Modeling hierarchical relationships in Hinkle's implications grid data

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Abstract
There have been few attempts to devise suitable methods of analysis for the implications grid devised by Hinkle (1965). As Hinkle noted (Hinkle, 1965, p. 63), there are three implications needed to define a hierarchical relationship (A \(\rightarrow\) B, B \(\rightarrow\) C, and A \(\rightarrow\) C). Hinkle did not attempt to test this requirement, as neither did the only other published use of the technique (Fransella, 1972). Subsequently, Caputi, Breiger, and Pattison (1990) published a technique that explicitly sought to model implications data with respect to this requirement. In this study we use this technique to both (a) evaluate some of the choice points in the technique using data from the 28 implications grids collected by Hinkle and published as an appendix to his thesis, and (b) subsequently analyze this data to examine the hierarchical relationships as defined above. Our evaluation of the choice points showed that the joint modification approach worked best and that there was a clear cut-off to most fully represent the relationships in the raw data. Our analysis via the modeling approach found that there was no difference between the mean number of transitive superordinate constructs implied by subordinate constructs and the mean number of transitive subordinate constructs implied by superordinate constructs in the modeled data, suggesting that the laddered constructs in this study were not necessarily superordinate to the generating constructs.

Keywords
hinkle, data, relationships, hierarchical, modeling, implications, grid

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Modelling Hierarchical Relationships in Hinkle’s Implications Grid data with the Caputi et al. (1990) approach.

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Abstract

There have been few attempts to devise suitable methods of analysis for the implications grid devised by Hinkle (1965). As Hinkle noted (Hinkle, 1965, p.63) there are three implications needed to define a hierarchical relationship (A -> B, B -> C, and A -> C). Hinkle did not attempt to test this requirement as neither did the only other published use of the technique (Fransella, 1972). Subsequently Caputi, Breiger & Pattison (1990) published a technique which explicitly sought to model implications data with respect to this requirement. In this study we use this technique to both (a) evaluate some of the choice points in the technique using data from the twenty eight implications grids collected by Hinkle and published as an appendix to his thesis, and (b) subsequently to analyze this data to examine the hierarchical relationships as defined above. Our evaluation of the choice points showed that the joint modification approach worked best and that there was a clear cut-off to most fully represent the relationships in the raw data. Our analysis via the modelling approach found that there was no difference between the mean number of transitive superordinate constructs implied by subordinate constructs and the mean number of transitive subordinate constructs implied by superordinate constructs in the modelled data, suggesting that the laddered constructs in this study were not necessarily superordinate to the generating constructs.
Modelling Hierarchical Relationships in Hinkle’s Implications Grid data with the Caputi et al. (1990) approach.

In his theory of personal constructs George Kelly (1955) posited that relationships among constructs were of a superordinate-subordinate kind. This corollary was first considered empirically by Dennis Hinkle (1965), who proposed both a method of eliciting superordinate constructs and a method of testing whether the relationships among elicited constructs were in fact of a superordinate-subordinate kind.

The former technique is now more generally known as laddering (Walker & Crittenden, 2012). There are a number of forms of this procedure but in essence is as follows. The laddering is built around an implicative relationship derived from the preference of one pole (of the generating construct) over the other, followed by the query "Why?" The response provided is one pole of the theoretically superordinate construct. The contrast pole is then elicited and the preference and reason procedure repeated to elicit a further superordinate construct. Hinkle had 28 subjects generate ten ordinary constructs by the triadic elicitation method, and these ten constructs were then used to generate ten further constructs by the laddering procedure. All 20 constructs were then employed in the implications grid technique. Again variations in this procedure exist but in essence it is a series of yes/no responses to queries of the kind "If you were at pole Y of this construct, would you be at pole X of that construct?" for each possible pair of constructs. For Hinkle each of his 20 constructs were compared with the other 19 constructs, both those elicited by triadic elicitation and those laddered. He counted the number of implications (yeses) for each construct and compared the totals for the triadic and laddered constructs. He concluded that constructs elicited by laddering were indeed superordinate as they had more implicative
relationships. However Hinkle based this on all links, both one-way and reciprocal, including those between constructs of the same kind (e.g., super-ordinate implies other superordinate construct). Bell (2014) re-examined Hinkle’s data, focussing on just those implicative links between the subordinate and superordinate constructs, and found subordinate was just as likely to imply superordinate as the reverse. However that analysis could be said to suffer from similar problems to Hinkle’s analysis. A problem is that in order to demonstrate a hierarchical relationship between constructs - as noted by Hinkle (1965, p.63) - three construct relationships are needed. If construct A implies construct B, and construct B implies construct C, then construct A must also imply construct C. Those three implications logically define a hierarchical relationship, which can be tested. Hinkle did not attempt to test it since a computer is really required as there are 20 x 19 x18 = 6840 triples of constructs each of which have over 16 possible patterns of transitivity - from none (no implications) through partial or weak (construct A implies construct B, and construct B implies construct C) to strong transitivity (construct A implies construct B, construct B implies construct C, and construct A implies construct C). He did suggest however that it might be possible to devise an index of logical consistency based on this approach.

Another problem was that both the Hinkle and Bell analyses were based on the observed implicative links. It could be that an individual response to a possible link between two constructs was in error (i.e., saying an implication was there when it wasn’t, or the reverse). One would hope such errors would average out across all links as is implicit in all summed scoring in standard tests. Within a personal construct grid context the use of only a few factors or dimensions likewise focuses on major relationships and ignores the minor perturbations caused
by random errors of response. Considering triples of relationships (as in properly testing for hierarchical relationships) magnifies this problem as any error link will affect many triples. An approach that overcomes this problem is the modelling approach proposed by Caputi, Breiger, and Pattison (1990). That approach seeks to model the transitivity in an implications grid by iteratively altering the values of the links so as to increase local transitivity in the vicinity of the links. For those interested a brief outline of the procedure is given below. However, the following two paragraphs are not essential to an understanding of the main thrust of the results.

The key to modelling implication relationships lies in the triad of directed relationships: if construct A implies construct B, and construct B implies construct C, then construct A must also imply construct C. If all relationships in an implications grid R satisfy this relationship, then the following relationship must also hold. If $R$ is the matrix of implicative relationships (i.e., the implications grid) then the following relationship holds: matrix when multiplied by itself in a Boolean product is equal to the matrix

$$ R \circ R = R $$

where $\circ$ denotes the Boolean product

An implications grid which is fully transitive and satisfies this condition is unlikely to be obtained in practice. The Caputi et al. approach is to modify the matrix by considering related links outside the A, B, C triad and adjusting the links in the triad on the basis of this information so as to increase the degree of transitivity in the grid. How well this works is measured by the fit of the matrix of adjusted links in the Boolean product relationship against the generating grid. This can be carried out in an iterative fashion, assessing the current modification against the preceding modification until no further improvement in transitivity can be made.
There are different possible ways of making the adjustments. Choice of method (and criterion) is a common issue in modelling grid data. In factor analysis we choose the number of factors and the method of obtaining them; in cluster analysis we choose a measure of association and a method of clustering; and in the implicative dilemma procedure of Feixas, Saúl, & Sánchez Rodríguez (2000), there is a choice of magnitude of discrepancy between self and ideal and a choice of level of correlation between the relevant constructs. Here the method choices refer to the Boolean product, $R \circ R$. Boolean operations are logical rather than arithmetic, thus normal matrix multiplication operations do not hold here, rather the operations are comparisons of data through a proportional index (see Caputi, Breiger, and Pattison, 1990; or for more detail, Caputi, 2004). The way in which these comparisons are made differs for the left and right versions of the $R$ matrix in $R \circ R$. Thus there are three possible ways of adjusting implicative links, focussing on the left $R$, the right $R$, or both. A further decision point is to decide at what criterion level of adjusted implicative link a link in the final model should be defined. The initial paper by Caputi, Breiger, and Pattison (1990) was an introductory description and did not examine either of these issues. One aim of this paper was thus to provide some empirical information about these two choice points. We proposed to evaluate both the method of adjustment and choice of transitivity level in the Caputi et al. modelling procedure by analysing the Hinkle data with the three methods of adjustment described above and considering various levels of criterion from a weak transitivity level of 0.65 through to a very strong 0.95. (The generating Caputi et al. article took 0.90 as indicating a strong level of transitivity.) To assess the impact of choosing different criterion levels of linkage, we correlated the linkages in the generating data with the linkages at the various criterion levels in the modelled data. At weak levels of implication, we expected
poorer correlation because the model would contain numerous linkages not present in the generating data. We also expected poorer correlations at very strong levels of implication since the model would contain far fewer implicative links than in the generating data. We hoped that there might be some optimal intermediary level of criterion which would capture as much of the generating implications while at the same time ensuring that the linkages satisfied as much as possible the transitive requirement.

We also sought to replicate Hinkle’s original findings (1965) and Bell’s re-analysis (2014) by examining the nature of the links, not in the raw Hinkle grids, but in the iteratively modelled version of the grids. The information in a cell of an implication grid can indicate one of four possible relationships between a pair of constructs, one being in the rows of the grid, the other in the columns.

i. Null – no relationship between row and column

ii. Row implies column (but not the reverse)

iii. Column implies row (but not the reverse)

iv. Row and column imply each other.

Within 'subordinate' and within 'superordinate' constructs there is no useful distinction between ii, 'Row implies Column' and iii. 'Column implies Row': hence in these cases these two categories can be collapsed to that of a unidirectional relationship. Where rows are 'subordinate' and columns 'superordinate' (or the reverse) all four categories above have meaning. Thus in Hinkle's grids there are ten types of implication of interest (as detailed in Table 1).
An important outcome of the Caputi et al. (1990) procedure is the ability to examine the residuals from fitting the model to the data. In Hinkle’s data there are two classes of constructs; generating (subordinate) and laddered (superordinate). There are four groups of residuals of interest. Those due to

1. Relationships among subordinate constructs
2. Prediction of superordinate constructs by subordinate constructs
3. Prediction of subordinate constructs by superordinate constructs
4. Relationships among superordinate constructs

Neither theory nor previous research (there is none) suggests any hypotheses about these kind of residuals although the null hypothesis might suggest no differences.

**Methodology**

**The Data**

The raw implication grids were transcribed from the thesis of Hinkle. Links were coded as 1 for both Hinkle’s ‘x’ (one-way link) and ‘r’ (reciprocal link).

**The Analyses**

A purpose written fortran program\textsuperscript{ii} was written to carry out the modelling procedure. SPSS was used to carry out the statistical testing.

**Results**

Methodological (Evaluating choice-points in the Caputi et al. procedure)
Our comparison of the differences between the three methods was based in part on the fit of such models to the data. A mixed model analysis of variance approach was used to test for differences in fit for the three methods. There was no significant difference between the sums of residuals for the three methods ($F = 0.447, df=2,81, p=0.64$).

In order to assess cutting point choice we plotted the correlation between data links and model links for each of the methods at each of seven levels of criterion between 0.65 and 0.95 for each of the 28 Hinkle implication grids. Figure 1 shows the average results.

*Insert Figure 1 about here*

It can be seen that although there was no significant difference between the three methods in terms of residuals, however the joint proportional index method (IRL) shows higher correlations between data links and model links. As expected lower criterion levels were associated with poorer average correlations and there was some decline in correlation at higher levels of criterion. The best results were obtained for the joint proportional index method at a criterion level of 0.80, and accordingly these were used in producing the substantive results that follow.

**Substantive (testing Hinkle's hypothesis of implication)**

Means and standard deviations for Hinkle’s ‘subordinate’ and ‘superordinate’ construct relationships are shown in Table 1.

*Insert Table 1 about here*

There are clearly substantial differences among the 28 implications grids as evidenced by the high standard deviations (relative to the means). We can see that modelling reduces
relationships (and thus increases null relationships) in all three groups of constructs. The changes are all significant except for directed relationships within laddered constructs ($F=0.36$, $df=1,54$, $p=.554$). Comparisons among the raw grid means were discussed by Bell (2014) and here we focus on comparisons among the modelled implications.

We used a mixed model approach (as we did with all statistical testing in this paper in order to account for the within grid nature of the data) to the testing of significant differences between types of relationship across the three kinds of construct relations (within generating constructs (subordinate), within laddered constructs (superordinate), and between generating and laddered constructs).

Testing the percentage of null or no relationship is equivalent to jointly testing both mutual and directed links. There were significant differences between the three means ($F = 14.29$, $df = 2,81$, $p<.001$) but LSD post hoc tests showed only the within laddered constructs (mean 41.67) to significant differ from both within generating constructs (mean 79.13) and between generating and laddered constructs (mean 68.75). Thus there were more links (either directed or reciprocal) within the laddered constructs. With respect to mutual or reciprocal relationships, the same pattern emerged; there was significant variation among the three ($F = 5.31$, $df = 2,81$, $p=.007$) with only within laddered constructs differing from both within generating constructs and between generating and laddered constructs. Simple comparisons between among generating and among laddered constructs show that for both directed and mutual cases, laddered constructs have more links (respectively; $F = 15.56$, $df = 1,54$, $p<.001$ for directed and $F = 8.36$, $df = 1,54$, $p<.001$ for mutual).
p=.006 for mutual). Within the set of generating constructs there were significantly fewer mutual relationships than directed relationships (F=7.51, df=1,54, p=.008).

For the purpose of this study when we considered directed (implicative) links we did not differentiate between row implies column and column implies row for within both generating and laddered constructs since these are by definition homogeneous groups of constructs. But in the set of links between generating and laddered constructs, there are possibly meaningful differences between generating implies laddered constructs, and the converse, laddered implies generating constructs, hence there were four means overall. It is apparent (from Table 1) that null relationships between generating and laddered constructs predominated. Accordingly we focussed on the two directed links and mutual links. There was no significant variation among the means (F = 0.174, df = 2,81, p=.0.84). The key comparison for Hinkle's superordinate hypothesis was the post-hoc test between mean directed links from generating to laddered constructs (11.07) and mean directed links from laddered to generating constructs (10.96). There was no significant difference (F = 1.08, df = 2,81, p=.345).

In terms of subsets of residuals (for construct implications of different kinds as outlined above), means and standard deviations for residuals in these four categories are shown in Table 2.

*Insert Table 2 about here*

Although laddered constructs showed a smaller residual in fitting the model to data, there was no significant variation among means (mixed model F=0.94, df=3,108, p=.423).
Discussion

The modelling procedure devised by Caputi, Breiger and Pattison (1990) is the only approach to date that can properly test implication grid data for superordinate structures. Like many grid data modelling procedures (such as principal components or clustering) it requires several choices to be made by the researcher. This study of a small published data set (albeit a famous one) provides some guidance as to the choosing of a method and establishing a cutoff, with the joint approach and a criterion of 0.80 providing the closest fitting of the final model to the raw implications data. Although the modelling discards a number of raw implicative links, it is encouraging to note that substantial amounts of superordinate structures in the raw implications grid, particularly those of laddered constructs, are preserved. This suggests that in practical situations users can interpret the hierarchical structures in the raw grid with some confidence (although ideally a definitive modelling should be preferred).

In the substantive examination of the implications grid data of Hinkle (1965), it was found that laddered constructs had more relationships with each other, both directed and mutual, than the generating set of constructs. This was in accord with the conclusion of Hinkle - although this study omits his inclusion of the confounding generating and laddered links. Although there were less mutual relationships than directed ones in the laddered constructs, there were significantly more than were found in the other two segments of the data, both within the generating set of constructs and between the generating and laddered sets. Directed relationships between generating and laddered constructs indicate both that the less superordinate construct implies the more superordinate and the more superordinate implies the less one. There were also substantial
mutual relationships (an issue noted by Hinkle) between generating and laddered constructs. This seems to conform with the laddering observation of Butt (1995, p.229) that “It is my experience that this procedure frequently produces snakes as well as ladders, going both up and down the system in a looping and circular fashion.” However Butt is not the only one to note this. Hinkle (1965, p.59) observed “that occasionally a specific construct label would be given at several different levels in the hierarchy, e.g., if constructs A, B, and C imply X, and X implies D, E, and F, then occasionally D, E, and F would imply X again, and this would, in turn, imply G, H, and I.”. This would suggest that rather than a restricted hierarchical organization of laddered constructs, there is a more general network structure of such constructs which can involve both directed and mutual relationships. This possibility has been examined in a market research oriented laddering context by van Rekom & Wierenga (2007), who suggested that network modelling may be an appropriate way to investigate both directed and mutual implications.

The generating constructs were not significantly different in terms of model fit residuals but had far fewer raw implications (both directed and mutual) retained in the final modelled implications. In fact within the set of generating constructs there were fewest mutual relations. This is perhaps surprising given the relatively routine finding of substantial mutual relationships (as correlations) in repertory grid research. For example, Bell (2004, p.292) found that in an analysis of 400 grids over three studies, there was a preponderance of symmetric relations over asymmetric relations (between 3 and 5 times more). Why were there so few mutual relationships among these constructs generated by triadic elicitation as in repertory grids in comparison with repertory grid research? The answer might lie in the fact that while both here and in research
using repertory grids, triadic elicitation is used, in other repertory grid research the elicited construct is then used to construe a common and larger set of elements which provides the data for the construct correlations that define symmetry. The repertory grid constructs are thus more firmly anchored as “ways of anticipating events” in a broader sense, which may explain the contrasting results here.

The major focus of this study was on the third segment of the implications grid, the interface between the generating and laddered constructs. If there was a strict hierarchical structure of relationships between generating and laddered constructs then there should have been significantly more instances of generating construct implying laddered construct than laddered construct implying generating construct. But there weren’t: there were no significant difference between the modelled implications in either direction.

Given the differences in results for the generating constructs (elicited from elements) and laddered constructs (elicited from constructs) the possibility that these two kinds of constructs are qualitatively different must be considered. An early controversial issue in construct theory was whether or not constructs could function as elements within the range of convenience of other constructs. Bannister and Mair (1968, p.126) defined elements as “constructs within the range of convenience of a superordinate construct.” This treating of elements as constructs was contested; Slater (1969) was concerned about the confusion between the roles of constructs as both operators and operands and the role of elements, as was Ryle (1975, pp. 121–122). However laddered constructs are derived precisely in this way – with reference to generating constructs rather than events or objects. Since all constructs are mental abstractions one would
expect that constructs of constructs would be more abstract – as indeed found by Neimeyer, Anderson & Stockton (2001).

**Summary and Conclusions**

The first focus of this paper was concerned with evaluating the transitive oriented approach of Caputi et al, in modelling implications grid data. Perhaps (pleasingly) similar results were obtained to earlier analysis of the raw implications grid data. It may well be that simple methods of directly analysing raw grid data will suffice in some instances.

The second focus of this paper is about testing a hypothesis implicit in the laddering technique devised by Hinkle with another technique he devised, the implications grid. An implicative link is not, in and of itself, sufficient to claim a hierarchical relationship. There must be a particular arrangement of implicative links for a hierarchical relationship to be demonstrated. This study showed that laddered constructs were not necessarily superordinate to constructs used to generate them – generating constructs were not significantly less likely to be superordinate to laddered constructs.

This is a global finding only as the only information about the laddering was that provided by the implications grid. Information about the sequences of laddering, which would allow for a more fine-grained analysis, was not included in Hinkle's thesis. The recently devised procedure of consistent laddering (Korenini, 2014) would preserve such information and further address the issue of how laddered constructs relate to the elements at the basis of the construal processes.

**References**


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\(^{i}\) The structure we refer to is known technically as a "directed acyclic graph". It is a requirement for a hierarchical order. If we only know that A predicts B and B predicts C, then it would be possible that C would predict A. This would be a closed loop (a "directed cyclic graph") and no inference of hierarchy could be made. Thus we need the third relationship A predicts C to rule the alternative possibility out.

\(^{ii}\) Readers interested in obtaining a copy of the program should contact the first author.
Table 1. Mean percentages of types of construct links for Generating and Laddered constructs as raw data and as modelled data.

<table>
<thead>
<tr>
<th></th>
<th>Raw Grid</th>
<th>Modelled Grid</th>
<th>Pairwise Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std.Dev</td>
<td>Mean</td>
</tr>
<tr>
<td><strong>Within generating constructs (subordinate)</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>No relationship</td>
<td>59.68</td>
<td>18.08</td>
<td>79.13</td>
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<tr>
<td>Mutual relationship</td>
<td>14.52</td>
<td>11.97</td>
<td>5.63</td>
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<td>Directed relationship</td>
<td>25.79</td>
<td>11.73</td>
<td>15.24</td>
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<tr>
<td><strong>Within laddered constructs (superordinate)</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>No relationship</td>
<td>22.94</td>
<td>20.90</td>
<td>41.67</td>
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<tr>
<td>Mutual relationship</td>
<td>37.30</td>
<td>22.56</td>
<td>21.90</td>
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<tr>
<td>Directed relationship</td>
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<td>16.67</td>
<td>36.43</td>
</tr>
<tr>
<td><strong>Between generating and laddered constructs</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>No relationship</td>
<td>46.82</td>
<td>16.82</td>
<td>68.75</td>
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<tr>
<td>Mutual relationship</td>
<td>19.18</td>
<td>14.11</td>
<td>9.21</td>
</tr>
<tr>
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<td>17.11</td>
<td>10.06</td>
<td>11.07</td>
</tr>
<tr>
<td>Directed (Laddered implies Generating)</td>
<td>16.89</td>
<td>8.03</td>
<td>10.96</td>
</tr>
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</table>

* df= 1,54
Figure 1. Tracelines of average correlations between data and model for three methods of modifying model and seven levels of criterion.
Table 2. Means and standard deviations for residuals according to four type of links between constructs.

<table>
<thead>
<tr>
<th>Residual</th>
<th>Mean</th>
<th>Standard Deviation</th>
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<tr>
<td>Relationships among subordinate constructs</td>
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<td>.11</td>
</tr>
<tr>
<td>Prediction of superordinate constructs by subordinate constructs</td>
<td>.35</td>
<td>.10</td>
</tr>
<tr>
<td>Prediction of subordinate constructs by superordinate constructs</td>
<td>.36</td>
<td>.11</td>
</tr>
<tr>
<td>Relationships among superordinate constructs</td>
<td>.31</td>
<td>.11</td>
</tr>
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