2012

Experimental and theoretical approached for AC losses in practical superconducting tapes for engineering applications

Khay Wai See
University of Wollongong

Recommended Citation
UNIVERSITY OF WOLLONGONG

COPYRIGHT WARNING

You may print or download ONE copy of this document for the purpose of your own research or study. The University does not authorise you to copy, communicate or otherwise make available electronically to any other person any copyright material contained on this site. You are reminded of the following:

Copyright owners are entitled to take legal action against persons who infringe their copyright. A reproduction of material that is protected by copyright may be a copyright infringement. A court may impose penalties and award damages in relation to offences and infringements relating to copyright material. Higher penalties may apply, and higher damages may be awarded, for offences and infringements involving the conversion of material into digital or electronic form.
Faculty of Engineering

EXPERIMENTAL AND THEORETICAL APPROACHES FOR AC LOSSES IN PRACTICAL SUPERCONDUCTING TAPES FOR ENGINEERING APPLICATIONS

Khay Wai See

This thesis is presented as part of the requirement for the Award of the Degree of

DOCTOR OF PHILOSOPHY

from

University of Wollongong

March 2012
DECLARATION

I, Khay Wai See, declare that this thesis, submitted in fulfillment of the requirements for the award of Doctor of Philosophy, in the Faculty of Engineering, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. The document has not been submitted for qualification at any other academic institution.

Khay Wai See
March 2012
ACKNOWLEDGEMENTS

This Ph.D. thesis contains the results of three and a half years of comprehensive research, which has been performed at the Laboratory of Institute of Superconducting and Electronic Materials (ISEM) under the Faculty of Engineering, University of Wollongong. I would like to express my sincere thanks to my supervisors, Professor Chris Cook and Professor Shi Xue Dou, for relentlessly supporting and supervising during the course of my study. I also would like to thank Associate Professor Dr. Joseph Horvat for the continuous advising and assistance during the years of the laboratory works. Special thanks are given to Dr Frank Darmann from Zenergy for invaluable discussions and provision of superconducting tapes.

Thanks are also due to Dr. Xun Xu, Mr. Ron Kinnell and Mr. Robert Morgan from ISEM for their invaluable assistance in the instrumentation and experimental development. Likewise, I wish to express my gratitude to the University of Wollongong’s Research Centre for providing the scholarship and living allowances during the course of my study in Australia. I gratefully acknowledge the advice and discussion which I have received from Professor Toby Norris and the late Professor Ernst Helmut Brandt concerning the theoretical approaches of ac losses which I have presented in the thesis. Similarly I thank Dr. Su Mei Goh for the generous help and advice she gave for the numerical computation part of the thesis.

Last but not least, the love, patience and understanding of my parents Kok Wah See and Cuu Hock Fong and my siblings were of great help in completing my study.
ABSTRACT

Practical superconducting wires and tapes have significant potential for use in electrical engineering applications. The work presented in this thesis is focused on developing new measurement and theoretical techniques to more accurately model and measure ac losses in superconducting tapes in order to contribute to the skills, tools and understandings needed to assist in their utilization. Investigations of both BSCCO and MgB$_2$ superconducting tape and wire are described.

A review of the literature with special emphasis on the mechanism of AC losses and theoretical approaches is presented. Some of the widely used analytical expressions for losses are presented and existing experimental techniques are reviewed and discussed in detail. Major instrumentation methods generally used in the measurement of ac losses are described.

An experimental chamber for low temperature measurements has been assembled comprising cryogenic parts, electronic and electrical instrumentation and software written to provide a comprehensive data acquisition system. Detailed measurements are made on the effects of magnetic fields, temperature, current and mechanical strain on superconducting tape performance.

A new method for AC loss measurement of BSCCO and MgB$_2$ superconducting tape at low temperatures (< 50 K) using a calorimeter has been investigated and demonstrated. Calorimeter techniques have been widely accepted and have the advantage of being able to measure the total losses in a superconducting wire. The calorimeter presented here allows the measurement to be performed at low temperatures from 10 K to 50K as compared with previously reported calorimeters that only measure at liquid nitrogen temperature at 77 K. This temperature range is of interest since the rapid development of cryogen-free chambers now makes this operating range important for practical applications of superconducting materials. In addition, the calorimeter was built to include a superconducting solenoid coil so that measurements on the effect of both external DC and AC magnetic fields on the superconducting sample can be undertaken. This method has been used to measure losses of BSCCO tape exposed perpendicularly to both DC and AC magnetic fields at various
temperatures. Comparison between measurement results and theoretical computations verified the accuracy and reliability of the method. Moreover, AC loss measurement has also been performed on MgB₂ tape but with different external conditions in order to measure transport AC current with applied DC transverse field. The tape is mounted distinctively with a sharp bending edge and several calibration and stability tests carried out to assure the validity of measurement results. Losses were found to be higher than the theoretical predictions because the metallic parts of the tape contribute quite significantly to the total losses.

Transport critical current measurement for MgB₂ wire and tape has been investigated with two different techniques, the DC four-probe arrangement and pulsed current with varying rate. The former method suffers from inevitable heating effect when measuring wire with high critical current value. This effect is more pronounced for the commonly used measurement of short samples. The pulsed method on the other hand has no significant heating effect but the critical current can depend on the rate of the current change \(\frac{dI}{dt}\) in the pulse. This method is particularly useful at low field regions which are often inaccessible using conventional DC methods. A finite element method (FEM) analysis is also performed to provide further evidence of the limitation of the DC method in obtaining high transport critical current. It is shown that the best way to accurately measure \(I_c\) at low fields is to extrapolate the values of \(I_c\) measured at different \(dI/dt\) to \(dI/dt = 0\). This overcomes problems caused by the effects of heating (introduced in DC measurements) or effects of vortex dynamics (introduced in pulsed measurements).

A new direct analytical method of solving the nonlinear integral equation that gives the current as a function of the applied external field or of the transport current is also presented. This method solves the singularity problem in thin superconductor and provides analytical expressions for both the sheet current and field distribution under self and external field conditions. In addition, a new numerical scheme is introduced using a two dimensional nonlinear magnetic diffusion equation model to find AC losses with finite thickness.


K W See, X Xu, J Horvat, C D Cook and S X Dou, “Calorimetric AC Loss Measurement of MgB$_2$ Superconducting Tape in Alternating Transport Current and Direct Magnetic Fields,” *submitted to Supercondor Science and Technology*. 
# TABLE OF CONTENTS

ACKNOWLEDGMENTS iii
ABSTRACT iv
PUBLICATIONS BASED ON RESEARCH WORK FOR THIS THESIS vi
TABLE OF CONTENTS vii
LIST OF FIGURES x
LIST OF TABLES xvii
LIST OF PRINCIPAL SYMBOLS xviii

## CHAPTER 1: INTRODUCTION 1

1.1 Superconducting Materials 3
1.2 Alternating Transport Current and Magnetic Field: - Loss Mechanisms 5
1.3 Theoretical Progress – Past and Present 7
1.4 Alternating Current Loss Measurements 11
1.5 Contributions of This Thesis 12
1.6 Structure of The Thesis 14

## CHAPTER 2: EVOLUTION OF SUPERCONDUCTING THEORIES – FIELD, CURRENT AND LOSSES DISTRIBUTIONS 16

2.1 Three Critical Quantities (T_c, H_c and I_c) 16
2.2 Perfect Conductivity and Perfect Diamagnetism (Meissner Effect) 18
2.3 Type I and Type II Superconductors 19
2.4 Penetration Model (London Model) 20
2.5 Current and Field Distributions for Simple Geometries 21
2.6 Critical State Model (Bean Model) 23
2.7 Current in Superconductors 26
2.8 Losses in Superconductors 28
2.9 Conclusion 33
CHAPTER 3: THEORETICAL APPROACHES TO FINDING FIELD AND CURRENT DISTRIBUTIONS AND AC LOSSES 34
3.1 Introduction 34
3.2 Transport Current without Applied Magnetic Field (ie in Self field) 35
3.3 Transverse Direction of Applied Magnetic Field 42
3.4 Losses Computation and Distribution – Analytical Approach 46
3.5 Variational Iteration Method for Nonlinear Magnetic Diffusion Model for AC Losses 50
3.6 Conclusion 58

CHAPTER 4: DEVELOPMENT OF EXPERIMENTAL APPROACHES AND LOW TEMPERATURE ENGINEERING 60
4.1 Introduction 60
4.2 Progress in Measurement Techniques 60
4.2.1 Electrical (electromagnetic) Method 61
4.2.2 Calorimetric Method 64
4.3 Cryogenic Instrumentations 67
4.3.1 Supporting Structure – Sample and Coil 71
4.4 Electrical and Acquisition System Design 73
4.5 System Limitations and Solution Studies 79
4.6 Conclusion 81

CHAPTER 5: TRANSPORT CRITICAL CURRENT MEASUREMENT: PULSED CURRENT OF VARYING RATE COMPARED TO DIRECT CURRENT METHOD 83
5.1 Introduction 83
5.2 Finite Element Method (FEM) Analysis 85
5.3 Experimental Details 91
5.4 Results and discussions 95
5.5 Conclusion 100
CHAPTER 6: INNOVATIVE CALORIMETRIC AC LOSS MEASUREMENT OF HIGH TEMPERATURE SUPERCONDUCTOR IN PERPENDICULAR APPLIED MAGNETIC FIELD WITHOUT TRANSPORT CURRENT

6.1 Introduction 101
6.2 Experimental Procedure 102
6.3 AC Solenoid Coil Characteristic 105
6.4 Results and Discussion 106
6.5 Conclusion 113

CHAPTER 7: INNOVATIVE CALORIMETRIC AC LOSS MEASUREMENT OF MgB₂ SUPERCONDUCTING TAPE CARRYING ALTERNATING TRANSPORT CURRENT WITH APPLIED DIRECT CURRENT MAGNETIC FIELDS

7.1 Introduction 115
7.2 Experimental Procedure 116
7.3 Design Stability and Measurement Considerations 118
7.4 Results and Discussion 121
7.5 Conclusion 128

CHAPTER 8: SUMMARY AND CONCLUSIONS

8.1 Finding new Mathematical Models for Field and Current Distribution and AC Losses 129
8.2 Experimental and Low Temperature Engineering 130
8.3 Method of Transport Critical Current Measurement 131
8.4 Calorimetric Measurement of Losses in HTSC Tape 132
8.5 Calorimetric Measurement of Transport Losses in MgB₂ Tape 132
8.6 Recommendations for Further Research 133

REFERENCES 135

APPENDIX A: Fourier Series with Homogenous and Nonhomogenous Boundary Conditions 152
LIST OF FIGURES

Figure 1.1. Critical state parameter requirements for several electric power applications. Data extracted from [4]………………………………………………………..2

Figure 1.2. The discovery of superconductivity materials over the years with their respective critical temperature. Black – elemental superconductors, Blue – LTS, Red – HTS, Green – MgB2. Data from reference 9…………………..4

Figure 1.3. (a) Screening currents in the white region shield the interior (grey) from the magnetic field. (b) Distribution of magnetic field \( \overline{B} \) inside and around the sample…………………………………………………………………6

Figure 2.1. The phase diagram of a typical type II superconductor………………………………………………………..17

Figure 2.2. The difference between materials that become perfect conductors or perfect diamagnets below a critical temperature \( T_c \)………………………………………………………..19

Figure 2.3. (a) An infinite slab of width 2a with an applied magnetic field \( H \) parallel to the long side of the slab. (b) Magnetic field and current distributions in the slab for an applied magnetic field \( H \)…………………………………...25

Figure 2.4. (a) Magnetic field and current distribution in the slab when the applied magnetic field has decreased, the applied field \( H=0 \). (b) The typical hysteresis loop occurs when the mean magnetic flux density \( \overline{B} = \frac{a}{2a} \int_{-a}^{a} h(x)dx \) in the superconductor is plotted versus the applied magnetic field \( H \) [7,8]………………………………………………………..25

Figure 3.1. Schematic view of the thin strip superconductor with width 2a and the coordinate system used for the current and field distribution studies. The transport ac current flows along the \( z \) axis. The region with width 2b is the field free region and the region in \( b < |x| < a \) is the saturation region with current flow equal to \( I_c \)………………………………………………………..37

Figure 3.2. Plot of (a) current density \( J_z(x) \) distribution from equation (3.21) and (b) normal component of magnetic field \( H_n(x) \) from equation (3.23) in a superconducting thin strip of width 2a carrying a transport current. The
penetration width \( b \) for \( I/I_c = 0.8 \) is indicated at the top figure (a) and \( a \) as the half width unit length.

Figure 3.3. Plot of (a) current density \( J_z(x) \) distribution from equation (3.33) and (b) normal component of magnetic field \( H_y(x) \) from equation (3.34) in a superconducting thin strip of width \( 2a \) in a perpendicular magnetic field \( H_a \) which increased from zero. The distribution profiles are for \( H_a/H_c = 0.5, 1.0, 1.5, 2.0 \) and the penetration width \( b \) is obtained from (3.32).

Figure 3.4. AC losses for both transport current (a) and applied transverse field (b) at various critical current for superconducting strip. The shaped markers are from the variational iteration method and the lines are from analytical expression from equations (3.41) and (3.46).

Figure 4.1. Conceptual drawing of the system used for studying superconducting samples in a magnetic field at varying temperatures.

Figure 4.2. Superconducting magnet cryostat system with multilayer super-insulated and liquid nitrogen shielded. The magnet is capable of providing dc magnetic field up to 15 T in vertical axis and operating temperature range from 2 K to 300 K inside the insert.

Figure 4.3. Variable temperature insert for practical superconducting wires. Left: Cylindrical tubular shape of supporting structure. Right: Internal configuration for sample cooling, mounting and testing.

Figure 4.4. Left: Supporting frame for sample and coil placed inside the variable temperature insert. The entire frame is made of G10 phenolic material with nylon screw as the fastener. Right: Photograph of the constructed frame.

Figure 4.5. The configuration for sample and external ac coil with the supporting frame mounted on a copper disk with nylon screw. Copper disk is fixed to the removable rod and the assembly can be withdrawn from the chamber as a whole. Inset: Photograph of the rod with HTS current leads and mounting copper disk.

Figure 4.6. A GPIB topology with daisy chaining of the electrical/electronic instruments to the computer. The computer is installed with LabVIEW software to
transform the assembly into a complete data acquisition, analysis, and presentation tool...............................................................79

Figure 4.7. Photograph of the integrated measurement system with data acquisition system for both transport critical current and ac losses measurements......80

Figure 5.1. Geometry used in the FEM model, consisting of the three layers of elements shown in the inset.................................................86

Figure 5.2. Electric potential along the wire behind the current contacts ($x < 0 \text{ m}$ and $x > 0.02 \text{ m}$) and along the sample between the contacts ($0 \text{ m} < x < 0.02 \text{ m}$). The two copper blocks at which the electrical contacts to the sample are made at $-0.01 \text{ m} < x < 0 \text{ m}$ and $0.02 \text{ m} < x < 0.03 \text{ m}$. Inset shows the potential distribution along the sheath material from the contact to the voltage level of negligible resistivity.................................................................87

Figure 5.3. a) Simulated temperature evolution in MgB$_2$ filament with respect to time due to heating by the current contacts, for $J/J_c = 0.7$. The sample ends are at 0 and 0.02 m, respectively. Each curve represents the temperature at a time step of 0.001s and the last (top) curve gives the temperature at 0.01s after switching the DC current on. The input parameter in the simulation for critical current density is $10^8 \text{ A/m}^2$, giving the density of $7 \times 10^7 \text{ A/m}^2$. The base temperature is 20K. b) Increase of the temperature at the middle of the sample vs. the sample current normalized to its $I_c$, at 1ms and 1s after the DC current was switched on. The base temperature is 20K.........................89

Figure 5.4. a) Simulated temperature evolution in MgB$_2$ filament with respect to time due to heating by the current contacts, for $J/J_c = 0.7$. The physical descriptions of the figure are similar to figure 5.3, except the base temperature is 25K. b) Increase of the temperature at the middle of the sample vs. the sample current normalized to its $I_c$, at 1ms and 1s after the DC current was switched on. The base temperature is 25K.........................90

Figure 5.5. a) Simulated temperature evolution in MgB$_2$ filament with respect to time due to heating by the current contacts, for $J/J_c = 0.7$. The physical descriptions of the figure are similar to figure 5.3, except the base temperature is 30K. b) Increase of the temperature at the middle of the sample vs. the sample current normalized to its $I_c$, at 1ms and 1s after the DC current was switched on. The base temperature is 30K.........................90
sample vs. the sample current normalized to its $I_c$, at 1ms and 1s after the DC current was switched on. The base temperature is 30K………………..90

Figure 5.6. Transverse cross section view of (a) Columbus Superconductors tape and (b) Hyper Tech wire. The Nb chip, produced when cutting the wire, covers part of the round MgB$_2$ core…………………………………………………….91

Figure 5.7. Schematic showing principal of experimental setup for transport measurement with DC current source and modified pulse current source…93

Figure 5.8. A typical snapshot of voltage vs. current waveforms for MgB$_2$ composite wires, for three different ramp rates of the current. The temperature and field were 20K and 2T respectively…………………………………………………………94

Figure 5.9. Field dependence of $I_c$ at 30 K for MgB$_2$ tape conductor with field perpendicular to the width. Generally good agreement is found between the DC transport measurement and pulse current measurements up to about 120 kA/s………………………………………………………………………95

Figure 5.10. Field dependence of $I_c$ at 20K for MgB$_2$ tape conductor with field perpendicular to the width. The visibility of DC measurement is 100A and below to avoid excessive heating and further damage on the wire………..96

Figure 5.11. Field dependence of $I_c$ at 30K for MgB$_2$ wire conductor with field perpendicular to the wire axis. A low value of $I_c$ (high field region) is easily obtained from DC transport current while a high value of $I_c$ is more accurately obtained with a pulse of rate up to 16kA/s……………………..97

Figure 5.12. Field dependence of $I_c$ at 20K for MgB$_2$ wire conductor with the field perpendicular to the wire axis. The drop-off of DC transport $I_c$ at low field region indicates the significance of the heat load from the contact leads….97

Figure 5.13. Current ramp rate dependence of the measured $I_c$ for the pulsed current measurement method for the MgB$_2$ tape conductor, with the applied field perpendicular to the tape face………………………………………………..99

Figure 6.1. The sample is placed symmetrically inside the AC superconducting coil with G10 phenolic sample holder. The coil is separated from the sample space by G10 insulating bobbin…………………………………………………………103
Figure 6.2. Schematic sample holder, heater and sensors circuit. The heater is wound axial symmetrically around the sample and properly sealed to ensure the total heat produced is measure by the sensor. The inset shows the photograph of the set-up…………………………………………………..104

Figure 6.3. Voltage and magnetic field measurement at 77K for the AC solenoid superconducting coil. The voltage was measured at the end terminals by applying the DC current and magnetic field generated in the center was measured by a Hall effect probe……………………………………………..106

Figure 6.4. Resistance readings from the sensor associated with the ac losses in the sample tape exposed to an ac applied field perpendicular to the tape face at 40K with and without dc background field. The resistances are plotted against the rms applied magnetic field at 50Hz frequency………………..108

Figure 6.5. Calibration curve relating the coil dissipation to measured sensor reading in resistance. The line is a linear fit to the data……………………………………109

Figure 6.6. AC losses as a function of the external transverse field rms at 40 K and frequency 50 Hz with different DC background magnetic field. The result at 77 K from Sumitomo without background field is also shown for comparison. Theoretical curve for respective $I_c$ at 40 K with applied perpendicular field of 0 T, 0.5 T and 1.0 T are plotted. The solid lines connect the points obtained by theoretical expression. ..................110

Figure 6.7. AC losses as a function of the external transverse field rms at 30 K and frequency 50 Hz with different DC background magnetic field. Theoretical curve and results from VIM method for respective $I_c$ at 30 K with applied DC transverse field of 0.5 T and 1.0 T are plotted. The solid lines connect the points obtained by theoretical and numerical calculations………………..111

Figure 6.8. AC losses as a function of the external transverse field rms at 50 K and frequency 50 Hz with different DC background magnetic field. Theoretical curve for $I_c$ at 30 K with applied DC transverse field of 0 T, 0.5 T and 1.0 T are plotted. The results from VIM method are shown only for $I_c = 343$ A. The solid lines connect the points obtained by theoretical and numerical calculations…………………………………………………..112
Figure 7.1. Schematic arrangement of the tape sample subjected to external transverse field and alternating transport current. The voltage taps are attached to the bending edge and the straight portion for monitoring the voltages during the transport current. Temperature sensors are mounted on the insulation surface and the tape surface.

Figure 7.2. Variation of transport critical current along the superconducting tape configured as in figure 7.1. The electric field is the potential reading of unit length measured by the respective tap position on the tape surface. The measurement temperature is maintained at 30 K with DC magnetic field of 0.5 T applied longitudinally to the axis of the chamber.

Figure 7.3. Variation of transport critical current along the superconducting tape configured as in figure 7.1. The electric field is the potential reading of unit length measured by the respective tap position on the tape surface. The measurement temperature is maintained at 20 K with DC magnetic field of 2.0 T applied longitudinally to the axis of the chamber.

Figure 7.4. Variation of transport critical current along the superconducting tape configured as in figure 7.1. The electric field is the potential reading of unit length measured by the respective tap position on the tape surface. The measurement temperature is maintained at 20 K with DC magnetic field of 1.6 T applied longitudinally to the axis of the chamber.

Figure 7.5. Plots of temperature profile of the superconducting tape against the DC transport current. The temperature is measured by taking the difference in the sensors reading. The sensors are thermally attached to the tape surface and the insulation surface. Measurements are taken which correspond to the condition as shown in the voltage-current curve in figure 7.2 to figure 7.4.

Figure 7.6. Resistance readings obtained by subtracting the value of the sensors placed on the insulation surface from the one on the tape surface. These readings are associated with the ac losses in the tape subjected to an AC transport current with external DC transverse field at 30K. The resistances are plotted against the peak transport current at 50Hz frequency.
Figure 7.7. Calibration curve showing sensor response after passing DC current to the tape as a function of DC power dissipation. The line is a linear fit to the data with the calculated slope as 1.334 (W/m)/ohm. The slope is the calibration constant that will be used to calculate losses in figure 7.6.

Figure 7.8. AC transport losses as a function of the peak transport current for MgB$_2$ tape subjected to external DC transverse magnetic field. The frequency of the applied current is 50 Hz and measurements performed at temperature 30 K. Theoretical curves are taken for $I_c$ at 30 K with external field of 0.5 T and 0.7 T, a value extracted from figure 5.9.

Figure 7.9. AC losses as a function of the peak transport current for MgB$_2$ tape subjected to external DC transverse magnetic field of a) 0.5 T and b) 0.7 T. The frequency of the applied current is 50 Hz and measurements performed at temperature 30 K. The results from VIM computational are shown respectively for each case. The solid lines connect the points obtained by theoretical and numerical calculations.

Figure 7.10. AC transport losses as a function of the peak transport current for MgB$_2$ tape subjected to external DC transverse magnetic field. The frequency of the applied current is 50 Hz and measurements performed at temperature 20 K. Theoretical curves are taken for $I_c$ at 20 K with external field of 1.6 T and 1.8 T, a value extracted from figure 5.9.

Figure 7.11. AC losses as a function of the peak transport current for MgB$_2$ tape subjected to external DC transverse magnetic field of a) 1.6 T and b) 1.8 T. The frequency of the applied current is 50 Hz and measurements performed at temperature 20 K. The results from VIM computational are shown respectively for each case. The solid lines connect the points obtained by theoretical and numerical calculations.
LIST OF TABLES

Table 1.1. Superconducting wires presently available on the market. MTS- Middle Temperature Superconductor. Data extracted from reference 12……………4
Table 5.1. Physical parameters used to simulate the superconductor in FEM model…88
**LIST OF PRINCIPAL SYMBOLS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi$</td>
<td>Magnetic flux (Tm²)</td>
</tr>
<tr>
<td>$\Phi_0$</td>
<td>Superconducting flux quantum (Wb)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Resistivity (Ωm)</td>
</tr>
<tr>
<td>$\rho_n$</td>
<td>Normal state resistivity for superconductor (Ωm)</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>Resistivity of superconductor (Ωm)</td>
</tr>
<tr>
<td>$e$</td>
<td>Charge of an electron (C)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Scalar potential (V)</td>
</tr>
<tr>
<td>$h$</td>
<td>Planck’s constant (Js)</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass of an electron (kg)</td>
</tr>
<tr>
<td>$n$</td>
<td>Index of transition or a measure of sharpness of transition ( )</td>
</tr>
<tr>
<td>$n_s$</td>
<td>Number of super-electrons per unit volume (m⁻³)</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>Permeability of a vacuum (H/m)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Penetration depth (m)</td>
</tr>
<tr>
<td>$\lambda_L$</td>
<td>London penetration depth (m)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Electrical conductivity (1/Ωm)</td>
</tr>
<tr>
<td>$\sigma_n$</td>
<td>Conductivity of normal electron (1/Ωm)</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>Material electrical conductivity (1/Ωm)</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>Electrical conductivity of superconductor (1/Ωm)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular frequency (rad/s)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Skin depth (m)</td>
</tr>
<tr>
<td>$d$</td>
<td>Thickness of a superconductor with geometry of slab (m)</td>
</tr>
<tr>
<td>$w$</td>
<td>Width of a superconductor with geometry of slab (m)</td>
</tr>
<tr>
<td>$\mathcal{L}$</td>
<td>Linear operator ( )</td>
</tr>
<tr>
<td>$\mathcal{N}$</td>
<td>Nonlinear operator ( )</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>Change in temperature (K)</td>
</tr>
<tr>
<td>$A$</td>
<td>Vector potential (Vs/m)</td>
</tr>
<tr>
<td>$A_z$</td>
<td>$Z$ component of the vector potential (Vs/m)</td>
</tr>
</tbody>
</table>
\( \vec{B} \)  Magnetic field strength (T)
\( \vec{B}_a \)  Applied magnetic field (T)
\( \vec{B}_p \)  Full penetration of applied transverse field (T)
\( C_p \)  Specific heat capacity (J/kg/K)
\( \vec{E} \)  Electric field (V/m)
\( E_c \)  Electric field at the critical current density of a superconductor (V/m)
\( \vec{F}_p \)  Pinning force (N)
\( \vec{F}_L \)  Lorenz force (N)
\( F \)  Ratio of applied current to the critical current of a superconductor ( )
\( \vec{H} \)  Magnetic intensity (A/m)
\( H_a \)  Applied magnetic field (A/m)
\( H_{dc} \)  Applied Direct Current magnetic field (A/m)
\( H_y \)  Magnetic field component perpendicular to the superconducting strip (A/m)
\( \vec{H}_0 \)  Applied magnetic field parallel to the plane \( x = 0 \)
\( H_c \)  Critical magnetic field of a superconductor (A/m)
\( H_c(T) \)  Temperature dependent critical field intensity of a superconductor (A/m)
\( H_c(0) \)  Critical field intensity of a superconductor at temperature equal to 0 K (A/m)
\( I \)  Transport current (A)
\( I_m \)  Peak current flow
\( I_c \)  Critical current of a superconductor (A)
\( \vec{J} \)  Current density (A/m²)
\( J_z \)  Z-component of the current density (A/m²)
\( \vec{J}_s \)  Super-Current density (A/m²)
\( \vec{J}_n \)  Normal current density (A/m²)
\( \vec{J}_c \)  Critical current density in a superconductor (A/m²)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_c(B)$</td>
<td>Magnetic field dependent critical current density in a superconductor (A/m$^2$)</td>
</tr>
<tr>
<td>$k$</td>
<td>Thermal conductivity (W/m/K)</td>
</tr>
<tr>
<td>$\bar{M}$</td>
<td>Magnetization (A/m)</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature (K)</td>
</tr>
<tr>
<td>$T_c$</td>
<td>Critical temperature of a superconductor (K)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Flux velocity (m/s)</td>
</tr>
<tr>
<td>V</td>
<td>Voltage (V)</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

The term superconductivity is used in reference to materials that have conductivity beyond the normal. It was introduced by Kamerlingh Onnes, three years after he liquefied helium in his laboratory in 1908. He discovered the resistance of mercury, Hg, dropped to zero at temperatures below 4.19K. The resistance not only decreases with temperature but has a sudden drop at some critical temperature $T_c$. He called this a superconductivity state in contrast to a normal state, and materials that exhibit such behaviour are consequently called superconductors. Subsequently other characteristics were established. For example, that superconductivity can be destroyed not only by heating of the sample, but also by applied magnetic field and current.

It has been the dream of physicists and engineers that the “perfect conductivity” nature of superconductivity could be utilized for practical applications. However, this dream was delayed till the 1960s with the discovery and maturation of type II superconductors, capable of sustaining electric current in high magnetic fields. These developments paved the way for the commercialization of magnetic resonance medical imaging (MRI), the most ubiquitous application of superconductivity visible to the public at large today. The advent of superconductivity has expanded the possible range of application to electric industry power devices, and a rapidly growing program of prototype development and demonstration projects are currently underway worldwide [Grant97, Grant07].

Some perhaps optimistic estimates are that in the year 2020 up to 80% of new power transformers, 75% of motors, 40% of generators and 35% of transmission cables will be using high temperature superconductors instead of copper and aluminium [Sheahen02]. Hence the expectations are rather high in the power-engineering sector. Most of the existing power grid uses a sinusoidal voltage operating at 50 or 60 Hz. Some other applications demand a frequency of about 20 Hz (transformers for trains) and of 400 Hz (aero-space). Consequently, the electromagnetic behavior of these superconductors carrying alternating
transport current, exposed to alternating magnetic and electric fields and operated at a certain frequency and temperature must be well understood and described.

Figure 1.1 illustrates a number of applications with the current density requirements that have to be sustained in the presence of their various operation ranges for magnetic field and temperature [Blaugher00]. Typically, values of critical current density $J_c$ above 10kA/cm$^2$ are required for most applications.

![Figure 1.1: Critical state parameter requirements for several electric power applications. Data extracted from [Blaugher00].](image)

It is important to stress that each of the applications addressed in figure 1.1 individually comprises significant engineering undertakings and a wide variety of specification. Performance tradeoffs can be entertained, and thus individual entries have wide associated margins. Furthermore, two of the applications considered, cables and transformers, can involve significant line frequency ac losses, as opposed to other applications. Nevertheless,
even these are susceptible to some induced ac loss due to pickup and, in the case of Superconducting Magnetic Energy Systems (SMES), charging and discharging. The only power application relatively free from transient loss would be supply/load balanced dc transmission line.

1.1 Superconducting Materials

Superconductors are generally divided into two main groups according to the value of their critical temperature $T_c$: low temperature superconductors (LTS) and high temperature superconductors (HTS). Obviously, LTS were discovered earlier than HTS and they compose simpler phases then it is in case of HTS. The maximal critical temperature of the LTS group approaches approximately 25K and typically the materials representing this group are Nb$_3$Sn [Matthias54] or NbTi [Hulm61]. On the other hand, HTS were found much later after 1986 [Bednorz86]. Their structure is much more complicated than LTS material and this is reflected in their manufacturing difficulty and production price. The lowest critical temperature of HTS materials ranges from approximately 50K [http://org]. For instance, most promising HTS materials for practical applications are YBa$_2$Cu$_3$O$_7$ and Bi$_2$Sr$_2$Ca$_n$Cu$_{n+1}$O$_{2n+6}$ ($n=1$ or 2) which have critical temperatures 92, 87 and 110K respectively. Figure 1.2 shows the critical temperature evolution of selected superconducting materials discovered over the years [Holubek08]. In year 2001, a group at Aoyama-Gakuin University in Japan [Nagamatsu01] noticed the trace of superconductivity at 39K in an impurity phase of MgB$_2$ while investigating the properties of a titanium-magnesium-boron ternary compound in search of magnetic behavior. This immediately triggered the superconductivity community around the world and perhaps most astounding of all were the almost immediate prospects for realizing practical lengths of MgB$_2$ wire. In fact there was even an indication of possible superconductivity in MgB$_2$ that went unrecognized in 1957 in published low temperature specific heat measurements [Swift57]. Simple structure, high $T_c$ around 39K (within standard commercial cryocooler), relatively cheap and a common abundance of magnesium and boron gave this material many advantages over other superconductors. Table 1.1 presents the superconducting wires currently available on the market along with several key characteristics [Grasso11].
Figure 1.2: The discovery of superconductivity materials over the years with their respective critical temperature. Black – elemental superconductors, Blue – LTS, Red – HTS, Green – MgB$_2$. Data from reference [Holubek08].

Table 1.1: Superconducting wires presently available in the market. MTS- Middle Temperature Superconductor. Data extracted from reference [Grasso11].

<table>
<thead>
<tr>
<th>Wire type</th>
<th>NbTi</th>
<th>Nb$_3$Sn</th>
<th>MgB$_2$</th>
<th>Bscco</th>
<th>YBCO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_c$ (K)</td>
<td>9 K</td>
<td>18 K</td>
<td>39 K</td>
<td>108 K</td>
<td>90 K</td>
</tr>
<tr>
<td>$B_{c2}$ (T)</td>
<td>10 T</td>
<td>28 T</td>
<td>&lt;70 T</td>
<td>&gt;100 T</td>
<td>&gt;100 T</td>
</tr>
<tr>
<td>Operation in LN$_2$</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>&lt; 1T</td>
<td>&lt; 2T</td>
</tr>
<tr>
<td>Ductile compound</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Flexible wires</td>
<td>YES</td>
<td>NO</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Superconducting splices</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>NO</td>
<td>NO</td>
</tr>
<tr>
<td>Low cost</td>
<td>YES</td>
<td>≈YES</td>
<td>YES</td>
<td>NO</td>
<td>Not yet</td>
</tr>
</tbody>
</table>

LTS  MTS  HTS
1.2 Alternating Transport Current and Magnetic Field: - Loss Mechanisms

Limitations on the use of superconductors are posed by factors such as their critical temperature, ability to carry transport current, mechanical strength, stability against a quench, and alternating current (ac) loss. The latter, which has been a major limitation, will be briefly explained in this section and more details are provided in Chapter 2. Since the 1960s, the issue of the behavior of a superconductor in a magnetic field has become the subject of systematic study. It was found in particular that losses in such superconductors may become quite large, especially at finite frequencies, unless special precautions are taken. Because of this loss problem most of the applications for superconductors in the past have been in a mainly dc magnetic environment. But even in these cases magnetic field changes and/or ac ripples are present which require loss analysis. Superconductors in general are subject to losses where electric fields can be produced inside a superconductor. In many applications the electric field results from applying a transverse alternating magnetic field, and the loss it produces is called the transverse applied field loss, with the energy coming from the source of the applied field. However, one can also produce a loss by connecting normal leads to a superconducting wire which are then connected to a power supply. When an alternating current is established in an isolated superconducting wire in this fashion (with peak current below a critical value) the loss is called an alternating current loss. The energy for the alternating current loss comes from the power supply attached to the leads, and the loss occurs in the same way as the applied field loss, with the self field of the wire substituting for the applied field. For the case of a superconducting wire connected to the leads of a dc power supply a dc loss occurs in the wire. This is small if the current is less than a critical value, but can become quite large if the dc current considerably exceeds the critical value. Obviously, for an ac power supply which produces a current with peak value well above the critical value a similar large loss would be observed in parts of the ac cycle, in addition to the loss due to the changing self field. To determine which is dominant in this case one must compare the electric field produced by the changing magnetic field with that produced by the potential difference between the leads.
Figure 1.3a displays a sample of superconductor placed in an external magnetic field. The magnetic field penetrates the material in the form of flux lines. If the magnetic field is changed, the flux line pattern and internal magnetic field change as well. The magnetic field variation inside the material induces an electric field $\vec{E}$ according to Faraday’s law. Subsequently the field drives a current (screening currents) in the material as indicated by arrows in the figure. The grey volume is screened by currents in the white volume, flowing at a density of practically the critical current density. The screening currents determine the magnetic field distribution in the superconductor according to Ampere’s Law and dissipate energy at a local power density given by $E \cdot J$. The energy is delivered by the external magnetic field and is supplied by the power source of the magnet that generates the field. This energy is required for depinning and moving the flux lines, which is a dissipative process, and so is converted into heat that must be removed by the cooling system. AC loss is therefore an undesirable phenomenon.

Figure 1.3: (a) Screening currents in the white region shield the interior (grey) from the magnetic field. (b) Distribution of magnetic field $\vec{B}$ inside and around the sample.

In figure 1.3b, the distribution of the magnetic field $\vec{B}$ inside and around the sample is illustrated for a time shortly after the beginning of the magnetic-field sweep. Assuming the
generated magnetic intensity $\overline{H}$ is homogenous, then the lines of $\overline{H}$ are straight verticals and the local magnetic field is given by $\mu_0(\overline{H} + \overline{M})$ where $\overline{M}$ is the magnetization caused by the screening currents. The interior region of the sample is completely shielded from the magnetic field which means the rate of change $\frac{dB}{dt}$ is zero in the grey region in figure 1.2. Magnetic field, screening currents and energy dissipation are present only in the outer (white) region. If the magnetic field increases further, the outer region penetrates deeper into the sample and eventually reaches its centerline. The state where the screening currents fill the whole sample volume is called the fully penetrated state and is described as in section 2.8. The AC loss due to the screening currents in a superconductor is hysteretic in nature and thus can be calculated as in the magnetization loss method by integrating the product $\overline{B}d\overline{m}$ over a single magnetic field cycle [Carr83, Wilson83, Seeber98].

The way of looking at the loss for the case of a superconductor carrying transport current is the same as above. The transport current generates a magnetic field around the conductor, which is called the self-field. With an alternating transport current, the alternating self-field penetrates the superconductor during each current cycle. Even if there is no external magnetic field, the variation of the self-field inside the material causes a hysteresis loss, which is called self-field loss [Norris69].

1.3 Theoretical Progress – Past and Present

In the past several years many research groups have investigated and focused on experimental and theoretical analyses of the AC loss. One of the main complications in experimental work arises from the low losses of superconductor that may easily be overwhelmed by other external loss factors, such as lead and wiring connections that make accurate measurement difficult. On the other hand, theoretical work is difficult in the sense that superconductors have a sudden change of resistivity near the transition point. This change is often modeled in various ways by researchers. The most commonly used approximations are the critical state model from Bean C.P. [Bean62] and the nonlinear E-J
relation that is also well known as the ‘n-power law electric field-current density relations’ [Amemiya97, Rhyner98].

The theoretical analyses of AC loss in superconductors have been made through several stages of development. The first computation of the loss in a superconductor carrying alternating transport current was made by London [London63] who extended a calculation made for a slab in a parallel applied magnetic field to that for a circular wire carrying a transport current. London made an assumption that the loss in a cyclic applied transport current follows from the loss in a cyclic applied magnetic field with the self field replacing the applied field. Later in 1969, an analytical method was proposed by Norris [Norris69] to estimate the AC loss in a self field problem for a uniform superconducting strip carrying transport current. In his approach the strip was imagined to be infinite in length and broken up into ‘fibres’ for which the loss was computed using circuit theory, the Bean model, and a starting expression given in terms of the flux between the fibres. Shortly afterwards in 1974 Carr [Carr74] solved Maxwell equations for a long wire of radius $R_0$ and twist length $L$ to calculate the ac losses in a twisted filamentary superconductor. The approach is based on the assumption of a continuum model with anisotropic conductivity where a voltage-current relation is obtained from the current distribution of a filament. Along with the rapid development of practical high temperature superconductors, Wilson and Carr’s [Wilson83, Carr83] contribution to ac loss used a theoretical analysis of the problem mostly based on the critical state model (CSM) proposed by Bean that assumes the superconductor screens changes in magnetic fields by setting up screening currents always flowing at their critical value.

In early 1990s, Rhyner [Rhyner93] proposed a theory for ac losses in superconductors with smooth current-voltage characteristic, which is not based on the concept of a critical current. This theory contains the normal ohmic conductor and the perfect (critical state) superconductor as limiting cases. In the same year, Brandt and Indenbom [Brandt93] delivered an analytical expression for the loss in thin superconductor in a perpendicular field or with transport current. Their paper shows that the current and field profiles in such geometry differ from the Bean model for longitudinal geometry in many ways. In the late 1990s, Yazawa [Yazawa98] developed a numerical model to calculate the ac loss in high
temperature superconductor with finite thickness based on the equation of motion for the current density developed by Brandt [Brandt96]. The loss obtained from this model has found to be in good agreement with the experimental data and as well the effects of some physical parameters on ac loss are pointed out. A different computational model for ac loss has also been formulated by Paasi and Lahtinen [Paasi97] based on the magnetic diffusion model. It takes into account the real current density-electric field characteristic of the superconductor and the spatial dependence of the current density. More recently in 2002, Rhyner [Rhyner02] has published a simplified loss calculation based on vector potential formulation together with the Bean model but his calculation is limited to the special case of peak current equal to the critical current.

In nearly every case tacit approximations are used and it is often difficult to know the approximations used. Even the term Bean model means different things to different authors. A new approach from W.J. Carr in 2004 [Carr04] on computing the loss in a coated superconductor carrying ac transport current in zero applied magnetic field has revealed the surface charge theory not mentioned in the London and Norris work. It is based on a rigorous solution of Maxwell’s equations which correctly treats the problem as three dimensional, having a time-dependent charge on the surface of the superconductor, and having the electric field described by both a vector and scalar potential. Carr has also pointed out again the importance of the existence of surface charge in computing the loss in a coated conductor, but now subject to a combined applied magnetic field and transport current [Carr06]. Apart from the single conductor, John R Clem [Clem07] has extended the AC loss studies from Muller [Muller99] to a finite stack of thin superconducting tapes with each carrying a fixed transport current. Before, Muller has considered this problem in the limit of infinite stack and obtained analytic expression for the fields and currents. Since it is now possible to wind pancake coils from long lengths of high temperature superconducting tape, the Clem approach has inevitably contributed to coil geometry. Following the recent development of fabricating a non-inductive coil with bifilar windings using 2G superconducting tapes for use in superconducting fault current limiters [Noe07], Clem further contributes by presenting analytic expressions for a bifilar stack of superconducting
strips from which the losses can be estimated, but this time with an infinite dimension of the stack [Clem08].

Finite element methods have also been intensively applied to develop the numerical solutions of AC loss problems in superconductors. Some 2D and 3D FEM models, based on the critical-state model, have been proposed to find the AC loss in superconducting cables [Amemiya98]. A slightly different approached from Hong Z. et al. [Hong06] has been proposed. They utilized the E-J constitutive law together with an H-formulation to calculate the current distribution and electromagnetic field in superconductors and hence the AC loss. This numerical method is based on solving the time-dependent partial differential equations which has the advantage of modeling the superconductivity in a more simple, flexible and extendable way. The method was then extended by the same group to estimate the loss in MgB$_2$ wires in self field condition [Hong08]. The results demonstrate that the multifilamentary MgB$_2$ wire has a lower AC loss than a monocore one when carrying the same amount of current. Numerical solutions of the critical state in superconductors have played an important part in the understanding of their properties, and in particular, computing of AC loss. Recently, a new numerical method for solving the critical state was suggested by A.M Campbell [Campbell07]. This method was based on the force-displacement curve of the flux lines in which the corresponding equation can be expressed in terms of the vector potential and can be solved by relatively inexpensive commercial software on a desktop computer. The vector potential expression avoids some of the numerical problems that occur if one uses the E-J curve of the form $E \propto J^n$ with large value of $n$. Wolsky has extended this model to a more complicated three-dimensional current densities and fields for all shapes [Wolsky08]. Most recently, Clem [Clem09] presented comprehensive studies for ac loss in superconducting strips by exploring the questions on how the transport alternating current losses of a superconducting strip depend upon the ratio $I/I_c$, and how this functional dependence can be affected by the superconductor’s cross sectional shape, aspect ratio and $J_c(B)$.

Given the number of techniques in calculating the ac loss in different geometries and conditions, there are still shortcomings in the generality of each method. In most cases, the Norris’s equation or Brandt’s equation is employed to estimate theoretical values even
though these equations are particularly derived for thin strip geometry. Numerical methods are practical if the computation time and cost are reasonable while considering accuracy and precision. In short, calculation of ac loss is most favorable if analytical expressions exist. Numerical methods are needed when analytical solutions are not available in calculating ac losses.

1.4 Alternating Current Loss Measurements

There are generally three methods of measuring AC losses in superconductors: electric, magnetic and calorimetric. The properties, advantages and drawbacks of each method are briefly described in this section.

The loss in superconductors due to ac transport currents is commonly measured by electrical methods with voltage taps on the conductor. It is necessary that the measured oscillating voltage along the superconductor must be in-phase with the ac transport current in order to calculate the energy losses. Since the energy dissipated during each cycle is supplied from the transport current source, it is also frequently called the transport current loss. Although voltage measurements are sensitive, the presence of a very large reactive component causes complications than can become increasingly severe as the current is reduced. Furthermore, it has been pointed out [Campbell95, Ciscek94] that this voltage measurement is very dependent on the position of the contacts, and surprisingly the measurement circuit must encompass a substantial area outside the superconductor in order to obtain a true measurement of the loss.

The alternating current loss generated in a superconductor due to screening currents is measured with magnetic methods via the change of magnetization of the sample [Rabber02, Yamagishi06]. It commonly consists of a system of pickup coils around the sample (to measure the changes in magnetic moment) and is particularly suitable in situations where the loss is generated by external field only. The loss component represents the energy delivered by the magnetic field that induces the screening currents in the conductor. One major drawback of this technique is the requirement of compensation for the pickup coil that requires sophisticated instrumentation due to low voltages.
The total energy dissipated in the superconductor regardless of magnetization and/or transport current losses can be measured with the calorimetric method by means of measuring the temperature increase of the superconductor or the amount of cryogen boil-off [Okamoto07, Dolez96, Magnusson98]. The sensitivity and speed of this method are less than other approaches due to the complications from the bath superheating and thermal effects from eddy current loss or contact ohmic loss. In addition, it does not distinguish between the magnetization loss and the transport current loss. Hence less information is obtained by comparing the results to the predictions of theoretical AC-loss models. Nevertheless, the calorimetric method is not impaired by the disturbances due to alternating currents or magnetic fields, which are inherent in the electric and magnetic methods.

1.5 Contributions of This Thesis

In this thesis, measurement and theoretical work on AC losses in practical superconductor tapes and wires (MgB$_2$ and BSCCO) have been undertaken. The aim of this study is to further develop the skills, tools and understanding required to assist in practical use of these superconductors in electrical engineering applications.

A comprehensive magnetic system comprising cryogenic parts, transport probe, instrumentation and comprehensive data acquisition systems has been designed and developed to perform experimental work on the physical characterization and ac loss measurements of superconductors. Measurements to validate the research contributions are made at various operating temperatures from 20 K to 77 K and applied magnetic fields up to 15 T for direct current (DC) measurement and 100 mT (r.m.s.) for alternating current (AC) measurement.

A calorimetric method for AC loss measurement of short superconductor tapes and wires has been investigated and demonstrated with the superimposed DC and AC magnetic fields likely to be experienced in practical devices such as Fault Current Limiters. In addition, transport current loss in MgB$_2$ tapes have also been obtained with similar techniques at various operating temperatures. The calorimetric method presented here is different from
previous work conducted by other groups such as Dolez P. et al and Ashworth S.P. et al, where liquid nitrogen of temperature 77K was used. Instead, the calorimetric system developed here provides a wide range of temperature with reasonable stability and accuracy in addition to the cryogenic magnet system that was capable of providing both AC and DC magnetic fields simultaneously or independently. Furthermore, very few studies over the range 25 to 45K have been conducted even though this is one of the favored temperature ranges for cost-effective applications of superconductor wires.

A hybrid transport critical current ($I_c$) measurement system that consists of both DC and pulsed methods have also been developed along with the AC loss measurement system. The transport critical current value of any practical superconductor is one of the most basic and important parameters to be considered before any further design work. It is equally applied to AC loss studies since any theoretical ground-work for AC loss relies on current distribution analysis that is based mostly on the critical state model where the current takes only zero or critical current value. Therefore, a reliable and practical system has to be developed to carry out this measurement task accurately and repeatedly. Since the measurement is usually made over a relatively short section of a wire or tape due to space constraints and stability requirements, the conventional four-probe DC method suffers from inevitable heating effects from the contact when high currents are used. Hence the pulsed method currently used has the advantage of negligible heating effect owing to the fast ramping rate of applied current. However, the measured critical current can depend on the rate of the current pulse applied and also the background inductive voltage may introduce some complications. More details on this subject will be presented in Chapter 5 where these problems are solved.

Apart from this experimental work, comprehensive and rigorous new theoretical work on AC loss distribution in superconductors will be presented and derived with analytical and numerical approaches. The analytical section consists of systematic derivation on hysteresis losses for a superconductor tape sample carrying alternating current and in perpendicular applied magnetic field respectively. The approach is based on solving the nonlinear integral equation that describes the current density and magnetic field distribution which has been proposed by Brandt. The method developed avoids any vague assumptions used in
numerical computation to circumvent the logarithmic singularity or divergence of the diagonal terms in the kernel. On the other hand, the numerical section consists of solving the ac losses as a nonlinear diffusion process. The technique is based on solving the two dimensional vector potential formulations with variational iteration method (VIM). This method is a powerful scheme for handling linear and nonlinear diffusion equations while taking into consideration time dependent factors. Moreover, this is the first application of VIM in solving ac losses for practical superconductor wires. The results were validated by comparing with the measurement results.

1.6 Structure of the Thesis

This thesis consists of eight chapters with the arrangement as follows. Chapter 2 presents the evolution of superconducting theories for field, current and losses distributions. It gives an overview of superconducting theories macroscopically and gathers related information and formulations from various group of researchers and engineers around the world from the past till present. In Chapter 3, the theoretical approaches for field, current and losses distribution will be discussed systematically for two independent cases: transport current without applied magnetic field, and transverse direction of applied magnetic field. Analytical derivation and numerical formulation will be comprehensively studied and developed.

Chapter 4 discusses the development of experimental approaches and the integration with instrumentation. The advantages and limitations of the design configuration will be discussed in detail and the requirements for different measurement conditions will be explained and demonstrated. Chapter 5 discusses the transport critical current measurement for both wires and tapes with pulse current of varying rate and direct current method. Comparison between these two techniques is investigated and the most positive features from the two will be exploited as the transport measurement system. Finite element analysis is also performed to obtain the time dependent heat distribution in the superconductor due to electric potential produced at the current contacts and its gradient.
Chapter 6 presents the AC loss results on superconductor tape under the applied alternating and direct magnetic field. The losses are investigated at various operating temperatures with and without applied direct magnetic field. Theoretical comparison and validity is discussed in the last section of this chapter. Chapter 7 has the same structure as in the previous chapter except that the losses now are for wires carrying alternating current. A summary of conclusions and recommendations for further research are given in Chapter 8.
CHAPTER 2

EVOLUTION OF SUPERCONDUCTING THEORIES – FIELD, CURRENT AND LOSSES DISTRIBUTIONS

Two important phenomena characterize superconductivity in solid conductors: close to zero electrical resistance and the Meissner effect. The former means zero voltage across a superconductor when a certain transport current passes through it, while the latter means induced surface currents in the superconductor shield the interior to some degree from the external magnetic field. The superconductivity of a certain material exists in a range of parameters such as temperature, electric and magnetic field, density of electric current and straining everything together form a multidimensional volume of the parameters. The outer surface of the volume, usually taken at a certain value of the electric field, is called the critical surface. It separates superconducting and non-superconducting states and links the critical parameters.

2.1 Three Critical Quantities (Tc, Hc and Ic)

It has been found that perfect conductivity (zero resistance) is not the only characteristic of a superconductor. Meissner and Ochsenfeld noted in 1993 that the magnetic flux is expelled from the interior of a superconductor, which cannot be fully explained by perfect conductivity [Muller97]. They verified that perfect diamagnetism (expulsion of magnetic flux) is a fundamental property of superconductors. The physical picture is that screening super-currents flow in a thin surface layer of a sample, exactly cancels the external field. As a result, the magnetic field inside a superconductor is zero. It has later been confirmed through theories and experiments that there is a third basic feature of superconductors, that is, magnetic flux passing through a superconducting coil can only take certain values of quantified flux, \( \Phi = n \Phi_0 \) [Kresin90]. All these features are, however, destroyed when the temperature surpasses the critical temperature \( T_c \) or when the magnetic field exceeds a temperature-dependent critical magnetic field \( H_c(T) \).
Another important characteristic of a superconductor is the maximum possible transport current which can flow without dissipation, the \textit{critical current} $I_c$. The value of the critical current depends on the sample geometry and sample quality. According to Silsbee’s criterion, a superconductor loses its zero resistance when it is at any point on the surface. Its total magnetic field strength, due to transport current and applied magnetic field, exceeds the critical field strength $H_c$. This quantity $I_c$ is called the thermodynamic critical current or the depairing current and depends on the external magnetic field and is typically of the order of $10^7$-$10^8 \text{ Acm}^{-2}$.

The $I_c$ in most practical superconductors is much smaller than the thermodynamic critical current due to the penetration of magnetic flux into a superconductor at magnetic field lower than $H_c$. In this respect, according to Abrikosov (1952), superconductors are classified into two kinds: type I and type II superconductors. Siblee’s criterion of depairing current holds only for type I superconductors, whereas for type II superconductors, the complete flux expulsion at $H < H_c$ does not take place. This can be shown in a typical $H$-$T$ phase diagram for a type II superconductor in figure 2.1. The complete flux expulsion (Meissner phase) occurs only for weak fields $H < H_{c1}$ and for $H > H_{c1}$, magnetic flux penetrates the superconductor but penetration is incomplete.

![Figure 2.1: The phase diagram of a typical type II superconductor.](image)
Complete penetration of a flux takes place at a much higher field $H_{c2} > H_c$ which is called the upper critical field. The curve $H_{c2}(T)$ on the phase diagram is the line of the second order phase transition between superconducting and normal states. This second order transition is in contrast to the first order phase transition of a type I superconductor placed in a magnetic field. In the field range $H_{c1} < H < H_{c2}$ a superconductor is in a mixed state [Tinkham96].

2.2 Perfect Conductivity and Perfect Diamagnetism (Meissner Effect)

Perfect conductivity (zero resistance) implies that a change in magnetic flux enclosed in the material is not possible, due to one of Maxwell’s equation [Iskandar92] as below:

\[
\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0
\]

(2.1)

\[
\frac{\partial \vec{B}}{\partial t} = 0
\]

(2.2)

are for zero resistivity ($\rho E = \rho J$), where $\vec{E}$ = electric field, $\vec{B}$ = magnetic field density and $\vec{J}$ = current density.

So, when taking a material with zero resistance below a critical temperature $T_c$ or lowering the temperature below this $T_c$ and thereafter applying a magnetic field, screening current will be induced in order to have an unchanged magnetic field in the interior of the material as shown in figure 2.2(a). If, however, the magnetic field were applied when the material is in normal state (before the temperature was lowered below the critical temperature) then the magnetic field is trapped inside, and this is illustrated in figure 2.2(b). This is quite different from how a diamagnetic material performs [Ketterson97]. In the first case when the magnetic field was applied after cooling below the critical temperature, there is no difference since the magnetic field is zero for both the materials. In the second case, the magnetic field is expelled from the interior of the material as it is cooled below the critical temperature. The Meissner effect is the expulsion of any magnetic field from the interior of
a superconductor, whether it was there before the specimen became superconducting or not (figure 2.2c).

Figure 2.2: The difference between materials that become perfect conductors or perfect diamagnets below a critical temperature $T_c$

Perfect diamagnetism is an intrinsic property of a superconductor which, however, is only valid if the temperature and the magnetic field are below their critical values everywhere, where the latter depends on the temperature as follows:

$$H_c(T) = H_c(0) \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]$$ \hspace{1cm} (2.3)

where $H_c$ = critical field intensity and $T_c$ = critical temperature.

2.3 Type I and Type II Superconductors

In type I superconductors, the magnetic field $H<H_c$ is completely screened due to the Meissner effect and zero resistance is preserved in the field up to $H_c$. In many real situations geometrical effects may cause magnetic field exceeding $H_c$ in some parts of the sample. As a result, a field smaller than $H_c$ can, in principle, penetrate into type I
superconductor due to a large demagnetization factor. For instance, in a thin superconducting film in a perpendicular magnetic field, a transition from the superconducting to a normal state can occur in some volume fraction of the sample. As such, the sample is in the so-called intermediate state.

Type II superconductors are characterized by incomplete flux expulsion, even in a small magnetic field, which is a fundamental property of these materials, regardless of shape-dependent effects. Magnetic field penetrates type II superconductors in the form of superconducting vortices. Each vortex carries a magnetic flux equal to a superconducting flux quantum \( \Phi_0 \)

\[
\Phi_0 = \frac{hc}{2e} \approx 2.07 \times 10^{-15} \text{Wb}
\]  

where \( h \) is Planck’s constant \( 6.6262 \times 10^{-34} \text{Js} \), and \( e \) is the charge of an electron \( 1.602 \times 10^{-19} \text{C} \).

If magnetic vortices are present in a sample, they start to move under external current and as a result, electric field is generated. Therefore, a truly zero-resistance state does not generally occur in a sample due to the motion of the magnetic vortices. Most practical superconducting metals and alloys are type II superconductors. It is important to note that the zero-resistance (or extremely small resistance) state is still possible in these materials, provided the magnetic flux pattern interacts with the crystal lattice and therefore cannot move. This effect is called vortex pinning and is very crucial for practical applications.

### 2.4 Penetration Model (London Model)

The fundamental electrodynamics relations describing the magnetic field distribution inside a superconductor were well expressed by the brothers F. and H. London (1935) with two equations dealing with the electric and magnetic fields [Zhou91]. Within the London model, the set of Maxwell’s equations:
\[ \text{curl} \vec{H} = \vec{J}_s \]  
\[ \vec{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla \phi \]  
(2.5)  
(2.6)

should be completed by the equation of motion of superconducting electrons:

\[ \vec{E} = \frac{m}{e^2 n_s} \frac{\partial \vec{J}_s}{\partial t} = \mu_0 \lambda_{ls}^2 \frac{\partial \vec{J}_s}{\partial t} \]  
(2.7)

and by the following equation describing the relation between the magnetic field and current density in a superconductor:

\[ \lambda_{ls}^2 \text{curl} \vec{J}_s + \vec{H} = 0 \]  
(2.8)

Here, \( \text{curl} \vec{A} = \vec{H} \) and \( \phi \) are vector and scalar potentials respectively, \( n_s \) is the number of super-electrons per unit volume, \( \vec{J}_s \) is the super-current density, \( m \) and \( e \) their mass and electric charge respectively, and \( \mu_0 \) the permeability in a vacuum. The parameter \( \lambda_{ls} \) introduced above has the dimensions of length and is called the London penetration depth

\[ \lambda_{ls} = \sqrt{\frac{m}{\mu_0 n_s e^2}} \]  
(2.9)

In the most general case, the total current density \( \vec{J} \) is the sum of a normal current density \( \vec{J}_n \) and a super-current density \( \vec{J}_s \). This relation should be added to the above set of equations

\[ \vec{J} = \vec{J}_n + \vec{J}_s = \sigma_n \vec{E} + \vec{J}_s \]  
(2.10)

where \( \sigma_n \) is a conductivity associated with normal electrons. From equations (2.7)-(2.10), one can work out the distributions of field and currents in a superconducting specimen under various conditions. The static distribution of a magnetic field and current within a superconductor of arbitrary shape can be found, as well as the response of a superconductor to an external high-frequency electromagnetic field.
The application of Maxwell’s equations to the second London equation leads to the

equation describing the penetration of a magnetic field into a superconductor

\[ \lambda_L^2 \nabla^2 \vec{H} - \vec{H} = 0 \]  

(2.11)

This equation can be used in principle to find the distributions of flux density within any
superconducting body. One needs to apply boundary conditions to the solutions of this
equation which follow the shape of the body. For instance, if a superconductor occupies the
semi-space \( x > 0 \) and a magnetic field is applied parallel to the plane \( x = 0 \), by taking into
account the symmetry, the equation in one-dimensional form

\[ \frac{\partial^2 \vec{H}}{\partial x^2} = -\frac{\vec{H}}{\lambda_L^2} \]  

(2.12)

with the boundary conditions \( \vec{H}(0) = \vec{H}_0 \), \( \vec{H}(\infty) = 0 \), has the solution

\[ \vec{H}(x) = \vec{H}_0 e^{-x/\lambda_L} \]  

(2.13)

Equation (2.13) shows that the magnetic field \( \vec{H} \) decays exponentially upon penetrating
into a superconducting specimen in accordance with the Meissner effect.

2.5 Current and Field Distributions for Simple Geometries

The penetration of a magnetic field into a superconducting specimen is most easily
illustrated by reference to the case of a parallel-sided plate with a magnetic field \( \vec{H}_0 \)
applied parallel to the surfaces of the plate. The length of the plate is in the direction of the
field and its width is assumed to be much larger than its thickness \( d \). If the direction normal
to the surfaces of the plate is chosen as the x-direction, the variation of the flux density with
‘x’ is easily found from the London equation (2.11) with the boundary conditions
\( H(\pm d / 2) = H_0 \)
\[ H(x) = H_0 \frac{\cosh(x/\lambda_s)}{\cosh(d/2\lambda_s)} \quad (2.14) \]

The super-current density is found by applying Maxwell’s equation and is given by:

\[ J_s(x) = -H_0 \frac{\sinh(x/\lambda_s)}{\lambda_s \cosh(d/2\lambda_s)} \quad (2.15) \]

Another example is the distribution of the magnetic field and the current in a parallel-sided current-carrying plate. The plate has a width \( w \) and the linear current density equals \( J = I / w \) where \( I \) is the total current. The direction normal to the surfaces of the plate is chosen as the x-direction and the thickness equals \( d \). The solution of the problem is found from the London equation with the boundary conditions \( H(\pm d / 2) = H_j \) at the surfaces of the plate with the magnetic field \( H_j = J / 2 \) parallel to the surfaces:

\[ H(x) = -H_j \frac{\sinh(x/\lambda_s)}{\sinh(d/2\lambda_s)} \quad (2.16) \]

The super-current density is found by applying Maxwell’s equations:

\[ J_s(x) = \frac{H_j \cosh(x/\lambda_s)}{\lambda_s \sinh(d/2\lambda_s)} \quad (2.17) \]

### 2.6 Critical State Model (Bean Model)

Bean presented in 1962 [Bean62] a model that was based on experimental observations. It was noted that the current density \( J \) takes only the value of zero, where the perfect diamagnetism property holds, or a critical value \( J_c \) in the mixed state:

\[ J = \begin{cases} 0 & E < E_c \\ J_c & E > E_c \end{cases} \quad (2.18) \]

where \( E_c \) is the electric field at the critical current density.
The model allows only two states, perfect diamagnetism or mixed state, with a sharp transition and it has thus been named the critical state model (CSM). The CSM is therefore stating that the currents in regions of changing magnetic field (or finite electric field $E$) are given by $\pm J_c$ or zero otherwise. The general validity of the appropriate but simple CSM is mainly, but not only, due to the fact that the induced currents are usually not considerably larger than $J_c$.

The model has later been explained as a consequence of having an equally strong pinning force, using the following arguments: when a magnetic field $H$ is applied to a superconductor specimen that has no prior magnetic flux, there will be shielding currents induced on the superconductor surface in order to expel the magnetic field from the inside of the specimen. These currents can be very large, which then means that the Lorenz-force at the surface becomes very large too. If it is larger than the pinning force, pinned vortices are displaced inwards, into the specimen from the surface. Hence, the surface current is spread out over a larger area, and the current density is diminished. This displacement of vortices continues until the Lorenz-force is smaller than the pinning force $F_L < F_p$, which occurs when the shielding current has a density equal to $J_c$ everywhere. For the example of an infinite slab of width $2a$ with a magnetic field applied parallel to the long side of the slab as shown in figure 2.3(a), the macroscopic magnetic field distribution $h(x)$ and the current distribution $j(x)$ (both as functions of the slab width) take the values shown in figure 2.3(b). The vortices in the specimen are now pinned and they remain therefore unchanged unless they are forced to move by a Lorenz-force. Suppose now that the magnetic field decreases from the situation in figure 2.3(b). A surface current in the opposite direction is then induced, which forces the flux vortices with opposite direction to enter into the specimen, and these replace the formerly pinned vortices. The flux density and the current distribution in the slab are as shown in figure 2.4(a). The superconductor thus has a memory of formerly applied magnetic field, which is erased by a changed field. If the mean magnetic flux density in the specimen $\overline{B} = \mu_0 / 2a \int_{a}^{a} h(x)dx$ is plotted versus applied field $H$ with amplitude $H_0$, a typical loop of a hysteresis as in figure 2.4(b) is obtained. Hence, it is understood that the critical state model exhibits hysteresis.
Figure 2.3: (a) An infinite slab of width 2a with an applied magnetic field H parallel to the long side of the slab. (b) Magnetic field and current distributions in the slab for an applied magnetic field H.

Figure 2.4: (a) Magnetic field and current distribution in the slab when the applied magnetic field has decreased, the applied field H=0. (b) The typical hysteresis loop occurs when the mean magnetic flux density $B = \frac{\mu_0}{2a} \int_{-a}^{a} h(x) \, dx$ in the superconductor is plotted versus the applied magnetic field H [Prigozhin97, Badia02].
2.7 Current in Superconductors

The current distribution in superconductors is generally related to the field distribution as discussed above. At low frequencies and below the thermodynamic critical magnetic field in type I superconductors or below the lower critical magnetic field in type II superconductors, only the Meissner shielding currents are induced when the applied magnetic field or electric field are changed. These currents flow without resistivity and are determined by the geometry of the superconductor and the applied magnetic field. In type II superconductors, the situation is different above the lower critical field where the lattice of quantized flux lines forms the so-called mixed state. The description of the inhomogeneous type II superconductors within the framework of the critical state model (CSM) is usually sufficient for nearly all electromagnetic properties, mainly for hysteresis losses.

In normal conductors, the current distribution is determined by Maxwell’s equation:

\[ \nabla \times \vec{E} = \frac{\partial \vec{B}}{\partial t} \]  

(2.19)

and the material equation \( \vec{j}_n = \frac{\vec{E}}{\rho_n} \), where \( \rho_n \) is the normal state resistivity. In a homogenous material, inserting \( \vec{B} = \nabla \times \vec{A} \) will have the usual coulomb gauge \( \rho_n \vec{j}_n = -\frac{\partial \vec{A}}{\partial t} \). These equations determine the penetration of the magnetic flux into a normal conductor, as well as into different loops consisting of superconducting and normal parts in superconducting composites. The corresponding losses are of ohmic nature and the loss density is given by \( \vec{j}_n \cdot \vec{E} \). In particular, when the current \( I \) is applied to a nonmagnetic wire with radius \( a \), the applied field inside (\( B_i \)) and outside (\( B_o \)) the wire is given by:

\[ B_i = \mu_0 \frac{I}{2\pi r} = \mu_0 \frac{ja^2}{2r} \]  

(2.20)

\[ B_o = \mu_0 \frac{Ir}{2\pi a^2} = \mu_0 \frac{jr}{2} \]  

(2.21)

At higher frequencies, when the induced currents create a magnetic field comparable with the applied one, the fields are partially shielded from the conductor and they penetrate
effectively to a distance $\delta = (2 \rho_n / \mu_0 \omega)^{1/2}$ only (the skin depth), where $\omega$ is the angular frequency of the applied field.

In the framework of the London models (sec 2.4), there are different contributions to the current at changing magnetic fields or currents: the super, normal and displacement currents. Their absolute values are related by:

$$j_s : j_n : j_d = 1 : \sigma \Lambda \omega : \Lambda \omega^2 / \mu_0 \omega^2$$  \hspace{1cm} (2.22)

where $\Lambda = m / n_e e^2$ ($n_e$ is the density of superconducting electrons, $e$ and $m$ their charge and mass, $\sigma$ the normal state conductivity and $\omega$ the angular frequency of the field). This means that for frequencies below $10^{12}$ Hz or wavelengths above 1 mm, the normal and displacement currents can be neglected. Then, the magnetic field penetrates into type I superconductors below the thermodynamic critical magnetic field only to the penetration depth $\lambda = (\Lambda / \mu_0)^{1/2}$ from the surface of the specimen as described by equation (2.13). The connection of the super-current with vector potential $\vec{A}$ is different from that for the normal current, namely $\vec{J}_s = -\vec{A} / \Lambda$. The solution for a half-space in accordance to equation (2.8) is:

$$\vec{J}_s = (B_0 / \mu_0 \lambda) \exp(-x / \lambda)$$  \hspace{1cm} (2.23)

where $B_0$ is the field induction at the surface. Therefore, the superconducting shielding currents are flowing effectively in the distance $\lambda$, too.

As discussed earlier, the magnetic flux penetrates into type II superconductors in the form of quantized flux lines (FLs). The microscopic currents, connected with them, flow essentially at a length $\lambda$ around their core, whereas the macroscopic currents are the result of the spatial gradients in the density of FLs or due to their curvature. These spatial gradients are, of course, possible only due to surfaces or some inhomogeneities in the volume of the superconductors, called pinning centres. They can compensate the Lorentz force $\vec{F}_L = [\vec{J}_c \times \vec{B}]$ acting on the flux line lattice. The resulting pinning force $\vec{F}_p$ on the
flux line lattice then determines the ideal critical current density $\overline{J}_c$ given by the condition $\overline{F}_p = \overline{F}_L$. This ideal value of $\overline{J}_c$ cannot be determined exactly as due to some excitations of the FLs [9], the flux line lattice or part of it moves far below the $\overline{J}_c$ leading to dissipation and voltage (flux creep). There are different models for including this effect, the best known of them are in the exponential form [Anderson64] and the power-like form for the current voltage characteristics [Campbell72] as such:

$$\frac{\overline{E}}{\overline{E}_0} = \exp[-\alpha(\overline{J}_c - \overline{J})]$$

(2.24)

$$\frac{\overline{E}}{\overline{E}_0} = (\overline{J} / \overline{J}_c)^n$$

(2.25)

The determination of $\overline{J}_c$ is therefore somewhat arbitrary, depending on the choice of the corresponding electric field $\overline{E}_0$ at this current density. Generally, the most widely used criterion of $\overline{E}_0 = 1 \mu\text{Vcm}^{-1}$ is well suited for practical superconducting samples. For large values of $n$ and $\alpha$, the CSM is adequate, stating that the currents in regions of changing $\overline{B}$ or finite electric field $\overline{E}$ are given by $\pm \overline{J}_c$, being zero otherwise. The limit $n = 1$ corresponds to a purely resistive material (ohmic case) where the conductivity is given by $\sigma = j_0 / E_0$. Thus the power law equation allows for a macroscopic description of normal and superconductors on an equal footing.

2.8 Losses in Superconductors

The understanding of loss mechanisms in superconductors is central for practical applications, for example, power transmission cables, motors, transformers and other devices that work under alternating current or field conditions. Before going into details, it is worth stating that some general principles are derived from electromagnetic theory. If a body is placed in an oscillating magnetic field there are two extreme possibilities. One is that the field is completely excluded and the other is that it completely penetrates the sample. The exclusion of field may be due to any combination of super-currents and eddy...
currents but in both cases, the loss per cycle is zero since the magnetization curve is a straight line. Intermediate cases occur when eddy currents and super-currents are such that the external field can just penetrate to the centre of the sample. For ohmic materials, the penetration is skin depth and is independent of amplitude but dependent on frequency. In the superconductor, the depth is of the form $\frac{B_0}{\mu_0 J_c}$, which is dependent on amplitude but independent of frequency. In the case for wire carrying transport current, the flux penetrates from outside but the current flow is now parallel to the wire’s axis and the magnetic field is circumferential.

The idea of the depth of field penetration is central to loss calculation. For small penetrations, the loss is proportional to surface area and decreases with increasing $J_c$. For fields much larger than that needed to penetrate the sample fully, the loss depends on sample volume and sample size and increases with increasing $J_c$. The field which just penetrates to the centre of the conductor, called the penetration field, is a useful parameter with which to characterize the sample. For fields parallel to the surface of a slab with thickness $d$, the penetration occurs at $\mu_0 J_c d / 2$ [Ciszek95b], whereas for the full penetration of a field directed perpendicular to the surface or width $a$ of the sample occurs at a field $\overline{B_p}$ [Campbell95]:

$$\overline{B_p} = (\mu_0 J_c d / \pi) (z \arccot(z) + 0.5 \ln(1 + z^2))$$

(2.26)

where $z = 2a / d$. In either case, the critical current density completely fills the sample.

It can be seen that an external magnetic field flux enters the superconductor from the surfaces and meets in the middle. At this point, flux is stationary so that the electric field is zero and this is called the electric centre of the conductor. If a transport current is flowing less than the critical value, the electric centre is displaced towards the edge of the superconductor, but there is still a line along the conductor along which $E = 0$ and no flux crosses this line. Only when the transport critical current is exceeded, there is no line with zero electric field. In this case either flux moves continuously across the superconductor from one side to the other.
Expressions for the loss are different for different wire geometries and for different conditions under which the superconductors are subjected. In most cases, assumptions are made such that the length of the wire extends to infinity and with current-voltage characteristics of an ideal superconductor (critical state model). The former is practically valid since the length is often much larger in dimension as compared to the cross section of the wire. While this assumption is in principle a severe restriction, it allows for the calculation and tractable analysis of many important situations.

For the simplest one-dimensional geometry of slab with thickness \(d\) and width \(w \gg d\) in a sinusoidal external magnetic field applied parallel to the slab plane, the loss per unit volume per cycle \(\dot{Q}_m\) is given by [Wilson83, Rhyner02]:

\[
\dot{Q}_m = \frac{4B_0^3}{3\mu_0^2 J_c d} \quad \text{(J/m}^3/\text{cycle)}
\]

for incomplete penetration of amplitude \(\overrightarrow{B_0}\) field, \((\overrightarrow{B_0} < \mu_0 J_c d / 2)\) and:

\[
\dot{Q}_m = \overrightarrow{B_0} J_c d \left(1 - \frac{d\mu_0 J_c}{3B_0}\right) \quad \text{(J/m}^3/\text{cycle)}
\]

for complete penetration of amplitude \(\overrightarrow{B_0}\) field, \((\overrightarrow{B_0} > \mu_0 J_c d / 2)\).

The losses for the case of transport current (self-field) can be calculated in the same way as for an oscillating external field. Transport currents, like induced currents, flow on the surface to whatever depth is needed to screen the magnetic field from the interior without exceeding \(J_c\) at any point. Unlike the distribution shown in Figure 2.2, the field is now asymmetric and the current is symmetric about the central plane. The analytical expression for the self-field losses in thin strip geometry with alternating current of amplitude \(I_o\) was calculated by Norris [Norris69] as:

\[
\dot{Q}_i = \frac{\mu_0 I_c^2}{\pi} \left[(1 - F)\ln(1 - F) + (1 + F)\ln(1 + F) - F^2\right] \quad \text{(J/m/cycle)}
\]
where $F = I_0 / I_c$.

Brandt [Brandt93] further extended the work for field and current profiles in a thin strip in a perpendicular field with zero transport current and the hysteresis losses are given as:

$$Q = \mu_0 J_c \overline{H_0} d^2 \left[ 2 \frac{H_c}{H_0} \ln \left( \cosh \frac{H_0}{H_c} \right) - \tanh \left( \frac{H_0}{H_c} \right) \right] \quad \text{(J/m/cycle)}$$

(2.30)

where $\overline{H}_0$ is the applied magnetic field amplitude and $\overline{H}_c = J_c / \pi$.

In reference [Rhyner02], the expression for losses per cycle is obtained for a rectangular cross section with width $w$ and thickness $d$ in self field condition. The formulation used was in terms of vector potential with critical state model $(E - (\overline{J} / \overline{J}_c)^n, n \to \infty)$ approach and was given as:

$$Q_{oc} = \frac{\mu_0 I_c^2}{\pi} \Theta(\xi) \quad \text{(J/m/cycle)}$$

(2.31)

with $\Theta(\xi)$ defined as the shape function for the aspect ratio $\xi = w / d$. The shape function has the form:

$$\Theta(\xi) = \frac{1}{6} \left[ -7 + 6 \log 4 - \xi^2 \log(1 + \xi^2) - \xi^{-2} \log(1 + \xi^{-2}) + 2(\xi \arctan \xi^{-1} + \xi^{-1} \arctan \xi) \right]$$

(2.32)

where $\Theta(1) \approx 0.512$ for square cross-section conductor and $\Theta(\infty) \approx 0.386$ for thin strip geometry as in (2.8c) for $I_0 = I_c$.

The total alternating current loss for an infinite slab with a transport current of amplitude $I_t$ in a parallel applied magnetic field with amplitude $\overline{B}_0$ is presented in reference [Carr79, Rabber99] for three different cases as follows:

$$Q_{\text{oc}} = \frac{2 \overline{B}_0}{3 \mu_0} \left( i^3 + 3 \beta^2 i \right) \quad \text{for } \beta < i$$

(2.33)
\[ Q_{\text{out}} = \frac{2B_{p}^{2}}{3\mu_{0}} \left( \beta^3 + 3\beta i^2 \right) \text{ for } i < \beta < 1 \]  

(2.34)

\[ Q_{\text{out}} = \frac{2B_{p}^{2}}{3\mu_{0}} \left( \beta\left(3+ i^2\right) - 2\left(1-i^3\right) + 6i^2 \frac{(1-i)^2}{(\beta-i)} - 4i^2 \frac{(1-i)^3}{(\beta-i)^2} \right) \text{ for } \beta > 1 \]  

(2.35)

where the normalized magnetic field, \( \beta = \frac{\vec{B}}{\vec{B}_p} \), and normalized transport current, \( i = I_t / I_c \), are utilized.

As can be seen from the above discussions, in many cases, the losses for superconductors can advantageously be described by analytical expressions for experimental and practical studies in many practical engineering applications. In the situation where the analytic solution is not possible or difficult, a numerical approach has to be implemented to find the losses. A number of numerical methods have been used to solve such problems in superconductors and the necessary first step is always devoted to field and current distributions [Brandt96, Brandt98, Rhyner98, Yazawa98, Amemiya97, Nguyen07]. Most of these methods start by solving the time-dependent and highly non-linear partial differential equation for the vector potential. The vector potential has greater significance in superconductors that in normal materials since it can usually be directly related to the distance moved by flux lines. This can be seen from the fact that \( \dot{A} = \vec{E} = \vec{B} \nu \) where \( \nu \) is the flux velocity. Since \( \nu = d\vec{y}/dt \) where \( \vec{y} \) is the flux displacement, the vector potential is a direct measure of how far the flux has moved. A number of equations that start with the expression for the current density are directly related to the vector potential:

\[ \vec{J} = \text{curl} \vec{H} = \nabla \times \nabla \times \vec{A} / \mu_0 = -\nabla^{2} \vec{A} / \mu_0 \]  

(2.36)

For a conductor (\( \vec{J} = \sigma \vec{E} = j\omega \sigma \vec{A} \)), the eddy current equation (\( -\nabla^{2} \vec{A} / \mu_0 = j\omega \sigma \vec{A} \)) is obtained. In a superconductor obeying the London equations, \( \nabla^{2} \vec{A} / \mu_0 = \Lambda^{2} \vec{A} \) with \( \Lambda \) defined in (2.22), and for a superconductor obeying the Bean model, \( \nabla^{2} \vec{A} / \mu_0 = \pm \vec{J}_c \) or zero. Thus, the technique used for eddy current solutions can be readily adapted for superconductors. Given the amount of research effort which has gone into numerical
methods by several groups and experts, it is clear that such methods will play an increasing role in the development of practical devices.

2.9 Conclusion

This chapter presents the macroscopic point of view of the distribution of the current and field in superconductors. These distributions are important in understanding the loss mechanisms and in deriving the loss expressions for superconductors. Practical applications required an accurate knowledge of these losses. As discussed in Chapter 3, the theoretical approaches in finding the distributions are realized by first solving the vector potential equation (2.36), followed by the integral form current equation. The solutions are derived analytically for thin strip geometry in self and applied field conditions respectively. In addition, equation (2.36) is also employed in numerical implementation for solving the ac losses for superconductor with finite thickness and nonlinear voltage-current relationship. The equation is expressed in a nonlinear magnetic diffusion form with the material’s law incorporated (equation 2.25). The generality of the differential potential equation, which is derived from Maxwell’s equations provides an excellent platform for solving the macroscopic property of superconductors.
CHAPTER 3
THEORETICAL APPROACHES TO FINDING FIELD AND CURRENT DISTRIBUTIONS AND AC LOSSES

3.1 Introduction

The development of theoretical efforts in determining or estimating the ac losses in superconductors has been the subject of much research. Nevertheless theoretical calculations or models of ac losses are often not in accurate accordance with measurements. The anisotropic nature and natural variance in materials as well as inevitable experimental error can explain such discrepancies. However, the approximations of losses are adequate for most practical design purposes and in some simple cases analytical expressions are conveniently derived.

In ac losses calculations, it is necessary to first find the current distribution inside the conductor and hence the field distribution and so the losses. Exact expressions for both current and field distributions under transport current or applied field are presently only available for the case of thin superconducting strip, where the width is several magnitudes greater than the strip thickness. For arbitrary thickness to width ratio with partial penetration, numerical techniques are necessary to calculate the losses’ distribution. One of the earliest works on finding the current distribution was in 1968 by Swan [Swan68] in a paper which successfully solved the problem analytically in a long superconducting strip with small thickness after a rise from zero to a peak less than the saturation current. Later, Norris [Norris69] calculated the hysteretic ac losses in type-II superconductors for a variety of systems, including ellipse and thin strip by using the conformal transformations to the critical state model for the first time. The results were then extended by Brandt and Indenbom [Brandt 96, Brandt93] to include the perpendicular geometry that was not considered in the Norris and Swan papers.

In the work presented here, current and field distributions are found by the application of the Poisson equation obtained from Maxwell’s equations and the general solution is represented by an integral equation that gives the current as a function of the applied...
external field or of the transport current. This chapter will also describe a direct analytical method of solving this nonlinear integral equation in thin superconductors to provide analytical solutions to the sheet current and field distribution under self and external field conditions. The presented method avoids the need to make simplifying assumptions commonly required in numerical computation to circumvent the logarithmic singularity or divergence of the diagonal terms in the kernel.

In subsequent sections, the variational iteration method (VIM) is introduced to solve the problem with finite thickness which involves the two dimensional Poisson equation with a constitutive relation, in this case the Bean model. The versatility of this approach also provides an insight on solving many non-superconductor physical problems described by similar differential equations.

### 3.2 Transport Current without Applied Magnetic Field (Self field)

An alternating current that flows in a superconductor strip will generate an alternating magnetic field that cuts into the plane of the strip. The current will try to distribute itself across the width to make this field equal to zero. By the properties of the superconductor, the current per unit width for a thin strip is limited to a critical value $I_c$ and always occupies a portion of the strip from each edge inward. This saturated portion increases with current towards the center of the strip until it reaches $I_c$ where the entire cross section of the strip is filled. At any instance of time the total current flowing through the superconductor should be equal to the applied alternating transport current $I_{tr}(t) = I_m \cdot \sin(\omega t)$, where $I_m$ is the amplitude of the input current and $\omega$ is the angular frequency. If the current is flowing in the $z$ direction which is taken as the longitudinal direction, the corresponding Poisson equation obtained from Maxwell’s equations and with introduction of a vector potential $\vec{A}$ is written as;

$$\nabla^2 A_z = -\mu_0 J_z$$  \hspace{1cm} (3.1)
where \( J_z \) is the \( z \) component of the current density, \( A_z \) is the \( z \) component of the vector potential produced by the current inside the conductor, and \( \mu_0 \) is the vacuum permeability. Equation (3.1) is derived from the assumption of material laws \( \vec{B} = \mu_0 \vec{H} \) where \( \vec{B} = \nabla \times \vec{A} \) and by taking the coulomb condition or coulomb gauge \( \nabla \cdot \vec{A} = 0 \). Thus, from Ampere’s circuit law, \( \nabla \times \vec{H} = \vec{J} \) and by neglecting the displacement current density, \( \vec{J} = \mu_0^{-1} \nabla \times (\nabla \times \vec{A}) = -\mu_0^{-1} \nabla^2 \vec{A} \) is obtained. The displacement current is negligible because at low frequencies (< kHz), the ratio of this current to the conduction current, \( \vec{J} \) is in the order of \( 10^{-15} \).

The general solution of equation (3.1) can be written in the integral form as;

\[
A_z(r) = -\mu_0 \int_S Q(r, r') J_z(r')d^2r'
\]

(3.2)

where \( Q(r, r') \) is the integral kernel and the integration is over the cross section \( S \) of the superconductor. This integral form solution is not in the simplest form of integral equations where the integration is made without any consideration of possible singularity in the kernel. Since the logarithmic kernel becomes singular for \( r = r' \), a direct analytical solution is cumbersome but not for the case of a thin strip of width \( 2a \) where the current is invariant along the thickness of the strip and consequently the conductor can be effectively treated as a 1-Dimensional object with half width \( a \), as represented in figure 3.1. The utilized coordinate system is also shown and the ac transport current flows in the \( z \)-direction.
Figure 3.1: Schematic view of the thin strip superconductor with width \(2a\) and the coordinate system used for the current and field distribution studies. The transport ac current flows along the \(z\) axis. The region with width \(2b\) is the field free region and the region in \(b < |x| < a\) is the saturation region with current flow equal to \(I_c\).

An alternative solution to the singular integral equation is to use numerical approaches (e.g. finite difference or finite element methods) and one that has been widely implemented is Brandt’s method [Yazawa98, Nguyen07, Brandt94]. Brandt circumvented this problem by substituting for the variables an odd function with a new variable. Then the integral is transformed into discrete forms with the new variable discretized to a number of equidistant points. In the final expression the logarithmic kernel matrix is defined separately for both \(r = r'\) and \(r \neq r'\) cases.

A more direct analytical approach based on the solution of a Cauchy-type integral is employed here. In the thin strip limit and using the symmetrical current distribution \(J_z(x) = J_z(-x)\), the kernel \(Q(r,r')\) in equation (3.2) can be rewritten as;

\[
Q_{xx} = \frac{\ln\left(\frac{a^2 - x^2}{a^2 + x^2}\right)}{2\pi} \quad \text{for} \quad 0 < x < a
\]  

(3.3)
\[ Q_{xc} = \frac{\ln\left(\frac{x-x'}{x'}\right)}{2\pi} \quad \text{for} \quad (-a < x < a) \quad (3.4) \]

Substituting the above kernel (3.4) to the integral solution of equation (3.2) and differentiating with respect to \( x \) to obtain the Cauchy-type integral equation;

\[-\frac{1}{\mu_0} \frac{dA_z(x)}{dx} = \frac{1}{2\pi} \int_{-a}^{a} \frac{J_z(x')}{x-x'} \, dx' \quad (-a < x < a) \quad (3.5)\]

By equating the right hand side (r.h.s) of the equation and the induction \( \vec{B} = \nabla \times \vec{A} \) law, the following equation:

\[ H_y(x) = \frac{1}{2\pi} \int_{-a}^{a} \frac{J_z(x')}{x-x'} \, dx' \quad (3.6) \]

is obtained using \( H_y = B_y / \mu_0 = H_y(x) \). Equation (3.6) is a singular integral equation with a Cauchy kernel for the quantity \( J_z(x') \) and the general solution of this type can be found in various literatures [Estrada89, Chakrabarti02, Harry73] and after some manipulation is:

\[ J_z(x) = \frac{2}{\pi} \int_{-a}^{a} \left( \frac{a^2-x^2}{a^2-x'^2} \right)^{1/2} \frac{H_y(x')}{x-x'} \, dx' + \frac{2c}{\left[ \left( a^2-x^2 \right) \right]^{1/2}} \quad (3.7) \]

where \( c = \int_{-a}^{a} J_z(x') \, dx' \) is the total current through the strip.

Notice that the first term in equation (3.7) originates from the shielding current induced by the field \( H_y(x') \) or by any applied magnetic field if it exists, and the second term comes from an external transport current that makes the solution unique. Taking the strip with saturation current density per unit width, \( J_c \), and \( 2b \) as the width of field free region (shielding current), as in figure 3.1, then:

\[ J_z(x) = \begin{cases} 
J_c & b < |x| < a \\
J_{zh}(x) & |x| < b 
\end{cases} \quad (3.8) \]
$J_{zb}(x)$ is solved by first calculating the field $H_y(x')$ due to the constant current density in the regions $-a < x < -b$ and $b < x < a$, then substituting the expression into equation (3.7) with opposite polarity to account for field expulsion. From Ampere’s law, the magnetic field $H_y(x')$ is expressed as

$$H_y(x') = \frac{J_c}{2\pi} \left\{ \int_{-b}^{-a} \frac{1}{x-x'} \, dx + \int_{b}^{a} \frac{1}{x-x'} \, dx \right\} = \frac{J_c}{2\pi} \left\{ \ln(x-x') \bigg|_{-b}^{-a} + \ln(x-x') \bigg|_{-a}^{b} \right\}$$

$$= \frac{J_c}{2\pi} \left\{ \ln\left[ \frac{-b-x'}{-a-x} \right] + \ln\left[ \frac{a-x'}{b-x} \right] \right\}$$

$$= \frac{J_c}{2\pi} \ln \left[ \frac{(a-x')(b-x')}{(b-x')(a-x')} \right] \quad \text{for} \quad -b < x' < b \quad (3.9)$$

Subsequently the current density $J_{zb}(x)$ over this central region is calculated as;

$$J_{zb}(x) = \frac{J_c}{\pi^2 \sqrt{b^2-x^2}} \int_{-b}^{b} \ln \left[ \frac{(a-x')(b+x')}{(b-x')(a+x')} \right] \sqrt{b^2-x^2} \, dx - \frac{2c}{\left[ (b^2-x^2)^{\gamma/2} \right]} \quad (3.10)$$

Remarkably, the indefinite integral of equation (3.10) can be solved analytically by first taking the derivative with respect to a constant in the integrand, $a$ and then integrate back the differential equation with the same constant to obtained the final expression for $J_{zb}(x)$.

First, let

$$J_{zb}(x) = \frac{J_c}{\pi^2 \sqrt{b^2-x^2}} Y(x) - \frac{2c}{\left[ (b^2-x^2)^{\gamma/2} \right]} \quad (3.11)$$

where

$$Y(x) = \int_{-b}^{b} \ln \left[ \frac{(a-x')(b+x')}{(b-x')(a+x')} \right] \frac{\sqrt{b^2-x^2}}{x-x'} \, dx' \quad (3.12)$$

Because of symmetry $J_{zb}(-x) = J_{zb}(x)$, the above integral reduces to
\[ Y(x) = \int_{0}^{b} \ln \left[ \frac{(a - x')(b + x')}{(b - x')(a + x')} \right] \frac{2x' \sqrt{b^2 - x'^2}}{x^2 - x'^2} \, dx' \]  \hspace{1cm} (3.13)

Thus from (3.11),

\[ \frac{dJ_{a}(x)}{da} = \frac{J_{c}}{\pi^2 \sqrt{b^2-x^2}} \frac{dY(x)}{da} \]  \hspace{1cm} (3.14)

with

\[ \frac{dY(x)}{da} = \int \frac{4x^2 \sqrt{b^2-x^2}}{(x^2-x'^2)(a^2-x'^2)} \, dx' \]

\[ = \frac{2}{a^2-x^2} \left( 2x^2 \arctan \left( \frac{x'}{\sqrt{b^2-x^2}} \right) - x \sqrt{b^2-x^2} \ln \left( \frac{b^2+x' + \sqrt{b^2-x^2} \sqrt{b^2-x'^2}}{x+x} \right) \right) \]

\[ - 2a^2 \arctan \left( \frac{x'}{\sqrt{b^2-x^2}} \right) - a \sqrt{b^2-a^2} \ln \left( \frac{b^2-x'a + \sqrt{b^2-a^2} \sqrt{b^2-x^2}}{x-a} \right) \]

\[ + x \sqrt{b^2-x^2} \ln \left( \frac{b^2-x' + \sqrt{b^2-x^2} \sqrt{b^2-x'^2}}{x'-x} \right) \]

\[ + a \sqrt{b^2-a^2} \ln \left( \frac{b^2+x'a + \sqrt{b^2-a^2} \sqrt{b^2-x^2}}{x+a} \right) \]  \hspace{1cm} (3.15)

The singularity in the indefinite integral of (3.15) signifies that the Cauchy’s principal value of the integral has to be taken. Putting the integrand in equation (3.15) as \( I_{ind} \), the following relation applies;

\[ \frac{dY(x)}{da} = I_{ind}^{x_{c}} + I_{ind}^{x_{c}} = I_{ind}^{x_{c}} - I_{ind}^{x_{c}} \]  \hspace{1cm} (3.16)

and after some manipulation and taking \( \varepsilon \to 0 \), the final reduction is:

\[ \frac{dY(x)}{da} = \frac{2\pi a \sqrt{a^2-b^2}}{x^2-a^2} \]  \hspace{1cm} (3.17)

Integrating equation (3.14) with respect to \( a \) on both sides with equation (3.17) gives:
\[ J_{zh}(x) = \frac{J_c}{\pi \sqrt{b^2 - x^2}} \sqrt{x^2 - b^2} \ln \left( \frac{2b^2 - 2\sqrt{x^2 - b^2} \sqrt{a^2 - b^2} - x^2 - a^2}{a^2 - x^2} \right) \]  

(3.18)

By using the following identity:

\[ \arctan(x) = \frac{1}{2} \left( \ln(1 - ix) - \ln(1 + ix) \right) \]  

(3.19)

equation (3.18) reduced to

\[ J_{zh}(x) = \frac{2J_c}{\pi} \arctan \left( \frac{a^2 - b^2}{b^2 - x^2} \right)^{1/2} \]  

(3.20)

Substituting the above expression into equation (3.8), the final result is:

\[ J_z(x) = \begin{cases} 
J_c & b < |x| < a \\
\frac{2J}{\pi} \arctan \left( \frac{a^2 - b^2}{b^2 - x^2} \right)^{1/2} & |x| < b 
\end{cases} \]  

(3.21)

The total current in the strip is obtained by integrating this to give:

\[ I_{\text{total}} = 2J_c \sqrt{a^2 - b^2} \]  

(3.22)

where at full penetration \( b = 0 \), \( I_p = 2J_c a \).

Notice that equation (3.21) is in accordance with Norris’s result for a thin strip of finite width which was obtained by using a conformal mapping technique.

The magnetic field component perpendicular to the strip associated with this current is found by inserting into equation (3.6) and with the technique used earlier, a similar set of equations follow;
Both sets of equations (3.21) and (3.23) are plotted in figure 3.2 across the strip of width $2a$ carrying a transport current $I$ after an initial rise from zero to $0.2\, I_c$, $0.5\, I_c$, and $0.8\, I_c$ respectively. The penetration depth $b$ from the edges is shown with the increase of transport current and towards the center of the strip. When the current is reduced after its initial maximum, the region $-b < x < b$ does not shrink or expand but instead a region of opposite polarity starts to grow inward from each edge. The magnetic field profiles in this case will have a trapped region in the inner part of the strip and this trapped flux is typical of a hysteresis effect. Thus the loss is indeed a hysteresis loss.

\[
H_y(x) = \begin{cases} 
0 & |x| < b \\
\frac{J_c x}{\pi |x|} \arctanh \left( \frac{x^2 - b^2}{a^2 - b^2} \right) & b < |x| < a \\
\frac{J_c x}{\pi |x|} \arctanh \left( \frac{a^2 - b^2}{x^2 - b^2} \right) & |x| > a
\end{cases}
\]  
\tag{3.23}

Figure 3.2: Plot of (a) current density $J_z(x)$ distribution from equation (3.21) and (b) normal component of magnetic field $H_y(x)$ from equation (3.23) in a superconducting thin strip of width $2a$ carrying a transport current. The penetration width $b$ for $I/I_c = 0.8$ is indicated at the top figure (a) and $a$ as the half width unit length.
3.3 Transverse Direction of Applied Magnetic Field

The field and current profiles for a thin superconducting strip in an applied perpendicular field and with zero transport current can be computed analytically in a similar way as for the transport current case (section 3.2). With reference to figure 3.1, the magnetic field is applied along the $y$ axis and the induced current flows along the $z$ axis. Beginning with equation (3.2) but now the kernel $Q_{xx}$ is replaced with

$$Q_{xx} = \frac{1}{2\pi} \ln \left| \frac{x-x'}{x+x'} \right| \text{ for } (0 < x < a)$$

(3.24)

to take into account the symmetrical distribution $J_z(-x) = -J_z(x)$ and choosing the half width $a$ again. In the ideal Meissner state where $H_y(\{x\} < a) = 0$, the responsible shielding current $J_s(x)$ is found by solving equation (3.2) with (3.24) and the solution simplifies to:

$$J_s(x) = \frac{4xH_a}{\pi \sqrt{a^2-x^2}} \int_0^a \sqrt{\frac{a^2-x'^2}{x'^2-x^2}} \, dx' \quad |x| < a$$

(3.25)

where $H_a$ is the applied magnetic field. Inserting the limits and integrating in the sense of principal value, the final $J_s(x)$ in $|x| < a$ is then:

$$J_s(x) = \frac{2xH_a}{\sqrt{a^2-x^2}}$$

(3.26)

When the applied field $H_a$ is increased the shielding current will saturate near the edges of the strip to $J_c$ (critical current density) at one side and $-J_c$ on the other side. If these regions are denoted as $b \leq |x| \leq a$ the current distribution in the region $|x| \leq b$ can then be obtained by first finding the magnetic field produced by a pair of odd currents $I = J_c dx$ flowing along $z$ at the edges and then substituting into the $H_a$ term in equation (3.25). The solution of the integral must then be added with the shielding current (3.26) to compensate for $H_a$ in $|x| < b$. This gives
\[ J_{zh}(x) = \frac{2J_x}{\pi^2 \sqrt{b^2 - x^2}} \int_0^b \ln \left( \frac{b^2 - x^2}{a^2 - x^2} \right) \frac{\sqrt{b^2 - x^2}}{x^2 - x^2} \, dx - \frac{2xH_x}{\sqrt{b^2 - x^2}} \quad |x| < b \] (3.27)

Using the technique in section 3.2 by taking the first derivative of the integral solution with respect to \( a \) and performing the integration as usual:

\[
\frac{dJ_{zh}(x)}{da} = \frac{-2J_x}{\pi^2 \sqrt{b^2 - x^2}} \int_0^b \frac{2a}{(a^2 - x^2) (x^2 - x^2)} \, dx' \]

\[
= \frac{-2J_x}{\pi^2 \sqrt{b^2 - x^2}} \left( \frac{1}{(a^2 - x^2)} \right) \left[ x \sqrt{b^2 - a^2} \ln \left( \frac{b^2 - x a + \sqrt{b^2 - a^2} \sqrt{b^2 - x^2}}{x' - a} \right) \right] \]

\[
- a \sqrt{b^2 - x^2} \ln \left( \frac{b^2 - x' x + \sqrt{b^2 - x^2} \sqrt{b^2 - x^2}}{x' - x} \right) \]

\[
- x \sqrt{b^2 - a^2} \ln \left( \frac{b^2 + ax' + \sqrt{b^2 - a^2} \sqrt{b^2 - x^2}}{x' + a} \right) \]

\[
+ a \sqrt{b^2 - x^2} \ln \left( \frac{b^2 + ax' + \sqrt{b^2 - x^2} \sqrt{b^2 - x^2}}{x' + x} \right) \left|_0^b \right. \]

(3.28)

Inserting the limits and taking the principal value, the second and last terms inside the bracket are both taken out and the final solution after integrating back with respect to \( a \) gives:

\[
J_{zh}(x) = \frac{-2J_x}{\pi \sqrt{b^2 - x^2}} \left( \ln \left( a + \sqrt{a^2 - b^2} \right) - \sqrt{x^2 - b^2} \ln \left( \frac{-b^2 + ax + \sqrt{x^2 - b^2} \sqrt{a^2 - b^2}}{a - x} \right) \right) \]

\[
+ \frac{\sqrt{x^2 - b^2}}{2x} \ln \left( \frac{-b^2 - ax + \sqrt{x^2 - b^2} \sqrt{a^2 - b^2}}{a + x} \right) \]

(3.29)

Applying the identity in (3.19) as before and knowing that

\[
\sinh \ln(x) = \frac{1}{2} \left( x - \frac{1}{x} \right), \quad (3.30)\]

equation (3.29) can be re-written as
\[ J_{zb}(x) = \frac{2J_c}{\pi} \text{arctan} \left( \frac{x\sqrt{a^2 - b^2}}{a\sqrt{b^2 - x^2}} \right) \]  
\[ (3.31) \]

for \(|x| < b\). After substituting (3.31) into equation (3.27) and taking into account a constant term that needs to compensate the shielding current term, this yields the relationship between \(b\) and \(H_a/J_c\) as

\[ b = \frac{a}{\cosh \left( \frac{H_a}{H_c} \right)} \]  
\[ (3.32) \]

with \(H_c = J_c/\pi\) being the critical field. The complete profile of the current distribution in the strip of width \(2a\) in a perpendicular field is then:

\[ J_z(x) = \begin{cases} 
\frac{J_c}{\pi} \frac{x}{|x|} & b < |x| < a \\
\frac{2J_c}{\pi} \text{arctan} \left( \frac{x\sqrt{a^2 - b^2}}{a\sqrt{b^2 - x^2}} \right) & |x| < b 
\end{cases} \]  
\[ (3.33) \]

using equation (3.8). The magnetic fields associated with these currents are found as in section 3.2 by inserting into equation (3.6) and these give:

\[ H_y(x) = \begin{cases} 
0 & |x| < b \\
\frac{J_c}{\pi} \text{arctanh} \left[ \frac{a\sqrt{x^2 - b^2}}{|x|\sqrt{a^2 - b^2}} \right] & b < |x| < a \\
\frac{J_c}{\pi} \text{arctanh} \left[ \frac{|x|\sqrt{a^2 - b^2}}{a\sqrt{x^2 - b^2}} \right] & |x| > a 
\end{cases} \]  
\[ (3.34) \]

The current density \(J_z(x)\) and magnetic field \(H_y(x)\) distribution in a superconducting thin strip of width \(2a\) in transverse applied magnetic field \(H_a\) are depicted in figure 3.3. The profiles shown are for \(H_a\) from zero (virgin state) to 0.5\(H_c\), 1.0\(H_c\), 1.5\(H_c\) and 2.0\(H_c\) respectively with \(a\) indicated as a unit length width. The penetration width \(b\) is calculated from equation (3.32) with the \(H_a/H_c\) values inserted.
Figure 3.3: Plot of (a) current density $J_z(x)$ distribution from equation (3.33) and (b) normal component of magnetic field $H_y(x)$ from equation (3.34) in a superconducting thin strip of width $2a$ in a perpendicular magnetic field $H_a$ which increased from zero. The distribution profiles are for $H_a/H_c = 0.5, 1.0, 1.5, 2.0$ and the penetration width $b$ is obtained from (3.32).

3.4 Losses Computation and Distribution – Analytical Approach

Once the field and current distribution inside the conductor is known, the loss can be calculated based on the flux crossing in within the non-free field region. Given the magnetic flux as the integrated field across the width, then the corresponding flux per unit length in thin superconducting strip is:

$$\Phi = \int_{-a}^{a} B_y(x) \, dx$$

with $B_y(x)$ taken from equation (3.23) and equation (3.34) for self field and applied field respectively. For the case of self field,
Thus the loss per cycle per unit length can be estimated as:

\[ L_{cycle} = \int_{b}^{a} 4J_{c} \Phi(x) \, dx \]  \hspace{1cm} (3.37)

according to Norris. Putting equation (3.36) into (3.37) and inserting the limits after performing the integration, the following loss expression is obtained after some manipulation:

\[ L_{cycle} = \frac{4 \mu_{0}J_{c}^{2}}{\pi} \left( -\frac{a^{2} - b^{2}}{2} - a^{3} \ln \left( \frac{a}{b} \right) + a^{2} \sqrt{a^{2} - b^{2}} \ln \left( \frac{a + \sqrt{a^{2} - b^{2}}}{b} \right) \right) \]

Using the relationship \( I_{c} = 2J_{c}a \) for the critical current instead, the loss is expressed as:

\[ L_{cycle} = \frac{\mu_{0}J_{c}^{2}}{2\pi} \left( -\frac{a^{2} - b^{2}}{a^{2}} - \frac{1 - \sqrt{a^{2} - b^{2}}}{a} \ln \left( \frac{a^{2}}{b^{2}} \right) + 2 \sqrt{a^{2} - b^{2}} \ln \left( 1 + \frac{\sqrt{a^{2} - b^{2}}}{a} \right) \right) \]

Since the loss is more conveniently expressed in terms of the total current flows in the strip and if the peak current flow is \( I_{m} \) then from equation (3.22):

\[ I_{m} = 2J_{c} \sqrt{a^{2} - b^{2}} = I_{c} \frac{\sqrt{a^{2} - b^{2}}}{a} \]

Setting \( F = \frac{\sqrt{a^{2} - b^{2}}}{a} \) and so \( F = \frac{I_{m}}{I_{c}} \), then

\[ L_{cycle} = \frac{\mu_{0}J_{c}^{2}}{\pi} \left( (1 - F) \ln (1 - F) + (1 + F) \ln (1 + F) - F^{2} \right) \]

\( (3.41) \)
taking into account both edges on the strip. Expanding the $L_{\text{cycle}}$ in a series of $F$,

$$L_{\text{cycle}} = \frac{\mu_0 J_c^2}{\pi} \left( \frac{F^4}{6} + \frac{F^6}{15} + \ldots \right)$$

(3.42)

Equation (3.42) conveniently estimates the loss for transport current of small ($F \ll 1$) amplitude and if $F = 1$ expression (3.41) gives the exact loss calculation of the state. Noticed that (3.41) is the same as in equation (2.29) which has been obtained by Norris.

In the case of applied transverse field and zero transport current, equation (3.35) and (3.34) give

$$\Phi = \int_b^a \frac{\mu_0 J_c}{\pi} \arctanh \left( \frac{a \sqrt{x^2 - b^2}}{x \sqrt{a^2 - b^2}} \right) dx$$

$$= \frac{\mu_0 J_c}{\pi} \int_b^a x \arctanh\left( \frac{a \sqrt{x^2 - b^2}}{x \sqrt{a^2 - b^2}} \right) - a \arctanh\left( \frac{\sqrt{x^2 - b^2}}{\sqrt{a^2 - b^2}} \right)$$

(3.43)

Substituting this again in equation (3.37) for the loss per cycle of unit length,

$$L_{\text{cycle}} = \frac{4 \mu_0 J_c^2}{\pi} \int_b^a \left( \arctanh \left( \frac{a \sqrt{x^2 - b^2}}{x \sqrt{a^2 - b^2}} \right) - a \arctanh \left( \frac{\sqrt{x^2 - b^2}}{\sqrt{a^2 - b^2}} \right) \right) dx$$

(3.44)

where $L_{\text{cycle}}$ is used here to avoid confusion from the self field loss expression. After some manipulation the solution to the integral is:

$$L_{\text{cycle}} = \frac{4 \mu_0 J_c^2}{\pi} \left( \frac{1}{2a \sqrt{a^2 - b^2}} \ln \left( \frac{a + \sqrt{a^2 - b^2}}{b} \right) + \frac{a^2}{2} \ln \left( \frac{a^2}{b^2} \right) \right)$$

(3.45)

With the following relations $H_a = -H_c \ln \left( \frac{a + \sqrt{a^2 - b^2}}{b} \right)$ and $\tanh \left( H_a / H_c \right) = \frac{\sqrt{a^2 - b^2}}{a}$ from (3.32) and knowing that $H_c = J_c / \pi$, the above loss expression reduces to:
\[ L_{\text{acyle}} = 4J_c \mu_0 a^2 H_a \left( \frac{2}{z} \ln \cosh z - \tanh z \right) \]  

(3.46)

where \( H_a \) = applied magnetic field, \( z = H_a / H_c \) taking into account both edges on the strip. The critical current density \( J_c \) is expressed as the current of unit length. Expanding the \( L_{\text{acyle}} \) in a series of \( z \):

\[ L_{\text{acyle}} = 4J_c \mu_0 a^2 H_a \left( \frac{1}{6} z^3 - \frac{47}{360} z^5 + \ldots \right) \]  

(3.47)

The above expression conveniently estimates the loss for small amplitude of applied magnetic field (\( z \ll 1 \)). Also, notice that equation (3.46) is the same as in equation (2.30) which Brandt had calculated by using the area of hysteresis loop.

It is desirable for both practical purposes and to gain greater understanding of physical properties that the expression for losses be derived via an analytical solution wherever possible. In addition to the two solutions above (3.41 and 3.46) for self and applied field respectively, solutions for cases that involved finite thickness and elliptical cross-sections were also found. Except for the latter, the analytical method only exists for transport current equal to \( I_c \) (equation 2.31) and for applied magnetic field much larger than the field of full penetration [Banno99, Rabber04]. If the dependence of \( J_c \) on the magnetic field is negligible, the hysteresis loss for a conductor with elliptical cross section in perpendicular applied magnetic field with amplitude \( B_a \) and frequency \( f \) is:

\[ L(\theta_c = \theta_p) = \frac{16 \lambda J_c}{\pi E_c^{1/n}} \frac{n}{1 + 3n} (aB_a)^{1+1/n} (2\pi f)^{1/n} \]  

(3.48)

and

\[ L(\theta_c = \theta_p, h, e) = \frac{2a \lambda J_c B_a}{(1 + 1/2n)} \left[ \frac{2\pi \hat{a} B_a}{E_c} \right]^{1/n} \]  

(3.49)
for a rectangular cross section. Here the $n$ represents the $E$-$J$ power law factor, $a$ is the half width cross section, $E_c$ is the critical electric field (typically $10^{-4}$ V/m) and $\lambda$ is the fraction of the superconductor.

Analytical solutions for losses that involved not only arbitrary cross section thickness, but also with arbitrary transport current ($I < I_c$) or transverse field amplitude ($H_a < H_p$) have not yet been found. For such conditions numerical calculations have to be performed and will be discussed in the subsequent section.

3.5 Variational Iteration Method for Nonlinear Magnetic Diffusion Model for AC Losses

The low frequency limit of Maxwell’s equations that described the macroscopic electromagnetic fields is described as follows:

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$  \hspace{1cm} (3.50)

$$\nabla \times \vec{H} = \vec{J}$$  \hspace{1cm} (3.51)

where $\vec{H}$ is the magnetic field, $\vec{J}$ is the current density, $\vec{E}$ is the electric field and $\mu_0$ is the vacuum permeability. Taking the curl of both sides of equation (3.51),

$$\nabla \times \nabla \times \vec{H} = \nabla \times \vec{J}$$  \hspace{1cm} (3.52)

is obtained. Knowing the relation between the current density and electric field as:

$$\vec{J} = \sigma_m \vec{E}$$  \hspace{1cm} (3.53)

with $\sigma_m$ being the material electrical conductivity, equation (3.52) can be rewritten after substituting from (3.50) as

$$\nabla \times \nabla \times \vec{H} = -\sigma_m \mu_0 \frac{\partial \vec{H}}{\partial t}$$  \hspace{1cm} (3.54)
Substituting $\tilde{B} = \mu_0 \tilde{H}$ into (3.54), the same form of equation is obtained:

$$\nabla \times \nabla \times \tilde{B} = -\sigma_m \mu_0 \frac{\partial \tilde{B}}{\partial t}$$

(3.55)

Similarly taking the curl of equation (3.50) and repeating the steps:

$$\nabla \times \nabla \times \tilde{E} = -\sigma_m \mu_0 \frac{\partial \tilde{E}}{\partial t}$$

(3.56)

and

$$\nabla \times \nabla \times \tilde{J} = -\sigma_m \mu_0 \frac{\partial \tilde{J}}{\partial t}$$

(3.57)

are obtained. Notice that the four equations (3.54-3.57) are essentially the same with the understanding that $\tilde{J}, \tilde{E}, \tilde{H}$, or $\tilde{B}$ can be replaced by one another. Applying the vector identity

$$\nabla \times \nabla \times \tilde{J} = \nabla (\nabla \cdot \tilde{J}) - \nabla \times \nabla \times \tilde{J} = -\nabla \times \tilde{J}$$

(since $\nabla \cdot \tilde{J} = 0$)

(3.58)

to equation (3.57) s the following magnetic diffusion equation:

$$\nabla \times \nabla \times \tilde{J} = \sigma_m \mu_0 \frac{\partial \tilde{J}}{\partial t}$$

(3.59)

where $\tilde{J}$ as before, can be replaced by any of the other three terms. Diffusion equations describing the non-linear magnetic or conducting materials and magnetic or electric fields are also mathematically similar when the vector potential $\nabla \times \tilde{A} = \tilde{B}$ is used in a similar way. In equation (3.59), the electrical conductivity $\sigma_m$ is defined individually by the material property and for a superconductor, the combination of a power law and Ohm’s law is used:

$$\sigma_s = \frac{\tilde{E}^{(1-n)/n}}{E_c^{1/n}} \cdot \tilde{J}_c$$

(3.60)
where $\sigma$, denotes the electrical conductivity of the superconductor. Generally, the magnetic diffusion equation can be solved in several different software packages but require computation time and cost. An alternative approach, called the variational iteration method (VIM), will be employed here for solving the two dimensional diffusion problem, in regions where its analytical solutions are not known. This semi-analytical technique provides a continuous representation of the approximate solutions for both homogeneous and nonhomogeneous problems with minimal computing affort in generating rapid results. A recent paper [See12] has shown that VIM is able to overcome the shortcomings of fourier series method in generating results that involves more than one nonzero boundary conditions.

To understand the basic concepts of the VIM, consider the following nonlinear differential equation:

$$
\mathcal{L}V(x,t) + N\dot{V}(x,t) = g(x,t)
$$

(3.61)

where $x \in \{x_1,x_2,\ldots\}$, $\mathcal{L}$ is a linear operator, $N$ a nonlinear operator, and $g(x,t)$ an inhomogeneous term (or a known analytical function). According to the VIM [He05a, He05b], a correction functional can be constructed as follows:

$$
\nu_{n+1}(x,t) = \nu_n(x,t) + \int_{0}^{t} \lambda \left\{ \mathcal{L}v_n(x,s) + N\tilde{v}_n(x,s) - g(x,s) \right\}ds
$$

(3.62)

where $\lambda$ is a general Lagrange multiplier [Inokuti78], which will be identified using the variational theory, $n$ denotes the $n$th approximation and $\tilde{v}_n$ is considered as a restricted variation, i.e. $\delta \tilde{v}_n = 0$.

The iterative scheme in equation (3.62) will lead to the exact solution as $n \to \infty$, i.e.

$$
V(x,t) = \lim_{n \to \infty} \nu_n(x,t).
$$

(3.63)

In the theory of variations, or better known as calculus of variations, the Lagrange multiplier plays a key role in obtaining a solution to a linear or nonlinear problem. A variational principle is sought for any quantity given as a function of the solution of that
equation. The equation may involve algebraic, differential, integral, or finite-difference operators, or even any combination of them. This technique may be viewed as a generalization of the use of the Lagrange multiplier. The notion of an adjoint equation, satisfied by the multiplier is the essence of this framework. We seek the most optimized multiplier to enable fast convergence of solutions and thus the multiplier is often approximated. The Lagrange multiplier is commonly a function rather than a number. This concept was well documented by Inokuti et. al. [Inokuti78]. This theory of variations was later adapted to the technique called variational iteration method (VIM), which also utilizes the science of Lagrange multipliers. The choice of Lagrange multiplier used in VIM is often approximated but nonetheless it is always a guided guess. The idea is to find the most optimum multiplier possible to assist in solving the equations. The steps to obtain the multiplier is further explained below. Refer also to equations (3.84).

The Lagrange multiplier can be easily and precisely obtained for linear problems. However, for nonlinear problems, it is not as trivial. The nonlinear terms are treated as restricted variations such that the Lagrange multiplier can be determined as a simpler form. This is done optimally via integration by parts.

Once the Lagrange multiplier is identified, the successive approximations $v_{n+1}(x,t)$, of the solution $V(x,t)$, will be generated by carefully choosing a suitable initial approximation function, $v_0(x,t)$. This is sometimes known as the trial function. The initial values, $v(x,0)$ and $v_j(x,0)$, along with its boundary conditions are usually used for selecting the zeroth approximation, $v_0(x,t)$.

Consider the diffusion problem in equation (3.59) with substitution of vector potential, $\tilde{A}$ in the region $0 \leq x \leq a$ and $0 \leq y \leq b$,

$$\mu_0 \left( \sigma, \frac{\partial \tilde{A}}{\partial t} - J \right) = \frac{\partial^2 \tilde{A}}{\partial x^2} + \frac{\partial^2 \tilde{A}}{\partial y^2}, 0 < x < a, 0 < y < b, t > 0,$$  

(3.64)
where \( \sigma \) is the conductivity of the superconducting wire from equation (3.60) and \( J = J_0 \sin(2\pi ft) \) corresponds to the alternating transport current. For magnetization loss, this value is equal to 0. An appropriate boundary condition should be set at the domain boundaries and both Dirichlet (\( \tilde{A} = \tilde{A}_0 \sin(\omega t) \)) and Neumann (\( \partial \tilde{A} / \partial n = 0 \)) conditions are specified at their respective boundary for two separate cases, transport and magnetization. Equation (3.64) is assumed to be subjected to the following boundary conditions:

\[
\tilde{A}(0, y, t) = t_1 \\
\tilde{A}(a, y, t) = t_2 \\
\tilde{A}(x, 0, t) = t_3 \\
\tilde{A}(x, b, t) = t_4
\]  
(3.65)

where \( t_1, t_2, t_3 \) and \( t_4 \) are the corresponding Dirichlet and Neumann condition. The initial condition is:

\[
\tilde{A}(x, y, 0) = f(x, y)
\]  
(3.66)

By the method of separation of variables, the general solution is obtained as:

\[
\tilde{A}(x, y, t) = (A \cos px + B \sin px)(C \cos qy + D \sin qy) E e^{\frac{s^2}{\sigma \mu_0}}
\]  
(3.67)

However, substituting the boundary conditions, (3.65), into the general solution above will not assist in obtaining direct solutions to the constants. Based on physical observations, the potential field \( \tilde{A}(x, y, t) \) will approach a steady-state potential function \( \phi(x, y) \) as \( t \to \infty \). Therefore, assuming that the solution of (3.64) can be expressed in the form:

\[
\tilde{A}(x, y, t) = w(x, y, t) + \phi(x, y),
\]  
(3.68)

where \( w(x, y, t) \), the transient solution which vanishes as \( t \) approaches infinity. Next, take

\[
\mu_0 \left( \sigma \tilde{A}_t - J \right) = w_t,
\]  
(3.69)

\[
\tilde{A}_{xx} = w_{xx} + \phi_{xx},
\]  
(3.70)
\[ A_{yy} = w_{yy} + \phi_{yy}. \] (3.71)

Substituting (3.69)-(3.71) into (3.64),
\[ w_t = w_{xx} + w_{yy} + \phi_{xx} + \phi_{yy} \] (3.72)

where the boundary conditions are:
\[ w(0, y, t) = t_1 - \phi(0, y), \]
\[ w(a, y, t) = t_2 - \phi(a, y), \] (3.73)
\[ w(x, 0, t) = t_3 - \phi(x, 0), \]
\[ w(x, b, t) = t_4 - \phi(x, b), \]

and the initial condition as:
\[ w(x, y, 0) = f(x, y) - \phi(x, y) \] (3.74)

By taking the limit \( t \to \infty \) of equation (3.72), the transient terms in the differential equation and boundary conditions will all vanish, leaving a steady-state problem:
\[ \phi_{xx} + \phi_{yy} = 0, \] (3.75)

where the boundary conditions are:
\[ \phi(0, y) = t_1, \]
\[ \phi(a, y) = t_2, \] (3.76)
\[ \phi(x, 0) = t_3, \]
\[ \phi(x, b) = t_4. \]

The steady-state problem above is the exact Laplace’s problem with its solution as derived in Appendix A. The remaining transient problem deduced from (3.72) is
\[ w_t = w_{xx} + w_{yy} \] (3.77)

subject to the boundary conditions:
\[ w(0, y, t) = 0, \]
\[ w(a, y, t) = 0, \]
\[ w(x, 0, t) = 0, \]
\[ w(x, b, t) = 0, \]

with the initial condition as in (3.74). Notice that the transient solution \( w(x, y, t) \) satisfies a diffusion problem where the boundary conditions are homogeneous. Therefore, its solution is given by

\[ w(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} F_{mn} e^{-\beta^2 t} \sin \left( \frac{m \pi x}{a} \right) \sin \left( \frac{n \pi y}{b} \right), \tag{3.79} \]

where \( \beta^2 = \pi^2 \left( \frac{m^2}{a^2} + \frac{n^2}{b^2} \right) \). The initial condition is given as:

\[ w(x, y, 0) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} F_{mn} \sin \left( \frac{m \pi x}{a} \right) \sin \left( \frac{n \pi y}{b} \right), \tag{3.80} \]

where

\[ F_{mn} = \frac{4}{ab} \int_0^a \int_0^b (f(x, y) - \phi(x, y)) \sin \left( \frac{m \pi x}{a} \right) \sin \left( \frac{n \pi y}{b} \right) \, dx \, dy. \tag{3.81} \]

The \( f(x, y) \) and \( \phi(x, y) \) are defined in (A3) and (A14) from Appendix A. Therefore, the complete solution of the boundary value problem (3.64) is

\[ \tilde{w}(x, y, t) = w(x, y, t) + \phi(x, y), \tag{3.82} \]

with \( \phi(x, y) \) and \( w(x, y, t) \) are found in appendix A and (3.79), respectively.

To solve equation (3.64) by VIM, a correction functional on the diffusion problem using (3.62) is constructed:

\[ \bar{A}_{n+1} = \bar{A}_n + \int_0^t \lambda(s) \left[ \frac{\partial \tilde{A}_n(x, y, s)}{\partial s} - \frac{1}{\mu \sigma^2} \left( \frac{\partial^2 \tilde{A}_n(x, y, s)}{\partial x^2} + \frac{\partial^2 \tilde{A}_n(x, y, s)}{\partial y^2} \right) - \frac{J}{\sigma^2} \right] \, ds, \tag{3.83} \]
where $\lambda$ is a general Lagrange multiplier [Inokut i78], which will be identified using the variational theory, $n$ denotes the $n$th approximation and $\bar{A}_n$ is considered as a restricted variation, i.e. $\delta \bar{A}_n = 0$. By taking variation with respect to the independent variable $\bar{A}_n$ and making the correction functional stationary, we can therefore obtain the stationary conditions for $\lambda(s)$ as follows:

$$1 - \lambda(t) = 0, \quad \lambda'(s) = 0. \quad (3.84)$$

Solving for the Lagrange multiplier in (3.84) yields $\lambda(s) = -1$. Substituting the multiplier into the correction functional (3.83) results in the following iteration formula:

$$\bar{A}_{n+1} = \bar{A}_n + \int_0^t \left\{ \frac{\partial \bar{A}_n(x,y,s)}{\partial s} - \frac{1}{\mu_0 \sigma_s} \left( \frac{\partial^2 \bar{A}_n(x,y,s)}{\partial x^2} + \frac{\partial^2 \bar{A}_n(x,y,s)}{\partial y^2} \right) - \frac{J}{\sigma_s} \right\} ds. \quad (3.85)$$

and using $\bar{A}_0(x,y,t) = \bar{A}(x,y,0) = \omega(x,y,0) + \phi(x,y)$ as the initial approximation function, equation (3.64) is solved.

The AC loss finally can be obtained by integrating the electric field and the current density over the conductor cross section during one cycle:

$$L = \int \int_{V_f S} E \cdot J dS dt \quad (3.86)$$

where the electric field $\vec{E}$ is given as the time derivative of the magnetic vector potential $\vec{A}$. The loss calculation is carried out by comparing with the analytical results for thin strip limit ($b \gg a$) equation (3.41) for transport current and equation (3.46) for applied transverse field. Figure 3.4 shows the comparison results for AC losses at different critical current value as a function of transport current and applied magnetic field respectively. The shaped markers depicted in Figure 3.4 are data evaluated by variational iteration method and the lines show loss results from the analytical solution. Results from the variational iteration method shows good agreement with the analytical results. Further comparison results with the experimental data are presented in Chapter 6 and Chapter 7.
Figure 3.4: AC losses for both transport current (a) and applied transverse field (b) at various critical current for superconducting strip. The shaped markers are from the variational iteration method and the lines are from analytical expression from equations (3.41) and (3.46).

3.6 Conclusion

The present chapter presents new analytical methods for solving the current and field distribution in thin superconducting strip. This approach solved the singularity problem that arises from the green function solution of the vector potential equation. These methods of solution can be applied to many other physical problems that contain similar integral singularity without having to make any assumptions otherwise required for numerical computation.

Subsequently, in the case of superconductor problems, the losses can be calculated by substituting the field distribution into the magnetic flux expression. The ac loss for strip with small amplitude of transport current was found to be proportional to the fourth power of the peak applied current. This is true also for the strip in perpendicular field with applied field replacing transport current.

In addition an effective semi analytical technique has been developed to solve the two dimensional problem of ac losses which utilizes the nonlinear magnetic diffusion model.
The computation method was further verified by comparison with the known analytical solution of the thin strip limit and good agreement was obtained between them.

The results have also demonstrated the feasibility and practicability of this computational scheme for calculating losses in the case of arbitrary aspect ratios of the conductor’s cross section.
4.1 Introduction

The experimental methods in ac losses for superconductors have been developed over nearly five decades and there is now a rich literature concerning this measurement. The interest in this topic may be gauged from the fact that ac losses are particularly important to high Tc superconductors for very practical reasons – applications in superconductor often experienced time varying magnetic field and transport current and the energy dissipated needs to be taken into account during the design of the device. As mentioned in section 1.4, the ac losses of superconductors have been measured based on two principles; electrical (measuring m-H loops using some form of magnetometer, or measuring voltages developed along the conductors carrying transport currents) or thermometric (bolometry or calorimetry). In the presence of a variety of experimental configurations and techniques, decisions on the choices can be rather subjective. Electrical (with voltage taps) methods used to determine the energy dissipation due to the self-field of a varying transport current alone are adequate in situations where the effect of an external alternating field on the conductor can be ignored. On the other hand, magnetic methods that are used to measure the dissipation due to screening (induced) currents only are suitable for situations in which the loss is mainly determined by the external field. Finally the thermometric techniques are suitable to obtain the total dissipated energy and they are mainly used for large volumes or with complicated geometries in which other methods would be impractical.

4.2 Progress in Measurement Techniques

AC losses in superconductors have been the subject of intense experimental investigations since the early 1980s for low temperature superconductors [Ogasawara79, Reuver85a, Dragomirecki86, Seibt84, Reuver85b, Schmidt90, Dubots85, McConnell75a] and have
been developed for high temperature superconductors in the 1990s. The complexity and accuracy of the experimental set-up are the main two factors that most researchers have to trade off. In many circumstances, accuracy has to be reduced to obtain a cost and time effective solution. In the past two decades there are a number of publications that detail the comparison of the electric (electromagnetic) and calorimetric (thermometric) loss measurement on HTS conductors mainly with BSCCO tapes [Hughes97, Coletta99, Magnusson00, Tsukamoto06] and some YBCO coated conductors [Ashworth01].

4.2.1 Electrical (electromagnetic) Method

The first data on losses subjected to ac transport current in BSCCO conductors were published in 1994/95 by Ashworth [Ashworth94] and Fukunaga [Fukunaga95]. These were at 77K using an ac transport method measuring resistive voltage under ac transport current over a range of frequency and current amplitudes. The ac voltage was recorded either by a low noise lock-in amplifier or via an instrument amplifier to a digital oscilloscope. Since the measurements were made under zero external field, the magnetic field generated by the sample is the self-field related to ac transport current and hence is the measured self-field loss. Later in the year with further experimental [Ciscek94, Fleshler95] and theoretical [Ciszek95b, Campbell95] work, an understanding was gained on how to measure ac transport losses and rather surprisingly that the measurement circuit must encompass a substantial area outside the superconductor in order to return the correct measurement of the loss voltage.

The early publications on magnetization loss for high temperature superconductor, mainly on BSCCO tape at 77K, were made by several groups [Zanella93, Oota95, Kwasnitza94] and the references therein. The technique is based on the evaluation of a magnetic loop of the magnetization curve. It utilizes a set of similar coils that consist of pick-up coil and compensation (cancel) coil, apart from the coil used to generate the ac magnetic field. Some are placed in a clear bore of a superconducting magnet that is wound from many fine filaments of a niobium-titanium (NbTi) alloy embedded in a copper matrix. Besides employing coils, Kwasnitza et al. [Kwasnitza94] used a double Hall sensor method to
measure the signal that is proportional to the conductor magnetization and the losses were then calibrated by the virgin magnetization curve.

The study of ac losses were also extended to the case of combined external field and transport current, in this context the field and current can be alternating (ac) or direct (dc). In the case for ac transport current and applied external magnetic field (either ac or dc), Ciszek and co-workers [Ciszek95a, Ciszek96] used the four probe technique with potential taps on the edge and a single layer of pick-up coil wound in the central part of the sample. The measurements were all carried out at 77K with the applied magnetic field parallel to the sample surface.

This has also been extended to the case of different orientations of applied external dc magnetic field and its influence on the transport ac losses [Ciszek97]. In subsequent years several groups have studied loss measurement under conditions which the superconductors are most likely to be exposed to in power applications such as coils and cables. Such applications can expose the material to external ac magnetic fields and ac or dc transport currents.

In reference [Rabber98], an ‘8’ shaped arrangement of the voltage leads is proposed to minimize the inductive signal set up by an external magnetic field. The length of the leads is taken to a distance of three times the half width of the tape, as have been suggested by previous references. Another inventive method to minimize the disturbing signals on the voltage taps caused by external fields is with the leads wound on a cylindrical surface enclosing the tape [Fukui98]. In this way, the spurious loss caused from the wide voltage loop can be avoided and as an added benefit more room in the sample space is made available.

As an alternative to measuring ac losses using a measurement loop to find the rate of change of flux in the sample, either with pick-up coils or voltage taps, a rather simpler technique has been reported [Ashworth99b, Ashworth99c] that considered only the energy losses in power supplies (one each for transport current and applied field respectively) instead of the sample. However, experimentally this still presents a challenge due to the larger inductive voltages that must be nulled.
Some researchers argue that a direct transport current with ac applied field would give a more reliable measurement result compared with the model predictions [Oomen99]. In this case the direct current experiences a dynamic resistance which increases with magnetic field amplitude. In practical electrical applications the magnetic field can have different angles with respect to the tape surface, particularly for three-phase cables and motors. Measurement of losses in such conditions has gained considerable attention with the effects of all possible angles of the applied alternating magnetic field fed simultaneously with an alternating transport current [Rabber99b, Rabber99a, Rabber01, Rabber02] being examined.

It is well-known that in the past decade, attention has been paid to MgB$_2$ superconductors to support its rapid development for practical and potential applications. One of the early studies published in reference [Glowacki02] discussed the feasibility, reliability and some computations on ac losses of MgB$_2$ conductors for dc and ac applications. Preliminary results on ac loss measurement of MgB$_2$ wire in an applied magnetic field were first presented in [Yang05] and later extended to transport losses at different temperatures [Majoros08] and frequencies [Young07].

Generally the electrical method of measurement used in different laboratories can be divided into the following clusters:

i) Power supply – Provides transport current through the sample via the output of a power ac amplifier or an isolating transformer. Also a solenoid coil is energized to provide external magnetic field for both dc and ac applications.

ii) Current monitoring – Transport current is commonly monitored using a non-inductive resistance shunt or a current transformer. The rms (root mean square) value of the current is measured with a digital voltmeter or a digital recording oscilloscope. A Hall’s probe current monitor can also be used. In certain circumstances an auxiliary lock-in amplifier is used for precise current and phase shift measurements. The grounding on the sample can be omitted if using a contactless method to monitor the current (current transformer, Hall probe based current transducer).
iii) Current control – The frequency and amplifier of the transport current can be controlled using an external function generator or sine output of a digital lock-in amplifier.

iv) Phase setting – This can be adjusted manually or using the autophase procedure. The reference channel phase can be adjusted using a non-inductive resistor or using a Rogowski coil mutually coupled to the current lead.

v) Grounding – This is one of the crucial factors influencing the measurement accuracy. The rule of thumb is the fewer the number of electronic instruments involved in the setup the better because every instrument has its own ground at different potentials. Also it is crucial to create only one ground point to which all instruments are connected, along with electrostatic screen of the isolating transformer (if used for power supply) and the sample center.

vi) Lock-in amplifier – This is a standard technique for self-field ac loss measurement which enables one to measure very small ac signals down to the nano-volt level. It uses a sine output to drive the power amplifier that is connected to an isolation transformer which provides ac electric current flowing through the sample. Two typical input connection setups are: First, with differential floating input between the sample and the compensating coil; Second, single input connection with sample voltage taps connected in series with a compensating coil.

4.2.2 Calorimetric Method

The restricted conditions of the electrical techniques [Ashworth00], the influence of the pick-up coil position [Yang04, Schmidt08] and the position dependent voltage taps render these electrical methods of little practical use, particularly for complex configuration or conditions. Hence calorimetric methods have been developed, either by monitoring cryogen boil-off or sample’s temperature changes, so that ac loss measurement is now more achievable. At an early stage calorimetric techniques were only used at liquid helium temperature and for the losses of superconducting coil wound of hundreds of meters of tape.
or wire and were usually unable to measure the losses of short samples. The calorimetric method for short samples measurement on LTS was first published by Schmidt and Specht [Schmidt90] and further simplified by replacing the vacuum insulation with a material of low thermal conductivity [Schmidt94]. Additionally a carbon resistor was used as the thermometer to detect the losses at 4.2 K under applied ac magnetic field. This resistor was chosen because of its high sensitivity at 4.2 K and low self-heating. A similar technique was adopted for the measurement of high-T_c tapes at 77 K [Schmidt00], but with a copper wire as the thermometer instead.

Boil-off calorimetric methods have proven to be useful on wire samples, cable sections and for coils [Dubots85, Okamoto07]; however in circumstances that involved measurement at liquid nitrogen, this method is less useful due to the larger latent heat of liquid nitrogen compared to helium and the smaller ratio between the densities of liquid and gas at room temperature. Adiabatic techniques, where the sample changes in temperature as the energy is dissipated, have been widely used and adopted at wide ranges of temperatures. Different thermal insulations and thermo sensing materials have been used. Besides carbon resistors, some employed Ge thermometers to measure temperature increases [McConnell75b] while many favor chromel constantan thermocouples for flexibility and robustness [Dolez96, Magnusson98, Ashworth99a]. Vacuum vessels or insulation have been used in the past to achieve high sensitivity but they are cumbersome and inconvenient. It was later shown that replacing the vacuum atmosphere with medium to low conductivity insulators (e.g. styrofoam, teflon, polyamide, PVC, epoxy) is acceptable for measurement of the losses for most applications [Hardano99].

A rather successful technique based on temperature difference is the bolometric approach [Chakraborty00] that uses the sheath material (e.g. Ag) as one of the conductors to form a thermocouple. In this way, the sample will not have an issue of thermal contact with the electrically insulated thermocouples since one point is itself acting as part of the thermocouple. This method has also been extended to measure losses on HTS tapes in the presence of both ac transport current and ac magnetic field [Polichetti03].

Generally the calorimetric method of measurement used in different laboratories can be divided into the following clusters:
i) Thermal Insulation – Material or room with low thermal conductivity to minimize heat flow and leakage to and from the sample. Often the tendency has been to use quite sophisticated set-ups to produce a significant (or easily measurable) temperature rise, but most prefer to compensate with a more advanced and flexible measurement and noise reduction system.

ii) Heaters – This consists of high-resistive wire usually wound in the form of a coil with inputs connected to the dc power source. They are mainly used in cryogenic experiments for temperature stabilization and variation. For calorimetric ac loss measurement the heater is placed adjacent to the sample with electrically insulated and high thermal connectivity for calibration purposes. This means the temperature rise of the sample caused by the ac loss is measured and calibrated by temperature rise caused by the heater.

iii) Sample holder – This is a supporting structure or frame for the sample’s assembly which includes the housing for the insulation, sensors and calibration heater. Ideally the holder should be made from material that has excellent thermal conductivity, electrical insulation and physical stability that can withstand a low temperature atmosphere. Generally the design of the sample holder takes into account the sample’s size/shape, the accessible space in addition to current leads and magnet, the orientation of the sample and the cooling region of the test-rig.

iv) Thermometry – This comprises of thermometer circuit and data acquisition system. A number of temperature sensors (usually 3-4) are required to measure the temperature at different points on the sample and its surroundings. Low temperature sensors are typically resistance based sensors with fixed excitation current. In order to have good sensitivity and reproducibility, they must be mounted properly with the four electrical leads tightly wrapped and thermally anchor to a heat sink. The data acquisition system manages the sensors’ data and provides the graphical representation on the computer that will greatly simplify the analysis and calibration.

v) Power Supply – This provides current through the sample and coil for ac loss measurement and also provides external magnetic fields for both dc and ac application.
4.3 Cryogenic Instrumentations

The core of cryogenic engineering systems is the low temperature chamber that is capable of maintaining a wide range of temperature, preferably from liquid helium to half of room temperature. In cases that involved only an in liquid nitrogen bath (77 K), the set-up is simpler as any thermal insulating containers could house the sample assemblies and the external magnetic field is supplied by a solenoid coil that can be placed either externally or inside the bath. However, for measurements that require low temperatures (< 77 K) and continuous variation of temperature, the chamber will need to have certain insulation criteria and a cooling mechanism. This can be achieved by standard cryo-cooler systems or helium vapor combined with high vacuum insulation space. The external magnetic field is supplied by a set of solenoid coils that are immersed inside the cryogen or vacuum with conduction-cooling through bus bars. Such chambers are mostly incorporated with a superconducting magnet that is wound using low temperature superconductor (e.g. NbTi) and operated in liquid helium bath at 4.2K or in vacuum without the need of liquid helium. In this case, the chamber is capable of providing continuous and intense dc magnetic field while efficiently controlling the temperature in the sample space. The temperature variation is realized by controlling the amount of liquid helium flow from the magnet reservoir to the sample and the power to the heater. An ac application would required an ac current source to energize the superconducting magnet but this will in turn cause some complications relating to ac losses and eddy current generation. Furthermore, most commercial superconducting magnet systems do not accommodate applications that involve an ac field. However an additional external coil can be placed inside the sample space if sufficient room is allowed and if its thermal stability can tolerate the increased thermal mass and heat load. In short, the feasibility and complexity of a chamber for any measurement circumstance largely depends on the methodology, field requirements, coil design, space availability and operating temperature range and stability.

The low temperature experimental rig employed in this research for the measurements of ac losses of practical superconducting wires and for the transport critical current under various operating temperatures and applied magnetic fields is a 15 T superconducting magnet cryostat system. A partial conceptual drawing of the system is shown in Figure 4.1.
Figure 4.1: Conceptual drawing of the system used for studying superconducting samples in a magnetic field at varying temperatures.

The cryostat consists of a variable temperature insert used for applications that require continuous variation of temperature on the sample. Cooling inside the space is achieved by withdrawing a controlled amount of liquid helium from the magnet reservoir through a valve capillary tube located at the bottom (not shown). The helium’s flow path is from the bottom of the access, over the column and extracted at the top of the dewar. The temperature within is varied by heating the helium with an electrical heater placed on the bottom adjacent to the capillary tube. With this arrangement the chamber is capable to achieved operating temperatures from 2K to 300K. The cryostat is also multilayer superinsulated and liquid nitrogen shielded to reduce heat load and consumption of the liquid helium.

The superconducting magnets are wound using multifilamentary Nb$_3$Sn for high field (>10T) applications (inner coil) and NbTi in the outer coil for field up to 9T. Electrical insulation is provided by the insulation on the wire and by the epoxy between each turn. The former on which the magnet is wound is constructed of aluminum. The magnet is mounted in the dewar by means of a support stand having a top plate, radiation baffles and a magnet mounting plate. Figure 4.2 illustrates the integrated magnet cryostat system which houses the sample inside the variable temperature insert.
Figure 4.2: Superconducting magnet cryostat system with multilayer super-insulated and liquid nitrogen shielded. The magnet is capable of providing dc magnetic field up to 15 T in vertical axis and operating temperature range from 2 K to 300 K inside the insert.

The variable temperature insert has a cylindrical tubular structure that is inserted into the bore of a superconducting magnet. The sample is mounted onto a removable rod placed inside the insert. As mentioned before the temperature can be varied by flowing cold helium liquid or vapor and this is achieved through a capillary tube controlled by the throttle valve located on the top plate. A detailed picture of the sample insert system is shown in Figure 4.3. The temperature stability is attained by the vaporizer which is the main heater (bottom) and another secondary heater placed 120 mm above the former. The sample is placed in between these two heaters with appropriate design of supporting structure. In addition, a Cernox RTD type temperature sensor is employed together with each heater and calibrated for temperature range from 1.4 K to 325 K.
In order to minimize the thermal loads and helium losses, the sample current leads as shown in the figure are made from two portions, resistive upper portion and high temperature superconductor lower portion. The upper portion is a stainless steel tube which connects between cryostat and the warm terminal for connection to current source, whereas the HTS lower portion is sandwiched between copper strips connected to the magnet. The leads are vapor cooled and any heat generated is removed by the exhaust helium vapor flowing through them.
4.3.1 Supporting Structure – Sample and Coil

The design and construction of a sample holder is a matter of broad discussion and generally varies from one design to another dependent on several factors such as the cooling rates, accessible dimensions, application, cost and flexibility. Hence it is crucial to identify precisely the measurement method and conditions the sample will be subjected to prior to construction. Similarly this determines the choice of materials to use for the holder and the integrated frame. As a good comparability, oxygen free copper is an excellent candidate for the holder in transport current measurement due to its high electrical and thermal conductivities. On the other hand, calorimetric ac losses measurement requires a good insulating material to be employed. An insulating material can have a wide range of choices and simple styrofoam pieces are widely used due to low cost and availability. However for rigid mechanical support and insulation, teflon or phenolic assemblies are more expeditiously employed. In some circumstances within the supporting frame, sapphire rods are made to provide good thermal contact in addition to electrical insulation and mechanical support.

The structure for sample ac loss measurement mounted in between the vaporizer and secondary heater in the rig shown in figure 4.3 is made entirely from G10 phenolic material. The framework is also capable of supporting an ac solenoid coil in which the ac magnetic field is generated simultaneously with the dc magnetic field transverse to the sample. The detailed construction of the ac solenoid coil will be discussed in Chapter 6. Electrical insulating properties and good physical toughness of G10 phenolic avoid any induced current from the applied ac field and are durable in a helium environment. Figure 4.4 illustrates the drawing and photograph of the supporting frame and its various dimensions. It comprises of several parts that can be readily dismantled and assembled by means of the nylon screws located on the several positions shown. The sample is firmly attached in between the middle rings of the frame and the solenoid coil is fitted onto the frame with the 40 mm diameter supporting base.
Figure 4.4: Left: Supporting frame for sample and coil placed inside the variable
temperature insert. The entire frame is made of G10 phenolic material with
nylon screw as the fastener. Right: Photograph of the constructed frame.

The chassis is then mounted onto a solid copper disk that is fixed with the removable rod
inside the variable temperature insert. Attached to the copper disk are the secondary heater
and temperature sensor which are used for temperature variation and monitoring. The
configuration of the assembly is well demonstrated in figure 4.5 with the inset showed the
photograph of the end segment of the sample insert. The coil current leads are soldered to
the HTS leads using the PbSn solder with controlled amount of heat applied. Additional
sensors and electrical leads (Hall probe, temperature sensor) will be placed inside the
solenoid coil and directly attached to the sample surface as sufficient room is allowed in the
middle and designed for this purpose. All connected electrical leads are tightly twisted and
neatly wound around the insulating frame to minimize the induced signal from the applied
field. Besides the vaporizer and the secondary heaters, sample temperature stability is also
influenced by the rate of flow of helium vapor and the insulating layer between the aperture
on the base and top portion of the coil. The latter can improve the stability by preventing
the irregular ingress of helium vapor or liquid to the sample area that frequently occurs
during the application of dc magnetic field. Non-conducting sealant like silicon rubber for
instance can be used as the layer that shields the apertures. This material is abundantly
available and durable in a low temperature environment. However after sometime it tends to become crystalline and has a reasonable thermal conductivity that might influence the measurement sensitivity and accuracy. Thus the amount of usage is always as little as possible and replacement is made regularly.

Figure 4.5: The configuration for sample and external ac coil with the supporting frame mounted on a copper disk with nylon screw. Copper disk is fixed to the removable rod and the assembly can be withdrawn from the chamber as a whole. Inset: Photograph of the rod with HTS current leads and mounting copper disk.

4.4 Electrical and Acquisition System Design

Although the low temperature chamber is the heart of any cryogenic engineering system, the electrical and acquisition systems are its essential complementing components which
must be integrated to form a complete measuring system. The electrical system in this context means the instrumentation that is placed externally to the chamber responsible for supplying and acquiring the power and signals to and from the chamber via wiring configurations. Most of these instruments are uniquely designed for cryogenic applications which required high sensitivity and stability due to the low signal to noise ratio and extreme atmosphere. The electrical wiring is made from cryogenic wire which has a much lower thermal conductivity and higher electrical resistivity than ordinary copper wire to minimize heat leak into the attached sensing elements or devices. Phosphor bronze wire is one of the most common types for such applications along with manganin for nonmagnetic applications, nichrome for heater requirements and heavy duty copper wire for current leads to resistance heaters. The electric/electronic instruments are connected to a data acquisition system which consists of a personal computer and interface configuration for control and processing reasons.

The electrical instrumentation employed along with the superconducting magnet chamber for transport critical current and calorimetric ac losses measurements are delineated as follows;

i) Magnet Power Supply – This supplies energy to the superconducting magnets and are usually low voltage and high current systems. At one volt, the magnet is typically in the 60-90 ampere range and can be charged to the rated fields in ten to fifteen minutes or less. For protective measure against the quench of magnet, a large rectifier and heat sink rated for continuous operation at full output current of the supply are included. The power supply is a four quadrant power supply which can change the polarity of the magnetic field and is rated at ± 6 Volts, ± 125 Amperes.

ii) Direct Current Power Supply – This is used for feeding transport current through the superconducting wire and tape samples, usually for critical current measurement with four probe configuration at varying applied magnetic fields and operating temperatures. Resembling the magnet power supply, the dc power supply is also the current source type but with higher rated current and voltage to provide the high critical current value and contact resistivity of practical superconducting conductors. One important feature that is incorporated into the device is the programmable sweep
rate range. This allows the possibility of ramping the current through the samples slower or faster in a particular range of interest, especially near the transition to normal state. A fast ramp up to 10 amperes/second and output current up to 200 amperes are available.

iii) Alternating Current Power Supply - In contrast to direct current, an ac power source provides an alternating environment to the samples at the frequency of choice. The alternating magnetic field applied to the sample surface is generated from the alternating current through the tailor made solenoid coil and alternating current through the sample is produced via the current leads connection from the output of power source to the ends of the sample. A programmable controller which configures the power source not only allows control of voltage and frequency, but also is able to simulate virtually any transient and waveform required for sample characterization. The output current from the source is measured using a closed loop current transducer that is fastened onto one of the current leads and connected to an oscilloscope.

iv) Low Voltage Meter (Nano-voltmeter) – This is an essential part in instrumentation for characterizing low resistance materials (e.g., superconductor) reliably and repeatedly. Four-point probe techniques are commonly used for resistivity measurement of samples and inevitably for transport critical current of superconducting wires and tapes a nanovoltmeter is required to measure low voltage levels as noiselessly as possible. Such an arrangement involves sending a known amount of current through the two outer probes and measuring the voltage through the inner probes. A typical range of the voltage signal during the transport measurement is from $10^{-8}$ V to $10^{-6}$ V, depending on the noise and solder points.

v) Digital Multimeter (DMM) – This is similar to the nanovoltmeter system but provides a lower sensitivity and broader usage. DMM is generally considered adequate for measurements at a signal level greater than one milivolt and up to 1000 Vdc. Besides dc measurements, DMM is also commonly used for ac applications which output the root mean square (r.m.s) value of the signal. For delicate and computer-controlled experiments, a programmable bench-top multimeter is with improved resolution and durability is satisfactory. One distinctive feature of such a
multimeter is it can provide four input connections making it possible to measure temperature with resistance temperature detector (RTD) sensors or thermocouples. These sensors generate the voltage signal according to a certain function (usually linear) with respect to temperature to be measured and the function is then store inside the meter. In most cases with common sensors, the functions are pre-loaded inside the meter for convenience and adaptability.

vi) Cryogenic Temperature Controller – This is used to monitor the change of temperature in low temperature experiments and the passage of heat energy into heaters that are attached to samples or near samples to achieve a desired average temperature. The passage of heat energy is adjusted by using a programmable PID feedback control loop. The control algorithm attempts to keep the load at exactly the desired temperature by using feedback from the control sensor to calculate and actively adjust the heating output. The temperature controller comes with dual control loop features and each loop is responsible for one heater inside the chamber. The loops are completely independent and either heater can be controlled by either sensor input at the same time.

vii) Temperature Monitor – This is a simplified version of the temperature controller with only the monitoring purpose. The bench-top DMM could in fact play the same role but with only one sensor input. A dedicated temperature controller can be used with nearly any diode or resistive temperature sensor and can be read more quickly than other scanning monitors which have to wait for current source switching.

viii) Gaussmeter – This measuring instrument is used along with Hall sensors or probes to measure the strength or direction of a magnetic field produced during the experimental process. Static and slowly changing fields are measured in dc mode while periodic ac fields are measured in ac r.m.s measurement mode. The gaussmeter includes the zero probe function for use in compensating the zero offset of the probe or existence of small magnetic fields like the earth’s local field. For large magnetic fields, the use of relative mode is helpful especially in the case for simultaneous application of both dc and ac magnetic fields.
ix) Liquid Helium Level Instrument – This provides a continuous readout of the liquid helium level during the filling/refilling and measurement process. The level sensor is a fine superconducting wire in a non-conductive tube connected to the level monitor via electrical wiring through the vacuum feed-through. During operation a small current is conducted along the wire causing it to be resistive in the helium gas and superconducting in the liquid. This results in a voltage being established along the sensor that is proportional to the length of the wire above the liquid helium, and thus provides continuous measure of the helium depth.

x) Resistance Temperature Sensors – These are commonly known as RTD sensors that are based on the change of resistance with temperature. Two different types of sensor belong to this group, positive temperature coefficient (PTC) and negative temperature coefficient (NTC). A typical PTC is made from metallic material particularly platinum and has a fairly linear temperature resistance response. NTCs are normally semiconductors with very strong temperature dependence resistance and highly nonlinear response. Therefore NTC sensors are much more sensitive to temperature change and the most common types are germanium, cernox and ruthenium oxide. Cernox is a trade name for zirconium oxy-nitride produced by Lake Shore Cryotronics, Inc and is extensively used in cryogenic experiments because of its high sensitivity over a wider range and resistant toward induced errors in magnetic field environment. Measuring temperature with an RTD is most commonly done with two-wire or four-wire methods. However, the latter should be preferably used to avoid uncertainties associated with lead resistance as experienced by the former.

xi) Cryogenic Hall Sensors – This is a solid state sensor which provides an output voltage proportional to magnetic flux density. The sensor is a four leads device with current and voltage leads arranged in a manner based on the Hall effect concept. The Hall effect is the development of a voltage across a sheet of conductor when current is flowing and the conductor is placed in a magnetic field. A typical Hall sensor can be assembled in either axial or transverse configurations depending on the direction of magnetic field to be measured with respect to the sensor placement. With the use of a protective ceramic case as the housing for the Hall plate which is made of
A semiconductor material, the sensor is capable of being used at cryogenic temperatures. The Hall sensor is connected to the gaussmeter which outputs the field magnitude in the unit of interest and is regularly calibrated in a zero gauss chamber for accuracy.

The effort in developing any experimental system in any laboratory will not be fruitful without incorporating a reliable and versatile acquisition system. Data acquisition (DAQ) is the process of measuring an electrical or physical phenomenon such as voltage, current, temperature, magnetic field, pressure, frequency or sound. All of these are converted from the signal that originates from its respective sensor and instrument. With a standard computer and appropriate software, the acquisition process that comprises of hardware and software transforms the experimental system into a desired control and measurement system.

The research described here uses communication and control between the instruments and computer realized using an instrumentation bus under the name GPIB (General Purpose Interface Bus). It is an international standard governed by IEEE-488.1 and IEEE-488.2 in the United States and IEC 60625-1 and IEC 60625-2 internationally. GPIB is a byte serial, bit parallel bus that uses a three-wire handshake and can connect up to 15 instruments to one computer. There are three topologies (star, daisy-chain and mixed) to connect the multiple instruments to a computer and the daisy chain topology was employed here for reason of space and GPIB’s conductors length.

Figure 4.6 illustrates the daisy chain topology with each device corresponding to one measuring or electrical instrument responsible for signal acquisition from the chamber. The DAQ software in the communication process is called LabVIEW (Laboratory Virtual Instrumentation Engineering Workbench), a visual programming language from National Instruments. LabVIEW is a versatile and effective platform commonly utilized for designing and deploying measurement, test and control systems. It uses graphical icons and wires that resemble a flowchart and allows non-programmers to quickly configure different devices by dragging and dropping virtual representations of devices. In addition, many experimental instruments contain the LabVIEW driver software that facilitates the register-level programming or complicated commands to access the hardware functions. This
shortens the acquisition development time allowing more effort to be concentrated on the development of application software. The application software adds analysis and presentation capabilities to driver software. It is solely based on the desired acquisition process and pattern, the amount of time available to develop the application, and the complexity of the application. Obviously the transport critical current measurement program is less complicated compared to ac losses measurement due to the lower number of equipments involved.

![Figure 4.6: A GPIB topology with daisy chaining of the electrical/electronic instruments to the computer. The computer is installed with LabVIEW software to transform the assembly into a complete data acquisition, analysis, and presentation tool.](image)

**4.5 System Limitations and Solution Studies**

A complete photograph of the integrated cryogenic system for measurement of alternating current losses and transport critical current for practical superconducting wires and tapes is shown in figure 4.7. The system consists of an acquisition program and instrumentation entity that had been described in section 4.2 and 4.3. For any developed laboratory or
experimental system, there are always some disadvantages or limitations imposed by the system. However, compromises that will not severely influence the experimental objectives are usually available. The cryogenic measurement system shown in figure 4.7 is not an exceptional case and one of the restrictions is the narrow clear bore access. The size of the bore access is only 40 mm in diameter and measurement with sample length greater than 40 mm will need to be mounted in a vertical position with magnetic field direction parallel to its long surface. For transverse direction, the bore however could accommodate a sample length of 30 mm or less depending on the current leads design and dimensions. With appropriate techniques and analysis, such length is nevertheless sufficient in most of the sample characterization experiments.

Figure 4.7: Photograph of the integrated measurement system with data acquisition system for both transport critical current and ac losses measurements.

Since the superconducting solenoid coil contained by the cryogenic chamber caters only for dc applications, an additional coil needs to be incorporated for ac applications. The ac coil will be inserted into the clear bore access and consequently increase the space constraint. Furthermore the main concern that arises is the increase of heat load cause by the Ohm’s
law heating for high field applications. Therefore the magnitude of the magnetic field and the thermal stability of the system have to be considered accordingly to gain the optimum outcomes. Taking this into such consideration, and to increase flexibility, the ac coil is entirely made of high temperature superconductor with phenolic bobbin and the current leads are mounted onto a thermal sink made of oxygen free copper.

The sample current leads as had been mentioned in section 4.2, could only carry continuous current up to the maximum value of 200 dc ampere. This rated current is incapable of obtaining a broad transport characterization for practical superconducting conductor particularly in low field and temperature regions. Moreover the inevitable heating consequences from the current contact between the leads and sample are intolerable at this amount of current for short samples. Therefore the pulsed measurement method for transport critical current characterization has been developed and thoroughly investigated for its feasibility and reliability. This subject of interest will be discussed in detail in the following chapter.

4.6 Conclusion

The experimental system developed consists of the integration of hardware and software and is capable of measuring ac losses under various operating temperatures and magnetic field conditions. The applied magnetic field is achieved by having two sets of superconducting coils (for dc and ac respectively) wound in a solenoid form. One of the coils that is used for dc applications is made of low temperature superconductors (Nb₃Sn and NbTi) with attainable magnetic field up to 15 T and the other coil is made of high temperature superconductor (BSCCO) for ac magnetic field. These coils together with the tailor-made sample holder are assembled and housed in a multilayer superinsulated cryogenic chamber.

Various electrical and physical sensors, heaters and current leads responsible for measurement and characterization of practical superconducting wires and tapes are provided. The instrumentation system uses an instrumentation bus under the name GPIB for
control, acquisition, analysis and presentation purposes. The programming software that ties these processes is called LabVIEW. Besides ac losses, the built system also measures transport critical current for superconductors over a broad range of operating temperatures and magnetic fields.
CHAPTER 5
TRANSPORT CRITICAL CURRENT MEASUREMENT: PULSED CURRENT OF VARYING RATE COMPARED TO DIRECT CURRENT METHOD

5.1 Introduction

The rapid development of MgB$_2$ wires as a practical superconductor has been well documented over the last few years [Tomsic07, Penco07, Yao10, Wang09]. The transport current measurement is one of the most basic characterization techniques for the wire and is the most important measure of the wire’s capability. This measurement is usually made over a relatively short section (30mm-50mm) of a wire or tape, using a four-probe arrangement with voltage taps 5-10mm apart in the centre of the sample. Since practical MgB$_2$ superconductor consists of a metallic matrix or sheath on the outer perimeter, the transport current obviously has to pass through this resistive path from the current lead to the MgB$_2$ filaments. The transfer of current from the outer metallic material to the superconductor does not happen in a straight-forward fashion; instead the current penetrates over some length from the lead contact to the superconductor, depending on the physical and electrical properties of the sheath and current contact. This length of concern had its definition given by Ekin [Ekin78a, Ekin78b] as the current transfer length (CTL) and several researchers have studied experimentally CTL in Ag-sheathed BSCCO [Polak97] and MgB$_2$/Fe mono-core wire [Holubek05]. A more recent publication [Holubek07] related to CTL is also on MgB$_2$ wire but with different sheath materials in a higher field. A generalized transfer model described by the equation 5.1 has been developed [Wilson83, Lucas65]

\[
V_n = -E_{so} \lambda \frac{\sinh \left[ L - \frac{2x}{2\lambda} \right]}{\cosh \left[ L / 2\lambda \right]} \quad (5.1)
\]
where $V_n$ represents the voltage measured at the distance $x$ from the current joint, $E_{n0}$ represents the electric field at the current joint, $L$ is the length of the sample and $\lambda$ is the current transfer length.

The existence of contact potential and its gradient introduces a direct heat flux from the sheath matrix to the superconductor filament, which results in higher temperature on the superconductor itself. This is particularly true for measurements at low field region that usually involve a high current density. In addition, recently developed MgB$_2$ composite wires that consists of Fe, Nb, Ni or stainless steel as the sheath materials [Stenvall06, Martinez08] have higher resistivity than Ag or Cu sheaths, which are commonly used for high temperature superconductors. Therefore transport measurement of MgB$_2$ at low field region has always been a challenge for most systems in laboratories; either the sample is damaged by overheating or a large mass of copper current leads are incorporated in the system. Massive copper leads have large thermal conductivity and capacity, thus acting as thermal stabilizers. The latter requires space and a considerable amount of liquid helium or cooling power each time the measurement is performed. The inevitable heating effect on the current contact can only be reduced if the way of feeding the current through the contact is changed. This is a problem for all short sample measurement in particular for MgB$_2$ which has a sharp transition from superconducting to resistive state.

In previously published work [Horvat03, Horvat08], the pulsed current method was used in studies of magnetic shielding on MgB$_2$ superconductor and for comparison of transport and magnetic critical currents. The pulse was obtained by either discharging a capacitor or ramped using a programmable AC power source. The pulsed current method could not be used successfully for many high-temperature superconductors due to strong vortex relaxation effects. Vortex relaxation in MgB$_2$ is much weaker, which enables the use of the pulsed current method. Good agreement between the critical current ($I_c$) obtained by pulsed current and DC method was obtained if the DC method was used for measuring small $I_c$’s (in high fields) and pulsed method for large $I_c$’s (in low fields) [Horvat05a].

The upper limit of the current giving accurate DC measurements of $I_c$ depended strongly on the exact experimental set-up used. Continuous flow cryostats and sample holders with
large thermal mass enabled the use of up to 100A of DC current. The cryostats using low-pressure helium as heat exchange gas and sample holder with low thermal conductivity and capacity resulted in a temperature increase and deterioration of measured I_c even with DC current of 1A [Horvat05b]. However, large discrepancies between the pulsed current and DC method were always obtained at low fields and temperatures, where the values of I_c were large, unless the sample was in a liquid helium bath. This was ascribed to heating effects with the DC method. Further, the apparent values of I_c obtained by the pulsed current method seemed to depend on the rate of changing the current. Only low ramp rates of current provided values of I_c that were in agreement with the ones obtained by DC method. This warrants further research to ascertain which current ramp rates give acceptable accuracy.

The accuracy in the pulsed current method is inherently lower than with the DC method due to the background voltage induced in the voltage taps by a fast changing pulse of current. This necessitates the use of higher voltage ranges of the measuring instruments thus resulting in worse signal-to-noise ratio. Further, the effects of re-magnetization of the Fe sheath by the self-field of the current pulse need to be taken into account [Paasi97].

This work is extended here by studying the effects of the current ramp rate on the values of I_c. Also the CTL and associated heating effects are examined and more accurately described by numerical modeling. First the finite element method (FEM) analysis of the thermal distribution on the superconductor due to the contact potential and current transfer length that is described analytically by equation (5.1) is presented. Then, a direct experimental comparison between the DC and pulsed current methods is presented. The advantages and conditions under which the pulsed method can provide the true transport current are identified.

5.2 Finite Element Method (FEM) Analysis

A FEM model of the transport measurement was constructed by a combination of the heat transfer and the conductive electric current partial differential equations coupled in two
dimensions. The simplified geometry of the tape sample on its holder that is used in the model is shown in figure 5.1. It comprises of an oxygen free high conductivity (OFHC) copper holder, a sheath matrix element and an MgB$_2$ filament. To simplify the modeling, the contact potential and the CTL’s effect was assumed the same for every filament present in a composite wire. The thickness of the matrix layer was taken as 0.2 mm and that of the superconducting filament as 0.1 mm. The electric conductivity of the MgB$_2$ for current exceeding $I_c$, $\sigma_{sc}$, is represented by a power law relationship:

$$\sigma_{sc} = \frac{1}{\rho_{sc}} = \frac{1}{n^{-1}} \left( \frac{E_c}{J_c} \right)^n + \rho_0$$

(5.2)

where $E_c$ is the critical electric field, usually taken as 1 $\mu$Vcm$^{-1}$, $J_c$ is the critical current density, $n$ is the power law index and $J$ is the applied current density. The resistivity bias term, $\rho_0$, is used for simulation stability with the value of $10^{-18}$ $\Omega$m in the model.

Figure 5.1: Geometry used in the FEM model, consisting of the three layers of elements shown in the inset.

The electrical conductivity, thermal conductivity and heat capacity of the OFHC copper was taken from the literature [Iwasa94] at the temperature of interest. These parameters were considered to be independent of the temperature for the copper since the copper slabs are the current leads that contribute most of the thermal mass to the system. In contrast, the temperature dependence of these parameters was taken into account for the sheath matrix,
as obtained from the fit to experimental data [Kemp55]. The thermal properties of MgB$_2$ were published in [Bauer01] and again we employed curve fitting of the experimental data for our simulation model.

In order to demonstrate the heat progression in the conductor, we first solve the evolution of potential along the surface of the sheath material in the FEM model, as shown in figure 5.2. The CTL is obtained from this figure, as the length over which the electric potential decays from its maximum value (at the copper holder) to zero. The inset illustrates the potential distribution in the vicinity of the contact. The typical input parameters used for MgB$_2$ in the simulated distribution are listed in table 5.1.

Figure 5.2: Electric potential along the wire behind the current contacts ($x < 0$ m and $x > 0.02$ m) and along the sample between the contacts ($0$ m $< x < 0.02$ m). The two copper blocks at which the electrical contacts to the sample are made at $-0.01$ m $< x < 0$ m and $0.02$ m $< x < 0.03$ m. Inset shows the potential distribution along the sheath material from the contact to the voltage level of negligible resistivity.
Table 5.1: Physical parameters used to simulate the superconductor in FEM model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical current density</td>
<td>$J_c$</td>
<td>$1 \times 10^8$ A/m$^2$</td>
</tr>
<tr>
<td>Critical electric field strength</td>
<td>$E_c$</td>
<td>$1 \times 10^{-4}$ V/m</td>
</tr>
<tr>
<td>Power law index</td>
<td>$N$</td>
<td>30</td>
</tr>
<tr>
<td>Critical temperature</td>
<td>$T_c$</td>
<td>38 K</td>
</tr>
<tr>
<td>Current density</td>
<td>$J$</td>
<td>$7 \times 10^7$ A/m$^2$</td>
</tr>
</tbody>
</table>

For a different transport current (current density), the potential distribution will be similar but with a different value of voltage and a slight variation in the current transfer length, until the critical current is reached. Then, the filament becomes highly resistive. With these distributions, we investigated the heat transfer from the contact to the filament with respect to time. The equation incorporated in the model is represented as follows:

$$\rho C_p \frac{\partial T}{\partial t} + \nabla \cdot (-k \nabla T) = Q_{dc}$$

(5.3)

where $k$ is the thermal conductivity, $C_p$ is the specific heat capacity and $\rho$ is the material density. $Q_{dc} = \bar{E} \cdot \bar{J}$ is the heat source term that couples to the conductive current equation as represented below:

$$\nabla \cdot (-\sigma E - \bar{J}^\ast) = \nabla \cdot (\sigma \nabla V - \bar{J}^\ast) = 0$$

(5.4)

where $\sigma$ is the electric conductivity of the material, $V$ is the electric potential, $E$ is the electric field strength, and $\bar{J}^\ast$ is the current density induced by the change of magnetic field, which is always set to zero in the model.

Figure 5.3(a) shows the simulation of the temperature variation along the sample for different times after a DC current was switched on. The critical current density $J$ was taken as $10^8$ A m$^{-2}$ and the figure shows the results for 0.7 $J_c$. The base temperature is 20 K. The temperature is the highest at the current contacts to the sample. The voltage is measured in the middle of the sample, where the temperature increase above the base temperature is the lowest. Nevertheless, there is a significant temperature increase at the time-scales of the DC
measurements. The temperature in the middle of the sample increases by 0.264, 2.624 and 3.015 K for times of 1 ms, 10 ms and 1 s, respectively at \( I/I_c = 0.7 \). The temperature increase is even larger as the sample current approaches \( I_c \) (figure 5.3(b)). There is a strong increase of temperature as the current exceeds \( I/I_c = 0.8 \), which will probably cause a quench. The value of \( I_c \) decreases with temperature and lower currents are used in the measurements at higher temperatures. This will give smaller heating effects. For examples, figure 5.4 and figure 5.5 showed the temperature changes along the sample as in figure 5.3 but with base temperature of 25 K and 30 K respectively. The temperature variation has a similar trend as before but with a smaller temperature difference for the same time steps and fraction of \( I_c \).

Figure 5.3: a) Simulated temperature evolution in MgB\(_2\) filament with respect to time due to heating by the current contacts, for \( J/J_c = 0.7 \). The sample ends are at 0 and 0.02 m, respectively. Each curve represents the temperature at a time step of 0.001s and the last (top) curve gives the temperature at 0.01s after switching the DC current on. The input parameter in the simulation for critical current density is \( 10^8 \) A/m\(^2\), giving the density of \( 7 \times 10^7 \) A/m\(^2\). The base temperature is 20K. b) Increase of the temperature at the middle of the sample vs. the sample current normalized to its \( I_c \), at 1ms and 1s after the DC current was switched on. The base temperature is 20K.

These simulations show that \( I_c \) cannot be measured accurately with the DC method at low temperatures and fields, where the values of the critical current are high. The sample will quench due to the heating at the current contacts before the current reaches \( I_c \). The pulsed
current method with a pulse duration of 1 ms is going to suffer from only slight heating effects, giving a more reliable value of $I_c$.

**Figure 5.4:**  a) Simulated temperature evolution in MgB$_2$ filament with respect to time due to heating by the current contacts, for $J/J_c = 0.7$. The physical descriptions of the figure are similar to figure 5.3, except the base temperature is 25K. b) Increase of the temperature at the middle of the sample vs. the sample current normalized to its $I_c$, at 1ms and 1s after the DC current was switched on. The base temperature is 25K.

**Figure 5.5:**  a) Simulated temperature evolution in MgB$_2$ filament with respect to time due to heating by the current contacts, for $J/J_c = 0.7$. The physical descriptions of the figure are similar to figure 5.3, except the base temperature is 30K. b) Increase of the temperature at the middle of the sample vs. the sample current normalized to its $I_c$, at 1ms and 1s after the DC current was switched on. The base temperature is 30K.
The heating at high temperatures and fields is much smaller, giving only a fraction of a Kelvin temperature increase using the DC method. The accuracy of the model has been verified by placing a temperature sensor directly in contact with the composite wire and feeding the sample with a DC source at $1 \text{ A s}^{-1}$. An abrupt increase of temperature was observed when the applied DC current was close to $I_c$ of the sample.

### 5.3 Experimental Details

The main results of this study are based on pulsed current measurements of numerous samples published since 2002 [Horvat05b]. They included pure, nano-SiC and carbon nanotube doped MgB$_2$ samples, sheathed in Fe, Cu and combination of both. These measurements were performed with slow current ramp rates, which gave a very good agreement with the DC measurements at high field or temperatures. However in this chapter two samples were used, for which a systematic set of measurements was performed specifically to point to the limits of the pulsed current method and its accuracy when used with MgB$_2$ wires. Two different commercially available MgB$_2$ composite wires were used in this study.

Figure 5.6: Transverse cross section view of (a) Columbus Superconductors tape and (b) Hyper Tech wire. The Nb chip, produced when cutting the wire, covers part of the round MgB$_2$ core.
One of the samples from Columbus Superconductors has a rectangular cross section of 0.65 mm $\times$ 3.6 mm width. It contains 99.5% pure nickel matrix and the core is made up of 99.5% OFHC 10100 Copper. In order to avoid the diffusion of the copper inside the 12 MgB$_2$ filaments a 99.5% pure iron barrier is used. The filling factor is 9 % and the ratio between copper and superconductor is 1.67. The second wire was made by Hyper Tech Research Inc. and it has a circular cross section and matrix elements with fabrication details presented elsewhere [Hossain09]. The wire cross section is 0.83mm in diameter with Nb barrier and Monel as sheath material. Figure 5.6 shows the transverse cross section view of the Columbus Superconductors’ tape and Hyper Tech’s wire. All experimental results obtained were in agreement with measurements performed on MgB$_2$ samples made for over the last 10 years, adding the weight to the results presented here for the two samples of different origin.

The MgB$_2$ samples were mounted onto two oxygen free high conductivity (OFHC) copper blocks, which served as a heat reservoir and low-resistance current leads to the sample. Two ends of the sample were soldered onto two separate copper blocks, with the soldered length of the sample being 10 mm. The sample length between the two copper blocks was 20 mm. The other ends of the copper blocks were soldered directly to the high-temperature superconductor (HTS) leads that extend 2m down to the chamber, positioning the sample in the middle of a DC superconducting magnet. The magnet system explained in chapter 4 is capable of producing a magnetic field up to 15T and is constructed with a continuous flow of pre-heated helium gas in the sample chamber, allowing the control of temperature to better than 0.1K. An additional heater was on the sample holder for better temperature control. The magnetic field was perpendicular to the tape plane and to the round wire.

Two pairs of voltage leads were soldered 5mm apart on the wire or tape with total length of 35mm. One of the pairs served as the voltage probes for DC transport measurement and was attached to a nano-voltmeter outside the chamber. The other pair was used for pulse transport measurement of voltage, simultaneously with the pulsing current using a 2-channel digital oscilloscope. The current through the circuit was measured via the voltage drop on series connected resistive elements that were calibrated to give precisely 0.001
Ohm. Since both channels of the oscilloscope had the same ground, the voltage taps had to be decoupled from the current loop to avoid creation of ground loops and parasitic voltages in the system. This was achieved by first feeding the voltage signal to a transformer preamplifier that amplified the voltage 100 times, thus improving the sensitivity of the experiment as well as decoupling the voltages. Figure 5.7 shows the experimental arrangement.

![Figure 5.7: Schematic showing principal of experimental setup for transport measurement with DC current source and modified pulse current source.](image)

A Pacific Power AMX3120 programmable AC current source was used to generate the required waveform of the current through the sample. The duration of the waveform could be adjusted. One cycle of a triangle waveform was edited to a sawtooth form, which provides a linear increase of current with time. Figure 5.8 shows typical voltage vs. current plots for three different ramp rates of the current. A background voltage is induced in the voltage tap by the time-varying current, proportional to dI/dt. At I = Ic, there is an apparent increase of voltage. Ic was obtained as the current where the increase of this voltage starts deviating from the induced voltage plus noise, as described in [Horvat03]. The voltage
criterion for obtaining $I_c$ was not as significant an issue as for high-temperature superconductors, since the voltage increases quite abruptly at $I = I_c$ for MgB$_2$, except at high fields and temperatures. The thus obtained $I_c$ apparently increased with $\text{d}I/\text{d}t$, which cannot be explained simply by the increase of the induced voltage in the voltage taps and consequent decrease of sensitivity of the measurements (figure 5.8).

![Figure 5.8](image.png)

**Figure 5.8:** A typical snapshot of voltage vs. current waveforms for MgB$_2$ composite wires, for three different ramp rates of the current. The temperature and field were 20K and 2T respectively.

The maximum peak current was limited to 200A with the available programmable current source and the rate of change of current could vary from 16kA/s to 800kA/s. A bipolar power source from Cryomagnetics was used as the current source for the DC transport measurement. The current ramping rate was fixed at 0.5A/s for currents up to 60A and 1A/s thereafter. The critical current, $I_c$ was obtained as the current giving a voltage of $1\mu\text{V/cm}$ of sample length. All measurements were carried out on the composite wires at different temperatures and applied magnetic fields.
5.4 Results and discussions

The field dependence of transport critical current $I_c$ for the multi filamentary tape (from Columbus) at 30K with DC and several ramp rates of the current are shown in Figure 5.9. In the figure, the legend shows the rates at which the pulse current is applied to the wire. The inset indicates the transverse field direction on the composite tape. The field dependence of $I_c$ for the same wire at 20K is shown in Figure 5.10. There is a good agreement between the DC and pulsed measurements up to a certain current ramp rate at low temperatures and fields. At higher fields or temperatures, there is an increased dependence of the measured $I_c$ on the current ramp rate. The same results are also shown for a round wire (from Hyper Tech) in Figures 5.11 and 5.12 for 30 and 20 K, respectively.

![Figure 5.9: Field dependence of $I_c$ at 30 K for MgB$_2$ tape conductor with field perpendicular to the width. Generally good agreement is found between the DC transport measurement and pulse current measurements up to about 120 kA/s.](image)

The round (Hyper Tech) wire had a smaller $I_c$ than the tape (Columbus) and it was difficult to obtain the measurements at high ramp rates due to the large voltage induced in the
voltage taps. The difference between the $I_c$ obtained by pulsed current method at ramp rates of 16 and 80 kA/s is small, especially at 20K. For the round (Hyper Tech) wire, however, the DC measurements gave the value of $I_c$ that deviated from the pulsed measurements already for currents exceeding 20A, regardless of the current ramp rate. This was different from what was obtained for the tape (Columbus) sample, where reasonably good agreement with pulsed measurements at small ramp rates was obtained up to about 80A. The difference between the results for the two samples is due to the heat transfer from the current contacts to the sample to the OFHC copper blocks.

In particular, the Columbus tape had much larger contact area with the OFHC copper blocks than the Hyper Tech round wire due to geometrical differences. This highlights the importance of transferring the heat produced on the current contacts to high thermal capacity and conductivity heat reservoirs, i.e OFHC copper blocks. If lower purity copper blocks were used, in low-pressure helium as a heat exchange gas in the cryostat, the DC measurements would give substantially lower $I_c$ than the pulsed current at currents of 1A [Horvat05a].
Figure 5.11: Field dependence of $I_c$ at 30K for MgB$_2$ wire conductor with field perpendicular to the wire axis. A low value of $I_c$ (high field region) is easily obtained from DC transport current while a high value of $I_c$ is more accurately obtained with a pulse of rate up to 16kA/s.

Figure 5.12: Field dependence of $I_c$ at 20K for MgB$_2$ wire conductor with the field perpendicular to the wire axis. The drop-off of DC transport $I_c$ at low field region indicates the significance of the heat load from the contact leads.
Because the pulsed current method does not suffer from the heating effects, it is expected to provide more accurate values of \( I_c \) than the DC method at fields and temperatures for which the \( I_c \) is large enough to cause substantial heating for a particular experimental set-up. However, the results show that the pulsed current \( I_c \) is dependent on the rate of the current change (figures 5.8-5.12). This difference in the obtained \( I_c \) is partly due to the voltage induced in the voltage taps by the fast changing current through the sample. Larger current ramp rates give large induced voltages in the voltage taps, requiring the use of large sensitivity range of the oscilloscope that measures the instantaneous voltage. This lowers the signal to noise ratio, making it difficult to distinguish the current at which the voltage just starts increasing beyond the induced background voltage. This effect is especially pronounced at high fields and temperatures, where the V-I characteristics are rounded at \( I = I_c \). However, the measured \( I_c \) increases with the current ramp rate even at low fields and temperatures, where the voltage increases very abruptly with current at \( I_c \). This is not due to the measuring electronics because otherwise, the abrupt increase of voltage at \( I_c \) would be distorted into a more rounded shape. Hence the vortex dynamics is suggested to also play a role in the pulsed current measurements. The change of the vortex profile in the sample with fast current pulse can become so rapid that the effects of thermal excitation on the vortex profile become negligible. This would lead to the measured \( I_c \) being larger than in the DC measurements. Such a scenario is supported by the measurements (figures 5.9, 5.10), where the discrepancy between the DC and fast pulsed measurements increases as the importance of thermal excitations increases, i.e. at higher temperatures and fields.

These effects raise the question of the accuracy of the pulsed current method itself. To obtain the value of \( I_c \) that would correspond to DC conditions, the pulsed \( I_c \) is measured at each temperature and field with several different pulsed current rates, \( dI/dt \). Extrapolating the obtained values of \( I_c \) to \( dI/dt = 0 \) should correspond to \( I_c \) obtained with DC method. Figure 5.13 shows the dependence of the pulsed \( I_c \) on \( dI/dt \) for the tape sample, with two different combinations of field and temperature. The pulsed \( I_c \) depends linearly on \( dI/dt \), making it easy to extrapolate to \( dI/dt = 0 \). This gave a value of \( I_c \) which was still larger than the \( I_c \) obtained with the DC method (open symbols in figure 5.13) due to heating effects in the DC method, as also predicted by the modeling (figures 5.3-5.5). This difference
between the $I_c$ obtained by pulsed and DC methods decreases as the value of $I_c$ decreases, so that at 2T and 20K it is within the experimental uncertainty of the pulsed current method. The uncertainty of each of the pulsed measurements will depend on the sample used, $dI/dt$, the magnetic field, the temperature and the resistivity of the metallic sheath of the sample. High quality samples with iron sheath will have a very abrupt increase of voltage at $I = I_c$ [Horvat05b] at low temperatures and fields. A measurement uncertainty that is better than 5% can be obtained under these conditions. If, however copper sheath is used, the voltage will increase more gradually at $I = I_c$. Similar V-I characteristics will be also obtained at large fields and temperatures due to the vortex dynamics. With such V-I characteristics, measurement uncertainty will be much worse and it can exceed 15 %, depending on the exact combination of the above factors.

![Figure 5.13: Current ramp rate dependence of the measured $I_c$ for the pulsed current measurement method for the MgB$_2$ tape conductor, with the applied field perpendicular to the tape face.](image)

The $n$-value of V-I characteristics ($V \propto I^n$ for $I > I_c$) is another important parameter. However, the V-I characteristics are also affected by the ramp rate of the current. The
correct value of \( n \) can be obtained by extrapolating each point of the V-I characteristics to \( \frac{dI}{dt} = 0 \).

5.5 Conclusion

The transport critical current for MgB\(_2\) superconducting wires is one of the most important parameters that must be known before employing the wires for practical use. In this chapter, a new flexible and practical method of obtaining this essential parameter over a wide range of applied magnetic fields, including the self-field condition is presented. Values of \( I_c \) were easily obtained for both MgB\(_2\) composite tape and wire from two different companies and comparisons were made between conventional DC measurement and the new method. It was shown that employing the pulsed current method with a suitable rate of current ramp (\( dl/dt \)) enables measurements of \( I_c \) in substantially greater range of fields and temperatures in comparison to the DC method. The acceptable range of \( dl/dt \) should be obtained for each measuring system and for each sample. The FEM simulations were performed to provide an estimation of the inevitable direct heat flux from the contact potential and its gradient. It was shown that the increase of temperature at high values of transport current caused by conventional DC method is very likely to increase the sample temperature and therefore give an underestimate of \( I_c \). This is more pronounced if the superconducting wire has large critical current of hundreds of amperes. The pulsed current method incorporating measurements at different \( dl/dt \) provides a way of avoiding these problems. The best way to accurately measure \( I_c \) is to extrapolate the values of \( I_c \) measured at different \( dl/dt \) to \( dl/dt = 0 \). The thus obtained \( I_c \) corresponds to true \( I_c \), without the effects of heating (introduced in DC measurements) or effects of vortex dynamics (introduced in pulsed measurements).
CHAPTER 6
INNOVATIVE CALORIMETRIC AC LOSS MEASUREMENT OF HIGH TEMPERATURE SUPERCONDUCTOR IN PERPENDICULAR APPLIED MAGNETIC FIELD WITHOUT TRANSPORT CURRENT

6.1 Introduction

This chapter details the investigation into calorimetric AC loss measurement on short HTS tapes. It describes the design, operation and results of an experimental calorimeter. The cryogenic chamber and supporting frame has already been well describe in Chapter 4 and will be incorporated along with the measurement technique here to investigate the losses under applied transverse magnetic field at various temperatures. With the recent significant progress in HTS practical applications particularly in the area of fault current limiters, the results at the end of this chapter will be of great assistance in the development of such devices.

In the past few years, the superconducting fault current limiter (SFCL) has been used in power grids. Generally, SFCL’s can be divided into two primary classes based on their working principle, the resistive limiter and the inductive limiter. Both of these, however, have to cope with the quenching of the superconductor and thus can suffer from troublesome long recovery times. The solution was first suggested by B. P. Raju et al. in early 1980’s [Raju82] by introducing a superconducting dc bias. Later developments included a discussion in 1998 by T. Verhaege and Y. Laumond [Verhaege98] on a saturated iron core concept for a fault current limiter (SICFCL). SICFCL does not need the quench of superconductivity to create sufficient impedance for fault current limiting and the only superconducting portion is a dc coil which is used to supply permanent ampere turns to magnetize the iron cores. The novel concept of this design has been extended in recent simulation work for Medium Voltage distribution systems by S. B. Abbott et al. [Abbott06]. Despite the growing interest in this application, AC loss studies on superconductors subject to similar condition have yet to be done, particularly in the
temperature range of interest. This is unfortunate as accurate knowledge of AC loss from such a device is necessary for optimizing the dimensions and the cryogenic costs of the superconducting part. The superconducting coil in a SICFCL will be simultaneously exposed to both DC self field and AC magnetic field from the AC winding. Hence, the experimental conditions considered in this chapter will be set to resemble the conditions in a SICFCL in order to allow an accurate estimation of the AC loss. It will include both the experimental data and will be supported by theoretical studies which are demonstrated to provide a close approximation of the actual losses. This will begin with the worst case condition: i.e. the sample will be exposed perpendicularly to both DC and AC external magnetic fields at the intermediate temperatures of 30 K, 40 K, and 50 K which is within the typical range for practical applications of superconducting materials.

The vigorous development of the (Bi,Pb)$_2$Sr$_2$Ca$_2$Cu$_3$O$_{10}$ (Bi-2223) superconductor by various research groups has resulted in significant progress in fabricating a conductor with high critical current and a long length by use of the silver sheath method. Hence, the commercially produced Bi-2223 tape provided by Sumitomo Electric [Ayai06] is used throughout this chapter. The existence of a variety of calorimetric techniques [Magnusson98, Magnusson00] to determine the ac losses have been thoroughly discussed in Chapter 4. Most of these are carried out in a liquid nitrogen bath at 77 K with either transport current or applied field only, or with both simultaneously. In the measurement technique used here a new calorimetric apparatus for short samples (2.5 cm) with adequate accuracy of AC loss measurement has been developed.

6.2 Experimental Procedure

The measuring system consists of two parts, the loss measurement and the calibration system. The measurement technique used for losses is to detect the temperature increment on the sample by using commercially available Cernox’s RTD sensor. This sensor measured the change in resistance that corresponds to a change in temperature. There is no need to calibrate the sensor because only the changes of the resistance with the calibration
system need to be acquired and compared with the loss measurement. The superconducting magnet cryostat system that is used in this measurement system has been well described in figure 4.2.

The sample is placed inside the sample holder with transverse field direction from both DC and AC superconducting magnetic coils. To achieve a variable temperature, the bottom and top heater have to be varied accordingly and at the same time the throttle needle valve on the top flange needs to be adjusted. This valve serves as a channel between the helium reservoir and the sample space. The sensor on the respective heater shows the temperature in the chamber and hence measures the temperature stability inside the sample space. The current leads to both DC and AC coils are made of high temperature superconductor to reduce the joule heating that could affect the sensitivity of the measurement. The sample holder is made entirely from G10 phenolic material to avoid any induced current from the applied AC field as has been discussed in section 4.2.1. Figure 6.1 shows the sketch and photograph of the AC solenoid (6.0 cm in height and an inner diameter of 3.0 cm) with the sample placed inside the insulating bobbin. Apart from supporting the solenoid, the bobbin is also used to enclose the sample to restrict any heat transfer.

![Image](image_url)

Figure 6.1: The sample is placed symmetrically inside the AC superconducting coil with G10 phenolic sample holder. The coil is separated from the sample space by G10 insulating bobbin.
Included in the package that is not shown in Figure 6.1 is a small heater coil made of high resistance wire and temperature sensors. The uniqueness of this measurement system is that the calibration method does not need the sample to be in contact with the current lead or any conducting elements. This is to eliminate the end effects associated with short length samples and also the uncertainties of the thermal properties of the materials involved (if any) which are usually poorly known at low temperatures. The calibration process is done in exactly the same configuration as the ac loss measurement except for the changing of current lead between the ac and dc power supply. This change will not affect the result as it only takes place outside the insulated sample space. Figure 6.2 shows the schematic arrangement and placement of the sensors (temperature and gauss), heater and sample. The heater is wound around a thin layer of nylon and is placed symmetrically around the sample. Heat loss from the heater to the ambient is restricted as it is enclosed between the G10 bobbin and the nylon layer. As the sample is closest to the heater, the total heat or energy produced will be instantaneously measured by the sensor. This is because the heating wire has extremely low heat capacity at low temperature.

![Schematic sample holder, heater and sensors circuit.](image)

Figure 6.2: Schematic sample holder, heater and sensors circuit. The heater is wound axial symmetrically around the sample and properly sealed to ensure the total heat produced is measure by the sensor. The inset shows the photograph of the set-up.
Two sensors are used to measure the changes of temperature from the sample and the insulated region. The sample is again thermally insulated in the centre region with a block of G10 material, thereby producing a measurable and sensitive temperature change under the applied field condition. As the calibration and the loss measurements are made at the same temperature, the sensors need not to be calibrated because the sensitivity of the sensors (Ω/K) varies according to the change of temperature. Before taking the measurement, the sample is replaced with a dummy insulating material to investigate the response of the sensors under the applied field condition and to measure any additional heat flow in or out of the system.

6.3 AC Solenoid Coil Characteristic

The purpose of this work is to determine the AC superconducting coil performance by measuring the magnetic field generated in the center of the coil as a function of the current flowing in it. Concurrently the voltage drop at the terminals of the coil is also measured to allow the amount of energy or heat dissipated to be known at the lead contact during the application of current. This is useful as the heat generated might influence the measurement results or quench (overheat) the field coil during operation. The solenoid coil has 8 layers wound around the bobbin in total with 12 turns in the first 7 inner layers and 9 turns in the outer layer which give a total of 93 turns. The inner diameter of the coil with the bobbin is 30 mm wide and 33.4 mm without the bobbin (inner diameter winding). The outer diameter of the winding is 39.1 mm and the height is 60 mm in length. The coil’s wire is made of first generation Bi-2223 from American Superconductor and possessed critical current of approximately 148 A. The wire’s dimensions are 4.3 mm wide and 0.27 mm thick with 8 microns thick paper insulation wrapped around them. The coil is vacuum pressure impregnated (VPI) with 4 component epoxy resin and total length of wire used is approximately 10.6 m. Figure 6.3 illustrates the terminal voltage drop and magnetic field at the center of coil as a function of coil current.
Figure 6.3: Voltage and magnetic field measurement at 77K for the AC solenoid superconducting coil. The voltage was measured at the end terminals by applying the DC current and magnetic field generated in the center was measured by a Hall effect probe.

The voltage taps were soldered to the end terminals of the coil and DC current was applied to measure the voltage drop with a multimeter. The magnetic field generated by the coil in the center was measured with a Hall probe and recorded by a gaussmeter. The measurements were taken in a liquid nitrogen open bath (77 K) to provide a considerable margin for the AC losses measurements which will be performed at a lower operating temperature. This has also to protect the coil from overheating and subsequently from any further damage. It is desirable to keep the voltage drop over the coil as low as possible throughout the operation and the data from figure 6.3 shows that the 1 mV limit is the maximum dissipation that the coil could handle. Moreover the maximum ac(r.m.s.) field that is required for the losses measurement is below 140 mT.

6.4 Results and Discussion

The silver sheathed Bi-2223 tape with dimensions 4.3 mm width and 0.23 mm thickness was cut into 2.5cm length and loaded into the sample holder as described above. The sample was first cooled down to liquid helium temperature (~4.2K) and then heated to
measurement temperatures by using the bottom and top heater. The power supply to the
heaters is manually controlled by the temperature controller and it determines the level of
stability at the desired temperature. A good stability will have a temperature variation of
0.3K or less at the set point. With good thermal insulation, the sample temperature will vary
steadily and slowly until it reaches equilibrium with the heat transfer within the heater and
the helium input from the reservoir. Since the heaters and the amount of helium input can
be varied, the system is capable of achieving any desired temperature from 10K to 100K
with reasonable stability.

Once the sample stabilizes at the respective temperature, pulses of ac current with variable
frequency, amplitude and duration can be applied through the ac solenoid coil to generate
the magnetic field. Results of ac loss for applied sinusoidal field at 50Hz with and without
DC background field will be presented. It is noted that the field is applied in the
perpendicular direction to the tape flat face. The response of the temperature sensors in the
applied field without the presence of the sample shows no observable temperature changes
and hence gives confidence that no error is introduced during the ac loss measurement.
Figure 6.4 shows the loss measurement conducted on the sample as a function of the
magnetic field at 0T, 0.5T and 1.0T DC background field at 40 K. The resistance value
corresponds to the temperature rise on the sample and is directly related to the ac losses of
the HTS.
In exactly the same way and arrangement as the loss measurement, a dc current is passed through the heating coil for 60 seconds and the changes of the sensor reading are recorded for calibration purposes. The total power supply to the heating coil is measured by placing an external resistive shunt connected to the dc circuit and the voltage measured across the heating coil terminal inside the chamber. During the calibration experiment, the sample is in the superconducting state and not exposed to any external applied magnetic field. In doing so, all the changes from the sensor will be from the calibration coil alone. Figure 6.5 contains a typical set of calibration data at 40K with the dissipation shown in J/m. The calibration constant is obtained from the changes in temperature measured by the changes in resistance. The calibration constant as calculated from the slope is 6 (mJ/m)/ohm.
The results shown in figure 6.6 are the ac losses in J/m/cycle obtained from the calibration constant. The figure also shown the result extracted from Sumitomo [Ayai06] at 77 K without applied DC field for comparison. At higher fields, the losses are lower for the case with applied DC background field as opposed to the lower applied field. When applied to a DC background field, the critical current density of the superconductor is reduced and consequently affects the loss behavior depending on the critical field.

To better understand the behavior, theoretical expression from equation 3.46 (also referred as Brandt curve) are plotted with three different critical currents, $I_c$. These values are corresponding to the critical properties of the tape at temperature 40 K and with DC field of 0.5 T and 1.0 T normal to the tape surface. The critical magnetic field $H_c$ is obtained from the relation $H_c = J_c / \pi$ and using $\bar{B} = \mu_0 \bar{H}$, the critical magnetic field at 40 K is calculated as 43.4 mT. It is to be noted that the critical current of the tape is 434 A at 40 K without background field and width 4.3 mm. Figure 6.6 shown the cross-over of the measured value of losses at this applied magnetic field and according to equation 3.47, the
Hysteresis losses are in inverse proportion to the magnitude of critical current for low applied field and direct proportion relation for high applied magnetic field. This behavior has actually been reported by Y. Fukumoto [Fukumoto95] in the past but with a longitudinal applied magnetic field instead. Apart from having much higher ac losses with perpendicular alignment, the losses behavior exhibited similar trends as discussed and in accordance to critical state model [Wilson83].

Figure 6.6: AC losses as a function of the external transverse field rms at 40 K and frequency 50 Hz with different DC background magnetic field. The result at 77 K from Sumitomo without background field is also shown for comparison. Theoretical curve for respective $I_c$ at 40 K with applied perpendicular field of 0 T, 0.5 T and 1.0 T are plotted. The solid lines connect the points obtained by theoretical expression.

Using exactly the same measurement and calibration techniques, the experiment is repeated for temperature at 30 K and 50 K with DC magnetic field applied perpendicularly to the wide surface as before. The results along with the theoretical curves are presented in figure 6.7 for 30 K and figure 6.8 for 50 K respectively. The critical currents of the tape are shown
inside the individual legend except for the self field condition at temperature of 30 K. This is because the critical current value is near or above 500 A at this state and for the reason of thermal consequences, no actual data is available.

![Figure 6.7: AC losses as a function of the external transverse field rms at 30 K and frequency 50 Hz with different DC background magnetic field. Theoretical curve and results from VIM method for respective $I_c$ at 30 K with applied DC transverse field of 0.5 T and 1.0 T are plotted. The solid lines connect the points obtained by theoretical and numerical calculations.](image)

In figure 6.7, the results from variational iteration method are included for the case of $I_c = 252$ A and 210 A that correspond to the applied field of 0.5 T and 1.0 T. The measured losses are close to the calculated values and exhibited similar trends throughout the applied magnetic field region. The differences are due to the finite thickness of the superconducting tape and the irregularity of the critical current along the conductor. Figure 6.7 also shows that the calculated VIM results lie in between the experimental measured losses and the thin strip limit curve from equation 3.46 (Brandt). This is reasonable considering the fact that the tape is treated as a two dimensional problem and with voltage-current relationship
defined by the n-value. Note that for losses below $10^{-6}$, experimental data are not obtainable due to the limited sensitivity and high fluctuations of the sensor reading in this range. However, the results are excellent over most of the range of values likely to be experienced for practical HTS applications.

Figure 6.8: AC losses as a function of the external transverse field rms at 50 K and frequency 50 Hz with different DC background magnetic field. Theoretical curve for $I_c$ at 30 K with applied DC transverse field of 0 T, 0.5 T and 1.0 T are plotted. The results from VIM method are shown only for $I_c = 343$ A. The solid lines connect the points obtained by theoretical and numerical calculations.

Similar to figure 6.7 is figure 6.8 that presents the same type of losses as a function of the applied transverse field but at a temperature of 50 K. The ac losses calculated by VIM have the same trend as before and for this reason figure 6.8 only shows the results for 0 T DC background field so as to allow clearer representation of the losses. At the temperature of 50 K, the critical field $H_c$ is obtained as 34.3 mT and for low applied magnetic field, the loss is proportional to $J_c^2$ which means that the tape with lower critical current will have
higher losses instead. This is again clearly shown in the loss figures above for operating
temperature at 30 K, 40 K and 50 K. The consideration of eddy current loss in the sheath
material of this tape which is silver is taken to have negligible effect at this frequency and
the main ac loss component comes from the superconducting portion of the tape supporting
the hysteresis loss-type behaviour. The eddy current loss strongly depends on the cross
sectional ratio of the superconducting core to the normal matrix as well as the operating
frequency [Ishii96, Fukumoto95] and become dominant only if the ratio is much smaller
than 0.4. On the other hand, the coupling loss contribution in this geometrical arrangement
might explain the differences between the measured losses and the calculated value from
VIM. The study of the coupling current loss involves the detailed investigation of
geometrical coefficient, coupling relaxation time, effective electrical resistivity between
filaments and twist pitch length. However the study on this loss portion is not in the scope
of the present thesis. Moreover, almost every application of HTS conductor focuses on the
total ac loss, which is the major concern for device designers.

6.5 Conclusion

The apparatus presented for ac loss measurement has proved to be capable of giving results
for superconducting wire exposed to conditions that have rarely been investigated
previously. This technique in principle provides reasonable accuracy over wide ranges of
temperatures, magnetic fields and frequencies. The technique is appropriate for short
samples at approximately 30 mm in length or less, and provides a good measure of the
performance of newly developed or prototype conductors. The small volume of the
chamber offers great stability in temperature and homogeneity in applied magnetic field.

The data presented in section 6.4 are consistent with the anticipated losses as described in
the theoretical curves and calculation that have been thoroughly discussed in Chapter 3.
While the results do not match precisely, the differences are within the predictable range
and therefore can provide a good approximation for design purposes. These results show
that the theoretical method provides a useful, accurate and versatile approach to estimating the ac losses in real applications without requiring complex experimental set-ups.

The hysteresis losses in the tape sample for applied field transverse to the wide surface of the tape are found to be dependent on the 4th power of B at small fields and above \(B_c\), and they become proportional to B. The definition of \(B_c\) used here refers to the critical magnetic field derived for the one dimensional limit as \(\mu_0 J_c / \pi\) where \(J_c\) is the sheet critical current density in \(A/m\).

The critical current density is also another factor influencing the hysteresis losses. At a particular operating temperature and in low applied fields, tape exposed to a DC background magnetic field (lower critical current) will have higher losses as opposed to the tape without any applied background field due to the \(J_c^{-2}\) dependency. This situation changes in high external field regions as the dependency becomes proportional to \(J_c\).
CHAPTER 7

INNOVATIVE CALORIMETRIC AC LOSS MEASUREMENT OF MgB$_2$ SUPERCONDUCTING TAPE CARRYING ALTERNATING TRANSPORT CURRENT WITH APPLIED DIRECT CURRENT MAGNETIC FIELDS

7.1 Introduction

The discovery of MgB$_2$ compound as a superconducting substance triggered the development and manufacture of conductors for direct current and alternating current energy device applications such as MRI, SMES magnets, NMR, transformers, fault current limiters and electric cables. AC loss is obviously one of the key issues to study and to be given considerable attention when employed in applications. The MgB$_2$ conductors are generally available in the forms of tape and wire cross section. As in other superconducting wires (BSCCO and LTS), MgB$_2$ can also be made in either multi-filamentary strands or mono-filament sheathed conductor. Taking the consideration of stability and practicability, MgB$_2$ conductor has always been made in multi-filamentary architecture with iron or nickel cladding and oxygen free copper as the stabilizing constituent.

This chapter presents the similar calorimetric method as in Chapter 6 for ac loss measurement of the MgB$_2$ multi-filamentary tape conductor with alternating transport current and external DC transverse magnetic field. It includes the sample positioning design, operation, calibration procedures and results from both experimental and theoretical data. The scope of the investigation here is to determine the total transport losses generated by the tape sample that come primarily from the MgB$_2$ filaments, sheath material and matrix elements (stabilizer and barrier). Therefore the tape sample does not require electrolytic etching to remove the outer sheath material for detailed study of loss components. Usually in the transport current case, the ac losses are driven by the magnetic field associated with the transport current and for high resistive matrices and sheath material, the loss contribution from eddy currents is small and can be neglected [Majoros08]. However in certain circumstances and particular wire architecture, losses from magnetic sheath or barrier are significant and could dominate the loss behavior of the
conductor. A detailed discussion on this will be included in the results section of this chapter.

### 7.2 Experimental Procedure

In the similar methodology with Chapter 6 the MgB$_2$ ac loss measurement also consists of two systems, one for acquisition and one for calibration. The distinctive aspects are the sample mounting configuration and the source of the sample ac loss. The MgB$_2$ tape is mounted on a holder with DC magnetic field transverse to the surface of the tape and subjected to alternating transport current through the tape. Figure 7.1 illustrates the schematic arrangement of the tape sample and the position of the temperature sensors and voltage taps.

![Schematic arrangement of the tape sample subjected to external transverse field and alternating transport current. The voltage taps are attached to the bending edge and the straight portion for monitoring the voltages during the transport current. Temperature sensors are mounted on the insulation surface and the tape surface.](image)

Figure 7.1: Schematic arrangement of the tape sample subjected to external transverse field and alternating transport current. The voltage taps are attached to the bending edge and the straight portion for monitoring the voltages during the transport current. Temperature sensors are mounted on the insulation surface and the tape surface.
As shown in the figure the MgB$_2$ tape is bent into a symmetrical $\perp$-shaped configuration with voltage tap placed in three different regions:- vertical, bending edge and horizontal part. The current leads soldered to both ends of the tape are exposed to the chamber ambient temperature which is cooled by the helium vaporizer. The external DC magnetic field is supplied by the superconducting magnet mounted permanently inside the liquid helium chamber. When the alternating transport current is applied to the superconducting tape, the losses are measured by the temperature difference from the two Cernox’s sensors and the calibration is attained by feeding a known amount of DC current through the tape. The attainable transport current is determined by the critical current $I_c$ of the tape and the heat dissipation on the bending edge. Since the bending $I_c$ is comparatively smaller, larger currents will flow through the resistive matrix element of the tape and voltage drop will be observed as a linear function with transport current. Depending on the dissipation energy the surrounding ambient can absorb at a constant tape temperature, the configuration is useful for transport ac loss measurement of the tape with reasonable accuracy and over significant current ranges.

The calibration is important for making certain the results are highly reliable and reproducible. Therefore the sample arrangement and setup are identically maintained throughout the calibration and measurement process with the only changes being the external room temperature current leads. By passing a DC transport current along the tape and measuring the temperature and voltages variation on the tape, the applicable range of the transport current for ac loss measurement will be determined. A tube-shaped insulating bobbin made of G10 material is employed to enclose the tape sample (excluding the leads contact) and the related attached sensors from the helium vapor ambient. The cooling of the tape is achieved by conduction from the leads and the diffusion from the vapor ambient through the insulating bobbin to the tape. The desired operating temperatures will be achieved and stabilized at some later time when the thermal equilibrium condition is reach between the vapor ambient and the insulated region. Therefore a set of heaters which are already placed in the insert chamber are used to control and attain the desire measurement temperature. The current leads shown in the figure 7.1 are soldered to the MgB$_2$ tape using low temperature solder and secured to a copper stage (not shown) for heat sink. A layer of
cigarette paper with vacuum grease is used in between the leads surface and the copper for electrical insulation.

7.3 Design Stability and Measurement Considerations

Prior to the transport AC loss measurement the variation of voltage and the temperature along the tape with applied DC current are investigated. The voltage taps shown in figure 7.1 record the voltage at the respective position and the Cernox’s sensors record the temperature variation on the tape surface and the ambient. Figure 7.2 illustrates the E-I curves for operating temperature of 30 K and with applied dc field of 0.5 T. The figure corresponds to the tap potential in figure 7.1 and the bending edge obviously has the lowest critical current follow by the horizontal and the vertical portion. Similar behavior is observed for measurement at different operating temperatures and applied magnetic fields as shown in figure 7.3 and 7.4.

![Figure 7.2: Variation of transport critical current along the superconducting tape configured as in figure 7.1. The electric field is the potential reading of unit length measured by the respective tap position on the tape surface. The measurement temperature is maintained at 30 K with DC magnetic field of 0.5 T applied longitudinally to the axis of the chamber.](image-url)
The variation of DC transport critical current along the MgB$_2$ tape is not surprising since the tape has been placed into a sharp bending edge and exposed to different direction of external DC magnetic field with respect to the wide surface. Nevertheless the results showed that the straight portion of the tape is not adversely affected by the bending effect as long as the heat load is thermally balanced with the exchange rate of the vapor cooling. This is seen by observing the transport critical current values on the horizontal portion of the tape from figure 7.2 to figure 7.4. The results are consistent with the values presented in Chapter 5 and in reference [See11] for the tape sample at their respective temperature and applied magnetic field.

Figure 7.3: Variation of transport critical current along the superconducting tape configured as in figure 7.1. The electric field is the potential reading of unit length measured by the respective tap position on the tape surface. The measurement temperature is maintained at 20 K with DC magnetic field of 2.0 T applied longitudinally to the axis of the chamber.

The temperature profile on the tape surface that corresponds to the sensors position in figure 7.1 is shown in figure 7.5. The vertical axis on the figure represents the difference in temperature measured at two different positions, the tape surface and the insulation surface.
The latter indicates the ambient or tape surrounding temperature and the former gives the direct dissipated energy from the tape carrying current. The figure is plotted against the DC current flowing through the superconducting tape with three different conditions that correspond to figure 7.2, figure 7.3 and figure 7.4 respectively.

Notice in the figure that the temperature on the tape and the ambient is in thermal equilibrium until a certain transport DC current is reached. This value of DC current is close to critical current of the horizontal portion. Thereafter the temperature difference increases abruptly due to the heat generated from the current flowing through the resistive fraction of the tape. The increase causes the tape’s portion to quench and propagate throughout the length of the tape. Depending on the rate of the applied transport current, the temperature increment rate will be affected and hence the quench propagation rate.

However the purpose of this investigation is to determine the applicable range of transport...
current that will be used in the measurement of the ac transport losses in MgB$_2$ tape. As shown in figure 7.5, there is no significant thermal problem caused by the bending effect on the edges that have lower critical current and earlier transition to ohmic state. Therefore the ac losses measurement will not be influenced by the bending edge effect for the transport current with average amplitude lower than the critical current of the horizontal portion. Any differences in temperature observed are strictly due to the total ac losses from the tape.

![Figure 7.5](image)

**Figure 7.5:** Plots of temperature profile of the superconducting tape against the DC transport current. The temperature is measured by taking the difference in the sensors reading. The sensors are thermally attached to the tape surface and the insulation surface. Measurements are taken which correspond to the condition as shown in the voltage-current curve in figure 7.2 to figure 7.4.

### 7.4 Results and Discussion

The MgB$_2$ superconducting tape previously employed for transport critical current measurement in Chapter 5 will be the sample for the loss results presented in this section. Once the sample stabilizes at the respective temperatures, pulses of ac current with variable frequency, amplitude and duration can be applied through the current leads soldered to the tape sample.
The results of ac transport loss for applied sinusoidal current at 50Hz with external DC field at 30 K and 20 K will be presented. It is noted that the field is applied in the transverse direction to the tape flat face as shown in figure 7.1. The response of the temperature sensors under the applied DC transport current with and without the presence of the external DC field returns no observable temperature changes. This investigation has been illustrated in the previous section. Hence it gives confidence that no error is introduced during the ac loss measurement.

Figure 7.6 shows the loss measurement conducted on the MgB$_2$ tape as a function of the peak transport current at 0.5 T and 0.7 T of external DC transverse field at 30 K. The resistance value corresponds to the differential value of the sensors that indicate the temperature increment on the sample and it is directly related to the ac losses of the tape.

![Figure 7.6: Resistance readings obtained by subtracting the value of the sensors placed on the insulation surface from the one on the tape surface. These readings are associated with the ac losses in the tape subjected to an AC transport current with external DC transverse field at 30K. The resistances are plotted against the peak transport current at 50Hz frequency.](image-url)
In exactly the same way and arrangement as the loss measurement, a DC current is passed through the MgB$_2$ tape with applied DC field and the changes of the sensor reading are recorded for calibration purposes. The DC magnetic field is applied for the reason of suppressing the critical current of the tape and hence avoiding the heating effect during the passing of DC current. The total current supply to the tape is measured by placing an external resistive shunt connected to the dc circuit and the voltage measured across the shunt terminal. During the calibration experiment, the tape sample is driven out from superconducting state and dc voltage is generated that gives a direct indication of losses in the tape. The dc losses caused the temperature rise in the tape in just the same way as the ac losses. Throughout the calibration process, the tape temperature at maximum dissipation was kept below 0.5 K from the starting measurement temperature. As such no corrections for the data are necessary for significant changes in the critical current of the tape sample during the measurement. Figure 7.7 contains a typical set of calibration data at 30K and DC magnetic field of 1 T with the dissipation shown in W/m. The calibration constant is obtained from the slope of the linear fit curve that gives 1.334 (W/m)/ohm.

The results shown in figure 7.8 are the tape sample losses in J/m/cycle subjected to ac transport current at 50 Hz under external dc magnetic field at 30 K. The loss figures are obtained by multiplying the calibration constant from figure 7.7 with the corresponding resistances in figure 7.6. The critical current $I_c$ of the tape was averaged from figure 5.9 as 100 A and 65 A for 0.5 T and 0.7 T of applied DC magnetic field respectively. Analytical expression from Norris’s ellipse and thin strip superconductors are plotted with their respective $I_c$ for comparison studies. The thin strip expression can be obtained from equation (3.41) and the conductor with elliptical cross section is as follows:

$$L_{ellipse} (J/m/cycle) = \frac{H_c I_c^2}{\pi} \left[ \frac{1}{I_c} \ln \left( \frac{1 - I_p}{I_c} \right) + \frac{I_p}{I_c} + \frac{1}{2} \left( \frac{I_p}{I_c} \right)^2 \right]$$  

(7.1)
Figure 7.7: Calibration curve showing sensor response after passing DC current to the tape as a function of DC power dissipation. The line is a linear fit to the data with the calculated slope as 1.334 (W/m)/ohm. The slope is the calibration constant that will be used to calculate losses in figure 7.6.

In separate case studies, the above results are plotted independently in figure 7.9 for each condition and incorporated within the VIM computation of ac losses. The loss trend behavior of the measured MgB$_2$ tape closely resembles the behavior exhibited by the theoretical estimation. The quantitative differences observed are caused by several factors that are determined by the critical current, sheath material and matrix material of the tape. The Norris’s calculated curve shows purely the hysteresis losses in superconductors whereas the measured losses include the ferromagnetic loss, eddy current loss and coupling loss in addition to the superconductor loss. The results computed by VIM (section 3.5) by taking into account the finite thickness and voltage current relationship are relatively insignificant compared with those measured experimentally. Such high losses are very unlikely to originate from the superconducting portion alone and studies have been made [Young07] which show that the loss contribution might be dominated by the ferromagnetic portion of the tape conductor.
Figure 7.8: AC transport losses as a function of the peak transport current for MgB$_2$ tape subjected to external DC transverse magnetic field. The frequency of the applied current is 50 Hz and measurements performed at temperature 30 K. Theoretical curves are taken for $I_c$ at 30 K with external field of 0.5 T and 0.7 T, a value extracted from figure 5.9.

Similar to the experimental and calibration process at 30 K, the loss measurements are repeated for temperature at 20 K. Figure 7.10 presents the losses for the tape at 20 K as a function of the peak transport current for applied DC transverse field of 1.8 T and 1.6 T. The applied transport current is the same frequency as before at 50 Hz. The critical current of the tape at 20 K is 90 A and 180 A for the applied field of 1.8 T and 1.6 T respectively. Even though the tape has the critical current of 180 A, the temperature profile shown in figure 7.5 indicates that the acceptable amount of current passing through the tape must be less than 90 A for a negligible heating effect. This is in fact shown in figure 5.10 where the data for transport current measurement by DC method at 20 K and 1.6 T is not obtainable due to the heating problem. Therefore in figure 7.10 the losses measurement are depicted for transport current with peak of up to 90 A for tape subjected to 1.6 T DC magnetic field. Subsequently in figure 7.11 the results are plotted individually for each applied field.
Figure 7.9: AC losses as a function of the peak transport current for MgB$_2$ tape subjected to external DC transverse magnetic field of a) 0.5 T and b) 0.7 T. The frequency of the applied current is 50 Hz and measurements performed at temperature 30 K. The results from VIM computational are shown respectively for each case. The solid lines connect the points obtained by theoretical and numerical calculations.

The results shown in figure 7.11 show similar behavior as observed for the losses at 30 K. The experimental losses are apparently higher than the theoretical curves and also the VIM computed curve. Such discrepancy as had been explained previously and is no surprise since the MgB$_2$ multifilament tape includes a nickel sheathed copper stabilized component that contributes significantly to the total loss of the tape when subjected to alternating current or field applied to the tape. The individual component of losses can be studied comparatively if the outer sheath is removed by electrolytic etching.
Figure 7.10: AC transport losses as a function of the peak transport current for MgB$_2$ tape subjected to external DC transverse magnetic field. The frequency of the applied current is 50 Hz and measurements performed at temperature 20 K. Theoretical curves are taken for $I_c$ at 20 K with external field of 1.6 T and 1.8 T, a value extracted from figure 5.9.

Figure 7.11: AC losses as a function of the peak transport current for MgB$_2$ tape subjected to external DC transverse magnetic field of a) 1.6 T and b) 1.8 T. The frequency of the applied current is 50 Hz and measurements performed at temperature 20 K. The results from VIM computational are shown respectively for each case. The solid lines connect the points obtained by theoretical and numerical calculations.
7.5 Conclusion

Work on ac transport losses under applied DC magnetic field for MgB$_2$ superconductor by other research groups has been mainly carried out in liquid neon baths with a Rogowskki coil for current reference. The method presented in this chapter simply uses a couple of temperature sensors and adequate thermal insulation materials to measure the losses at a reasonable sensitivity and range. The design and construction of the measurement rig presented here together with the sample configuration has shown that this calorimetric method is an innovative and feasible approach.

While the chamber offers great stability in temperature and homogeneity in applied magnetic field, the setup configuration gives an ideal platform for different types of physical and electrical studies on the sample to be performed in addition to ac loss measurement. These include the quench, bending critical current, normal zone propagation and bi-directional field dependent critical current. The measurement results presented are the total ac loss of the MgB$_2$ multifilament tape in 30 K and 20 K subjected to alternating transport current in external DC transverse magnetic field. Prior to these measurements, the variation of critical current along the tape sample is investigated and the DC dissipation profiles are studied to ensure the results of ac losses are not influenced or affected by the resistive dissipation itself.

The losses are compared with the theoretical and computational values and the differences are caused by the contribution from the magnetic sheath and barrier. Despite this additional loss, the total ac loss behavior of the tape is as expected from knowledge of the theoretical and computed loss figures. Generally the superconducting tape losses under applied alternating transport current at practical frequency varies according to $I_c^{-1}$ for peak currents less than the critical current of the tape.
CHAPTER 8
SUMMARY AND CONCLUSIONS

The main intention of the measurement and theoretical work on practical superconducting tape and wire presented here was to develop innovative skills, new and more useful mathematical models and understandings required to assist the practical use of these superconductors in electrical engineering applications. Contributions were made in the practical measurement of the characterization of short and long superconductors along with the integration, control and acquisition of instrumentation systems. Contributions to the analytical approach of solving the one-dimensional Poisson equation for field and current distributions in superconductors leading to the solution of singularity problems were also described. In addition, a new method exploiting Fourier series with the variational iteration method was developed to solve the two dimensional nonlinear magnetic diffusion equations and hence find the losses for superconductors with finite thickness. These new mathematical methods were validated using the experimental techniques developed.

8.1 Finding new Mathematical Models for Field and Current Distribution and AC Losses

In chapter 3, derivations were presented for solving the current and field distribution in thin strip superconductor by using the Poisson’s form of vector potential equation as the first stage. This then leads to the general solution in integral form with logarithmic kernel that contains a singularity problem. A new direct analytical approach was presented to circumvent this problem and provide an analytical current density solution. The final current distribution was then solved by introducing a differential technique to reduce the equation to a solvable type. Once the current distribution is found the field and hence the losses distributions are calculated. Section 3.2 presents the distribution profiles for the case of transport current without external field and Section 3.3 presents the profiles for external
field in transverse direction to the tape’s wide face. The analytical loss expression for thin strip geometry was derived in the subsequent section.

Prior to the concluding remarks of Chapter 3, an effective computational method based on variational iteration method was introduced to compute the ac losses of superconductors with finite thickness and voltage-current relation law. The computational scheme was employed to solve the two dimensional nonlinear magnetic diffusion models that represent the diffusive nature of current density and magnetic field inside the superconductor. From this, the losses for the conductor can be found. These numerically computed results showed excellent agreement with the derived analytical expression when the geometry reduces to 1-dimensional strip problem.

8.2 Experimental and Low Temperature Engineering

In Chapter 4, a comprehensive description of the development of a new experimental configuration, instrumentation and data acquisition system was described. The developed system consists of the superconducting magnet system that provides DC magnetic fields up to 15 T and housed the sample assembly system with excellent temperature homogeneity and stability. The sample assembly system comprises of sample holder, insulating bobbin, high temperature superconducting coil for AC magnetic field, current leads, multiple sensors and sample heater. All of these components were well integrated and connected to the external communication circuitry for data acquisition purposes. The data acquisition system was designed and programs were written using the LabVIEW platform environment and instruments were connected via the GPIB instrumentation bus to the personal computer. The descriptions of each electrical and electronic instrument used were presented.

The fully operated integrated systems are versatile and capable of measuring or acquiring most physical signals from any practical sample at temperature ranges from 2 K to 300 K. The accessible dimensions for the sample are 40 mm in diameter and 100 mm in height. In this thesis, the system was employed to measured ac losses under various external
conditions and transport critical current for superconducting wires and tapes over a broad range of operating temperatures and magnetic fields. The theoretical and computational estimations were verified by comparison with the measured results.

8.3 Method of Transport Critical Current Measurement

Chapter 5 presents the transport critical current measurement on MgB$_2$ wire and tape sample with two different techniques. The first used a general four-probe technique which consists of a current probe on the two outer leads and voltage probe on the middle two leads. A constant direct current was applied through the sample until the critical current of the sample is reached when the voltage drop was recorded as 1 $\mu$V for separation leads of 1 cm. The second method employed the pulsed current method with a suitable rate of current ramp and the results were recorded in a digital oscilloscope. The waveform produced showed the simultaneous voltage-current measurement on the wire and tape sample. The distinctive feature of the pulsed method is to eliminate or minimize the heating effect frequently affecting the DC four-probe method. However the critical current measured by the pulse technique was proven to be dependent on the rate of current ramp in the pulse. It was shown that the best way to accurately obtained the critical current measured in a short sample configuration is by extrapolating the values measured at different rates to $\frac{dI}{dt} = 0$. This technique greatly extends the ranges over which superconducting properties can be measured.

In Section 5.2, a finite element model of the transport measurement was constructed to demonstrate the heat progression in the conductor and the simulated temperature profile was found to increase abruptly for transport current near the critical transition. The results demonstrated that the DC four-probe technique has a tendency of overheating from the contact resistances particularly for samples at with high values of critical current. The subsequent sections in Chapter 5 present the experimental procedures, results and discussions.
Chapter 6 presents an innovative approach for measuring ac losses of silver sheathed Bi-2223 superconductor under applied transverse magnetic field. The approach employed a calorimetric technique that measured the temperature rise from the sample subjected to transverse magnetic field. The calibration method was implemented through a resistive coil wound around the sample and enclosed within the insulating layer. Any heat generated on the coil will be conducted through the sample and measured by the temperature sensor. The heat generated from the ac loss is taken in the same way as the heat conducted through the sample from the calibration coil. The results presented were the total ac losses in external transverse magnetic field at 50 Hz with additional applied DC magnetic field. Such conditions resemble those in a saturated iron core fault current limiter application. The experimental loss data were shown to be closely aligned with the calculated theoretical value. Generally the superconductor losses that are at a particular operating temperature have the $J_0^{-2}$ dependency in low applied field regime and $J_c$ dependency in high applied field regime, which is in accordance with the theoretical expression.

In Chapter 7, ac loss measurement of the MgB$_2$ multi-filamentary tape conductor in alternating transport current with external DC transverse magnetic field was demonstrated. The distinctive features of the measurement here are the mounting procedure where sample was bent into a symmetrical $\perp$-shaped configuration. The stability and the temperature variation on the sample during applied transport current were thoroughly studied prior to the transport ac loss measurement of the tape sample. In spite of the bending effect it was confirmed, after conducting several calibration experiments and stability studies, that the method presented was feasible and reliable in measuring transport losses on superconducting tape.

Section 7.2 detailed the experimental procedures for the arrangement of the tape sample and the insulating architecture of the assemblies. The subsequent section presents the calibration and variation of the E-J curve measured throughout the tape sample. The
bending edge has the lowest measured critical current and the heat developed will not severely affect the loss measurement if the applied peak transport current is below the critical current value measured on the horizontal segment. The temperature profiles were illustrated and followed by results and discussion. The results of AC transport losses under applied DC magnetic field were plotted against the peak transport current at 30 K and 20 K. The measured losses were compared with the theoretical and computational values and the differences were caused by factors contributed by the sheath, barrier and stabilizer materials. Typically the transport current losses at low frequency were shown to be of $I_c^{-1}$ dependence for the peak current less than the critical current of the tape.

8.6 Recommendations for Further Research

Further research is recommended for both the experimental techniques and theoretical work to help enhance the analysis of practical superconducting wires and tapes for electrical engineering applications. The following are some recommendations for further work.

1. The experimental rig that consists of a superconducting magnetic coil and variable temperature insert (VTI) system are operating under the liquid helium bath. The high consumption of liquid helium in maintaining the operation system incurred a considerable cost in running the system for extended periods. Therefore upgrading the whole system to a cryogen free conduction cooled system or a recondensing system will enormously enhance the research feasibility and versatility. Also the clear bore size of the sample chamber is worthwhile to be considered and studied as it determines the flexibility of the sample housing design and architecture. The more space allocated to the sample space the more variation of physical measurement can be performed on the sample. However it is crucial to acknowledge that the temperature and magnetic field stability will be influenced by the size of the chamber.

2. The methods for measurement of losses in superconducting tape and wire under the application of DC and AC magnetic fields at temperatures lower than 77 K were successfully demonstrated in this thesis. These methods could potentially be applied to a
superconducting coil, particularly MgB$_2$ coil, with dimensions that fit into the chamber. Stability and quench studies could also be performed on the coil or wire sample with the attachment of several voltage taps and temperature sensors on the superconductors, as it was shown in Chapter 7 that the performance of tape samples subjected to bending stress and exposed to variations of voltages and temperatures under applied external current and field can be successfully recorded.

3. It has been shown in Chapter 5 that the transport critical current measurement of MgB$_2$ superconductor can be measured using two different techniques. The pulse current technique offers advantages over the four probe DC technique in obtaining a more accurate $I_c$ over a greater range of fields and temperatures. Therefore it is suggested that the pulse measurement be further developed by replacing or upgrading the current power source to a greater voltage and current limit. At the same time, the pre-amplifier circuitry has to be improved as well to further enhance the signal to noise ratio and to suppress unwanted spurious signals during the injection of pulse waveform.

4. Finally, finding ways to develop the theoretical computation of the ac losses in practical superconducting wire or tape would be an advantage, in order to account for the geometrical effect, loss contributions from other composite materials and external conditions imposed on the conductor. Analytical expressions for ac losses are most desirable in every circumstance but it is very unlikely that the complications that arise in real-world applications are able to be represented by just one equation. The work in this thesis could be extended to include a more comprehensive numerical algorithm to represent a superconducting coil and to accurately predict the behavior of the prospective coil for engineering applications. Furthermore, it is also constructive to find a numerical method that includes all the components of losses in a practical superconducting tape and wire to accurately compute the total losses. Ultimately it would be desirable to be able to estimate the ac losses that incurred on the conductor in all forms of practical usage.
REFERENCES


[Ishii96] H. Ishii et al., ”The AC losses in (Bi,Pb)2Sr2Ca2Cu3Ox silver-sheathed superconducting wires,” Cyogenics, vol. 36, no. 9, pp. 697-703, 1996


Fourier Series with Homogeneous and Nonhomogeneous Boundary Condition

Diffusion Equation

Consider a rectangular object which has boundaries $0 \leq x \leq a$ and $0 \leq y \leq b$. Temperatures at the boundaries are maintained at zero and a heat source with an initial temperature is prescribed at a point within the rectangle. Such a heat diffusion problem can be described by:

$$
\frac{\partial v}{\partial t} = \alpha \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right), 0 < x < a, 0 < y < b, t > 0, \tag{A1}
$$

subject to the boundary conditions:

$$
v(0, y, t) = 0, \tag{A2}
$$

$$
v(a, y, t) = 0, \tag{A2}
$$

$$
v(x, 0, t) = 0, \tag{A2}
$$

$$
v(x, b, t) = 0, \tag{A2}
$$

with initial condition as:

$$
v(x, y, 0) = f(x, y). \tag{A3}
$$

By applying the method as in Section 3.1, the final series solution is given by:

$$
v(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} e^{-\alpha \beta^2 t} \sin \left( \frac{m \pi x}{a} \right) \sin \left( \frac{n \pi y}{b} \right), \tag{A4}
$$

where $\beta^2 = \pi^2 \left( \frac{m^2}{a^2} + \frac{n^2}{b^2} \right)$, and the $A_{mn}$ is

$$
A_{mn} = \frac{4}{ab} \int_0^a \int_0^b f(x, y) \sin \left( \frac{m \pi x}{a} \right) \sin \left( \frac{n \pi y}{b} \right) \, dx \, dy. \tag{A5}
$$
Laplace's Equation

Consider a two--dimensional Laplace's equation in a rectangular column:

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0, \quad 0 < x < a, 0 < y < b, \quad (A6)
\]

satisfying the boundary conditions:

\[
\phi(0, y) = 0, \\
\phi(a, y) = 0, \quad (A7) \\
\phi(x, 0) = 0, \\
\phi(x, b) = t_4,
\]

where \( t_4 \) is a positive constant.

The general solution for the Laplace's problem is given as

\[
\phi(x, y) = \sum_{m=1}^{\infty} D_m \sin \left( \mu_m x \right) \sinh \left( \mu_m y \right), \quad (A8)
\]

where \( \mu_m = m\pi / a \). Using the fourth boundary condition in (19), we get \( D_m \) as the Fourier sine coefficients of \( t_4 \),

\[
D_m = \frac{2}{a \sinh (\mu_m b)} \int_0^a t_4 \sin (\mu_m x) dx. \quad (A9)
\]

We can now extend the solution to include the possibilities of having more than one nonzero boundary condition for the Laplace's equation. Using the same equation, (18), we look at four nonzero boundary conditions as such:

\[
\phi(0, y) = t_1, \\
\phi(a, y) = t_2, \quad (A10) \\
\phi(x, 0) = t_3, \\
\phi(x, b) = t_4,
\]

where \( t_1, t_2, t_3 \) and \( t_4 \) are all positive constants.
By denoting (20) by \( \phi_4(x, y) \) and interchanging the roles of \( x \) and \( y \), we can similarly obtain \( \phi_4(x, y) \), the solution of the problem with \( t_1, t_3 \) and \( t_4 \) replaced by zero as below:

\[
\phi_2(x, y) = \sum_{m=1}^{\infty} B_m \sinh(\gamma_m x) \sin(\gamma_m y), \quad \gamma_m = \frac{m\pi}{b}.
\]  

(A11)

The remainder of the solution can be obtained by substituting \( y \) with \( b-y \) in \( \phi_4(x, y) \) and \( x \) with \( a-x \) in \( \phi_4(x, y) \), as follows:

\[
\phi_3(x, y) = \sum_{m=1}^{\infty} C_m \sinh(\mu_m (b-y)) \sin(\mu_m x), \quad \mu_m = \frac{m\pi}{a},
\]

(A12)

\[
\phi_4(x, y) = \sum_{m=1}^{\infty} A_m \sinh(\gamma_m (a-x)) \sin(\gamma_m y), \quad \gamma_m = \frac{m\pi}{b}.
\]

(A13)

Adding all four functions of \( \phi \) gives a solution for the Laplace's equation which satisfies all four nonzero boundary conditions. The complete solution, \( \phi(x, y) = \phi_1(x, y) + \phi_2(x, y) + \phi_3(x, y) + \phi_4(x, y) \), is summarized as follows:

\[
\phi(x, y) = \sum_{m=1}^{\infty} \left\{ A_m \sinh(\gamma_m (a-x)) + B_m \sinh(\gamma_m x) \right\} \sin(\gamma_m y) + \sum_{m=1}^{\infty} \left\{ C_m \sinh(\mu_m (b-y)) + D_m \sinh(\mu_m x) \right\} \sin(\mu_m x),
\]

(A14)

where the coefficients \( A_m \), \( B_m \), \( C_m \) and \( D_m \) are given as follows:

\[
A_m = \frac{2}{b \sinh(\gamma_m a)} \int_0^b t_1 \sin(\gamma_m y) dy,
\]

(A15)

\[
B_m = \frac{2}{b \sinh(\gamma_m a)} \int_0^b t_2 \sin(\gamma_m y) dy,
\]

(A16)

\[
C_m = \frac{2}{a \sinh(\mu_m b)} \int_0^a t_3 \sin(\mu_m x) dx,
\]

(A17)

\[
D_m = \frac{2}{a \sinh(\mu_m b)} \int_0^a t_4 \sin(\mu_m x) dx.
\]

(A18)