regions 1 and 2, power extracted from them can be calculated by summation of the vertical force exerted on each blade multiplied by its vertical velocity (equation 3.21). The vertical force exerted on the blades \( (F_y) \) have been calculated using \( F_L \) and \( F_D \) in equation 3.17 and 3.18, and vertical velocity of the blades has been calculated from equation 3.20.

\[
Power = \sum_{i}^{N} \left( \frac{\rho B}{2} \right) V_{rel, i}^2 \left[ C_L \cos \beta - C_D \sin \beta \right] (V_{wind} \lambda)
\] (3-21)

\( V_{rel} \) shows relative wind velocity to a blade and can be calculated in equation 3.22 \((i = 1 \text{ for front blades, } 2 \text{ for rear blades})\).

\[
V_{rel, i}^2 = V_{y,relative, i}^2 + V_{x,relative, i}^2
\] (3-22)

Considering that very low distance between front and rear blades of PowerWindow does not allow wind to expand when passing regions 1 and 2 (in Figure 3.8), its horizontal velocity does not change in the sections 2, 3, 4 and 5 in Figure 2.7 (a) and (b) have same average velocity when facing the front and rear blades. On the other hand the blades have no horizontal velocity through their path in regions 1 and 2. Hence this velocity would be equal to the relative horizontal velocity of the wind to both the front and the rear blades in these regions, which are stated in equation 3.23. But the relative vertical velocity of the wind when facing the rear blades is not equal to the relative vertical velocity of the wind when facing the front blades, because the relative vertical of the wind when facing the rear blades is not
only dependent on their downward vertical velocity, but is also affected by the vertical velocity of the front blades which are moving on the opposite direction and lead the flow upward and also their design angle which lead this flow downward. Hence those have to be calculated separately.

\[ V_{1x} = V_{2x} = V_{\text{wind}} (1 - a) \quad (3-23) \]

Considering that \( V_1 \) in momentum theory equals to \( V_{\text{wind}} \) (wind velocity far from the PowerWindow before entering its inlet region), air axial velocity affecting the front blades was achieved in equation 3.23 and its tangential velocity equals to the PowerWindow blades absolute velocity which is shown in equation 3.24.

\[ V_{1y} = \lambda V_{\text{wind}} \quad (3-24) \]

By the definition of \( \beta \) and \( \alpha \), \( \beta_1 \) defines the angle between the drag force exerted on the front blades and the horizontal axis (and/or between the lift force exerted on the front blades and the vertical axis) and \( \alpha_1 \) is the angle of attack of the wind created the front blades when the design angle is \( \theta_0 \). These angles are shown in Figure 3.8. Using equations 3.23 and 3.24, \( \beta_1 \) and \( \alpha_1 \) can be written as:

\[ \beta_1 = \tan^{-1} \left( \frac{V_{1y}}{V_{1x}} \right) = \tan^{-1} \left( \frac{\lambda}{1 - a} \right) \quad (3-25) \]

\[ \alpha_1 = \theta_0 - \tan^{-1} \left( \frac{\lambda}{1 - a} \right) \quad (3-26) \]

Using the velocities achieved in the equation 3.22, 3.23 and 3.24, and the angles achieved in the equations 3.25 and 3.26 in the equation 3.21 gives equation 3.27 which calculates the power generated by the front blades of the PowerWindow.
\begin{align*}
P_{PW,front} &= \sum N \left( \frac{\rho B}{2} \right) V_{wind}^2 \left( (1 - a)^2 + \lambda^2 \right) \left[ C_{L_{air}} \cos \left( \tan^{-1} \left( \frac{\lambda}{(1-a)} \right) \right) - ight. \\
& \left. C_{D_{air}} \sin \left( \tan^{-1} \left( \frac{\lambda}{(1-a)} \right) \right) \right] \lambda V_{wind} \\
&= N \left( \frac{\rho B}{2} \right) V_{wind}^3 \lambda((1 - a)^2 + \lambda^2) \left[ C_{L_{air}} \cos \left( \tan^{-1} \left( \frac{\lambda}{(1-a)} \right) \right) - ight. \\
& \left. C_{D_{air}} \sin \left( \tan^{-1} \left( \frac{\lambda}{(1-a)} \right) \right) \right] \quad (3-27)
\end{align*}

Assuming that uniform aerodynamic force is exerted on every blade, the power exerted from the entire front blades can be obtained by simplifying this equation to equation 3.28. The power extracted by each blade is not practically equal, however this assumption is reasonably acceptable if the PowerWindow is set up in suspended position.

\begin{align*}
P_{PW,front} &= N \left( \frac{\rho B}{2} \right) V_{wind}^3 \lambda((1 - a)^2 + \lambda^2) \left[ C_{L_{air}} \cos \left( \tan^{-1} \left( \frac{\lambda}{(1-a)} \right) \right) - ight. \\
& \left. C_{D_{air}} \sin \left( \tan^{-1} \left( \frac{\lambda}{(1-a)} \right) \right) \right] \quad (3-28)
\end{align*}

As mentioned before the axial component of the wind velocity affecting the blades in the rear region is equal to the front region (as shown in equation 3.23), but its vertical component changes when passing the front blades. Assuming a high \(\sigma\) for the PowerWindow such that the front blades can influence the entire flow passing them, this flow turns upward with the same velocity as the vertical velocity of the front blades and then downwarded with the vertical velocity due to their design angle. Hence the vertical component of the wind velocity in the middle of PowerWindow upstream of the rear blades shown in Figure 3.8 can be calculated as written in equation 3.29.

\begin{align*}
V_{zy,absolute} &= \lambda V_{wind} - V_{wind}(1 - a)\tan\theta_0 = V_{wind}(\lambda - (1 - a)\tan\theta_0) \\
&= V_{wind}(\lambda - (1 - a)\tan\theta_0) \\
&= (3-29)
\end{align*}
Considering that the rear blades also have a downward vertical velocity equal to the upward vertical velocity of the front blades, the relative vertical velocity of the wind facing the rear blades shown in Figure 3.5 can be achieved from equation 3.30.

\[ V_{2y} = V_{2y,\text{absolute}} + \lambda V_{\text{wind}} = V_{\text{wind}}(2\lambda - (1 - a)\tan \theta_0) \]  

(3-30)

Considering equations 3.23 and 3.30, \( \beta_2 \) and \( \alpha_2 \) shown in Figure 3.9 can be calculated as written in equations 3.31 and 3.32.

\[ \beta_2 = \tan^{-1} \left( \frac{V_{2y}}{V_{2x}} \right) = \tan^{-1} \left( \frac{2\lambda - (1 - a)\tan \theta_0}{1 - a} \right) \]  

(3-31)

\[ \alpha_2 = \theta_0 - \tan^{-1} \left( \frac{(2\lambda - (1 - a)\tan \theta_0)}{(1 - a)} \right) \]  

(3-32)

Using the velocities achieved by equation 3.22, 3.23 and 3.30, and the angles of the equations 3.31 and 3.32 in the equation 3.21 gives equation 3.33 which calculates the power generated by the rear blades of the PowerWindow.

\[ P_{\text{pw,\text{rear}}} = \sum \left( \frac{\rho \beta_2}{2} \right) V_{\text{wind}}^2 \left( (1 - a)^2 + (2\lambda - (1 - a)\tan \theta)^2 \right) \left[ C_{\text{az}_2} \cos \left( \tan^{-1} \left( \frac{(2\lambda - (1 - a)\tan \theta_0)}{(1 - a)} \right) \right) \right] - \]

\[ C_{\text{Daz}_2} \sin \left( \tan^{-1} \left( \frac{(2\lambda - (1 - a)\tan \theta_0)}{(1 - a)} \right) \right) \left( \lambda V_{\text{wind}} \right) \]  

(3-33)

Assuming uniform aerodynamic force to be exerted on every blade, the power extracted by each blade would be equal in equation 3.33, and it simplifies this equation to equation 3.34. As mentioned before, although the power extracted from every blade is not practically equal, this assumption is reasonably acceptable if the PowerWindow is set up in suspended position.
\[ p_{pw, \text{rear}} = \]
\[ N \left( \frac{\rho \theta}{2} \right) V_{\text{wind}}^3 \lambda ((1 - a)^2 + (2\lambda - (1 - a)\tan\theta)^2) \left[ C_{l_{ax}} \cos \left( \tan^{-1} \left( \frac{(2\lambda + (1 - a)\tan\theta_0)}{(1 - a)} \right) \right) - C_{D_{ax}} \sin \left( \tan^{-1} \left( \frac{(2\lambda + (1 - a)\tan\theta_0)}{(1 - a)} \right) \right) \right] \]

\( (3-34) \)

However if the entire flow passed the front blades and facing the rear blades is not affected by the vertical velocity and design angle of the front blades, equations 3.29-3.32 give different velocity and angles. When \( \sigma \) is very low \((\sigma \ll 1)\), or incoming wind velocity is very high or flow is very turbulent, etc. the front blades might only affect the air adjacent to their boundaries. The area within the dashed line in Figure 3.8 shows such an area. In this condition an Affected Flow Ratio (AFR), \( \varepsilon \) is defined which presents the proportion of the flow affected by the front blades to the entire incoming flow. This changes equations 3.29, 3.30, 3.31 and 3.32 to equations 3.35-3.38.

\[ V_{yz, \text{absolute}} = \varepsilon V_{\text{wind}}(\lambda - (1 - a)\tan\theta_0) \]  
\( (3-35) \)

\[ V_y = \varepsilon V_{yz, \text{absolute}} + \lambda V_{\text{wind}} = V_{\text{wind}}((\varepsilon + 1)\lambda - \varepsilon(1 - a)\tan\theta_0) \]  
\( (3-36) \)

\[ \beta_z = \tan^{-1} \left( \frac{V_{zy}}{V_{zs}} \right) = \tan^{-1} \left( \frac{(\varepsilon + 1)\lambda - \varepsilon(1 - a)\tan\theta_0}{(1 - a)} \right) \]  
\( (3-37) \)

\[ \alpha_z = \theta_0 - \tan^{-1} \left( \frac{(\varepsilon + 1)\lambda - \varepsilon(1 - a)\tan\theta_0}{(1 - a)} \right) \]  
\( (3-38) \)

Replacing above velocity and angles in equation 3.34 modifies this equation to equation 3.39.
\[ P_{PW,\text{rear}} = N \left( \frac{\rho B}{2} \right) V_{\text{wind}}^3 \lambda \left( \frac{(1 - \alpha)^2 + \varepsilon^2}{((\varepsilon + 1) - \varepsilon(1 - \alpha)\tan\theta_0)^2} \right) \left[ C_{L_{\alpha_2}} \cos \left( \tan^{-1} \left( \frac{(\varepsilon + 1) - \varepsilon(1 - \alpha)\tan\theta_0}{1 - \alpha} \right) \right) - C_{D_{\alpha_2}} \sin \left( \tan^{-1} \left( \frac{(\varepsilon + 1) - \varepsilon(1 - \alpha)\tan\theta_0}{1 - \alpha} \right) \right) \right] \]

(3.39)

Magnitude of \( \varepsilon \) depends on \( \sigma \) and the incoming wind condition and is needed to be investigated in further studies. However if \( 1 \geq \sigma \) the flow passed the front blades and facing the rear blades is highly affected by the front blades and it can be assumed that \( \varepsilon \approx 1 \) and the power captured by the rear blades can be calculated using equation 3.34.

Power generated by each blade of PowerWindow blades when located in regions 3 and 4 can also be calculated by multiplying the linear velocity magnitude of the blade to the force exerted on the blade, parallel to this velocity. Using equations 3.17 and 3.18, Equation 3.40 shows the power generated by the PoweWindow blades when locating at the top/bottom of PoweWindow.

\[ \text{Power} = \left( \frac{\rho B}{2} \right) V_{\text{rel}}^2 \left[ C_L \cos \beta - C_D \sin \beta \right] (V_{\text{wind}} \lambda \cos \varphi) + \left[ C_L \sin \beta + C_D \cos \beta \right] (V_{\text{wind}} \lambda \sin \varphi) \]

(3.40)

The difficulty in calculating power generation of the top/bottom blade via the BEM model is that there are many items \( (\lambda, \alpha, \varepsilon) \) in Equation 3.40 which are dependent on \( \varphi \) value. Therefore the power generated in those regions is obtained exclusively using CFD simulations. However, the results showed that the power extracted from the top blade when located at its optimum \( \varphi \) value (on the margin of region 3 and 4) might have a considerable (maximum 11%) contribution in the total power generation, but the undesirable force simultaneously exerted on the bottom blade counteracts a significant (70-80%) part of this power generation.
Therefore power generation of the top/bottom blade of PowerWindow has not been further investigated in this study.

### 3.2.3 Derivation of BEM Formulation

When $\varepsilon \cong 1$ the power captured by the PowerWindow can be calculated by equating the total power with summation of the captured power by the front blades using equation 3.26 and captured power by the rear blades using equation 3.32, which results equation 3.38.

\[
\frac{1}{2} \rho AV_{\text{wind}}^3 4a(1-a)^2 = B \left( \frac{\rho C_p}{2} \right) V_{\text{wind}}^3 \lambda \left[ (1-a)^2 + \lambda^2 \right] C_{D_{a1}} \cos(\tan^{-1} \lambda) - \\
C_{D_{a1}} \sin(\tan^{-1} \lambda)] + \\
((1-a)^2 + (2\lambda - (1-a)\tan \theta)^2) \left[ C_{L_{a2}} \cos \left( \tan^{-1} \left( \frac{(2\lambda - (1-a)\tan \theta) - \varepsilon}{(1-a)} \right) \right) - \\
C_{D_{a2}} \sin \left( \tan^{-1} \left( \frac{(2\lambda - (1-a)\tan \theta) - \varepsilon}{(1-a)} \right) \right) \right] \right] \\
(3-41)
\]

If that $\varepsilon < 1$ equation 3.41 would be modified to equation 3.42.

\[
\frac{1}{2} \rho AV_{\text{wind}}^3 4a(1-a)^2 = B \left( \frac{\rho C_p}{2} \right) V_{\text{wind}}^3 \lambda \left[ (1-a)^2 + \lambda^2 \right] C_{D_{a1}} \cos(\tan^{-1} \lambda) - \\
C_{D_{a1}} \sin(\tan^{-1} \lambda)] + \\
\left( (1-a)^2 + ((\varepsilon + 1)\lambda - \varepsilon(1-a)\tan \theta)^2 \right) \left[ C_{L_{a2}} \cos \left( \tan^{-1} \left( \frac{((\varepsilon + 1)\lambda - \varepsilon(1-a)\tan \theta)}{(1-a)} \right) \right) - \\
C_{D_{a2}} \sin \left( \tan^{-1} \left( \frac{((\varepsilon + 1)\lambda - \varepsilon(1-a)\tan \theta)}{(1-a)} \right) \right) \right] \right] \\
(3-42)
\]
The approach to solving equation 3.41 (or 3.42) is to fix a value for $\lambda$ and solve the equation to find $a$ in that $\lambda$. Once $a$ is achieved, the total power captured by the PowerWindow and its front and rear blades can be achieved using equations 16, 28 and 3.34 (or 3.39) respectively. The considerable point about this model is that as the power exerted from every blade is assumed to be equal, this model can predict the flow mechanism and power captured by the PowerWindow in suspended position, not in landed position. MATLAB is used in this study for solving equation 3.41 (or 3.42), calculating $a$ and its relevant powers in every $\lambda$, and plotting them against $\lambda$. The $\lambda$ in which the maximum total power is captured by the PowerWindow, would be its operating condition and that power is ideally its power generation.

Appendix C contains the modified BEM model programed for a PowerWindow prototype configuration (which has been investigated in this study), when 5 blades are located on each side ($N = 5$), each blade and PowerWindow swept area equals $0.3 \, m^2$ and $3.35 \, m^2$ ($B = 0.3 \, m. \, s^{-1}, A = 3.35 \, m. \, s^{-1}$ ), which results in a solidity of $0.428$ ($\sigma = 0.428$), design angle of $18^\circ$ ($\theta_0 = 12^\circ$), when incoming wind velocity is $8 \, m. \, s^{-1}$ ($V = 8 \, m. \, s^{-1}$) and AFR is $0.5$ ($\varepsilon = 0.5$).

Power extraction by the PowerWindow front and rear blades and the relevant $a$, $\alpha_1$ and $\alpha_2$ in every $\lambda$ calculated by the modified BEM model when $\sigma = 0.428$, $\varepsilon = 0.5$ and $1$, $\theta_0 = 12^\circ$, $14^\circ$, $16^\circ$ and $18^\circ$ are reported in the tables in appendix D.
CHAPTER 4. POWERWINDOW COMPUTATIONAL FLUID DYNAMIC MODEL

To assess the validity of the analytical (BEM) model derived for the PowerWindow, a 2D CFD model of the PowerWindow in suspended position has been created. A 2D model of the PowerWindow in landed position has also been created using CFD simulations in Fluent (ANSYS). The modified BEM model cannot predict the flow mechanism and power generation of PowerWindow in landed position, so the result achieved by an experimental test has been compared with CFD results. It should be mentioned that experimental prototyping (with minimum errors) might be very reliable compared to analytical or CFD models, but CFD simulations are much more desirable than experimental prototyping since the time they need and the costs they have are much less. Nevertheless accurate analytical models need practical parameters to express different effects unique to ducted flow.

4.1 Solution Method

The flow over the PowerWindow blades can be subject to significant region of laminar-turbulence transition and the transition process can affect the separation behavior of the boundary layer on the blade surface. Hence selecting an appropriate model for CFD simulation is very important because not every model can predict precisely separation phenomenon. Separation from the blade surface greatly affects the performance of the PowerWindow. SST model has extensively been validated for separating 2D flows with Reynolds-averaged Navier–Stokes (RANS) models [77]. Menter, F.R. [78] showed the SST model outperformed the 4-equation, $v^2$-f (transition SST) model in velocity profiles separation prediction for the NACA 4412 airfoil case. El-Behery, S.M. and M.H. Hamed [79] compared the SST, 7-equation
Reynolds stress model (RSM), v2-f and low-Re k-ε models using the Buice, C.U. and J.K. Eaton [80] data for an asymmetric planar diffuser. The v2-f model gave the best prediction of the separation point. Menter, F.R. [78] suggested that flow over the rotor blades can be subject to significant region of laminar-turbulence transition and because the transition process can affect the separation behavior of the boundary layer on the blade surface, it is agreed that the v2-f model is the best model in case of separation. Therefore v2-f model is selected and used for this study.

4.2 Mesh Structure, Quality and Boundary Conditions

Reliability of the CFD result is dependent on the mesh structure and quality, and also setting appropriate boundary conditions which are discussed in this study.

4.2.1 Mesh Structure

Higher mesh quality is mostly achievable using finer and structured mesh. The number of grids are higher, the simulations speed is slower. In order to make a balance between solution accuracy and calculation time, a combination of structured and unstructured mesh was used. Although structured mesh is the preferred; it is difficult to generate a high quality structured mesh in the entire domain. This technique assists to decreasing number of grid while having a high quality mesh around the body [66]. Therefore rows of very fine structured rectangular grids were generated adjacent to the blade surface as shown in Figure 4.1 (c). The optimum number of these grids is investigated later. This structured region was connected to the structured regions via unstructured triangular grids as shown in Figure 4.1 (b) structured coarser mesh was used around this region in the entire domain which can be seen in this Figure 4.1 (a). Gambit is used for mesh generation in this study.
Figure 4-1 (a). Structured mesh generated around the unstructured region in the entire domain. (b). Combination of structured and unstructured mesh around the blades. (c). Very fine structured rectangular grids adjacent to the blade surface.
4.2.2 Mesh Independence Study

As mentioned before, increasing the number of grids results in decreasing solution speed. Hence there is a challenge in mesh generation between mesh quality (using finer grids) and simulation speed (using coarser grids). Mesh independence study assists to find the optimum grid structure over object boundaries. In this study the mesh quality around the PowerWindow airfoil has been increased in three steps. 50 cells (6 mm) were initially generated around each blade. This number is increased to 100 (3 mm), 200 (1.5 mm) and 400 (0.75 mm) in the next steps. Figure 4.2 (a), (b), (c), and (d) shows the grid structure and size around the airfoil in each step.

![Mesh generated around the PowerWindow airfoil with (a) 50 cells, (b) 100 cells, (c) 200 cells and (d) 400 cells.](image)

The main purpose of this part of mesh independence study is to investigate the effect of mesh quality on the result of the simulations, regardless of their consistency with the experimental or analytical models. Therefore the pressure distribution is investigated on the PowerWindow blades in this study. Figure 4.3 shows pressure
value created over one PowerWindow blade with different number of mesh grids over its boundary. This figure shows that the mesh with 50 and 100 cells on the blades are not reliable enough as the results are very far from the results achieved with 200 and 400 cells, while the result achieved by 200 cells is very close to 400 cells. This means that increasing the number of grids on the blades further than 200 does not improve the simulation results significantly. Hence using such a mesh structure preserves the result accuracy while saves its the calculation time.

![Graph showing pressure created over PowerWindow airfoil](image)

Figure 4-3. pressure created over PowerWindow airfoil when surrounded by different number of cells.

Appendix E contains figures showing (a) the mesh generated around the front, bottom blade of PowerWindow, (b); the pressure contours around this blade (c); and the pressure created over the five front blades of PowerWindow along their cords.
4.2.3 Boundary Conditions

The frame of the hybrid region containing the front and rear blades were set as Multiple Reference Frame (MRF). MRF computational method is a steady-state approximation. In this method individual cell zones can move with different rotational and/or translational speeds. Interfaces are defined between cell zones of a local reference frame and the adjacent cell zones to enable computing (interpolating) flow variables in between them. In the MRF approach relative motion of a moving zone is not accounted respecting the adjacent zones (moving or stationary); but the grid are fixed for the computation. Fluent enforces the continuity of the absolute velocity at the interfaces between two cell zones.

In the PW CFD model, the inlet boundary condition is constant free stream velocity of $8 \text{m.s}^{-1}$ and outlet boundary is set to atmospheric pressure. The top and bottom wall were set as stationary walls for the landed PowerWindow, but moving wall with the same velocity as incoming wind for the suspended PowerWindow. This condition assists to create more uniform flow (as expected in practice) over the PowerWindow blades in suspended position, since as shown in Figure 4.4 (a) and (b), boundary layers of the stationary walls at the top and bottom of the wind tunnel create a non-uniform flow over the PowerWindow blades. The dashed line shows the uniform flow in the simulated wind tunnel which can be achieved by setting moving walls with the same velocity as incoming wind.
Figure 4-4. Non-uniform flow due to boundary layers of the stationary walls at top and bottom of the wind tunnel over PowerWindow in (a). suspended and (b). landed position.
CHAPTER 5. RESULTS AND DISCUSSION

The modified BEM model obtained in this study might be the best tool to analyze and enhance the performance of PowerWindow in further studies. Effect of solidity and design angle on the performance of PowerWindow is investigated and discussed using this model in this study.

However, in a prior step, the modified BEM model is used to investigate its validity by comparing its results with the ones obtained by CFD and experimental model. Hence PowerWindow efficiency is also investigated in term of coefficient of performance ($C_p$) using all three approaches and been compared in this study. $C_p$ shows the ratio of the power captured by the generator to the entire wind energy passing through its swept area. Equation 5.1 gives $C_p$ definition.

$$C_p = \frac{P_{\text{measured}}}{P_{\text{wind}}} = \frac{T \omega}{0.5 \rho A U^3}$$  \hspace{1cm} (5-1)

5.1 Coefficient of Performance by BEM, CFD and Experimental Model

Power generation of PowerWindow, operating in suspended and landed position achieved by BEM, CFD and experimental model has been investigated in this part in term of $C_p$ in order to: (i) validate its CFD results by the BEM results in suspended position; (ii) verify its CFD results by the experimental result in landed position; and also (iii) find the flow mechanism and power generation of the PowerWindow front and rear blades in those positions.
5.1.1 Suspended Position

Figures 5.1 (a) and (b) (extracted from Tables D.1-D.8 in appendix D) show \( a \) calculated by the modified BEM model of the suspended PowerWindow when \( \sigma = 0.428 \) against \( \lambda \) when \( \theta_0 = 12^\circ, 14^\circ, 16^\circ \) and \( 18^\circ \). Figure 5.1 (a), shows \( a \), calculated with \( \varepsilon = 0.5 \) and Figure 5.1 (b) shows \( a \), calculated with \( \varepsilon = 1.0 \).
Both Figures 5.1 (a) and (b) show that there is a maximum $\alpha$ for each $\theta_0$ which increases and occurs in higher $\lambda$ by increasing $\theta_0$. Considering equations 2.15 and 3.5, the maximum power generation occurs when $\alpha = 1/3 \approx 0.33$, which results in $C_p = 59\%$ (Betz limit). Since the maximum $\alpha$ achieved in these operating conditions is $\alpha = 0.175$ (belongs to $\theta_0 = 18^\circ$), the $C_p$ is not expected to be very high. However this shows that increasing $\theta_0$ may assist to optimize PowerWindow efficiency. This is more investigated later in this study.

Comparing Figures 5.1 (a) and (b) also shows that when $\varepsilon = 1.0$, higher $\alpha$ is predicted by the modified BEM model rather than to when $\varepsilon = 0.5$. Moreover as mentioned before, as $\sigma$ is higher $\varepsilon$ is closer to 1.0. This shows that increasing $\sigma$ may assist to optimize PowerWindow efficiency. This is also more investigated later in this study.

Figures 5.2 (a), (b), (c) and (d) show the power captured by the suspended PowerWindow in term of $C_p$ when $\sigma = 0.428$ against $\lambda$ when $\theta_0 = 12^\circ, 14^\circ, 16^\circ$ and $18^\circ$ achieved by BEM and CFD models. Since the exact value of $\varepsilon$ has not been investigated in this study, $C_p$ of the suspended PowerWindow achieved by BEM is plotted once when $\varepsilon = 1.0$, and once when $\varepsilon = 0.5$. However the exact value of $\varepsilon$ is needed to be found based on incoming wind velocity, operating $\lambda$ and the $\sigma$ of PowerWindow in further studies. It should be mentioned that $P_{\text{front}}, P_{\text{rear}}$ and $P_{\text{total}}$ refer to the power generated by the front blades, rear blades and the total power captured by PowerWindow. These figures show that the power generation achieved by the CFD models, are within the range predicted by the modified BEM model.
Figure 5-2. $C_P$ of PowerWindow in suspended position in an ideal wind tunnel when $\sigma = 0.428$ against $\lambda$ when (a). $\theta_0 = 12^\circ$, (b). $\theta_0 = 14^\circ$, (c). $\theta_0 = 16^\circ$, and (d). $\theta_0 = 18^\circ$ using BEM and CFD models.
Figures 5.2 (a), (b), (c) and (d) show that both BEM and CFD model agrees that the power captured by the rear blades is greater than the front blades in the major range of $\lambda$. Comparing equations 3.26 and 3.32 (or 3.38 when $\epsilon \neq 1$) shows that $\alpha_2$ is higher than $\alpha_1$ when $\lambda < (1 - a)\tan\theta_0$. Although there is a limited range in which $\alpha_2$ is higher than $\alpha_1$, BEM results shows that in these configurations and operating $\lambda$, $\alpha_2$ is higher than $\alpha_1$ and they both are between $0^\circ$ and $20^\circ$ ($0^\circ < \alpha_1 < \alpha_2 < 20^\circ$). The reason is that the vertical velocity of the flow increases when passes the front blades and creates a higher $\alpha$ over the rear blades. Figure 3.6 (a) shows that for $0^\circ < \alpha < 20^\circ$, increase in $\alpha$ results in increase in $C_\ell$. Hence vertical force exerted on the rear blades, and consequently their power generation are expected to be greater than the front blades. As a result the front blades assist the rear blade to capture greater power.

Lee et al. [11] showed that using a counter rotating rotor in the counter rotating HAWTs enhances the $C_p$, but not in every design condition. For example using a counter rotating rotor with the same radius, rotating speed and pitch as the front one behind it, has decreased $C_p$ from 0.448 ($C_p$ of the relative single rotor) to 0.445, but by optimizing the pitch difference between the front and rear blades, $C_p$ has increased to 0.457. Similar optimization can also be done on the PowerWindow by creating a design angle difference between its front and rear blades in further studies. Although $C_p$ of the counter rotating HAWT was not significantly greater than its single rotor, this design is considerably more efficient than a single rotor design, because two turbines have been installed on one structure and operating with a slightly higher $C_p$ in this design.
A major difference in $C_p$ calculation of the PowerWindow and counter rotating HAWT is that in the counter rotating HAWT, $C_p$ shows the ratio of the total power captured by the front and rear rotor to their total swept area, but in the PowerWindow, $C_p$ shows the ratio of the total power captured by the front and rear rotor to their common swept area. Because as mentioned before, in the counter rotating HAWT, the downstream flow of the front rotor would be fully developed before reaching its rear rotor inlet, while in PowerWindow, power is assumed to be extracted during one flow expansion and deceleration.

Another point which Figures 5.2 (a), (b), (c) and (d) show is that an advantage of PowerWindow is that it can operate with a 0.11 efficiency in very low linear velocity ratios ($0.1 < \lambda < 0.2$) which is not achievable with conventional wind turbines. It can also be seen that increasing $\theta_0$ results in greater power extraction from the PowerWindow and shifts the optimum $\lambda$ (the operation point) to higher values. Equations 3.26 and 3.32 (or 3.38 when $\varepsilon \neq 1$) show that increasing $\theta_0$ generally increases $\alpha$ and as discussed before increasing $\alpha$ results in increasing the lift force exerted on the blades and capturing greater power from wind energy. This shows that performance of the PowerWindow is greatly affected by $\theta_0$ which can be increased and optimized in further studies.

5.1.2 Landed Position

As mentioned before, BEM model cannot predict the flow mechanism and power generation of PowerWindow in landed position, because the aerodynamic forces exerted on the PowerWindow blades are very non uniform in this position. Nevertheless the PowerWindow prototype (configured in $\sigma = 0.428$ and $\theta_0 = 16$)
has been experimentally tested in landed position, the measured data compared to CFD simulation results. As shown in Figure 5.3 (a) the prototype is placed on the ground in a wind tunnel and a small ramp is embedded at its inlet bottom which helps to accelerate overall magnitude and increase vertical component of wind when entering PowerWindow. Wind velocity is 8 m/s in the wind tunnel. The power generated in the generators has been recorded against time and shown in Figure 5.3 (b). As can be seen the maximum power generated by PowerWindow with this configuration is around 140 W which results in $C_p = 0.1467$. Linear velocity of the PowerWindow blades has been measured within 1.5 m/s and 2.0 m/s in this condition which results in $0.1875 < \lambda < 0.25$. It should be mentioned that the total captured power is greater than this since some power will always be wasted due to transmission loss.

![Figure 5-3 (a). Prototype of the PowerWindow operating in landed position when $\sigma = 0.428$ and $\theta_0 = 16$ in wind tunnel, and (b) its power generator recorded results against time.](image)

For coordinating the CFD model boundary conditions to the PowerWindow prototype, its CFD model was located in landed position in the wind tunnel and a
small ramp was embedded at its inlet bottom. The top and bottom walls were also set to stationary walls. The rest of the variables in this CFD model were exactly similar to the CFD model of the suspended PowerWindow generated for the BEM model. Figures 5.4 shows the total power captured by the PowerWindow prototype (when $\theta_0 = 16^\circ$) and the landed PowerWindow achieved by the CFD model when $\theta_0 = 12^\circ, 14^\circ, 16^\circ$ and $18^\circ$ and $\sigma = 0.428$ in term of $C_p$ against $\lambda$. The reason for investigating $C_p$ in $\theta_0 = 14^\circ, 16^\circ$ and $18^\circ$ is that although in the prototype $\theta_0 = 16^\circ$, it was seen that the prototype belts were slightly bent due to the wind axial force during the experimental test, which should have created a slight deviation in $\theta_0$ when transferring the blades from bottom to top and vice versa.

Figure 5-4. $C_p$ of the PowerWindow prototype (when $\theta_0 = 16^\circ$) and the landed PowerWindow achieved by the CFD model when $\theta_0 = 14^\circ, 16^\circ$ and $18^\circ$ and $\sigma = 0.428$ against $\lambda$.

Figure 5.4 shows that the operating $\lambda$ and the maximum $C_p$ of the PowerWindow prototype is very close to the values expected by the CFD model (when $\theta_0 = 16^\circ$), which assists to validated the results obtained by the CFD model. Comparing Figure
5.4 and Figures 5.2 (b), (c) and (d) also shows that a greater $C_p$ (50-70%) is achieved by the PowerWindow in landed position rather than suspended position. The reason is discussed later.

5.2 Effect of Installation Configuration on the Coefficient of Performance

Comparing Figure 5.4 and Figures 5.2 (b), (c) and (d) shows that $C_p$ of the PowerWindow has increased from 0.065, 0.08 and 0.1 in suspended position to 0.12, 0.14 and 0.17 in landed position, which means that the performance of the PowerWindow installed in the landed position (in this boundary and operating conditions) is around 70% higher than the same PowerWindow installed in the suspended position. Figure 5.5 (a), (b), (c) and (d) show the power captured by the suspended and landed PowerWindow in term of $C_p$ when $\sigma = 0.428$ against $\lambda$ when $\theta_0 = 12^\circ, 14^\circ, 16^\circ$ and $18^\circ$ achieved by the CFD models. In these figures $P_{\text{Suspended\_front}}$, $P_{\text{Suspended\_rear}}$ and $P_{\text{Suspended\_total}}$ refer to the power generated by the front blades, rear blades and the total power captured by the suspended PowerWindow and $P_{\text{Landed\_front}}$, $P_{\text{Landed\_rear}}$ and $P_{\text{Landed\_total}}$ refer to the power generated by the front blades, rear blades and the total power captured by the landed PowerWindow.
Figure 5-5. $C_p$ of the PowerWindow in suspended and landed position when $\sigma = 0.428$ against $\lambda$ when (a) $\theta_0 = 12^\circ$, (b) $\theta_0 = 14^\circ$, (c) $\theta_0 = 16^\circ$, and (d) $\theta_0 = 18^\circ$ obtained by the CFD models.
Figure 5-6. $R_V$ contours of the wind passing (a) the suspended PowerWindow and (b) the landed PowerWindow, with the ramp located at its inlet bottom ($\sigma = 0.428$ and $\theta_0 = 16^\circ$)
Figure 5-7. Velocity vectors around (a) suspended; and (b) landed PowerWindow bottom blades when $\sigma = 0.428$ and $\theta_0 = 12^\circ$
Figure 5-8. Velocity vectors around (a) suspended; and (b) landed PowerWindow bottom blades when $\sigma = 0.428$ and $\theta_0 = 14^\circ$
Figure 5-9. Velocity vectors around (a) suspended; and (b) landed PowerWindow bottom blades when $\sigma = 0.428$ and $\theta_0 = 16^\circ$
Figure 5-10. Velocity vectors around (a) suspended; and (b) landed PowerWindow bottom blades when $\sigma = 0.428$ and $\theta_0 = 18^\circ$
As can be seen in Figure 5.5 (a), (b), (c) and (d) a significant difference between the suspended PowerWindow and landed PowerWindow is that on the contrary of the suspended PowerWindow, the power captured by the front blades are greater than the power captured by the rear ones in landed PowerWindow. Total power captured by the landed PowerWindow is also higher than suspended PowerWindow, and the operation point of the PowerWindow has shifted to higher $\lambda$s in the landed position. For example the maximum $C_p$ of the suspended PowerWindow when $\theta_0 = 16^\circ$ is 0.08, in $\lambda = 0.175$, while the maximum $C_p$ of the landed PowerWindow is 0.14, in $\lambda = 0.2$. The reason can be found by investigating the flow mechanism when passing both models. Figure 5.6 (a) and (b) show the velocity ratio ($R_V$) contours of the wind passing the suspended PowerWindow and the landed PowerWindow with the ramp located at its inlet bottom. $R_V$ shows the local air velocity ratio to the wind velocity in these figures (equation 5.2).

$$R_V = \frac{v_{local}}{v_{wind}}$$  \hspace{1cm} (5-2)

As can be seen in Figure 5.6 (a) in the suspended PowerWindow, the maximum $R_V$ is around 1.2 within the blades, while as can be seen in in Figure 5.6 (b) in landed PowerWindow it is almost 1.5. This is because: (i) air flow cannot expand from bottom side of the landed PowerWindow; and (ii) the ramp at the bottom of landed PowerWindow inlet leads more air to its swept area which creates higher velocity there. Increase in the velocity magnitude in the PowerWindow results in increase in its dynamic pressure and as the dynamic pressure of the upstream flow of the PowerWindow increases, higher pressure would be exerted on the pressure side of its blades. On the other hand, vertical velocity of the wind increases when passing above
the ramp. Increase in wind vertical velocity at the upstream of the front blades results in increase in $\alpha_1$ and decrease in $\alpha_2$. This is investigated in Figures 5.7-10 (a) and (b) which show the velocity vectors around (a) suspended; and (b) landed PowerWindow bottom blades when ($\sigma=0.428$ and) $\theta_0 = 12^\circ$, $14^\circ$, $16^\circ$ and $18^\circ$. As a result, $P_2 - P_3 > P_4 - P_5$ in this position, while in suspended position, $P_4 - P_5 > P_2 - P_3$. $P_2, P_3, P_4$ and $P_5$ refer to the static pressure of air in the sections shown in Figure 5.11. This can more clearly be seen in the pressure contours around the suspended and landed PowerWindow.

![Pressure Contours](image)

Figure 5-11. Air static pressure when passing PowerWindow.

The regions within the dashed line in Figures 5.7-10 (a) and (b) indicate the separation point on the suction sides of the blades. These figures show that the separation point has slightly shifted to the trailing edge of the blades using the ramp at the bottom of the landed PowerWindow. Therefore using the ramp can change the stall condition and its effect can be investigated in further studies.
Figure 5-12. $C_{SP}$ contours of the wind passing (a) the suspended PowerWindow and (b) the landed PowerWindow, with the ramp located at its inlet bottom in $\lambda = 1.0$ ($\sigma = 0.428$ and $\theta_0 = 16^\circ$).
Figure 5-13. $C_{SP}$ contours of the wind passing (a) the suspended PowerWindow and (b) the landed PowerWindow, with the ramp located at its inlet bottom in $\lambda = 1.5$ ($\sigma = 0.428$ and $\theta_0 = 16^\circ$).
Figure 5-14. $C_{SP}$ contours of the wind passing (a) the suspended PowerWindow and (b) the landed PowerWindow, with the ramp located at its inlet bottom in $\lambda = 2.0$ ($\sigma = 0.428$ and $\theta_0 = 16^\circ$).
Figure 5-15. $C_{SP}$ contours of the wind passing (a) the suspended PowerWindow and (b) the landed PowerWindow, with the ramp located at its inlet bottom in $\lambda = 2.5$ ($\sigma = 0.428$ and $\theta_0 = 16^\circ$).
Figure 5-16. $C_{SP}$ over the bottom front and rear blades of the suspended and landed PowerWindow (when $\sigma = 0.428$, $\theta_0 = 16^\circ$) in different $\lambda$. 

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Figure 5-17. $C_{SP}$ over the front and rear blades of the suspended and landed PowerWindow (when $\sigma = 0.428$, $\theta_0 = 16^\circ$) in operating condition.
To have deeper insight over the pressure distribution on the front and rear blades of the suspended and landed PowerWindow, their $C_{sp}$ value is investigated and plotted through their cord line using CFD simulation tools. Figures 5.12 -15 show $C_{sp}$ contours around the front and rear blades of the suspended and landed PowerWindow when $\sigma = 0.428$, $\theta_0 = 16^\circ$ in $\lambda = 0.1$, $0.15$, $0.2$, and $0.25$. Figures 5.16 (a), (b), (c) and (d) also show $C_{sp}$ value over the front and rear blades of the suspended and landed PowerWindow when $\sigma = 0.428$, $\theta_0 = 16^\circ$ in $\lambda = 0.1$, $0.15$, $0.2$, and $0.25$. These figures show that increasing $\lambda$ has remarkably decreased $C_{sp}$ over the suspended and landed PowerWindow front and rear blades.

Figures 5.17 (a), (b), (c) and (d) show $C_{sp}$ value over the front and rear blades of the suspended and landed PowerWindow (when $\sigma = 0.428$, $\theta_0 = 16^\circ$) in operating condition ($\lambda = 0.15$ for the suspended PowerWindow and $\lambda = 0.2$ for the landed PowerWindow). In Figures 5.17 (a) and (b), $C_{sp}$ curves are very close together. This shows that almost a uniform force is exerted on suspended PowerWindow front and rear blades, which confirm the assumption considered for the BEM model. Moreover $C_{sp}$ curves over the suction side of the rear blades of the suspended PowerWindow are lower than front blades, hence the distance between the $C_{sp}$ curves over the pressure and suction side of the suspended PowerWindow rear blades, plotted in Figure 5.17 (b), are greater than the distance between the $C_{sp}$ lines of its front blades, shown in Figure 5.17 (a). This confirms the BEM model result which has calculated a greater power generation by the rear blades of the suspended PowerWindow compared to its front blades.
On the contrary of the $C_{SP}$ distribution over the suspended PowerWindow blades in Figure 5.17 (a) and (b) in Figures 5.17 (c) and (d), $C_{SP}$ distributions over the blades in landed PowerWindow are very non-uniform, and $C_{SP}$ lines over the suction side of its front blades are achieved lower than the $C_{SP}$ lines of its rear blades. In this figure, pressure lines show the $C_{SP}$ value on the blades, which have been named respectively from the bottom to the top of the landed PowerWindow shown in Figures 5.14 (b). Pressure lines show that as the blades are closer to the ramp (bottom of the landed PowerWindow) the sub-atmospheric pressure on their suction side is lower, so that as can be seen in Figure 5.17 (b), the minimum $C_{SP}$ is almost -2 which belongs to the suspended PowerWindow rear blades, while as can be seen in Figure 5.17 (c) it is almost -4, which belongs to the bottom blade among the landed PowerWindow front blades.

A noticeable point in Figures 5.16 and 5.17 (a), (b), (c) and (d) is that the $C_{SP}$ line crosses the zero pressure line on the pressure side of the blades, so that $C_{SP}$ has a positive value at the leading edge of the blades but a negative value at their trailing edge. The reason is that the flow strangely decelerates when reaching the leading edge and its pressure increases almost to the stagnation pressure. While the flow velocity increases to a value greater than the wind incoming velocity when reaches the trailing edge. As a result the pressure decreases to a value lower than the incoming wind (atmospheric) pressure.
5.3 Effect of Solidity on the Coefficient of Performance

As discussed before, increasing $\sigma$ may results in increase in the efficiency of PowerWindow. Therefore an investigation has been done to understand effect of $\sigma$ on the $C_p$ of using the modified BEM model for suspended PowerWindow. Figure 5.18 shows $C_p$ of the suspended PowerWindow against $\lambda$ when $\theta_0 = 16^\circ$ when $\sigma = 0.428$, 0.857 and 1.714. The assumptions considered for $\varepsilon$ is based on value of $\sigma$. As mentioned before if $\sigma > 1$ it might be accepted to assume $\varepsilon \approx 1.0$, but as $\sigma \ll 1$, $\varepsilon$ would be closer to zero ($\varepsilon \rightarrow 0$). Hence $\varepsilon$ is approximated $\varepsilon = 0.5$, 0.75 and 1.0 for $\sigma = 0.428$, 0.857 and 1.714 respectively.

![Figure 5.18](image)

Figure 5-18. $C_p$ of the suspended PowerWindow against $\lambda$ when $\theta_0 = 16^\circ$ (and $\varepsilon = 0.5$) when $\sigma = 0.428$ (and $\varepsilon = 0.75$), 0.857 and 1.714 (and $\varepsilon = 1.0$).

Figure 5.18 shows that the optimum $\lambda$ is not dependent on the $\sigma$ value. This figure also shows that increasing $\sigma$ from 0.428 to 0.857 has greatly enhanced the $C_p$, but increasing $\sigma$ from 0.857 to 1.714 has not remarkably enhanced it. The reason can be
investigated in Figure 3.5 (a). It is shown that as the $\sigma$ is higher, in a same $\alpha$, $C_L$ is lower, which results in decrease in vertical force exerted on each blade. However increase in $\sigma$ also results in increase in projection area which increases the total force exerted on all the blades. Hence there is always an optimum $\sigma$ for a $\theta_0$ in which the maximum $C_p$ would be gained and further the $C_p$ decreases. More extensive studies are needed to investigate the optimum $\sigma$ for each $\theta_0$.

Accuracy of the modified BEM model prediction on $C_p$ is also highly dependent on $\varepsilon$, and $\varepsilon$ is a function of $\sigma$ and $\lambda$. Hence further investigations are needed to find the relation between $\varepsilon$ and $\sigma$ and $\lambda$.

5.4 Effect of Design angle on the Coefficient of Performance

As discussed before, increasing $\theta_0$ may results in increase in the efficiency of PowerWindow. Therefore an investigation has been done to understand effect of $\theta_0$ on the $C_p$ of using the modified BEM model for suspended PowerWindow. Figure 5.19 shows $C_p$ of the suspended PowerWindow against $\lambda$ when $\theta_0 = 6^\circ, 12^\circ, 18^\circ$ and $24^\circ$ when $\sigma = 0.428$ and assuming $\varepsilon = 0.5$. This figure shows that increasing $\theta_0$ has significantly enhanced the $C_p$ and shifted the optimum $\lambda$ (the operation point) to higher values. As mentioned before equations 3.26 and 3.32 (or 3.38 when $\varepsilon \neq 1$) show that increasing $\theta_0$ generally increases $\alpha$ and as discussed before increasing $\alpha$ results in increasing the lift force exerted on the blades and capturing greater power from wind energy. Hence further studies can be done on optimization of the PowerWindow performance based on $\theta_0$ adjustment.
In this study $C_p$ has not been calculated for $\theta_0 > 24^\circ$, since stall is highly possible in further $\theta_0$, while its possibility is dependent on $\sigma$ for the front and rear blades and also $\varepsilon$ for the rear ones. As mentioned before $\varepsilon$ is a function of $\sigma$ and $\lambda$. Hence further studies are needed to investigate effect of $\sigma$ and $\lambda$ over $\varepsilon$ and also $\sigma$ and $\varepsilon$ over $C_L$ and $C_D$.

5.5 Effect of the Blades Position on the Coefficient of Performance

For investigating how locating the rear blades next to the front ones impacts on the $C_p$ of PowerWindow, the CFD model of the prototype ($\sigma = 0.428$ and $\theta_0 = 16^\circ$) has been built and tested in four different poses shown in Figure 5.20. In each pose $L_0$ shows the elevation of the front blade to its adjacent rear blade. The $L_0/C$ ratio is $2/3, 1/3, 0$ and $-1/3$ respectively in pose A, B, C and D while the cord length of each blade is 15cm ($C = 15cm$). This approach allows investigating the relation
between the power generation performance of PowerWindow and position of the front blades to the rear ones. Figure 5.21 shows the $C_p$ achieved by the front, rear and the total blades of the CFD model versus $\lambda$. The arrow within the horizontal solid lines shows the $C_p$ achieved by the prototype in the experimental test. The curved lines show the $C_p$ predicted by the CFD model in different $\lambda$s. The operating condition of the CFD model is the $\lambda$ in which the maximum $C_p$ is achieved ($\lambda = 0.2$). Figure 5.21 demonstrates that the fluctuation in the maximum $C_p$ of the PowerWindow prototype was also expected by the CFD model.

Figure 5-20. Front and rear blades of the PowerWindow CFD model in poses A: $L_0/C = \lambda$, B: $L_0/C = 1/3$, C: $L_0/C = 0$ and D: $L_0/C = -1/3$ ($C = 15\text{ cm}$).

Figure 5.21 shows that the $C_p$ of PowerWindow slightly depends on how the rear blades are located next to the front ones which results in a sinusoidal (quasi-steady) $C_p$. The reason has been investigated via the velocity and pressure distribution analysis. The sinusoidal (quasi-steady) $C_p$ is not desirable, as it can result in noise creation and fatigue failure. However, increasing $\sigma$ might result in decreasing the fluctuation, which needs more investigations in further studies.
Figure 5-21. $C_p$ of the PowerWindow prototype in operating condition (shown by the arrow between two horizontal solid lines in $\lambda \approx 0.2$), and the CFD model in A, B, C and D poses when $\theta_0 = 16^\circ$ and $\sigma = 0.428$ versus $\lambda$. 
Figures 5.22 (a), (b), (c) and (d) show the velocity ratio \( R_v \) the contours of the wind passing the PowerWindow model when \( \sigma = 0.428 \) and \( \theta_0 = 16^\circ \) in operating condition \( (\lambda = 0.2) \) respectively in poses A, B, C and D. A comparison between Figures 5.22 (a), (b), (c) and (d) show that in pose C and D the rear blades are located in the lower velocity area of the downstream of the front blades which not only adversely affect the wind flow over the rear blades but also blocking the downstream flow of the front blades might decrease the overall pressure drop of the flow passing PowerWindow.

Figure 5.23 (a), (b), (c) and (d) show the streamlines around the PowerWindow blades in \( \lambda = 0.2 \) (the operating condition) when \( \sigma = 0.428 \) and \( \theta_0 = 16^\circ \) in pose A, B, C and D. This figure shows that in pose A and B the front blades adding a downward velocity to the incoming wind, lead the upstream flow of the rear blades toward them, while in pose D the front blades have almost no effect of the upstream flow of the rear blades. It can also be seen that in pose C the front blades have slightly blocked the upstream flow of the rear blades. Hence the rear blades in pose C, are expected to have a lower \( C_p \).

Figures 5.24 (a), (b), (c) and (d) show the pressure contours of the wind passing over the middle blades of the PowerWindow model when \( \sigma = 0.428 \) and \( \theta_0 = 16^\circ \) in operating condition \( (\lambda = 0.2) \) respectively in poses A, B, C and D. Figure 5.25 (a) and (b) show the \( C_{sp} \) value over these blades. Figure 5.24 and 5.25 agree that the power generated by PowerWindow in pose A and B is higher than C and D due to the higher pressure created on their pressure side and lower pressure created on their suction side.
Figure 5-22 $R_v$ contours of the wind passing the PowerWindow CFD model in $\lambda = 0.2$ (the operating condition) when $\sigma = 0.428$ and $\theta_0 = 16^\circ$ in (a) pose A, (b) pose B, (c) pose C and (d) pose D.
Figure 5-23 Streamlines around the middle blades of PowerWindow model in $\lambda = 0.2$ (the operating condition) when $\sigma = 0.428$ and $\theta_0 = 16^\circ$ in (a) pose A, (b) pose B, (c) pose C and (d) pose D.
Figure 5-24 Air static pressure over the middle blades of PowerWindow CFD model in $\lambda = 0.2$ (the operating condition) when $\sigma = 0.428$ and $\theta_0 = 16^\circ$ in (a) pose A, (b) pose B, (c) pose C and (d) pose D.
Figure 5-25 $C_{SP}$ distribution over the (a) front and (b) rear middle blades of PowerWindow CFD model, when $\sigma = 0.428$, $\theta_0 = 16^\circ$ and $\lambda = 0.2$ though the cord length.
CHAPTER 6. CONCLUSION AND PROSPECTS

An analytical model based on the modified BEM theory successfully predicted the performance of a suspended PowerWindow. CFD model of the suspended PowerWindow has also been created and its results have been validated by the BEM model. CFD model of a landed PowerWindow was also created and an analogy has been done between this type and the suspended one. Validity of the CFD model of the landed PowerWindow was confirmed by an experimental test from the prototype operating in same condition.

Investigations on the aerodynamic mechanism of the PowerWindow showed that this device can generate electricity with an acceptable efficiency (around 15%) in very low linear velocity ratios (around 0.2) which is not achievable with most of the common wind turbines. Studies showed that front blades of PowerWindow can increase the angle of attack of the rear blades and consequently raise their performance significantly. It was also shown that using a ramp at the bottom of the PowerWindow inlet can greatly increase the power captured by the PowerWindow by increasing its inlet velocity magnitude and its direction. By using the ramp for the prototype the coefficient of performance of the PowerWindow with solidity of 0.428 was enhanced from 5%, 6.5%, 8% and 10% to 9.5%, 12%, 14.5% and 17% (which means 70 – 100% increase) respectively.

The BEM and CFD model developed in this study assist to analyze PowerWindow mechanism and performance, and can be very useful for optimization of the device and enhancing its performance. Effect of solidity and design angle on
performance of the suspended PowerWindow was also investigated in this study and it was shown that:

(i) Performance of the PowerWindow can be enhanced by increasing the design angle.

(ii) Increasing the design angle further than an optimum point, increases possibility of stall on PowerWindow blades which eventually results in decrease in its performance.

(iii) Increasing the solidity of PowerWindow, decrease possibility of stall and increases the angle in which stall happens.

(iv) Increasing solidity of the PowerWindow till an optimum point enhances its performance.

(v) Performance of the rear blades of PowerWindow is highly dependent of affected flow ratio, which is dependent on the solidity of the PowerWindow and its operating linear velocity ratio.

(vi) Coefficient of performance of PowerWindow slightly depends on how the rear blades are located next to the front ones, which results in a sinusoidal power generation, and is not desirable, as it results in noise creation and fatigue failure.

(vii) Increasing the solidity might assists to minimize the fluctuation of PowerWindow.
REFERENCES


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Figure A-1. A number of commercial airfoils with different applications


B. APPENDIX B

$C_L$ and $C_D$ data file of the isolated airfoil and cascade configuration of the

the PowerWindow blade.

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C. APPENDIX C

Modified BEM model programed for a PowerWindow prototype configuration (which has been investigated in this study), when 5 blades are located on each side \((N = 5)\), each blade and PowerWindow swept area equals 0.3 \(m^2\) and 3.35 \(m^2\) \((B = 0.3 \, m.s^{-1}, A = 3.35 \, m.s^{-1})\), which results a solidity of 0.428 \((\sigma = 0.428)\), design angle of 18° \((\theta_0 = 12°)\), when incoming wind velocity is 8 \(m.s^{-1}(V = 8 \, m.s^{-1})\) and AFR is 0.5 \((\epsilon = 0.5)\).

```matlab
syms i
N=5;
A=3.35;
B=0.30;
S=(N*B)/A;
e=0.5;
T=12*(pi/180);
V=8;
counter=0;
deg=11;
format long
Coef_Cl=polyfit(a,Cl,deg)
deg=5;
format long
Coef_Cd=polyfit(a,Cd,deg)
for y=0.025:0.025:0.225
    counter=counter+1;
    Y
    b1=atan(y/(1-i));
    a1=(T-b1)*(180/pi);
    Cl1= poly2sym(Coef_Cl,a1);
    Cd1= poly2sym(Coef_Cd,a1);
    b2=atan(((1+e)*y-(1-i)*e*tan(T))/(1-i));
    a2=(T-b2)*(180/pi);
    Cl2= poly2sym(Coef_Cl,a2);
    Cd2= poly2sym(Coef_Cd,a2);
    P1= V*y*.5*1.2*V^2*N*B* (1-2*i+i^2+y^2 )*(Cl1*cos(b1) - Cd1*sin(b1));
    P2= V*y*.5*1.2*V^2*N*B*(1-2*i+i^2+((1+e)*y-e*(1-i))*tan(T))^2*(Cl2*cos(b2) - Cd2*sin(b2));
    Pt=P1+P2;
    Pw=0.5*1.2*V^3*A* 4*i*(1-i)^2 ;
    x=solve (Pw - Pt == 0, i)
% digits(1000);
x=double(x);
    b1=atan(y/(1-x));
    a1=(T-b1)*(180/pi)
    Cl1= polyval(Coef_Cl,a1);
    Cd1= polyval(Coef_Cd,a1);
    b2=atan(((1+e)*y-(1-x)*e*tan(T))/(1-x));
    a2=(T-b2)*(180/pi)
    Cl2= polyval(Coef_Cl,a2);
    Cd2= polyval(Coef_Cd,a2);
```
\begin{align*}
P_1 &= V*y^*.5*1.2*V^2*N*B*(1-2*x+x^2+y^2)*(C1*cos(b1) - C1*sin(b1)) \\
P_2 &= V*y^*.5*1.2*V^2*N*B*(1-2*x+x^2+(1+e)*y-e*(1-x)^2)*tan(T))^2)*(C12*cos(b2) - C12*sin(b2)) \\
Pt &= P1 + P2 \\
Pw &= 0.5*1.2*V^3*A*4*x*(1-x)\^2; \\
fP1(counter) &= P1; \\
fP2(counter) &= P2; \\
fPt(counter) &= Pt; \end{align*}

\textbf{Note 1:} $\sigma, \epsilon, \theta_0, \alpha, \beta, \lambda, P_{front}, P_{rear}$ and $P_{total}$ are replaced with $S, e, T, a, b, i$ (or $x$), $y, P_1, P_2$ and $P_t$ in the program.

\textbf{Note 2:} $C_L$ and $C_D$ data file for the relevant cascade configuration (reported in appendix B), should be opened before running the program.
D. APPENDIX D

Power extraction by the PowerWindow front and rear blades and the relevant \( \alpha \) and \( \alpha_2 \) in every \( \lambda \) calculated by the modified BEM model when \( \sigma = 0.428 \), \( \epsilon = 0.5 \) and \( \theta_0 = 12^\circ, 14^\circ, 16^\circ \) and \( 18^\circ \)

Table D.1. Modified BEM results when \( \sigma = 0.428 \), \( \epsilon = 0.5 \) and \( \theta_0 = 12^\circ \)

<table>
<thead>
<tr>
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<th>( P_{\text{total}}(W) )</th>
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<th>( P_{\text{front}}(W) )</th>
<th>( \alpha_2 )</th>
<th>( P_{\text{rear}}(W) )</th>
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<td>13.75897</td>
<td>17.82107</td>
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<td>0.009803</td>
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<td>7.668545</td>
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Table D.2. Modified BEM results when \( \sigma = 0.428 \), \( \epsilon = 1 \) and \( \theta_0 = 12^\circ \)

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<th>( P_{\text{total}}(W) )</th>
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<th>( P_{\text{front}}(W) )</th>
<th>( \alpha_2 )</th>
<th>( P_{\text{rear}}(W) )</th>
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Table D.3. Modified BEM results when \( \sigma = 0.428 \), \( \epsilon = 0.5 \) and \( \theta_0 = 14^\circ \)

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<th>( P_{\text{front}}(W) )</th>
<th>( \alpha_2 )</th>
<th>( P_{\text{rear}}(W) )</th>
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Table D.4. Modified BEM results when $\sigma = 0.428$, $\epsilon = 1$ and $\theta_0 = 14^\circ$

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<th>$P_{\text{total}}(W)$</th>
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<th>$P_{\text{front}}(W)$</th>
<th>$\alpha_2$</th>
<th>$P_{\text{rear}}(W)$</th>
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Table D.5. Modified BEM results when $\sigma = 0.428$, $\epsilon = 0.5$ and $\theta_0 = 16^\circ$

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<th>$\alpha_2$</th>
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Table D.6. Modified BEM results when $\sigma = 0.428$, $\epsilon = 1$ and $\theta_0 = 16^\circ$

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<th>$\alpha_2$</th>
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<td>-0.7198</td>
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Table D.7. Modified BEM results when $\sigma = 0.428$, $\varepsilon = 0.5$ and $\theta_0 = 18^\circ$

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<th>$P_{\text{front}}(W)$</th>
<th>$\alpha_2$</th>
<th>$P_{\text{rear}}(W)$</th>
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Table D.8. Modified BEM results when $\sigma = 0.428$, $\varepsilon = 1$ and $\theta_0 = 18^\circ$

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<th>$P_{\text{front}}(W)$</th>
<th>$\alpha_2$</th>
<th>$P_{\text{rear}}(W)$</th>
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Figure E-1. (a) Mesh generated around the front, bottom blade of PowerWindow. (b) Pressure contours around this blade. (c) Pressure created over the five front blades of PowerWindow along their cords.
Figure E-2. (a) Mesh generated around the front, bottom blade of PowerWindow. (b) Pressure contours around this blade. (c) Pressure created over the five front blades of PowerWindow along their cords.
Figure E-3. Mesh generated around the front, bottom blade of PowerWindow. (b) Pressure contours around this blade. (c) Pressure created over the five front blades of PowerWindow along their cords.
Figure E-4. Mesh generated around the front, bottom blade of PowerWindow. (b) Pressure contours around this blade. (c) Pressure created over the five front blades of PowerWindow along their cords.