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Yuanlong Fan
University of Wollongong, yf555@uowmail.edu.au

Yanguang Yu
University of Wollongong, yanguang@uow.edu.au

Jiangtao Xi
University of Wollongong, jiangtao@uow.edu.au

Huiying Ye
Zhengzhou University

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Abstract
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Keywords
optical, laser, semiconductor, limit, feedback, stability, influence, gain, nonlinear

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Influence of the nonlinear gain on the stability limit of a semiconductor laser with optical feedback

Yuanlong Fan, Yanguang Yu∗, Jiangtao Xi, Huiying Ye

School of Electrical, Computer and Telecommunications Engineering, University of Wollongong, Northfields Ave, Wollongong, NSW, 2522, Australia

School of Information Engineering, Zhengzhou University, P. R. China

ABSTRACT

This paper presents the results revealing the influence of the nonlinear gain on the stability limit of a semiconductor laser (SL) with external optical feedback (EOF). A new system determinant is derived from the original Lang and Kobayashi (L-K) equations. By making analysis on the locus of the roots of the system determinant, the stability limit of the system is obtained, from which a number of important and interesting phenomenon revealed by the nonlinear gain is uncovered. The correctness of results is verified by numerical simulations.

Keywords: semiconductor lasers, stability analysis, optical feedback, nonlinear gain

1. INTRODUCTION

A semiconductor laser (SL) with an external optical feedback (EOF) can exhibit a rich variety of dynamical behaviors, including stationary state, periodic oscillation, quasi-periodic oscillation and chaos [1-3]. In this paper, we mainly focus on the stability property of the steady state of an SL with EOF because the study of steady state is of great importance for the applications to the self-mixing interferometry sensing system which can be used for measuring metrological quantities, such as displacement, velocity, distance and laser parameters [4-11]. The first stability analysis for an SL with EOF was originated along with the development of Lang and Kobayashi (L-K) equations [12] in 1980 by using a small signal analysis method which examines the time development of small deviations of the L-K equations’ stationary solutions (or external cavity modes). A system determinant was derived from the L-K equations and equations’ stationary solutions. When all the roots of the system determinant have a negative real part, the fluctuations of small deviations attenuate with time and the corresponding stationary solution delegates a stable system, or otherwise an unstable system. For an unstable system, the relaxation oscillation of the SL with EOF becomes undamped, in other word, the laser output becomes non-constant. Based on the work done in [12], significant amount of researches [13-22] have been devoted to finding the stability limit of an SL with EOF system. However, an outstanding issue associated with the results reported in [13-22] is that, none of them considers the influence of nonlinear gain on the stability limit. As a matter of fact, nonlinear gain is an important factor in describing the dynamic behaviors of an SL with EOF [23-24]. Hence, in order to better describe the stability properties of the system, nonlinear gain effect should be considered.

In this paper, our analysis starts from the L-K equations. Firstly, a new and accurate system determinant is derived with the inclusion of nonlinear gain. Then by varying the parameters of the system, i.e., the feedback strength $k$, the nonlinear confinement factor $\Gamma$ and the external cavity round-trip time $\tau$, we observe the root locus of the system determinant, from which, the stability limit of an SL with EOF system is obtained. Finally, the influence of the nonlinear gain on the stability limit is investigated and a number of interesting discoveries are presented.

2. THE STABILITY LIMIT OF AN SL WITH EOF SYSTEM

2.1 System model of an SL with EOF

The schematic configuration of an SL with EOF system is shown in Fig.1. When the light emitted from the front facet of an SL hits the external target, there will be a portion of light reflected into the SL’s internal cavity. The reflected light (dashed line in Fig.1) consequently changes the SL’s complex electric field $E_i(t)$ and the carrier density $N_i(t)$, where
$t$ is the time series. The complex electric field is $E(t) = E(t)e^{[\omega_0 t + \phi(t)]}$, where $E(t)$ is the electric field amplitude, $\omega_0$ is the angular frequency for a solitary laser and $\phi(t)$ is the electric field phase.

Figure 1. Schematic configuration of an SL with EOF system.

The mathematical model for describing the system is the L-K equations, which is a set of coupled nonlinear Delayed Differential Equations (DDEs) (shown as Eqs.(1)-(4)) for $E(t)$, $\phi(t)$ and $N(t)$.

\[
\frac{dE(t)}{dt} = \frac{1}{2} \left[ G[N(t), E(t)] - \frac{1}{\tau_p} \right] E(t) + \frac{k}{\tau_m} E(t-\tau) \cdot \cos[\omega_0 \tau + \phi(t) - \phi(t-\tau)]
\]

\[
\frac{d\phi(t)}{dt} = \frac{1}{2} \alpha \left[ G[N(t), E(t)] - \frac{1}{\tau_p} \right] - \frac{k}{\tau_m} \frac{E(t-\tau)}{E(t)} \cdot \sin[\omega_0 \tau + \phi(t) - \phi(t-\tau)]
\]

\[
\frac{dN(t)}{dt} = J - \frac{N(t)}{\tau_s} - G[N(t), E(t)] E^2(t)
\]

where $G[N(t), E(t)]$ is the modal gain per unit time and is expressed as:

\[
G[N(t), E(t)] = G_N \left[ N(t) - N_0 \right] \left[ 1 - d^2 E^2(t) \right]
\]

where $G_N$ is the modal gain coefficient, $N_0$ is the carrier density at transparency, $\varepsilon$ is the nonlinear gain compression coefficient, and $\Gamma$ is the confinement factor. The nonlinear gain in Eq. (4) is represented by the gain reduction $-d^2 E^2(t)$ with the increasing of laser output intensity. In this paper, we use $\Gamma$ as a parameter to describe the effect of the nonlinear gain. The external cavity parameters for an SL with EOF are the injection current $J$, the feedback strength $k$ and the round-trip time of feedback light in the external cavity $\tau$. The feedback strength $k = (1-r_2^2) r_1 / (\tau_p r_2)$, where $r_1$ is the reflectivity of laser facet, $r_2$ is the reflectivity of the external target and $\tau_2$ is the laser internal cavity round-trip time. The round-trip time in the external cavity $\tau = 2L/c$, where $L$ is the external cavity length, $c$ is the speed of light. $\tau_p$ and $\tau_s$ are the photon lifetime and carrier lifetime respectively.

2.2 Determining the stability limit

The stability of a system is usually analyzed based on the system determinant. For an SL with EOF, its system determinant is obtained by linearizing the L-K equations near the stationary solutions. The stationary solutions can be obtained from Eqs. (1)-(4), shown as follows:

\[
\omega_0 = \omega_s + \frac{k}{\tau_m} \sqrt{1 + \alpha^2 \sin(\omega_s \tau + \arctan \alpha)}
\]

\[
N_s = N_0 + \frac{1}{\tau_p G_N} - \frac{2k \cos(\omega_s \tau)}{\tau_m G_N}
\]
\[ E_s^2 = \frac{J - N_s/\tau_s}{G_s(N_s - N_h)} \]  

(7)

Assuming that \( E(t), \phi(t) \) and \( N(t) \) exhibit small deviations from the stationary state and these deviations are denoted by \( \delta \_E(t), \delta \_\phi(t) \) and \( \delta \_N(t) \), a set of linear differential equations from the L-K equations (1)-(4) can be derived. Based on the assumptions made in \(^{13,19} \), i.e., \( \omega_p^2 k/\tau_m \tau_R \) and \( \omega_R^2 (k/\tau_m)^2 \) where \( \omega_p^2 \) and \( \tau_R \) are the relaxation resonance angular frequency and the damping time of the relaxation oscillation for a solitary laser respectively, the Laplace transform of the linear differential equations gives the following system determinant:

\[
D(s) = s^3 + s^2 \left[ \frac{2k}{\tau_m} \cos(\omega_p \tau)(1 - e^{-\tau}) + \omega_R^2 + \frac{1}{\tau_R} \right] + s \left[ \frac{2}{\tau_m} + \omega_R^2 \right] \left( \cos(\omega_p \tau) - \alpha \omega_R^2 \sin(\omega_p \tau) \right) (1 - e^{-\tau}) + \frac{1}{\tau_R} \omega_R^2 + \omega_R^2 \left( \cos(\omega_p \tau) - \alpha \sin(\omega_p \tau) \right) (1 - e^{-\tau}) \right] \]

(8)

where \( \tau_n^s = \tau_n + G_N E_n^2 (1 - \delta \_E^2) \), \( \omega_R^2 = \omega_R^2 G_N E_n^2 \), \( \omega_R^2 = \omega_R^2 \delta \_I \left( G_N (1 - \delta \_E^2) \right) \) and \( \omega_R^2 = \omega_R^2 (1 - 2\delta \_E^2) \). \( E_{\text{sol}} \) is the field amplitude of the solitary laser and it is given by:

\[
E_{\text{sol}} = \frac{\tilde{J} \tau_p \tau_m G_s - N_s \tau_p G_N - 1}{\tau_n \tau_m G_N} \]

(9)

where \( \tilde{J} \) is the injection current for a solitary laser, and it is set as \( \tilde{J} = 1.17 \times 10^{13} \text{m}^{-3} \text{s}^{-1} \) in this paper. With the system determinant in Eq. (8), we are able to work out the locations of zeros of \( D(s) \) on the \( S \)-plane. The zeros of \( D(s) \) are defined as the roots of \( D(s) = 0 \), which are usually complex numbers and can be found using various techniques. Note that for a fixed set of parameters, Equation (8) has multiplicity roots. Among the roots, the rightmost root (denoted as \( s_o \)) can be used to determine the stability of the system. The system is stable if \( s_o \) lies in the left half side of the \( S \)-plane. The stability limit is reached when \( s_o \) is on the imaginary axis. Figure 2(a) shows the locus of \( s_o \) by varying the confinement factor \( \Gamma \) when \( \omega_R \tau_p (2\pi) = 0.5 \), \( J/J_{\text{th}} = 1.3 \) and \( k = 0.005 \), where \( J_{\text{th}} \) is the injection current at threshold. Note that \( \tau \) is normalized in the same way used in \(^{19} \). The variation range of \( \Gamma \) is from 0.00 to 1.50 with 31 samples. All the other parameters are adopted from \(^{19} \). From Fig. 2(a), we can see that from \( \Gamma = 0.05 \) to \( \Gamma = 0.80 \), the locus of \( s_o \) moves horizontally from the right half side of the \( S \)-plane to the imaginary axis, which means the variation of \( \Gamma \) in this range does not change the relaxation oscillation frequency. Keep increasing \( \Gamma \) from 0.80 to 1.50, \( s_o \) moves to the left half side of the \( S \)-plane and the relaxation oscillation becomes damped, therefore leading to a stable laser output. Figure 2(b)-(d) shows the corresponding normalized laser intensity output obtained by numerically solving Eqs.(1)-(4) for \( \Gamma = 1.50 \), \( \Gamma = 0.80 \) and \( \Gamma = 0.05 \) respectively, which verifies the correctness of the system determinant we derived. The method for solving Eqs.(1)-(4) is 4\textsuperscript{th} order Runge-Kutta integration method. The intensity of the SL output \( I(t) \) is calculated as \( I(t) = E^2(t)/E_{\text{sol}}^2 \). With the locus of \( s_o \), we are able to present the stability limit in terms of system parameters. In this paper, we choose \( k \), \( \Gamma \) and \( \tau \) as our variation parameters to determine the stability limit.

3. INFLUENCE OF THE NONLINEAR GAIN ON THE STABILITY LIMIT

In this section, the stability limit is constructed in a two dimensional plane of \( k \) and \( \Gamma \) (shown as in Fig. 3) by observing the locus of \( s_o \) for two different values of \( \tau \), i.e., \( \omega_R \tau_p (2\pi) = 0.3 \) and \( \omega_R \tau_p (2\pi) = 0.6 \). Figure 3 shows the unstable region. The variation ranges for \( k \) and \( \Gamma \) are chosen as \( k \in [0.000, 0.015] \), \( \Gamma \in [0.0, 1.2] \). For each of the parameters, we take 200 samples by equally-spaced sampling. From Fig. 3, we can see that, with the increase of \( \Gamma \), the value of \( k \) for guaranteeing a stable system also increases. This phenomenon just coincides with the results reported in \(^{20} \).
When $\tau$ increases, the relationship of $k$ and $\Gamma$ changes from linear to distorted while keeps the size of the unstable region almost unchanged.

Figure 2. (a) The locus of $s_0$ on the $S$-plane by varying $\Gamma$ when $\tau = 0.21 \text{ns}$, $J/J_{\alpha} = 1.3$ and $k = 0.005$, (b)-(d) are the corresponding normalized laser intensity for $\Gamma = 1.50$, $\Gamma = 0.80$ and $\Gamma = 0.05$ respectively.

Figure 3. The stability limit of $k$ and $\Gamma$ for different values of $\tau$ when $J/J_{\alpha} = 1.3$, (a) $\omega_{*}\tau/(2\pi) = 0.3$, (b) $\omega_{*}\tau/(2\pi) = 0.6$.

Similarly, we are able to construct the stability limit in the plane of $k$ and $\tau$ when $\Gamma = 1.5$ and $J/J_{\alpha} = 1.3$, shown as in Fig.4 where the shaded area is still the unstable region. The number of samples in the variation ranges of $k$ and $\tau$ is also 200. In Fig. 4, we define the critical feedback (denoted as $k_c$) as the minimum value of $k$ appeared in the unstable region. In fact, the value of $k_c$ increases proportionally to the value of $\Gamma$ which has been verified in \textsuperscript{26}. It is interesting to notice that the relationship of $k$ and $\tau$ for describing the stability limit is not continuous. The shape of the stability limit shows sawtooth like fringes.
For a relative short external cavity, the stable region always pertains to a very high feedback strength. If we define the critical external cavity round-trip time, denoted as \( \tau_c \), as for a stable region being pertained to a high feedback strength, dependence of \( \tau_c \) can be evaluated as:

\[
\tau_c = 2\pi\omega R^{-1} \cdot F(\Gamma)
\]

where function \( F(\Gamma) \) is presented in Fig. 5, which is obtained from the numerical simulation.

4. CONCLUSION

Influence of the nonlinear gain on the stability limit is investigated in this paper. Starting from the L-K equations, a new system determinant is derived from which we are able to observe the root locus of the system determinant, and therefore determining the stability limit of the system. The correctness of the system determinant is verified by the numerical simulation of the L-K equations to observe the status of the SL with EOF in the time domain. In this paper, the stability limit is constructed in terms of the feedback strength, the nonlinear gain confinement factor and the external cavity round-trip time. Finally, from the presented stability limit, a number of interesting discoveries revealed by the nonlinear gain is found.
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