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Modelling, dynamic stability analysis and control of an omni-directional road vehicle

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Modelling, dynamic stability analysis and control of an omni-directional road vehicle

A thesis submitted in fulfilment of the requirements
for the award of the degree of

Master of Engineering – Research

by

Boyuan Li

from

Faculty of Engineering, University of Wollongong

March 2012

Wollongong, New South Wales, Australia
CERTIFICATION

I, Boyuan LI, declare that this thesis, submitted in partial fulfilment of the requirements for
the award of Master of Engineering – research, in the School of Mechanical, Materials and
Mechatronic, Faculty of Engineering, University of Wollongong, is wholly my own work
unless otherwise referenced or acknowledged. The document has not been submitted for
qualifications at any other academic institution.

Boyuan LI

March 1st, 2011
ABSTRACT

In a modern society, traffic congestion is a major problem in every metropolis. To solve the problem of traffic congestion an innovative omni-directional vehicle is proposed. The most important advantage of an omni-directional vehicle is that by steering all four wheels it can turn in a small radius. This means that a smaller area is required for turning or parking. Lots of researches have focused on the use of an omni-directional vehicle for in-door situations, but less research has focussed on its use as an on road vehicle.

Therefore, this research has mainly focussed on developing the comprehensive vehicle dynamics model for an omni-directional road vehicle for high velocity conditions, transferring loads and the effect of traction and braking. This research consists of the following three parts:

A model was developed for an omni-directional vehicle that mainly focussed on vehicle dynamics in the yaw plane because the side slip angle and yaw rate are the two most important outputs of a dynamics system and determine its total stability. An innovative four wheel, independent steering system was incorporated into this vehicle dynamics model and the effect of roll motion on the yaw plane was examined.

Secondly, the stability of the vehicle in the yaw plane was analysed based on the response of the side slip angle, as first suggested. Then in the actual simulation, and based on the stability criterions, vehicle stability was determined under different scenarios. The first scenario was large and small radius turning, followed by simulating a combination of turning, braking, and traction. This was followed by an analysis of the disturbance of vehicle stability, including the bank angle, the effect of a lateral wind and the coefficient of friction. Finally, the effect of roll motion such as load transfer and roll steer, and how an independent driving system would affect vehicle stability is presented.

An active steering PID controller was suggested and which, according to the results of the simulation, improved the yaw plane stability (side slip angle).
ACKNOWLEDGEMENTS

I wish to thank my supervisor, A/PR. Weihua Li, for his enthusiastic support, professional direction, and constant encouragement that inspired me to overcome the challenges on my road of life and study.

A sincere appreciation is due to my co-supervisor Dr. Oliver Kennedy for his constant support and help. Particular thanks are also extended to my colleagues, Qing MENG, Gangrou PENG, and Yanhao ZHANG for their kind assistance, discussion, and encouragement.

Finally, I would like to thank my parents for their love, education, encouragement, and financial support, and wish them happiness and good health every day.
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<th>Description</th>
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</thead>
<tbody>
<tr>
<td>( m )</td>
<td>The vehicle total mass</td>
</tr>
<tr>
<td>( m_s )</td>
<td>The vehicle sprung mass</td>
</tr>
<tr>
<td>( W_s )</td>
<td>The vehicle sprung weight</td>
</tr>
<tr>
<td>( \dot{y} )</td>
<td>Vehicle lateral acceleration</td>
</tr>
<tr>
<td>( v )</td>
<td>Longitudinal velocity of centre of gravity (CG)</td>
</tr>
<tr>
<td>( u )</td>
<td>Lateral velocity of CG</td>
</tr>
<tr>
<td>( r )</td>
<td>Yaw rate of CG</td>
</tr>
<tr>
<td>( I )</td>
<td>Vehicle moment of inertia about yaw axis</td>
</tr>
<tr>
<td>( l_f )</td>
<td>Front wheel base</td>
</tr>
<tr>
<td>( l_r )</td>
<td>Rear wheel base</td>
</tr>
<tr>
<td>( T_f )</td>
<td>Front track width</td>
</tr>
<tr>
<td>( T_r )</td>
<td>Rear track width</td>
</tr>
<tr>
<td>( x, y; (i = fl, fr, rl, rr) )</td>
<td>The position of the wheel centre in the coordinate system</td>
</tr>
<tr>
<td>( x_{ICR}, y_{ICR} )</td>
<td>The position of ICR in the coordinate system</td>
</tr>
<tr>
<td>( \delta; (i = fl, fr, rl, rr) )</td>
<td>The steering angle of front left (fl), front right(fr), rear left(rl) and rear right(rr) wheel</td>
</tr>
<tr>
<td>( D_t )</td>
<td>The value will be either 1 (anticlockwise turning) or -1 (clockwise turning)</td>
</tr>
<tr>
<td>( \alpha_{sl} )</td>
<td>The critical side slip angle in the tire brush model</td>
</tr>
<tr>
<td>( \theta_y )</td>
<td>Constant parameter in the tire brush model</td>
</tr>
<tr>
<td>( c_{py} )</td>
<td>Lateral stiffness of the trend element</td>
</tr>
<tr>
<td>( a, L )</td>
<td>Contact length between the wheel and the ground</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Friction coefficient between the tire and the road</td>
</tr>
<tr>
<td>( F_z )</td>
<td>Vertical load in tire brush model</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>The side slip angle in tire brush model</td>
</tr>
<tr>
<td>( v_r )</td>
<td>Relative velocity between the tire and the ground in the lateral direction</td>
</tr>
<tr>
<td>( \sigma_0 )</td>
<td>The stiffness coefficients for the zero strain operating point</td>
</tr>
<tr>
<td>( \sigma_1 )</td>
<td>The damping coefficients for the zero strain operating point</td>
</tr>
<tr>
<td>( z )</td>
<td>Average horizontal bristle deflection</td>
</tr>
<tr>
<td>( \kappa(v_r), g(v_r) )</td>
<td>The characteristic lumped-model parameters</td>
</tr>
<tr>
<td>( \omega )</td>
<td>The rotation speed of the wheel shaft</td>
</tr>
<tr>
<td>( R )</td>
<td>The radius of the wheel</td>
</tr>
<tr>
<td>( F_y )</td>
<td>Tire lateral force</td>
</tr>
<tr>
<td>( C_\alpha )</td>
<td>The lateral stiffness of the tire</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>Road adhesion reduction factor</td>
</tr>
<tr>
<td>$\Delta W_f$</td>
<td>Front wheel load transfer</td>
</tr>
<tr>
<td>$\Delta W_r$</td>
<td>Rear wheel load transfer</td>
</tr>
<tr>
<td>$K_{sf}$</td>
<td>Front roll axis torsional stiffness</td>
</tr>
<tr>
<td>$K_{sr}$</td>
<td>Rear roll axis torsional stiffness</td>
</tr>
<tr>
<td>$h_s$</td>
<td>Distance of the CG of the sprung mass from the roll axis</td>
</tr>
<tr>
<td>$h_f$</td>
<td>The height of the front roll centre</td>
</tr>
<tr>
<td>$h_r$</td>
<td>The height of the rear roll centre</td>
</tr>
<tr>
<td>$\alpha_f$</td>
<td>Front roll angle</td>
</tr>
<tr>
<td>$\alpha_r$</td>
<td>Rear roll angle</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Vehicle roll angle</td>
</tr>
<tr>
<td>$\frac{\partial \alpha_f}{\partial \phi}$</td>
<td>Front roll steer coefficient</td>
</tr>
<tr>
<td>$\frac{\partial \alpha_r}{\partial \phi}$</td>
<td>Rear roll steer coefficient</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Side slip angle of CG point</td>
</tr>
<tr>
<td>$K_f$</td>
<td>Front tire lateral stiffness</td>
</tr>
<tr>
<td>$K_r$</td>
<td>Rear tire lateral stiffness</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$F_{ix}(i = fl, fr, rl, rr)or(i = 1,2,3,4)$</td>
<td>The longitudinal tyre force of each wheel</td>
</tr>
<tr>
<td>$Y_{ix}(i = fl, fr, rl, rr)or(i = 1,2,3,4)$</td>
<td>The lateral tyre force of each wheel</td>
</tr>
<tr>
<td>$u_i (i = fl, fr, rl, rr)or(i = 1,2,3,4)$</td>
<td>The velocity of the tyre contact point with the ground of each wheel</td>
</tr>
<tr>
<td>$F_{ix}(i = fl, fr, rl, rr)or(i = 1,2,3,4)$</td>
<td>The longitudinal force in the x direction</td>
</tr>
<tr>
<td>$F_{iy}(i = fl, fr, rl, rr)or(i = 1,2,3,4)$</td>
<td>The lateral force in the y direction</td>
</tr>
<tr>
<td>$I_{xxx}$</td>
<td>Vehicle moment of inertia about roll axis</td>
</tr>
<tr>
<td>$p$</td>
<td>The vehicle body rolling rate</td>
</tr>
<tr>
<td>$I_{szs}$</td>
<td>Sprung mass product of inertia</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration of gravity</td>
</tr>
<tr>
<td>$K_\phi$</td>
<td>Roll axis torsional stiffness</td>
</tr>
<tr>
<td>$C_\phi$</td>
<td>Roll axis torsional damping</td>
</tr>
<tr>
<td>$C_s$</td>
<td>Longitudinal stiffness of the tyre</td>
</tr>
<tr>
<td>$I_{w}$</td>
<td>Wheel moment of inertia</td>
</tr>
<tr>
<td>$\omega_i (i = fl, fr, rl, rr)or(i = 1,2,3,4)$</td>
<td>The rotation speed of each wheel</td>
</tr>
<tr>
<td>$T_{i}(i = fl, fr, rl, rr)or(i = 1,2,3,4)$</td>
<td>The total torque applied on each wheel</td>
</tr>
<tr>
<td>$F_{iz}(i = fl, fr, rl, rr)$</td>
<td>The vertical load of each wheel without the effect of load transfer</td>
</tr>
<tr>
<td>$F_{iz}'(i = fl, fr, rl, rr)$</td>
<td>The vertical load of each wheel with the effect of load transfer</td>
</tr>
<tr>
<td>$\alpha_i (i = fl, fr, rl, rr)$</td>
<td>The side-slip angle of each wheel</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------------------------------------------------</td>
</tr>
<tr>
<td>$a_y$</td>
<td>Lateral acceleration</td>
</tr>
<tr>
<td>$S_i (i = fl, fr, rl, rr)$</td>
<td>Tire relative longitudinal slip</td>
</tr>
<tr>
<td>$Y_b$</td>
<td>Lateral force caused by bank angle</td>
</tr>
<tr>
<td>$T_{di} (i = fl, fr, rl, rr)$</td>
<td>Driving torque</td>
</tr>
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<td>$T_{bi} (i = fl, fr, rl, rr)$</td>
<td>Braking torque</td>
</tr>
<tr>
<td>$Y_w$</td>
<td>The lateral wind force</td>
</tr>
<tr>
<td>$N_w$</td>
<td>The yaw moment</td>
</tr>
<tr>
<td>$C_y$</td>
<td>Lateral force coefficient</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Air density</td>
</tr>
<tr>
<td>$S$</td>
<td>Vehicle frontal area</td>
</tr>
<tr>
<td>$v$</td>
<td>Vehicle velocity</td>
</tr>
<tr>
<td>$W$</td>
<td>Lateral wind velocity</td>
</tr>
<tr>
<td>$l_w$</td>
<td>The distance between aerodynamics centre and vehicle centre of gravity</td>
</tr>
<tr>
<td>$\beta_w$</td>
<td>Airflow side slip</td>
</tr>
</tbody>
</table>
CHAPTER 1  INTRODUCTION

1.1  BACKGROUND AND MOTIVATION

In a modern society, traffic congestion is a major problem in every metropolis. The population of Shanghai, Tokyo, and London is more than 10 million and consequently traffic congestion is a big issue, particularly in peak hours. According to Schrank et al. [1], the cost of delays induced by congestion has risen from $24 billion in 1982 to $115 billion in 2009 in US, which is a serious economic cost. Verhoef and Rouwendal [2] suggested that traffic congestion impact on vehicle speed, traffic safety, and time costs. Shinar [3] also showed that driver aggression is increasing because of traffic congestion.

To solve this problem, several innovative ideas and technologies have been proposed. One possible solution is improving public transportation rather than personal automobiles. Dixon et al. [4] proposed using ultra-capacitors as the energy source because they are fast charging and recharging but have less travel distance, which could successfully meet the requirement of public transportation in large cities. Another possible solution is to minimise the space required for personal automobiles. Barker et al. [5] proposed a narrow, tilting, three wheeled vehicle which could save a significant amount of space on the street.

In this project, an innovative omni-directional vehicle is suggested as a means of solving the problem of traffic congestion. The concept of an omni-directional vehicle is from the research area of advanced robotics. The vehicle has four independent steering wheels, which means that the steering angle of each wheel is different.

The concept of an omni-directional vehicle has already been widely applied because it can turn in a small radius by steering all four wheels, which means that a smaller area is required when turning or parking. Lots of researches have focused on the use of an omni-directional vehicle in an in-door situations [6,7,8] but here its velocity is slow and the ground is quite smooth, without bumps. However, less research has focussed on using an omni-directional vehicle on the road. In this condition, the velocity of the vehicle is relatively high and the load transfer should also be considered while the vehicle is turning.
This research focuses on developing a comprehensive dynamics model for an omni-directional road vehicle that considers the effect of high velocity, load transfer, and traction and braking.

1.2 AIMS AND OBJECTIVES

This project aims to develop a dynamics model for an omni-directional road vehicle and present the results of a simulated stability analysis. The specific objectives include:

1. Develop a comprehensive vehicle dynamics yaw plane model, including a novel four wheel independent steering and driving system.

2. Analyse the stability and different scenarios such as turning, braking, and traction, according to the vehicle dynamics model by Matlab Simulink, and also show the effect that roll motion and driving torque distribution has on vehicle stability.

3. Develop a direct yaw moment controller to improve vehicle stability, and also compare the performance of an active steering controller with the direct yaw moment controller as regards vehicle stability.

1.3 THESIS OUTLINE

Chapter 2 presents a comprehensive literature review of vehicle dynamics system and the vehicle stability controller. Chapter 3 describes the modelling of the vehicle dynamics model, including the novel independent steering and driving system. Chapter 4 addresses the stability analysis of the vehicle dynamics performance with the help of Matlab Simulink. Chapter 5 proposes two kinds of vehicle stability controllers to improve vehicle stability. The conclusion and proposal for future work are summarised in the last part.
CHAPTER 2 LITERATURE REVIEW

2.1 INTRODUCTION

The aim of this project is to develop a vehicle dynamics model and analyse the stability of an omni-directional vehicle. A stability controller was developed to improve the stability of the vehicle. Therefore, this literature review incorporates reviews of the vehicle dynamics model and the stability controller.

Firstly, a review of the vehicle dynamics model; this includes the steering model, the traction/brake model, the load transfer model, the tire model, and the vehicle body model. Each sub-system of the vehicle dynamics model will be reviewed in this chapter.

The second part is a review of the vehicle stability controller that primarily focuses on yaw plane stability, because the side slip angle and yaw rate are the two most important values required to determine the yaw plane stability of the vehicle. To control the stability of the vehicle, the direct yaw moment controller and active steering controller are reviewed.

2.2 A REVIEW OF THE VEHICLE DYNAMICS MODEL

The full vehicle dynamics model has six degrees of freedom (Fig. 2.1)

1. Vertical motion in the z direction
2. Left and right motion in the y direction
3. Longitudinal motion in the x direction
4. Rolling motion about the x axis
5. Pitching motion about the y axis
6. Yawing motion about the z axis

![Diagram of vehicle dynamics model]

Fig 2.1 The vehicle dynamics model

The focus in this project is on the vehicle dynamics behaviour in the yaw plane, and therefore the yaw motion about the z axis, left and right motion in the y direction, and longitudinal motion in the x direction are addressed. In addition, roll motion about the x axis can cause load transfer, which also affects performance in the yaw plane.

![Diagram of basic vehicle dynamics system]

Fig 2.2 Basic vehicle dynamics system

The inputs of the vehicle dynamics system are the steering angle (from the steering system) and velocity (from the traction/brake system), and outputs are the side slip angle and yaw rate around the CG.
According to Fig. 2.3, the inputs from the tire dynamics system are the steering angle (from the in wheel steering motor system), wheel velocity (from the in wheel traction/brake system), and feedback signals from the body dynamics model (side slip angle of the CG and lateral force), and the load transfer mode (vertical load). The output of the tire dynamics model is the lateral force of each wheel, which is an input into the vehicle body dynamics model needed to calculate the side slip angle and yaw rate of the CG. In addition, the side slip angle and yaw rate of the CG are also feedback signals which can be sent back into the tire dynamics model. Finally, according to the output of the vehicle body dynamics model, the load transfer model can calculate the actual vertical load of each wheel and send this information back into the tire dynamics model.

2.2.1 The vehicle body dynamics model

Abe [9] developed the basic equations of motion based on a coordinate system fixed on the body of a moving vehicle:
These equations describe the force and moment equivalence of the yaw plane. There are two degrees of freedom in this set of equations: the side slip angle of the CG point and the yaw rate of the vehicle, which focus on the lateral stability of the vehicle. However, these equations only use $K_f$ and $K_r$ to show the linear relationship between the side slip angle and tire lateral force, which cannot describe the non-linear tire behaviour. In addition, these equations only consider the steering of a 2 front wheel steering system, which means that the steering behaviour of a 2 rear wheel steering and 4 wheel steering system must be considered in the vehicle dynamics model.

Karnopp [10] also developed the basic equations of motion based on the coordinate system fixed on the body of a moving vehicle. This set of equations also has two degrees of freedom compared with Abe’s equation: lateral acceleration and the yaw rate, but these equations focus on the lateral motion (lateral velocity and lateral acceleration) of a vehicle. Similar to Abe’s equation, this set of equations still only considers linear tire and 2 front wheel steering behaviour.

In this research the vehicle dynamics model should consider four wheel independent steering, which means that the steering angle of each wheel is different. In addition, since an omni-directional vehicle may generate large steering angles, the vehicle dynamics model should consider tire nonlinear behaviour. This means the simple tire linear equation is not enough.

Lam et al. [11] proposed a vehicle dynamics equation for an omni-directional vehicle:

\[
\begin{align*}
\sum_{i=1}^{4} F_{xi} &= m \ddot{x}_i - u \omega \\
\sum_{i=1}^{4} F_{yi} &= m \ddot{y}_i + v \omega \\
\sum_{i=1}^{4} (\dot{\theta}_i x_i - F_{xi} y_i) &= I \dot{\omega}
\end{align*}
\]
The force $F_{xi}$ and $F_{yi}$ is related to the traction and tire lateral force of each wheel, so the steering angle and tire force of each of the four wheels are independent.

$$
F_{xi} = F_i \cos \delta_i - Y_i \sin \delta_i
$$

$$
F_{yi} = F_i \sin \delta_i + Y_i \cos \delta_i
$$

(2.3)

Lam et al. [11] used the Magic formula tire model to describe the non-linear relationship between lateral tire force and the side slip angle of each wheel. This vehicle dynamics model has three degrees of freedom to describe the longitudinal, lateral, and yaw motions.

Boada et al. [12] also developed a more complicated vehicle dynamics model which considered both the yaw and the roll motion effect on the vehicle model. Thus this model has 8 degrees of freedom. The vehicle dynamics model used the Dugoff tire model to describe the non-linear relationship between the tire force and side slip angle.

Although Boada’s vehicle model is more comprehensive than Lam’s model, like considering the yaw motion, roll motion, and the traction and braking of the vehicle, Boada’s model is still a conventional 2 front wheel steering vehicle, which cannot apply to an omni-directional vehicle like Lam’s model.

Therefore, a more comprehensive vehicle dynamics model applied to an omni-directional vehicle must be developed.

With the development of model control theory, lots of researches focus on yaw stability control. Direct yaw moment control is a widely used yaw stability control strategy that is achieved by re-distributing the driving torque between the driving wheels [13]. An electrical vehicle can apply this control strategy easier by independent control of the in-wheel driving motor. Raksincharoensak et al. [14] proposed a direct yaw moment control system based on the recognition of driver behaviour, while Boada et al. [12] developed a Fuzzy-logic direct yaw moment control system. In addition, an active steering control is another widely used strategy to control yaw stability. In some researches, the direct yaw moment control and active steering control is integrated together to control yaw stability [15,16]. Li et al. [17] developed an integrated vehicle chassis control, including direct yaw moment control, active steering control, and active stabilisation.
2.2.2 Omni-directional steering system

The steering angle of a conventional vehicle is constrained by the mechanical linkages between each wheel. According to the literature, the steering angle for a traditional front wheel steering vehicle is between $+35^\circ$ and $-35^\circ$ [10]. However, an omni-directional vehicle has an independent steering system in each wheel to give better manoeuvrability, so without mechanical linkages between each wheel, the steering angle has a much larger theoretical constraint, from $+90^\circ$ to $-90^\circ$ [18]. In the area of advanced robotics the steering angle can be greater than $90^\circ$ [19].

The concept of an omni-directional steering system was developed from the area of robotics, indeed a wheeled mobile robot (WMR) is the original vision of an omni-directional vehicle, but is has not been used as a road vehicle.

2.2.2.1 Classifying a Wheeled Mobile Robot

Mobile robots have two types of wheels; conventional wheels and Swedish wheels.

(1) **Conventional wheels**, where the wheel works in a pure rolling motion, without any slip [19, 20].

(2) **Swedish wheels**, with a series of rollers attached to a conventional wheel, allowing it to move in any direction [21, 22].

Moreover, a WMR can also be classified as a holonomic mobile robot and the non-holonomic mobile robot.

(1) A **holonomic mobile robot** can move in an arbitrary direction and reach its destination immediately. The Swedish wheels with three degrees of freedom can perform this task. This kind of WMR has better mobility but the wheel mechanism is damaged [23, 24].

(2) A **non-holonomic mobile robot** moves with the steering of each wheel, but it cannot move arbitrarily. Conventional wheels are used in this kind of mobile robot. A non-holonomic mobile robot has better stability, but worse mobility [25, 26].
2.2.2.2 The application of WMR steering in a road vehicle

In recent years, in the specific area of non-holonomic mobile robots with conventional wheels, Lam et al. [11] proposed a kinematic constraint of the steering angle for an omni-directional vehicle to be used as a road vehicle.

The kinematic constraint of the steering angle of each wheel is determined by the following equation:

\[
\delta_i = a \tan 2(D^T (x_i - x_{ICR}), -D^T (y_i - y_{ICR})) \tag{2.4}
\]

2.2.3 The tire model

In the literature there are mainly two kinds of tire models; a physical tire model and an empirical tire model. The physical tire model can theoretically describe the tire characteristics, but it lacks accuracy, while an empirical tire model is the curve fitting results of the actual tire data quite accurately.

2.2.3.1 The physical tire model

The brush model is a simple physical tire model which consists of a row of elastic bristles called the tread element. The top of the tread element is attached to the base of the wheel and the bottom one is attached to the ground. The relative slip between the top and bottom of the element can generate the lateral slip and longitudinal forces [27].

Deur [28] proposed the LuGre tire model, which is a more complicated tire model that considers the decreasing effect of friction [29].

Boada, et al. [12] used the Dugoff tire model to analyse the stability of the vehicle.
\[ F_y = C_\alpha \tan \alpha f(\lambda) \]

\[ f(\lambda) = \begin{cases} 
\lambda (2 - \lambda) & \text{if } \lambda < 1 \\
1 & \text{if } \lambda > 1 
\end{cases} \]

\[ \lambda = \frac{\mu F_z - e_v \mu \tan \alpha}{2C_\alpha \tan \alpha} \] (2.5)

If \( \lambda < 1 \), \( f(\lambda) = \lambda (2 - \lambda) \), \( F_y = C_\alpha \tan \alpha \lambda (2 - \lambda) \). The tire is in the non-linear region of the adhesive region, or in the sliding region.

If \( \lambda > 1 \), \( f(\lambda) = 1 \), \( F_y = C_\alpha \tan \alpha \). The tire is in the linear region of the adhesive region.

The Dugoff tire model can describe the decreasing effect in the sliding region and clearly divide the curve of the tire lateral force into the linear region of the adhesive region, the non-linear region of the adhesive region, and the sliding region, as shown in Fig. 2.4.

Fig. 2.4 The relationship between the lateral force and side-slip angle of Dugoff tire model

In addition, Guo and Lu [30] proposed a UniTire model for the vehicle dynamics model. This is a unified non-steady and non-linear semi-physical tire model. Based on the UniTire model, Xu et al. [31] suggested a combined tire model for cornering and braking.
Zhou et al. [32] developed a three plane multi-spoke tyre model for transient tyre behaviour, while Lacombe [33] suggested an on-road analytical model to describe the tire force and moment.

An innovative non-linear tire observer was proposed by Canudas-de-Wit et al. [34] to describe the road friction. Yamazaki et al. [35] also estimated the friction between the tire and the road based on the tire brush model.

Oosten [36] introduced accurate tire modelling using Adams Software with a TMPT benchmark program. Mundl and Duvernier [37] described the measurement of the tire data in an experiment which is used for the TMPT.

To analyse vehicle performance during a crash, a new detailed model has been evaluated [38]. In addition, to effectively analyse vehicle behaviour during extreme cornering, Lu et al. [39] proposed a tire model that considers the effect of camber.

### 2.2.3.2 An empirical tire model

The Magic formula model is a widely used empirical tire model [27]. Like other empirical tire models, the Magic Formula model is the curve fitting results of actual data that can accurately describe the behaviour of a certain type of tire. However, if the type of tire has been changed, the coefficient of the Magic Formula model (B, C, D and E) should be changed. In addition, the Magic Tire model cannot describe how a tyre’s physical parameters affect its performance.

In addition, the semi-empirical tire model was developed to simulate normal driving [40]. Guo and Ren [41] developed the Unified Semi-Empirical High accuracy tire model, but it has less parameters. Best [42] also provided a tyre model from the test data by the Extended Kalman Filter.

Dihua et.al [43] proposed corning models with parameters obtained from the experiment. The cornering force was also estimated from the test data by the Extended Kalman Filter [44].

The coefficient of friction between the tire and the road has been explored by the curve fitting method [45]. Guan and Fan [46] modelled the vertical property of tires using the experimental model parameters.
Brach and Brach [47] used mathematical functions like the Bakker-Nyborg-Pacejka equations and exponential or piecewise linear functions, to describe the combined motion of braking and steering. A Slip Circle tire model has been used to model the cornering and braking tire force [48].

### 2.2.4 Vehicle load transfer

Load transfer is caused by a vehicle rolling. Rolling motion is critical to the stability of a vehicle if its centre of the gravity is high. Figure 2.6 shows the free body diagram of vehicle roll motion.

![Free Body Diagram of Vehicle Roll Motion](image)

**Fig. 2.5 Vehicle body roll motion**

According to Fig. 2.5, when a vehicle body is rolling, a rolling moment is applied onto the centre of the vehicle. To ensure the equilibrium of the force and the moment, there must be another moment in the opposite direction to the rolling moment. In this way the vertical load should be increasing on one wheel and decreasing on another. This is called the load transfer effect when a vehicle body is rolling. If the load transfer is too large, the vertical load of one wheel will decrease too much and become negative, which causes the vehicle to roll over.
Abe [9] proposed a mathematical model of load transfer:

\[
\Delta W_f = \frac{\ddot{y}W_f}{T_f} \left[ \frac{K_g h_s}{K_g + K_{\phi} - W_s h_s} + \frac{l_f}{l} h_f \right]
\]

(2.6)

\[
\Delta W_r = \frac{\ddot{y}W_r}{T_r} \left[ \frac{K_g h_s}{K_g + K_{\phi} - W_s h_s} + \frac{l_f}{l} h_f \right]
\]

According to the above equation, the load transfer is determined by lateral acceleration.

In addition, the rolling motion also affects the steering angle of the vehicle, which is called roll steer. According to Abe [9], the roll angle can be calculated:

\[
\phi = \frac{\ddot{y}W_s h_s}{K_g + K_{\phi} - W_s h_s}
\]

(2.7)

According to the rolling angle, the additional steering angle of each wheel (roll steer) can be calculated:

\[
\alpha_f = \frac{\partial \alpha_f}{\partial \phi} \phi
\]

(2.8)

\[
\alpha_r = \frac{\partial \alpha_r}{\partial \phi} \phi
\]

Some literatures focused on the effect of the load transfer on vehicle motion. Clover and Bernard [49] analysed the effect of load transfer on the direction in which a vehicle was travelling whereas the steady state non-linear cornering behaviour focused on the transfer of lateral loads [50]. Doumiati et al. [51] finished their estimation of lateral load transfer and related vertical tire force using cheap sensors.

Various design factors affect the roll stability and load transfer of a vehicle: like the track width of the vehicle, the height of the CG, and the vertical load. Around a 1% increase in the track width can raise the roll stability limit by 3%. In addition, a heavy road vehicle with a high CG has less roll stability limit than light vehicles [52]. An anti-roll bar can improve roll stability but the ride and comfort will be degraded because of the increase in vehicle mass [53]. With the development of modern control strategies, lots of literatures focused on the dynamic analysis and control of roll stability. The control of semi-active or active anti-roll systems for heavy vehicles has also been developed [54] [55]. Miege and Cebon [56] also
suggested an optimal roll control for articulated heavy vehicles. In addition to heavy vehicles, the anti-roll system and roll stability of light vehicle has also been proposed [57] [58].

2.3 VEHICLE STABILITY AND CONTROL

Over the past decades, as technology has developed, electronic controls have been widely used in the automobile industry. Stability control systems have been developed to improve vehicle stability [59]. This system consists of a steering wheel angular sensor, a yaw rate sensor, a lateral accelerometer, and wheel speed sensors.

A steering wheel angle estimator is used to measure the steering angle of a vehicle. There are two types of steering angle estimators, an absolute sensor and a relative sensor. An absolute sensor uses added hardware to estimate the steering angle while the relative sensor uses software to learn the position of the steering wheel.

The yaw rate sensor gets the actual value of the yaw rate and compares it with the desired yaw rate. The desired yaw rate is calculated according to the simple bicycle linear vehicle dynamics model. In this way the desired yaw rate depends on the driver’s steering input and velocity of the vehicle.

The prevention of excessive deviations between the intended and actual lateral response of the vehicle is also important in the vehicle’s stability control system. The lateral response mainly includes the yaw rate, side slip angle, side slip gradient, and path radius of curvature.

Raksincharoensak et al. [60] presented the individual wheel torque distribution control of an in-wheel driving motor to control the direct yaw moment and improve vehicle stability.

The 2 degrees of freedom vehicle dynamics model is suggested as follows:

\[
mv(\dot{\beta} + r) = 2K_f (\delta_f - \frac{I_f}{v} r - \beta) + 2K_r (\frac{I_r}{v} r - \beta)
\]

\[
I\dot{r} = 2l_f K_f (\delta_f - \frac{I_f}{v} r - \beta) - 2l_r K_r (\frac{I_r}{v} r - \beta) + M
\]

(2.9)

In this equation M denotes the yaw moment to implement the direct yaw moment control which is generated by the difference of the driving force of left and right wheel:
\[ M = \frac{d}{2} (-F_{ax} + F_{ar}) \] (2.10)

In this research the longitudinal slip was neglected and therefore the longitudinal force is the driving torque divided by the radius of the wheel.

The direct yaw moment control system is usually implemented by the active braking or driving system to generate braking or driving torque, and additional yaw moment. Ackermann et al. [61] developed an alternative active steering system to generate additional yaw moment. This research found that an active steering system has an added advantage over the emergency braking system, particularly regarding safety and comfort.

The active steering control is implemented by adding an additional steering angle. Abe [9] suggested active rear wheel steering in this vehicle dynamics equation:

\[
\begin{align*}
mv(\dot{\beta}) + 2(K_f + K_r)\beta + (mv + \frac{2}{v}(l_f K_f - l_r K_r)) r &= 2K_f \delta_f + 2K_r \delta_r, \\
2(l_f K_f - l_r K_r)\beta + I \frac{d\theta}{dt} + \frac{2(l_f^2 K_f + l_r^2 K_r)}{v} r &= 2l_f K_f \delta_f - 2l_r K_r \delta_r,
\end{align*}
\] (2.11)

In this equation the rear wheel steering angle \( \delta_r \) is used to provide additional yaw moment. The rear wheel steering angle is calculated to ensure that the vehicle has a zero side slip angle response.

In the literature, lots of researches focused on analysing the stability of the vehicle. Horiuchi et al. [62] proposed constrained bifurcation and continuation methods to analyse the stability of accelerating and braking. Zboinski and Dusza [63] developed a method to analyse the non-linear lateral stability of a railway vehicle. Marghitu et al. [64] suggested an analytical methodology based on Poincad maps and Floquet theory to analyse the dynamic stability of a vehicle system. The stability moment, which is a new stability metric, was developed to measure the stability of high speed vehicles [65]. In addition, a measurement of the rollover stability of a high speed mobile vehicle has already been proposed [66]. Minaker and Rieveley [67] presented a novel non-holonomic equation of motion to analyse vehicle stability. Shen [68] performed a non-linear stability analysis of the plane motion of a vehicle, while Börner and Isermann [69] designed a velocity stability indicator for a passenger car.
2.4 CONCLUSIONS

In this chapter, with regards to an omni-directional vehicle, the vehicle dynamics model and vehicle stability controller have been reviewed. This is the knowledge background of my thesis.
3.1 INTRODUCTION

The main contribution of the vehicle dynamics model in this research is to incorporate a novel omni-directional steering system into a comprehensive vehicle yaw plane. In addition, a combination of the vehicle dynamics body dynamics systems, and the steering and traction/braking systems in the omni-directional behaviour, including the interaction between each model, can be evaluated. Furthermore, although the vehicle dynamics model is the yaw plane model, it also includes the effect of roll motion. Therefore, the vehicle dynamics model proposed in this research can describe the stability of an omni-directional vehicle (Fig. 3.1) more comprehensively.
3.2 A NOVEL OMNI-DIRECTIONAL STEERING TRACTION/BRAKE SYSTEM

Unlike the traditional two wheel steering system, the novel omni-directional steering and traction/brake system proposed here consists of four separate modules, one for each wheel.

![Diagram of omni-directional steering system]

Fig. 3.2 An omni-directional steering system
According to Fig. 3.2 and Fig. 3.3, the whole vehicle dynamics system includes four steering modules and four traction/braking modules.

### 3.2.1 A novel omni-directional steering system

With four different steering modules, the vehicle dynamics system can have four different steering angles which can cause different side slip angles, velocity, and vertical load on each wheel. Therefore, there could be a problem of misarrangement between four different steering angles, especially when the velocity of the vehicle is relatively high and the difference between the steering angle of each wheel is great.

To solve this problem, certain constraints are suggested here to prevent any misarrangement between the four wheels.

First, according to the geometric equation (2.10) proposed by Lam et al. [17], the lateral axis of the tire plane should converge into the same point. In this way, the four wheels are likely to have virtual linkages between each wheel, which would prevent any misarrangement between them.
In addition, because this dynamics model is analysing the lateral performance of the vehicle, the steering angle of the vehicle cannot be more than $90^\circ$. Otherwise the vehicle will rotate around itself and the lateral dynamics will be difficult to analyse.

Therefore, if we use four wheel steering, the minimum turn centre is $(0, 0.5*td)$. If $td=1.436$, the minimum turn centre is $(0, 0.718)$.

### 3.2.2 A novel omni-directional traction/brake system

With the four wheel traction/brake system, the torque at each wheel will be different, and so too the angular velocity of the wheel shaft. Normally, the driving torque can be distributed equally between 4 wheels (four wheel driving), the two front wheels (front wheel driving), and two rear wheels (two rear wheel driving). However, the driving/braking torque of the left and right wheel is different when the vehicle needs a different longitudinal slip force to stabilise the yaw moment. Therefore, there is also a problem of interaction between the four different traction/brake torques. In the simulation part of this research, the stability of different kinds of the traction/brake torque will be compared to find its optimal distribution.
3.3 COMBINING ALL THE SUB-SYSTEMS OF THE WHOLE VEHICLE DYNAMICS MODEL

Because the novel omni-directional steering and traction/brake system is included in this research, the interaction between the vehicle body model, the tire model, the steering model, and traction/brake model is quite different from a traditional vehicle dynamics system (Fig. 3.4).

3.3.1 Vehicle body dynamics model

In this research the inputs from the vehicle body dynamics model are the four lateral and longitudinal forces from the tires which are calculated from the tire model. Because the steering angles of the each wheel are different, the directions of the lateral and longitudinal forces of each tire are different. Therefore, we need to project all the lateral and longitudinal forces in the $x$ and $y$ directions of the vehicle body coordinate system.

Fig. 3.5 A projection of the lateral and longitudinal forces of the tires into the vehicle body coordinate system (XOY)

The block diagram of the vehicle body model in this project is shown below:
In this research the side slip angle and yaw rate is the CG point of the vehicle. In addition, the acceleration and velocity are also the values of the CG point. If we need the side slip angle and velocity of each wheel, the values of CG point must also be projected into the specific wheel.

### 3.3.2 Tire model

In this research the four wheel independent Dugoff tire model was used. Therefore, the inputs of each tire model (yaw rate, side slip angle, velocity and vertical load) are different, which causes a different output from the lateral and longitudinal tire force from each wheel. (Fig 3.6)

In addition, the Dugoff tire model is a non-linear model with a linear and non-linear region. The different lateral force from each wheel can cause the tires to work in different regions, which means that some wheels may be in a linear region and some in a non-linear region. This may cause a problem with stability when the velocity is high and side slip angle large. Therefore, a stability analysis of the whole vehicle system is extensively related to the working region of each tire.
3.3.3 A combination of all the sub-systems of the vehicle dynamics model

The basic vehicle dynamics equation developed in this research is as follows:

\[
\begin{align*}
    m \left( \frac{d\beta}{dt} + r \right) &= -Y_f \cos \delta_f - Y_r \cos \delta_r - Y_l \cos \delta_l - Y_n \cos \delta_n = 0 \\
    I \frac{dr}{dt} &= l_f Y_f \cos \delta_f + l_r Y_r \cos \delta_r - l_l Y_l \cos \delta_l - l_n Y_n \cos \delta_n \\
    &+ 0.5T_f Y_f \sin \delta_f - 0.5T_f Y_r \sin \delta_f - 0.5T_f Y_l \sin \delta_l - 0.5T_f Y_n \sin \delta_n \tag{3.1}
\end{align*}
\]

Fig 3.7 A block diagram of the vehicle tire model
Table 3.1 The list of symbols in equation 3.1

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>The vehicle total mass</td>
</tr>
<tr>
<td>$\beta$</td>
<td>The side slip angle of vehicle CG</td>
</tr>
<tr>
<td>$Y_i$ (i=fl, fr, rl, rr)</td>
<td>Vehicle lateral force of the four wheels</td>
</tr>
<tr>
<td>$v$</td>
<td>Longitudinal velocity of centre of gravity (CG)</td>
</tr>
<tr>
<td>$\delta_i$ (i=fl, fr, rl, rr)</td>
<td>The steering angle of each wheel</td>
</tr>
<tr>
<td>$r$</td>
<td>Yaw rate of CG</td>
</tr>
<tr>
<td>$I$</td>
<td>Vehicle moment of inertia about yaw axis</td>
</tr>
<tr>
<td>$l_f$</td>
<td>Front wheel base</td>
</tr>
<tr>
<td>$l_r$</td>
<td>Rear wheel base</td>
</tr>
<tr>
<td>$T_f$</td>
<td>Front track width</td>
</tr>
<tr>
<td>$T_r$</td>
<td>Rear track width</td>
</tr>
</tbody>
</table>

The following diagram shows the relationship of the two main parts of the vehicle dynamics model in this research: the vehicle body dynamics model and the tire model.

![Diagram of vehicle dynamics model and tire model](image)

**Fig 3.8 A block diagram of the tire model and vehicle body model**

According to Fig 3.8, the input of the vehicle body model is the output of the tire model and the output of the vehicle body model is the input of the tire model. In this way, the tire model and vehicle body model form a feedback loop. This system can be considered as a closed loop feedback system. To stabilise the whole system, the output from each tire model should
converge. In the traditional model, only the two-wheel tire model needs to be considered, but in this research for an omni-directional vehicle, all four wheel models should be considered. This means the system is more complex and its stability is closely related to the stability of each wheel sub-system and the working region of each tire (linear or non-linear).

### 3.4 INCLUDING ROLL MOTION INTO THE VEHICLE DYNAMICS MODEL

In this research, roll motion caused by the load transferring between the left and right wheel should be considered because the vehicle is omni-directional, with four wheel independent steering and driving. In the vehicle dynamics model of this research, two sub-systems are suggested to describe the effect of roll motion: the load transfer model and roll steer model.

#### 3.4.1 The load transfer model

The load transfer model in this research is to describe all the vertical load of the four wheels.

![Fig 3.9 A block diagram of the load transfer model](image)

According to Fig 3.9, the vertical load from each wheel is input into the tire model to affect the working region and lateral force. For the omni-directional vehicle model in this research, each wheel has its individual vertical load to determine the individual working region and
lateral force of each wheel, which increases the complexity of the whole system. The stability of the whole vehicle is also related to the vertical load of each wheel because this can affect the working region of each tire. In an extreme condition such as very high speed, the vertical load of the vehicle may be even less than zero which means the vehicle is turning over.

3.4.2 The roll steer effect

The roll steer effect caused by the roll motion can affect the dynamic motion of the vehicle by adding an additional steering angle to each wheel.

3.5 CONCLUSION

In this chapter, modelling a novel omni-directional vehicle dynamics model was proposed. Unlike the traditional vehicle model, the model in this research included independent tire models of four wheels with different side slip angles, velocities, vertical loads, and working regions, which greatly increase the complexity of the whole system. Therefore, a stability analysis of the whole vehicle system based on each tire model is critical for researching an omni-directional vehicle, and will be presented in the following chapter.
CHAPTER 4  STABILITY ANALYSIS UNDER DIFFERENT WORKING CONDITIONS

4.1  INTRODUCTION

In the vehicle dynamics, the stability analysis of the yaw plane in a dynamic condition focused on the response of the side slip angle of each tire. Therefore, the following analysis is mainly about the side slip angle of each tire under different conditions.

If the vehicle needs to turn around the centre of rotation in a limited space like a car parking area, the vehicle is performing small radius turning, which is the main feature of an omni-directional vehicle. In this condition the velocity and turning radius of the vehicle are all relatively small. Therefore, the following analysis is also focusing on the scenario of vehicle small radius turning.

In this chapter, five sets of simulations are performed under dynamic conditions with the help of Matlab Simulink, to finalise the stability analysis of the omni-directional vehicle.

(a) Simple turning.
(b) Combined traction/brake and turning
(c) Vehicle motion under disturbance
(d) The effect of roll motion
(e) The effect of four wheel independent driving system

4.2  SIMPLE TURNING – SCENARIO ONE

In this set of simulations, the steering angle and the velocity are the constant values. The effect of traction/brake will be considered in section 4.3.

In the condition of small radius turning, the velocity cannot be too high and the turn radius must be relatively small, so in this set of simulations the velocity was lower than 20 km/h and the turn radius smaller than 10 m.
Case 1

If the turning radius was 2 m and the vehicle velocity was 6 km/h, the side slip angle response of each wheel is obtained.

To generate the 2 m turn radius turning, for the omni-directional vehicle, the turn centre is assumed as (0, 2) in the vehicle body coordinate system. The CG point in this coordinate system is (0, 0).

According to equation (2.4):

$$\delta_i = a \tan \left(2(D^T(x_i - x_{ICR}),-D^T(y_i - y_{ICR})) \right)$$

(2.4)

$$x_{ICR} = 0, y_{ICR} = 2$$ and $$x_i, y_i$$ are the position of each wheel centre in the coordinate system.

Therefore the steering angle of each wheel can be calculated:

![Diagram showing steering angle response of each wheel](image)

Fig.4.1 The steering angle of the four wheel independent steering
Case 2

If the turning radius was 2 m and the vehicle velocity was 10 km/h, the side slip angle response of each wheel is obtained. The steering angle is the same as Fig.4.2.

Fig.4.3 The tire side slip angle
Case 3

If the turning radius was 2 m and the velocity was 14 km/h, the side slip angle response of each wheel is obtained. The steering angle is the same as in Fig. 4.1.

![Graph](image)

**Fig. 4.4 The tire side slip angle**

According to the above three cases, with the increasing of the vehicle velocity, the side slip angle of each tire is increasing with longer settling time and larger overshoot. Therefore, the vehicle is more unstable. Particularly, the front left tire has the worse side slip angle response compared with other tire.

In this way the simulations under different velocities and turning radii can be finalised.
Fig. 4.5 The side slip angle of front left tire under different turn radii and velocity

Fig. 4.6 The side slip angle of front right tire under different turn radii and velocity
Fig. 4.7 The side slip angle of rear left tire under different turn radii and velocity

Fig. 4.8 The side slip angle of rear right tire under different turn radii and velocity
According to Fig.4.5 – Fig.4.8, when the velocity increased and the turning radius decreased, the side slip angle of each tire is increasing and the vehicle is more unstable.

The above simulations only consider the vehicle velocity as the constant value. However, the vehicle velocity is decreasing when turning due to the tire lateral force. The tire lateral force has both longitudinal component and lateral component in the vehicle body coordinate system. The longitudinal component can affect the longitudinal velocity of the vehicle and the lateral component can affect the lateral velocity of the vehicle.

If the turning radius was 5 m and the vehicle velocity was initially 6.2 m/s, the vehicle velocity decreases when the vehicle is turning:

![Graph](image)

**Fig. 4.9 The velocity of the vehicle when turning (turn radius = 5m)**

According to the Fig.4.9, the velocity of the vehicle decrease obviously. If the turn radius increases into 10 m, the decreasing effect of the vehicle velocity can be alleviated (Fig.4.10).
This velocity decreasing effect can be summarised below:

Fig. 4.10 The velocity of the vehicle when turning (turn radius = 10m)

Fig. 4.11 The vehicle velocity decreasing ratio under different turning radii and initial velocity
According to Fig. 4.1, the velocity decreasing ratio is the ratio between the velocity of the vehicle after turning for 10 s and the initial velocity of the vehicle. With the increasing of the vehicle velocity and the decreasing of the turn radius, the velocity decreasing effect is more obvious.

### 4.3 COMBINED TRACTION/BRAKE AND TURNING – SCENARIO TWO

In this section the effect of traction and braking will be considered. When the vehicle is turning, the driver may push the brake pedal to decrease the velocity and maintain stability, or they may increase acceleration to experience a higher speed. Therefore, this set of simulations will be classified combined traction and turning and combined braking and turning.

#### 4.3.1 Combined traction and turning

In the first 10s the vehicle is moving along the straight line with the constant velocity of 11.16 km/h. From 10s to 20s, the vehicle will turn around the turning centre with the turn radius of 5 m. From 15s to 17s, the traction force of 500N is applied for 2 s on the driving wheel and the velocity of the vehicle at 17s is increasing.

To generate the 5 m turn radius turning, for the omni-directional vehicle, the turn centre is assumed as (0, 5) in the vehicle body coordinate system. The CG point in this coordinate system is (0, 0).

According to equation (2.4):

\[
\delta_i = a \tan 2(D^T(x_i - x_{ICR}) - D^T(y_i - y_{ICR}))
\]

\(x_{ICR} = 0, y_{ICR} = 5\) and \(x_i, y_i\) are the position of each wheel centre in the coordinate system.
The steering angle of each wheel:

Fig 4.12 The steering angle of each wheel

The results of side slip angle of each tire:
According to Fig 4.13, there is no side slip angle in the first 10 s because there is no turning motion. From 10s to 15s, the vehicle is turning with a radius of 5 m and consequently each tire has side slip angle. From 15s to 17s there is a traction force applied on the vehicle while the vehicle is still turning. Therefore, the side slip angle of each wheel increase and the vehicle is more unstable (longer settling time and larger overshoot). Particularly, the front left and front rear tire have worse side slip angle response compared with other tires.

The vehicle velocity can also be obtained:
According to Fig. 4.14, in the first 4 s, the vehicle is still. From 4s to 6s, the velocity of the vehicle is increasing to 3.1 m/s. From 10s to the end, the vehicle is turning with the constant turn radius and the velocity decrease a bit because of the velocity decreasing effect while turning which has discussed in section 4.2. From 15 s, the velocity of the vehicle is increasing sharply because of the traction force. However, the vehicle velocity still decreases after that because of the velocity decreasing effect.

In this simulation, the value of maximum tire lateral force is also affected and the lateral force and the traction/brake force form a circular relationship, which is the friction circle.

If the traction/brake force is zero, the maximum lateral force is:

\[ Y_{\text{max}} = \mu F_z \]  

(4.1)

While the traction/brake force is nonzero, the maximum lateral force is:

\[ Y_{\text{max}} = \sqrt{\mu^2 F_z^2 - T^2} \]  

(4.2)

These equations are considered in the tire model.
The side slip angle of vehicle front left tire when considering the friction circle or not considering the friction circle is compared. According to Fig. 4.15, from 15s to 17s the vehicle is in the situation of combined turning and traction and the side slip angle is increase when considering the effect of friction circle. This is because the maximum lateral force is decreasing when considering the traction force and consequently the actual side slip angle will increase.

![Graph showing side slip angle comparison](image)

Fig. 4.15 The side slip angle of vehicle front left tire when considering the friction circle or not considering the friction circle

### 4.3.2 Combined brake and turning

In the first 10s the vehicle is moving along the straight line with the constant velocity of 22.32 km/h. From 10s to 20s, the vehicle will turn around the turning centre with the turn radius of 5 m. From 15s to 17s, the brake force of 500N is applied for 2 s on the driving wheel and the velocity of the vehicle at 17s is decreasing.
The steering input of each wheel is the same as Fig. 4.12. The simulation results of side slip angle of each wheel:

![Graph showing side slip angle response](image)

**Fig 4.16** The side slip angle response

According to Fig 4.16, there is no side slip angle in the first 10 s because there is no turning motion. From 10s to 15s, the vehicle is turning with a radius of 5 m and consequently each tire has side slip angle. From 15s to 17s there is a brake force applied on the vehicle while the vehicle is still turning. Therefore, the side slip angle of each wheel decrease and the vehicle is more stable (less overshoot and settling time).
According to Fig 4.17, in the first 4 s, the vehicle is still. From 4s to 6s, the velocity of the vehicle is increasing to 6.2 m/s. From 10s to the end, the vehicle is turning with the constant turn radius and the velocity decrease a bit because of the velocity decreasing effect while turning which has discussed in section 4.2. From 15 s, the velocity of the vehicle is decreasing sharply because of the brake force.

The front left tire side slip angles when consider the effect of friction circle or not are also compared below:
According to Fig. 4.18, from 15s to 17s the vehicle is in the situation of combined turning and brake and the side slip angle is increase when considering the effect of friction circle. This is because the maximum lateral force is decreasing when considering the brake force and consequently the actual side slip angle will increase.

### 4.4 VEHICLE MOTION BY DISTURBANCE – SCENARIO THREE

The above sections considered that the vehicle was moving in ideal conditions on a flat road with no wind. In this section the effects of the road bank angle, lateral wind force, and coefficient of friction on the stability of the vehicle are discussed.

#### 4.4.1 Bank angle

Part of the vehicle’s weight will change into a lateral force when the road has the bank angle.
According to equation [9]:

\[ F_z = mg \cos \alpha \]
\[ Y_b = mg \sin \alpha \]  
(4.3)

When the bank angle is small the above equation can be simplified as:

\[ F_z = mg \]
\[ Y_b = mg \alpha \]  
(4.4)

The vehicle dynamics equation can be modified as:

\[
mv \left( \frac{db}{dt} + r \right) - Y_b - Y_f \cos \delta_f - Y_r \cos \delta_r - Y_{fl} \cos \delta_{fl} - Y_{fr} \cos \delta_{fr} - Y_{rl} \cos \delta_{rl} - Y_{rr} \cos \delta_{rr} = 0
\]
\[
I \frac{dr}{dt} - l_f Y_f \cos \delta_f - l_f Y_r \cos \delta_r + l_r Y_r \cos \delta_r + l_f Y_{fl} \cos \delta_{fl} + l_f Y_{fr} \cos \delta_{fr} - 0.5T_f Y_f \sin \delta_f + 0.5T_f Y_r \sin \delta_r - 0.5T_r Y_r \sin \delta_r = 0
\]

(4.5)

To do the simulation we assumed the vehicle is moving along the straight line with the constant velocity. From 10 s to 12 s, the disturbance of bank angle is applying on the vehicle body.
Fig. 4.19 The tire side slip angle with the positive bank angle of 0.01 rad and velocity of 20 km/h

Fig. 4.20 The tire side slip angle with the negative bank angle of 0.01 rad and velocity of 20 km/h
Fig. 4.21 The tire side slip angle with the positive bank angle of 0.01 rad and velocity of 80 km/h

Fig. 4.22 The tire side slip angle with the negative bank angle of 0.01 rad and velocity of 80 km/h
Comparing the side slip angle response in different vehicle velocity (same bank angle) in Fig. 4.19-4.22, the high velocity situation has worse side slip response, like larger overshoot and longer settling time. However, the response value of side slip angle in high velocity condition is similar to the value in small velocity condition. In addition, the front left and front right tire has larger side slip angle comparing with the rear left and rear right tire.

![Graph showing side slip angle response](image1)

Fig. 4.23 The tire side slip angle with the positive bank angle of 0.02 rad and velocity of 20 km/h

![Graph showing side slip angle response](image2)

Fig. 4.24 The tire side slip angle with the negative bank angle of 0.02 rad and velocity of 20 km/h
Comparing the side slip angle response in different bank angle (same velocity) in Fig. 4.19-4.20 and Fig. 4.23-4.24, the large bank angle causes the large response side slip angle. However, with the increase of the bank angle, the slip angle responses like overshoot and the settling time remain stable.

### 4.4.2 Lateral wind

The wind scale as described by the Beaufort number [70]:

<table>
<thead>
<tr>
<th>Beaufort number</th>
<th>Wind speed (m/s)</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>B0, B1</td>
<td>0-3</td>
<td>Sub-critical</td>
</tr>
<tr>
<td>B2, B3</td>
<td>3-7</td>
<td></td>
</tr>
<tr>
<td>B4, B5</td>
<td>7-12</td>
<td></td>
</tr>
<tr>
<td>B6, B7</td>
<td>12-18</td>
<td></td>
</tr>
<tr>
<td>B8, B9</td>
<td>18-25</td>
<td>Weak</td>
</tr>
<tr>
<td>B10, B11</td>
<td>25-33</td>
<td></td>
</tr>
<tr>
<td>B12, B13</td>
<td>33-42</td>
<td></td>
</tr>
<tr>
<td>B14, B15</td>
<td>42-51</td>
<td></td>
</tr>
<tr>
<td>B16, B17</td>
<td>51-61</td>
<td>Strong</td>
</tr>
<tr>
<td>B18, B19</td>
<td>61-71</td>
<td></td>
</tr>
<tr>
<td>B20, B21</td>
<td>71-82</td>
<td></td>
</tr>
<tr>
<td>B22, B23</td>
<td>82-93</td>
<td></td>
</tr>
<tr>
<td>B24, B25</td>
<td>93-105</td>
<td>Violent</td>
</tr>
<tr>
<td>B26, B27</td>
<td>105-117</td>
<td></td>
</tr>
<tr>
<td>B28, B29</td>
<td>117-130</td>
<td></td>
</tr>
<tr>
<td>B30, B31</td>
<td>130-143</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1 The level of wind under different velocities

The lateral wind force can be calculated as [9]:

$$ Y_w = C_y \frac{D}{2} S \left( c^2 + W^2 \right) $$  \hspace{1cm} (4.6)

The yaw moment caused by a lateral wind force:

$$ N_w = Y_w l_w $$ \hspace{1cm} (4.7)

In addition, the value of $C_y$ is also determined by the value of $\beta_w$ according to the following figure:
The lateral force coefficient $w$ can be calculated as:

$$
\beta_w = \tan^{-1}\left(\frac{W}{v}\right)
$$

(4.8)

If the aerodynamics centre (AC) of the vehicle is behind the centre of gravity (CG), the yaw moment caused by the lateral wind is clockwise (negative).
If the aerodynamics centre (AC) is in front of the centre of gravity (CG), the yaw moment caused by a lateral wind is anti-clockwise (positive).

The dynamics equation of the vehicle can be modified as:
\[ mv \left( \frac{d\beta}{dt} + r \right) - Y_w Y_f \cos \delta_f - Y_{fr} \cos \delta_{fr} - Y_{rl} \cos \delta_{rl} - Y_{rr} \cos \delta_{rr} = 0 \]

\[ l \frac{dr}{dt} - l_f Y_f \cos \delta_f - l_{fr} Y_{fr} \cos \delta_{fr} + l_{rl} Y_{rl} \cos \delta_{rl} + l_{rr} Y_{rr} \cos \delta_{rr} - 0.5 T_f Y_f \sin \delta_f + 0.5 T_{fr} Y_{fr} \sin \delta_{fr} \]

\[-0.5 T_{rl} Y_{rl} \sin \delta_{rl} + 0.5 T_{rr} Y_{rr} \sin \delta_{rr} = \pm l_w Y_w \]

(4.9)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w )</td>
<td>2 m/s (wind level 2)</td>
</tr>
<tr>
<td>( \rho )</td>
<td>1.2 kg/m(^3)</td>
</tr>
<tr>
<td>( v )</td>
<td>5.556 m/s</td>
</tr>
<tr>
<td>( S )</td>
<td>1.2 m(^2)</td>
</tr>
</tbody>
</table>
| \( l_w \) | 1)0.1 m  
2)-0.1 m  
3)0 m |
| \( \beta_w \) | 0.3455 rad |
| \( C_y \)  | 0.58 |

Table 4.2 The parameters used in this simulation

In this simulation, the vehicle is assumed to move along the straight line with the constant velocity. From 10 s to 12 s, the disturbance of lateral wind is applying on the vehicle body.

4.4.2.1 The aerodynamics centre is the same as the vehicle CG point
Fig. 4.28 The tire side slip angle with the disturbance of positive lateral wind force and velocity of 20 m/s

Fig. 4.29 The tire side slip angle with the disturbance of negative lateral wind force and velocity of 20 m/s
According to the results in Fig. 4.28 - 4.30, the vehicle side slip angle response is more unstable with the increasing of velocity, which has larger response side slip angle, larger overshoot and longer settling time.
4.4.2.2 The aerodynamics centre is not the same as the vehicle CG point

If the aerodynamics centre is not the same as the vehicle CG point, the lateral wind not only affects the lateral force but also affects the moment in yaw plane.

![Graph showing side slip angle with different aerodynamics centre](image)

**Fig. 4.32 The side slip angle of the front left wheel with different aerodynamics centre**

In Fig. 4.32, ‘lw=0’ means that the aerodynamics centre is the same as the CG. When ‘lw=0.1(m)’, the aerodynamics centre is in front of the vehicle CG and the wind force produces the positive yaw moment; when ‘lw=-0.1(m)’, the aerodynamics centre is in the back of the CG and the wind force produces the negative yaw moment.

From Fig. 4.32, the positive ‘lw’ increases the tire side slip angle and the vehicle is more unstable, while the negative ‘lw’ decreases the tire side slip angle and the vehicle is more stable.
4.4.3 The effect of the coefficient of friction

If a vehicle is travelling on the normal dry road the coefficient of friction coefficient is 0.9, which is the default value of the above analysis but if it’s travelling on snow, it should be revised to 0.5.

If the road friction is decreasing, the total lateral force that a tire can generate will decrease, and therefore the side slip angle will increase and make the vehicle more unstable.

The vehicle is turning in a 10m radius at a velocity of 20 km/h (constant velocity). To generate the 10m turn radius turning, for the omni-directional vehicle, the turn centre is assumed as (0, 10) in the vehicle body coordinate system. The CG point in this coordinate system is (0, 0).

According to equation (2.4):

$$\delta_i = a \tan 2(D^T(x_i - x_{ICR}), -D^T(y_i - y_{ICR}))$$ \hspace{1cm} (2.4)

$x_{ICR} = 0, y_{ICR} = 10$ and $x_i, y_i$ are the position of each wheel centre in the coordinate system.

The steering angle of each wheel:

![Fig. 4.33 The steering angle of each wheel](image-url)
Fig. 4.34 The side slip angle of front left tire under different coefficients of friction

Coefficient of friction = 0.9 (red line);

Coefficient of friction = 0.5 (blue line);

Therefore, according to Fig. 4.34, as the coefficient of friction decreases, the side slip angle of the vehicle will increase, overshoot is larger and the settling time is longer. Therefore, the vehicle is more unstable.

4.5 THE EFFECT OF ROLL MOTION – SCENARIO FOUR

In this research the roll motion can affect the yaw plane motion by causing a load transfer and roll steering effect. In the above simulations the vehicle dynamics model included the effect of roll motion, but this is not clear without a comparison, therefore the yaw plane response of the vehicle dynamics model with and without the effect of roll motion will be compared in this section.
4.5.1 Load transfer effect

This simulation is based on the scenario of simple turning with different turning radius and velocity and focuses on the side slip angle of each tire considering the effect of load transfer or not.

![Graph showing the side slip angle of front left tire](image1)

Fig. 4.35 The side slip angle of front left tire when considering the load transfer effect and not considering the load transfer effect (V=10km/h)

![Graph showing the side slip angle of front right tire](image2)

Fig. 4.36 The side slip angle of front right tire when considering the load transfer effect and not considering the load transfer effect (V=10km/h)
Fig. 4.37 The side slip angle of rear left tire when considering the load transfer effect and not considering the load transfer effect (V=10km/h)

Fig. 4.38 The side slip angle of rear right tire when considering the load transfer effect and not considering the load transfer effect (V=10km/h)
According to Fig.4.35-4.38, when the turn radius is relatively large, the load transfer effect on the tire side slip angle can be neglected. However, when the turn radius is small (like 2 m) the load transfer effect is relatively important.

Fig. 4.39 The side slip angle of front left tire when considering the load transfer effect and not considering the load transfer effect (V=20km/h)

Fig. 4.40 The side slip angle of front right tire when considering the load transfer effect and not considering the load transfer effect (V=20km/h)
Fig. 4.41 The side slip angle of rear left tire when considering the load transfer effect and not considering the load transfer effect (V=20km/h)

Fig. 4.42 The side slip angle of rear right tire when considering the load transfer effect and not considering the load transfer effect (V=20km/h)
Comparing Fig. 4.39-4.42 with Fig.4.35-4.38, with the increasing of the vehicle velocity (from 10 km/h to 20 km/h), the load transfer effect on the tire side slip angle is more obvious.

### 4.5.2 The roll steer effect

This simulation is based on the scenario of simple turning with different turning radius and velocity and focuses on the side slip angle of each tire considering the effect of roll steer or not.

Fig. 4.43 The side slip angle of front left tire when considering the roll steer effect and not considering the roll steer effect (V=10 km/h)
Fig. 4.44 The side slip angle of front right tire when considering the roll steer effect and not considering the roll steer effect (V=10km/h)

Fig. 4.45 The side slip angle of rear left tire when considering the roll steer effect and not considering the roll steer effect (V=10km/h)
According to Fig. 4.43-4.46, when the turn radius is relatively large, the roll steer effect on the tire side slip angle can be neglected. However, when the turn radius is small (like 2 m) the roll steer effect is relatively important.

Fig. 4.46 The side slip angle of rear right tire when considering the roll steer effect and not considering the roll steer effect (V=10km/h)

Fig. 4.47 The side slip angle of front left tire when considering the roll steer effect and not considering the roll steer effect (V=20km/h)
Fig. 4.48 The side slip angle of front right tire when considering the roll steer effect and not considering the roll steer effect (V=20km/h)

Fig. 4.49 The side slip angle of rear left tire when considering the roll steer effect and not considering the roll steer effect (V=20km/h)
Fig. 4.50 The side slip angle of rear right tire when considering the roll steer effect and not considering the roll steer effect (V=20km/h)

Comparing Fig. 4.43-4.46 with Fig. 4.47-4.50, with the increasing of the vehicle velocity (from 10 km/h to 20km/h), the roll steer effect on the tire side slip angle is more obvious.

4.6 A FOUR WHEEL INDEPENDENT DRIVING SYSTEM – SCENARIO FIVE

In this set of simulations the dynamic performances of different driving styles such as front wheel drive, rear wheel drive, and four wheel drive, were analysed.

4.6.1 The straight line acceleration

The vehicle remained motionless for the first 4 seconds, but from 4s to 6s a driving force was applied and the vehicle velocity increase.

During front wheel driving and rear wheel driving, the driving force is only applied onto the two front wheels and two rear wheels. For four wheel driving the driving force is applied equally onto the four wheels.
In this simulation, for the two front/rear wheel driving, the driving force of 1000N is applied equally on the two front/rear wheels from 4s to 6s. For the four wheel driving, the driving force of 1000N is equally applied on the four wheels from 4s to 6s. The final vehicle of each driving style is showing:

![Graph showing vehicle velocity under different traction style](image)

**Fig. 4.51** The vehicle velocity under different traction style

### 4.6.2 The combined turning and traction

In this simulation, the vehicle is performing the motion of constant radius turning in the first 15s, while the driving force (500N) is applied on the vehicle with front/rear wheel driving or four wheel driving from 15s to 17s.

Assume the turn radius is 5m in this simulation, the steering of each wheel is:
Fig. 4.52 The steering angle of each wheel

The simulation results:

Fig. 4.53 The side slip angle of front left wheel in different traction styles

According to Fig. 4.53, from 15s when the driving force is applied, the four wheel driving vehicle has large tire side slip angle response compared with the two wheel driving vehicle. Therefore, the four wheel driving vehicle is more unstable.
According to Fig. 4.54, from 15s when the driving force is applied, the four wheel driving vehicle can have large vehicle velocity compared with the two wheel diving style because the four wheels all have traction force.

Therefore, the four wheel driving vehicle can have larger vehicle velocity but the vehicle is more unstable when turning compared with the two wheel driving vehicle.

4.7 CONCLUSION

In this chapter a simulation of vehicle turning and combined turning and traction/braking was addressed. In addition, external disturbances such as the bank angle, lateral wind and the coefficient of friction were also presented. Finally the effect of roll motion on vehicle stability and how the driving system affects vehicle velocity and vehicle stability was discussed.
5.1 INTRODUCTION

In Chapter 4 the dynamics performance of a vehicle was analysed. The side slip angle of each tire is critical to the stability of the vehicle. One advantage of the omni-directional vehicle is to realize the zero side slip angle condition by independently control the steering angle of each wheel.

5.2 IMPLEMENTATION AND SIMULATION OF PID CONTROLLER

To realize the zero side slip angle condition, a PID controller is needed to reduce the side slip angle of each tire. According to the actual side slip angle of each tire, this PID controller can calculate the additional steering angle of each wheel needed to reduce the vehicle’s tire side slip angle (Fig. 5.1).

Fig. 5.1 The overview of the controller
According to Fig 5.2, the target tire side slip angle is assumed to be zero for an ideal condition and the actual tire side slip angle is obtained from the output of the vehicle dynamics model. A different signal between the target side slip angle and the actual side slip angle is then sent into the PID controller to calculate the required steering angle to improve vehicle stability.

To examine the effective of the proposed side slip angle controller, the simulation of the simple turning is carried out. In this simulation, the vehicle is assumed to turn with the turn radius of 5m and velocity of 15 km/h. To have the 5m radius turning, the steering angle of each wheel initially (without the controller) should be:
In this simulation, only the P control gain is used and are set as (5,5,0,0) in front left wheel, front right wheel, rear left wheel and rear right wheel. The side slip angle of each wheel with the controller and without the controller is compared:

Fig. 5.4 The front left tire side slip angle

Fig. 5.5 The front right tire side slip angle
According to Fig. 5.4-5.7, the side slip angle is much improved in front left wheel, front right wheel and rear left wheel both in the steady-state value, the overshoot and the settling time. Only the side slip angle response is similar in rear right wheel.
However, this side slip controller need the additional steering angle and consequently the actual steering angle and the vehicle trajectory is changed greatly which is not desired.

![Vehicle Trajectory](image)

**Fig. 5.8 The vehicle trajectory**

Therefore, the P control gain need to be adjusted to prevent this situation. The advantage for the omni-directional vehicle is that it can have different steering angle for the left and right wheel. According to this, the P control gain for the left wheel and right wheel can be different and the value can be updated as (5,1,0,0) which stands for the front left wheel, the front right wheel, the rear left wheel and the rear right wheel. The simulation result of the adjusted controller is showing:
Fig. 5.9 The front left tire side slip angle (control gain revised)

Fig. 5.10 The front right tire side slip angle (control gain revised)
Fig. 5.11 The rear left tire side slip angle (control gain revised)

Fig. 5.12 The rear right tire side slip angle (control gain revised)
According to Fig. 5.9-5.12, the side slip angle is much improved in front left wheel and rear left wheel with the revised controller both in the steady-state value, the overshoot and the settling time. The side slip angle response is similar in rear right wheel, but only the front right wheel has larger side slip angle response with controller. Therefore, the vehicle stability is improved with the revised controller. In addition, according to fig. 5.13 and fig. 5.8, the vehicle trajectory with the revised controller is quite close to the original trajectory without the controller, which proves that the revised controller has better control performance than the first controller.

5.3 CONCLUSION

In conclusion, to improve vehicle stability the side slip angle controller is implemented on each wheel of the vehicle and the simulation results prove that this controller can improve the vehicle stability performance.
CHAPTER 6  CONCLUSION AND FUTURE WORK

The main objective of this thesis was to propose a vehicle dynamics model for an omni-directional vehicle. A simulation was then conducted using a vehicle dynamics model to analyse the stability of the yaw plane of an omni-directional vehicle, and a PID side slip angle controller was developed to improve the yaw stability of the vehicle. The main contribution of this thesis is summarised in part one of this chapter and future work is presented in part two.

6.1 SUMMARY

In chapter 3 a vehicle dynamics model was developed for an omni-directional vehicle. This model focussed on the dynamic behaviour of the yaw plane, which meant that the side slip angle and yaw rate were the two most important outputs of the dynamics system and determined its inherent stability. An innovative four wheel independent steering system was included in the vehicle dynamics model and the effect of roll motion on the yaw plane was considered. All the sub-systems in the vehicle dynamics model were linear, apart from the four independent tire model systems which can occasionally alternate from linear into non-linear systems. Therefore, the stability of the whole vehicle was determined by the four tire model systems.

In chapter 4 a stability analysis of the vehicle’s yaw plane was emphasised in different scenarios. The first scenario was simple radius turning. The side slip angles of four tires are compared with different turn radii and velocity. Then the simulation of combined turning and traction/braking was shown and the side slip angle of each wheel is also presented. The effect of disturbance on vehicle stability was analysed, which included the bank angle, lateral wind and the coefficient of friction. Finally, the effect of rolling motions such as load transfer, roll steer, and the performance of an independent driving system were presented.

In chapter 5 a side slip angle controller of each wheel is proposed. According to the actual side slip angle of each wheel, the controller provides the additional feedback steering angle to
each wheel to decrease the actual side slip angle of each wheel. However, the control system can change the vehicle trajectory because of the changing of the steering angle. Therefore, the suitable control gain is required to have the best control performance.

6.2 FUTURE WORK

This research only focussed on the vehicle dynamics yaw plane model and roll motion model because they are related to vehicle stability. The pitching motion which is related to passenger comfort was not addressed. In addition, this research only consists of mathematical modelling and simulations, it does not include the detailed steering mechanism, suspension system, or driving system. The effect of power systems such as electrical or hybrid on the vehicle dynamics was not included.

An omni-directional steering system is steering-by-wire technology which is controlled by electronic devices like a PC and a bus line. Detailed design of this novel electronic steering control system was not addressed in this research. A more advanced control strategy for direct yaw moment control and active steering control instead of a simple PID controller needs to be developed.

An omni-directional vehicle is novel technology in the automobile industry and may need a long time to be commercialised. This thesis only focussed on a mathematical model of the vehicle dynamics system and the simulation results are only based on computer software. Therefore, the results would need to be verified experimentally or by building and testing a real, commercialised omni-directional vehicle.
REFERENCES


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APPENDIX A:

The basic vehicle parameters used in the simulation [12]:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle total mass</td>
<td>1298.9 kg</td>
</tr>
<tr>
<td>Vehicle sprung mass</td>
<td>1167.5 kg</td>
</tr>
<tr>
<td>Distance of c.g. from the front axle</td>
<td>1 m</td>
</tr>
<tr>
<td>Distance of c.g. from the rear axle</td>
<td>1.454 m</td>
</tr>
<tr>
<td>Front track width</td>
<td>1.436 m</td>
</tr>
<tr>
<td>Rear track width</td>
<td>1.436 m</td>
</tr>
<tr>
<td>Height of the sprung mass c.g.</td>
<td>0.533 m</td>
</tr>
<tr>
<td>Distance of the sprung mass c.g. from the roll axis</td>
<td>0.4572 m</td>
</tr>
<tr>
<td>Vehicle moment of inertia about yaw axis</td>
<td>1627 kg.m²</td>
</tr>
<tr>
<td>Vehicle moment of inertia about roll axis</td>
<td>498.9 kg.m²</td>
</tr>
<tr>
<td>Sprung mass product of inertia</td>
<td>0 kg m²</td>
</tr>
<tr>
<td>Wheel radius</td>
<td>0.35 m</td>
</tr>
<tr>
<td>Wheel moment of inertia</td>
<td>2.1 kg m²</td>
</tr>
<tr>
<td>Cornering stiffness of one tyre</td>
<td>30000 N/rad</td>
</tr>
<tr>
<td>Longitudinal stiffness of one tyre</td>
<td>50000N/unit slip</td>
</tr>
<tr>
<td>Front roll steer coefficient</td>
<td>-0.2 rad/rad</td>
</tr>
<tr>
<td>Rear roll steer coefficient</td>
<td>0.2 rad/rad</td>
</tr>
<tr>
<td>Ratios of front roll stiffness to the total roll stiffness</td>
<td>0.552</td>
</tr>
<tr>
<td>Roll axis torsional damping</td>
<td>3511.6 Nm/rad/sec</td>
</tr>
<tr>
<td>Roll axis torsional stiffness</td>
<td>66185.8 Nm/rad</td>
</tr>
<tr>
<td>Road adhesion reduction factor</td>
<td>0.015 s/m</td>
</tr>
<tr>
<td>Acceleration of gravity</td>
<td>9.81 m/s²</td>
</tr>
<tr>
<td>Stability factor</td>
<td>0.005</td>
</tr>
<tr>
<td>Nominal friction coefficient between tyre and ground</td>
<td>0.9 and 0.5</td>
</tr>
</tbody>
</table>
APPENDIX B: IMPLEMENTING THE VEHICLE DYNAMICS MODEL INTO THE MATLAB SIMULINK

The vehicle dynamics model was the most important part in my project and an innovative omni-directional vehicle dynamics model is proposed here. According to Fig. 3.3, there are four major parts in the dynamics model: the tire model, the vehicle body model, the load transfer model, and the traction/brake model.

B.1 The tire model

The tire model describes the lateral and longitudinal forces generated by the tire. The Dugoff tire model is a physical tire model which can convey the actual physical meaning of tire parameters. In addition the Dugoff tire model is quite easy to analyse the three different regions: the linear region, the non-linear region, and the decreasing region. By finding the critical point between these regions, the vehicle dynamics stability analysis can easily be performed. Therefore, the Dugoff tire model has been used as part of the vehicle dynamics equations in my project.

The Dugoff tire model was revised to only consider lateral behaviour, so consequently, longitudinal velocity is a constant value without traction and braking:

The non-dimension parameter $\lambda$ can be calculated:

$$\lambda = \frac{uF_y(1 - \varepsilon_v \cdot \text{abs}(\tan \alpha))}{2 \cdot \text{abs}(C_\alpha \tan \alpha)}$$  \hspace{1cm} (B.1)

If $\lambda < 1$, the tire lateral force

$$F_y = C_\alpha (\tan \alpha) \lambda(2 - \lambda)$$  \hspace{1cm} (B.2)

If $\lambda > 1$, the tire lateral force

$$F_y = C_\alpha (\tan \alpha) \lambda$$  \hspace{1cm} (B.3)
The tire model was modelled by Matlab Simulink.

In the Matlab the tire model was edited as the m-function. The input value was the side-slip angle, the vertical load was the longitudinal velocity (the velocity was constant only if the tire lateral behaviour was considered). The output value was the tire lateral force. In this way the tire model could only be considered as a function file in the Matlab Simulink Block diagram because it was easier to organise into the whole vehicle dynamics model.

According to equation (B.1-B.3), the Matlab m-function file of the tire model is:

```matlab
function [ Fy0 ] = Dugoff( afa, Fz0, v )
% DUGOFF
% % Detailed explanation goes here
u=0.9;
Ca=30000;
xv=0.015;
lamada=u*Fz0*(1-ex*v*abs(tan(afa)))/(2*abs(0.001+Ca*tan(afa)));
if lamada<1
    flamada=lamada*(2-lamada);
else
    flamada=1;
end
Fy0=Ca*tan(afa)*flamada;
end
```

Fig. B.1 The Matlab m-function file of the tire model

**B.2 The load transfer model**

When the vehicle was turning the body will generate a roll angle to balance the centrifugal force. The roll angle will cause a load transfer of the vertical load, and there will be an additional steering angle caused by the roll angle. All the modelling was completed with the help of Matlab Simulink.

Firstly the vertical load without load transfer was determined by the front wheel base and the rear wheel base:
\[ F_{yf} = F_{yf} = \frac{1}{2} mg \frac{l_r}{l_f + l_r} \]
\[ F_{yl} = F_{yl} = \frac{1}{2} mg \frac{l_f}{l_f + l_r} \]

Therefore the constant value \( C \) in Fig. B.2 was set as \( '0.5mgl_f/(l_f + l_r)' \) for the front wheel and \( '0.5mgl_f/(l_f + l_r)' \) for the rear wheel. All the values of the parameters are according to Appendix A.

![Vehicle load transfer model in Simulink](image)

**Fig. B.2 Vehicle load transfer model in Simulink**

Next, the vertical load should add or subtract the front load transfer or rear load transfer. If the vehicle was turning left, the vertical load of the left wheel should decrease and the vertical load of right wheel should increase. However, if the vehicle was turning right, the
vertical load of the left wheel should decrease and the vertical load of the right wheel should increase.

If we assume that ‘turning left’ is positive

\[
\begin{align*}
F'_{cf} &= F_{cf} - \Delta W_f \\
F'_{cfr} &= F_{cfr} + \Delta W_f \\
F'_{crl} &= F_{crl} - \Delta W_r \\
F'_{cr} &= F_{cr} + \Delta W_r
\end{align*}
\]

The load transfer \(\Delta W_f\) and \(\Delta W_r\) can be calculated according to equation:

\[
\begin{align*}
\Delta W_f &= \frac{\dot{y} W_s}{T_f} \left[ \frac{K_\phi h_s}{K_\phi + K_\psi - W_f h_s} + \frac{l_f}{l} h_f \right] \\
\Delta W_r &= \frac{\dot{y} W_s}{T_r} \left[ \frac{K_\phi h_s}{K_\phi + K_\psi - W_r h_s} + \frac{l_f}{l} h_r \right]
\end{align*}
\]

Therefore, in the Fig. B.2, the gain value ‘dWf’ has been set as

\[
0.5 / (m/lbf) * [Kfai * hs / (2 * Kfai * ms * g * hs) + lr * (hcg - hs) / (lf + lr)]
\]

The gain value of ‘dWr’ has been set as

\[
0.5 / (m/lbr) * [Kfai * hs / (2 * Kfai * ms * g * hs) + lr * (hcg - hs) / (lf + lr)]
\]

Actually the value of lateral acceleration was not included in the ‘dWf’ and ‘dWr’. The value (labelled as the red dotted line) was obtained from the vehicle body dynamics model which will be discussed in detail in the following.

Then in Fig B.2, if the lateral acceleration was positive the vehicle was turning left and the value of load transfer was positive. Therefore, the vertical load of the left wheel should remove the load transfer and the vertical load of the right wheel should add the load transfer. If the lateral acceleration was negative and the vehicle was turning right, then the load transfer is negative. Therefore, the vertical load will change.

In addition, the roll angle also caused an additional steering angle, which is called roll steer. Firstly the roll angle should be calculated according to equation:
\[
\phi = \frac{\ddot{y}W_i h_i}{K_{\phi} + K_{\psi} - W_s h_s}
\]  
(B.7)

Then the additional steering angle can be calculated according to equation:

\[
\alpha_f = \frac{\partial \alpha_f}{\partial \phi} \phi
\]

\[
\alpha_r = \frac{\partial \alpha_r}{\partial \phi} \phi
\]  
(B.8)

Therefore, in Fig.B.3, the ‘roll effect’ constant should be set as:

Constant value:

\[
ms*hs*w/(kfr*ms*gs*hs)
\]

Then the ‘roll effect’ constant was multiplied by the yaw rate from the vehicle body dynamics model to get the roll angle. Finally the additional steering angle can be obtained by multiplying the roll angle by the roll steer coefficient, which can be found from Appendix A.

![Vehicle roll steer model in the Matlab Simulink](image_url)

Fig. B.3 Vehicle roll steer model in the Matlab Simulink
B.3 The vehicle body dynamics model

The vehicle body dynamics model combined the tire model, load transfer model, and the traction/brake model, together with the yaw plane model, to get important overall information like the yaw rate and side slip angle of the CG point and the lateral/longitudinal acceleration, which is critical to the stability analysis of the vehicle in the yaw plane.

If the effect of traction and brake was considered, the longitudinal velocity can no longer be considered as the constant value. Therefore, the 2 DOF yaw plane model is no longer suitable for this situation and a 3 DOF yaw plane model is proposed [12]:

\[
\begin{align*}
    m \left( \frac{dy}{dt} - ur \right) &= F_{sfl} + F_{sfr} + F_{srl} + F_{srr} \\
    m \left( \frac{du}{dt} + vr \right) &= F_{yfl} + F_{yfr} + F_{yrl} + F_{yrr} \\
    l \frac{dr}{dt} &= I_f (F_{sfl} + F_{sfr}) - I_r (F_{yrl} + F_{yrr}) + 0.5T_f (F_{sfl} - F_{sfr}) + 0.5T_r (F_{srl} - F_{srr})
\end{align*}
\] (B.9)

This set of equations considered the force equilibrium in the X direction of the yaw plane, the force equilibrium in the Y direction of the yaw plane, and the moment equilibrium of the yaw plane. In this way the traction and brake dynamics of the vehicle in the yaw plane can successfully be described.
According to equation (B.9), a Matlab Simulink model was developed. (Fig. B.4) The output of the dynamics model in Fig.B.4 is the longitudinal velocity, the lateral velocity and the yaw rate of the vehicle. The input of the model is the longitudinal force ‘$F_x$’ and lateral force...
‘\(F_y\)’, which is not exactly the traction force ‘\(F_i\)’ and tire lateral force ‘\(Y_i\)’. Equation (B.10) transfer the ‘\(F_i\)’ and ‘\(Y_i\)’ into the ‘\(F_x\)’ and ‘\(F_y\)’.

\[
\begin{align*}
F_x &= F_i \cos \delta_i - Y_i \sin \delta_i \\
F_y &= F_i \sin \delta_i + Y_i \cos \delta_i
\end{align*}
\] (B.10)

According to equation (B.9) in the Matlab Simulink:

![Diagram](image)

Fig. B.5 The transfer model of equation (B.9) (front left wheel)

Now a model of the tire lateral force and traction force needs to be developed.
Fig. B.6 The model of the tire lateral force in 3 DOF yaw plane model

Fig. B.6 shows the model of the tire lateral force, which is similar to the lateral tire force model of 2 DOF yaw plane model.

The Dugoff tire model was still used:

MATLAB function:

```
Dugoff(u(1),u(2),u(3))
```

There are three inputs in the model in Fig. B.6: the side-slip angle, the vertical load and vehicle velocity.

**B.3.1 Side slip angle**

The side slip angle of each wheel can be calculated according to the following equation, where the yaw rate and vehicle velocity are all from the output of the 3 DOF yaw plane model:
\[
\alpha_{fl} = \delta_{fl} - \arctan \left( \frac{u + l_f r}{v - 0.5T_f r} \right) \\
\alpha_{fr} = \delta_{fr} - \arctan \left( \frac{u + l_f r}{v + 0.5T_f r} \right) \\
\alpha_{rl} = \delta_{rl} + \arctan \left( \frac{l_r r - u}{v - 0.5T_r r} \right) \\
\alpha_{rr} = \delta_{rr} + \arctan \left( \frac{l_r r - u}{v + 0.5T_r r} \right) \\
\]

(B.11)

In the Matlab Simulink:

![Matlab Simulink diagram](image)

Fig. B.7 A model of side slip angle (front left wheel)

The additional side slip angle caused by the rolling still needs to be considered. The input of roll steer was obtained from the roll steer model.
B.3.2 The vertical load

The vertical load $F_z$ also used the load transfer model in Fig. B.2, but it does need the value of lateral acceleration, which can be obtained from the model in Fig. B.8.

The lateral acceleration can be calculated:

$$a_y = \sum F_{yi} / m \quad \text{(B.12)}$$

Fig. B.8 Part of the model in Fig. B.4