2011

Improving the performance of optical feedback self-mixing interferometry sensing

Lu Wei

University of Wollongong, lwei@uow.edu.au

Recommended Citation

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IMPROVING THE PERFORMANCE OF OPTICAL FEEDBACK SELF-MIXING INTERFEROMETRY SENSING

A thesis submitted in fulfillment of the requirements for the award of the degree of

DOCTOR OF PHILOSOPHY

from

UNIVERSITY OF WOLLONGONG

by

LU WEI
B.E., Beijing Jiaotong University, China, 1994
M.Tech., University of Wollongong, Australia, 2004

School of Electrical, Computer and Telecommunications Engineering
2011
Dedicated to my family
CERTIFICATION

I, Lu Wei, declare that this thesis, submitted in fulfillment of the requirements for the award of Doctor of Philosophy, in the School of Electrical, Computer and Telecommunications Engineering, University of Wollongong, is wholly my own work unless otherwise referenced or acknowledged. The document has not been submitted for qualifications at any other academic institution.

Lu Wei
29 March 2011
Acknowledgements

First of all, I would like to thank my supervisors, Professor Jiangtao Xi and Professor Joe F. Chicharo for their invaluable guidance and support throughout my PhD study. My special gratitude goes to Professor Jiangtao Xi for our enlightening discussions both in regard to my research and on the value of pursuing personal fulfillment. I am extremely grateful for the constant encouragement that he has given me. I have found him to be very inspiring as a mentor and to be a great role model due to his indefatigable efforts in life.

I would like to sincerely thank Dr. Yanguang Yu for the valuable discussions in my research and the warm encouragement in finishing my thesis. I also deeply appreciate the assistance from the people in the School of Electrical, Computer and Telecommunications Engineering at the University of Wollongong.

I would like to thank Australian government for the awarding of postgraduate scholarship (APA) throughout this work. In particular, I appreciate the kind help from Professor Joe F. Chicharo and Professor Jiangtao Xi for the extension of the scholarship.

I wish to sincerely thank my husband Yue Zhao for his complete support and understanding throughout my study. My daughter Kelly and my son Henry have been the untapped source of energy for me to finish this thesis.

Finally, I am deeply grateful to my parents, who have offered persistent encouragement in my study and endless support in every way that is needed.
Abstract

This thesis explores two measurement applications of optical feedback self-mixing system with semiconductor lasers. One is the displacement measurement of a remote target; the other is the measurement of the linewidth enhancement factor which is a characteristic specification of a semiconductor laser.

The above mentioned two types of measurements are performed based on the self-mixing signals acquired with an optical feedback system set-up. Apparently better signal quality will facilitate the proposed algorithms and yield improved accuracies. Hence, the issue of signal pre-processing is addressed before the measurements are conducted by means of digital signal processing techniques. The experimental signals are firstly processed with median filter to achieve basic level of data smoothing and the elimination of the impulsive noises. A high level of data smoothing is achieved by the employment of an artificial neural network. A good accordance is found between the pre-processed noisy self-mixing signal and its clean counterpart in computer simulations.

In order to investigate the displacement measurement with the laser self-mixing system, the analytical solution for the displacement of an external moving target is firstly examined with the Lang-Kobayashi theory. One difficulty that is associated with direct utilization of this solution is the wrapped phase values between 0 and \( \pi \) as a result of the inverse cosine function that is involved in recovering the phases of the reflected light. The basic idea of the phase unwrapping algorithm is to locate the points where the phase is wrapped and thus recover it to its real value by adding or subtract multiple numbers of \( 2\pi \) for the target movement away and towards the laser respectively. The accuracy of this phase unwrapping algorithm under difference levels of laser feedback was then investigated by computer simulations. It was shown that the target
displacement can be reconstructed with the accuracy of $\lambda/25$ under weak feedback regime and $\lambda/20$ under moderate feedback regime.

The measurement of the linewidth enhancement factor is based on the data-to-model fitting paradigm. That is, the experimental self-mixing data is applied to the theoretical model of the feedback interferometry system and a few model parameters are identified accordingly so that a pre-defined cost function is minimized. In order for the method to be applicable under a universal situation, a genetic searching approach is proposed to locate the global minimum so as to yield the estimation of a set of parameters in the theoretical model including the linewidth enhancement factor. The thorough investigation on the error surface revealed multiple local minima in the cost function and uneven sensitivities of the cost function with respect to different parameters. Thus a global searching procedure is proposed by performing multiple rounds of genetic algorithm with the later round concentrating the searching within the areas that is obtained from the previous round of running the algorithm. It is shown with computer simulations and experiments that the proposed algorithm achieves the accuracy of 3.8% for the measurement of the linewidth enhancement factor.
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Chapter 1  Preliminaries
1.1 Introduction

Traditionally, optical interferometry was established on the principles of the light wave interference resulted from superposition of two coherent optical wavefronts which have propagated through different paths. The measurement with a conventional two-beam interferometer is achieved either by observing interference fringes which appears as the alternating light and dark bands or by detecting the intensity of the super-positioned signal with a photo diode. The optical interferometry possesses a series of advantages such as high sensitivity, noncontact operation and fast measurement so that it has been used in a vast range of areas. Some examples of the major applications are found in metrology, velocimetry and vibrometry, surface profiling, interference spectroscopy, mechanical modal analysis and on-line quality control.

The optical self-mixing or feedback interferometry is a relatively new and significantly simple implementation of the optical sensing system owing to the triumphal invention of the highly intensive and coherent laser sources in the 1960’s. In this configuration, the light emitted from a laser source is reflected by a remote target and re-enters the laser active cavity, causing modulation of both laser output frequency and intensity. Measurements of the information about the target or the optical system parameters are performed by monitoring the laser intensity fluctuations with a photo diode. The self-mixing interferometry inherently possesses the superiority of self-alignment, mechanical stability and exemption from necessitating costly high precision optical apparatus. As a result, it has been used for the measurement of displacement, vibration, distance and velocity of a target.

This thesis deals with improvement of performance of optical feedback interferometry in two application areas. One is to improve the accuracy in the
measurement of a target displacement. This is achieved through signal pre-processing to eliminate the noises contained in the interferometric signals by employing an artificial neural network. The other aim of the research is to estimate the linewidth enhancement factor (LEF) of a semiconductor laser with improved accuracy and efficiency which is achieved by a novel implementation of the genetic searching scheme.

This chapter is organized as follows: Section 1.2 gives a brief historical review of optical interferometry techniques. The background of this research, i.e., the self-mixing interferometry is given in Section 1.3. The research issues are presented in Section 1.4. Approaches and contributions of this thesis are described in Section 1.5.

1.2 A Brief Historical Review

1.2.1 Traditional Optical Interferometry

The first experiment that observed the interference phenomenon in light was performed by British physicist Thomas Young in 1805 known as the double slit experiment. In his experiment, a single light source is split to two sources by allowing it to pass a barrier with a pair of slits. The two light sources are called coherent sources because they have identical frequencies and a constant phase difference. The light from the coherent sources interferes and forms interference pattern of bright and dark bands on a screen behind them as shown in Fig. 1.1. As the explanation to the origin of the interference pattern, Young proposed two of the most fundamental principles in the fields of optics and interferometry, i.e., the principle of coherence and principle of superposition. The principle of coherence states only coherent lights that have the same phase history are able to combine and produce interference effects. Principle of superposition indicates that if two waves arrive at a spot in phase, the waves interfere
constructively with new amplitude twice that of their initial ones. On the other hand, if the phase difference is somewhere between $0$ and $\pi$, the resultant wave will have amplitude somewhere between zero and twice the initial amplitude of the two waves. Young's experiment had profound influence in embarking the exploration about the interference phenomena in light and its applications for measurement purposes.

![Diagram of Young's double-slit experiment and interference fringes](image)

Figure 1.1 Thomas Young double-slit experiment and the interference fringes [1]

The first optical interferometer was successfully constructed in 1891 by American physicist Albert Abraham Michelson, who was later awarded Nobel Prize in 1907 primarily for this work. The principle of Michelson interferometer and an example of the interference fringes are shown in Fig. 1.2. The beam from the light source is split into two parts with a semitransparent mirror (called a beam splitter). The two beams are then reflected by two mirrors positioned at unequal distances from the beam splitter and recombined to form the light and dark band interferometric fringes. The power or the spatial shape of the resulting interference patterns is observed to perform measurements. Initially this interferometer was used in the famous Michelson-Morley experiment to
test the special theory of relativity, i.e., to show the constancy of the speed of light across multiple inertial frames. Nevertheless, this configuration has later found many other applications such as (1) Measurement of distance and displacement; (2) Measurement of the wavelength of a laser; (3) Measurement of the refractive index of a medium; (4) Modulation of the power or phase of the laser beam; (5) Measurement of the chromatic dispersion of optical components.

Figure 1.2 Schematic diagram of a Michelson interferometer [2] and an example of the interference fringe [3]

A variety of interferometers based on the same principle were proposed soon after. The arrangements are well-known as Mach-Zehnder interferometer[4, 5], Fabry-Perot interferometer[6, 7] and Sagnac Interferometer[8, 9]. A schematic illustration of these interferometers is shown in Fig. 1.3.
In the early days, the optical interferometers mainly employed white light as the light source. However, as white light has a large linewidth comprising wavelength from...
400nm to 700nm, the time period over which the light maintains perfect temporal coherence (known as the coherence time) is on the order of $10^{-12}$ second. Hence the coherence length is about 1 millimeter if the coherence time is multiplied with the speed of light ($3\times10^8$ m/s). This implies if the two arms of the interferometer differs by that tiny distance, no interference patterns will be visible. Therefore the optical interferometry had remained a laboratory technique for many decades due to the critical requirement for highly precise alignment. This difficulty was not eased until the advent of laser in 1960s. Being an intense source of highly chromatic and very directional light, laser is a remarkable coherent source. On one hand, its long coherence length had made it significantly easier to observe the interference patterns in the double-beam interferometers. On the other hand, being a highly controllable light source, laser had also offered a great opportunity for the optical interferometry to develop in a much less expensive and less complicated way, i.e., the so-called self-mixing interferometry.

1.2.2 Laser Self-mixing Interferometry

The principle of laser self-mixing interferometry is illustrated in Fig. 1.4. That is, the light emitted from a laser source is reflected by a remote target, re-enters the laser active cavity and causes modulation of both laser output frequency and intensity. When the external target is subject to a movement, the laser output power fluctuates for each half-wavelength movement of the target along the laser axis.
Chapter 1 Preliminaries

Figure 1.4 Schematic arrangement of self-mixing interferometry with a semiconductor laser

The laser self-mixing effect, or equally known as injection modulation, was first observed from a continuously operated He-Ne laser by King and Steward in 1963 [10]. They found the variation of laser intensity was dependent on the phase or the optical path of the laser beam that is injected back into the laser cavity. In the same year, Ashby and Jephcott [11] successfully measured the plasma density using an interferometer based on this observation, as shown in Fig. 1.5. In 1968, Rudd [12] reported a laser Doppler velocimeter employing similar experimental arrangements as shown in Fig. 1.6. The high output of a He-Ne laser is pointed to a moving target which reflects a small portion of light back into the laser cavity. The returned light, whose frequency and wavelength differ slightly from the original light, is mixed with the original light inside the cavity. The output of the laser as a whole is modulated at Doppler frequency and can be picked up on a photodetector. This velocimeter possesses significant advantages in terms of great simplicity, requiring the minimum number of optical components and not critical for alignment.
Meanwhile, other types of lasers were also employed to observe this effect such as in CO$_2$ lasers [13, 14] and semiconductor lasers (or laser diode, LD) [15-25]. The latter is of particular interest due to its advantages as follows (1) simplicity and compactness in that a laser diode and a focusing lens are the only components for the laser head, (2) self-aligning due to a single optical axis, (3) larger modulation because the laser active medium serves as an amplifier for the reflected light, and (4) directional discrimination by observing the inclinations of the interferometric signal waveforms.

At the early stage, the external feedback of a laser diode was found to be useful with regard to enhancement of single longitudinal mode operation, the spectral linewidth narrowing, improving frequency stability and the wavelength tenability.
[26-29]. The sensing application based on self-mixing effect with laser diode was subsequently reported by Shinohara et. al. in 1986 where a laser Doppler velocimeter was developed with a commercial Fabry-Perot (FP) laser diode [24]. In the same year, a device capable of measuring a target velocity and range was proposed by modulating a laser diode’s optical frequency and measuring the change in the phase of the light reflected back into the laser [25]. A variety of sensing applications based on the laser diode self-mixing arrangement had been found since then such as in velocimetry of a moving target [17-20, 22, 24, 30, 31], distance measurement [23, 25, 32-34], vibrometry and displacement measurement [35-38], 3-dimensional object imaging [39, 40], and recently for the measurement of laser linewidth and linewidth enhancement factor [41-44]. In addition, it has been applied for certain biomedical measurement purposes such as cardiovascular diagnoses, measurements of arterial pulse wave shape and skin vibration and intra-arterial laser Doppler velocimetry [21, 22, 45-49].

The exploration for a theoretical explanation of laser feedback effect began soon after the phenomenon was discovered in gas lasers. Clunie and Rock (1964) [50] were the first to explain the effect on the basis of Fabry-Pèrot interference where the external mirror together with the laser mirror nearer to it was regarded as a Fabry-Pèrot resonator. Such a composite mirror is then considered as one of the mirrors of the laser cavity whose transmission was examined as a function of the external cavity optical path length. Subsequently, more comprehensive analysis that are modeled on the basis of laser cavity subject to a variable loss were conducted by Gerado and Verdeyen (1964) [51], Hooper and Bekefi (1966) [52], Uchida (1967) [53] and Potter (1969) [54]. In 1972, Spencer and Lamb [55] put forward the most systematic theory for the effect which showed that the optical feedback gives both amplitude and frequency modulation. The driving term of the modulation is the optical pathlength \(2ks\) of light to the remote
target and back, where \( k = \frac{2\pi}{\lambda_0} \), \( \lambda_0 \) is the emission wavelength of the unperturbed laser and \( s \) is the distance between laser and the target. At very weak level of feedback, the modulation indexes are in quadrature, i.e., \( \cos 2ks \) for the amplitude component and \( \sin 2ks \) for the frequency component. By using a dual-frequency Zeeman He-Ne laser to recover the frequency modulation component by heterodyne detection with the second fixed frequency mode, Donati (1978) [56] demonstrated the principle of the first complete self-mixing interferometer for measurement of arbitrary displacement waveform in 1978. Nevertheless, in the cases where laser diode was adopted as the light source, this theory was found not pertinent to explain the phenomenon owing to the unique characteristics of semiconductor lasers. Firstly, the excessive frequency linewidth of laser diode made it impossible to perform frequency demodulation by heterodyne detection. This implies only the intensity modulation channel is available in laser diode self-mixing configuration in contrast to the gas laser feedback interferometers which provide two interferometric signals that are in quadrature. Secondly, as the laser diode active medium couples both optical gain and refractive index to the injected carrier density, the amplitude modulation term is different from the cosine function.

The widely accepted theory about the dynamics and stability of laser diode with optical feedback was conducted by Lang and Kobayashi yielding a set of fundamental rate equations to simulate the system [57]. The Lang-Kobayashi equations depicted the modulation of laser output intensity resulted from the variations in the optical path-length. It also coupled in the impact of the non-linear nature of the laser diode active medium by introducing the linewidth enhancement factor (LEF) into the model. The modulated laser intensity can be monitored with the photo diode enclosed in a typical laser diode package and is known as the self-mixing signal. According to the
Lang-Kobayashi theory, different feedback regimes are defined with a feedback level parameter $C$ [58, 59] featuring distinctive shaped self-mixing signal waveforms. For $C \ll 1$, there is very weak feedback regime. The self-mixing signal responds to the phase change in the same way as that of the conventional interference and takes the form of a cosine signal. For $C \leq 1$, the system works at weak feedback regime with one stable state. The self-mixing signal waveforms appear as distorted sinusoids. For $1 < C < 4.6$, known as moderate feedback regime, the system is bistable with two stable states and one unstable state corresponding to multimode lasing operation of the laser diode. This appears as the hysteresis with abrupt transitions in the output power of a single mode laser diode. For $C > 4.6$, i.e., strong feedback regime, multi-stability is predicted by the theory. In some cases, the laser enters mode-hopping regime and the interferometric measurement is no longer possible.

### 1.3 Background to This Thesis

This thesis focuses on two of the sensing applications of laser diode self-mixing interferometry among the others, that is, the target displacement measurement and the measurement of the LEF of a laser diode. Therefore, it is necessary to provide an overview of the relevant background knowledge as a corner stone for this thesis.

#### 1.3.1 Displacement Measurement

As has been mentioned in Section 1.2.2 and will be investigated in detail in Chapter 2, the laser output power with external feedback is modulated as a cosine function of the reflected laser phase. Thus one fringe of the self-mixing signal waveform corresponds to $2\pi$ change of the laser phase, i.e., half wavelength displacement of the external target. By counting the number of interference signal fringes, the displacement of a
moving target can be measured with a basic accuracy of $\lambda/2$ (i.e., 340nm for a laser diode of 780nm wavelength).

Numerous attempts have been made to improve the above measurement accuracy. The existing methods can be categorized based on their operating conditions. Under very weak optical feedback ($0 < C \ll 1$), it was found the laser output shows the same response to the phase variation as that of conventional interference. The phase measurement techniques that are widely used in conventional interferometers are also proposed to increase the measurement accuracy beyond $\lambda/2$ for self-mixing interferometry. Yoshino et al. [16] achieved a precision of 10-60 nm by detecting the optical phase change of a feedback injection current for an LD, which was supplied to keep constant intensity of undulation caused by the mirror displacement. However the measurement range was limited within several tens of micrometers by mode hopping of the LD. A phase-locked self-mixing interferometer which fixed the phase change of laser diode was proposed by Suzuki et al. [60] to measure displacement between -98 to +98 nm with accuracy better than 50 nm. Kato et al. [37] extend the measurable range to several tens of centimeters and improve the measuring accuracy below 25 nm by means of a phase detection method based on pseudo-heterodyne scheme. Wang [61] proposed a Fourier transform method to detect the initial phase of self-mixing signal and achieved the precision of 15 nm for a target displacement of 0.3 $\mu$m at the distance of 15 cm. Two years later, he applied the same technique to a dual external cavity system and improved the measurement resolution to 10 nm. However the measurable range was limited between -195 to +195 nm or $-\lambda/4$ to $\lambda/4$ due to the difficulties associated with phase unwrapping when the phase variations beyond $-\pi$ and $+\pi$.

As the optical feedback increases ($0 < C < 1$), the cosine interferometric signal is progressively distorted and the inclination of the signal fringes makes it possible for the
directional discrimination in displacement measurement. Merlo et al. [62] attempted the reconstruction of a target displacement waveform by solving the Lang-Kobayashi equations with known optical feedback level factor \( C \). A resolution on the order of tens of nanometers for a target displacement of a few micrometers was reported once the value of \( C \) is calibrated with good accuracy. Servagent et al. [63] designed a sensor which couples a phase shifter with a displacement sensor. A resolution better than 65nm for a target 10cm away from the sensor was reported for reconstructing the displacement of a target without prior information about feedback level factor \( C \). This setup was demonstrated to be well adapted for use with non-cooperative targets which was the main interest of such devices for industrial applications.

Meanwhile, The first arbitrary form displacement measurement performed under moderate external optical feedback ( \( 1 < C < 4.6 \) ) was proposed by Donati [38], where the two-level hysteresis is found in the amplitude modulated signal and allows the recovery of displacement without sign ambiguity from a single interferometric signal. The accuracy of their measurement was reported to be \( \lambda/2 \) for a target distance of 2.5m. The prototype instrument based on this technique was presented in [64] incorporating a PC interface to compensate for the temperature stabilization of the laser source. Later on, Servagent et al. [65] increased the resolution to \( \lambda/12 \) (65nm) by a signal processing technique of linear interpretation between optical power and displacement variations. Bes et al. [66] proposed an autoadaptive signal processing scheme and improved the accuracy to 40 nm for both harmonic and aleatory displacement of a remote piezoelectric actuator.

1.3.2 Linewidth Enhancement Factor (LEF) Measurement
Another important application of OFSMI discovered in recent years is to measure a characteristic parameter of semiconductor lasers – Linewidth Enhancement Factor (LEF, \(\alpha\)). LEF is responsible for the enhancement of the laser linewidth, affecting the frequency chirp, the modulation response, the injection-locking range, and the effect of external feedback [67]. The second part of this thesis deals with LEF measurement with OFSMI.

The definition of the LEF was firstly proposed by Henry [68] in order to explain the observed broadening of spectral linewidth in semiconductor lasers compared with other laser systems. He believed the excessive linewidth was due to the fluctuations in the phase of the optical field which arose from spontaneous emission. Following his pioneer work, considerable attention has been drawn to study this important phenomenon. A more often used explanation ascribes the excessive linewidth to the variations in the carrier density altering the imaginary part of the susceptibility (gain), which in turn causes changes in its real part (refractive index) via the Kramers-Kronig relations. The ratio of the partial derivatives of the real and complex parts of the complex susceptibility with respect to carrier density is defined as the LEF, i.e.,

\[
\alpha = -\frac{\frac{d}{dn} \text{Re}\{\chi(n)\}}{\frac{d}{dn} \text{Im}\{\chi(n)\}},
\]

where \(\chi\) is the complex susceptibility, \(n\) is the carrier density. It is also equivalently expressed as

\[
\alpha = -2k \frac{d\mu}{dn} \frac{dg}{dn},
\]

where \(k\) is the free-space wave vector, \(\mu\) is the refractive index and \(g\) is the electronic gain per length. The linewidth was found to be enhanced by a factor of \(1 + \alpha^2\).

Extensive research efforts have been devoted to the measurement of this significant parameter and a broad classification of the proposed techniques can be made as: 1) methods based on sub-threshold gain and refractive index measurements to yield the “material” LEF; 2) methods performed above threshold to measure the “device” LEF.
The latter is more closely matched to the behavior of lasers in operating conditions and hence can account for more practical situations. The pioneer work of Class 1 methods was carried out by Henry et al. [69]. The approximate value of $\alpha$ was deduced from analysis of spontaneous emission in buried hetero-structure lasers by measuring the refractive index and gain change within the active layer. This approach relies on an accurate knowledge of the carrier concentration in the active layer, which is not easily determined due to the uncertainties in the layer thickness, lifetime etc. Another technique that measures the “material” LEF relies on direct measurement of $dg$ and $d\mu$ when the carrier density is varied by an unknown amount $dn$ by slightly changing the current of a Semiconductor Laser (SL) in sub-threshold operating condition. Since this technique operates below threshold, it can only measure the “material” LEF as a function of injected current and photon energy. Hence there is no chance to measure the possible dependence of $\alpha$ on the optical power [70].

In contrast, a lot more approaches were proposed to allow the measurement of the “device” LEF under operating conditions. The FM/AM modulation method [71] is based on current modulation of a high frequency SL, which will in turn result in both amplitude modulation (AM) and frequency (FM) modulation in the laser. The ratio of the FM over AM components allows a direct measurement of the linewidth enhancement factor. The amplitude modulation term can be directly detected by means of a high speed photodiode, whereas the frequency modulation term is measured using a high resolution Fabry-Perot filter as it is related to laser sidebands intensity. This method is based on the hypothesis that the susceptibility is linear and the carrier density is longitudinally uniform [72]. In addition, the modulation frequency ($f_{\text{mod}}$) must be larger than the laser relaxation frequency ($f_R$) due to the dependence of FM/AM ratio on the frequency in the case of $f_{\text{mod}}<f_R$. This requirement poses experimental difficulties
for lasers with high relaxation frequency as very high speed RF generators and instrumentation is necessitated.

Based on the same principle as the FM/AM modulation, the FM/AM noise method [73] relies on the measurement of the phase correlation and the ratio between the spectral dependence of semiconductor laser FM noise and AM excess noise [74]. The AM noise can be measured by direct detection and RF spectrum analysis, while the FM noise is measured by Fabry-Perot filters or other techniques. This method requires complex experimental implementation, but is relaxed from active current modulation.

The methods based on injection locking are another category of approach that measures the value of $\alpha$ above threshold [75-79]. The principle of injection locking is that the injection of light from a master LD into a slave LD causes locking of the slave LD’s lasing frequency to be that of the master’s. The locking region is typically characterized by the injection level and the asymmetrical frequency detuning, due to the non-zero $\alpha$ factor. This category of methods are based on the complex theory of injection locking dynamics, however, simplified analytical dependence of the measured quantities such as asymmetric detuning range can be established on $\alpha$ factor. These techniques are of complicated experimental implementation, and the accuracy of measurement is dependant on the availability of knowledge about the injection level whose measurement is generally very difficult.

Similar to the injection locking phenomena, the behavior of LD subject to optical feedback exhibits some dependence on the value of the LEF as well as the optical feedback strength. Hence Yu et al. [42] proposed to determine the LEF by developing a relationship between the LEF and the interferometric waveforms without necessitation to measure the feedback level. However due to some special requirement on the shape of the interferometric waveforms, this method has to be performed under moderate
feedback levels. Xi and Yu et al. [43, 44] then proposed a data-to-model fitting approach to extend the measurement under weak feedback conditions by identifying a set of optimal parameters in the theoretical model including the LEF that yield the best agreement between the experimental observations and the theoretical predictions. The OFSMI based approaches have shown some superiority over the other methods in terms of the simple implementation without additional optical instrumentations and comparably high measuring accuracy.
1.4 Research Issues

1.4.1 Pre-processing of Self-Mixing Signals

For the OFSMI method, the quality of the experimentally acquired self-mixing signals plays an important role in improving the accuracy of sensing and measurement. In practice, there are always uncontrollable factors such as ambient light and noises, fluctuation in temperature and the LD driving source, together with the electronic interferences due to the electronic components used in the data acquisition process. These adverse experimental conditions will inevitably result in the self-mixing signals to be distorted and contain unwanted noises. Hence before the precision of measurement can be improved, the prominent issue of the data quality has to be addressed.

Noise elimination is a common topic in the digital signal processing context, however the attempts on removing noisy data from the self-mixing signals is seldomly investigated in the literatures. The research still remains open for finding the competent signal processing techniques to retrieve the “clean” data from the noisy experimental self-mixing signals.

The motivation of Chapter 3 is to find such a technique that is capable of effectively eliminating the noises contained in the self-mixing signals.

1.4.2 Displacement Measurement

Much has been done in the literature on the topic of measurement of the displacement of a moving reflector with OFSMI system operating under different feedback strength conditions. Under very weak feedback regime, the employment of phase measurement techniques has achieved very high measuring accuracy up to 10nm ($\lambda/80$) in [16, 33, 37, 60, 61]. However, there are three major problems associated with
the phase measurement techniques that operate under very weak feedback regime. Firstly, the signal-to-noise ratio is low since the feedback signal is very weak. This will in turn result in the degradation in the measurements. Secondly, the directional ambiguity still exists as the self-mixing signal is sinusoidal and no directional discrimination can be achieved without other manipulations. Thirdly, either costly high precision optical apparatus or complicated algorithms have to be involved in order to retrieve the phase information of reflected light.

On the other hand, under weak and moderate feedback conditions, the self-mixing signal offers significant convenience for the directional discrimination with its sawtooth-like waveform. Measurement accuracy up to 65nm and 40nm has been reported under weak feedback and moderate feedback condition respectively [38, 62, 63, 65, 66]. Whereas, these techniques did not offer a mechanism to combat the noises possibly contained in the self-mixing signals due to the interference in a not-so-ideal environment that the measurement is carried out.

Chapter 4 attempts to recover the displacement of a moving target under weak and moderate feedback conditions using the noisy self-mixing signals that have been pre-processed with the techniques presented in Chapter 3.

### 1.4.3 LEF Measurement

The linewidth enhance factor (LEF) has been recognized as a significant parameter dominating a few characteristic properties for a semiconductor laser. The measurement of the LEF with OFSMI offers remarkable simplicity and accuracy in terms of required optical components and system implementation. The first successful measurement was reported by Yu et al. [42] where the relationship between observed OFSMI signal and the LEF was developed. However, due to the special requirement on the shape of the
interferometric waveforms, it can only be performed when $C$ falls in a small range (i.e., $1 < C < 3$) for the LEF between 3 and 5 which poses some difficulties on the adjustment of the feedback in practice. Moreover, since only the information about four data points on a period of the OFSMI waveform is used in doing the measurement, this approach is highly susceptible to noises and other interferences in the process of acquiring the self-mixing signals.

The other class of OFSMI based method for LEF measurement was working under weak feedback regime ($0 < C < 1$) based on the data-to-model fitting paradigm [43, 44]. As a few segments of the self-mixing signal data samples are employed in this class of methods, they are more robust and immune to noises. Nonetheless, the method in [43] assumes that the external target motion law is known in advance. This requires high precision apparatus to control the movement of external target. The approach in [44] addresses the situation that the external target is driven by a sinusoidal signal with unknown frequency and amplitude, the employed gradient based optimization algorithm requires the initial searching parameter values to be close to their real counterparts in order to prevent the algorithm from converging to a local minimum.

Chapter 5 endeavors to find a more generally applicable solution for the LEF measurement with data-to-model fitting technique.
1.5 Approaches and Contributions

The aim of the work in this thesis is two-fold. In the first half, it attempts to recover the displacement of a moving target with improved accuracy. The second half of this thesis is dedicated to improve the effectiveness and accuracy of the Linewidth Enhance Factor (LEF) measurement of a laser diode with the OFSMI setup. In order to achieve both aims, the self-mixing signals are firstly pre-processed with suitable techniques for the purpose of eliminating noisy data that may interfere with the measurements.

To this end, Chapter 3 firstly discerns the characteristics of the noises contained in the self-mixing signals by looking at a few segments of typical experimental waveforms under different feedback conditions. Two types of noises were subsequently recognized. One is the data that is significantly deviate from the adjacent data samples. This kind of noises includes the “spikes” and can be effectively removed with a median filter. The other kind of noise was found to take the form of fast varying random noise in contrast to the self-mixing fringe frequency. In order to eliminate this sort of noise, a curve fitting approach is employed by means of artificial neural networks (ANN). Remarkable improvement is shown in the data quality as a result of numeral simulations and with experimental self-mixing signals. The pre-processed data was then used in carrying out the following measurements.

Chapter 4 attempts to reconstruct a target displacement by configuring the OFSMI system to exhibit the unsymmetrical sawtooth-like self-mixing waveforms under weak and moderate feedback regimes. The analytical solution for target displacement is derived by solving the Lang-Kobayashi equations. Since the operation of inverse cosine is involved in obtaining optical phase changes from the self-mixing signal, a phase unwrapping technique is employed to recover the phase to its real values beyond \([0, \pi]\) which is the principal interval of the inverse cosine function. The target displacement
has shown to be recovered with accuracy of 31.2nm (\(\lambda/25\)) for weak feedback and 39nm (\(\lambda/20\)) for moderate feedback condition with computer simulations. These accuracies are slightly better or comparable with those offered by other approaches under similar feedback conditions. However, owing to the signal pro-processing procedures, our approach shows advantages when the measurement is retrieved from the noisy self-mixing signals with signal to noise ratio up to 20 dB.

Chapter 5 endeavors to estimate the linewidth enhancement factor of an LD with data-to-model fitting technique under weak feedback regime OFSMI system setup. For this purpose, a thorough investigation about the properties of the cost function is firstly carried out with regard to the whole possible parameter ranges. It was found the cost surface exhibits a few local minima and a unique global minimum where all parameter values correspond to their real counterparts. Based on this observation, a genetic searching scheme is then introduced to locate the global minimum of the cost function. However owing to the uneven sensitivity of the cost function with respect to different parameters in the theoretical model, the standard genetic algorithm (GA) fails to yield a fair estimation for the less sensitive parameter – the LEF. Hence a multi-staged GA is proposed to refine the searching by employing a strategy to repeat the algorithm in a reduced version with smaller population and shorter chromosomes and concentrate the searching area in the neighborhood of solutions produced in the previous steps. Computer simulations and experiments have shown that good measurement results are achieved after performing the modified GA for two or three rounds.

The proposed approach allows measurement of the LEF with comparably good accuracies to the existent methods. Moreover, no a priori information about the target movement is required as the other approaches do.
Chapter 2

Background about Optical Feedback Self-mixing System
2.1 Introduction

The primary objective of this chapter is to describe the fundamental concepts and the theoretical model of an optical feedback self-mixing interferometry (OFSMI) system. Section 2.2 introduces the general structure of an OFSMI system. Section 2.3 reviews the derivation of the famous Lang-Kobayashi model which laid the foundation for the sensing applications of OFSMI. In Section 2.4, the behavior of OFSMI system is investigated in terms of the presentation of interferometric waveforms under different feedback level regimes. Section 2.5 summarizes this chapter.

2.2 Structure of OFSMI System

A typical self-mixing interferometer comprises of three parts – the optical head, the electronic controllers and data acquisition unit as shown in Figure 2.1. The optical head is made up of a single mode laser diode, the trans-impedance amplifier, the focusing lens and variable attenuator (optional). The DC-coupled trans-impedance amplifier amplifies and converts the current generated by the photo diode in the laser package into voltage. The focusing lens focuses the laser spot onto the distant target while the attenuator serves to adjust the strength of the feedback signal to obtain different feedback conditions. The electronic controllers contain the LD power supply, the temperature controller to prevent laser mode hopping due to temperature fluctuation, and the device (e.g., a piezoelectric transducer driver) to generate the driving signal for the external target movement. The data acquisition and signal processing can be accomplished with a DAQ card connected to a PC or hardware circuits.
2.3 Theoretical Model of a Semiconductor Laser with External Feedback

2.3.1. Phase Condition [59]

In order to understand a semiconductor laser system with external feedback (or known as external cavity laser), we start by considering the lasing conditions for a laser without external cavity.

The field amplitude for both forward and backward directions is considered as seen in Figure 2.2.
Chapter 2 Background about Optical Feedback Self-mixing System

Figure 2.2 Power flow in forward and backward direction in a semiconductor laser with facet reflectivities $R_1$ and $R_2$ [59]

The forward traveling complex electrical field $E_f(z)$ is expressed as

$$E_f(z) = E_{f0} \exp(-j\beta z + \frac{1}{2}(g - \alpha_s)z)$$  \hspace{2em} (2.1)

$g$ is the linear gain per unit length due to the stimulated emission inside the laser cavity, $\alpha_s$ is assumed the optical loss per unit length within the cavity, $\beta$ is the phase constant denoted as

$$\beta = \frac{4\pi\nu\mu_e}{c}$$  \hspace{2em} (2.2)

with $\nu$ being the optical frequency, $\mu_e$ being the effective refractive index and $c$ being the speed of light in vacuum.

Similarly, we have backward traveling wave amplitude:

$$E_b(z) = E_{b0} \exp(-j\beta(L-z) + \frac{1}{2}(g - \alpha_s)(L-z))$$  \hspace{2em} (2.3)

The relationship between $E_f$ and $E_b$ is established according to

$$E_f(z=0) = E_{f0} = \eta E_b(z=0) = \eta E_{b0} \exp(-j\beta L + \frac{1}{2}(g - \alpha_s)L)$$  \hspace{2em} (2.4)
Chapter 2 Background about Optical Feedback Self-mixing System

\[
E_{z=0} = r_z E_f(z = L) = r_z E_{f0} \exp(-j\beta L + \frac{1}{2}(g - \alpha) L)
\]  
(2.5)

with \(R_1 = |r|^2\), \(R_2 = |r_e|^2\).

By substituting Eq. (2.4) into Eq. (2.5), the condition for a stationary laser oscillation is obtained:

\[
r_1 r_2 \exp(-2j\beta L + (g - \alpha) L) = 1
\]  
(2.6)

The real part of Eq. (2.6) yields an amplitude condition for the threshold gain \(g_{th}\), while the imaginary part of Eq. (2.6) yields a condition for the phase constant \(\beta\). Assuming \(r_1\) and \(r_2\) are real, \(r_1 = \sqrt{R_1}\), \(r_2 = \sqrt{R_2}\), we obtain

\[
g_{th} = \alpha_s + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right)
\]  
(2.7)

and

\[2\beta L = 2m\pi, \quad m = \text{integer}.
\]  
(2.8)

In the case of a laser with external optical feedback, a portion of the light emitted from a laser source is reflected by an external reflector such as a mirror or a diffused target, re-enters the laser cavity and mixed with the light inside the cavity. A schematic arrangement is shown in Figure 2.3.

---

Figure 2.3 Schematic arrangement of a laser diode cavity with external feedback
$r_1$ and $r_2$ denote the amplitude reflection coefficients of the laser facets $F_1$ and $F_2$ respectively. $r_3$ represents the amplitude reflection coefficient of the external reflector $F_3$. In order to apply the above developed lasing condition equations to laser with external cavity, we introduce an effective reflection coefficient $r_{2e}$ at $z = L$, i.e.,

$$r_{2e}(\nu) = r_2 + (1 - |r_2|^2)r_3 \exp(-j2\nu\tau_L)$$  \hspace{1cm} (2.9)

Where $\nu$ denotes the optical frequency, $\tau_L$ denotes the round trip delay through the external cavity of length $L$. $1 - |r_2|^2$ represents the light transmission through the laser facet $F_2$. Multiple reflections within the external cavity is neglected in Eq. (2.9) because in general, the external reflection coefficient $r_3$ is much less than the reflection coefficient $r_2$ of the laser facet, i.e., $|r_3| \ll |r_2|$. If $r_{2e}$ is written as a function of amplitude and phase as

$$r_{2e}(\nu) = |r_{2e}| \exp(-j\phi_r)$$ \hspace{1cm} (2.10)

Eq. (2.9) will yield

$$|r_{2e}| = r_2 (1 + \kappa_{ext} \cos(2\nu\tau_L))$$ \hspace{1cm} (2.11)

with $\kappa_{ext}$ measuring the coupling strength (coefficient) between the two cavities as

$$\kappa_{ext} = \frac{r_3}{r_2} (1 - |r_2|^2)$$ \hspace{1cm} (2.12)

and

$$\phi_r = \kappa_{ext} \sin(2\nu\tau_L)$$ \hspace{1cm} (2.13)

for $\kappa_{ext} \ll 1$.

Since the round trip phase within the compound laser cavity also must meet the phase condition for the solitary laser according to Eq. (2.8), that is,
\[ 2\beta L + \phi_r = 2\pi m, m = \text{integer} \]  

(2.14)

By substituting \( \beta \) with the denotation in Eq. (2.2), the phase condition in Eq. (2.14) may be rewritten as

\[ \frac{4\pi \mu \nu L}{c} + \phi_r = 2\pi m \]  

(2.15)

The emission frequency without optical feedback \( \nu_{th} \) can be obtained by solving Eq. (2.15) by letting \( \phi_r = 0 \). In the presence of optical feedback, the emission frequency \( \nu \) and the threshold gain may change, resulting in variation in the refractive index, thus the change of \( \mu, \nu \) can be written as

\[ \Delta(\mu_e \cdot \nu) = \nu_{th} \Delta e + (\nu - \nu_{th}) \mu_e \]  

(2.16)

This may be expressed in the phase condition as

\[ \Delta \phi_L = \Delta(\mu_e \nu)4\pi L / c + \Delta \phi_r = (4\pi L / c)[\nu_{th} \Delta e + \mu_e (\nu - \nu_{th})] + \phi_r \]  

(2.17)

where \( \Delta \phi_L \) corresponds to the phase change in the round trip. Since the change in the effective index \( \mu_e \) may be expressed as

\[ \Delta \mu_e = \frac{\partial \mu_e}{\partial n} (n - n_{th}) + \frac{\partial \mu_e}{\partial \nu} (\nu - \nu_{th}) \]  

(2.18)

with the carrier density \( n_{th} \) denoting the threshold carrier density without feedback. Inserting Eq. (2.18) into Eq. (2.17), we obtain

\[ \Delta \phi_L = \frac{4\pi L}{c} \left[ \nu_{th} \frac{\partial \mu_e}{\partial n} (n - n_{th}) + \mu_e (\nu - \nu_{th}) \right] + \phi_r \]  

(2.19)

where the effective group refractive index \( \mu_e \) is denoted as

\[ \mu_e = \mu_e + \nu \frac{\partial \mu_e}{\partial \nu} \]  

(2.20)

For the purpose of depicting the wave propagation in the laser medium, a complex refractive index is defined as
\[ \mu = \mu' - j\mu'' \]  
(2.21)

\( \mu' \) and \( \mu'' \) are affected by the stimulated emission and are linked by the Kramers-Kroenig relations \([80]\). The coupling between \( \mu' \) and \( \mu'' \) is expressed by a parameter \( \alpha \), known as the Linewidth Enhancement Factor (LEF)

\[ \alpha = \frac{\Delta \mu'}{\Delta \mu''} \]  
(2.22)

The stimulated gain \( g_s \) is related to the imaginary part of the refractive index \( \mu'' \) by

\[ \mu'' = \frac{g_s c}{4\pi\nu} \]  
(2.22a)

Thus the variation of the refractive index with respect to varying carrier density corresponds to gain variations via the parameter \( \alpha \), yielding

\[ \frac{\partial \mu_c}{\partial n} = \alpha \frac{\partial \mu'_c}{\partial n} = \alpha \frac{\partial \mu''_c}{\partial n} = -\alpha \frac{\partial g}{\partial n} \frac{c}{4\pi\nu_{th}} \]  
(2.23)

Hence

\[ \frac{\partial \mu_c}{\partial n}(n-n_{th}) = -\frac{\alpha c}{4\pi\nu_{th}} \frac{\partial g_c}{\partial n}(n-n_{th}) = -\frac{\alpha c}{4\pi\nu_{th}} (g - g_{th}) \]  
(2.24)

Since \( g \) must satisfy the amplitude condition with \( g = g_c \), where \( g_c \) is the threshold gain for the compound cavity, Eq. (2.19) can be written as

\[ \Delta \phi_c = -\alpha (g_c - g_{th}) L + \frac{4\pi\mu_c L}{c} (\nu - \nu_{th}) + \phi_c \]  
(2.25)

where \( g_{th} \) is the threshold gain without optical feedback as is defined in Eq. (2.7).

Thus the threshold gain difference \( (g_c - g_{th}) \) due to feedback can be inferred from the amplitude condition Eq. (2.6) and Eq. (2.7) as

\[ (g_c - g_{th}) = -\frac{\kappa_{ext}}{L} \cos(2\pi\nu\tau_{ext}) \]  
(2.26)

The round trip phase change \( \Delta \phi_c \) is obtained by inserting Eqs. (2.13) and (2.26) into Eq.
Chapter 2 Background about Optical Feedback Self-mixing System

(2.25) as

$$\Delta \phi_L = \frac{4 \pi \mu L}{c} (v - v_{th}) + \kappa_{ext} \left[ \sin(2 \pi \nu \tau_L) + \alpha \cos(2 \pi \nu \tau_L) \right]$$  \hspace{1cm} (2.27)

By introducing round trip delay of laser cavity $\tau_d = 2 \mu L / c$, Eq. (2.27) reads

$$\Delta \phi_L = 2 \pi \tau_d (v - v_{th}) + \kappa_{ext} \sqrt{1 + \alpha^2} \sin(2 \pi \nu \tau_L + \arctan \alpha)$$
$$= \frac{\tau_d}{\tau_L} \left[ 2 \pi \tau_L (v - v_{th}) + \frac{\tau_d}{\tau_L} \kappa_{ext} \sqrt{1 + \alpha^2} \sin(2 \pi \nu \tau_L + \arctan \alpha) \right]$$  \hspace{1cm} (2.28)

When the phase condition for compound cavity laser is satisfied, i.e., $\Delta \phi_L = 0$, Eq. (2.28) may be written as

$$2 \pi \tau_L v = 2 \pi \tau_L v_{th} - C \cdot \sin(2 \pi \nu \tau_L + \arctan \alpha)$$  \hspace{1cm} (2.29a)

or

$$\phi_p(\tau_L) = \phi_o(\tau_L) - C \cdot \sin[\phi_p(\tau_L) + \arctan \alpha]$$  \hspace{1cm} (2.29)

with the following denotations:

- The phase of light without external feedback $\phi_o(\tau_L) = 2 \pi v_{th} \tau_L$;
- The phase of light with external feedback $\phi_p(\tau_L) = 2 \pi v \tau_L$;
- The feedback level factor (also called feedback coefficient)

$$C = \frac{\tau_L}{\tau_d} \kappa_{ext} \sqrt{1 + \alpha^2}$$

2.3.2 Intensity Modulation with External Feedback [81]

According to the rate equation [57], the dependence of the optical gain $G$ on the injection carrier density $N$ due to the stimulated emission in the laser active cavity can be described by

$$\frac{d}{dt} N = \frac{J}{e} - \kappa_i N - G(N) I$$  \hspace{1cm} (2.30)

where $J$ stands for the injection current, $e$ represents the electric charge, $\kappa_i$ is the
inverse spontaneous lifetime of the excited carriers, and $I$ represents the photon density in the active layer. $I$ can be normalized to make the optical intensity $P = |E|^2$, where $E$ represents the amplitude of the electric field inside the cavity.

Assuming that the steady state laser without and with external feedback is characterized by optical intensities $P_0$ and $P$ and carrier densities $N_0$ and $N$ respectively. Eq. (2.30) can be written for conditions without and with feedback as

$$\frac{J}{e} = \kappa_i N_0 + G(N_0)P_0$$  \hspace{1cm} (2.31)
$$\frac{J}{e} = \kappa_i N + G(N)P$$  \hspace{1cm} (2.32)

By linearizing $G(N)$ around the center of $N_0$, the following is obtained

$$G(N) = G(N_0) + \Delta G = G_0 + \kappa_2 N_0$$ \hspace{1cm} (2.33)

where $G_0 = G(N_0)$ and $\Delta G = \kappa_2 N_0$ with $\kappa_2 = dG/dN$.

Substituting Eqs. (2.31) and (2.33) into Eq. (2.32) with the assumption of $\Delta G \ll G_0$, the first order approximation for the optical intensity $P$ can be resulted as

$$P = P_0 (1 - \kappa_4 \Delta N)$$ \hspace{1cm} (2.34)

where $\kappa_4 = \frac{1}{G_0} (\kappa_2 + \frac{\kappa_i}{P_0})$ is a coefficient that is related to the gain $G_0$ and the optical intensity $P_0$ without optical feedback.

Since it is also known that the amount of linear gain $g$ produced by the laser active gain medium is determined by the carrier density $N$ [82], hence the following arises

$$\Delta N = \kappa_4 \Delta g$$ \hspace{1cm} (2.35)

where $\kappa_4$ is a constant.

Substituting Eqs. (2.26) and (2.35) into Eq. (2.34), we can obtain
\[ P(\tau_L) = P_0[1 + m \cos(\phi_E(\tau_L))] \]  
\[ (2.36) \]

where \( m = \frac{K_2}{G_0} \left( \kappa_0 + \frac{K_1}{P_0} L \right) \) is termed the modulation coefficient of the self-mixing interference. \( \kappa_{\text{ext}} \) measures the coupling strength between the laser active and external cavities as was defined in Eq. (2.12).

Obviously the intensity modulation is a periodic function of the external cavity laser phase with the period of \( 2\pi \) radians. Eq. (2.36) together with Eq. (2.29) is well-known as the Lang-Kobayashi equations or the theoretical model for the self-mixing interference in a single-mode diode laser with external optical feedback.

### 2.4 Influence of Feedback Level Factor (C) on the Self-mixing Waveforms

The properties of the above theoretical model have been investigated exhaustively which reveals that the OFSM system behaves very distinguishingly when \( C \) takes differently values [57, 58, 83-85]. These results have been in good agreement with the experimental observations. This section revisits how the feedback regimes are classified based on the value of \( C \) and the characteristics of the corresponding interferometric waveforms of each feedback regime.

#### 2.4.1 Division of Feedback Level Regimes

It has been seen that Eq. (2.29) reveals the dependence of external reflected laser phase \( \phi_E \) on the laser phase without external feedback \( \phi_0 \) and is rewritten as follows:

\[ \phi_E(\tau) = \phi_0(\tau) - C \cdot \sin[\phi_E(\tau) + \arctan \alpha] \]

where \( \tau \) is the round trip delay through the laser external cavity.

To help elucidate its physical meaning, this relationship is depicted in a graphical
form as in Fig. 2.4 where \( \alpha = 4 \) for both bases.

![Illustration of the relationship between \( \phi_p(\tau) \) and \( \phi_0(\tau) \)](image)

As is seen, when \( C \) takes a small value such as 0.7, the relationship between \( \phi_p(\tau) \) and \( \phi_0(\tau) \) is simple monotonic. Whereas when \( C \) equals 3, there are certain areas where one \( \phi_0(\tau) \) value corresponds to three \( \phi_p(\tau) \) values as marked points A, B and C.

In order to fully investigate the properties of the function given in Eq. (2.29), the derivatives of \( \phi_0(\tau) \) with respect to \( \phi_p(\tau) \) are studied. For the purpose of simplifying the expressions, we rewrite Eq. (2.29) as

\[
y(x) = x - C \sin(y + k)
\]

where \( x = \phi_0(\tau) \), \( y = \phi_p(\tau) \) and \( k = \arctan \alpha \). Thus the first derivative of \( x \) with respect to \( y \) is as follows:

\[
\frac{dx}{dy} = 1 + C \cdot \cos[y + k]
\]

In the case \( 0 < C < 1 \) (denoted as weak feedback regime), the above derivative is always positive which implies the phase with external feedback versus the phase without feedback is a monotonic function. Hence Eq. (2.29) will result in only one
solution and thus a single emission frequency as can be deduced from Eq. (2.29a).

The complexity arises in the case $C > 1$ where the mapping from $y$ to $x$ becomes uncertain. By letting the first derivative to zero, i.e.,

$$\frac{dx}{dy} = 1 + C \cdot \cos[y + k] = 0 \quad (2.39)$$

It is seen that within each $2\pi$ interval, there are two extrema for $x$ when

$$y_1 = 2n\pi + \varphi - k$$
$$y_2 = 2(n + 1)\pi - \varphi - k$$

where $n = 1, 2, 3, \ldots, \varphi = \arccos\left(-\frac{1}{C}\right)$ and $0 < \varphi < \pi$. By taking the second derivative of $x$ with respect to $y$, it can be found

$$\frac{d^2x}{dy^2} \bigg|_{y=y_1} = -C \cdot \sin(y_1 + k) = -\sqrt{C^2 - 1} < 0 \quad (2.40)$$
$$\frac{d^2x}{dy^2} \bigg|_{y=y_2} = -C \cdot \sin(y_2 + k) = \sqrt{C^2 - 1} > 0 \quad (2.41)$$

Thus $x$ exhibits a relative maximum $x_1$ at $y_1$ and a relative minimum $x_2$ at $y_2$ given by

$$x_1 = 2m\pi + \varphi - k + \sqrt{C^2 - 1}$$
$$x_2 = 2(m + 1)\pi - \varphi - k - \sqrt{C^2 - 1}$$

For illustrative purpose, Figure 2.5 (a) and (b) indicate the relationship between $y$ and $x$ with marked extrema when $C = 3$ and $k = \arctan(4)$ [86].
It is seen that there are three possible $y$ values whilst $x_{m,2} < x_{m,1} < x_{m+1,2}$, equivalent
to the following:

$$2\pi - \varphi - \sqrt{C^2 - 1} - k < \varphi + \sqrt{C^2 - 1} - k < 4\pi - \varphi - \sqrt{C^2 - 1} - k$$

Solving this inequity for $C$ yields

$$1 < C < 4.6$$

This feedback level interval corresponds to moderate feedback regime where there are three possible lasing modes. However, extensive experimental work has proven that in practice only one mode can be excited due to mode competition. This can be understood more easily with Figure 2.6. When $\phi_b(\tau)$ increases continuously with time from point B to A, $\phi_r(\tau)$ also tends to increase in a continuous manner until at point A where $\frac{d\phi_r(\tau)}{d\phi_b(\tau)} = \infty$, any further increase of $\phi_b(\tau)$ will cause $\phi_r(\tau)$ to present an abrupt upward switching (or a jump). On the contrary, when $\phi_b(\tau)$ decreases from point A’ to B, a downward switching (or a drop) will occur at point B where $\frac{d\phi_r(\tau)}{d\phi_b(\tau)} = -\infty$. That is, $\phi_r(\tau)$ evolves along different route for increasing and decreasing $\phi_b(\tau)$.

This mismatching in $\phi_r(\tau)$ in turn results in the hysteretic self-mixing waveforms in the moderate feedback regime which will be discussed in the following text.
Similarly, if $x_m$ falls between $x_{m+1,2}$ and $x_{m+2,2}$, there can be five possible modes for the laser. Solving the inequity

$$2\pi - \varphi - \sqrt{C^2 - 1} - k < \varphi + \sqrt{C^2 - 1} - k < 4\pi - \varphi - \sqrt{C^2 - 1} - k$$

gives the range for $C$ as $4.6 < C < 7.79$. Generally the feedback level factor beyond 4.6 is classified as strong feedback regime. And the behavior of self-mixing system operating under strong feedback becomes very complicated and chaotic so that it is infeasible for the sensing applications to be performed in this feedback region.

### 2.4.2 Characteristics of Self-mixing Waveforms under Weak and Moderate Feedback Regimes

Up to now, the self-mixing effect has been considered in the case that the external target is static at the distance $L$ from the front facet of the laser diode. If the target moves along the laser emission axis, then the distance from the laser to the target becomes a function of time $L(t)$, and $\tau$ varies with time as
\[ \tau(t) = \frac{2L(t)}{c} \] (2.42)

Substituting Eq. (2.42) into Eq. (2.36), laser intensity also fluctuates with time as

\[ P(t) = P_0[1 + m \cos(\phi_f(2L(t)/c))] \] (2.43)

The self-mixing signal (or interferometric signal) is obtained by monitoring the laser intensity fluctuations with the photodiode enclosed at the rear mirror in a typical diode laser. By introducing the expression

\[ G(t) = \cos(\phi_f(2L(t)/c)) \] (2.44)

we obtain from (2.43) that

\[ G(t) = \frac{P(t) - P_0}{mP_0} \] (2.45)

Obviously \( G(t) \) represents the influence of external cavity length to the laser intensity. Typically the modulation coefficient \( m \) approximates \( 10^{-3} \) [43, 87]. When the target vibrates along the laser axis, \( G(t) \) also varies periodically with time \( t \). Since \( G(t) \) exhibits the same changing law as that of the laser intensity \( P(t) \), the properties of \( G(t) \) will be studied in the following text and equivalently referred as self-mixing signal.

1. Weak Feedback Regime \((0 < C < 1)\)

As was discussed in Section 2.3.1, under weak feedback regime, the laser operates in single frequency and \( \phi_f \) varies monotonically with \( \phi_0 \). Thus Eq. (2.44) will also yield a monotonic function for \( G(t) \). To give a few illustrative examples of the self-mixing waveforms, \( G(t) \) is plotted for different feedback level factors assuming the external target is driven by a sinusoidal signal and \( \alpha = 4 \) as shown in Fig. 2.7. It is clearly seen for a small value of \( C \) such as 0.01 and 0.2 the self-mixing waveform is sinusoidal as in
the conventional interference, but for larger values of C such as 0.5 and 0.8 the self-mixing waveforms become asymmetrical or sawtooth-like. Another important feature of self-mixing waveforms is the inclination of the fringes. When the external reflector changes its direction of movement, the sawtooth-like waveforms also change the direction of their fringe inclination. This feature is very useful in terms of directional discrimination for displacement measurement as will be elaborated in Chapter 4.

Figure 2.7 A theoretical simulation of self-mixing interference waveforms. From top to bottom: (1) Displacement of external target (2) C=0.1 (3) C=0.2 (4) C=0.5 (5) C=0.8

As \( G(t) \) is a cosine function of \( \phi_r(\tau(t)) \), one fringe of the self-mixing waveform should correspond to \( 2\pi \) change in the external reflection phase. Thus the target displacement \( \Delta L \) for one self-mixing fringe is [88]
\[ |\Delta(2\pi n_L)| = 2\pi \iff |\Delta L| = \frac{c}{2n_{\text{th}}} = \frac{\lambda_0}{2} \]

In other words, one fringe in the self-mixing waveform corresponds to a half wavelength displacement at the external target. Although this conclusion is the same as in conventional interference waveform, along with the observation of the fringe inclinations, it is readily to get the target displacement readout with $\lambda_0/2$ resolution without directional ambiguity in the self-mixing interferometry in contrast to conventional interferometer.

2. Moderate feedback regime ($1 \leq C < 4.6$)

We had seen that there are three possible modes existing in this feedback regime. However, only one mode can be excited for a single mode laser diode in practice. We now look at how this is interpreted theoretically.

By taking cosine function of the discontinuous phase signal $\phi_0(\tau)$ as in Figure 2.6, the resultant self-mixing waveform is shown in Figure 2.8. When $\phi_0(\tau)$ increases from point O, $G(\tau)$ evolves along the theoretical waveform until reaches point A, where a downward transition to point A’ occurs. As $\phi_0(\tau)$ increases again, the next transition will occur at point C. On the other hand, when $\phi_0(\tau)$ decreases from point C’ and reaches point B, an upward transition to B’ takes place. This accounts for the hysteresis presented in a typical moderate feedback self-mixing waveform. As an example, Figure 2.9 gives a few simulated self-mixing waveforms when $C$ takes different values between 1 and 4.6, $\alpha = 3$ and the external target is subject to a sinusoidal movement. Clearly the waveforms appear as sawtooth-like with sharp switchings. The larger the $C$ value, the more inclined the self-mixing signal will lose its zero-crossing point, i.e., all the upper part of the signal are larger than zero and the lower part of signal less than
zero. It is also seen that the self-mixing fringe inclination reverses when the external target alters its direction of movement similar as in the weak feedback regime. Likewise in the weak feedback regime, one fringe of the self-mixing waveform also corresponds to $\frac{\lambda_0}{2}$ displacement in the target movement under moderate feedback level.

Figure 2.8 Discontinuity in the self-mixing waveform under moderate feedback regime

Figure 2.9 Simulated self-mixing waveforms in moderate feedback regime.

From top to bottom: (1) Displacement of external target (2) $C=2$ (3) $C=3$ (4) $C=4$
2.5 Conclusions

This chapter considers the OFSMI system from a theoretical perspective. The development of the theoretical model is reviewed firstly, yielding the following equations

\[ \phi_f(\tau_L) = \phi_0(\tau_L) - C \cdot \sin[\phi_f(\tau_L)] + \arctan \alpha \]  

(2.29)

and

\[ P(\tau_L) = P_0[1 + m \cos(\phi_f(\tau_L))] \]  

(2.36)

where

- \( \tau_L \) denotes the round trip delay through the external cavity of length \( L \);
- \( \phi_0 \) and \( \phi_f \) denote the phase of light without and with external feedback respectively;
- \( P_0 \) and \( P \) represent the laser intensity without and with external feedback respectively;
- \( C \) represents the feedback level factor;
- \( \alpha \) is the linewidth enhancement factor
- \( m \) is the modulation coefficient (typically \( \approx 10^{-3} \))

The theoretical model is then analyzed and three different feedback regimes are defined as a result. The self-mixing signal waveforms appear distinctively under different feedback regimes. To sum up, under very weak feedback regime \( (0 < C \ll 1) \), the self-mixing waveform appears sinusoidal. As the feedback level increases \( (0 < C < 1) \), the self-mixing waveforms exhibit distortion and become sawtooth-like. Further increase of the feedback level \( (1 < C < 4.6) \) will result in hysteresis in the interferometric waveforms and fast upward and downward switchings characterize the waveforms.
It should be noted that the inclination of the sawtooth-like self-mixing waveforms under weak and moderate feedback conditions plays a key role in determining the moving direction of an external target. This is a remarkable feature that an OFSMI system offers for the measurement of the displacement of a moving target as opposed to the conventional interferometers that do not possess such an advantage.
Chapter 3 Pre-processing of the Self-mixing Signals in the

OFSM Interferometry
3.1 Introduction

An OFSMI system comprises of both optical and electronic components. Hence it is susceptible to the disturbances such as fluctuation in temperature and LD driving source, ambient light and the electronic interference in the data acquisition process. This will inevitably introduces noises into the observed self-mixing signals and result in degradation in the sensing and measurement performed based on the self-mixing signals. It is especially the case when the sensing has to be done in an adverse environment other than a well-controlled lab situation. Hence the removal of the noises is an important research issue for getting improved accuracy of measurement with an OFSMI system. Traditionally, the OFSMI system is optimized in terms of optical structure and power detection circuits in order to increase the signal to noise ratio (SNR) of the self-mixing signals. However, the data quality is still far from “clean” and further improvement is unlikely to be achieved with the manipulations in the optical system design and implementation.

The purpose of this chapter is to address this issue from the perspective of pre-processing the self-mixing signals with appropriate digital signal processing techniques. Section 3.2 characterizes the noises contained in the self-mixing signals by observation of the self-mixing waveforms obtained under different feedback conditions with an experimental OFSMI system setup. Section 3.3 depicts the signal pre-processing procedure for removing the above noises. A curve-fitting approach employing the artificial neural network is presented subsequently. Section 3.4 and 3.5 give the simulation and experimental results for the proposed signal pre-processing procedure. Finally Section 3.6 concludes the chapter.
3.2 Features of the Noises in the Experimental Self-mixing Signals

For the purpose of showing the necessity of signal pre-processing and discerning the characteristics of the noises contained in the self-mixing signals, an OFSMI experimental system is set up as shown in Fig. 3.1. A GaAlAs laser diode of 780nm wavelength (HL7851) is used in the experiment. The laser diode is biased with a dc current of 80 mA with a commercial laser controller and operates at single mode. A metal plate is used as the target which is made to vibrate harmonically by placing it a few centimeters away from a loudspeaker driven by a sinusoidal signal. The temperature of the LD is maintained at $25^\circ C \pm 0.1^\circ C$ by the temperature controller. The self-mixing signal is detected by the monitor photodiode enclose in the same LD package and is amplified by a trans-impedance amplifier. The amplified signal is then acquired with a DAQ card (NI PCI6251). The target distance from the LD is slightly adjusted in order to get different feedback levels.

Examples of the resultant self-mixing signals are shown in Fig. 3.2 (a)-(d). The waveforms in Fig. 3.2 (a)-(c) are obtained with driving signal of 200 Hz frequency and peak-to-peak amplitude of 10 V. The sampling frequency is 200 KHz. The waveform in Fig. 3.2 (d) is obtained with driving signal amplitude of 18V and sampling frequency of 400 KHz. The enlarged one period of the signals in Fig. 3.2 (a)-(d) are presented in Fig. 3.3 (a)-(d) respectively.
Figure 3.1 (a) Schematic arrangement of the OFSMI system

(b) Experimental system setup
Chapter 3 Pre-processing of the Self-mixing Signals in the OFSM Interferometry

(a)

(b)
Figure 3.2 Examples of experimental self-mixing signal waveforms
Chapter 3 Pre-processing of the Self-mixing Signals in the OFSM Interferometry

Figure 3.3 Enlarged one period of the self-mixing signals in Fig. 3.2.

It can be seen that the most obtrusive noisy data samples are those that are significantly inconsistent with the surrounding samples. This kind of impulsive noise is referred as spikes. The impulsive noise appears more remarkable as the optical feedback becomes stronger as shown in Fig. 3.2 and 3.3 (c) & (d). Another kind of significant noise is found to take the form of fast varying random noise.

On one hand, these noises make it impossible to locate the characteristic points (such as the zero-crossing points and the peak points) accurately which is crucial for
some existent measurement approaches [42, 89]. On the other hand, they are also
detrimental to the performance of the measurement approaches based on the techniques
of data-fitting and phase unwrapping as will be discussed in the following chapters.
Hence it is essential that the self-mixing signals are pre-processed to remove the above
two types of noises before the measurements are performed.

3.3 Data pre-processing

Given the features of the noises and in order to facilitate the measurement
approaches for displacement and LEF measurement that are investigated in this thesis
work, the following requirement should be met in selecting the data pre-processing
method:

(1) The “spikes” should be removed while the “peaks” of the signals ought to be
preserved. This is because all the measurements are performed based on the
normalized self-mixing signals. The peak values are essential for doing the signal
normalization;

(2) The smooth self-mixing signal is retrieved from the experimental waveforms that are
buried in random noises.

3.3.1 Data Filtering with Median Filters

An easy and effective way of removing the impulsive noises is by means of a
nonlinear median filter. The main idea of the median filter is to run the signal through a
window and replace each signal sample with the median value of the neighbouring
samples within the window. The size of the window should be selected with care in
order to ensure both filter efficiency and preservation of the boundary information.
In order to remove the “spikes” from the OFSMI signal, we firstly pass the experimental data through a median filter. As an example, the typical OFSMI signals under weak and moderate feedback condition as shown in Fig. 3.2 (a) and (c) respectively are considered. There are 1000 samples in each period of the OFSMI signal. For the self-mixing waveform under weak feedback, the spikes are not significant and the span is less than 5 points. The filtering results with median filters of 5 points, 25 points and 35 points are presented in Fig. 3.4. In the case of fringe A, the filters yield very similar result. However for the short fringes like fringe B, the 5 points median filter best preserves the peak information of the fringe which will allow doing signal normalization more accurately.
Figure 3.4 The median filter result for the experimental OFSMI signal under weak feedback regime. (a) The raw signal (b) The filtered signal using a 5 points median filter (c) Enlarged view of fringe A processed with different size median filters (d) Enlarged view of fringe B processed with different size median filters.

Under moderate feedback regime, the spikes in the OFSMI signals become more remarkable. As the span of the spikes is less than 15 points, the self-mixing signal is processed with 15 points median filter. As a comparison, filters of 37 points and 57
points are also tried and the results are presented in Fig. 3.5. Similarly to the result under weak back, the filter size does not have notable impact on fringe A. Whereas, longer size median filters tend to lose the peak information of fringe B.
Figure 3.5 The median filter result for the experimental OFSMI signal under moderate feedback regime. (a) The raw signal (b) The filtered signal using a 15 points median filter (c) Enlarged view of fringe A processed with different size median filters (d) Enlarged view of fringe B processed with different size median filters.

The above results have shown that the median filters are capable of removing the impulsive noises effectively and achieving signal smoothing to a basic extent. However, the random noise is still significant in both cases after the filtering. Therefore, the data should be further processed.

To summarize, the procedure for removing impulsive noise with a median filter involves the following steps:

Step 1: Observe the OFSMI signal to find the duration of the spikes;
Step 2: Determine the filter length based on the spike duration;
Step 3: Apply the median filter to the raw signal to eliminate the spikes.
3.3.2 Data Smoothing with Artificial Neural Network

Recent development of neural network has recognized itself a powerful tool in a range of disciplines such as system modeling, pattern recognition and signal processing etc [90]. It represents a computing paradigm that learns through experience by mimicking the operation of human brains which differs from the conventional algorithmic approach. The basic processing element of a neural network is known as a neuron. A neural network employs a massive weighted interconnection of neurons that are organized in the form of layers. Each layer of neurons is associated with a so-called activation function, which introduces nonlinearity into the network and thus makes it more powerful.

Radial Basis Functions (RBF) is a category of functions that are particularly efficient for interpolation and smoothing of data [91] and is consequently selected to be used for our work. The Gaussian bell function is the most commonly used basis function. A typical radial basis network comprises of a hidden layer of neurons with radial basis function and a linear output which is a sum of the weighted output from hidden layer as shown in Fig. 3.6.

![Illustration of a neural network](image)

Figure 3.6 Illustration of a neural network

If we define \( x \in (x_0, x_n) \) as network input which is the distorted self-mixing signal, the network output \( \hat{y} \) can be expressed as
\[
\hat{y} = \sum_{j=1}^{N} w_{ij} \sum_{i=1}^{N} w_{ji} g_i(X)
\]  

(3.1)

where \(w_{ij}\) and \(w_{ji}\) are the weights of the network, \(g_i\) is the radial basis function for hidden neurons. Mathematically, \(g_i\) can be described by equation

\[
g_i(X) = \exp(-\sum_{j=1}^{N} \frac{|x_j - c_i|^2}{2\sigma_i^2})
\]

(3.2)

where \(c_i^T = [c_{i1}, c_{i2}, ..., c_{in}]\) is the centre of the receptive field and \(\sigma_i\) is the width (or called spread) of the Gaussian function. The output of the neural network is found by summing the output of the radial basis functions (RBF) multiplied by the weights of each neuron as indicated in Fig. 3.7 [92]. In particular, the dashed lines represent the input of the neural network and the solid line is the output of the RBF. The larger the spread is, the smoother the fitted function will be. However, too large a spread will result in a lot of neurons to be required to fit a fast changing function. Too small a spread requires many neurons to fit a smooth function, and the network may over-fit the function. Therefore, different spread should be tried for the best value for a given problem.

Figure 3.7 Weighted sum of radial basis functions [92]
Chapter 3 Pre-processing of the Self-mixing Signals in the OFSM Interferometry

The neural network is then trained to determine the following parameters:

1. The number of neurons in the hidden layer;
2. The spread for the radial basis functions;
3. The weight of each RBF output to pass to the summation layer.

Specifically the network is trained using a set of data containing N input-output pairs \((x_i, y_i)\) \((i = 1, 2, \ldots, N)\) with \(y_i\) representing the desired undistorted self-mixing fringes such as the theoretical calculated output for a controlled target movement. The performance of the network is evaluated by calculating the root-mean-square error, that is,

\[
RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2}
\]

During training, the number of hidden neurons, spreads and weights of the radial basis functions are updated in a way so that the RMSE is minimized.

3.4 Computer Simulations

The neural network approach for noise elimination in the OFSMI signals is firstly tested with computer simulations. The external target is assumed to be subject to a harmonic vibration which can be represented as \(L(t) = L_0 + \Delta L \cos(2\pi f t + \theta_0)\), where \(L_0\) is the initial distance between the laser front facet and the target, \(f\) is the vibration frequency, \(t\) is time variable, \(\theta_0\) is the initial phase of target movement. The laser phase without feedback is then calculated as

\[
\phi_0(t) = \frac{4\pi L(t)}{\lambda_0} = \frac{4\pi L_0}{\lambda_0} + \frac{4\pi \Delta L}{\lambda_0} \cos(2\pi f t + \theta_0)
\]

When sampled with frequency of \(f\), the discrete version of Eq. (3.4) can be written
as

\[ \phi_n(t) = \frac{4\pi n L_0}{\lambda_0} + \frac{4\pi \Delta L}{\lambda_0} \cos\left(\frac{2\pi n f_s}{f_v} + \theta_0\right) \]  

(3.5)

If we assume \( f = 200 \text{Hz} \), \( f_v = 200 \text{kHz} \), \( L_0/\lambda_0 = 25000 \) and \( \Delta L/\lambda_0 = 2.6 \), \( \theta_0 = -\frac{\pi}{2} \), the self-mixing signal can be generated using (2.29), (2.44) and (3.5) with \( C = 0.8 \), \( \alpha = 4 \) for weak feedback and \( C = 3 \), \( \alpha = 4 \) for moderate feedback regime. A small random noise with signal-to-noise ratio (SNR) of 20 dB is then added to emulate the practical situation. The signal waveform is shown in Figure 3.8.

![Figure 3.8 Simulated SM signals with SNR = 20dB.](image)

- (a) Moving track of external target
- (b) Under weak feedback with \( C = 0.8 \) and \( \alpha = 4 \)
- (c) Under moderate feedback with \( C = 3 \) and \( \alpha = 4 \).

Radial basis neural network is employed to fit the noisy self-mixing signals. The neural network comprises of two layers, i.e., one layer of neurons with Gaussian functions and one layer of linear output neuron as was depicted in Section 3.3.2. In training the network, one RBF neuron is added each time and the root-mean-square
error (RMSE) is calculated accordingly. The training is stopped until the RMSE is not showing improvement by adding more neurons.

In order to find the optimal spread for the radial basis functions, we tried different spread values 5, 20 and 40. The results for fitting the simulated noisy self-mixing signal under weak feedback condition whose waveform is seen in Fig. 3.8 (b) are presented in Fig. 3.9. The enlarged view of three typical fringes is shown in Fig. 3.10.

![Figure 3.9 The clean self-mixing waveform and the neural network fitted waveforms from the noisy SM signal. The dotted line: the clean self-mixing signal. The dash-dotted line: fitted curve with spread of 5. The solid line: fitted curve with spread of 20. The dashed line: the fitted curve with spread of 40.](image)

(a) Enlarged view of Fringe A
Figure 3.10 Enlargement of the waveform in Fig. 3.9. The thick solid line: clean self-mixing signal. The dash-dotted line: fitted curve with spread of 5. The thin solid line: fitted curve with spread of 20. The dashed line: fitted curve with spread of 40.
Table 3.1 The parameters of the neural network for fitting OFSMI signal under weak feedback regime

<table>
<thead>
<tr>
<th>RBF Spread</th>
<th>RMSE</th>
<th>Number of neurons</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.002</td>
<td>190</td>
</tr>
<tr>
<td>20</td>
<td>0.003</td>
<td>167</td>
</tr>
<tr>
<td>40</td>
<td>0.02</td>
<td>225</td>
</tr>
</tbody>
</table>

According to the above results, we summarize the observations as follows:

1. A small spread value like 5 can cause over-fitting of the noisy OFSMI signal. This appears as the small ripples in the fitted curve seen in Fig. 3.10 (a) and (c). Although the performance of the neural network seems good in terms of the RMSE, the fitted curve is not very smooth.

2. A large spread value like 40 can not yield as good performance as a smaller spread value in that the RMSE is almost ten times larger. This is obviously seen in Fig. 3.10 (a)-(c) as the fitted curves deviate significantly from the real self-mixing signal.

3. The intermediate spread value 20 is capable of fitting a curve that is in good agreement with the original clean signal waveform. The RMSE is as small as 0.003 which shows high performance of the neural network.

As a result, the fitted curve by the neural network with the spread of 20 and 167 neurons is shown in Fig. 3.11 and difference between original and fitted curves is shown in Fig. 3.12. It is seen the random noise has been effectively eliminated. The difference between the fitted waveform and the original clean signal is found to be within $\pm 0.04$ mostly, which indicates good accordance is achieved between the two signals that are
normalized between -1 and 1.

![Figure 3.11](image1.png) Neural network fitted curve under weak feedback regime

![Figure 3.12](image2.png) The difference between the fitted curve and original OFSM signal under weak feedback regime

In contrast, the performance of the neural network for fitting the OFSMI signals under moderate feedback condition degrades significantly due to the existence of the abrupt transitions. The results for fitting the self-mixing signal in Fig. 3.8 (c) with different spread values are presented in Fig. 3.13 and Table 3.2. It is seen in all cases the
fitted curves have remarkable ripples. Small spread (e.g., spread=5) tends to preserve the abrupt switchings better than large spread values. Whereas, the fitted curve has significant ripples around the switching points that will increase (or decrease) the peak value (or bottom point value) in the fitted waveform. Large spread values (20, 40) tend to broaden the jumping and dropping points in the signal and compromise the abruptness of the fast switchings.

To solve this problem, we segmented the waveforms at the switching points and performed fitting for each segment. As a result, each segment is fitted with significantly improved smoothness. The fitted curve is presented in Fig. 3.14. The difference between original and fitted curves as shown in Fig. 3.15 is generally within ±0.04 as was similar to the case of weak feedback condition. However, this is achieved with the time consuming work of signal segmentation at the fast switching points. In the case of real practice, it is not feasible that the experimentally acquired self-mixing signals have such ideal and explicit switching points, thus higher error ought to be expected between the fitted curve and its original counterpart around the switching points.
Figure 3.13 The neural network fitting results for the OFSMI signal under moderate feedback. (a) fitted curves with different RBF spread values (b) enlarged view of the fringes enclosed in the dashed line box.

Table 3.2 The parameters of the neural network for fitting OFSMI signal under moderate feedback regime

<table>
<thead>
<tr>
<th>RBF Spread</th>
<th>RMSE</th>
<th>Number of neurons</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.01</td>
<td>300</td>
</tr>
<tr>
<td>20</td>
<td>0.02</td>
<td>235</td>
</tr>
<tr>
<td>40</td>
<td>0.05</td>
<td>95</td>
</tr>
</tbody>
</table>
Chapter 3 Pre-processing of the Self-mixing Signals in the OFSM Interferometry

3.5 Experimental Results

The RBF neural network is also verified for further processing the experimental OFSMI signals as was shown in Fig. 3.2 (a) and (c) after they are processed with median filters. The network that is used to fit the weak feedback self-mixing signal comprises of 295 neurons. The spread for the radial basis functions is 20. The network training is stopped until no further improvement is achievable by adding more neurons.
The RMSE was found to be 0.15 as a result. The experimental OFSMI signal under moderate feedback condition is fitted within each segment by segmenting the signal at the jumping and dropping points. The signal waveforms after performing curve fitting for both feedback conditions are shown in Fig. 3.16 (b) and Fig. 3.17 (b), in comparison to the median filtered signal waveforms in Fig. 3.16 (a) and Fig. 3.17 (a). It can be found the random noise is removed more effectively after the median filtered signal is further processed with the neural network.

The pre-processed OFSMI signal will then be used for displacement reconstruction and LEF measurement in the following chapters.

Figure 3.16 Experimental weak feedback self-mixing signal waveforms after being processed with (a) Median filtered (b) Median filter and neural network
3.6 Conclusions

This chapter concerns with developing a signal pre-processing procedure that is capable of removing the noises contained in the experimental OFSMI signals. To this end, several typical waveforms obtained under different optical feedback levels are firstly observed in order to determine the characteristics of noises. Two types of noises are discerned subsequently, that is, the impulsive noise and the random noise.

The impulsive noise was able to be eliminated effectively with a median filter that is configured according to the duration of the impulsive spikes. The random noise is removed by the artificial neural network on the basis of radial functions.

It was clearly shown by computer simulations that the neural network is capable of fitting out a smooth curve with the noisy data input of SNR 20 dB by selecting suitable parameters for the network. The fitted curve is in good accordance with its counterpart before the random noise is added in. The difference between the simulated original
OFSMI signals and fitted signals under both weak feedback and moderate feedback regimes is generally within a small range of ±0.04 for the OFSMI signals normalized between -1 and 1.

It is also noted that a one-step fitting can be achieved with satisfaction for the continuously varying OFSMI signals under weak feedback regime. However, the satisfactory fitting for the OFSMI signals under moderate feedback level can only be obtained within each segment of signal between the two abrupt transition points in the signal.

The proposed pre-processing procedure was also tested with experimental OFSMI signals. It can be found the random noise is removed effectively by the neural network after the impulsive noise is eliminated with median filters.
Chapter 4 Displacement Measurement with OFSMI system
4.1 Introduction

This chapter concerns the displacement measurement of a target with the optical feedback system setup. As has been stated in Chapter 2, the laser intensity in the presence of external feedback is modulated by the reflected laser phase. The quantitative relation between the laser intensity and reflected phase is depicted in the theoretical model of the OFSMI system which is known as the Lang-Kobayashi equations taking the following forms:

\[
\phi_f(\tau) = \phi_0(\tau) - C \cdot \sin[\phi_f(\tau) + \arctan \alpha] \tag{4.1}
\]

\[
P(\tau) = P_0[1 + mG(\tau)] \tag{4.2}
\]

\[
G(\tau) = \cos(\phi_f(\tau)) \tag{4.3}
\]

where \( \phi_0(\tau) \) and \( \phi_f(\tau) \) are the laser phases without and with feedback respectively. \( \tau = 2L/c \), is the round trip time between the LD and the external target with \( L \) being the distance between the LD and the target, \( c \) being the speed of light. \( C \) is the feedback level factor and \( \alpha \) is the linewidth enhancement factor of the LD. \( P(\tau) \) and \( P_0 \) denotes the laser power with and without feedback respectively. \( m \) is called modulation index (typically \( m=10^{-3} \)).

Apparently, once \( P(\tau) \) is measured in an OFSMI experimental setup, the unperturbed laser phase \( \phi_0(\tau) \) can be retrieved from solving Equations (4.1)-(4.3) as

\[
G(\tau) = [P(\tau)/P_0 - 1]/m \tag{4.4}
\]

\[
\phi_f(\tau) = \arccos[G(\tau)] \tag{4.5}
\]

\[
\phi_0(\tau) = \phi_f(\tau) + C \cdot \sin[\phi_f(\tau) + \arctan(\alpha)] \tag{4.6}
\]

According to the relationship
\[ \phi_0(\tau(t)) = \frac{4\pi L(t)}{\lambda_0} \]  

(4.7)

The target instant distance \( L(t) \) from LD can be obtained readily and hence the target displacement if the equilibrium position of the movement is known. However, since \( \phi_p(\tau) \) is computed from an inverse cosine function in Eq. (4.5) which always produces values in the interval \([0, \pi] \), it has to be recovered to its real values.

Section 4.2 elaborates on a phase unwrapping technique to recover \( \phi_p(\tau) \) from \( G(\tau) \) and thus the displacement of the target. Computer simulation was employed to verify the fundamental viability of the phase unwrapping algorithm in Section 4.3. A random noise is also added to the simulated self-mixing signal and pre-processed with the procedure that was developed in Chapter 3. In Section 4.4, experiments were conducted to verify the proposed method. Finally Section 4.5 concludes this chapter.
4.2 Displacement Reconstruction via a Phase Unwrapping Technique

For the purpose of gaining some insight about the problem of phase unwrapping, we illustrate the waveforms of a few relevant signals (i.e., $\phi_b(\tau), \phi_f(\tau)$ and $G(\tau)$) in Eqs. (4.4) - (4.6). For simplicity, the external target is assumed to be driven by a sinusoidal signal. Two periods of the discrete samples of these signals are shown in Fig. 4.1. The self-mixing waveforms under weak feedback and moderate feedback regimes are generated with $C = 0.8, \alpha = 4$ and $C = 3, \alpha = 4$ respectively.

Theoretically, the laser phase with external feedback $\phi_f(n)$ can be computed by taking inverse cosine of $G(n)$. However, the mathematical nature of the arc cosine function causes the returned phase values to be always between 0 and $\pi$ as seen in Fig. (4.1) (d) and (f). In order to reconstruct the target displacement, the phase $\phi_f(n)$ must be recovered (or unwrapped) to its real values as in Fig. (4.1) (b).
In order to develop a phase unwrapping algorithm, the relationship between $G(n)$ and $\phi_r(n)$ is firstly considered and the following observations were drawn:

1. One fringe in the $G(n)$ waveform corresponds to $2\pi$ change in $\phi_r(n)$, thus $\text{acos}(G(n))$ should be manipulated to produce values between $-\pi$ and $\pi$ for each fringe in the self-mixing waveforms;

Figure 4.1 Waveforms of (a) $\phi_0(n)$; (b) $\phi_r(n)$; (c) and (d) $G(n)$ and $\text{acos}(G(n))$ under weak feedback regime; (e) and (f) $G(n)$ and $\text{acos}(G(n))$ under moderate feedback regime
2. The inclination of the fringes of $G(n)$ complies with movement direction of $\phi_x(n)$. Thus with the help of the interferometric fringes, it can be discerned whether the target is moving away from or towards the diode laser. Multiple of $2\pi$ is added when the target is moving away from the LD or subtracted when the target is moving towards the LD.

Based on the above observations, the following phase unwrapping algorithm is employed to reconstruct $\phi_x(n)$ as

$$\phi_x(n) = (-1)^{M_1} \arccos(G(n)) + M_2 \cdot 2\pi$$

(4.8)

where $M_1$ is accumulated by one when the self-mixing signal reaches peak and bottom points; $M_2$ is incremented by one when the self-mixing signal reaches the bottom points for the case that the target is moving away from the LD or decremented by one when the target is moving towards the LD. Note the starting point can be chosen as either ‘A’ or ‘D’ as seen in Fig. 4.1 (c) and (e) corresponding to different starting value of $M_1=1$ and $M_1=0$ respectively. The bottom point refers to the points such as ‘B’ and peak point refers to the points like ‘C’ in Fig. 4.1 (c) and (e).

It is apparent that the location of the above characteristic points on the self-mixing signal waveform $G(n)$ is highly susceptible to the noises contained in the signal. Hence, before the algorithm can be carried out, the raw experimental data should be pre-processed with the procedures developed in Chapter 3. The peak and bottom points are subsequently located based on three measures:

1. Since theoretically the peak value is 1 and valley value is -1 for the self-mixing signal $G(n)$, threshold values close to 1 and -1, such as 0.9 and -0.9 can be set to determine the peak and bottom points after the experimental data is normalized between -1 and 1;
(2) Each self-mixing data sample is compared with the adjacent few samples before and after it. The peak points are found of the greatest value among the neighboring samples, and the bottom points are the smallest;

(3) The slope for a valid self-mixing fringe is unsymmetrical in contrast to the symmetrical incomplete fringes where point ‘A’ and ‘D’ are located as seen in Fig. 4.1 (c). This feature is useful for excluding the turning points of target movement like ‘A’ and ‘D’ from being counted as peak and bottom points.

For computing $\phi_0(\tau)$ with Eq. (4.6), we need to determine the parameters $C$ and $\alpha$ in advance which entails discerning the grade of feedback level firstly upon obtaining an OFSMI signal sample. This can be achieved either by observation of the shapes of the signal due to their distinctive characteristics under different feedback regimes as was discussed in Chapter 2 or a systematic method for transition detection of the OFSMI signals can be found in reference [93].

- Under weak feedback regime, the feedback level factor $C$ is related to the asymmetry of the self-mixing waveform $G(\tau)$ [62], thus it can be determined from

$$\frac{t_i}{t_f} = \frac{\pi - 2C}{\pi + 2C}$$

(4.9)

where $t_i$ and $t_f$ are the time duration of increasing and decreasing part of self-mixing waveform. The linewidth enhancement factor can be estimated with the method in [94]. Assuming a simulated self-mixing waveform as shown in Fig. 4.2, on which the peak and bottom points are marked as $X_{M1}$ and $X_{M2}$, the zero-crossing points are marked as $X_{Z1}$ and $X_{Z2}$. By defining $k_1 = (X_{M2} - X_{M1})/2\pi$, $k_2 = (X_{Z2} - X_{Z1})/2\pi$ and measuring $k_1, k_2$ on the self-mixing waveform, $\alpha$ is calculated with

$$\alpha = \frac{0.5 - k_1}{0.5 - k_2}$$

(4.10)
For moderate feedback condition, \( C \) and \( \alpha \) can be obtained simultaneously with the method in [42]. Considering a self-mixing signal in Fig. 4.3, where \( \phi_1 \) and \( \phi_4 \) correspond to the phase values of the zero-crossing points of the waveform, \( \phi_2 \) and \( \phi_3 \) are the phase values corresponding to the points with infinite slope. \( C \) and \( \alpha \) are then calculated with

\[
\phi_{13} = \sqrt{C^2 - 1} + \frac{C}{\sqrt{1 + \alpha^2}} + \arccos\left(-\frac{1}{C}\right) - \arctan(\alpha) + \frac{\pi}{2}
\]

\[
\phi_{24} = \sqrt{C^2 - 1} - \frac{C}{\sqrt{1 + \alpha^2}} + \arccos\left(-\frac{1}{C}\right) + \arctan(\alpha) - \frac{\pi}{2}
\]
4.3 Computer Simulations

Simulations were firstly carried out on computers to test the effectiveness of the phase unwrapping algorithm for displacement reconstruction with OFSM signals. The assumption used in this simulation is that the external target is subject to a harmonic vibration which can be represented as

\[ L(t) = L_0 + \Delta L \cos(2\pi ft + \theta_0), \]

where \( L_0 \) is the initial distance between the laser front facet and the target, \( f \) is the vibration frequency, \( t \) is time variable. The laser phase without feedback is calculated as

\[ \phi_0(t) = \frac{4\pi L(t)}{\lambda_0} = \frac{4\pi L_0}{\lambda_0} + \frac{4\pi \Delta L}{\lambda_0} \cos(2\pi ft + \theta_0) \]  

(4.9)

4.3.1 Simulation with clean self-mixing signal

The weak feedback self-mixing signal is firstly generated and plotted in Fig. 4.4 using Eqs. (4.1) – (4.3) with \( \alpha = 4 \) assuming \( f = 20 \) Hz, \( \theta_0 = -\frac{\pi}{2} \), \( L_0/\lambda_0 = 20000 \) and \( \Delta L/\lambda_0 = 1.8 \). The target displacement is reconstructed as show in Fig. 4.5 and the error between reconstructed displacement and its real counterpart is shown in Fig. 4.6. Since the simulated self-mixing signal is free of noises, the peak and valley points are easily located by comparing each data samples with the two samples before and after it. It is seen the target displacement is recovered almost perfectly with the phase unwrapping algorithm. The minor difference between reconstructed and real displacement trace could be cause by the computational error in doing the simulation.
Figure 4.4 Simulated weak-feedback self-mixing signals with $C = 0.8$, $\alpha = 4$

(a) $\phi_0(t)$ (b) $\phi_r(t)$ (c) $G(t)$

Figure 4.5 The recovered $\phi_0(t)$ using the self-mixing signal $G(t)$ in Figure 4.2 (c)
Figure 4.6 Error between the reconstructed and real displacement under weak feedback.

The feasibility of recovering a target displacement was also investigated with moderate feedback self-mixing signals. To this end, the self-mixing signal was generated as seen in Fig. 4.7 with $C = 3$, $\alpha = 4$ with the other conditions remain the same as in the weak feedback example. The error between real and reconstructed displacement is shown in Fig. 4.8. Obviously the error is much remarkable compared with that under the weak feedback condition. This is due to the incomplete self-mixing signal fringes as a result of the abrupt switchings. Hence it is evident that the proposed approach will yield better measurement accuracy under the weak feedback levels than in the higher feedback levels.
Figure 4.7 Simulated self-mixing signal with $C = 3, \quad \alpha = 4$  
(a) $\phi_h(t)$  
(b) $\phi_f(t)$  
(c) $G(t)$

Figure 4.8 Error between reconstructed and real displacement under moderate feedback
4.3.2 Simulation with noisy self-mixing signal

In order to test the proposed method under the situation that is more representative to that in practice, a small white noise with the signal-to-noise ratio (SNR) of 20 dB is added to the self-mixing signals. The noisy data is firstly pre-processed with the procedures introduced in Chapter 3. One period of the corrupted and pre-processed signal waveforms under weak feedback condition are presented in Fig. 4.9. It is seen some small ripples still exist in the pre-processed signal. The span of the ripples is found within eight points upon observation. Thus the location of the peak and valley points are determined by comparing each data sample with the neighbouring eight samples before and after it to allow combat these ripples. The values of +0.85 and -0.85 are also set as another measure of discerning peak and bottom points respectively. The error between reconstructed displacement with the pre-processed data and the real displacement is shown in Fig. 4.10. The accuracy is found to be within $\lambda/25$ as a result.

Similarly the results with simulated signal under moderate feedback regime are presented in Fig. 4.11 and 4.12. Although the accuracy of $\lambda/20$ is seen as only a little degradation compared with that of $\lambda/25$ under the weak feedback level, this is achieved at the cost of the tedious and time consuming job to segment the self-mixing signal and perform curve fitting within each intervals between two fast switching points.
Figure 4.9 (a) The simulated noisy self-mixing signal waveform
(b) The signal waveform after pre-processing
Figure 4.10 Error between reconstructed displacement and real displacement with corrupted self-mixing signal under weak feedback level
Figure 4.11 Corrupted and pre-processed moderate feedback self-mixing signal waveforms
4.4 Experiments

The proposed approach is finally tested with experimental setup as was described in Section 3.2. The target distance from the LD is slightly adjusted in order to get different feedback levels. As an example, the resultant self-mixing signal at weak and moderate feedback levels are shown in Fig. 4.13.

Fig. 4.14 (a) and (b) show the median filter and mean filter processed one period of the self-mixing (SM) signals in weak and moderate feedback conditions respectively. It can be found that the random noise present in the moderate feedback SM signal is still significant. Fig. 4.14 (c) and (d) show further neural network processed SM signals for
weak and moderate feedback levels. It is clearly seen that the noises have been effectively eliminated.

Fig. 4.15 (a) presents the target displacement recovered from self-mixing signals under weak feedback condition. It appears harmonic, which reveals a good recovery of the movement of the target. Fig. 4.15 (b) shows the target displacement recovered from self-mixing signals under moderate feedback condition. Significant spikes are found along the recovered movement track. This is due to the broadened jumping and dropping areas in the experimental self-mixing signals waveforms which is an inevitable result of the data sampling process.

Figure 4.13 Experimental SMS waveforms for
(a) weak feedback and (b) moderate feedback
Figure 4.14 Self-mixing signal waveforms after pre-processing

(a) Median filtered SM signal under weak feedback condition (b) Median filtered SM signal under moderate feedback condition (c) Median filter and neural network processed SM signal under weak feedback condition (d) Median filter and neural network processed SM signal under moderate feedback condition
Figure 4.15 Recovered target displacement with self-mixing signals acquired under different feedback conditions (a) weak feedback (b) moderate feedback

4.5 Conclusions

This chapter intends to reconstruct a target displacement by means of a phase unwrapping approach. As the optical phase without external feedback can be solved analytically once the phase with external feedback is known, the target displacement is also obtained readily according to the relationship between the distance of the external target from the laser diode and the corresponding optical phase.

Based on this observation, the major task of the job falls in solving the equation \( \phi_f(\tau) = \arccos[G(\tau)] \) for \( \phi_f(t) \), where \( G(\tau) \) is the experimental self-mixing signal. However, this task is hindered by the nature of the reverse cosine function which always produces values within the interval of 0 and \( \pi \). As a solution, a phase unwrapping algorithm is employed to recover the optical phase to its original values. As has been stated in Chapter 2, one complete fringe in the self-mixing signal waveforms
corresponds to $2\pi$ change in the optical phase, hence the essence of the algorithm is to add or subtract multiple number of $2\pi$ to the result given by the inverse cosine function when the target is moving from or towards the laser diode respectively. The direction of target movement can be determined by the fringe inclination when the OFSM system is adjusted to work under the feedback levels which are characterized by the exhibition of sawtooth-like self-mixing signal waveforms.

Both computer simulation and experiments show the method is effective in reconstructing the target displacement when the OFSM system is operating under weak feedback regime before the abrupt switchings start to appear in the observed self-mixing waveforms. Numerically, the accuracy of $\lambda/25$ is obtained when a 20 dB white noise is added to the simulated self-mixing signals. Note the distinguishment of the characteristic points in the signal waveforms is greatly facilitated by means of the pre-processing procedures developed in Chapter 3. On the other hand, the measurement accuracy of $\lambda/20$ is shown with computer simulation under moderate feedback. However, this is achieved at the cost of time-consuming manual segmentation of the self-mixing signals in doing the signal pre-processing due to the existence of the abrupt switchings. Experiments under moderate feedback condition revealed remarkable degradation as significant spikes are seen in the reconstructed target displacement. This is due to the broadened jumping the dropping area in the sampled experimental signal as opposed to the ideal abrupt switchings in computer simulations.
Chapter 5

Linewidth Enhancement Factor Measurement Based on Optical Feedback Self-Mixing Effect:

A Global Optimization Approach
5.1 Introduction

A recent advent in the application of OFSMI is the measurement of the linewidth enhancement factor (LEF) of a semiconductor laser. The LEF is a significant device and operation parameter that is responsible for many characteristic features of semiconductor lasers such as enhancement of the laser linewidth, the frequency chirp, the modulation response, the injection-locking range, and the response to external feedback in contrast to other types of lasers. It is defined as the ratio of the partial derivatives of the real and complex parts of the complex susceptibility with respect to carrier density, i.e.,

$$\alpha = -\frac{d[\text{Re}(\chi(n))] / dn}{d[\text{Im}(\chi(n))] / dn}.$$  

The existent methods for measuring this parameter have been reviewed thoroughly in the first chapter. The novelty of OFSM based methods lies in the simplicity in the system implementation yet is capable of achieving comparable accuracy with the other approaches.

The first successful measurement of the LEF of a diode laser with an OFSMI setup was conducted by Yu et al. [42]. The measurement was performed under moderate feedback regime by solving the Lang-Kobayashi equations for the phase differences between the zero-crossing point and the point with infinite slope for the two directional self-mixing waveform fringes respectively. A relationship is then established for these points between the analytical solutions and the geometric measurements on the experimental self-mixing waveforms subject to a few conditions, that is,

- Firstly the measurement is performed where good linearity is presented in the waveform and the external target is moving at a constant speed. In the case that the
external target is driven by a sinusoidal signal, this condition is only met when the target is close to the equilibrium position of target movement.

- Secondly the OFSM system needs to be adjusted so that the self-mixing waveforms appear sawtooth like with hysteresis and the zero-crossing points can be discerned explicitly. This restricted the measurement to be workable only when $C$ and $\alpha$ fall in the small range of $1 < C < 3$ and $3 < \alpha < 5$ which relies on careful adjustment of the external optical feedback.

Later on, Xi et al. [43] achieved the measurement under weak optical feedback level ($0 < C < 1$) by searching the optimal solutions for a few parameters in the theoretical model including the LEF that yield the best agreement between the model and the experimental data. This method is known as data-to-model fitting. It is assumed in [43] that the external target moving law is known a priori, leaving only optical feedback level factor $C$ and the LEF to be determined by optimizing the cost function. However, this requires precise control of target movement. The method in [44] addresses the situation where the target is subject to a simple harmonic vibration with unknown frequency and amplitude, leaving four variables to be identified in the cost function including $C$, the LEF, target vibration amplitude and equilibrium position. However, since gradient-based algorithm was employed to locate the minimum of the error surface, initial parameter values close to the global minimum are required in order to prevent the algorithm from converging to a local minimum.

The purpose of this chapter is to address the above limitations associated with the data-to-model fitting approach. In Section 5.2, the principle of data-to-model fitting approach is briefly reviewed. In Section 5.3, a comprehensive study on the features of the cost function is conducted. Section 5.4 introduces the basics of genetic searching algorithm and its application to the OFSMMI parameter estimation. In Section 5.5, the
algorithm is tested with computer simulations to verify its validity. A modified genetic algorithm (GA) is then proposed that improves the efficiency of the standard GA and is capable of locating the global minimum of the cost function. Section 5.6 presents the testing results with experimental OFSMI data. And Section 5.7 concludes this chapter.
5.2 OFSMI Model Parameter Estimation

The data-to-model fitting (or model calibration) is a widely used method to identify the unknown parameters in a theoretical model by modifying the input parameters to the model until the output from the model matches an observed set of data. This is usually achieved by minimizing a cost function that is defined as the discrepancy between model output and the observed data. The cost function for identifying LEF with OFSM system was initially defined in [43] as

\[
F(\phi_0, \Delta \phi, \hat{C}, \hat{\alpha}) = \frac{1}{N} \sum_{i=1}^{N} [G_i - \hat{G}_i(\phi_0, \Delta \phi, \hat{C}, \hat{\alpha})]^2
\]  

(5.1)

where \(G_i\) is the experimentally tested OFSMI signal values, \(\hat{G}_i(\phi_0, \Delta \phi, \hat{C}, \hat{\alpha})\) are the theoretically calculated signal values by incorporating the estimated variables \((\phi_0, \Delta \phi, \hat{C}, \hat{\alpha})\) with Eqs. (2.29) and (2.44). \(N\) is the number of samples within one period of target movement. When the external target is driven by a simple harmonic vibration signal defined as \(L(t) = L_0 + \Delta L \cos(2\pi ft + \theta_0)\), with \(f\) being the vibration frequency and \(\theta_0\) being the initial phase, we have:

\[
G(t) = \cos(\varphi_0 + \Delta \varphi \cos(2\pi ft + \theta_0) - C \cdot \sin[\phi_f(\tau(t)) + \arctan(\alpha)])
\]  

(5.2)

where \(\varphi_0 = 4\pi L_0/\lambda_0\) and \(\Delta \varphi = 4\pi \Delta L/\lambda_0\) are the phases of backreflected light, \(L_0\) and \(\Delta L\) represent the vibration equilibrium position and vibration amplitude respectively. \(\lambda_0\) is the laser wavelength without external feedback. The unknown model parameters \((\varphi_0, \Delta \varphi, C, \alpha)\) in Eq. (5.2) are identified by searching the optimal values which minimizes the cost function in Eq. (5.1).

In order to calculate \(G(t)\) with Eq. (5.2), \(f, \theta_0\) and \(\phi_f(\tau(t))\) have to be known in advance. The vibration frequency can be determined by calculating the auto-correlation
function given a number of \( M \) data samples with the following equation

\[
 r_g(m) = \frac{1}{M} \sum_{n=0}^{M-1-m} g_M(n) \cdot g_M(n+m) \tag{5.3}
\]

where \( M \) is number of data samples used for calculating the auto-correlation function. \( m \) is the time delay index which varies from \(-(M - 1)/2\) to \((M - 1)/2\). The number \( N_0 \) of data sample in a fundamental period of vibration can be obtained by detecting the positions of the peaks in the auto-correlation function. Given the sampling frequency of \( f_s \), the vibration frequency can be obtained by \( f = f_s/N_0 \). The determination of \( \theta_0 \) is realized by using the sliding window to pick up a segment of \( N_0 \) samples as

\[
 g_w(n, j) = \begin{cases} 
 g(n + j), & \text{for } n = 0,1,...,N_0 - 1 \\
 0, & \text{otherwise}
\end{cases} \tag{5.4}
\]

where \( j \) is the starting point for the sliding window, so that \( g(n) = g(N_0 - n) \) for \( n = 0,1,...,N_0 - 1 \). is satisfied. This way, we will have \( \theta_0 = -\pi/2 \). Since there is no analytical solution for \( \phi_p(\tau(t)) \) as can be seen from Eq. (2.29), an iterative operation can be employed as follows:

\[
 f_j(\tau_i) = \phi_0(\tau_i) - C \sin(f_{j-1}(\tau_i) + k) \tag{5.5}
\]

\( f_j(\tau_i) \) is updated iteratively until a steady state is reached which is tested by

\[
 |f_j(\tau_i) - f_{j-1}(\tau_i)| < \delta \quad (\text{where } \delta \text{ is a small positive number}).
\]

The phase \( \phi_p(\tau(t)) \) is obtained as the steady state value of \( f_j(\tau_i) \). More details for the relative information for determining these three arguments can be found in References [43, 44].
5.3 Cost Function Analysis

The mathematical way to find the minimum of the cost function in Eq. (5.1) is to find zeros of the function derivatives. However, we found it too difficult to solve this mathematically because we are dealing with a non-linear multidimensional cost function. Nevertheless, an optimization method can always offer an alternative solution.

Among a great number of optimization algorithms, some are capable of locating a local minimum such as the gradient based method employed in References [43, 44]; others are designed to identify the global minimum for the best solution amid multiple local minima such as genetic algorithm. In order to develop an effective technique for solving the optimization problem defined in Section 5.2, it is necessary to examine the characteristics of the cost function such as the number of local minimum and the influences of variables on the cost function (i.e., variable sensitivities). Hence, we undertake a graphical study on the shape of the cost surface, which is constructed by all the possible cost function values with respect to the four variables $(\phi_0, \Delta \phi, \hat{C}, \hat{\alpha})$.

5.3.1 Variable ranges

The study of a cost surface and searching the minimum for the cost surface will be greatly facilitated if the variables have some constraints. Hence, firstly we examine the maximum variation ranges for the four cost function variables $(\phi_0, \Delta \phi, \hat{C}, \hat{\alpha})$.

The parameter range for $\alpha$ is $(0, 9)$ for most laser diodes [68, 70, 73, 77, 95-98], and in weak feedback OFSMI system $C$ is within the range $(0, 1)$ [38, 81].

With respect to $\phi_0$, as is seen in Eq. (5.2), $\phi_0 + 2k\pi$ will yield the same $G(t)$ for any integer value of $k$ due to cosine function. In other words, $\phi_0$ can only be identified at the interval of $[0, 2\pi]$, which is consequently the range of interest when we
study the cost surface.

The range for $\Delta \phi$ is determined by the vibration amplitude and can be very wide. However, an approximation of the target displacement with $\lambda/2$ resolution is easily obtained by means of fringe counting [38]. Assuming that there are $K$ fringes in the self-mixing signal during the half target vibration period, $\Delta \phi$ should fall within the range $[K\pi, K\pi + 2\pi]$. Note that this range is much wider than what was considered in Reference [44].

### 5.3.2 Cost surface analysis

In order to reach a general conclusion about the features of the cost surface, we choose 20 sets of parameter values scattered all over parameter spaces. We generate a $G_i$ with each set of parameters using Eq. (5.2) as the tested data, and for each $G_i$ we calculate the cost function values (referred as costs in short) with respect to different $\hat{\phi}_0, \Delta \hat{\phi}, C$ and $\alpha$ using Eq. (5.1) and plot the costs versus these variables (i.e., cost surfaces) for illustrative purposes. We found the cost surfaces exhibit similar characteristics over all data sets. Hence without loss of generality, we present the results of a typical example where $C = 0.6$, $\alpha = 4$, $\lambda_0 = 785nm$, $L_0 = 40cm$ (thus $\phi_0 = 1.76rad$), $\Delta L = 1.6\mu m$ (thus $\Delta \phi = 25.61rad$), $f = 200Hz$ and $\theta_0 = -\pi/2$. The OFSMI waveform is shown in Fig. 5.1. As the number of fringes in half target vibration period is 7, the variable range for $\Delta \hat{\phi}$ is $[7\pi, 9\pi]$. 


Since there are four parameters in the cost function and it is not possible to reveal the surface shape in one plot, the parameters are divided into two groups, i.e., $\hat{C}$ and $\hat{\alpha}$ in one group, $\phi_0$ and $\Delta \phi$ in another group. The influence of $\hat{C}$ and $\hat{\alpha}$ on the cost function has been investigated in [43, 44]. It was shown the error surface exhibited unimodal with minimum at the location where $\hat{C}$ and $\hat{\alpha}$ equal to their true values, when $\hat{\phi}_0$ and $\Delta \hat{\phi}$ also take their true values. Nevertheless, when $\hat{\phi}_0$ and $\Delta \hat{\phi}$ deviate from their true values, the cost surface minimum occurs at a location other than the crossing of true $\hat{C}$ and $\hat{\alpha}$ values. As an example, when $\hat{\phi}_0$ and $\Delta \hat{\phi}$ take a small deviation from their true values in the following cases (a) $\Delta \hat{\phi} = \Delta \phi + 1.6$, $\hat{\phi}_0 = \phi_0 + 0.4$ (b) $\Delta \hat{\phi} = \Delta \phi - 1.6$, $\hat{\phi}_0 = \phi_0 - 0.4$ (c) $\Delta \hat{\phi} = \Delta \phi + 2.5$, $\hat{\phi}_0 = \phi_0 - 0.8$ (d) $\Delta \hat{\phi} = \Delta \phi - 2.5$, $\hat{\phi}_0 = \phi_0 + 0.8$.

The cost surface minima appear at (a) $\hat{C} = 0.02$, $\hat{\alpha} = 1$ (b) $\hat{C} = 0.17$, $\hat{\alpha} = 1$ (c) $\hat{C} = 0.95$, $\hat{\alpha} = 1$ (d) $\hat{C} = 0.02$, $\hat{\alpha} = 1$. This has indicated that small deviation of $\phi_0$ and $\Delta \phi$ will result in significant changes in the cost surface, thus accurate estimation of $\hat{C}$ and $\hat{\alpha}$ is only viable when $\phi_0$ and $\Delta \phi$ are known with a reasonably good
precision.

We then look into the error surface with respect to $\hat{\phi}_0$ and $\Delta \phi$ while leaving $\hat{C}$ and $\hat{\alpha}$ constant.

- Firstly, we consider the case when $\hat{C}$ and $\hat{\alpha}$ take their true values, i.e., $\hat{C} = C = 0.6$ and $\hat{\alpha} = \alpha = 4$. The cost surface is shown in Fig. 5.2. It is found that the cost surface has one global minimum corresponding to the true values of $\hat{\phi}_0$ and $\Delta \phi$, i.e., $\hat{\phi}_0 = \phi_0 = 1.76rad$ and $\Delta \phi = \Delta \phi = 25.61rad$. Besides, there are a number of local minima on the cost surface.

- Secondly, we look at the cost surface under the condition that $\hat{C}$ and $\hat{\alpha}$ deviate from their true values. Fig. 5.3 shows the cost surfaces for the following cases: (a) $\hat{C} = 0.3$ and $\hat{\alpha} = 6$, (b) $\hat{C} = 0.1$ and $\hat{\alpha} = 2$, (c) $\hat{C} = 0.1$ and $\hat{\alpha} = 9$, (d) $\hat{C} = 0.8$ and $\hat{\alpha} = 2$, (e) $\hat{C} = 0.8$ and $\hat{\alpha} = 8$. It is easily found that they are all similar to Fig. 5.2 in terms of the existence of multiple local minima, and the global minima slightly deviate from the crossing of true $\phi_0$ and $\Delta \phi$ values. In specific, $\hat{\phi}_0 = 1.70rad$ and $\Delta \hat{\phi} = 25.64rad$ for case (a), $\hat{\phi}_0 = 1.70rad$ and $\Delta \hat{\phi} = 25.64rad$ for case (b), $\hat{\phi}_0 = 1.74rad$ and $\Delta \hat{\phi} = 25.68rad$ for case (c), $\hat{\phi}_0 = 2.01rad$ and $\Delta \hat{\phi} = 25.59rad$ for case (d), and $\hat{\phi}_0 = 1.70rad$ and $\Delta \hat{\phi} = 25.57rad$ for case (e). It is seen that $\hat{C}$ and $\hat{\alpha}$ has small impact on the cost function. Thus fair estimations for $\hat{\phi}_0$ and $\Delta \hat{\phi}$ are achievable even if $C$ and $\alpha$ are not known.

5.3.3 Variable sensitivity in cost function

Another issue that should be looked at is the sensitivity of the cost function with respect to each of the parameters. Fig. 5.4 shows the cost function variations versus...
each of the variables while leaving the others constant. It is clearly seen that the cost function is much less sensitive to the variance of $\hat{\phi}_b$ and $\Delta \hat{\phi}$ than that of $\phi_0$ and $\Delta \phi$. The result implies that identification of $\phi_0$ and $\Delta \phi$ is easier than $C$ and $\alpha$ by optimizing the cost function.

![Cost surface versus $\phi_0$ and $\Delta \phi$ with $\hat{C}$ and $\hat{\alpha}$ taking their true values.](image)

Figure 5.2  Cost surface versus $\phi_0$ and $\Delta \phi$ with $\hat{C}$ and $\hat{\alpha}$ taking their true values. The simulation parameters are $\hat{C} = C = 0.6$, $\hat{\alpha} = \alpha = 4$. 

![Diagram](image)
Figure 5.3 Cost surface versus $\varphi_i$ and $\Delta \varphi$ when $\hat{C}$ and $\hat{\alpha}$ deviate from their true values. The simulation parameters are (a) $\hat{C} = 0.3, \hat{\alpha} = 6$  (b) $\hat{C} = 0.1, \hat{\alpha} = 2$  (c) $\hat{C} = 0.1, \hat{\alpha} = 9$

(d) $\hat{C} = 0.8, \hat{\alpha} = 2$  (e) $\hat{C} = 0.8, \hat{\alpha} = 8$. 
Based on the same observation, Reference [44] reduce the dimension of cost function from four \((\hat{\phi}_0, \Delta \hat{\phi}, \hat{C}, \hat{\alpha})\) to two \((\phi_0, \Delta \phi)\) by setting two median values to \(C\) and \(\alpha\) within their variable ranges in order to create the quadratic condition to apply the gradient based method. Despite of this, the gradient based method may still converge to other local minima on the cost surface if initial values of \(\phi_0\) and \(\Delta \phi\) are not selected within the small area where the global minimum is located. This is a rather rigorous constraint for the method to be practical. Hence we are motivated to find a more effective approach to address these limitations. It is desirable that the algorithm allows multiple variables so that the deviation resulted from dimension reduction in the cost function can be avoided. In addition, the unequal sensitivity of different variables on the cost function implies some modifications are entailed on a standard optimization method so that the less sensitive variables can also be identified with high accuracy. Consequently, we propose to use genetic algorithm to address this issue due to its capacity to achieve minimization with respect to four variables simultaneously. As will be discussed in the following text, we also applied a multi-iteration strategy in implementing the algorithm in order to overcome the insensitivity of \(C\) and \(\alpha\) on the cost function.
5.4 Parameter Estimation Based on Genetic Algorithm

Genetic algorithm (GA) is a global optimization technique derived from mimicking the mechanism of natural selection in biological evolution. With GA, the parameters are encoded as genes and chromosomes, and optimal solution is reached by updating the chromosomes on generation-by-generation basis. The principle of natural selection is two-folded. Firstly, the offsprings possess many of the characteristics of their parents and there are variations in the characteristics between individuals. The survival of the individuals depends on their inherited characteristics, i.e., those well adapted to their environment will survive. Secondly, mutation may occur, that is, a random change occurs in the characteristics of a gene as a result of external factors. This change will then be passed along to the offsprings. In each generation, operations are carried out to the existing chromosomes, yielding a new generation of chromosomes. The operators include selection, crossover and mutation.

5.4.1 Genetic algorithm

The use of GA for estimating the OFSM parameters involves the following operations:

5.4.1.1 Encoding of the parameters

Firstly the parameters to be estimated are encoded into binary strings, known as genes. All genes are concatenated as a long binary string corresponding to a chromosome. For instance, assuming that we have a system modeled by a cost function or fitness function denoted as \( f(V) \), where \( V = [v_1, v_2, ..., v_M] \) is a set of variables. We want to optimize the system by finding values for the parameters \( V \) so that the cost
function is minimized. With GA, each individual variable is considered as a gene and encoded as a binary string, that is, \( v_i = [b_{i,1}, b_{i,2}, ..., b_{i,L}] \), where \( b_{i,j} \) are binary bits. By lining up all the genes, a chromosome is constructed as following:

\[
V = [v_1, v_2, ..., v_M] = [(b_{1,1}, b_{1,2}, ..., b_{1,L}), (b_{2,1}, b_{2,2}, ..., b_{2,L}), ..., (b_{M,1}, b_{M,2}, ..., b_{M,L})]
\]

Obviously \( V \) is a long binary string, the value of which corresponds to a solution to the optimization problem. The length of each gene is primarily determined by the desired accuracy of the parameters. For example, given the ranges of \( \hat{\phi}_0, \Delta \hat{\phi}, \hat{C} \) and \( \hat{\alpha} \) described above and if quantization errors are to be kept below 0.01, we select 6 bits for \( \hat{C} \), and 9 bits for \( \alpha, \phi_0 \) and \( \Delta \phi \). By lining them up, a chromosome consists of 33 binary bits.

5.4.1.2 Initial population selection

The exploration of error surface was carried out by a population of \( M \) chromosomes, denoted by \( V_i \) (\( i = 1, 2, ..., M \)), each corresponding to a possible solution for the parameter estimation problem. The underlying idea of choosing population size is always a trade-off between efficiency and effectiveness, as a too small population will not allow sufficient room for exploring the search space, while a too large population is unlikely to reach the optimal solution within reasonable number of computations. Theoretical attempts of examining the choice of population size has established the following formula for calculating the minimum population in order to ensure a meaningful search [99]

\[
N = \left[1 + \log\left(-\ell / \ln P'_2\right) / \log 2 \right]
\]

(5.6)

where \( \ell \) represents the chromosome length. \( P'_2 \) is the probability with which every point in the search space is reachable from the initial population by crossover only, and
typically $P_2$ should be chosen to exceed 99.9%. Hence, for the chromosome of 33 bits, the population size of 16 is the minimum.

On the other hand, despite of the fact that large populations tend to explore complicated cost surface more thoroughly, they are also notorious for causing computational stringency because of the large number of function evaluations. Fortunately, empirical results from numerous authors [100-102] suggest that population size between 30 and 40 combined with larger mutation rate is quite adequate in most cases. In consequence, we select population size of $M=36$.

Another issue needing some consideration is the initial population composition. Intuitively, an adequate sampling will reduce convergence time and prevent the algorithm from prematurely converge in a local minimum. For instance, the simplest way is to randomly sample the cost surface, however this may result in oversampling of some regions but sparsely sampling in others. The other side of the coin is to uniformly sample the error surface, whose drawback is the extreme complexity in the case of multi-dimensional optimization. Alternatively, we employ the scheme to randomly generate half of the population and take their complementary numbers as the other half. By doing this, every bit assumes both a one and a zero within the population to ensure diversity.

5.4.1.3 Survival

In each generation, a portion of the chromosome, say $L$, are selected to survive to the next generation, while the rest ($M-L$) are discarded. The survival of chromosomes is based on their “fitness” which is inverse proportional to the cost function values. In this work, we calculate the cost function values associated with all the $M$ chromosomes, and then choose $L=M/2$ chromosomes with less cost to survive to the next generation.
5.4.1.4 Parent selection

In order to keep the population number constant for each generation, we should generate \((M-L)\) chromosomes to replace those discarded. This is accomplished by selecting pairs of survived chromosomes as parent chromosomes, and each pair produces two off-springs by crossover. Parents are selected based on roulette wheel weighting. In particular, the mating probability for each chromosome is determined by its fitness rank where the best fitted individual is ranked the first, i.e.,

\[
p_n = \frac{L-n+1}{\sum_{i=1}^{n} n} = \frac{L-n+1}{L(L+1)}
\]

where \(p_n\) is the mating probability for the \(n\)th best fitted chromosome among all the \(L\) chromosomes. The \(j\)th chromosome will be selected as a parent if its cumulative probability is larger than a randomly generated number \(x \in (0,1)\), i.e., \(\sum_{i=1}^{j} p_i > x\).

\((M-L)/2\) parent pairs are selected accordingly and offsprings are produced via a crossover procedure.

5.4.1.5 Crossover

Essentially, crossover combines the features of the two “parent” chromosomes and produces two “child” chromosomes. As an example, we consider two parent chromosomes, denoted as strings \(V_A\) and \(V_B\), each consisting of 5 variables, i.e.

\[(v_{A,1}, v_{A,2}, v_{A,3}, v_{A,4}, v_{A,5}) \text{ and } (v_{B,1}, v_{B,2}, v_{B,3}, v_{B,4}, v_{B,5})\]

In simple crossover (also known as one point crossover), a crossover point is randomly selected between the first and last bits of the parents’ chromosomes. The first offspring is produced by taking the binary code to the left of the crossover point from
parent A and the binary code to the right of the crossover point from parent B. In a like
manner, the second offspring is produced by taking the binary code to the left of the
crossover point from parent B and binary code to the right of the crossover point from
parent A. As a result, the offsprings contain part of binary codes from both parents. In
the above example with crossover point between bits 2 and 3, the offsprings will be
\[
(v_{A,1}, v_{A,2}, v_{B,3}, v_{B,4}, v_{B,5}) \quad \text{and} \quad (v_{B,1}, v_{B,2}, v_{A,3}, v_{A,4}, v_{A,5})
\]

An alternative that adds some complication to simple crossover is two-point
crossover, where two crossover points are selected from the parents. The parents then
swap the bits between the two crossover points to produce two offsprings.

A more random crossover operation is called uniform crossover [102], which is
easily seen by observing the crossover operator as a binary string or a mask, represented
by a vector \(m \in \{0,1\}^\ell\) with \(\ell\) being the chromosome length. When the bit in mask is 1,
then the corresponding bit in parent A is passed to offspring A and the corresponding bit
in parent B is passed to offspring B. When the bit in mask is 0, the corresponding bit in
parent A is passed to offspring B and the corresponding bit in parent B is passed to
offspring A. Accordingly, the offsprings of parents \(V_A\) and \(V_B\) are
\[
m \oplus V_A \ominus \bar{m} \oplus V_B \quad \text{and} \quad m \oplus V_B \ominus \bar{m} \oplus V_A
\]
where \(\bar{m}\) is the complement of \(m\), and \(\oplus, \ominus\) denote bit-wise addition and multiplication
respectively.

In order to choose an optimal crossover operator, we run computer simulations for
single-point crossover, 2-point crossover and uniform crossover under the same
situation where all other parameters remain constant. The results are averaged over 15
times independent runs and presented in Table 5.1, indicating a slightly better
performance of uniform crossover operator over the other two and is consequently
chosen as our crossover operator.

<table>
<thead>
<tr>
<th>Crossover Operator</th>
<th>Average cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-Point Crossover</td>
<td>4.55</td>
</tr>
<tr>
<td>2-Point Crossover</td>
<td>3.28</td>
</tr>
<tr>
<td>Uniform Crossover</td>
<td>2.14</td>
</tr>
</tbody>
</table>

5.4.1.6 Mutation

Mutation introduces traits not contained in the current generation into the new generation so as to prevent the algorithm from converging prematurely. A single point mutation changes one bit from 1 to 0, and visa versa. The percentage of bits to be mutated in a population is defined as mutation rate. Larger mutation rate helps to increase the converging speed in early stages of searching; however, it also tends to distract the algorithm from converging to an optimal solution. Some studies have suggested changing the mutation rate adaptively so that both faster convergence and smaller error can be achieved simultaneously.

5.4.1.7 Convergence criterion

GA will keep running unless it is forced to stop, hence we should have a means to measure the performance in order to terminate the algorithm. As the cost function value also measures the accuracy of the parameter estimations, we will find the smallest cost $F_{\text{min}} = \min_{i=1,...,M} \{ F(V_i) \}$, in each generation and compare it against a threshold $F_{\text{th}}$. GA will be terminated when the following condition is met:
\[ F_{\text{min}} \leq F_{\text{th}} \]  

(5.8)

In order to determine \( F_{\text{th}} \), we studied the cost surface again and found that for a measuring accuracy of 1\% for \( \alpha \), the cost should be less than \( 2.5 \times 10^{-6} \). Consequently we set \( F_{\text{th}} = 2.5 \times 10^{-6} \).

It is worth noting due to the selection of GA control parameters including the population size, gene lengths, crossover and mutation rate etc, the above condition might never be met, hence GA will keep running. In order to solve this problem, we had another criterion for terminating the operation. That is, when the minimum cost is not showing improvement over 50 generations, we also terminate the execution of the algorithm.

As a whole, the above GA operations can be summarized in the following flow chart:
5.5 Computer Simulations

In order to test the proposed approach, computer simulations were conducted. We assume that the external target vibrates simple harmonically with vibration frequency $f = 200Hz$ and initial phase $\theta_0 = -\pi/2$. The unperturbed laser wavelength is $\lambda_0 = 785nm$. The simulations were carried out with the following procedure:

- **Step 1:** Given a set of true parameter values of $C, \alpha, L_0$ and $\Delta L$ within the parameter ranges, we generated a block of OFSMI signal using Eq. (5.2).
• Step 2: Divide the above signal block into 15 segments, each corresponding to a vibration cycle of the external target. Apply GA independently to each signal segment.

• Step 3: Evaluate the result by calculating the standard deviation over 15 estimations.

The cost function variable ranges are determined based on the discussion in Section 5.3.1. That is, \( C \in [0,1] \), \( \alpha \in [0,9] \), \( \phi_0 \in [0,2\pi] \) and \( \Delta \phi \in [K\pi,K\pi+2\pi] \), where \( K \) is the number of fringes in half target vibration period. The control parameters of GA are as follows: gene size of 6 bit for \( C \), and 9 bit for \( \alpha \), \( \phi_0 \) and \( \Delta \phi \). The initial population consists of 36 chromosomes. The selection rate is 0.5 and mutation rate is 0.7.

We repeat the simulation with different sets of true parameter values intentionally picked to cover entire parameter area. Similar performance in terms of the convergence and accuracy are observed for each signal. To sum up, we noticed that \( F_{\text{min}} \) decreases significantly at the beginning, but after 100 generations it stays on the order of \( 10^{-3} \) and can not be reduced further. In other words, the termination criterion in Eq. (5.8) can never be met. Therefore after observing \( F_{\text{min}} \) is not showing improvement over 50 generations, the algorithm is terminated.

Table 5.1 gives the results of the estimation after running GA for 150 generations. \( \hat{C}_{\text{min}} \) and \( \hat{C}_{\text{max}} \) represent the minimum and maximum estimations for \( C \) over 15 times running of GA, and the similar notations apply to the other three variables. The relative standard deviation of \( \alpha \) represented by \( \delta_\alpha/\hat{\alpha} \), where \( \delta_\alpha \) is the standard deviations of the estimations from their averages, is computed to reveal the consistency of the measurements. It can be noticed that good estimation for \( \phi_0 \) and \( \Delta \phi \) has been
obtained, as the minimum and maximum estimations are shown within a small range around their true values. However, estimates of $C$ and $\alpha$ still exhibit significant fluctuations which reveals the insensitivities of $C$ and $\alpha$ to the cost function as was discussed in Section 5.3.3.

Although the above simulations did not yield satisfactory estimations for the variables, the estimations for $\varphi_0$ and $\Delta \varphi$ show confinement within dramatically smaller area in contrast to their initial searching ranges. Hence inspired by the boosting theory [103] and microgenetic algorithm [104], we perform another iteration of GA by narrowing variable searching ranges to the neighbourhood of the solution from previous iteration. Given the results in table 5.1, the variable ranges are modified as $C \in [\hat{C}_{\min}, \hat{C}_{\max}]$, $\alpha \in [\hat{\alpha}_{\min}, \hat{\alpha}_{\max}]$, $\varphi_0 \in [\hat{\varphi}_{\min}, \hat{\varphi}_{\max}]$, and $\Delta \varphi \in [\Delta \hat{\varphi}_{\min}, \Delta \hat{\varphi}_{\max}]$. The GA control parameters are also modified as follows: gene size of 6, 9, 4 and 4 bits for $C$, $\alpha$, $\varphi_0$ and $\Delta \varphi$ respectively. The initial population consists of 24 chromosomes. Selection rate is still 0.5, but mutation rate is reduced to 0.3.

We found that $F_{\min}$ is able to reach the level of $10^{-5}$ in about 100 generations but no further improvement can be made after that. Hence we terminated the algorithm at generation 150 and the results are presented in table 5.2. Obviously, compared to the first iteration of GA, better results are achieved, although the estimates for $\alpha$ and $C$ are still not satisfactory.

In order to further improve the estimation accuracy, we narrowed the searching areas again based on the results from the second iteration and carried out another round of GA. The results are shown in table 5.3. It is seen that consistent estimations are obtained for all four parameters over the 15 times of tests. In particular, the minimum and maximum estimations for $\alpha$ and $C$ vary in a sufficiently small range with relative
standard deviation less than 1%. Therefore we calculate the average of the 15 estimations for each parameter, which are taken as the final estimates of $\hat{C}$, $\hat{\alpha}$, $\hat{\phi}_0$ and $\Delta \hat{\phi}$, as shown in table 5.4. Note that $\delta \hat{C}/\hat{C}$, $\delta \hat{\alpha}/\hat{\alpha}$, $\delta \hat{\phi}_{\delta}/\hat{\phi}_0$ and $\Delta \delta \phi/\delta \phi$ represent relative standard deviations where $\delta \hat{C}$, $\delta \hat{\alpha}$, $\delta \hat{\phi}_0$ and $\delta \Delta \phi$ are the standard deviations of the estimations from the true parameter values. We also noticed that after three iterations of GA, $F_{\text{min}}$ has reached the threshold and satisfactory estimates are obtained for all the four parameters, including $\alpha$.

Table 5.1 Simulation results for the first round of GA.
Table 5.2  Simulation results for the second round of GA.

<table>
<thead>
<tr>
<th>True Parameter Values</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>0.6</td>
<td>4</td>
</tr>
<tr>
<td>0.3</td>
<td>6</td>
</tr>
<tr>
<td>0.7</td>
<td>8</td>
</tr>
<tr>
<td>0.5</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 5.3  Simulation results for the third round of GA.

<table>
<thead>
<tr>
<th>True Parameter Values</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>0.6</td>
<td>4</td>
</tr>
<tr>
<td>0.3</td>
<td>6</td>
</tr>
<tr>
<td>0.7</td>
<td>8</td>
</tr>
<tr>
<td>0.5</td>
<td>3</td>
</tr>
</tbody>
</table>
Table 5.4 Parameter estimates.

<table>
<thead>
<tr>
<th>True Parameter Values</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>α</td>
</tr>
<tr>
<td>0.6</td>
<td>4</td>
</tr>
<tr>
<td>0.3</td>
<td>6</td>
</tr>
<tr>
<td>0.7</td>
<td>8</td>
</tr>
<tr>
<td>0.5</td>
<td>3</td>
</tr>
</tbody>
</table>

In order to study the impact of noise on the performance of the proposed algorithm, simulations with different level of signal-to-noise ratio (SNR) were also carried out. The results are shown in table 5 with true parameters being $C=0.6, \alpha=4, \varphi_0=1.76$ and $\Delta\varphi=25.61$. It is seen that the measuring accuracy is satisfactory when SNR is better than 10 dB. It is also noticed that noise impairs the performance of the multi-iteration GA significantly. The second and third iteration of GA hardly improve the result any further when the SNR is less than 5 dB.
Table 5.5  Effect of noise with the true parameters

\[ C = 0.6, \alpha = 4, \phi_0 = 1.76 \text{ and } \Delta \phi = 25.61 \, . \]

<table>
<thead>
<tr>
<th>SNR(dB)</th>
<th>( C )</th>
<th>( \delta_C/C )</th>
<th>( \hat{\alpha} )</th>
<th>( \delta_\alpha/\alpha )</th>
<th>( \hat{\phi}_0 )</th>
<th>( \delta_{\phi_0}/\phi_0 )</th>
<th>( \Delta \hat{\phi} )</th>
<th>( \delta_{\Delta \phi}/\Delta \phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0.5931</td>
<td>0.48%</td>
<td>4.0050</td>
<td>0.89%</td>
<td>1.7612</td>
<td>0.08%</td>
<td>25.6102</td>
<td>0.008%</td>
</tr>
<tr>
<td>30</td>
<td>0.5914</td>
<td>0.97%</td>
<td>4.0034</td>
<td>1.74%</td>
<td>1.7651</td>
<td>0.24%</td>
<td>25.6108</td>
<td>0.014%</td>
</tr>
<tr>
<td>20</td>
<td>0.6033</td>
<td>1.52%</td>
<td>3.9925</td>
<td>2.67%</td>
<td>1.7663</td>
<td>0.78%</td>
<td>25.6107</td>
<td>0.047%</td>
</tr>
<tr>
<td>10</td>
<td>0.5878</td>
<td>2.54%</td>
<td>4.0126</td>
<td>5.31%</td>
<td>1.7601</td>
<td>2.95%</td>
<td>25.6132</td>
<td>0.121%</td>
</tr>
<tr>
<td>5</td>
<td>0.6081</td>
<td>5.69%</td>
<td>3.9451</td>
<td>12.8%</td>
<td>1.7532</td>
<td>4.98%</td>
<td>25.6197</td>
<td>0.583%</td>
</tr>
</tbody>
</table>

To sum up, the multi-stage GA is described as follows:

- **Step 1:** Encode parameters as genes and chromosomes as described in Section 5.1.
- **Step 2:** Choose the initial population, selection rate and mutation rate as 36, 50% and 0.7 respectively.
- **Step 3:** Execute GA with a period of OFSMI signal data, using the termination criteria discussed above. If either of the criteria is met, terminate the algorithm. Repeat this 15 times using different data segments from the same OFSMI signal.
- **Step 4:** Evaluate the range of the 15 estimates for each parameter in Step 3. If the ranges are within the accuracy requirement associated with all the parameters, stop running the algorithm and take the average of the 15 estimates as the final results. Otherwise, modify parameter searching areas to
these ranges and change GA parameters including gene size, population and mutation rate and go back to Step3.

5.6 Experiments

The modified GA algorithm was also tested with the OFSMI experimental setup as was described in Section 3.2. A white paper is used as the reflective target which is made to vibrate harmonically by placing it on a loudspeaker driven by a sinusoidal signal. The distance between LD and target is kept about 10cm. The system is adjusted to work under weak feedback regime. As an example, an experimental self-mixing signal is given in Fig. 5.5. Four blocks of OFSMI signals were acquired corresponding to different $C$ levels by adjusting the experimental system. For each block of self-mixing signal, we chose fifteen segments, each corresponding to a vibration period of the target. The proposed algorithm is applied over the fifteen segments and averaged to yield the estimation results. Since ambient noise and electronic noise can be introduced due to data acquisition process, the signals are firstly processed with the procedure investigated in Chapter 3. Still, the relative standard deviations are computed to measure the consistency of the estimates. The results after performing the first iteration of GA for 150 generations are presented in table 5.6. Similar to computer simulations, the first iteration of GA yields estimations for $\phi_0$ and $\Delta \phi$ in a significantly smaller range, but not yet for $C$ and $a$.

We then refine the searching by modifying variable ranges and GA control parameters according to the result from the first iteration. The results after the second and third iterations of GA are given in table 5.7 and table 5.8 respectively. Unlike what happens in the simulation, the third iteration of GA does not yield much further improvement in measurement accuracy over the second iteration. This is easily
explained as a result of the noises contained in the experimental data. Since more
iterations of GA give no improvement, we terminate the operation after three iterations
and take the average of 15 results as the final parameter estimates, as shown in table 5.9.
We noticed that the relative standard deviations for the estimates are below 4.5%,
therefore we can say that good estimates are obtained. Our experiment was carried out
on the Intel® Core™2 Duo 3GHz personal computer. On average, it takes around 10
minutes to finish the three-staged searching for the optimized result.

In order to further verify the approach developed above, we also carried out a
comparison of the measured $\alpha$ with other OFSMI based approaches using the same SL.
As shown by table 5.10, our approach yielded close estimate to those by the approaches
in References [42] and [44] with much smaller standard deviation. Therefore we can
conclude that the proposed GA is effective.

![Experimentally measured self-mixing signal](image)

Figure 5.5 Experimentally measured self-mixing signal
### Table 5.6  Estimation result of first round of GA with experimental data.

<table>
<thead>
<tr>
<th>Data blocks</th>
<th>( \hat{C}_{\text{min}} )</th>
<th>( \hat{C}_{\text{max}} )</th>
<th>( \hat{\alpha}_{\text{min}} )</th>
<th>( \hat{\alpha}_{\text{max}} )</th>
<th>( \delta_{\alpha} / \bar{\alpha} )</th>
<th>( \hat{\phi}_{0,\text{min}} )</th>
<th>( \hat{\phi}_{0,\text{max}} )</th>
<th>( \Delta \hat{\phi}_{\text{min}} )</th>
<th>( \Delta \hat{\phi}_{\text{max}} )</th>
<th>Cost (( \times 10^{-3} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block1</td>
<td>0.31</td>
<td>0.70</td>
<td>1.8</td>
<td>6.8</td>
<td>58%</td>
<td>3.60</td>
<td>4.02</td>
<td>15.96</td>
<td>16.22</td>
<td>38.1</td>
</tr>
<tr>
<td>Block2</td>
<td>0.45</td>
<td>0.77</td>
<td>1.9</td>
<td>6.2</td>
<td>48%</td>
<td>2.32</td>
<td>2.81</td>
<td>16.01</td>
<td>16.24</td>
<td>43.2</td>
</tr>
<tr>
<td>Block3</td>
<td>0.50</td>
<td>0.86</td>
<td>1.4</td>
<td>5.7</td>
<td>51%</td>
<td>2.18</td>
<td>2.46</td>
<td>16.02</td>
<td>16.25</td>
<td>52.8</td>
</tr>
<tr>
<td>Block4</td>
<td>0.52</td>
<td>0.93</td>
<td>1.2</td>
<td>5.3</td>
<td>50%</td>
<td>1.81</td>
<td>2.12</td>
<td>15.94</td>
<td>16.15</td>
<td>53.4</td>
</tr>
</tbody>
</table>

### Table 5.7  Estimation result of second round of GA with experimental data.

<table>
<thead>
<tr>
<th>Data blocks</th>
<th>( \hat{C}_{\text{min}} )</th>
<th>( \hat{C}_{\text{max}} )</th>
<th>( \hat{\alpha}_{\text{min}} )</th>
<th>( \hat{\alpha}_{\text{max}} )</th>
<th>( \delta_{\alpha} / \bar{\alpha} )</th>
<th>( \hat{\phi}_{0,\text{min}} )</th>
<th>( \hat{\phi}_{0,\text{max}} )</th>
<th>( \Delta \hat{\phi}_{\text{min}} )</th>
<th>( \Delta \hat{\phi}_{\text{max}} )</th>
<th>Cost (( \times 10^{-3} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block1</td>
<td>0.45</td>
<td>0.50</td>
<td>2.1</td>
<td>4.8</td>
<td>16%</td>
<td>3.65</td>
<td>3.70</td>
<td>16.07</td>
<td>16.12</td>
<td>2.1</td>
</tr>
<tr>
<td>Block2</td>
<td>0.51</td>
<td>0.57</td>
<td>1.9</td>
<td>4.7</td>
<td>11%</td>
<td>2.47</td>
<td>2.52</td>
<td>16.09</td>
<td>16.13</td>
<td>3.3</td>
</tr>
<tr>
<td>Block3</td>
<td>0.58</td>
<td>0.66</td>
<td>2.2</td>
<td>5.6</td>
<td>13%</td>
<td>2.28</td>
<td>2.32</td>
<td>16.08</td>
<td>16.13</td>
<td>6.2</td>
</tr>
<tr>
<td>Block4</td>
<td>0.67</td>
<td>0.72</td>
<td>2.5</td>
<td>5.2</td>
<td>9.8%</td>
<td>1.95</td>
<td>2.02</td>
<td>16.08</td>
<td>16.11</td>
<td>6.8</td>
</tr>
</tbody>
</table>
### Table 5.8  Estimation result of third round of GA with experimental data.

<table>
<thead>
<tr>
<th>Data blocks</th>
<th>$\hat{C}_{\text{min}}$</th>
<th>$\hat{C}_{\text{max}}$</th>
<th>$\hat{\alpha}_{\text{min}}$</th>
<th>$\hat{\alpha}_{\text{max}}$</th>
<th>$\delta_{\alpha}/\hat{\alpha}$</th>
<th>$\hat{\phi}_{\text{max}}$</th>
<th>$\hat{\phi}_{\text{min}}$</th>
<th>$\Delta\hat{\phi}_{\text{min}}$</th>
<th>$\Delta\hat{\phi}_{\text{max}}$</th>
<th>Cost $(\times 10^{-3})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block1</td>
<td>0.46</td>
<td>0.49</td>
<td>3.01</td>
<td>3.46</td>
<td>4.5%</td>
<td>3.66</td>
<td>3.69</td>
<td>16.09</td>
<td>16.11</td>
<td>1.7</td>
</tr>
<tr>
<td>Block2</td>
<td>0.52</td>
<td>0.56</td>
<td>2.96</td>
<td>3.31</td>
<td>3.6%</td>
<td>2.49</td>
<td>2.52</td>
<td>16.10</td>
<td>16.11</td>
<td>2.5</td>
</tr>
<tr>
<td>Block3</td>
<td>0.59</td>
<td>0.63</td>
<td>3.15</td>
<td>3.39</td>
<td>4.1%</td>
<td>2.29</td>
<td>2.32</td>
<td>16.10</td>
<td>16.12</td>
<td>5.4</td>
</tr>
<tr>
<td>Block4</td>
<td>0.68</td>
<td>0.71</td>
<td>2.91</td>
<td>3.28</td>
<td>3.0%</td>
<td>1.96</td>
<td>2.02</td>
<td>16.10</td>
<td>16.11</td>
<td>5.2</td>
</tr>
</tbody>
</table>

### Table 5.9  Parameter estimates for experimental data.

<table>
<thead>
<tr>
<th>Data blocks</th>
<th>$\hat{C}$</th>
<th>$\delta_{\hat{C}}/\hat{C}$</th>
<th>$\hat{\alpha}$</th>
<th>$\delta_{\hat{\alpha}}/\hat{\alpha}$</th>
<th>$\hat{\phi}_{0}$</th>
<th>$\delta_{\hat{\phi}<em>{0}}/\hat{\phi}</em>{0}$</th>
<th>$\Delta\hat{\phi}$</th>
<th>$\delta_{\Delta\hat{\phi}}/\Delta\hat{\phi}$</th>
<th>Average Cost $(\times 10^{-3})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block1</td>
<td>0.48</td>
<td>2.5%</td>
<td>3.30</td>
<td>4.5%</td>
<td>3.68</td>
<td>0.9%</td>
<td>16.10</td>
<td>0.10%</td>
<td>1.7</td>
</tr>
<tr>
<td>Block2</td>
<td>0.54</td>
<td>1.9%</td>
<td>3.25</td>
<td>3.6%</td>
<td>2.50</td>
<td>0.7%</td>
<td>16.11</td>
<td>0.07%</td>
<td>2.5</td>
</tr>
<tr>
<td>Block3</td>
<td>0.60</td>
<td>2.1%</td>
<td>3.27</td>
<td>4.1%</td>
<td>2.31</td>
<td>0.9%</td>
<td>16.11</td>
<td>0.09%</td>
<td>5.4</td>
</tr>
<tr>
<td>Block4</td>
<td>0.69</td>
<td>1.4%</td>
<td>3.23</td>
<td>3.0%</td>
<td>1.98</td>
<td>1.0%</td>
<td>16.10</td>
<td>0.08%</td>
<td>5.2</td>
</tr>
</tbody>
</table>

### Table 5.10  Comparison of different approaches based on OFSMI for measuring LEF.

<table>
<thead>
<tr>
<th></th>
<th>[105]</th>
<th>[44]</th>
<th>Our approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\alpha}$</td>
<td>$3.5 \pm 7.6%$</td>
<td>$3.28 \pm 4.6%$</td>
<td>$3.26 \pm 3.8%$</td>
</tr>
</tbody>
</table>


5.7 Conclusions

This chapter presented genetic algorithm based method for estimating the linewidth enhancement factor (LEF) of semiconductor lasers using an optical feedback self-mixing system with an external target in simple harmonic vibration. Firstly a thorough investigation is carried out for the cost function on all possible parameter ranges, i.e., \( C \in [0,1], \quad \alpha \in [0,9], \quad \varphi_0 \in [0,2\pi] \) and \( \Delta \varphi \in [K\pi, K\pi + 2\pi] \), where \( K \) is the number of fringes in half target vibration period. Multiple local minima and a unique global minimum are found to be present on the error surface as a result. This implies a global optimization method should be employed to locate the global minimum for the cost function in order to achieve parameter estimation. Based on this observation, genetic algorithm (GA) is applied to search the minimum of the cost function. Whereas due to the uneven sensitivities of different parameters, the less sensitive parameters \( C \) and \( \alpha \) can not be identified with good accuracy in one step as the other two parameters. Hence a suitable procedure is proposed by performing another round of GA within the area that is obtained from the first round running of GA. Note the second and third round of GA is carried out with reduced parameters, i.e., shorter chromosomes, less population, thus to ensure fast convergence and effective identification of the less sensitive parameters in the cost function.

Compared to existing approaches, the proposed one does not suffer from the limitations on the requirement of knowledge regarding the movement of the external target. In other words, the proposed approach is effective without needing to know anything about vibration frequency, amplitude and the location of external target in terms of its distance to the laser. The proposed approach is tested with computer
simulation and experimental data which shows the relative standard deviations of 3.8% for the LEF measurements.
Chapter 6  Conclusions
Improvements of the efficiency and accuracy of two measurement applications of optical feedback self-mixing interferometry, namely measurement of the displacement of a moving target and measurement of the Linewidth Enhancement Factor (LEF) of a semiconductor laser has been achieved in this thesis work. Both measurement techniques are based on the mathematical model of the OFSM system. Hence the derivation of the theoretical model of the OFSM system and the dynamics of the model was firstly investigated (Chapter 2). For the purpose of facilitating the implementation of the measurement algorithms and improving the measurement accuracies, the noise contained in the self-mixing signals was eliminated with a signal pre-processing technique (Chapter 3). The displacement measurement was achieved with an analytical solution developed from the Lang-Kobayashi equations by means of a phase unwrapping technique (Chapter 4). The measurement of the LEF was accomplished with a data-to-model fitting technique via a multi-staged genetic searching algorithm (Chapter 5).

Chapter 2 reviews the phase condition and the intensity modulation of a semiconductor laser system with external feedback. The mathematical model (Lang-Kobayashi equations) is deduced as a result. The theoretical analysis of the Lang-Kobayashi equations indicated three feedback regimes (weak, moderate and strong feedback) featuring different shaped interferometric signal waveforms. Under weak feedback regime, the interferometric signal is continuous and can be adjusted to exhibit sawtooth-like shape when the feedback level factor is close to 1. This is a very useful feature for displacement measurement as the inclination of the interferometric fringes indicates the directional information of the target movement. Under moderate feedback regime, the self-mixing signal waveforms present abrupt switchings as a result.
of the hysteresis. As the hysteresis becomes stronger with the increasing feedback strength, the interferometric fringes become shorter until the zero-crossing points disappear. This poses difficulties in adjusting the experimental conditions for the LEF measurement approaches that relies on the location of this point. Under strong feedback regime, the self-mixing system exhibit chaotic behaviors which make it infeasible to perform reliable measurement with existent techniques.

The measurement performed with an OFSMI system relies solely on the observed self-mixing signals. Hence the data quality of experimentally obtained self-mixing signals plays a key role in achieving a measurement with high efficiency and good accuracy. Chapter 3 proposes a pre-processing procedure that is capable of eliminating the impulsive and random noises that were recognized as the main forms of noises contained in the self-mixing signals. In specific, the impulsive noise is firstly filtered with a nonlinear median filter. In order to achieve further data smoothing result which is essential for the performance of the proposed algorithms for displacement and LEF measurement in the following chapters, the artificial neural network approach is employed to eliminate the random noise at an advanced level. In particular, the radial function basis neural network is developed to fit the noisy self-mixing data input with a smooth output curve. Computer simulations revealed a perfectly smoothing result for the corrupted self-mixing signals under weak feedback level. The fitted curve is in good accordance with the original self-mixing waveform before the random noise with the signal-to-noise ratio of 20 dB is added in. Whereas, under moderate feedback level, satisfactory smoothing outcome can only be achieved by segmenting the self-mixing signal at the abrupt switching points and performing curve fitting within each segment of data between two switching points. In both cases, the error between fitted curve and the original clean signal is within the range of $\pm 0.4$ which indicates good consistency.
for the OFSMI signals that are normalized between -1 and 1. The verification of the proposed data pre-processing technique with noisy experimental data also indicated very effective data smoothing capacity.

The displacement reconstruction of a moving target is subsequently presented in Chapter 4. The analytical solution for the target displacement is solved from Lang-Kobayashi equations; however the direct recovery of displacement is hindered by the limitation of the principal value interval of the inverse cosine function in computing the laser phase with external feedback. The basic idea of phase unwrapping is to add or subtract multiple number of $2\pi$ to the result given by inverse cosine function when the target is moving from or towards the laser diode respectively. The directional discrimination of target movement is achieved by observing the fringe inclination of the sawtooth-like self-mixing waveforms. Computer simulations indicated measurement accuracy of $\lambda/25$ under weak feedback level and $\lambda/20$ under moderate feedback level, with the latter at a cost of the time consuming job to do data segmentation in performing signal pre-processing as was discussed in Chapter 3. Measurement with experimental data indicated good result under weak feedback regime. The significantly degraded measurement result under moderate feedback level is ascribed to the broadened jumping and dropping areas due to the data acquisition process in practice as opposed to the ideal abrupt switchings in theory.

Finally Chapter 5 presented a global searching method for the measurement of the linewidth enhance factor of a semiconductor laser. The measurement is performed based on the idea to find the best matching between theoretical model predicted output and the experimentally acquired data by updating a set of parameters in the model including the LEF. A thorough investigation on the error surface of the cost function revealed the presence of multiple local minima on the surface. This implied a global searching
algorithm is required to locate the global minimum among these local minima. Moreover, the investigation also found that the cost function responds to the change of each parameter in an unbalanced manner. In other words, the cost function is relatively insensitive to the change of the LEF. As a result, a multi-staged genetic algorithm is developed. This includes the running of a standard genetic algorithm in the first stage and performing another round of the algorithm by concentrating the searching within the area obtained from the previous round in the following stages. In particular, from the second round onwards, the searching for the more sensitive parameters is confined in a small area whereas the searching for less sensitive parameters such as the LEF varies in a bigger range so as to enlarge the influence of the less sensitive parameters on the cost function. The proposed approach is tested with computer simulation and experimental data which shows the relative standard deviations of 3.8% for the LEF measurements.

As an extension to this thesis work, the proposed method for target displacement measurement can be verified with the result obtained with other instruments. And the results can be prepared for publication in a journal.

The characteristics of self-mixing signal and system behavior under high level feedback regime can be further investigated in our future work. Other signal processing techniques such as wavelet transform filters can be explored for better pre-processing outcomes. An emerging interesting application of the self-mixing configuration to the signal encryption in the communication systems is also a very promising topic for our future work.
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List of Publications during PhD Study


