Analytical solutions for filtration process based on the constriction size concept

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Publication Details
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Abstract
In this paper, an analytical model was proposed to describe the filtration process applicable to a base soil-filter system. The Navier-Stokes equation for porous media was used to capture the hydrodynamic behavior, while numerically a new algorithm has been proposed to solve the Navier-Stokes equation in non-linear form. The various mixtures of base particles eroded and water flow within the system was computed using the work-energy principle incorporating the constriction size of the filter. The model can assess the filtration process through the flow rate, and the accumulation and erosion of base soil within the filter. By discretising the base soil and filter domains into discrete elements, the model can predict the time dependent particle gradation of the filter for each element. Laboratory tests reported in other studies and those conducted by the Authors have verified the validity of the model, in relation to other available models.

Keywords
filtration, process, solutions, constriction, concept, size, analytical

Disciplines
Engineering | Science and Technology Studies

Publication Details

This conference paper is available at Research Online: http://ro.uow.edu.au/eispapers/2508
Analytical Solutions for Filtration Process Based on the Constriction Size Concept

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ABSTRACT: In this paper, an analytical model was proposed to describe the filtration process applicable to a base soil-filter system. The Navier-Stokes equation for porous media was used to capture the hydrodynamic behavior, while numerically a new algorithm has been proposed to solve the Navier-Stokes equation in non-linear form. The various mixtures of base particles eroded and water flow within the system was computed using the work-energy principle incorporating the constriction size of the filter. The model can assess the filtration process through the flow rate, and the accumulation and erosion of base soil within the filter. By discretising the base soil and filter domains into discrete elements, the model can predict the time dependent particle gradation of the filter for each element. Laboratory tests reported in other studies and those conducted by the Authors have verified the validity of the model, in relation to other available models.

INTRODUCTION

Filters have been employed in earth structures to prevent the base soil erosion by seepage water. Since its first initiation by Terzaghi (Fannin 2008), filtration problems have been studied by numerous researchers using empirical and analytical techniques. Typical works using empirical method are those of Sherard et al. (1984), Sherard & Dunnigan (1989), Lafleur et al. (1989), Honjo & Veneziano (1989), Indraratna et al. (1996). Analytical models were based on probabilistic theory to establish the nature of the pore network or constriction sizes of filter media (Silveira et al. 1975, Kenney
et al. 1985, Humes 1996, Indraratna et al. 2007, Raut & Indraratna 2008, Indraratna et al. 2012, Nguyen et al. 2013). These have made major contributions towards the filter problems by recommending particle-size based or constriction-size based criteria. However, since filtration is a time dependent process, this mechanism needs to be studied in more detail.

In this paper, a numerical framework based on Patankar (1980) has been modified to incorporate the Navier-Stokes equations. The accumulation and erosion of base particles within the filter media can be evaluated using work-energy balance concepts that include the controlling constriction size concept (Indraratna et al. 2007).

**MATHEMATICAL FORMULATION**

The concept of one dimensional flow was assumed (Figure 1). Downward flow produces the velocity and pressure field that can apply hydraulic forces on base particles and transfer them into the filter. Based on the initial hydraulic conditions, base particles may be either captured within the filter at various depths or moved out of the system. The Navier-Stokes equation for porous media was used to consider the principles of hydrodynamics within the system (Bouillard et al. 1989):

\[
\rho_w \frac{\partial \varepsilon u}{\partial t} + \rho_w u \frac{\partial \varepsilon u}{\partial x} = -\varepsilon \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( \mu \frac{\partial \varepsilon u}{\partial x} \right) + f_b
\]

where \( \rho_w \) is water density; \( u, p, \varepsilon, \) and \( \mu \) are velocity, pressure, porosity and dynamic viscosity of water, respectively; and \( f_b \) is body force per unit volume.

The mass balance equation can be expressed as:

\[
\frac{\partial \varepsilon}{\partial t} + \frac{\partial \varepsilon u}{\partial x} = 0
\]  (2)

Water flow within the medium dislodges and transports base particles into the filter layer. Indraratna et al. (2007) showed out that the constriction sizes of filters play a key role in controlling the erosion of base soils. Accordingly, particles of base soil that are less than the controlling constriction size \( (D_c) \) are likely erodible. The procedure for computing the controlling constriction size was described elsewhere by Indraratna et al. (2007). Once particles of base soil are eroded and transported into the filter medium, the mixture of base grains and water flow can be considered as homogenous slurry. The hydrodynamic work-energy rule was used to evaluate the variation of the slurry density. The work done on a fluid system equals the change in the potential and kinetic energy of the system. Accordingly, the work-energy principle can be expressed as (Street et al. 1996):

\[
\frac{dW}{dt} = \frac{dE}{dt}
\]

(3)

where \( W \) is work done; and \( E \) is energy.

Applying the Reynolds Transport Theorem (Street et al. 1996) to evaluate the rate of change of energy of the system, one can have:

\[
\frac{dE}{dt} = \rho_{i+1} \left( g z_{i+1} + \frac{u_{i+1}^2}{2} \right) u_{i+1} \varepsilon_{i+1} - \rho_i \left( g z_i + \frac{u_i^2}{2} \right) u_i \varepsilon_i
\]

(4)
where $\rho$, $z$, $u$ and $\varepsilon$ are slurry density, height above the datum, velocity and porosity, respectively.

In this case the work done on the fluid system can be divided into two forms, namely pressure work and shear work:

$$\frac{dW}{dt} = \frac{dW_p}{dt} + \frac{dW_s}{dt}$$  \hspace{1cm} (7)

where $W_p$ is pressure work due to pressure forces via fluid entering and leaving the control volume; and $W_s$ is shear work done due to shear forces acting on the system at control surface.

Combining the above equations, the work-energy equation can be expressed as:

$$\rho_{i+1} \left( gZ_{i+1} + \frac{u_{i+1}^2}{2} \right) u_{i+1} \varepsilon_{i+1} - \rho_i \left( gZ_i + \frac{u_i^2}{2} \right) u_i \varepsilon_i = p_i \varepsilon_i u_i - p_{i+1} \varepsilon_{i+1} u_{i+1} - \frac{\mu_s}{\mu} u_{Li} \sum_{m=1}^{n_p} F_m$$  \hspace{1cm} (8)

where, $p_i$ and $p_{i+1}$ are pressures at Sections $i$ and $i+1$, respectively, $u_{Li}$ is velocity of water within the layer, $\mu$ and $\mu_s$ are its viscosity of water and slurry, respectively, $F_m$ is the hydrodynamic force on the $m^{th}$ particle within the layer.

**COMPARISON OF MODELS WITH EXPERIMENTAL DATA**

The aforementioned numerical procedures were integrated into a computational program using MATLAB. The proposed model was then compared with other existing filtration models by Indraratna & Vafai (1997), Locke et al. (2001), Raut & Indraratna (2008). For convenience, CM, IVM, LM, and RIM were designated for the current model, Indraratna & Vafai (1996) model, Locke et al. (2001) model, and Raut & Indraratna (2008) model, respectively.
Predictions of PSDs of filters

Locke et al. (2001) conducted comprehensive filtration experiments for their model verification. These models were compared with the PSDs of filters after the test to evaluate the accumulation of base soil within filters at various depths. The tests conducted considered uniform base and filter soils whose coefficients of uniformity were approximately 3 (Figure 2).

The base soils used in this study were uniform gradations while the filter was well-graded. After the tests, the filters were assessed to obtain the PSDs at 5cm and 20cm depths from the surface of the filter. Figure 4 presents the PSDs of filters as well as the predictions from CM, IVM, and LM. As expected, the PSDs of the filter elements shifted from the left, which represented an accumulation of base particles within the filter. IVM and LM employed mass balance concept to calculate the slurry density. The mass balance did not consider the effects of moving particles interaction and the characteristics of filter pore network. Two models assumed quite conservative erodible base particles compared to the laboratory observations. As shown out by Indraratna et al. (2007), for a given filter soil, the size of erodible base particles can be corresponded to the controlling constriction size ($D_c$) that can be assessed using the filter PSD. The size of erodible base particles used in IVM (i.e. the equivalent diameter of pore channel) and in LM (i.e. the constriction size at which 95% were finer) is about few times the controlling constriction size. A conservative assumption can contribute to a higher rate of erosion of base soil, as can the assumption that Darcy’s law normally provides a higher velocity field than one due to the turbulent effect. When all these factors are considered, the density of the slurry along the media can be overestimated, allowing particles of base soil to exist at any depth in the filters, even in effluent flow. For example, Fig. 4 shows the over predicted accumulations of base particles by IVM and LM at the 20cm depth for filters that were not observed in the tests. Moreover, the accumulation of base particles predicted by LM, were more than the others because of the assumption that once base particles are retained, they cannot move any more.
Additional tests were conducted to obtain the flow rate during filtration. All the tests used the same base soil (B). The test series was conducted using uniform filters (F1 and F2) (Fig. 5). The results then were compared with the predictions from CM, IVM, and LM (Fig. 6). The observations from the tests showed that combinations of B-F1 and B-F2, provide effective filters where the flow rate slowed initially and then became steady; a tendency that can be attributed to the formation of a stable, internal, self-filtering layer. The IVM and LM models appeared to have limitations in predicting the time dependent flow rate because the trend of the flow rates observed by these models were different from the laboratory observations. As mentioned earlier, all the simulations of IVM and LM showed there was a washout of base soil. Therefore, the flow rate predictions using IVM and LM were similar. At the initial stage, the flow rate increased due to base soil erosion, and then it becomes stable as erosion stopped.
FIG. 5. Particle size distributions of base and filters used in current study (data from Nguyen et al. 2013)

CONCLUSIONS

Filtration process is a time dependent process caused by factors such as the size of the base particles, constriction sizes of the filters, and the hydrodynamic conditions. This study presents an analytical model to capture these factors using the Navier-Stokes equation for porous media to capture the hydrodynamic flow. The work-energy equation incorporating the effect of the controlling constriction size could further elucidate the phenomena of accumulation and erosion of base soil in the filter.

A series of laboratory tests conducted in this study and data sourced from previous literatures were employed to confirm the model accuracy. By capturing the changes in particle size distributions within the filters, and the flow rates during the filtration process, the current model could offer reasonable predictions compared well with the experimental data. It was found that two filtration models proposed by Indraratna & Vafai (1997) and Locke et al. (2001) over-predict the PSDs of filters due to the simplified assumptions. These models cannot simulate the condition of water flow that changes due to the forming of self-filtering layers or temporary self-filtering layers within filter media. In contrast, the current model provides better predictions in terms of the accumulation of base soils within filters and the flow conditions during the filtration process.

ACKNOWLEDGMENTS

More elaborate details of the contents discussed in this paper can be found in previous publications of the first author and his research students in ASCE and Canadian Geotechnical Journals, since mid 1990’s.

REFERENCES