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Theoretical modelling and experimental investigation of the performance of screw feeders

Yongqin Yu

University of Wollongong

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THEORETICAL MODELLING AND EXPERIMENTAL INVESTIGATION OF THE PERFORMANCE OF SCREW FEEDERS

A thesis submitted in fulfilment of the requirements for the award of the degree of

Doctor of Philosophy

from

UNIVERSITY OF WOLLONGONG

by

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B.E. (SJTU), M.E. (SJTU)

Department of Mechanical Engineering

1997
DECLARATION

This is to certify that the work presented in this thesis was carried out by the author in the Department of Mechanical Engineering at the University of Wollongong and has not been submitted for a degree to any other university or institution.

Yongqin Yu
ACKNOWLEDGMENT

I would like to express my sincerest appreciation to Prof. Peter C. Arnold in the Department of Mechanical Engineering at the University of Wollongong for his supervision, encouragement and valuable advice during the course of this research program.

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SUMMARY

Screw feeders are being widely used in industry to feed reliably many kinds of bulk solids because of their simplicity in structure, good metering characteristics, total enclosure for safety, great flexibility of design and wide application. However, to date, the design of this type of equipment is still not precise due to an overall shortage of theoretical analyses and experimental data reported in the literature. For this reason, this thesis aims at formulating theoretical models to predict the performance of single and twin screw feeders. The models concentrate on the volumetric efficiency, draw-down performance and torque or power requirement of screw feeders.

The models are based on analyses of the relevant particulate mechanics. An element of bulk solid sliding on the helical surface of a screw flight will move in a direction related to the angle of friction between the bulk solid and the surface, and helical angle of the flight surface. An analytical solution to the integral equation to express the volumetric efficiency of screw feeders has been obtained. A theoretical model for the torque requirement is developed by analysing the stresses acting on the five confining surfaces surrounding the bulk solid contained within a pitch. This model allows the torque characteristics to be predicted. A criterion for uniform draw-down performance is presented based on the pitch characteristics of the screw.

To obtain detailed information on the interaction among the screw geometry, the operating conditions and properties of the bulk solid, a systematic experimental investigation is undertaken into the performance of screw feeders with different screw and trough configurations, three types of bulk material and different operating conditions. Experimental programs on the twin screw feeders are also conducted to observe the effect of operating conditions and geometric parameters. The factors affecting performance of the twin screw feeders also are analysed.
Experimental data are compared with the results obtained from the theoretical predictions. The volumetric efficiency, draw-down performance and torque requirements of screw feeders predicted by the theoretical models developed in this work are found to provide results which agree well with experimental data.

The performance prediction models are also applied in industrial practice. Some typical problems experienced by industry are analysed. New design of screw feeders are provided for four upgrade projects.

KEY WORDS: screw feeders, volumetric efficiency, bin flow patterns, torque characteristics
TABLE OF CONTENTS

ACKNOWLEDGMENTS i
SUMMARY ii
TABLE OF CONTENTS iv
LIST OF FIGURES ix
LIST OF TABLES xvii
NOMENCLATURE xviii

CHAPTER

1 INTRODUCTION 1

2 LITERATURE SURVEY 7

2.1 Introduction 7

2.2 Outline of Studies on Screws Applied in Bulk Materials Handling 8

2.2.1 Screw Conveyor and Elevator 8

2.2.2 Screw Extruder 14

2.2.3 Screw Feeder 18

2.3 Distinction between Screw Feeder and Other Screw Devices 23

2.4 Past Studies on the Performance of Screw Feeders 26

2.4.1 Volumetric Efficiency 26

2.4.2 Torque Requirement 28

2.4.3 Draw-Down Performance 29

3 THEORETICAL MODELLING OF SCREW FEEDER PERFORMANCE 32

3.1 Introduction 32

3.2 Volumetric Efficiency 34

3.2.1 Definition and Derivation 34

3.2.2 An Analytical Solution 36
3.2.3 Equivalent Helical Angles 38
3.2.4 Comparison between Calculated Results 40
3.2.5 Influence of Parameters 42
3.2.6 Effect of Clearance on Output 43

3.3 Torque Requirements 45
3.3.1 Feeder Loads 45
3.3.2 Pressure on Boundary Surfaces of Bulk Material 47
3.3.3 Pressure Distribution on Bulk Material in Lower Region 48
3.3.4 Forces Acting on Individual Surfaces 52
    3.3.4.1 Axial Force on Shear Surface 53
    3.3.4.2 Axial Force on Core Shaft 54
    3.3.4.3 Axial Force on Trailing Side of Flight 55
    3.3.4.4 Axial Force on Trough Surface 56
    3.3.4.5 Axial Force and Stress on Driving Side of Flight 56
3.3.5 Torque Requirement 57
    3.3.5.1 Torque Requirement in Feed Section 57
    3.3.5.2 Torque Requirement in Choke Section 60
3.3.6 Application of Equivalent Helical Angles in Torque Calculation 61
3.3.7 Torque Characteristics of a Screw Feeder 63
    3.3.7.1 Torque Components 63
    3.3.7.2 Influence of Clearance on Torque 65
    3.3.7.3 Influence of Trough Wall Friction Coefficient 66
    3.3.7.4 Influence of Effective Angle of Internal Friction 67
    3.3.7.5 Influence of Flight Friction Coefficient 68
    3.3.7.6 Influence of Ratio d/D 69
3.3.8 Power Efficiency 69
3.3.9 A Simplified Approach to Power Calculation 73

3.4 Draw-Down Performance 74
3.4.1 Average Effective Area 74
3.4.2 Criterion for Uniform Draw-Down Performance 75
3.4.3 Limitation of Some Design Methods 77
  3.4.3.1 Stepped Pitch 77
  3.4.3.2 Tapered Shaft, Uniform Pitch 79
  3.4.3.3 Tapered Screw Diameter, Uniform Pitch 80
3.4.4 Hopper Geometry Interfacing with Screw Feeder 81

4 EXPERIMENTAL FACILITY AND TECHNIQUES 87
  4.1 Introduction 87
  4.2 Description of Test Rig 87
    4.2.1 Hopper and Trough 90
    4.2.2 Dividing grid 91
    4.2.3 Test Screws 93
    4.2.4 Driving Unit 96
    4.2.5 Receiving and Weighing Silo 97
  4.3 Instrumentation and Data Acquisition 97
    4.3.1 Mass Output of Bulk Material Discharged 98
    4.3.2 Torque Requirement 98
    4.3.3 Rotating Speed of Screw 98
    4.3.4 Data Acquisition System 98
    4.3.5 Data Processing 99
  4.4 Calibration 100
    4.4.1 Load Cell Calibration 100
    4.4.2 Torque Transducer Calibration 101

5 TEST BULK SOLIDS AND PROPERTIES 104
  5.1 Introduction 104
  5.2 Particle Size and Distribution 104
  5.3 Density Analyses and Measurement 107
    5.3.1 Particle Density 107
    5.3.2 Bulk Density 109
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Caption</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>A screw feeder/discharger [72]</td>
<td>2</td>
</tr>
<tr>
<td>1.2</td>
<td>Direct end transfer to conveyor/elevator [9]</td>
<td>3</td>
</tr>
<tr>
<td>1.3</td>
<td>Double plug seal and side transfer to pneumatic duct [9]</td>
<td>3</td>
</tr>
<tr>
<td>1.4</td>
<td>Roll press feed system combined with screw feeder [34]</td>
<td>4</td>
</tr>
<tr>
<td>2.1</td>
<td>Screw conveyor [22]</td>
<td>9</td>
</tr>
<tr>
<td>2.2</td>
<td>Inclined screw conveyors [22, 26]</td>
<td>10</td>
</tr>
<tr>
<td>2.3</td>
<td>An experimental screw extruder [43]</td>
<td>14</td>
</tr>
<tr>
<td>2.4</td>
<td>View of velocities of viscous liquids in a screw extruder [19]</td>
<td>15</td>
</tr>
<tr>
<td>2.5</td>
<td><strong>Forces balance</strong> on a volume element for the channel region [53]</td>
<td>16</td>
</tr>
<tr>
<td>2.6</td>
<td>Cross section of a twin screw extruder [53]</td>
<td>17</td>
</tr>
<tr>
<td>2.7</td>
<td><strong>Bulk material transport</strong> in the channel of a twin screw extruder [53]</td>
<td>17</td>
</tr>
<tr>
<td>2.8</td>
<td>A single screw feeder as a separate unit [22]</td>
<td>18</td>
</tr>
<tr>
<td>2.9</td>
<td>A single screw feeder with extension conveyor [22]</td>
<td>18</td>
</tr>
<tr>
<td>2.10</td>
<td>A test rig for screw feeders [81]</td>
<td>19</td>
</tr>
<tr>
<td>2.11</td>
<td>A experimental set-up for screw feeders [77]</td>
<td>19</td>
</tr>
<tr>
<td>2.12</td>
<td>A twin screw feeder [22]</td>
<td>20</td>
</tr>
<tr>
<td>2.13</td>
<td>A typical form of screw feeder fitted with hopper</td>
<td>25</td>
</tr>
<tr>
<td>2.14</td>
<td>Assumed pressure distribution around boundary [65, 69]</td>
<td>28</td>
</tr>
<tr>
<td>2.15</td>
<td>Flow patterns with four screws and sodium perborate tetrahydrate [8]</td>
<td>30</td>
</tr>
<tr>
<td>2.16</td>
<td>Screw feeder geometry [40, 63, 64, 69]</td>
<td>30</td>
</tr>
<tr>
<td>2.17</td>
<td>Example for optimal pitch design along the screw axis [29, 29]</td>
<td>31</td>
</tr>
<tr>
<td>3.1</td>
<td>Description of objectives</td>
<td>33</td>
</tr>
<tr>
<td>3.2</td>
<td>Velocity and displacement diagram for element at radius r</td>
<td>35</td>
</tr>
</tbody>
</table>
3.3 Analysis of forces on an element
3.4 Effect of $P/D$ and $\mu_f$ on $\eta_v$ and $V_s$ ($R_c/R_o = 0.4$)
3.5 Effect of radial clearance on output (measurements from [14])
3.6 Bulk material boundary within a pitch: (a) Five boundary surfaces; (b) Two basic regions
3.7 Stress on an element at lower region of screw
3.8 Mohr circle representation of stresses in an element on wall
3.9 A material sector in a pitch
3.10 Forces on shear surface
3.11 Forces on shaft surface
3.12 Forces on trailing side of flight
3.13 Forces on driving side of flight
3.14 Variation of factors with ratio $P/D$
   ($\mu_d = 0.8, \mu_f = \mu_w = 0.5, \eta_v = 0.3$)
3.15 Influence of $c/D$ on $K$ ($\mu_d = 0.8, \mu_f = \mu_w = 0.5, c_d = 0.3$)
3.16 Influence of $\mu_w$ on $K$ ($\mu_d = 0.8, \mu_f = 0.5, c_d = 0.3$)
3.17 Influence of $\delta$ on $K$ ($\mu_f = \mu_w = 0.5, c_d = 0.3$)
3.18 Influence of $\mu_f$ on $K$ ($\mu_d = 0.8, \mu_w = 0.5, c_d = 0.3$)
3.19 Influence of $c_d$ on $K$ ($\mu_d = 0.8, \mu_f = \mu_w = 0.5$)
3.20 Variation of $\eta_p$ with $P/D$ and $\mu_f$
   ($\mu_d = 0.8, \mu_w = 0.5, c_d = 0.3, \eta_v = 1, \eta_l = 1$)
3.21 Screw geometry for draw-down performance
3.22 Screw configurations for increasing capacity
3.23 $f_p$ for a stepped pitch, uniform diameter screw
3.24 Variation of $f_{p2}$ with $L/D$ and $d_f/D$ for tapered shaft screw
3.25 Variation of $f_{p2}$ with $L/D$ and $D_f/D$ for tapered outside diameter screw
3.26 Hopper walls for uniform draw-down performance
3.27 Variation of $P'_1/P_1$ with $A_{al}/A_{a1}$ and $L/D$ 83
3.28 Variation of $P'_L/P_L$ with $c/D$ and $d/D$ 84
3.29 Inclined end wall angle for uniform flow pattern 84
3.30 Variation of $\alpha_I$ with $H/D$ and $L/D$ (P_I/D = 1, P_L/P_L = 0.3, A_{al}/A_{a1} = 0.5) 85
3.31 Variation of $\alpha_L$ with $H/D$ and $c/D$ ($d/D = 0.4, P_I/D = 1$) 86

4.1 Test rig for single screw feeder 88
4.2 Test rig for twin screw feeder 89
4.3 Trough for single screw feeder 90
4.4 Trough for twin screw feeder 90
4.5 Grid for single screw feeder 92
4.6 Grid for twin screw feeder 92
4.7 Configuration of four single screws 94
4.8 Configuration of screws for No. 2 twin screw feeder 95
4.9 Configuration of screws for No. 4 twin screw feeder 95
4.10 Driving unit for single screw feeder 96
4.11 Driving unit for twin screw feeder 97
4.12 Data acquisition system 99
4.13 Typical experimental results recorded by chart recorder 100
4.14 Calibration of load cells 101
4.15 Calibration for torque transducer 102
4.16 Calibration of torque transducer for single screw 103
4.17 Calibration of torque transducer for twin screw 103
5.1 Regular and irregular shaped particles 105
5.2 Particle size distribution 106
5.3 Schematic of stereo pycnometer 108
5.4 Different arrangement of particles 110
5.5 Jenike shear cell 111
5.6 Mohr circle and yield locus of cohesive material 112
5.7 Jenike-type Direct Shear Tester 114
5.8 Typical measured yield locus 116
5.9 Arrangement for wall yield locus test 117
5.10 Wall yield locus 117
5.11 Wall yield locus for semolina 118
6.1 Mass output versus rotating speed with white plastic pellets 122
   (P = 100 mm, c = 5 mm)
6.2 Mass output versus rotating speed with semolina 123
   (P = 100 mm, c = 5 mm)
6.3 Mass output versus rotating speed with cement 123
   (P = 100 mm, c = 5 mm)
6.4 Effect of P/D on volumetric efficiency (c = 5 mm, n_m = 20 rpm) 125
6.5 Effect of P/D on volumetric efficiency 126
   (c = 10 mm, n_m = 20 rpm)
6.6 Effect of P/D on volumetric efficiency 126
   (c = 20 mm, n_m = 20 rpm)
6.7 A cross-section of dead layer of bulk solid in choke section 127
6.8 Effect of c on V_o with white plastic pellets (k = 0) 129
6.9 Effect of c on η_v with white plastic pellets 129
6.10 Effect of c on V_o with semolina (k = 0) 130
6.11 Effect of c on η_v with semolina 130
6.12 Effect of c on V_o with cement 131
6.13 Effect of c on η_v with cement 131
6.14 Draw-down performance with No. 1 screw 134
   (white plastic pellets)
6.15 Volume withdrawn per revolution along No. 1 screw 134
   (white plastic pellets)
6.16 Draw-down performance with No. 1 screw (semolina) 135
6.17 Volume withdrawn per revolution along No. 1 screw (semolina) 135
6.18 Draw-down performance with No. 2 screw 136
(white plastic pellets)
6.19 Volume withdrawn per revolution along No. 2 screw 137
(white plastic pellets)
6.20 Draw-down performance with No. 2 screw (semolina) 137
6.21 Volume withdrawn per revolution along No. 2 screw (semolina) 138
6.22 Draw-down performance with No. 3 screw 139
(white plastic pellets)
6.23 Volume withdrawn per revolution along No. 3 screw 139
(white plastic pellets)
6.24 Draw-down performance with No. 3 screw (semolina) 140
6.25 Volume withdrawn per revolution along No. 3 screw (semolina) 140
6.26 Draw-down performance with No. 4 screw 142
(white plastic pellets)
6.27 Volume withdrawn per revolution along No. 4 screw 142
(white plastic pellets)
6.28 Draw-down performance with No. 4 screw (semolina) 143
6.29 Volume withdrawn per revolution along No. 4 screw (semolina) 143
6.30 Comparison of profile coefficient for No. 1 screw 145
6.31 Comparison of profile coefficient for No. 2 screw 145
6.32 Comparison of profile coefficient for No. 3 screw 145
6.33 Comparison of profile coefficient for No. 4 screw 146
6.34 Flow pattern without grid for No. 1 screw (white plastic pellets) 147
6.35 Flow pattern without grid for No. 1 screw (semolina) 148
6.36 Flow pattern without grid for No. 2 screw (white plastic pellets) 148
6.37 Flow pattern without grid for No. 2 screw (semolina) 149
6.38 Flow pattern without grid for No. 3 screw (white plastic pellets) 149
6.39 Flow pattern without grid for No. 3 screw (semolina) 150
6.40 Flow pattern without grid for No. 4 screw (white plastic pellets) 150
6.41 Flow pattern without grid for No. 4 screw (semolina) 151
6.42 Torque versus speed of rotation for No. 1 screw with cement 152
6.43 Torque versus speed of rotation for No. 1 screw with semolina 152
6.44 Comparison of torques for No. 2 single screw feeder 154
6.45 Comparison of torques for No. 4 single screw feeder 154
6.46 Comparison of power efficiency for No. 2 single screw feeder 155
6.47 Comparison of power efficiency for No. 4 single screw feeder 156
6.48 Comparison of power requirement for No. 2 single screw feeder ($n_m = 20$ rpm) 158
6.49 Comparison of power requirement for No. 4 single screw feeder ($n_m = 20$ rpm) 158
7.1 Comparison of volumetric output per revolution for No. 2 twin screw feeder 162
7.2 Comparison of volumetric output per revolution for No. 4 twin screw feeder 162
7.3 Volume withdrawn per revolution by each pitch of No. 2 twin screw (upward) 164
7.4 Volume withdrawn per revolution by each pitch of No. 2 twin screw (downward) 164
7.5 Volume withdrawn per revolution by each pitch of No. 4 twin screw (upward) 165
7.6 Volume withdrawn per revolution by each pitch of No. 4 twin screw (downward) 165
7.7 Mass output versus rotating speed with No. 2 twin screw feeder (semolina) 167
7.8 Mass output versus rotating speed with No. 4 twin screw feeder (white plastic pellets) 168
7.9 Increase output rate versus rotating speed 168
7.10 Draw-down performance with No. 2 twin screw (upward) 171
7.11 Volume withdrawn per revolution along No. 2 twin screw (upward) 171
7.12 Draw-down performance with No. 2 twin screw (downward) 172
7.13 Volume withdrawn per revolution along No. 2 twin screw (downward) 172
7.14 Draw-down performance with No. 4 twin screw (upward) 173
7.15 Volume withdrawn per revolution along No. 4 twin screw (upward) 173
7.16 Draw-down performance with No. 4 twin screw (downward) 174
7.17 Volume withdrawn per revolution along No. 4 twin screw (downward) 174
7.18 Increase in volume withdrawn by screws rotating in downward direction (white plastic pellets) 175
7.19 Comparison of torque requirements for No. 2 twin screw feeder 177
7.20 Comparison of torque requirements for No. 4 twin screw feeder 177
7.21 Torque versus screw rotating speed for No. 2 twin screw feeder with semolina ($l_t = 160$ mm) 178
8.1 Original screw feeder for 300T bath bin 182
8.2 Original screw used in feeder for 300T bath bin 183
8.3 Original screw feeder for 75T product bin 184
8.4 Proposed screw for 300T bath bin 185
8.5 Proposed screw for 75T product bin 186
8.6 Original screw feeder for bath crusher silo 189
8.7 Proposed screw for bath crusher silo 190
8.8 Rotary breaker discharger 193
8.9 Transfer screw conveyor 194
8.10 Proposed screw for rotary breaker discharger 195
8.11 Proposed screw for transfer conveyor 196
8.12 Arrangement of screw feeder in Case 4 200
8.13 Original screw geometry in Case 4 201
8.14 Proposed screw geometry in Case 4 202
9.1 Trough with curve bottom 209
9.2 Trough with pup tent between screws (from [20]) 210
A-1 Comparison of axial force on driving side 224
A-2 Comparison of torque requirement 224
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Caption</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Comparison of calculated volumetric efficiencies (R_c/R_o = 0.4)</td>
<td>41</td>
</tr>
<tr>
<td>3.2</td>
<td>Comparison of (K_s) and (K_{se}) (R_c/R_o = 0.4)</td>
<td>63</td>
</tr>
<tr>
<td>5.1</td>
<td>Physical properties of test bulk solids</td>
<td>119</td>
</tr>
<tr>
<td>8.1</td>
<td>Assumed parameters and calculated results for Case 1</td>
<td>181</td>
</tr>
<tr>
<td>8.2</td>
<td>Assumed parameters and calculated results for Case 2</td>
<td>188</td>
</tr>
<tr>
<td>8.3</td>
<td>Assumed parameters and calculated results for Case 3</td>
<td>192</td>
</tr>
<tr>
<td>8.4</td>
<td>Assumed parameters and calculated results for Case 4</td>
<td>199</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>$A$</td>
<td>cross-sectional area of screw [m²]</td>
<td></td>
</tr>
<tr>
<td>$A_a$</td>
<td>cross-sectional area of annulus between screw flight and trough in choke section [m²]</td>
<td></td>
</tr>
<tr>
<td>$A_c$</td>
<td>cross-sectional area of shear cell of Jenike-type Direct Shear Tester [cm²]</td>
<td></td>
</tr>
<tr>
<td>$A_d$</td>
<td>cross-sectional area of dead layer in choke section [m²]</td>
<td></td>
</tr>
<tr>
<td>$A_e$</td>
<td>effective area defined in Eq. (3.34) [m²]</td>
<td></td>
</tr>
<tr>
<td>$A_{ai}$</td>
<td>average effective area defined in Eq. (3.118) [m²] (i = 1, 2,...,L)</td>
<td></td>
</tr>
<tr>
<td>$A'_{ai1}$</td>
<td>equivalent area in Eq. (3.130) [m²]</td>
<td></td>
</tr>
<tr>
<td>$A'_{aiL}$</td>
<td>equivalent area in Eq. (3.136) [m²]</td>
<td></td>
</tr>
<tr>
<td>$B$</td>
<td>opening width of hopper outlet [m]</td>
<td></td>
</tr>
<tr>
<td>$B_r$</td>
<td>width of ribbon flight in Fig. 8.14 [m]</td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>clearance between trough and tip of flight [m]</td>
<td></td>
</tr>
<tr>
<td>$c_j$</td>
<td>integration constant in Eq. (3.50) [-]</td>
<td></td>
</tr>
<tr>
<td>$c_d$</td>
<td>ratio of core shaft diameter to screw diameter [-]</td>
<td></td>
</tr>
<tr>
<td>$c_i$</td>
<td>interparticle cohesion [-]</td>
<td></td>
</tr>
<tr>
<td>$c_p$</td>
<td>ratio of pitch length to screw diameter [-]</td>
<td></td>
</tr>
<tr>
<td>$c_t$</td>
<td>ratio of trough inside radius to screw outside radius [-]</td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>core shaft diameter [m]</td>
<td></td>
</tr>
<tr>
<td>$d_j$</td>
<td>end diameter of core shaft in Fig. 3.22 [m]</td>
<td></td>
</tr>
<tr>
<td>$d_p$</td>
<td>mean equivalent particle diameter [mm]</td>
<td></td>
</tr>
<tr>
<td>$D$</td>
<td>screw diameter [m]</td>
<td></td>
</tr>
<tr>
<td>$D_j$</td>
<td>end screw diameter in Fig. 3.22 [m]</td>
<td></td>
</tr>
<tr>
<td>$f_p$</td>
<td>profile coefficient defined in Eq. (3.124) [-]</td>
<td></td>
</tr>
<tr>
<td>$F$</td>
<td>resultant force on bulk material element [N]</td>
<td></td>
</tr>
</tbody>
</table>
\( F_a \)  
axial component of force  
\([N]\)

\( F_A \)  
total axial force in Eq. (2.17)  
\([N]\)

\( F_{AC} \)  
force to slide bulk material along trough surface  
\([N]\)

\( F_{AS} \)  
force to shear bulk material along shear surface  
\([N]\)

\( F_{ca} \)  
axial resisting force on shaft surface  
\([N]\)

\( F_d \)  
resultant force on driving side of flight  
\([N]\)

\( F_{da} \)  
axial component of \( F_d \)  
\([N]\)

\( F_{dt} \)  
tangential component of \( F_d \)  
\([N]\)

\( F_{la} \)  
axial resisting force on trailing side of flight  
\([N]\)

\( F_p \)  
peripheral component of force  
\([N]\)

\( F_{ia} \)  
axial resisting force on trough surface  
\([N]\)

\( F_{iaa} \)  
axial resisting force on upper region of screw  
\([N]\)

\( F_V \)  
resultant vertical load in Eq. (2.18)  
\([N]\)

\( H \)  
hopper height in Fig. 3.29  
\([m]\)

\( k \)  
coefficient of dead layer of bulk material at choke section  
\([-]\)

\( k_c \)  
factor given in Eq. (3.62)  
\([-]\)

\( k_l \)  
factor given in Eq. (3.65)  
\([-]\)

\( k_t \)  
factor given in Eq. (3.68)  
\([-]\)

\( k_u \)  
factor given in Eq. (3.58)  
\([-]\)

\( K \)  
factor given in Eq. (3.95)  
\([-]\)

\( K_1, K_2, K_3 \)  
pressure ratios determined by Eq. (2.20)  
\([-]\)

\( K_c \)  
factor given in Eq. (3.97)  
\([-]\)

\( K_l \)  
factor given in Eq. (3.98)  
\([-]\)

\( K_s \)  
factor given in Eq. (3.82)  
\([-]\)

\( K_{se} \)  
factor given in Eq. (3.94)  
\([-]\)

\( K_t \)  
factor given in Eq. (3.99)  
\([-]\)

\( K_u \)  
factor given in Eq. (3.96)  
\([-]\)

\( K_\sigma \)  
factor given in Eq. (3.71)  
\([-]\)
\( K_{oc} \) factor given in Eq. (3.86) [-]

\( l_c \) length in Fig. 4.15 [m]

\( l_i \) centre distance between two screws [m]

\( L \) length of feed section [m]

\( L_c \) length of choke section [m]

\( m \) hopper factor in Eq. (3.36) [-]

\( m_p \) mass of particles [kg]

\( M \) measured mass of discharged bulk solid [kg]

\( n \) number of screw revolutions per second [s\(^{-1}\)]

\( n_c \) number of pitches in choke section [-]

\( n_f \) number of pitches in feed section [-]

\( n_m \) rotational speed of screw [rpm]

\( n_p \) number of particles in a given mass [-]

\( N \) number of screw revolutions [-]

\( p \) normal stress on bulk material element in Fig. 3.3 [Pa]

\( p_2, p_3 \) air pressure in Eq. (5.2) [Psig]

\( P \) pitch length [m]

\( P_{ax} \) power for transport of bulk solids within a pitch in axial direction [Nm \( s^{-1} \)]

\( P_{axf} \) power for transport of mass output of a screw feeder in axial direction [Nm \( s^{-1} \)]

\( P_f \) power required for a single screw feeder [Nm \( s^{-1} \)]

\( P_i \) pitch length in feed section \((i = 1, 2, \ldots L)\) [m]

\( P_{sc} \) power for turning screw [Nm \( s^{-1} \)]

\( P_{scf} \) power for turning all pitches in feed section [Nm \( s^{-1} \)]

\( P'_1 \) length in Fig. 3.26 [m]

\( P'_L \) length in Fig. 3.26 [m]

\( q \) non-dimensional surcharge factor [-]
\( q_{\text{surf}} \) surcharge factor for flow condition based on \( \sigma_t \) [\( \text{[-]} \)]

\( Q \) feeder load exerted by bulk solids in hopper [\( \text{[N]} \)]

\( Q_{\text{max}} \) maximum flow rate per revolution in Fig. 2.17 [\( \text{[kg/rev]} \)]

\( Q_0 \) flow rate per revolution in Fig. 2.17 [\( \text{[kg/rev]} \)]

\( r \) radius of screw flight [\( \text{[m]} \)]

\( r_m \) radius of Mohr circle [\( \text{[-]} \)]

\( R_c \) radius of core shaft [\( \text{[m]} \)]

\( R_m \) mean radius of screw flight defined in Eq. (2.8) [\( \text{[m]} \)]

\( R_o \) outside radius of screw flight [\( \text{[m]} \)]

\( R_t \) inside radius of trough [\( \text{[m]} \)]

\( S \) shear force [\( \text{[N]} \)]

\( S', S'', S''' \) shear forces in Jenike Direct Shear Test [\( \text{[N]} \)]

\( S_a \) axial displacement of material element per revolution [\( \text{[m]} \)]

\( S_p \) peripheral displacement of flight per revolution [\( \text{[m]} \)]

\( S_{pr} \) prorated shear force in Jenike Direct Shear Test [\( \text{[N]} \)]

\( S_{sel} \) shear force selected for prorating raw test results in Jenike Direct Shear Test [\( \text{[N]} \)]

\( S_{\text{test}} \) test value of shear force \( S \) in Jenike Direct Shear Test [\( \text{[N]} \)]

\( t \) flight thickness [\( \text{[m]} \)]

\( T \) torque required for driving screw [\( \text{[Nm]} \)]

\( T_c \) torque required for a pitch in choke section [\( \text{[Nm]} \)]

\( T_{ct} \) torque required for total pitches in choke section [\( \text{[Nm]} \)]

\( T_f \) torque required for a pitch in feed section [\( \text{[Nm]} \)]

\( T_{ft} \) torque required for total pitches in feed section [\( \text{[Nm]} \)]

\( v \) actual velocity of material element [\( \text{[m s}^{-1}\text{]} \)]

\( v_a \) axial component of material element velocity [\( \text{[m s}^{-1}\text{]} \)]

\( v_{ax} \) velocity of bulk solid in axial direction of screw [\( \text{[m s}^{-1}\text{]} \)]
\( v_{axf} \) velocity of bulk solid within last pitch in axial direction of screw [m s\(^{-1}\)]

\( v_f \) velocity of screw flight at radius \( r \) [m s\(^{-1}\)]

\( v_p \) peripheral component of material element velocity [m s\(^{-1}\)]

\( v_r \) velocity of material element relative to flight surface [m s\(^{-1}\)]

\( V \) normal force in Jenike Direct Shear Test [N]

\( V', V'' \) normal forces in Jenike Direct Shear Test [N]

\( V''' \)

\( V_a \) added cell volume of stereo pycnometer [cm\(^3\)]

\( V_{ad} \) additional vertical forces due to shear lid, ring and mass of bulk solid contained within the ring in Jenike Direct Shear Test [N]

\( V_b \) added normal force in Jenike Direct Shear Test [N]

\( V_c \) sealed sample cell volume of stereo pycnometer [cm\(^3\)]

\( V_{con} \) volume conveyed per revolution within a pitch [m\(^3\)]

\( V_p \) powder sample volume in stereo pycnometer [cm\(^3\)]

\( V_o \) volumetric output per revolution of screw feeder [m\(^3\)]

\( V_s \) specific volume defined in Eq. (3.32) [-]

\( W_c \) mass for calibration [kg]

\( x \) coordinate [-]

\( x_i \) point on \( x \) coordinate in Fig. 3.21 (\( i = 1, 2, \ldots , L \)) [-]

\( X, Y \) factors in feed load equations, flow condition [-]

\( \alpha \) flight helical angle ['\(^\circ\)'

\( \alpha_l \) slope angle in Fig. 3.29 ['\(^\circ\)'

\( \alpha_c \) flight helical angle at core shaft ['\(^\circ\)'

\( \alpha_d \) arc subtended by dead layer of bulk solid in clearance [-]

\( \alpha_e \) equivalent flight helical angle given in Eq. (3.29) ['\(^\circ\)'

\( \alpha_h \) hopper half-angle ['\(^\circ\)']
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_L$</td>
<td>slope angle in Fig. 3.29</td>
</tr>
<tr>
<td>$\alpha_m$</td>
<td>mean helical angle in Eq. (2.10)</td>
</tr>
<tr>
<td>$\alpha_o$</td>
<td>flight helical angle at outside diameter</td>
</tr>
<tr>
<td>$\alpha_r$</td>
<td>flight helical angle at radius $r$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>helical angle of bulk material motion</td>
</tr>
<tr>
<td>$\beta_e$</td>
<td>equivalent helical angle of bulk material motion</td>
</tr>
<tr>
<td>$\beta_h$</td>
<td>angle determined by Eq. (3.38)</td>
</tr>
<tr>
<td>$\beta_m$</td>
<td>helical angle in Eq. (2.11)</td>
</tr>
<tr>
<td>$\beta_o$</td>
<td>helical angle in Eq. (2.6)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>effective angle of internal friction of bulk solid</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>bulk voidage of material</td>
</tr>
<tr>
<td>$\phi$</td>
<td>kinematic angle of internal friction</td>
</tr>
<tr>
<td>$\phi_f$</td>
<td>wall friction angle of bulk solid on flight surface</td>
</tr>
<tr>
<td>$\phi_h$</td>
<td>wall friction angle of bulk solid on hopper wall</td>
</tr>
<tr>
<td>$\phi_w$</td>
<td>wall friction angle between bulk solid and a confining surface, eg. trough or core shaft surface</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>specific weight of bulk solid $[\text{N m}^{-3}]$</td>
</tr>
<tr>
<td>$\eta_p$</td>
<td>power efficiency defined by Eq. (3.102) $[-]$</td>
</tr>
<tr>
<td>$\eta_v$</td>
<td>volumetric efficiency of screw feeders $[-]$</td>
</tr>
<tr>
<td>$\eta_{av}$</td>
<td>volumetric efficiency given by Eq. (2.14) $[-]$</td>
</tr>
<tr>
<td>$\eta_{pf}$</td>
<td>power efficiency for whole feed section $[-]$</td>
</tr>
<tr>
<td>$\eta_{ps}$</td>
<td>power efficiency based on simplified method $[-]$</td>
</tr>
<tr>
<td>$\lambda_s$</td>
<td>stress ratio of bulk solid sliding on a surface $[-]$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>friction coefficient of bulk solid $\mu = \tan\phi$ $[-]$</td>
</tr>
<tr>
<td>$\mu_d$</td>
<td>tangent of effective angle of internal friction, $\mu_d = \tan\delta$ $[-]$</td>
</tr>
<tr>
<td>$\mu_e$</td>
<td>equivalent friction coefficient of bulk solid, $\mu_e = (0.8\sim 1)\sin\delta$ $[-]$</td>
</tr>
<tr>
<td>$\mu_f$</td>
<td>wall friction coefficient between bulk solid and flight $[-]$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>$\mu_w$</td>
<td>wall friction coefficient between bulk solid and a confining surface</td>
</tr>
<tr>
<td>$\theta$</td>
<td>polar coordinate</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>loose poured bulk density</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>particle density</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>normal stress</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>major principal stress</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>minor principal stress</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>axial stress on driving side of flight</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>stress in Fig. 3.8</td>
</tr>
<tr>
<td>$\sigma_n$</td>
<td>normal stress coordinate</td>
</tr>
<tr>
<td>$\sigma_o$</td>
<td>stress exerted by bulk solids in hopper</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>axial stress in Fig. 3.7</td>
</tr>
<tr>
<td>$\sigma_w$</td>
<td>wall stress in Fig. 3.7</td>
</tr>
<tr>
<td>$\sigma_{wa}$</td>
<td>average wall stress defined in Eq. (3.52)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>shear stress</td>
</tr>
<tr>
<td>$\tau_n$</td>
<td>shear stress coordinate</td>
</tr>
<tr>
<td>$\tau_w$</td>
<td>shear stress on wall</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular velocity</td>
</tr>
<tr>
<td>$\zeta_c$</td>
<td>ratio of core shaft radius to pitch length in Eq. (2.13)</td>
</tr>
<tr>
<td>$\zeta_o$</td>
<td>ratio of outside radius of flight to pitch length in Eq. (2.13)</td>
</tr>
<tr>
<td>$\zeta_{av}$</td>
<td>average ratio of radius to pitch length in Eq. (2.13)</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

Many mechanical feeding devices are being used in bulk solids handling and processing to feed a wide range of bulk solids. Experience has demonstrated that any successful application lies in the proper design/selection of the hopper/feeder system. Therefore, numerous efforts have been made to investigate desirable feeding devices to cope with an increasing number of applications. As a result, different types of feeding devices are being developed continually. Among them, the most common ones are belt feeder, apron feeder, rotary feeder, plough feeder, vibratory feeder and screw feeder.

Some principles of bin and feeding system design for bulk solids have been summarised by Arnold and Roberts [5] and Bates [9, 10]. In the design of a mechanical feeding device it is important to consider the interfacing between the storage facility, such as silo, bin or hopper, with the feeder. There are two basic kinds of flow pattern developed during the discharge of bulk solids storage containers; these are normally termed Mass Flow (coined by Jenike [32]) and Funnel Flow (sometimes referred to as Internal Flow or Core Flow). Mass flow requires that the material slips on all contact surface of the container and significant velocity profiles may be present. This means that the material is in motion at substantially every point whenever any of the solids are discharged. On the contrary, funnel flow is a description of the cross section shape of the flow channel. The core region of flow takes place within the bulk solid as a confined channel, with the rest of the bulk solid outside this channel being stationary. Although screw feeders can be used for conical hoppers with round or square outlet openings, they are often fitted to the slotted outlets of mass flow bins with wedge or transition shaped hoppers.
The screw feeder, as shown in Fig. 1.1, has been employed in industry for many years and is a particularly useful feeding device. A screw feeder possesses the following advantages:

1. Simplicity and reliability: only few moving parts and compact cross section make the feeding system simpler in construction and more reliable in operation;

2. Good metering characteristics: it is an important feature for a feeder to obtain accurate control of the feeding rate. In the normal speed range (generally less than 80 rpm), the output of a screw feeder can be of good linear relationship with the rotating speed of the screw;

3. Total enclosure for safety: screw feeders can be constructed to be weatherproof and some problems, such as containment of dust, dust explosion hazards, can be avoided;

Fig. 1.1 A screw feeder/discharger [73]
4. **Great flexibility** of design: screw feeders can be designed to match hopper outlet dimensions (e.g., dual or multiple array for wide outlets), feeder discharge location (e.g., increasing conveying section to extend hopper outlet), alternative outlet position and inclination upwards or downwards, etc., so they can provide a more flexible choice in consideration of a feeding system. Fig. 1.2 shows two examples of such flexibility:

![Fig. 1.2 Direct end transfer to conveyor/elevator [9]](image)

5. **Wide application**: screw flights can be modified to suit different processing requirements, e.g., cut flights, folded flights, paddles (or a combination) can be used for prebreaking of lumps, mixing, blending or stirring bulk material. Screw feeders can also be used in a variety of conveying or feeding systems playing special roles, as shown in Figs 1.3 and 1.4.

![Fig. 1.3 Double plug seal and side transfer to pneumatic duct [9]](image)
Besides the advantages mentioned above, Carson [20] pointed out that a wedge hopper with a rectangular outlet is a good design choice for use with screw feeders because it reduces the minimum bin opening width by a factor of approximately two. If a certain bulk solid requires a minimum 600 mm wide outlet diameter to prevent arching in a conical hopper, the same bulk solid will only require a 300 mm wide (by at least 900 mm long) rectangular slot outlet to prevent arching. Wedge hoppers also allow mass flow to occur in bins with less steep hopper walls. The side walls of a conical hopper must be at least 10 to 12 degrees steeper than the side walls of a wedge or chisel shaped hopper to allow mass flow with the same wall surface and bulk solid.

Certainly screw feeders also have some disadvantages, such as:

1. Due to the relative rotation of the bulk solid within the screw, mechanical efficiency for transport is low;

2. Because of the existence of the clearance between the flight and the trough a screw feeder may not provide a self-cleaning action;
3. Hard foreign bodies can wedge into the space between the flight tip and the trough.

Although screw feeders have been used widely in industry for many years, research into their performance is only now becoming comprehensive. Some comments in the last decade reflect the state of art of the research on screw feeders:

- At present, dependable design equations for the calculation of power consumption and capacity of screw feeders do not exist -1987, Rautenbach et al. [55];

- Most screw feeders are custom-designed. Unfortunately, many designers don't understand material flow properties or the interaction mechanisms between a bin and a feeder. As a result, unnecessary problems occur -1987, Carson [20];

- However, no base for an optimal solution exists, so one has to rely on common sense and experience -1993, Haaker et al. [28, 29];

- ...no methodology is available so far for the design of screw feeders -1994, Roberts et al. [40].

Therefore, it is necessary to undertake comprehensive theoretical research and systematic experiments on the performance of screw feeders. The main aim of this thesis is the formulation of theoretical models to predict the performance of screw feeders. To achieve this aim, particular objectives of the work are listed below:

1. Review published literature to assess the current state of knowledge of screw conveying devices, with particular attention paid to screw feeders (Chapter 2);

2. Introduce and improve the theoretical modelling of screw feeder performance including volumetric efficiency, torque requirements and draw-down performance (Chapter 3);
3. Undertake experiments to investigate the influence of geometric parameters of the screw, the operating conditions and properties of the bulk solid on screw feeder performance (Chapter 4 and 5);

4. Analyse and discuss the experimental results to evaluate the factors affecting the performance of screw feeders and assess the accuracy of the theoretical models (Chapter 6 and 7);

5. Apply the performance prediction models to solve typical problems experienced in industry (Chapter 8);

Finally, concluding remarks based on the investigations and suggestions for further work are presented in Chapter 9.
CHAPTER 2

LITERATURE SURVEY

2.1 Introduction

A screw feeder is a special form of screw conveyor. In referring to screw feeders, it is instructive to trace the development of the screw conveyor. Screws used as transport devices date back more than 2000 years ago. The first screw conveyor as such was designed by Archimedes (287 to 212 BC) - Greek mathematician, physicist and inventor - for removing water from the hold of a ship built for King Hiero of Syracuse [22]. Until 200 years ago screws have not been used widely in industrial applications. The first mill built by Evans in 1785 employed screws of helically formed sheet metal sections mounted on a wooden core [22]. Since then this technology of mechanisation has been keeping up with the demands of bulk materials handling and processing.

The metal screw conveyor flights were originally of the sectional flight variety, formed from flat sheets cut in circular form with a hole in the centre then split on one side and the two edges pulled apart to form one flight section of a screw. Successive flights were then joined by riveting, shingle fashion, to make a continuous helix of whatever length was required. When the varying sizes of iron pipes became available the wooden core was replaced by an iron pipe.

The next technological advancement of importance in screw conveyor design was patented on March 29, 1898 by Frank C. Caldwell [22]. This was a continuous, one piece screw flight formed by rolling a continuous strip of steel in to a helix. This construction is now known as the "helicoid" flight, and it simplified manufacture and assembly by eliminating the joints in the sectional flight screws. Both types of screw are still produced.
Since the screw conveyor came into general use a little over a century ago for moving grains, fine coal and other bulk material of the time, it has come to occupy a unique place in the general field of material handling and processing.

Today, helical screws used to extract and/or transport bulk solids generally can be classified as a

- Screw conveyor
- Screw elevator
- Screw feeder
- Screw extruder

Although these four types of screw devices have individual characteristics in their particular applications, studies on these screw devices show that very close relationships exist among them.

2.2 Outline of Studies on Screws Applied in Bulk Materials Handling

2.2.1 Screw Conveyor and Elevator

In many studies there is no distinction between a screw conveyor and a screw elevator. In some literature screw conveyors are called as worm conveyors [23]; screw elevators are called vertical screw conveyors or vertical augers [25, 27, 48-51, 62, 72, 78]. Screw conveyors and screw elevators are also called augers or auger conveyors [30, 67, 68, 74]. Between the screw conveyor (normally operating in the horizontal direction) and the screw elevator (normally operating in the vertical direction) is the inclined screw conveyor [56]. CEMA include the screw feeder, inclined screw conveyor and vertical screw conveyor in their book on screw conveyors [22].

Fig. 2.1 shows a screw conveyor in a typical application, by which bulk material is transported from the inlet to the outlet.
Typical arrangements of inclined screw conveyors are shown in Fig. 2.2. Descriptions given for the arrangements depicted in Fig. 2.2 are:

**Top:** A typical coal handling screw conveyor, inclined 20°, delivering stoker coal from a bin into two stoker hoppers. It consists of a short pitch, tapered feeder section followed by a full diameter, normal pitch screw.

**Left bottom:** A general purpose 45° inclined screw conveyor. It consists of a full diameter, short pitch conveyor screw in a regular trough with a shroud cover plate.

**Right bottom:** A general purpose 60° inclined screw conveyor. The short pitch screw is in a split tubular casing and includes a feed hopper with a bar grating and an adjustable feed inlet gate.
Rehkugler [60] reviewed the studies of the performance of the screw conveyor and its conveying process during the 50’s to 60’s. Previous to that time, from his point of view, little was known about screw conveyors other than that they were devices simple to operate and easy to install for conveying granular materials. Although extensive data, both graphical and tabular, are available to describe screw conveyor performance, the usefulness of these data is limited because it has not been collected and correlated in a consistent manner. Each researcher has approached the program in a different manner and likewise has presented his results in a number of different ways.

Numerous studies have shown that the conditions existing at the inlet to the screw conveyor frequently govern performance of the conveyor [49, 50, 57, 62, 68, 71].
Each of the studies has examined some aspect of the influence of changes in geometry of the intake section of the conveyor on volumetric capacity and the required power. Several investigators [49, 57, 68, 70, 71, 83] have shown that an increase in the length of the exposed screw increased the output of the screw conveyor. It was indicated that there would be a unique maximum length of the intake for a given speed of a vertical conveyor, beyond which increasing intake length would be no longer effective in increasing the capacity of the conveyor.

O'Callaghan [50] examined the influence of intake length on the power requirement for vertical screw conveyors operating at different speeds. The rate of increase of power required with increased capacity (where capacity was increased by extending the intake length) was greater at the lower screw speeds than at the higher screw speeds. His results also illustrated that, with a vertical screw conveyor, it would be more efficient to obtain a given discharge at the lowest possible speed by increasing the intake length.

Screw pitch and diameter, shaft diameter and clearance between the screw and closing tube influence both capacity and power requirements. Theoretical analyses of the motion of granular material in the conveyor have been conducted by several investigators [7, 25, 27, 48, 62, 71, 78]. At least four different approaches have been used to describe the motion of the material and to predict either, or both, capacity and power required.

One approach has been the analysis of the motion of a single particle of material in contact with the surface of the screw and the enclosing tube. Gutyar [27], Baks et al. [7], Vierling et al. [78] and Ephremidis [25] have developed similar expressions for the critical speed of screw conveyors. Generally these expressions neglected the influence of the intake condition on the conveyor performance, although Vierling et al. [78] considered the degree of filling of the space between the flights based on an assumed profile of the material piled on the screw flighting. Therefore, analyses based on the equilibrium and motion of an individual particle in the screw conveyor, neglect
the interaction of other particles and the influence of inlet and discharge conditions which seriously affect the performance of this device.

Roberts [62] and Ross [71] considered the interaction of particles in the screw in two different ways. Roberts [62] studied the vortex motion of grain both theoretically and experimentally. Measured free vortex motion was closely correlated with a theoretically developed expression for vortex motion of the grain. However, with forced vortex motion the mass of grain behaved more like a solid and considerable hysteresis effect upon the vortex profile was observed. Roberts' analysis of auger performance data showed that actual vortex motion in the conveyor was not as great as free vortex motion, and that vortex motion is restricted by complete immersion of the screw at the inlet. He stated that the actual vortex motion of material in a screw conveyor changed from forced vortex at low speeds to a constant, tangential-speed vortex at higher speeds.

Ross [71] idealised the mass of grain in a screw conveyor by considering the stacking of perfect inelastic spherical particles and the interaction of forces between particles within the enclosing tube and the screw surface. He was able to calculate a theoretical path of an individual particle and then assumed all particles in a completely full conveyor would follow the same path. A partially experimental procedure enabled prediction of capacities with a maximum error of 25 percent. His analysis did not include the influence of inlet conditions upon the degree of filling and the path of motion of the particles. Therefore, the experimental portion of the analysis was used to establish the influence of all other parameters on the performance of the conveyor, including the effects of the inlet configuration upon the path of the material through the conveyor.

Nilsson [48] considered the pressures around a sector element in the axial plane of the screw. For self-feeding vertical conveyors it is difficult to get high transport capacities, as the centrifugal forces, caused by the rotation of the screw, throw the material away
before it enters the tube. With the development of a feeding apparatus, which was presented by Siversson et al. [48], it is possible to get a much better filling capacity even at high numbers of revolutions. A hydrostatic pressure distribution was accepted by Nilsson as a good approximation. Calculations for the conveyor, such as forces and torques, were based on the approximate pressure distribution.

Investigations on screw conveyors used as transportation equipment for granular materials also reported by Wada et al. [80]. The influence of the rotating speed of the screw, the size and the shape of a screw flight and the filling ratio of granules, on the transport efficiency of the screw conveyor was considered. The transportation mechanism of granules inside the screw conveyor was investigated using two parameters: the transport quantity and the leakage quantity in the stationary state.

Since granules transported by screw conveyors are discharged pulsatingly, studies on the control of outflow quantity from a screw conveyor and optimal control of a screw conveyor have been undertaken by Wada et al. [79, 82]. They considered that the optimal transport conditions could be realised by controlling both the transport quantity and the torque of the screw shaft at the same time. They applied a multi-input multi-output linear system to the control process of a screw conveyor. The effects of the control were examined by FFT analysis.

Rademacher [54] investigated inclined screw blades used for vertical grain augers. Due to modern technology, screw flights are often manufactured by rolling them out of one single strip of steel. When simultaneously some flight inclination is applied, less residual stresses and/or larger possible ratios between outer and shaft diameter are claimed. It is sometimes also claimed that the efficiency of the conveyors is increased by such an inclined flight. Rademacher modified previous investigations based upon the conveyance of a single particle for an inclined screw flight. He pointed out that such a flight had no significant advantages over a normal one, which was confirmed by a series of tests.
Peart et al. [52] recommended dimensional standards and performance-test procedures for screw conveyors. They described those dimensional standards as functional standards which allow much more knowledgeable application of the equipment to the desired function. The screw conveyor must serve the function of fitting in the allowable physical space, conveying the material from one given point to another given point, and conveying a given throughput. The performance test must be complete enough to give information over a range of applications, but it also must be simple enough to be feasible and reasonably economical. One standard test form for identifying conveyor and test conditions and one tabular form for displaying results of screw conveyor performance tests were recommended.

2.2.2 Screw Extruder

Screw extruders are mainly employed in the plastics industry. The whole screw channel can be basically divided into three zones [1]: solids feeding zone, melting zone and pumping or extruding zone. A typical experimental extruder is shown in Fig. 2.3.

![Screw Extruder Diagram](image)

Fig. 2.3 An experimental screw extruder [43]

A-solids feeding zone; B-melting zone; C-extruding zone
With extrusion becoming an increasingly important operation in the plastics industry, the basic principles of the extrusion process were studied regarding the design and operation of extruders in the early 50's [16-19, 33, 39, 43]. However, those studies were mainly concentrated on the flow behaviour of plastic melts in the channels of extruder screws, as shown in Fig. 2.4. Attempts to describe the motion of solid particles were also made at that time. It was assumed that if the solid particles were small relative to the depth of the screw threads - i.e., particle diameter was less than one tenth thread depth - the layer of particles in the screw is sheared much like a liquid, and the behaviour of solids flow in the screw extruder was similar to that described for the viscous liquids.

![Fig. 2.4 View of velocities of viscous liquids in an screw extruder [19]](image)

Compared to the extensive published work on polymer melt flow in extruder channels, little effort has been expended in analysing the solids conveying process in the feeding zone of an extruder. A plug flow model applied to the solids conveying zone was reported by Amellal et al. [1], Lovegrove et al. [36, 37] and Poltersdorf et al. [53]. But from the analysis of the force balance on a volume element for the channel region, the physical properties of bulk solids have not received enough attention and only friction forces acting on the contact surfaces have been considered, as shown in Fig. 2.5. In this case a volume element is more like a "solid" than a "bulk solid".
Conner et al. [21] developed a model for the feeder-extruder interactions. In the field of plastic extrusion, the accuracy of feeding materials to the extrusion process is an important issue. The end product quality is partially dependent upon the extruder receiving feed materials in the correct proportions. Therefore, the understanding of the relationship between the feeder and the extruder is very important. In their paper the feeder was considered to be a separate and distinct item from the extruder. An analysis on the effect of the feed materials and environmental conditions on the flow rate was presented.

Dec et al. [24] designed an experimental simulator to analyse the feeding-precompacting process for many options of screw feeders. This simulator was designed to separate screw feeder operation from the roll press in order to independently study the pressure build up and material flow pattern in the feed screw barrel. Experiments on the screw feeder simulator for optimum screw feeder design was presented by demonstrating the usage of the simulator.

Studies on twin screw extruders have been found in the literature [21, 53]. Fig. 2.6 shows a cross section of a twin screw extruder. Conner et al. [21] concentrated their study on the accurate feeding process. Poltersdorf et al. [53] established transport equations by means of a force and momentum balance on the volume element. These
equations served as a basis for describing the material feeding, i.e., the transition of the polymer from the feed hopper into the filling region of the screws. However, this force and momentum balance on the volume element were analysed based on the movement of a rigid plug, as shown in Fig. 2.7.

Fig. 2.6 Cross section of a twin screw extruder [53]

Zone 1—feeding bulk solids; Zone 2—screw channel facing the feed throat;
Zone 3—conveying channel

Fig. 2.7 Bulk material transport in the channel of a twin screw extruder [53]
2.2.3 Screw Feeders

A screw feeder or discharger is used to control the flow rate of a bulk material from a bin or hopper. Among screw feeders, single screw feeders are most commonly used. The single screw feeder may be a separate unit, as shown in Fig. 2.8, or it may be extended by sections of a normal screw conveyor to any practical length, as shown in Fig. 2.9.

![Fig. 2.8 A single screw feeder as a separate unit [22]](image)

![Fig. 2.9 A single screw feeder with extension conveyor [22]](image)

In some literature screw conveyors operated under conditions where they are flooded with the bulk solid, are also called screw feeders [15, 77, 81]. Fig. 2.10 and 2.11 show the schematic diagrams of the experimental devices.
Chapter 2

Literature Survey

Fig. 2.10 A test rig for screw feeders [81]

![Diagram of a test rig for screw feeders](image1)

H: Hopper level
Z: Axial direction of the screw

Fig. 2.11 A experimental set-up for screw feeders [77]

1. steel tube; 2. upper screw; 3. lower screw; 4. hopper; 5. coupling;

6. torque meter; 7. motor; 8. bearing; 9. tachometer.
The design flexibility of screw feeders allows them to be used in situations where no other feeder can be considered. One aspect of this flexibility is multiple screw applications, by which two or more screws arranged side by side in a feeder, can service a larger slot width to meet the design requirement of a hopper. A twin screw feeder is shown in Fig. 2.12.

![Fig. 2.12 A twin screw feeder [22]](image)

Experimental and theoretical studies on screw feeders appear to be reported from the 1960’s. Since then the performance characteristics of screw feeders, such as their transport capacity, torque or power requirements and entrainment patterns in a bin or hopper, have received extensive attention.
Metcalf [44] considered the mechanics of a screw feeder concentrating on the rate of delivery and the torque required to feed different types of coal. The model chosen was that of a rigid plug of bulk material moving in a helix at an angle to the screw axis. In the experiments two types of mining drill rods, with screws of 2\(\frac{1}{4}\) and 1\(\frac{5}{8}\) inch in nominal diameter, served as screws and concrete blocks, cast for the purpose, as tubes.

A detailed experimental investigation was conducted by Burkhardt [14]. The tests included the effect of the pitch, the radial clearance between screw flight and trough, the hopper exposure and the head of bulk solids contained in the hopper on the performance of a screw feeder. The range of screw speeds from 25 to 3300 rpm in the experimental investigations appears too wide so that the recorded results could not give a regular pattern.

Bates [3] studied the basic principles involved in screw feeders with regard to the motion of the particulate material as a function of the screw dimensions and the angle of friction of bulk solid on the flight face. In his paper a concept of "mean radius" was introduced for calculating the volumetric efficiency and transported volume. He also studied the flow patterns developed by a screw feeder in a hopper. A series of tests on each combination of screw and bulk solid was undertaken to compare the different flow patterns. In later papers [9,10] Bates reviewed the requirements for interfacing hoppers with screw feeders.

Carleton et al. [15] discussed both screw conveyors and feeders. Experiments were described in which the effects of screw geometry, screw speed, the degree of filling in the trough and bulk materials with different physical properties, were investigated. But from the experimental apparatus and results described in their paper there was more emphasis on screw conveyors rather than screw feeders.

Rautenbach et al. [55] carried out scale-up experiments with two geometrically similar screws of 50 mm and 144 mm in diameter. By dimensional analysis the relevant set of
dimensional numbers was derived for the calculation of power consumption and capacity, taking screw geometry and the properties of bulk materials into account. According to the authors, these correlations seem to be valid for scale up as well as scale down beyond the range of their experiments.

A wider range of theoretical and experimental studies on the performance of screw feeders has been undertaken by Roberts et al. [40, 63-65, 69]. Based on the analysis of the mechanics of screw feeder performance in relation to the bulk solid draw-down characteristics in the feed hopper, a criterion for uniform draw-down performance of screw feeders was proposed. Theories for predicting the volumetric and torque characteristics of screw feeders were established.

Haaker et al. [28, 29] discussed theoretical bases for the volumetric efficiency of screw feeders. Two theories were proposed for calculation of the volumetric efficiency, one was based on a deformation in the bulk solid and the other was based on plug flow of the bulk material. Several experiments were conducted on a test rig and the experimental results were compared with the predictions from the two theories. They also proposed a method for optimal pitch design for a screw with stepped pitches.

Wada et al. [81] investigated the dynamic characteristics and the transportation mechanism inside the screw feeder. First, the effects of the rotating speed of the screw shaft and the pressure of granular materials inside a feed hopper on the required power and quantity of discharge were examined. Secondly, the transporting state of granular materials inside the screw feeder was investigated. The transportation mechanism was discussed by analysing the motion of a tracer inserted into the screw feeder and by using a velocity diagram.

Experimental studies on the mixing of granular materials in a screw feeder were conducted by Tsai et al. [77]. It was observed that the granular flow in a screw feeder is similar to plug flow. The degree of mixing of granular materials in a screw feeder
was examined by using different screws, such as a tapered shaft screw, a screw with paddles, a cut-flight screw and a stepped-flight diameter screw, under different operating conditions.

Design procedures for screw feeders were reviewed by Carson [20]. The interaction mechanisms between proper bin design and optimum screw feeder design were also discussed in his paper. He concluded that efficient screw feeders could be designed if the properties of the bulk solid and the interaction between the bin and feeder geometry were thoroughly understood.

Maton [42] reviewed the hopper and screw geometry in order to predict the normal running power load requirements and in particular the initial loads to enable the screw to breakaway under the theoretical loads imposed by the bin geometry. In his paper both an empirical method and a more analytical method were discussed and a design example was provided to compare the empirical and testwork approach for designing screw feeders.

Mi et al. [45, 46] presented an analysis for the mechanical design optimization of screw feeders. A mathematical model of the optimization problem was established. The specific objective was a combination of weight and deflection minimization and discharge per revolution maximization. Optimization results were presented for 32 different situations corresponding to different thicknesses and widths of screw flight.

### 2.3 Distinction between Screw Feeder and Other Screw Devices

The screw feeder, using a screw as the only operating component, has some similarity with the screw conveyor, screw elevator or screw extruder in configuration. However, it is necessary to draw a distinction among these four types of helical screw equipment. Some helpful definitions were given by Bates and Andrews in a contribution to Metcalfe's paper [44] and by Bates to Carleton et al.'s paper [15].
The screw elevator can be used at any angle, but is most commonly used above 20 degrees to the vertical, at which angle the conventional screw conveyor in a U-trough has little elevating capacity. The speed of rotation is relatively high, since the conveying/elevating capacity is less than for the horizontal position, for a given rotational speed. The inlet for the screw elevator normally needs greater attention, as discussed in relation to Fig. 2.2.

The screw conveyor is used horizontal or slightly inclined. The cross-sectional loading of the screw is normally less than 50 percent so that the material is conveyed below shaft level. From a practical point of view this is desirable to reduce contamination in intermediate bearings and minimize torque. The low cross-sectional loading makes a circular casing unnecessary, so that the conventional U-trough can be adopted, giving easy access.

The screw extruder is used for the extrusion of both bulk solids and viscous liquids. The screw employed in an extruder is more like a threaded shaft, which is rotating in a fixed cylinder or barrel. The screw channel is very small compared with that of the screw conveyor, elevator or feeder which are mainly used for transporting bulk solids. Interaction between the screw and hopper need not be considered, as the inlet for the extruder is very small.

The screw feeder (as used in this study) is shown in Fig. 2.13. The screw feeder is fed from a hopper, so that the inlet is completely covered and the bulk material fills the available space. The speed of rotation is low, as this gives a stable output. The low speed means that centrifugal effects are small and therefore dilation of the bulk solid is negligible, thus maintaining virtually 100 percent "fullness" of the feeder. A choke section is adjacent to the hopper, at the beginning of the conveying section. The choke section is cylindrical and has the same radial clearance as the lower section of the trough. For effective flow control it should extend for at least one pitch [8] and, preferably, two standard pitches [22]. The single screw feeder may be a separate unit,
or it may be extended by sections of a normal screw conveyor to any practical length.
For this case it should be described in fundamental terms as a combination of the screw feeder and screw conveyor.

Fig. 2.13 A typical form of screw feeder fitted with hopper

Thus, compared with the screw elevator, screw conveyor and screw extruder, a screw feeder usually has some particular features. They are summarised as follows:

- The ratio of the length to width of the inlet is relatively large, so particular attention should be paid to the draw-down performance in a bin or hopper.

- The exposed inlet is "live", i.e. the feeding section should be completely covered and the bulk material fill the available space between screw shaft and trough.

- The choke section where the screw leaves the feeding section with the same radial clearance should extend for at least one pitch [8] or two standard pitches [22] for effective flow control purposes.

- The rotating speed of the screw is relatively low (normally less than 80 rpm [22, 29]) to obtain good metering characteristics.
2.4 Past Studies on the Performances of Screw Feeders

2.4.1 Volumetric Efficiency

A number of researchers [8, 28, 29, 65, 69, ] have indicated that the volumetric efficiency $\eta_v$ of a screw feeder can be defined as the bulk volume $V_{con}$ conveyed in one revolution, divided by the volume of one pitch length at that location

$$\eta_v = \frac{V_{con}}{\pi P \left( R_o^2 - R_c^2 \right)}$$

(2.1)

The volumetric efficiency of the material element conveyed can be expressed as

$$\eta_v = \frac{\tan \beta}{\tan \alpha + \tan \beta}$$

(2.2)

The relationship between $\alpha$ and $\beta$ is determined by

$$\alpha + \beta = 90^\circ - \varphi_f$$

(2.3)

Because the flight face varies in helix angle from a minimum at the outside radius to a maximum at the core shaft, within a pitch the bulk volume transported per revolution can be calculated from the following equation

$$\eta_v = \frac{2}{R_o^2 - R_c^2} \int_{R_c}^{R_o} \frac{\tan \beta}{\tan \beta + \tan \alpha} r \, dr$$

(2.4)

Haaker et al. [28, 29] considered that the integral given in Eq. (2.4) cannot be solved analytically. They also proposed another theory based on the plug flow of bulk solids where it is assumed that the internal friction is high enough to prevent internal shear in the bulk solid. The helical angle at the outer radius of the flight is chosen for the calculation of $\eta_v$

$$\alpha_o = \tan^{-1} \left( \frac{P}{2 \pi R_o} \right)$$

(2.5)

$$\beta_o = 90^\circ - \alpha_o - \varphi_f$$

(2.6)
The volumetric efficiency becomes

\[ \eta_v = \frac{\tan \beta_o}{\tan \alpha_o + \tan \beta_o} \]  
\[ (2.7) \]

Bates [8] proposed a concept of the “mean radius” \((R_m)\)

\[ \pi R_o^2 - \pi R_m^2 = \pi R_m^2 - \pi R_c^2 \]  
\[ (2.8) \]

The “mean radius” can be obtained from

\[ R_m = \sqrt{\frac{R_o^2 + R_c^2}{2}} \]  
\[ (2.9) \]

The “mean helix angle” is

\[ \alpha_m = \tan^{-1}\left( \frac{P}{2\pi R_m} \right) \]  
\[ (2.10) \]

\[ \beta_m = 90^\circ - \alpha_m - \varphi_f \]  
\[ (2.11) \]

The volumetric efficiency can be calculated using the constants \(\beta_m\) and \(\alpha_m\)

\[ \eta_v = \frac{\tan \beta_m}{\tan \alpha_m + \tan \beta_m} \]  
\[ (2.12) \]

Roberts et al. [65, 69] assumed an average ratio of radius to pitch to obtain the solution for Eq. (2.4)

\[ \zeta_{av} = \frac{\zeta_o + \zeta_c}{2} \]  
\[ (2.13) \]

where, \(\zeta_o = \frac{R_o}{P}, \zeta_c = \frac{R_c}{P}\). The volumetric efficiency can be obtained from

\[ \eta_{av} = 1 - \frac{1 + 2\pi \mu \zeta_{av}}{4\pi^2 \zeta_{av}^2 + 1} \]  
\[ (2.14) \]
2.4.2 Torque Requirement

Roberts et al. [65, 69] proposed a method by which the torque requirement can be calculated based on the axial forces generated. An obvious advantage of this method is that both torque and axial force can be obtained from one calculating process.

The loads acting on the feeder and pressure distributions around the screw are illustrated in Fig. 2.14.

![Fig. 2.14 Assumed pressure distribution around boundary [65, 69]](image)

The total torque acting over one pitch length of the screw is expressed as

\[
T = \frac{2\Delta F_A}{R_o^2 - R_c^2} \int_{R_c}^{R_o} r^2 \left( \frac{1 + 2\mu_f \frac{r}{P}}{2\pi \frac{r}{P} - \mu_f} \right) \, dr
\]

(2.15)

where \(\Delta F_A\) is the average force acting over one screw pitch.

\[
\Delta F_A = F_A \frac{P}{L}
\]

(2.16)

\(F_A\) is the total axial force.
\[ F_A = F_{AS} + F_{AC} \]  

(2.17)

\( F_{AS} \) is the force to shear the bulk solid along the shear surface and \( F_{AC} \) is the force to slide the bulk material along the trough surface.

The component \( F_{AS} \) is given by

\[ F_{AS} = \mu_e F_V \]  

(2.18)

\( \mu_e \) is the equivalent friction coefficient, \( \mu_e = (0.8 \sim 1) \sin \delta \). \( F_V \) is the resultant vertical load arising from the vertical pressure acting at the hopper outlet.

The component \( F_{AC} \) is estimated by

\[ F_{AC} = \mu_f F_V (K_1 + K_2 + K_3) \]  

(2.19)

where \( \mu_f \) is the friction coefficient on the trough surface. \( K_1, K_2 \) and \( K_3 \) are the pressure ratios. The values of the pressure ratios \( K_1, K_2 \) and \( K_3 \) are assumed to be

\[ 0.4 \leq (K_1 = K_2) \leq 1.0 \]
\[ 0.6 \leq K_3 \leq 1.0 \]  

(2.20)

2.4.3 Draw-Down Performance

Detailed experiments for the flow patterns in a hopper fed by a screw feeder were conducted by Bates [8]. A glass-sided container with vertical walls served as an observation hopper above a mounting for interchangeable screws. A vertical dividing grid of polished stainless steel was fitted above the screw and extended to the hopper top. The grid formed a central division above the axis of the screw and isolated each side into a number of equispaced divisions. Extraction results plotted for four screws with sodium perborate tetrahydrate (very free flowing bulk material) are shown in Fig. 2.15.
Fig. 2.15 Flow patterns with four screws and sodium perborate tetrahydrate [8]

A theory for the uniform draw-down of bulk solids from a hopper fitted with a screw feeder was established by Roberts et al. [40, 63, 64, 69].

Fig. 2.16 Screw feeder geometry [40, 63, 64, 69]
At location \( x \), as shown in Fig. 2.16, the volumetric feed rate is given by:

\[
Q(x) = A(x)v(x)\eta_v(x)
\]  

(2.21)

where, for location \( x \) in the feed section, \( A(x) \) is the cross-sectional area of the screw, \( v(x) \) is the axial feed velocity and \( \eta_v(x) \) is the volumetric efficiency.

According to Roberts, the required criterion for uniform draw-down performance is

\[
Q'(x) = \frac{dQ(x)}{dx} = \text{const.}
\]

(2.22)

Haaker et al. [28, 29] proposed a procedure for optimal screw design which mainly was concentrated on better draw-down performance. For the calculation of the optimum pitch value immediately before the choke section together with the other pitches an example is given as shown in Fig. 2.17.

Haaker et al. considered that the optimal flow rate per revolution \( Q_o \) can be chosen such that \( Q_o \) is equal to or less than 0.75\( Q_{\text{max}} \) (\( Q_{\text{max}} \) is the flow rate corresponding to the maximum value of the ratio of the pitch to screw diameter. Beyond this value increasing the pitch length will not provide an increase in the conveyed volume). This leads to the pitch value just before the choke section. According to their theoretical consideration and observation, the minimum value of the pitch to screw diameter ratio should not be less 0.25. This can be the regarded as lower bound, denoted as \( P_{\text{min}} \).

![Fig. 2.17 Example for optimal pitch design along the axis [28, 29]](image-url)
CHAPTER 3

THEORETICAL MODELLING OF SCREW FEEDER PERFORMANCE

3.1 Introduction

As mentioned in Chapter 1, the design of a screw feeder has a close relationship with the bin outlet design. Once the length and width of the bin outlet are determined based on the flow properties of the bulk solid and bin flow theory [2, 32], the screw diameter can be chosen depending on the width of the bin outlet. Further considerations for a screw feeder design include:

- Output or capacity which is related to the screw geometry, the rotating speed of the screw and the volumetric efficiency;

- Power requirement which is related to the rotating speed of the screw and the torque for turning the screw;

- Even draw-down performance which is mainly related to the length of the bin outlet and the screw geometry in the feed section.

Thus, the major objectives for the effective design of a screw feeder can be summed up as determination of the volumetric efficiency, the torque requirement and the screw geometry to achieve the desired bin flow pattern. These objectives are also shown in Fig. 3.1 to describe the desired outcomes of this research.
In this Chapter, theoretical modelling for the performance of screw feeders is presented. Two different procedures to estimate the volumetric efficiency are obtained. One is an analytical solution to the integral equation [Eq. (2.4)] while the other uses the concept of a equivalent helical angle. The calculated results based on equivalent helical angles are almost identical to those obtained from the analytical solution. The effect of the clearance between the screw flight and the inside surface of the trough on the output is considered. For predicting the torque requirement the load which is imposed on a screw feeder by the bulk solid in the hopper is assumed to be the flow load, determined on the basis of the major consolidation stress. Five boundaries around the bulk material within a pitch are considered and forces acting on these surfaces are analysed. Particular attention is paid to the pressure distribution in the lower region of the screw. An analytical procedure for the calculation of the torque requirement is determined. Equivalent helical angles for the screw flight and the movement of the bulk solid are also applied in the calculation of the torque requirement. A comparison of the two approaches indicates that the torque results
calculated based on the equivalent helical angles are very close to the values obtained from the analytical procedure; the maximum deviation is only 5%. A theoretical model for achieving a uniform flow pattern in the bin is proposed based on the pitch characteristic of the screw. The limitation of some traditional methods for increasing the screw capacity along its length are discussed.

3.2 Volumetric Efficiency

3.2.1 Definition and Derivation

Neglecting the thickness of the screw flight \( t \), the volumetric efficiency \( \eta_v \) of a screw feeder at a particular location along the screw can be defined as the bulk volume \( V_{\text{con}} \) conveyed in one revolution, divided by the volume of one pitch length at that location:

\[
\eta_v = \frac{V_{\text{con}}}{\pi P(R_o^2 - R_c^2)}
\]  

(3.1)

The volumetric efficiency is composed of two factors: the rotational effect and the fullness effect. For screw feeders operating at low speed, the fullness effect is negligible and the screw can be considered to be operating 100 percent "full". The dominant factor influencing the volumetric efficiency is the rotational motion which is a function of the screw geometry as well as the coefficient of friction between the bulk solid and the flight surface.

A bulk solid element sliding on the helical surface of a flight will move in a direction related to the angle of friction on the surface and helical angle of the flight face, as shown in Fig. 3.2. The axial and peripheral component of velocity of the element can be derived as follows

\[
v_f = v_p + v_a \cot \alpha
\]  

(3.2)

\[
v_a = v_p \tan \beta
\]  

(3.3)

\[
v_f = v_p(1 + \tan \beta \cot \alpha)
\]  

(3.4)
\[
\nu_p = \nu_f \frac{1}{1 + \tan \beta \cot \alpha} \tag{3.5}
\]

\[
\nu_a = \nu_f \frac{\tan \beta}{1 + \tan \beta \cot \alpha} \tag{3.6}
\]

The displacement of the element in the peripheral direction per revolution

\[
S_p = 2\pi r = \frac{P}{\tan \alpha} \tag{3.7}
\]

The axial movement of the element

\[
S_a = S_p \frac{\tan \beta}{1 + \tan \beta \cot \alpha} = P \frac{\tan \beta}{\tan \alpha + \tan \beta} \tag{3.8}
\]

Fig. 3.2 Velocity and displacement diagram for element at radius \( r \)

For one revolution the axial movement of the flight at radius \( r \) is the pitch \( P \). Thus, the volumetric efficiency of the conveyed material element can be calculated from

\[
\eta_v = \frac{S_a}{P} \tag{3.9}
\]

Substituting Eq. (3.8) into Eq. (3.9), the volumetric efficiency of the bulk solid element conveyed can be expressed as
\[ \eta_v = \frac{\tan \beta}{\tan \alpha + \tan \beta} \]  \hspace{1cm} (3.10)

The relationship between \( \alpha \) and \( \beta \) is determined by

\[ \alpha + \beta = 90^\circ - \varphi_f \]  \hspace{1cm} (3.11)

Because the flight face varies in helical angle from a minimum at the outside radius to a maximum at the core shaft, within a pitch the bulk volume transported per revolution can be calculated from the following equation, which was also obtained by Haaker et al. [28, 29]:

\[ \eta_v = \frac{2}{R_o^2 - R_c^2} \int_{R_c}^{R_o} \frac{\tan \beta}{\tan \beta + \tan \alpha} r \, dr \]  \hspace{1cm} (3.12)

### 3.2.2 An Analytical Solution

From Fig. 3.2 it can be seen that at radius \( r \) the following relationship exists

\[ \tan \alpha = \frac{P}{2\pi r} \]  \hspace{1cm} (3.13)

\[ \tan \beta = \tan \left[ \frac{\pi}{2} - (\alpha + \varphi_f) \right] = \cot(\alpha + \varphi_f) = \frac{2\pi r - \mu_f P}{2\pi \mu_f + P} \]  \hspace{1cm} (3.14)

Substituting Eqs. (3.13) and (3.14) into Eq. (3.12)

\[ \frac{\tan \beta}{\tan \beta + \tan \alpha} = 1 - \frac{1 + 2\pi \mu_f \frac{r}{P}}{1 + 4\pi^2 \left( \frac{r}{P} \right)^2} \]  \hspace{1cm} (3.15)

The volumetric efficiency can be expressed as

\[ \eta_v = \frac{2}{R_o^2 - R_c^2} \int_{R_c}^{R_o} [1 - \frac{1 + 2\pi \mu_f \frac{r}{P}}{1 + 4\pi^2 \left( \frac{r}{P} \right)^2}] r \, dr \]  \hspace{1cm} (3.16)

This is exactly the equation obtained by Roberts et al. [65, 69].
Let \( x = r/P \), then

\[
dr = P \, dx
\]

\( x = R_c/P \), at \( r = R_c \)

\( x = R_0/P \), at \( r = R_0 \)

Eq. (3.16) becomes

\[
\eta_v = \frac{2P^2}{R_0^2 - R_c^2} \left[ \int_{R_c/P}^{R_0/P} \frac{x}{1 + 4\pi^2 x^2} \, dx - \int_{R_c/P}^{R_0/P} \frac{\mu_f x}{1 + 4\pi^2 x^2} \, dx \right] + \frac{2\pi \mu_f x^2}{R_0^2 - R_c^2} \frac{R_c}{P} \]

\( 2P^2 \)

\( K_0 \)

\( p \)

\( f \)

\( L \)

\( -1/2 \)

\( A \)

\( K \)

\( P \)

\( R_0 \)

\( R_c \)

\( 2 - R_2 \)

\( J \)

\( 2\pi \mu_f x^2 \)

\( P \)

\( R_0 \)

\( R_c \)

\( 2\pi \mu_f x^2 \)

\( 1 + 4\pi^2 x^2 \)

\( 2nR_0 \)

\( 2nR_c \)

\( \tan \)

\( 2\pi R_0 \)

\( 2\pi R_c \)

\( 2\pi \mu_f x^2 \)

\( 1 + 4\pi^2 x^2 \)

\( 2\pi \mu_f x^2 \)

\( 1 + 4\pi^2 x^2 \)

\( 2n \)

\( R_0 \)

\( R_c \)

\( \tan^{-1} \)

\( 2\pi R_0 \)

\( 2\pi R_c \)

\( \tan^{-1} \)

\( 2\pi R_0 \)

\( 2\pi R_c \)

Substituting Eqs. (3.18), (3.19) and (3.20) into Eq. (3.17), the volumetric efficiency can be obtained
\[ \eta_v = 1 - \frac{P^2}{4 \pi^2 (R_o^2 - R_c^2)} \ln \frac{4 \pi^2 R_o^2 + P^2}{4 \pi^2 R_c^2 + P^2} - \frac{\mu f P}{\pi (R_o + R_c)} \]

\[ + \frac{\mu f P^2}{2 \pi^2 (R_o^2 - R_c^2)} \left[ \tan^{-1} \left( \frac{2 \pi R_o}{P} \right) - \tan^{-1} \left( \frac{2 \pi R_c}{P} \right) \right] \] (3.21)

By means of non-dimensional parameters

\[ c_d = \frac{d}{D} = \frac{R_c}{R_o} \]

\[ c_p = \frac{P}{D} \]

the volumetric efficiency can be expressed as

\[ \eta_v = 1 - \frac{c_p^2}{\pi^2 (1 - c_d^2)} \ln \frac{\pi^2 + c_p^2}{\pi^2 c_d^2 + c_p^2} - \frac{2 \mu f c_p}{\pi (1 + c_d)} \]

\[ + \frac{2 \mu f c_p^2}{\pi^2 (1 - c_d^2)} \left[ \tan^{-1} \left( \frac{\pi}{c_p} \right) - \tan^{-1} \left( \frac{\pi c_d}{c_p} \right) \right] \] (3.22)

3.2.3 Equivalent Helical Angles

Because of the complex shape of the flight surface and the internal stresses, the pressures on the flight face are not readily investigated. For this analysis, it is assumed that the pressure acting on the flight surface is constant within a pitch. The concept of an equivalent helix angle for the screw flight (\( \alpha_e \)) and for the material element motion (\( \beta_e \)) is proposed based on the analysis of the forces acting on the material element. It is considered that the direction of the resultant force acting on the material element should be consistent with the direction of movement of this element within a pitch. Fig. 3.3 shows a screw flight with a bulk solid element. Assuming that \( p \) is the normal pressure between the bulk solid and the flight, then the resultant force exerted on the element can be written as

\[ dF = \frac{pr}{\cos \alpha \cos \phi_f} d\theta \, dr \] (3.23)
The axial and peripheral components of the force are

\[
dF_a = dF \cos(\alpha + \varphi_f) = prd\theta dr(1 - \tan \alpha \tan \varphi_f) \tag{3.24}
\]

\[
dF_p = dF \sin(\alpha + \varphi_f) = prd\theta dr(\tan \alpha + \tan \varphi_f) \tag{3.25}
\]

Substituting \( \tan \alpha = \frac{P}{2\pi r} \) and \( \tan \varphi_f = \mu_f \) into equation (3.24) and (3.25) and integrating for \( r \)

\[
F_a = \left[ \frac{R_o^2 - R_c^2}{2} - \frac{\mu_f P}{2\pi} (R_o - R_c) \right] p \int d\theta \tag{3.26}
\]

\[
F_p = \left[ \frac{P}{2\pi} (R_o - R_c) + \frac{\mu_f}{2} (R_o^2 - R_c^2) \right] p \int d\theta \tag{3.27}
\]

In one pitch, \( \int d\theta = 2\pi \). The equivalent helical angle of motion of the material element can be expressed as

Fig. 3.3 Analysis of forces on an element
\[ \tan \beta_e = \frac{F_a}{F_p} = \frac{2\pi p \left[ \frac{R_o^2 - R_c^2}{2} - \frac{\mu f P}{2\pi} (R_o - R_c) \right]}{2\pi p \left[ \frac{P}{2\pi} (R_o - R_c) + \frac{\mu f}{2} (R_o^2 - R_c^2) \right]} \]

and the equivalent helical angle of the flight is

\[ \alpha_e = 90^\circ - \varphi_f - \beta_e \]  

The volumetric efficiency can be written as

\[ \eta_v = \frac{\tan \beta_e}{\tan \alpha_e + \tan \beta_e} \]  

### 3.2.4 Comparison between Calculated Results

The calculated results from Eq. (3.30) are identical with the results based on Roberts' Eq. (2.14).

A comparison of the volumetric efficiency calculated from Eq. (2.7) (Haaker et al. [28, 29]), Eq. (2.12) (Bates [8]), Eq. (2.14) (Roberts et al. [65, 69]), Eqs. (3.22) and (3.30) (Yu and Arnold [84, 90]) is shown in Table 3.1. It can be seen that the results obtained from Eqs. (2.14), (3.22) and (3.30) are nearly the same and results from Eq. (2.12) are also very close. For the smaller ratio of pitch to screw diameter, the results from all five equations are reasonably close. However, for the larger ratio of pitch to screw diameter \((P/D = 1.1)\) the results based on Eq. (2.12) are greater by 5% to 8% for \(\mu_f\) varying from 0.3 to 0.7, while the volumetric efficiencies based on Eq. (2.7) are greater by 17% to 29%.
Table 3.1 Comparison of calculated volumetric efficiencies \((R_c/R_o = 0.4)\)

<table>
<thead>
<tr>
<th>(\mu_f)</th>
<th>(P/D)</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>0.9</th>
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<tr>
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<td>.886</td>
<td>.821</td>
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<td>.680</td>
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<tr>
<td></td>
<td>Eq.(2.14)&amp;(3.30)</td>
<td>.941</td>
<td>.884</td>
<td>.820</td>
<td>.751</td>
<td>.682</td>
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<tr>
<td></td>
<td>Eq. (2.12)</td>
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<td>.840</td>
<td>.777</td>
<td>.712</td>
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<td>.867</td>
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<td>.672</td>
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</tbody>
</table>
3.2.5 Influence of Parameters

From Eq. (3.1) the conveyed volume of a screw feeder can be calculated analytically from

\[ V_{con} = \eta_v \pi P \left( R_o^2 - R_e^2 \right) \]  \hspace{1cm} (3.31)

The specific volume is defined by Bates [8]

\[ V_s = \frac{V_{con}}{R_o^3} \]  \hspace{1cm} (3.32)

Substituting \( V_{con} \) from Eq. (3.31), the specific volume becomes

\[ V_s = \eta_v \pi \left[ 1 - \left( \frac{R_e}{R_o} \right)^2 \right] \frac{P}{R_o} \]  \hspace{1cm} (3.33)

Fig. 3.4 gives values of the volumetric efficiency \( \eta_v \) [from Eq. (3.30)] and the specific volume \( V_s \) [from Eq. (3.33)] for variations in \( \mu_f \) and \( P/D \). It can be seen from Fig. 3.4 that at the point where the volumetric efficiency is nearly equal to 0.5, the specific volume begins to drop from the highest point. The corresponding value of \( P/D \) varies from 1.1 to 1.7 depending on the different friction coefficient of the bulk solid on the flight surface. Beyond these values increasing the pitch length will not provide an increase in the conveyed volume. It can also be seen that the effect of the wall friction coefficient of the bulk solid on the flight surface on the volumetric efficiency and the specific volume increases with larger values of the \( P/D \) ratio.
3.2.6 Effect of Clearance on Output

An important constructional feature affecting the output of a screw feeder is the clearance between the trough and the tips of the flight in the choke section. Clearance is necessary to prevent metallic contact from taking place during rotation due to various adverse factors such as shaft deflection, minor manufacturing eccentricities, and tolerance on the screw and the trough. It is also essential to avoid nipping or wedging of particles to prevent damage and the generation of extreme contact pressures and, hence, high torques resisting rotation.

A series of experiments to examine the effect of the radial clearance on the output of screw feeders were reported by Burkhardt [14]. In the experiments six screws, which were turned from a 2 inch solid aluminium bar, were employed. The ratios of P/D were 0.5, 0.675, 0.75, 0.875, 1.0 and 1.125 respectively. Six troughs were made of cold-drawn seamless steel tubing. The radial clearance approximated a geometric series, for ratio of c/D starting at 1/128 and doubling each step to 1/4. The test bulk
was barley. Because the screw speed range used in the experiments varied widely from 25 to 3300 rpm, the recorded results did not give a regular pattern. But interesting results can be observed for the low speed range (< 100 rpm), which are shown in Fig. 3.5. The plots are the experimental results from [14], and the lines are theoretical predictions based on Eq. (3.35), in which \( k = 0 \). The data for clearances \( c/d = 1/128 \) and \( c/D = 1/64 \) are almost identical and hence only five different ratios of \( c/D \) are shown in Fig. 3.5. It can be clearly seen that the observed values have a good consistency with the theoretical calculation and the increase of the output is proportional to the increase of the clearance.

![Fig. 3.5 Effect of radial clearance on output (measurements from [14])](image)

At the choke section the predicted capacity increases with an increase in the “effective” area. The effective area is dependent on the properties and conditions of the bulk solid being conveyed and on the geometry and the properties of the trough and flight surfaces. The effective area for a single screw feeder is defined as

\[
A_e = \pi \left[ (R_o^2 - R_c^2) + (1 - k) (2cR_o + c^2) \right] \quad (3.34)
\]
A coefficient \( k (k = 0 \sim 1) \) is proposed to account for the possible dead layer of material between the screw flights and the trough wall. \( k = 0 \) when there is full wall slip and no dead layer is left; the effective area is equal to the area of the trough at the choke section. \( k=1 \) when there is an annular layer formed in the choke section and the material will shear at the flight radius. Due to the relatively complex state of stresses around the periphery of a screw it is difficult to analyse and predict conditions in the choke section. \( k \) needs to be obtained by experiment or by experience.

The output per revolution of a single screw feeder can be expressed as

\[
V_o = \pi \eta_P \left[ \left( R_o^2 - R_c^2 \right) + (1 - k) \left( 2cR_o + c^2 \right) \right]
\]  

(3.35)

### 3.3 Torque Requirements

#### 3.3.1 Feeder Loads

The load which is exerted on a feeder by the bulk solid in a hopper was discussed in Ref. [38, 41, 66]. There are two main load conditions: the initial filling condition when the bin is filled from the empty state and the flow condition when discharge has occurred. Experimental evidence in this study suggests that the feeder load on a screw feeder can be considered to be that arising under flow conditions. Carson [20] also noted that the starting torque is close to the running torque for many bulk materials and situations, but warned that there would be exceptions, for example: bulk materials which adhere to surfaces after storage at rest, (effectively increasing wall friction angles); bulk materials which gain strength with storage time and require increased additional shear forces to commence flow; bins or hoppers which are vibrated during storage at rest (greatly increasing the vertical stress, ie feeder load). The recommended flow loads can be obtained by using methods proposed by Reisner et al. [61].
According to McLean and Arnold [38], the feeder load $Q$ acting at the outlet of the hopper is given by

$$Q = qyL^{1-m}B^{m+2}$$  \hspace{1cm} (3.36)

$q$ is a non-dimensional surcharge factor. $m$ is a hopper shape factor; $m = 1$ for axisymmetric flow or conical hopper, $m = 0$ for plane flow or wedge-shaped hopper.

The flow load on a screw feeder can be determined on the basis of the major consolidation stress $\sigma_f$. The non-dimensional flow surcharge factor may be expressed as

$$q_f\sigma_f = \left(\frac{\pi}{4}\right)^m \frac{Y(1 + \sin \delta)}{2(X - 1)\sin \alpha_h}$$  \hspace{1cm} (3.37)

where

$$\beta_h = \frac{1}{2}\left[\phi_h + \sin^{-1}\left(\frac{\sin \phi_h}{\sin \delta}\right)\right]$$  \hspace{1cm} (3.38)

$$X = \frac{2^m \sin \delta}{1 - \sin \delta}\left[\frac{\sin(2\beta_h + \alpha_h)}{\sin \alpha_h} + 1\right]$$  \hspace{1cm} (3.39)

$$Y = \frac{\{2[1 - \cos(\beta_h + \alpha_h)]\}^m(\beta_h + \alpha_h)^{1-m}\sin \alpha_h + \sin \beta_h \sin^{1+m}(\beta_h + \alpha_h)}{(1 - \sin \delta)\sin^{2+m}(\beta_h + \alpha_h)}$$  \hspace{1cm} (3.40)

In Eq. (3.40) both $\beta_h$ and $\alpha_h$ must be in radians for the numerator term $(\beta_h + \alpha_h)^{1-m}$.

For a hopper fitted with a screw feeder, in general, $m = 0$. Based on the flow case, the feeder load can be written as

$$Q = q_f\sigma_f yLB^2$$  \hspace{1cm} (3.41)

It is reasonable to assume that the feeder load is uniformly distributed over the hopper outlet. The resulting stress $\sigma_o$ can be obtained by
\[ \sigma_o = \frac{Q}{LB} = q_{fo} \gamma B \]  

(3.42)

3.3.2 Pressure on Boundary Surfaces of Bulk Material

Considering the bulk material boundary in a pitch, "pressures" are imposed on five surfaces, as indicated in Fig. 3.6 (a). Taking account of the boundary conditions applying to the bulk material moving within screw flights, two basic regions can be specified as: an upper region in which a "shear surface" exists between the bulk solid surrounding the screw and the bulk solid propelled by the screw and a lower region in

Fig. 3.6 Bulk material boundary within a pitch  
(a) Five boundary surfaces; (b) Two basic regions
which the bulk solid is moving within a limited space which comprises rigid surfaces, as shown in Fig. 3.6 (b). The five surfaces to which pressure is applied are:

- the “shear surface” on the upper region of the screw;
- the trailing side of the screw flight;
- the driving side of the screw flight;
- the outside surface of the core shaft;
- the inside surface of the trough.

3.3.3 Pressure Distribution on Bulk Material in Lower Region

Consider the bulk material axial cross section in the lower region in a pitch, as depicted in Fig. 3.7. The bulk material boundary in this region is composed of four sides: the trailing side and driving side of the screw flight, the inside surface of the trough and the outside surface of the core shaft. For simplicity this boundary can be assumed to be rectangular in shape and of unit thickness. Considering the forces acting on the material limited by this boundary the force due to gravity is neglected. Because the speed of rotation is relatively low, the centrifugal effects are also regarded as negligible.

![Fig. 3.7 Stress on an element in lower region of screw](image)
Stress $\sigma_w$ is the normal wall pressure acting perpendicularly to the wall of the trough and the core shaft. $\sigma_x$ is the axial compression stress. The ratio

$$\lambda_s = \frac{\sigma_w}{\sigma_x}$$

is known as the stress ratio of bulk material sliding on the confining surface (i.e., the trough and the core shaft surfaces). A general expression can be obtained from

$$\lambda_s = \frac{\sigma_w}{\sigma_x} = \frac{1}{1 + 2\mu_d^2 + 2\sqrt{(1 + \mu_d^2)(\mu_d^2 - \mu_w^2)}}$$

$\mu_d = \tan \delta$ and $\delta$ is the effective angle of internal friction of the bulk solid. $\mu_w$ is the wall friction coefficient between the bulk solid and a confining surface. The derivation of Eq. (3.44) is presented below. For free flowing materials, $\mu_d = \mu$, where $\mu$ is the friction coefficient of the bulk solid. Then, Eq. (3.44) has the same form as that given by Hong et al. [31].

The Mohr circle representing the stress of a bulk solid element on a confining surface is shown in Fig. 3.8.
From the right angle triangle ACD in Fig. 3.8

\[(\sigma_m - \sigma_w)^2 + \tau_w^2 = r_m^2 \tag{3.45}\]

Substituting \(\tau_w\) with \(\mu_w \sigma_w\)

\[(\sigma_m - \sigma_w)^2 + \mu_w^2 \sigma_w^2 = r_m^2 \tag{3.46}\]

The solution to Eq. (3.46) is

\[\sigma_w = \frac{\sigma_m - \sqrt{r_m^2 (1 + \mu_w^2) - \sigma_m^2 \tau_w^2}}{1 + \mu_w^2} \tag{3.47}\]

Since

\[\mu_d = \tan \delta\]

\[\sigma_m = \frac{r_m}{\sin \delta} = \frac{r_m \sqrt{1 + \mu_d^2}}{\mu_d}\]

and

\[\sigma_w + \sigma_x = 2\sigma_m\]

The stress ratio of the bulk solid sliding on a confining surface can be obtained by

\[\lambda_x = \frac{\sigma_w}{\sigma_x} = \frac{\sigma_w}{2\sigma_m - \sigma_w} = \frac{1}{1 + 2\mu_d^2 + 2\sqrt{(1 + \mu_d^2)(\mu_d^2 - \mu_w^2)}} \tag{3.48}\]

When a moving bulk solid reaches steady state, there is equilibrium between the driving force and the resisting force. Assuming the axial stress and the radial stress are functions of \(x\) only, as shown in Fig. 3.7, and the frictional coefficients between the bulk solid and the core shaft and trough are the same, the balance of forces acting on the element of length \(dx\) results in

\[\frac{2\mu_w \lambda_x}{R_t - R_c} \sigma_x - \frac{d\sigma_x}{dx} = 0 \tag{3.49}\]
A solution to Eq. (3.49) is

\[ \sigma_x = c_1 \cdot \exp \left( \frac{2\mu_w \lambda_s x}{R_t - R_c} \right) \]  

(3.50)

Where \( c_1 \) is an integration constant. For the purposes for this analysis, \( c_1 \) is determined by making the following simplifying assumption concerning the boundary condition

\[ \sigma_x = \sigma_o, \text{ at } x = 0 \]

\( \sigma_o \) is the stress exerted on the screw feeder by the bulk solid in a hopper. The solution to Eq. (3.49) can then be written as

\[ \sigma_x = \sigma_o \exp \left( \frac{2\mu_w \lambda_s x}{R_t - R_c} \right) \]  

(3.51)

To simplify the calculation an average radial stress along a pitch is introduced as

\[ \frac{\lambda_s}{P} \int_0^P \sigma_x \, dx \]  

(3.52)

Substituting \( \sigma_x \) from Eq.(3.51), the average radial stress becomes

\[ \sigma_{wa} = \sigma_o \frac{R_t - R_c}{2\mu_w P} \left[ \exp \left( \frac{2\mu_w \lambda_s P}{R_t - R_c} \right) - 1 \right] \]  

(3.53)

For convenience, non-dimensional parameters are used

\[ c_d = \frac{d}{D} = \frac{R_c}{R_o} \]

\[ c_p = \frac{P}{D} \]

\[ c_t = \frac{(D + 2c)}{D} = \frac{2R_t}{D} \]

Eq. (3.53) can be expressed as
\[
\sigma_{wa} = \sigma_0 \frac{c_t - c_d}{4\mu_wc_p} \left[ \exp \left( \frac{4\mu_w\Lambda c_p}{c_t - c_d} \right) - 1 \right]
\] (3.54)

In an actual application the wall friction between the bulk solid and the core shaft and the trough may be the same. If they are not identical, then, the wall friction coefficient between the bulk solid and the trough surface \(\mu_t\) should be chosen for \(\mu_w\), as the trough surface are dominates.

### 3.3.4 Forces Acting on Individual Surfaces

The analysis of the forces acting upon the bulk solid element on the individual surfaces when a non-cohesive bulk material is transported in a vertical screw conveyor, was made by Nilsson [48]. For screw feeders the surfaces upon which forces are exerted and the status of the acting forces, should be distinguished from those produced in a vertical screw conveyor.

A bulk material sector in a pitch is used for the calculation of the axial forces acting on individual surfaces, as depicted in Fig. 3.9.

![Fig. 3.9 A material sector in a pitch](image)

It is assumed that within the length of a pitch the forces acting on the individual surfaces are uniformly distributed (on the upper shear surface, trailing side and driving
side of flight) or these forces can be represented by average forces (on the outside surface of the core shaft and the inside surface of the trough).

### 3.3.4.1 Axial Resisting Force on Shear Surface

The axial resisting force acting on the element of the bulk solid on the shear surface, as shown in Fig. 3.10, is given by

\[
dF_{ua} = \mu_e \sigma_o R_o P d\theta \cos(\alpha_o + \phi_f)
\]  

(3.55)

\(\alpha_o\) is the helical angle of the flight at the outside radius. \(\mu_e\) is the equivalent friction coefficient. According to Roberts et al. [41, 66], \(\mu_e = (0.8-1.0)\sin\delta\).

![Fig. 3.10 Forces on shear surface](image)

The total axial force acting over a pitch length of the screw is

\[
F_{ua} = \mu_e \sigma_o R_o P \cos(\alpha_o + \phi_f) \int_0^\pi d\theta
\]  

(3.56)
After integration and by means of non-dimensional parameters, Eq. (3.56) can be written as

\[ F_{wa} = \frac{\pi}{2} \mu_c c_p \cos(\alpha_o + \phi_f) \sigma_o D^2 = k_u \sigma_o D^2 \]  

(3.57)

and

\[ k_u = \frac{\pi}{2} \mu_c c_p \cos(\alpha_o + \phi_f) \]  

(3.58)

### 3.3.4.2 Axial Resisting Force on Core Shaft

On the surface of the core shaft, the axial force acting on the element of the bulk solid is shown in Fig. 3.11. This axial resisting force is

\[ dF_{ca} = \mu_w \sigma_{wa} R_c P d\theta \sin \alpha_c \]  

(3.59)

\( \mu_w \) is the wall friction coefficient of the bulk solid on the core shaft. \( \alpha_c \) is the helical angle of the flight at the core shaft.

![Fig. 3.11 Forces on shaft surface](image)

The total axial force acting over a pitch length of the screw is

\[ F_{ca} = \mu_w \sigma_{wa} R_c P \sin \alpha_c \int_0^{2\pi} d\theta \]  

(3.60)
\( \sigma_{wa} \) can be obtained from Eq.(3.54). Integrating Eq. (3.60) leads to
\[
F_{ca} = \frac{\pi c_d (c_t - c_d)}{4} \sin \alpha_c \left[ \exp \left( \frac{4 \mu_w \lambda_s c_p}{c_t - c_d} \right) - 1 \right] \sigma_o D^2 = k_c \sigma_o D^2 \tag{3.61}
\]
and
\[
k_c = \frac{\pi c_d (c_t - c_d)}{4} \sin \alpha_c \left[ \exp \left( \frac{4 \mu_w \lambda_s c_p}{c_t - c_d} \right) - 1 \right] \tag{3.62}
\]

3.3.4.3 Axial Resisting Force on Trailing Side of Flight

The axial resisting force acting on the trailing side of the flight as presented in Fig. 3.12 is
\[
dF_{la} = \lambda_s \sigma_o \frac{rdrd\theta}{\cos \alpha_r \cos \phi_f} \cos (\phi_f - \alpha_r) = \lambda_s \sigma_o rdrd\theta (1 + \tan \alpha_r \tan \phi_f) \tag{3.63}
\]
Substituting \( \tan \alpha_r = \frac{P}{2\pi r} \) and \( \tan \phi_f = \mu_f \), and integrating for \( r \) from \( R_c \) to \( R_o \) and for \( \theta \) from 0 to \( 2\pi \)
\[
F_{la} = \lambda_s \left[ \frac{\pi}{4} \left( 1 - c_d^2 \right) + \frac{\mu_f c_p}{2} (1 - c_d) \right] \sigma_o D^2 = k_l \sigma_o D^2 \tag{3.64}
\]
and
\[
k_l = \lambda_s \left[ \frac{\pi}{4} \left( 1 - c_d^2 \right) + \frac{\mu_f c_p}{2} (1 - c_d) \right] \tag{3.65}
\]

![Fig. 3.12 Forces on trailing side of flight](image_url)
3.3.4.4 Axial Resisting Force on Trough Surface

From Fig. 3.10, the axial resisting force on the trough surface is given by

\[ dF_{ta} = \mu_\omega \sigma_{wa} R_t P d\theta \cos(\alpha_\omega + \phi_f) \]  

(3.66)

After integrating for \( r \) from \( R_c \) to \( R_o \) and for \( \theta \) from 0 to \( \pi \), the total axial force over a pitch is

\[ F_{ta} = \frac{\pi}{8} c_t (c_t - c_d) \cos(\alpha_\omega + \phi_f) \left[ \exp\left( \frac{4 \mu_\omega \lambda_s c_p}{c_t - c_d} \right) - 1 \right] \sigma_o D^2 = k_t \sigma_o D^2 \]  

(3.67)

and

\[ k_t = \frac{\pi}{8} c_t (c_t - c_d) \cos(\alpha_\omega + \phi_f) \left[ \exp\left( \frac{4 \mu_\omega \lambda_s c_p}{c_t - c_d} \right) - 1 \right] \]  

(3.68)

3.3.4.5 Axial Force and Stress on the Driving Side of a Flight

The axial force acting on the driving side of a flight is shown in Fig. 3.13. This force should be equal to the total resisting axial forces

\[ F_{da} = F_{ua} + F_{ca} + F_{ta} + F_{ta} \]  

(3.69)
It is assumed that the total force is uniformly exerted on the surface of the driving side.

The axial stress can be determined by

$$\sigma_a = \frac{F_{da}}{\pi \left( R_o^2 - R_c^2 \right)} = \frac{4(k_u + k_c + k_l + k_t)}{\pi \left( 1 - c_d^2 \right)} \sigma_o = K_\sigma \sigma_o \quad (3.70)$$

where

$$K_\sigma = \frac{4(k_u + k_c + k_l + k_t)}{\pi \left( 1 - c_d^2 \right)} \quad (3.71)$$

### 3.3.5 Torque Requirement

Most screw conveyors can be designed with little thought given to thrust as the thrust force or axial force is moderate and commonly used screw conveyor drives will accommodate thrust in either direction. However, in a screw feeder, especially with long inlet openings, axial force can be very significant. Thus, determination of the axial force is necessary for the design of a screw feeder.

Roberts et al. [65, 69] proposed a method by which the torque requirement can be calculated based on the axial forces. An obvious advantage of this method is that both torque and axial force can be obtained from one calculating process.

#### 3.3.5.1 Torque Requirement in Feed Section

Referring to Fig. 3.12, the tangential force on the bulk material element is

$$dF_{dt} = \sigma_a d\theta dr d\tau \tan (\alpha_r + \phi_f) \quad (3.72)$$

The torque required for turning screw is

$$T = 2\pi \sigma_a \int_{R_c}^{R_o} r^2 \tan (\alpha_r + \phi_f) dr \quad (3.73)$$

which becomes
\[ T = K_s \sigma_a D^3 \] (3.74)

and

\[ K_s = \frac{2\pi}{D^3} \int_{R_o}^{R_c} r^2 \tan(\alpha_r + \phi_f) \, dr \] (3.75)

Substituting \( \tan \alpha_r = \frac{P}{2\pi r} \) and \( \tan \phi_f = \mu_f \) into Eq. (3.75) gives

\[ K_s = \frac{2\pi}{D^3} \int_{R_o}^{R_c} r^2 \left( \frac{1 + 2\pi\mu_f \frac{r}{P}}{2\pi \frac{r}{P} - \mu_f} \right) \, dr \] (3.76)

Let \( x = \frac{r}{P} \), then

\[ dr = P \, dx \]

\( x = R_c / P \), at \( r = R_c \)

\( x = R_o / P \), at \( r = R_o \)

Eq. (3.76) becomes

\[ K_s = \frac{2\pi P^3}{D^3} \left[ \int_{\frac{R_o}{P}}^{\frac{R_c}{P}} \frac{x^2}{2\pi x - \mu_f} \, dx + \int_{\frac{R_o}{P}}^{\frac{R_c}{P}} \frac{2\pi\mu_f x^3}{2\pi x - \mu_f} \, dx \right] \] (3.77)

The solution to the first integral is

\[ \int_{\frac{R_o}{P}}^{\frac{R_c}{P}} \frac{x^2}{2\pi x - \mu_f} \, dx = \frac{1}{4\pi} \left[ \left( \frac{R_o}{P} \right)^2 - \left( \frac{R_c}{P} \right)^2 \right] + \frac{\mu_f}{4\pi^2} \left( \frac{R_o}{P} - \frac{R_c}{P} \right) \]

\[- \frac{3\mu_f^2}{16\pi^3} + \frac{\mu_f^2}{8\pi^3} \ln \left( \frac{2\pi \frac{R_o}{P} - \mu_f}{2\pi \frac{R_c}{P} - \mu_f} \right) \] (3.78)
The solution to the second integral is

$$\int_{\frac{R_o}{P}}^{\frac{R_c}{P}} \frac{2\pi \mu_f x^3}{2\pi x - \mu_f} \, dx = \frac{\mu_f}{3} \left[ \left( \frac{R_o}{P} \right)^3 - \left( \frac{R_c}{P} \right)^3 \right] + \frac{\mu_f}{2\pi} \int_{\frac{R_o}{P}}^{\frac{R_c}{P}} \frac{x^2}{\mu_f} \left( \frac{1}{x} - \frac{1}{2\pi} \right) \, dx$$

(3.79)

For the integral in Eq. (3.79), the solution is

$$\int_{\frac{R_o}{P}}^{\frac{R_c}{P}} \frac{x^2}{\mu_f} \left( \frac{1}{x} - \frac{1}{2\pi} \right) \, dx = \frac{\mu_f}{2} \left[ \left( \frac{R_o}{P} \right)^2 - \left( \frac{R_c}{P} \right)^2 \right] + \frac{\mu_f^2}{2\pi^2} \left( \frac{R_o}{P} - \frac{R_c}{P} \right)$$

(3.80)

Combining Eq. (3.78), (3.79) and (3.80), \(K_s\) can be expressed as

$$K_s = 2\pi \frac{p^3}{\rho^3} \left\{ \frac{\mu_f}{3} \left[ \left( \frac{R_o}{P} \right)^3 - \left( \frac{R_c}{P} \right)^3 \right] + \frac{1 + \mu_f}{4\pi} \left[ \left( \frac{R_o}{P} \right)^2 - \left( \frac{R_c}{P} \right)^2 \right] \right\}$$

$$+ \frac{\mu_f (1 + \mu_f^2)}{4\pi^2} \left[ \frac{R_o}{P} - \frac{R_c}{P} \right] - \frac{3\mu_f^2 (1 + \mu_f^2)}{16\pi^3}$$

$$+ \frac{\mu_f^2 (1 + \mu_f^2)}{8\pi^3} \ln \left( \frac{\frac{2\pi}{P} \frac{R_o}{P} - \mu_f}{\frac{2\pi}{P} \frac{R_c}{P} - \mu_f} \right)$$

(3.81)

Employing non dimensional parameters allows \(K_s\) to be expressed as

$$K_s = \pi c_p \frac{\mu_f}{12c_p} \left[ (1 - c_d) - \frac{1 + \mu_f}{8c_p^2} (1 - c_d^2) + \frac{\mu_f (1 + \mu_f^2)}{4\pi^2 c_p} (1 - c_d) \right]$$

$$- \frac{3\mu_f^2 (1 + \mu_f^2)}{8\pi^3} + \frac{\mu_f^2 (1 + \mu_f^2)}{4\pi^3} \ln \left( \frac{\pi - \mu_f c_p}{\pi c_d - \mu_f c_p} \right)$$

(3.82)

Substituting \(\sigma_d\) from Eq.(3.70), the torque requirement can be expressed as

$$T = K_s K_o \sigma_d D^3$$

(3.83)
Eq. (3.83) is a general expression for the torque required for one pitch in the feed section. For pitch $i$, Eq. (3.83) can be written as

$$T_{fi} = K_{si}K_{\sigma i}\sigma_c D^3$$  \hspace{1cm} (3.84)

The torque required for all pitches in the feed section is

$$T_{f} = \sum_{i=1}^{n_f} T_{fi}$$  \hspace{1cm} (3.85)

where $n_f$ is the number of the pitches in the feed section.

It can be seen from Eq.(3.83) that the required torque is proportional to the stress exerted on the feeder by the bulk solid in the hopper and to the third power of the screw diameter. An increase of 50% in screw diameter will result in a 50% increase in opening width of the hopper and a 50% increase in stress exerted by the bulk solid in the hopper from Eq. (3.42). According to Eq. (3.83) the total increase in torque will be 500%. This conclusion agrees with the dimensional analysis and experimental results reported by Rautenbach et al. [55].

A method for the calculation of the torque requirement of a screw feeder proposed by Roberts et al. is outlined in Chapter 2. An example, which was used by Roberts et al. in References [65, 69], is presented in Appendix A to compare the results calculated by Roberts’ method and the model developed in this study.

### 3.3.5.2 Torque Requirement in Choke Section

It is assumed that in the choke section the screw is operating 100 percent “full”. The shear surface which occurs in the feed section does not exist, but is replaced by a cylindrical sliding surface. Thus, in the choke section Eq. (3.71) can be replaced by

$$K_{oc} = \frac{4(k_c + k_j + 2k_i)}{\pi(1 - c_d^2)}$$  \hspace{1cm} (3.86)
A general expression for torque required for a pitch in the choke section is

\[ T_c = K_s K_{oc} \sigma_D D^3 \]  

(3.87)

Normally, pitches in the choke section have the same geometry. The total torque required for pitches in the choke section can be expressed as

\[ T_{ct} = n_c T_c \]  

(3.88)

where \( n_c \) is the number of the pitches in the choke section.

### 3.3.6 Application of Equivalent Helical Angles in Torque Calculation

In Eq. (3.83) factor \( K_s \) is derived from the tangent force acting on the driving side of the flight. It is only related to the screw geometry and the wall friction coefficient between the bulk solid and the flight. It can be seen from Eq. (3.75) that both \( r \) and \( \alpha_r \) are the variables in the integral. For a given screw geometry and wall friction coefficient between the bulk solid and the screw flight, the equivalent helical angle \( \alpha_e \) and \( \beta_e \) can be obtained from Eq. (3.28) and (3.29).

Substituting \( \alpha_r \) with \( \alpha_e \) into Eq. (3.75), an equation for \( K_s \) using the equivalent helical angle of the screw flight is

\[ K_{se} = \frac{2\pi}{D^3} \int_{R_c}^{R_o} r^2 \tan(\alpha_e + \phi_f)dr \]  

(3.89)

Both \( \alpha_e \) and \( \phi_f \) are constants, so Eq. (3.89) becomes

\[ K_{se} = \frac{2\pi}{D^3} \tan(\alpha_e + \phi_f) \int_{R_c}^{R_o} r^2 dr \]

\[ = \frac{2\pi}{3D^3} (R_o^3 - R_c^3) \tan(\alpha_e + \phi_f) \]  

(3.90)

From Eq. (3.29) and functions of angles in any quadrant in terms of angles in the first quadrant, the following relationship exists...
\[ \tan(\alpha_e + \phi_f) = \tan \left( \frac{\pi}{2} - \beta_e \right) = \cot \beta_e \]  

(3.91)

From Eq. (3.28), Eq. (3.91) becomes

\[ \cot \beta_e = \frac{P + \pi \mu_f (R_o + R_c)}{\pi (R_o + R_c) - \mu_f P} \]  

(3.92)

Incorporating Eq. (3.91) and (3.92) in Eq. (3.90) leads to

\[ K_{se} = \frac{2\pi}{3D^3} \left( R_o^3 - R_c^3 \right) \frac{P + \pi \mu_f (R_o + R_c)}{\pi (R_o + R_c) - \mu_f P} \]  

(3.93)

By means of non dimensional parameters, Eq. (3.93) becomes

\[ K_{se} = \frac{\pi}{12} \left( 1 - c_d \right)^3 \frac{2c_p + \pi \mu_f (1 + c_d)}{\pi (1 + c_d) - 2\mu_f c_p} \]  

(3.94)

A comparison of the results calculated from Eq. (3.82) and (3.94) is shown in Table 3.2. It can be seen that the results obtained are very close. The maximum error is only 5% for \( \mu_f \) varying from 0.3 to 0.7 and \( c_p \) varying from 0.3 to 1.1.
Table 3.2 Comparison of \( K_s \) and \( K_{se} \) \((R_e/R_o = 0.4)\)

<table>
<thead>
<tr>
<th>( \mu_f )</th>
<th>( P/D )</th>
<th>( 0.3 )</th>
<th>( 0.5 )</th>
<th>( 0.7 )</th>
<th>( 0.9 )</th>
<th>( 1.1 )</th>
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<td>( .139 )</td>
<td>( .168 )</td>
<td>( .198 )</td>
<td>( .231 )</td>
</tr>
<tr>
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<td>( .144 )</td>
<td>( .177 )</td>
<td>( .211 )</td>
<td>( .247 )</td>
<td>( .285 )</td>
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<td>( K_{se} )</td>
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<td>( .202 )</td>
<td>( .237 )</td>
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</tr>
<tr>
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<td>( .208 )</td>
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<td>( .287 )</td>
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<td>( .238 )</td>
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<tr>
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<td>( .240 )</td>
<td>( .282 )</td>
<td>( .328 )</td>
<td>( .378 )</td>
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<td></td>
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<td>( .270 )</td>
<td>( .321 )</td>
<td>( .381 )</td>
<td>( .453 )</td>
</tr>
</tbody>
</table>

3.3.7 Torque Characteristics of a Screw Feeder

3.3.7.1 Torque Components

From the analyses presented in Section 3.3.4, the torque requirement for a screw feeder is dependent on the resisting forces on four surfaces. When the stress exerted by the bulk solid in the hopper and the screw diameter are determined, the values of the resisting forces acting on individual surfaces can be predicted. For the sake of convenience, the equation for the calculation of the torque requirement is rewritten

\[
T = K_s K_o \sigma_o D^3
\]  

(3.83 repeated)

The factor \( K_o \) is related to the axial resisting forces acting on individual surfaces and comprises the components given in Eqs. (3.58), (3.62), (3.65) and (3.68). The factor
$K_s$ is derived from the tangential force acting on the driving side of a flight. To facilitate an examination of the contribution of the various components to the total torque requirement, let

$$K = K_t K_s = \frac{4K_s}{\pi(1-c_d^2)}(k_u + k_c + k_t + k_i) \quad (3.95)$$

Let

$$K_u = \frac{4k_u K_s}{\pi(1-c_d^2)} \quad (3.96)$$

$K_u$ reflects the torque contribution from the shear surface

$$K_c = \frac{4k_c K_s}{\pi(1-c_d^2)} \quad (3.97)$$

$K_c$ reflects the torque contribution from the core shaft surface

$$K_t = \frac{4k_t K_s}{\pi(1-c_d^2)} \quad (3.98)$$

$K_t$ reflects the torque contribution from the trailing side of the flight

$$K_l = \frac{4k_l K_s}{\pi(1-c_d^2)} \quad (3.99)$$

$K_l$ reflects the torque contribution from the trough surface

Fig. 3.14 shows the contribution of various components to the total torque. It can be seen that the major contribution is the resisting torque acting on the upper shear surface, varying from 43\% for $P/D = 0.3$ to 50\% for $P/D = 1$. When $P/D > 0.5$, factor $K_u$ contributes 50\% of the whole torque. The resisting torques on the trough surface and on the shaft surface increase with an increase of the $P/D$ ratio, varying from 13\% to 19\% and from 3\% to 11\%, respectively. Compared to the torques on the other three surfaces, the resisting torque on the shaft surface is low, even when the values of the ratio of the core shaft diameter to screw diameter increases up to 0.5. The contribution
of the torque on the trailing side of the flight decreases with an increase of the $P/D$ ratio, from 42% for $P/D = 0.3$ to 20% for $P/D = 1$.

![Graph showing variation of factors with ratio $P/D$](image)

Fig. 3.14 Variation of factors with ratio $P/D$

($\mu_d = 0.8$, $\mu_f = \mu_{iv} = 0.5$, $c_d = 0.3$)

### 3.3.7.2 Influence of Clearance on Torque

Fig. 3.15 shows the influence of the values of the $c/D$ and $P/D$ ratio on the torque requirement. The values of $c/D$ vary from 1/64 to 1/8. Results from the theoretical calculation show that there is no obvious increase in the factor $K$ with the ratio $c/D$ varying from 1/64 to 1/8 for the different values of the $P/D$ ratio. Although an increase in the clearance will result in some increase in the resisting torque (due to an increase in $R_i$), this is offset by a decrease in radial pressure [from Eq. (3.51)], which leads to a decrease in the resisting torque on the trough surface and shaft surface. However, from Eq. (3.42), the stress exerted by the bulk solid in the hopper is linearly proportional to the opening width of the hopper outlet. Normally, the opening width
of the hopper outlet \( B=2(D+2c) = 2R_t \), which means that the stress \( \sigma_o \) increases with an increase of the clearance. This influence of the clearance cannot be neglected in the calculation of the torque requirement.

![Graph showing the influence of \( c/D \) on \( K \).](image)

**Fig. 3.15 Influence of \( c/D \) on \( K \)**  
(\( \mu_d = 0.8, \mu_f = \mu_{vw} = 0.5, c_d = 0.3 \))

### 3.3.7.3 Influence of Trough Wall Friction Coefficient

Fig. 3.16 shows the influence of the trough wall friction coefficient on factor \( K \). It can be clearly seen that \( K \) increases with an increase in the trough wall friction coefficient \( \mu_t \). Such an increase in \( K \) is due to two influences. First, the radial pressure along the pitch length increases with the increase in \( \mu_t \). Secondly, the resisting torque increases with both the radial pressure and the friction coefficient, with the effect being stronger for a larger pitch length.
3.3.7.4 Influence of Effective Angle of Internal Friction

The influence of the effective angle of internal friction is related to the stress ratio, shear force on the upper part of the screw and the radial pressure on the lower part of the screw. The stress ratio and the radial pressure decrease with an increase of the effective angle of internal friction, although the increase of the shear force caused by the effective angle of internal friction coefficient is not sufficient to compensate for the decrease of the stress ratio and the radial pressure. Fig. 3.17 shows the influence of the effective angle of internal friction on the factor $K$. 

![Graph showing influence of $\mu_f$ on $K$]


Fig. 3.17 Influence of $\delta$ on $K$

$\mu_f = \mu_w = 0.5$, $c_d = 0.3$

### 3.3.7.5 Influence of Flight Friction Coefficient

Fig. 3.18 gives the results for $K$ with variation of $\mu_f$ from 0.3 to 0.7. It can be seen that $K$ becomes larger with an increasing flight friction coefficient. The forces acting on both the trailing side and the driving side of the flight increase with an increase of the friction coefficient between the bulk solid and the flight, resulting in an increase in the torque requirement.

Fig. 3.18 Influence of $\mu_f$ on $K$

$\mu_d = 0.8$, $\mu_w = 0.5$, $c_d = 0.3$
3.3.7.6 Influence of Ratio \( d/D \)

An increase of \( c_d \), the ratio of the core shaft diameter to the screw diameter, will result in an increase of the stress ratio, which affects the radial pressure, especially for longer pitch lengths. The resistant force and the torque on the shaft surface also increase with an increase of the ratio \( c_d \). Fig. 3.19 shows the effect of ratio \( c_d \) on \( K \).

![Fig. 3.19 Influence of \( c_d \) on \( K \)](image)

(\( \mu_d = 0.8, \mu_f = \mu_w = 0.5 \))

3.3.8 Power Efficiency

The power needed for the transport by the screw of the bulk solid in one pitch in the feed section can be calculated with the aid of Eq. (3.83)

\[
P_{sc} = T \omega = K_s K_\sigma \sigma_0 D^3 \omega
\]

(3.100)

where \( \omega \) is the angular velocity of the screw.

The power required for the transport of the bulk solid in the axial direction is
Chapter 3  Theoretical Modelling of Screw Feeder Performance

\[ P_{ax} = \frac{\pi}{4} \gamma \rho (D^2 - d^2) v_{ax} \]  
(3.101)

where \( v_{ax} \) is the axial velocity of the bulk solid.

The power efficiency for one pitch in the feed section is defined as

\[ \eta_p = \frac{P_{ax}}{P_{sc}} = \frac{\pi \gamma \rho (D^2 - d^2) v_{ax}}{4 K_s K_\sigma \sigma_o D^3 \omega} \]  
(3.102)

For one revolution the axial movement of the screw flight is \( P \), i.e. the pitch length. The axial movement of the bulk solid in the pitch is \( \eta_v P \), where \( \eta_v \) is the volumetric efficiency. If screw turns \( n \) revolutions per second, then, \( \omega = 2\pi n \) and \( v_{ax} = n \eta_v P \).

Substituting \( \sigma_o \) from Eq. (3.42), Eq. (3.102) can be expressed as

\[ \eta_p = \frac{\eta_v P^2 (D^2 - d^2)}{8 q_{f\sigma_1} B K_s K_\sigma D^5} \]  
(3.103)

As an example, let the volumetric efficiency \( \eta_v = 1 \), the non-dimensional surcharge factor \( q_{f\sigma_1} = 1 \) and the opening width of the hopper outlet \( B = D \). By non-dimensional parameters, Eq. (3.103) can be simplified to

\[ \eta_p = \frac{c_p^2 (1 - c_d^2)}{8 K_s K_\sigma} \]  
(3.104)

It can be seen from Eq. (3.104) that the power efficiency is proportional to the square of the ratio of \( P/D \), suggesting that the power efficiency increases with an increase in the pitch length. However, the factors \( K_s \) and \( K_\sigma \) also increase with an increase in the pitch length. Increases in the pitch length are limited by the screw geometry and properties of the bulk solid.

The upper limit for the pitch length can be calculated on the condition from Eq. (3.82). According the definition of a natural logarithm, the following condition has to be satisfied
\[ \frac{\pi - \mu_f c_p}{\pi c_d - \mu_f c_p} > 0 \] (3.105)

Since, in general, \( \pi - \mu_f c_p > 0 \), the above condition can be changed to

\[ \pi c_d - \mu_f c_p > 0 \] (3.106)

The upper limit of the pitch length is determined by

\[ c_p < \frac{\pi c_d}{\mu_f} \] (3.107)

![Graph showing variation of power efficiency with ratio P/D and friction coefficient](image)

Fig. 3.20 Variation of \( \eta_p \) with \( P/D \) and \( \mu_f \)

\( (\mu_d = 0.8, \mu_w = 0.5, \ c_d = 0.3, \ \eta_v = 1, \ \eta_{\alpha1} = 1) \)

Fig. 3.20 shows the variation of the power efficiency with the ratio of \( P/D \) and the friction coefficient between the bulk solid and the flight surface. A line in the figure indicates the points on the curves of maximum power efficiency. These calculations indicate that the values of \( c_p \) corresponding to the highest values of the power efficiency agree with the upper limit calculated from Eq. (3.107). It can be observed from Fig. 3.20 that for the shorter pitch length or the higher flight friction coefficient the power efficiency is very low, because, comparing to the power required for advancing the bulk solid in the feed direction, more power is required for rotating the
bulk solid in the pitch. Even when the ratio of $P/D$ is the standard value of 1, the power efficiency is still low, indicating an obvious weakness of screw feeders. In the above discussion the influence of the feeder load exerted by the bulk solid in the hopper and the volumetric efficiency are neglected ($q_{fM} = 1, \eta_v = 1$). Including these factors will decrease the power efficiency further. This discussion relates to only one pitch in the feed section.

Define the theoretical power to transport the mass output of a screw feeder in the axial direction as the power to transport in the axial direction the mass contained within the last or exit pitch, that is

$$P_{axf} = \frac{\pi}{4} \gamma P_L (D^2 - d^2) V_{axf}$$

(3.108)

where $P_L$ is the length of the last pitch in the feed section. The axial movement of the bulk solid per revolution is $\eta_v L P_L$, where $\eta_v L$ is the volumetric efficiency for the last pitch in the feed section. The axial velocity for the bulk solid in the last pitch $v_{axf} = n \eta_v L P_L$. Substituting into Eq. (3.108) leads to

$$P_{axf} = \frac{\pi}{4} n \eta_v L P_L^2 (D^2 - d^2) = n \gamma V_o P_L$$

(3.109)

where $V_o$ is the volumetric output per revolution of a screw feeder

$$V_o = \frac{\pi}{4} \eta_v L P_L (D^2 - d^2)$$

(3.110)

The power needed for the transport by the screw of the bulk solid in all pitches in the feed section is

$$P_{scf} = \omega \sum T$$

(3.111)

where $\sum T$ is the sum of the torques for all pitches in the feed section.

For all pitches in the feed section, the power efficiency is
\[ \eta_{pf} = \frac{P_{axf}}{P_{scf}} = \frac{\gamma N_o P_L}{2\pi \sum T} \]  \hfill (3.112) 

### 3.3.9 A Simplified Approach to Power Calculation

It can be seen from Eq. (3.112) that the power efficiency of a screw feeder can be obtained after calculating the torque of all pitches in the feed section. Screw feeders are commonly fitted to mass flow bins or hoppers with slotted outlets. This requires good geometric design of the outlet-feeder combination in order for the bulk solids to be entrained over the full length of the outlet. Thus, some methods to increase screw capacity in the direction of feed, e.g., stepped pitch, tapered shaft, tapered screw diameter, or a combination of these methods, are used. The changes of screw geometry result in a complicated calculation of torque for all pitches in the feed section. It is desirable to find a simplified method for the calculation of the power efficiency of a screw feeder.

As a simplifying assumption, let

\[ \eta_{ps} = \eta_{pl} \frac{P_L}{(L + L_c)} \]  \hfill (3.113)

where \( \eta_{ps} \) is the approximate power efficiency of a screw feeder; \( \eta_{pl} \) is the power efficiency of the last pitch in the feed section and can be calculated from Eq. (103); \( P_L \) is the last pitch length; \( L \) is the length of the feed section and \( L_c \) is the length of the choke section. This approximation means that the power efficiency for a screw feeder is proportional to the ratio of the last pitch length to the sum of the length of the feed section and the length of the choke section.

An equation for the power requirement for a screw feeder can be derived from Eq. (3.113)

\[ P_f = \frac{n \gamma N_o P_L}{\eta_{ps}} = \frac{n \gamma N_o (L + L_c)}{\eta_{pl}} \]  \hfill (3.114)
3.4 Draw-Down Performance

3.4.1 Average Effective Area

The screw geometry considered for the assessment of draw-down performance is shown in Fig. 3.21. At any location $x$ the relevant screw geometrical variables are assumed to be

$$P(x) = \text{pitch}$$

$$D(x) = \text{outside diameter of screw}$$

$$d(x) = \text{core or shaft diameter}$$

The volume withdrawn from the hopper by the screw within a pitch is

$$V(x) = A(x) \eta_s(x) P(x)$$

(3.115)

where $A(x)$ is the cross-sectional area of the screw at the location $x$.

$$A(x) = \frac{\pi}{4} [D^2(x) - d^2(x)]$$

(3.116)
For simplicity, the pitch of a screw can be regarded as varying incrementally rather than continuously. Hence, along the feeding length the pitch can be expressed as

\[ P_1 = x_1 \]

\[ P_2 = x_2 - x_1 \]

\[ \ldots \ldots \]

\[ P_L = \text{the pitch at the feeder outlet} \]

Within a particular pitch, Eq. (3.115) can be written as

\[ V_i(x) = P_i A(x) \eta_v(x) \]  

(3.117)

where: \( i = 1, 2, \ldots, L \). From Eq. (3.117), it can be seen that if a screw feeder has variable pitches, the transported volume is not a continuous function along the feed length but is also incremental. In considering the flow pattern in a hopper fitted with a screw feeder the withdrawn volume can be considered separately in each pitch. Within a pitch the average effective area is defined as

\[ A_{ai} = \frac{\int_{x_{i-1}}^{x_i} A(x) \eta_v(x) dx}{x_i - x_{i-1}} \]  

(3.118)

Utilising the average effective area, the volume transported per revolution in pitch \( i \) is

\[ V_i = P_i A_{ai} \]  

(3.119)

### 3.4.2 Criterion for Uniform Draw-Down Performance

For many processes, the bulk solid should be withdrawn uniformly from the entire hopper outlet area; this requires a screw capacity which increases in the direction of feed. To satisfy this requirement the volumetric transportation should be consistent with the following equations reported by Yu and Arnold [85–87]:
\[ V_2 = V_1 + P_2 A_{a1} = A_{a1} (P_1 + P_2) \]  
\[ (3.120) \]

or

\[ A_{a2} = A_{a1} \frac{P_1 + P_2}{P_2} \]  
\[ (3.121) \]

in general

\[ A_{a_i} = A_{a1} \frac{P_1 + P_2 + \cdots + P_i}{P_i} \]  
\[ (3.122) \]

and at the feeder outlet

\[ A_{aL} = A_{a1} \frac{L}{P_L} \]  
\[ (3.123) \]

Eq. (3.122) is the required criterion for uniform draw-down performance and Eq. (3.123) is the boundary condition. From Eqs. (3.122) and (3.123) it can be concluded that the average effective area in the first pitch has a very important influence on the performance of the whole screw.

Consider now the influence of a particular pitch \( P_i \) on the withdrawal performance of a screw. The volume conveyed from the adjacent upstream pitch \( P_{i-1} \) is \( A_{a_{i-1}} P_{i-1} \), which contributes \( A_{a_{i-1}} P_{i-1} / P_i \) to the average effective area of the (downstream) pitch \( P_i \). The net increment in the average effective area of pitch \( P_i \) is

\[ A_{ai} - A_{a_{i-1}} P_{i-1} / P_i \].

To allow the withdrawal performance of a particular screw to be compared to the ideal, a profile coefficient \( f_{pi} \) is introduced

\[ f_{pi} = \left( A_{ai} - A_{a_{i-1}} \frac{P_{i-1}}{P_i} \right) / A_{a1} \]  
\[ (3.124) \]

\( f_p = 1 \) for ideal uniform withdrawal performance and \( f_p = 0 \) for no volumetric increment in the pitch.
3.4.3 Limitations of Some Design Methods

In many instances, screw feeders incorporate a constant pitch screw in which the pitch is usually equal to the screw diameter. Unfortunately, a constant pitch screw is not suited for withdrawing bulk solids from a rectangular hopper outlet; it only withdraws material from approximately one pitch length near the rear end of the outlet. There are three main methods to increase screw capacity in the direction of feed, ie stepped pitch, tapered shaft or tapered screw diameter (Fig. 3.22). It is instructive to examine the effectiveness of each of these methods. The symbols in the following analyses are related to this Figure.

![Screw configurations for increasing capacity](image)

**Fig. 3.22** Screw configurations for increasing capacity

### 3.4.3.1 Stepped Pitch

As shown in Fig. 3.22 (a), along the whole feed length the cross sectional area is constant and within a pitch the volumetric efficiency is also constant, ie

\[ A(x) = A \]

\[ \eta_{vi}(x) = \eta_{vi} \]
According to Eq. (3.117) the volume transported per revolution in pitch $i$ is

$$V_i = P_i A \eta_{vi} \tag{3.125}$$

Comparing with the required criterion for uniform draw-down performance [Eq. (3.122) and (3.123)], it can be seen that a screw with stepped pitch and uniform diameter, theoretically speaking, cannot provide a uniform withdrawal flow pattern, because there is no increment in the average effective area. In application the variable pitch range of a screw is limited. Based on experimental investigations and experience, the minimum pitch should be not less than one third the screw diameter and the maximum pitch should be approximately one screw diameter for most bulk solids. If the pitch is too small the pocket between the shaft and the first two flights is so narrow that there is a danger that the bulk solid will compact and rotate with the screw. This results in no bulk solid being fed into the adjacent section of the screw. Above the upper bound of the pitch, the flight forces the bulk solid to rotate excessively with the screw rather than move toward the discharge end and the volumetric efficiency is greatly decreased.

As a simple case, one can assume a constant volumetric efficiency. The profile coefficient in pitch $i$ can be approximated by:

$$f_{pi} = 1 - \frac{P_{i-1}}{P_i} \tag{3.126}$$

The range of the profile coefficient is shown in Fig. 3.23.
3.4.3.2 Tapered Shaft, Uniform Pitch

The pitch is constant along the whole feed length as shown in Fig. 3.22 (b), ie

\[ P_1 = P_2 = \cdots = P_L = P \]

The volume withdrawn in the pitch \( i \)

\[ V_i = PA_{ai} \]  \hspace{1cm} (3.127)

Similar to the case in Section 3.4.3.1, the volumetric efficiency \( \eta_{vi} = 1 \) is assumed. As an example, the profile coefficient in the second pitch is considered in order to understand the effect of the feeding length \( L \) and the shaft diameter at the beginning of the first pitch \( d_1 \) on the profile coefficient. Their relationships are shown in Fig. 3.24. It can be seen that the profile coefficient is very sensitive to the ratio \( d_1/D \) which determines the volume withdrawn in the first pitch. When \( d_1/D \) is less than 0.8 the profile coefficient is less than 0.5.
3.4.3.3 Tapered Outside Diameter Screw, Uniform Pitch

The assumptions are the same as in Section 3.4.3.2; again the profile coefficient in the second pitch is considered. The relationships among the geometric variables are shown in Fig. 3.25. Because the narrow rear end of the screw is prone to having arches form over it and it is difficult to properly fabricate the screw trough and the interface with the hopper outlet, the tapered diameter screw is not recommended for most bulk solids.

In the above discussion the volumetric efficiency of the screw feeder is neglected. However, the volumetric efficiency decreases with an increase in the ratio of the pitch to the screw diameter. It also decreases with a decrease in the ratio of the shaft diameter to the screw diameter. The inclusion of volumetric efficiency would result in further reductions in the value of profile coefficient.
Fig. 3.25 Variation of $f_{p2}$ with $L/D$ and $D_1/D$ for tapered outside diameter screw

$$(P/D = 0.7, \ d/D = 1/3)$$

3.4.4 Hopper Geometry Interfacing with Screw Feeder

In order to eliminate the “end effects” and obtain uniform draw-down performance, the hopper walls at the front end and the rear end can be sloped, as shown in Fig. 3.26.

![Hopper Walls Diagram](image)

Fig. 3.26 Hopper walls for uniform draw-down performance

It is assumed that the bulk solid in the hopper is divided into “vertical channels”, which match the corresponding pitch length, as shown in Fig. 3.26. The volume transported in the first pitch is
\[ V_1 = A_{a1} P_1 \] (3.128)

Due to the volume being withdrawn from both \( P_1 \) and \( P'_1 \), an equivalent area is introduced for the first pitch. Then, Eq. (3.128) can be rewritten as

\[ V_1 = A'_{a1} (P_1 + P'_1) \] (3.129)

The equivalent area for the first pitch is

\[ A'_{a1} = A_{a1} \frac{P_1}{P_1 + P'_1} \] (3.130)

It is easy to apply the criterion for uniform draw-down to the equivalent area. The average effective area for the second pitch should satisfy by the following requirement

\[ A_{a2} = A'_{a1} \frac{P_1 + P_2}{P_2} = A_{a1} \frac{P_1(P_1 + P_2)}{P_2(P_1 + P'_1)} \] (3.131)

\[ \frac{P'_1}{P_1} = \frac{A_{a2} - A_{a2}}{A_{a2}} + \frac{A_{a1} P_1}{A_{a2} P_2} \] (3.132)

To obtain uniform draw-down performance along the whole feed section, the boundary condition should be satisfied according to Eq. (3.123)

\[ A_{aL} = A'_{a1} \frac{L}{P_L} = A_{a1} \frac{P_L}{P_L(P_1 + P'_1)} \] (3.133)

Eq. (133) can lead to

\[ \frac{P'_1}{P_1} = \frac{A_{a1} L}{A_{a1} P_L} - 1 \] (3.134)

For \( P_L = D \), the variations of \( P'_1 / P_1 \) with \( A_{aL} / A_{a1} \) and \( L/D \) are shown in Fig. 3.27.
For free-flowing granular materials, the volume transported in the last pitch, which is adjacent to the choke section, is determined by both the geometry of the screw and the clearance between the flight and the trough inside surface. According to the definition in Eq. (3.118), the average effective area in the last pitch is

\[ A_{al} = \frac{\pi}{4} \int_{x_{L-1}}^{x_L} \eta v_L(x) \left[D^2(x) - d^2(x)\right]dx \]  

(3.135)

After taking account of the effect of the clearance, the equivalent area is

\[ A'_{al} = \frac{\pi}{4} \int_{x_{L-1}}^{x_L} \eta v_L(x) \left[(D(x) + 2c)^2 - d^2(x)\right]dx \]  

(3.136)

To compensate for the effect of clearance in the last pitch the criterion for uniform hopper draw-down should be modified so that the following equations are satisfied

\[ A_{al}(P_L + P'_L) = A'_{al}P_L \]  

(3.137)

\[ \frac{P'_L}{P_L} = \frac{A'_{al}}{A_{al}} - 1 \]  

(3.138)

To investigate the effect of the clearance, assume \( D \) and \( d \) are constants in the last pitch and \( \eta_{al} = 1 \), then the variations of \( P'_L / P_L \) with \( c/D \) and \( d/D \) are shown in Fig. 3.28.

Fig. 3.27 Variations of \( P'_L / P_L \) with \( A_{al} / A_{ol} \) and \( L/D \)
Chapter 3  Theoretical Modelling of Screw Feeder Performance

Fig. 3.28 Variations of $P'_L / P_L$ with $c/D$ and $d/D$

In practical applications the front wall and the rear wall of a hopper can be inclined, as shown in Fig. 3.29. The slope angle of the front wall and the rear wall can be calculated by:

$$\alpha_1 = \tan^{-1}\left(\frac{2P'_L}{H}\right)$$  \hspace{1cm} (3.139)

$$\alpha_L = \tan^{-1}\left(\frac{2P'_L}{H}\right)$$  \hspace{1cm} (3.140)

Fig. 3.29 Inclined end wall angle for uniform flow pattern
Incorporating Eq. (3.128) and (3.132) the expressions for the slope angles become

\[
\alpha_1 = \tan^{-1}\left[\frac{2 \frac{P_1}{D} \left(\frac{A_{a1} L}{A_{al}} \frac{L}{P_L} - 1\right)}{H/D}\right] \\
\alpha_L = \tan^{-1}\left[\frac{2 \frac{P_L}{D} \left(\frac{A'_{aL}}{A_{al}} - 1\right)}{H/D}\right]
\]

(3.141) (3.142)

As an example, let \(P_I/D=1\), \(P_I/P_L=0.3\), \(A_{a1}/A_{al}=0.5\), then the variations of \(\alpha_I\) with \(H/D\) and \(L/D\) are shown in Fig. 3.30. The variations of \(\alpha_L\) with \(H/D\) and \(c/D\) for \(d/D=0.4\) are shown in Fig. 3.31. Comparing \(\alpha_I\) with \(\alpha_L\), it can be seen that the “end effect” in the first pitch on the flow pattern is larger than that in the last pitch, which is due only to the clearance.

![Fig. 3.30 Variations of \(\alpha_I\) with \(H/D\) and \(L/D\)

\((P_I/D = 1, P_I/P_L = 0.3, A_{a1}/A_{al} = 0.5)\)
Fig. 3.31 Variations of $\alpha_L$ with $H/D$ and $c/D$

$$(d/D = 0.4, P_{f/D} = 1)$$
4.1 Introduction

To assist in understanding the performance of screw feeders, experimental investigations are valuable. Despite some experimental work for screw feeders, there is still a shortage of systematic experimental data relating to the volumetric efficiency, the torque characteristics and the draw-down performance. Furthermore, past experimental studies focused attention on single screw feeders. The performance of multi screw feeders does not appear to have received extensive investigation. The equipment described in this Chapter was designed specifically for understanding the influence of geometrical parameters, properties of bulk material conveyed and operating conditions on single and twin screw feeder performance. The experimental work also was aimed at verifying the theoretical analyses and the performance prediction models developed in Chapter 3.

4.2 Description of Test Rigs

The screw feeder test rigs, as depicted in Figs 4.1 and 4.2, were designed and fabricated to monitor the performance of different types of screw (such as stepped pitch, tapered shaft and stepped shaft) operating with different bulk solids, rotating speeds and inside diameter of the troughs. The five primary components of each test rig are: the hopper and the trough, the dividing grid, the screws to be tested, the driving unit, the receiving and weighing silo.
Fig. 4.1 Test rig for single screw feeder

1-hopper; 2-trough; 3-test screw

4-driving unit; 5-receiving and weighing silo
Fig. 4.2 Test rig for twin screw feeder

1-hopper; 2-trough; 3-test screw

4-driving unit; 5-receiving and weighing silo
4.2.1 Hopper and Trough

The hopper consisted of two perspex plates, which were set vertically at the front and rear, and two metal plates, which were joined to the supporting bars and formed the sloping sides of the hopper. The configuration of the hopper was designed to be easily adjusted for different trough sizes and still operate as a plane mass flow hopper.

The inside radius \((R_t)\) of the trough for the single screw feeder were 80, 85 and 95 mm, giving radial clearances of 5, 10, and 20 mm respectively, as shown in Fig. 4.3.

Two troughs were used for the twin screw feeder, as shown in Fig. 4.4. Both had the same inside radius \((R_t = 85 \text{ mm})\) but different centre distances. The centre distance \((l_t)\) of 160 mm gave a 10 mm gap between the two screws, while the centre distance \((l_t)\) of 175 mm gave a 25 mm gap.

![Fig. 4.3 Trough for single screw feeder](image)

![Fig. 4.4 Trough for twin screw test](image)
The upper half part of all troughs at the choke section had the same geometry as the lower part.

### 4.2.2 Dividing Grid

To investigate the hopper flow pattern due to screw feeders and the volume withdrawn by individual flights along the whole feed section, a dividing grid was fitted over the screw to form a division along the axis of the screw and isolate each side into a number of divisions which matched the different pitches of the screw.

Fig. 4.5 shows the grid used for the single screw feeder. A plate was placed lengthwise along the axis of the screw. Two types of radial dividing plates were employed for two test materials (white plastic pellets and semolina). At first, only one type of radial dividing plate was used, as shown in the left bottom of Fig. 4.5. Because semolina is a very free flowing material and in order to minimise the effect of cross flow, the form of the radial dividing plate was improved, as shown in the right bottom of Fig. 4.5.

The dividing grid for the twin screw test is shown in Fig. 4.6. A dividing plate was placed lengthwise at the centre between the two screws and parallel to their axes. The radial dividing plates were across the whole screw width and also were positioned to match the different pitch lengths.
Fig. 4.5 Grid for single screw feeder

Fig. 4.6 Grid for twin screw feeder
4.2.3 Test Screws

All screws employed for the test programs, as shown in figs. 4.7 to 4.9, had the same outside diameter \( D = 150 \text{ mm} \). Four screws of different configuration were investigated on the single screw feeder test rig. All of them had an increase in the screw capacity in the feed direction. The general configuration of these screws are shown in Fig. 4.7.

The No. 1 screw had five different pitch lengths \( P = 50, 75, 100, 125 \text{ and } 150 \text{ mm} \). By changing the position of the choke section (i.e., changing the length of the feed section), five different ratios of the pitch to screw diameter were generated for investigating the volumetric efficiency and output on the single screw feeder test rig.

The two screws used in the twin screw feeder had the same configuration but with left hand and right hand flights. Two screws (No. 2 screw with tapered shaft and stepped pitch and No. 4 screw with stepped shaft and pitch) were chosen for the experiments in the twin screw feeder test rig. Figs. 4.8 and 4.9 show the configurations investigated. Apart from the hand of the flights the No. 2 screw and the No. 4 screw used in both test rigs had the same geometry.

“Left” or “right” screw in the twin screw feeder is distinguished in the following way: looking along the screw in the direction of the bulk solids fed, the screw at the left side is the left screw and the screw at the right side is the right screw. Two directions of rotation were investigated in the experimental program. The rotating direction of the screws is distinguished in the following way: “upward” refers to screws rotating upward near the trough walls and “downward” refers to screws rotating downward near the trough walls.

In the experimental programs for torque requirements and flow patterns a feed length of 550 mm was chosen on the single screw feeder test rig and a feed length of 700 mm on the twin screw feeder test rig.
Fig. 4.7 Configurations of four single screws
Fig. 4.8 Configuration of No. 2 screw in twin screw feeder

Fig. 4.9 Configuration of No. 4 screw in twin screw feeder
4.2.4 Driving Units

The driving unit used for the single screw feeder consisted of an electric motor, a V-belt drive and a shaft mounted speed reducer, as shown in Fig. 4.10. Direct mounting onto the screw shaft by simple and effective locking methods eliminated the necessity for an independent foundation, shaft coupling and conventional motor side-rails. The rotating speed of the electric motor was controlled by a MSC frequency controller. The screw speed or the amount of rotation was measured by a counter.

![Fig. 4.10 Driving unit for single screw feeder](image)

1- screw shaft, 2 - shaft mounted speed reducer, 3 - electric motor
4 - V-belt drive, 5 - torque arm anchor

Fig. 4.11 shows the arrangement of the driving unit for the twin screw feeder. In addition to the components in the driving unit for the single screw feeder, a pair of gear wheels and two pairs of HTD synchronous drives were added to the twin screw driving unit. The HTD synchronous drives consisted of two pulleys and a belt, on which there were teeth of the same pitch. They maintained a high transmission efficiency and provided a constant angular velocity to each screw.
4.2.5 Receiving and Weighing Silo

A receiving and weighing silo was mounted under the outlet of the screw (see Figs. 4.1 and 4.2). It was supported by load cells allowing the mass of the bulk material discharged from the screw feeder to be measured.

4.3 Instrumentation and Data Acquisition

The instrumentation was designed to measure directly the following parameters during the experiments on the screw feeder performance:

- mass output of the bulk material discharged by a screw feeder
- torque requirement
- rotating speed of the screw
4.3.1 Mass Output of Bulk Material Discharged

Shear-beam-type load cells supported the receiving and weighing silo (see Figs. 4.1 and 4.2). The mass of the bulk material discharged from the screw and loaded into this silo over a period of time or for a number of the rotations of the screw, was measured by these load cells. An average mass output of the bulk solid per revolution could be obtained by calculating the mass and the corresponding number of revolutions.

4.3.2 Torque Requirement

A force transducer and its connectors were used for the torque arm anchor to the shaft mounted speed reducer. The speed reducer, including the electric motor and V-belt drive which were mounted together, were set inclined vertically to a certain angle. The resulting offset centre of mass provided an initial tension in the force transducer.

4.3.3 Rotating Speed of Screw

A small magnet was fixed on a ring which was a part of the bearing seat and rotated with the screw shaft. A detector was fastened in a position where it could receive a pulse signal. For one revolution of the screw shaft the magnet gave a single pulse signal. According to the number of the pulse signals during a period of time, a counter displayed the rotating speed of the screw shaft.

4.3.4 Data Acquisition System

A data acquisition system, as shown in Fig. 4.12, was used to collect the experimental data. This system consisted of a YEW 3063 multi-pen chart recorder, a digital rotating speed display and a digital frequency display. The chart recorder captured the mass output and torque data. The digital display of the rotating speed of the screw and the frequency of the power supply were used for adjusting the required speed of the screw shaft.
4.3.5 Data Processing

The chart recorder gave a two dimensional graphic of time and displacement. The coordinate of time (vertical direction) was controlled by the chart speed (eg 6 cm/min). The coordinate of the displacement (horizontal direction) presented the value of the torque or the mass of the bulk material discharged. Typical graphical outputs generated by the chart recorder are repeated in Fig. 4.13.
To generate the performance characteristics of screw feeders, such as volumetric efficiency, the output of a screw, hopper flow patterns and the torque requirement, a commercial graph software package (Cricket Graph) was used.

4.4 Calibration

4.4.1 Load Cell Calibration

Load cells were used to support the receiving silo and monitor the mass of the bulk material discharged from the screw feeder. Calibration of load cells was carried out by putting a known mass into the silo. The detailed steps were:

(i) Put down the recorder pen on the zero line.

(ii) Load a given mass (say 5 kg) into the receiving silo. The recorder pen plots a value of displacement.

(iii) Repeat step (ii) until all the designated weights are loaded into the receiving silo.
The calibration line of the load cells is presented in Fig. 4.14. The calibration factor for the load cells of the transducer is 0.2408 kg/mm.

![Graph showing calibration data]

\[ y = 0.23259 + 0.24080x \quad R^2 = 1.000 \]

Fig. 4.14 Calibration of load cells

4.4.2 Torque Transducer Calibration

The torque transducers were calibrated by adding a known mass on the end of a cantilevered arm, as shown in Fig. 4.15. The calibration procedure can be summarised as follows:

(i) Fasten the arm rod on the screw shaft and keep it in the horizontal position.

(ii) Put down the recorder pen on the zero line.

(iii) Load a given mass (say 1 kg) on the pendant at one end of the arm rod. This mass gives a certain torque acting on the screw shaft depending on the mass and the distance between the shaft centre and the mass centre. A recorder pen plots a corresponding value of the displacement.
(iv) Repeat step (iii) until all the designated weights are loaded on the pendant.

Fig. 4.15 Calibration for torque transducer

1-shaft mounted speed reducer, 2-screw shaft, 3-transducer
4-arm rod, 5-mass

The calibration results of the torque transducers both for the single screw feeder and for the twin screw feeder are presented in Fig. 4.16 and 4.17. A quite good linearity can be seen from the results. The calibration factors of the transducers are 0.42809 Nm/mm and 0.42284 Nm/mm, respectively.
Fig. 4.16 Calibration of torque transducer for single screw feeder

\[ y = 0.42578 + 0.42809x \quad R^2 = 1.000 \]

Fig. 4.17 Calibration of torque transducer for twin screw feeder

\[ y = 9.2216 \times 10^{-2} + 0.42284x \quad R^2 = 1.000 \]

\[ y = 9.2216e-2 + 0.42284x \quad R^2 = 1.000 \]
5.1 Introduction

As analysed in Chapter 3, the performance of screw feeders is influenced by the properties of the bulk solid to be conveyed. Hence for a screw feeder to be designed to ensure satisfactory and efficient operation, the influence of the properties of the bulk materials must be considered properly.

In the case of bulk solids there are many terms that are used to describe their properties [6, 12]. Many of these properties are used in qualitative, descriptive and empirical ways. They are often difficult to define precisely and even more difficult to measure. In this work, attention is given particularly to the properties possibly affecting the screw feeder performance, such as particle size, density and flow properties. This chapter introduces these properties and their measurement method. In addition, three types of representative material were chosen and their properties measured and presented.

5.2 Particle Size and Distribution

Particle size and distribution are the most often used characteristics of a bulk material. However, it is often difficult to define particle size. For regular shaped particles such as spherical or cubic particles, see Fig. 5.1(a) and (b), the size can be defined easily as the largest linear dimension. For the spherical particle the size would be the diameter and for the cubic particle the corner-to-corner diagonal.
However, for irregular shaped particles, see Fig. 5.1(c), terms such as length, thickness and diameter have little meaning as many different values for each can be determined from each single particle. In an attempt to represent the size of an irregularly shaped particle by a single quantity, the term most often used is equivalent diameter. This refers to the diameter of a sphere that exhibits the same behaviour as the particle when subjected to the same sizing technique, eg the sphere that has the same projected area or mass or that just passes through a mesh aperture. Thus the measurement of the size (equivalent diameter) of particles is dependent on the method used to determine that parameter.

The particle size mentioned above actually indicates single particle size. The size and shape of particles that randomly make up a real bulk solid usually vary quite widely. In this case, a mean particle size is needed to represent the size nature of the bulk solid. Only after knowing the single particle size and distribution of a bulk solid, can the mean particle size (equivalent diameter) of the bulk solid be calculated by an appropriate method, such as the methods of arithmetic mean, geometric mean and log geometric mean, etc. Hence the size range (distribution) of the bulk solid also is an important parameter that defines the size nature of the bulk solid. There are many methods that can be used for determining the size distribution of particulate materials. These include:

- Mechanical sieving,
• Sedimentation,

• Optical methods (eg laser diffraction).

Mechanical sieving is the most widely used method for determining the size distribution of a bulk solid and is a process well known to most researchers and engineers, as it covers the range of particle sizes that are of considerable industrial importance. With this method, a bulk solid sample is placed on a nest of screens with precisely defined apertures. These sieves are either manually or mechanically shaken for a designated period of time, resulting in a proportion of granules being retained on each screen. The particle size and distribution, as measured by sieving, can be defined by quoting the aperture of the two screens, one through which the particles pass and the other on which they are retained.

![Particle size distribution graph](image)

**Fig. 5.2** Particle size distribution.

For applications, the most useful approach is to plot the data graphically, as shown in Fig. 5.2. This shows the particle size or equivalent diameter plotted against the mass...
percentage of the sample under a certain size. Such information gives an appreciation of the range of particle size constituting the bulk solid. A commonly used method for assigning a characteristic figure to this information is by quoting the median size. This is defined as the particle size which represents 50% of the sample by mass.

In the case of monosized or nearly monosized particles, mean equivalent spherical size by mass is often employed. For large size particles like polyethylene pellets, the mean equivalent spherical size can be determined by the following equation as the numbers of particles in a known mass can be counted,

\[ d_p = \sqrt[3]{\frac{6 m_p}{\pi \rho_s n_p}} \]  

where \( d_p \) is the mean equivalent particle diameter, \( m_p \) is the mass of particles, \( n_p \) is the number of the particles of the known mass, \( \rho_s \) is the particle density.

### 5.3 Density Analysis and Measurement

A bulk solid consists of many randomly grouped particles. This bulk material has an apparent bulk density, ie the mass of the bulk divided by the volume of the particles and voids contained in the bulk. Besides the bulk density, each particle that makes up the bulk solid has particle density. Generally speaking, the particle density is constant, the bulk density is not unique for a bulk material. It is dependent on the particle density, particle shape and how the particles are packed or positioned with respect to one another. The bulk density is a parameter of primary importance in the investigation of the screw feeder performance.

#### 5.3.1 Particle Density

Particle density can often be measured using an air comparison pycnometer or stereo pycnometer. In this study, a stereo pycnometer was used for most measurements of particle density. It employs Archimedes' principle of fluid displacement to determine
the volume of the solid objects. The displaced fluid is a gas which can penetrate the finest pores to assure maximum accuracy. A diagram displaying the principle of the stereo pycnometer is presented in Fig. 5.3.

![Schematic of stereo pycnometer](image)

**Fig. 5.3** Schematic of stereo pycnometer.

The device consists of two cells (i.e., the sealed sample cell and added cell) with volumes $V_c$ and $V_a$ connected through a selector valve. A pressure transducer is installed in the sample cell to allow accurate monitoring the system pressure. The basic operating procedures are stated below:

(i) Open the vent valve and selector valve to bring the system to ambient pressure, then close the selector valve carefully.

(ii) Place a known mass of bulk solid sample in the sample cell, then close the vent valve and seal the sample cell.
(iii) Open the gas flow valve and pressurise the sample cell to a designated pressure $p_2$ (e.g. 17 psig) above ambient.

(iv) Open the selector valve to connect the added cell with the sample cell, then the air in the sample cell flows into the added cell, the pressure will fall to a lower value $p_3$.

(v) Calculate the particle volume according to the following equation

\[ V_p = V_c + \frac{V_a}{1 - \frac{p_2}{p_3}} \]  

(vi) Determine the particle density by knowing the mass and the particle volume.

It should be noted that both the air comparison- and stereo-pycnometer only measure the average particle density of a bulk solid. The densities of different constituent particles in a blended product can be determined by measuring them before mixing. Also, the pycnometer yields the apparent particle density which is the mass of product divided by the occupied volume including closed pores but excluding open pores.

5.3.2 Bulk Density

Bulk density does not have a unique value for a particular bulk solid and it varies with the condition of the bulk solid. It is not always easy to determine the bulk density of a bulk solid under changing consolidation conditions. Only the loose-poured bulk density is discussed here.

Loose-poured bulk density usually can be obtained by the following steps:

(i) Pour carefully and gently a certain volume of bulk solid into a measuring cylinder. Note that the measuring cylinder must be held at an angle of 45° to the horizontal when pouring to avoid compaction.

(ii) Bring the cylinder upright and note the volume occupied by the bulk solid.
(iii) Weigh the cylinder and bulk solid and deduce the mass of the bulk solid.

(iv) Determine the loose-poured bulk density by knowing the mass and the poured volume of the bulk solid.

5.3.3 Bulk Voidage

The space occupied by a bulk material is not completely filled by the particles that make up the bulk material. Part of the space is filled by voids. The volume ratio of the total voids to the bulk material is defined as the bulk voidage of the material. The bulk voidage can be calculated theoretically by using geometry for a bulk material which only consists of mono-sized spherical particles. However, due to the different arrangements of the particles, the bulk voidage can vary from 0.26 to 0.47 [11] even though the particle size does not change, as shown in Fig. 5.4. Also for multi-sized particles, many other factors such as size, shape and distribution of particles and degree of consolidation affect the value of bulk voidage. Hence, it is virtually impossible to calculate bulk voidage directly from geometry.

Fig. 5.4 Different arrangements of particles [11].

However, bulk voidage has the following relationship with particle density and bulk density
\[ \varepsilon = 1 - \frac{\rho_b}{\rho_s} \]  
\[(5.3)\]

Hence the bulk voidage of a bulk solid is often calculated from the above equation after measuring the bulk density and particle density of the bulk solid.

### 5.4 Flow Properties of Bulk Solids

The flow properties of the bulk solids mainly include internal and effective friction angle and wall friction angle, which are of special interest to the screw feeder performance.

#### 5.4.1 Internal and Effective Friction Angle

Fig. 5.5 shows the cross-section of a Jenike shear cell [32]. The shear cell is composed of a base which is located on a frame, a shearing ring which rests on the top of the base, and a cover which has a loading bracket attached to it. The test apparatus encourages failure to develop only along the shear plane.

![Fig. 5.5 Jenike shear cell [32].](image-url)
A bulk solid is placed in the shear cell and loaded by a normal force \( V \). After applying a shearing force \( S \) through the shearing ring, the normal stress \( \sigma (= V/A_c) \) and shearing stress \( \tau (=S/A_c) \) are assumed to occur along the shear plane, where \( A_c \) is the cross-sectional area of the shear cell. If \( S \) is less than the maximum possible shearing force corresponding to a given normal force \( V \), then no continuous deformation occurs. However, slip takes place along the shear plane as soon as \( S \) reaches its maximum value. If a \( \sigma_n-\tau_n \) coordinate system is introduced to generate a shear-compressive stress diagram, the stresses on the shear plane at shearing can be represented by a Mohr circle with centre \( C \), as shown in Fig. 5.6.

\[
\begin{align*}
\tau_n & \quad \sigma_n \\
\phi & \quad \delta \\
O & \quad \sigma_2 \\
& \quad C \\
& \quad \sigma_1 \\
\end{align*}
\]

Fig. 5.6 Mohr circle and yield locus of cohesive material.

\( \sigma_1 \) and \( \sigma_2 \) are major and minor principal stress. If repeating the direct shear test for the sample with the same initial void ratio (ie same consolidating pressures) but with a range of constant \( V \) values, a yield locus (YL) can be obtained, see Fig. 5.6. For the values of the normal stress and shearing stress \( (\sigma, \tau) \) lying below that line, the solid can be considered to behave rigidly (or elastically), for the values lying on the line, failure or yield occurs. The locus usually can be represented approximately by a straight line. If the slope and intercept of the line are designated as \( \phi \) and \( c_i \), respectively, then the locus in Fig. 5.6 may be written as
\[
\tau = \sigma \tan \phi + c_i
\]  

The form of Eq. (5.4) was given by Coulomb in 1776 and hence, is called the Coulomb failure criterion. \( \phi \) and \( c_i \) in Eq. (5.4) are called the kinematic angle of internal friction and the cohesion of the bulk solid.

As shown in Fig. 5.6, the tangent to the Mohr circle passing through the origin of the \( \sigma_n - \tau_n \) coordinate system is called the effective yield locus (EYL). The angle \( \delta \) between the effective yield locus and \( \sigma_n \)-coordinate in Fig. 5.6 is called the effective angle of internal friction. Using angle \( \delta \), the following relationship exists between major and minor principal stresses [3],

\[
\frac{\sigma_1}{\sigma_2} = \frac{1 + \sin \delta}{1 - \sin \delta}
\]

For cohesionless solids, such as dry sand and plastic pellets, the yield locus passes through the origin of the normal stress and shear stress axes. This yield locus coincides with the effective yield locus (EYL). The kinematic angle of internal friction \( \phi \) is equal to the effective angle of internal friction \( \delta \).

The internal and effective friction angles can be measured by a Jenike-type direct shear tester shown in Fig. 5.7. The tester includes a shear cell of circular shape, a gravity vertical loading system which applies a normal force on the top of the shear cell, a driven loading stem which moves horizontally and generates the shearing action. The shear force is measured by a load cell and recorded by a chart recorder.
The test procedure is summarised below:

(i) Turn on the Jenike shear tester and chart recorder, then calibrate the chart recorder.

(ii) Undertake preconsolidation of the sample ie

- Put the shear cell ring with a mould ring on the frame of the tester and fill with the sample (do not press down on the sample).

- Place the twisting top on the mould ring, apply weight $W$ on the twisting top by a weight hanger and then twist the top for a given number of times.

- Take the hanger with weight and mould ring off, scrape off the excess sample flush with the shear ring.

(iii) Undertake shear consolidation of the sample ie

- Place the cover of the shear cell on the top ring, add shear weight $V_b$ by the hanger and then turn on the loading stem drive motor.
• Determine the additional force $V_{ad}$, above the shear plane. $V_{ad}$ is the vertical force due to the shear lid, the shear ring and the mass of the bulk solid contained within the ring. $V = V_{b} + V_{ad}$.

• Check the chart recorded results, turn off the loading stem drive motor and retract the loading stem.

• Repeat from step (ii) onwards adjusting preconsolidation weights and number of twists until the correct result is obtained (chart record of the maximum shear force flattens out). Record the shear value $S$.

(iv) Shear to failure

• Take off the shear consolidation weight and put on shear-to-failure weight $V_{b'}$.

• Turn on the loading stem motor and shear the sample, noting the peak shear value $S'$ on the chart recorder.

• Determine the total normal force. $V' = V_{b'} + V_{ad}$.

• Check that the shear plane is acceptable (i.e., approximately ideal shear plane).

(v) Repeat the preconsolidation and shear consolidation steps. Shear the new samples to failure under total vertical loads $V''$ and $V'''$ respectively. Record the corresponding values of $S''$ and $S'''$.

Note: when determining a yield locus, the value of shear force $S$ obtained during the shear consolidation process will usually show some scatter. The allowable deviation of $S$ from the intermediate value should be less than $\pm 5\%$ and the following relationship should be used to adjust or prorate the raw test results.

$$S'_{pr} = S' \frac{S_{sel}}{S_{test}}$$  \hspace{1cm} (5.6)
The value of $S_{sel}$ should be an intermediate value within the range of scatter of the $S_{test}$ values; an average value of all the $S_{test}$ values may be satisfactory. As a guide, when prorating $S'$ values for a family of yield loci, the $S_{sel}$ value should lie close to a straight line which passes through or above the origin.

(iv) Draw a straight line passing through points $(V', S')$, $(V'', S'')$ and $(V''', S''')$ and a semi-Mohr circle tangential to the straight line and passing through the point $(V, S)$, as shown in Fig. 5.8. Normally, the values of the forces determined in the laboratory are converted to shears by dividing by the cross-sectional area of the shear cell.

![Fig. 5.8 Typical measured yield locus.](image)

This straight line is a yield locus for the bulk solid at a particular consolidation level and the slope of the yield locus defines the kinematic angle of internal friction $\phi$ (for the particular consolidation level). If the bulk solid is cohesionless, the line will pass through the origin of the normal force and shear force axes, the angle between yield locus and V axis also is the effective angle of internal friction $\delta$ (for the particular consolidation level). The Mohr circle defines the major and minor principal forces $V_1$ and $V_2$, see Fig. 5.8. For cohesive bulk solids it is necessary to determine a “family”
of yield loci at different consolidation levels. The values of $\phi$ and $\delta$ may vary somewhat with consolidation.

5.4.2 Wall Friction Angle

![Diagram of Wall Yield Locus Test](image)

**Fig. 5.9** Arrangement for wall yield locus test.

![Diagram of Wall Yield Locus](image)

**Fig. 5.10** Wall yield locus.

The Jenike Direct Shear Tester also is used to determine the WYL; the set-up is depicted in Fig. 5.9. When a normal force $V_b$ is applied, failure occurs on the boundary of the bulk solid and wall as soon as $S$ reaches a particular value. Repeat the
shearing test for the sample with a range of $V_b$ values. After converting the forces to shears, a wall yield locus $WYL$ can be obtained on the $\sigma_n\tau_n$ coordinate system. The typical form of the WYL is shown in Fig. 5.10. As shown, the angle $\phi_w$ is defined by the straight line from the origin to the point $(\sigma, \tau)$. In general the wall friction test is repeated once or twice or until consistent results are obtained.

The wall yield locus for semolina on the cold rolled smooth mild steel is shown in Fig. 5.11. The small increment (0.15369) on the shear force axis has been ignored and the slope of the fitted line has been taken as the wall friction angle $\phi_w$.

![Fig. 5.11 Wall yield locus for semolina](image)

**5.5 Test Bulk Solids**

Three types of bulk solid were chosen to examine the influence of bulk solid properties on the performance of screw feeders. The major properties of the bulk materials have been measured with the methods mentioned in the previous sections of this chapter. Table 5.1 lists the physical properties of these materials.
Table 5.1: Physical properties of test bulk solids.

<table>
<thead>
<tr>
<th>Bulk Solid</th>
<th>$d_p$ (mm)</th>
<th>$\rho_s$ (kgm$^{-3}$)</th>
<th>$\rho_b$ (kgm$^{-3}$)</th>
<th>$\varepsilon$</th>
<th>$\phi_w^*$ ($^\circ$)</th>
<th>$\delta$ ($^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>White Plastic Pellets</td>
<td>3.66</td>
<td>893</td>
<td>530</td>
<td>0.406</td>
<td>17.0</td>
<td>44.5</td>
</tr>
<tr>
<td>Semolina</td>
<td>0.39</td>
<td>1459</td>
<td>736</td>
<td>0.496</td>
<td>27.5</td>
<td>31.0</td>
</tr>
<tr>
<td>Cement</td>
<td>0.028</td>
<td>2812</td>
<td>791</td>
<td>0.719</td>
<td>27.0</td>
<td>51.5</td>
</tr>
</tbody>
</table>

* Wall material was cold rolled smooth mild steel with RA = 0.55 $\mu$m.
EXPERIMENTAL INVESTIGATION INTO SINGLE SCREW FEEDERS

6.1 Introduction

In engineering practice, the determination of the volumetric efficiency, the torque or power requirement and the parameters that affect draw-down performance should be the first step in designing/selecting a screw feeder.

Despite some experimental work on screw feeders [8, 14, 15, 28, 29, 40, 44, 55, 63-65, 77, 81], to date the understanding of the influence of the screw geometry, the properties of the bulk solid and the operating conditions on the performance of screw feeders is far from adequate. The main reason is that there is a shortage of systematic experimental data relating to the volumetric efficiency, the torque or power requirement and the draw-down performance. As a result, the design of screw feeders still relies heavily on experience and trial-and-error, as mentioned in Chapter 1. Hence, it is desirable to investigate the effect of the screw geometry, the properties of the bulk solid and the operating conditions on the performance of screw feeders so that a design strategy can be formulated and options to improve their performance can be determined.

During the present research, a systematic experimental program has been designed and undertaken in the Bulk Solids Handling Laboratory at the University of Wollongong. The investigation was aimed at examining the factors that influence the performance of screw feeders. The results obtained from this work have been used to verify the theoretical models developed in Chapter 3.
6.2 Volume Withdrawn by Single Screw Feeders

6.2.1 Metering Characteristics

A ideal feeder should have a constant output per revolution. With such a system, output would be directly proportional to the speed of the screw. This is called the metering characteristic of a screw feeder.

Experiments on the metering characteristics were carried out on the test rig for single screw feeders depicted in Fig. 4.1. The No. 1 test screw was used for this test program. The length of the feed section was 550 mm, terminating in the region where the pitch length was 100 mm. Only one trough with the inside radius of 80 mm was used, giving the radial clearance of 5 mm. Three types of bulk material were used for the test program. Each material was loosely poured into the hopper to a desired level. Three different states of the bulk material in the hopper were examined: hopper filled from an empty state to a level of 600 mm (high initial); hopper filled from an empty state to a level of 300 mm (low initial); after some discharge the level of the bulk solid was reduced to 300 mm (low flow).

Experiments under the three different filling states and for the screw speed range from 20 to 80 rpm, were carried out. The experimental results for the metering characteristics did not show any obvious differences due to the filling states and were constant over the range of screw speeds. Figs. 6.1, 6.2 and 6.3 show the mass output versus the speed of rotation with three types of test bulk solids under three different states. In the legends H. I. is short for the high initial state, L. I. for the low initial state and L. F. for the low flow state.

It can be seen that a good linear relationship between the output and the speed of rotation was obtained for both the three loading states and the three bulk solids. The three different loading states had virtually no effect on the output of the test screw with white plastic pellets and semolina. Three lines representing the relationship between
the mass output and the rotating speed of the screw almost coincide. For cement there was a small slope deviation among the three lines. The main reason for this deviation is that cement is a fine bulk material which can have a high voidage variation. Its bulk density is sensitive to the consolidation stresses, which result from the filling state and the velocity of the movement, which is caused by the rotating speed of the screw. However, a reasonably good metering characteristic was still obtained.

![Graph showing mass output versus rotating speed with white plastic pellets](image)

Fig. 6.1 Mass output versus rotating speed with white plastic pellets

\[P = 100 \text{ mm, } c = 5 \text{ mm}\]
Fig. 6.2 Mass output versus rotating speed with semolina

\( (P = 100 \text{ mm}, c = 5 \text{ mm}) \)

Fig. 6.3 Mass output versus rotating speed with cement

\( (P = 100 \text{ mm}, c = 5 \text{ mm}) \)
6.2.2 Variation of Volumetric Efficiency with P/D

The experiments described in Section 6.2.1 under three different filling states and for the screw speed range from 20 to 80 rpm were carried out for the test screw with \( P = 100 \text{ mm} \) in the trough with clearance \( c = 5 \text{ mm} \). As discussed, the experimental results for the metering characteristics did not show any obvious differences due to the filling states and were constant over the range of screw speeds. Hence, the tests with the other pitches and clearances were conducted at 20 rpm only. Experiments for the variation of the volumetric efficiency with the ratio of the pitch to screw diameter were conducted with the No. 1 screw. The No. 1 screw had five different pitch lengths (\( P = 50, 75, 100, 125 \) and \( 150 \text{ mm} \)). By changing the length of the feed section, i.e., changing the position of the choke section, five different ratios of the pitch to screw diameter at the exit end of the feed section (0.33, 0.5, 0.67, 0.83 and 1) were generated for investigating the variation of the volumetric efficiency with P/D.

The experimental results of the volumetric efficiency were obtained by the following methods: for a known number of the screw revolutions \( N \), the mass output of the bulk solid \( M \) was measured by the shear-beam-type load cells, which supported the receiving and weighing silo. The observed volumetric output per revolution is

\[
V_o = \frac{M}{Np_b}
\]  

\( (6.1) \)

From Eq. (3.35)

\[
V_o = \pi \eta_v P \left[ R_o^2 - R_c^2 \right] + (1 - k) \left( 2cR_o + c^2 \right)
\]

\( (3.35 \text{ repeated}) \)

where \( k \) is the coefficient proposed to account for the possible dead layer of the bulk solid between the screw flights and the trough surface. The determination of \( k \) will be discussed further in Section 6.2.3.

The observed volumetric efficiency can be obtained from
\[ \eta_v = \frac{M}{\pi N \rho b P \left[ (R_o^2 - R_c^2) + (1 - k)(2cR_o + c^2) \right]} \]  

(6.2)

Figs. 6.4, 6.5 and 6.6 show the comparisons between measurements (plots) and theoretical predictions (lines) for the volumetric efficiency varying with the ratios of P/D and the clearance between the screw flight and the trough. It will be noted that the experimental results have no regular deviations from the theoretical predictions with the three test materials, although, generally, there is good consistency between the theoretical predictions and the experimental results. For white plastic pellets the two largest P/D ratios gave good consistency for three different clearances, while for semolina the three smallest P/D ratios give good consistency for three different clearances. For cement the three smallest P/D ratios give good consistency for the smallest clearance, while the three largest P/D ratios give good consistency for the two larger clearances.

![Graph showing the effect of P/D on volumetric efficiency for white plastic pellets, semolina, and cement.](image)

**Fig. 6.4** Effect of P/D on volumetric efficiency  
(c = 5 mm, \( n_m = 20 \text{ rpm} \))
Fig. 6.5 Effect of P/D on volumetric efficiency
(c = 10 mm, n_m = 20 rpm)

Fig. 6.6 Effect of P/D on volumetric efficiency
(c = 20 mm, n_m = 20 rpm)
6.2.3 Effect of Clearance on Output and Volumetric Efficiency

As mentioned in Section 3.2.6, the clearance between the trough and the tips of the flight in the choke section is an important constructional feature affecting the output of a screw feeder. To consider such an effect, an effective area is introduced

\[ A_e = \pi[(R_o^2 - c^2) + (1 - k)(2cR_o + c^2)] \]  

(3.34 repeated)

where a coefficient \( k \) is proposed to account for the possible dead layer of the bulk solid between the screw flights and the trough surface. In this study \( k \) is obtained by experiment. In practice, it could be estimated by experience.

![Diagram of screw feeder components](image)

**Fig. 6.7** A cross-section of dead layer of bulk solid in choke section

A cross-section of the choke section is shown in Fig. 6.7. The annular area between the screw flights and the casing is

\[ A_a = \pi(R_i^2 - R_o^2) \]  

(6.3)

The area of the dead layer is

\[ A_d = \frac{1}{2} \alpha_d (R_i^2 - R_o^2) \]  

(6.4)
where \( \alpha_d \) is the arc subtended by the dead layer of the bulk solid in the clearance (radians). The coefficient \( k \) can be calculated from

\[
k = \frac{A_d}{A_a} = \frac{\alpha_d}{2\pi}
\]  

(6.5)

Figs. 6.8 to 6.13 show the effect of the clearance between the trough and the tips of the flight in the choke section on the volumetric output and efficiency with the three bulk materials. Comparisons between the theoretical prediction (lines) and the results from experiments (plots) are also shown in these figures. For white plastic pellets and semolina the volumetric output increases with the increase in the clearance. Although some deviations can be seen in the volumetric output and the volumetric efficiency between the theoretical calculations and measurements, the same trends are evident for the three different clearances. Thus, the coefficient, \( k = 0 \) can be chosen for full wall slip between the screw flights and the trough surface for these two bulk materials. For cement, however, output is actually independent of clearance. The reason for this is that the arc length (\( \alpha_d \)) and the thickness of the dead layer between the screw flights and the trough wall increases with an increase in the radial clearance. The different values of the coefficient \( k \) should be chosen to account for the different dead layers of cement. The coefficient \( k \) for cement was obtained by taking off the upper part of the trough in the choke section and taking measurements of \( \alpha_d \) for the dead layer between the screw flights and the trough wall.
Fig. 6.8 Effect of c on $V_o$ with white plastic pellets ($k = 0$)

Fig. 6.9 Effect of c on $\eta_v$ with white plastic pellets
Fig. 6.10 Effect of c on $V_0$, with semolina ($k = 0$)

Fig. 6.11 Effect of c on $\eta_v$, with semolina
3.0
2.0
1.0
0.0

Output $V_o$ ($10^{-3}$ m$^3$)

$k=0.68$ (c=20 mm)

$k=0.45$ (c=10 mm)

$k=0$ (c=5 mm)

A c  = 20 mm

O c  = 10 mm

• c  = 5 mm

0.2 0.4 0.6 0.8 1.0 1.2

Ratio P/D

Fig. 6.12 Effect of c on $V_o$ with cement

1.0
0.8
0.6
0.4
0.2
0.0

Volumetric efficiency $\eta_v$

$\triangle$ c = 20 mm

$\bigcirc$ c = 10 mm

$\square$ c = 5 mm

0.2 0.4 0.6 0.8 1.0 1.2

Ratio P/D

Fig. 6.13 Effect of c on $\eta_v$ with cement
6.3 Draw-Down Performance

6.3.1 Method of Measurement

The experimental program concerned with draw-down performance of single screw feeders was undertaken using the four screws shown in Fig. 4.7 and the trough with clearance $c = 5$ mm. A dividing grid was fitted into the hopper and over the screw to form a central division above the axis of the screw and isolate each side into a number of divisions which matched the different pitches of each screw. Because it is difficult to take measurements on the surface of cement, only white plastic pellets and semolina were used in the experiments. The bulk solid was loosely poured into the hopper. After several revolutions of the screw the hopper was refilled and trimmed level to secure consistent conditions in the bulk material. Measurements of fall of level were then taken at each division after each two successive rotations of the screw. Preliminary trials confirmed that the flow pattern and total quantity withdrawn did not show a significant variation over a speed range from 10 to 80 rpm. Tests reported here were carried out at 20 rpm.

The observed volume withdrawn per revolution along the screw was also obtained by this experimental program. Some small variations existed in the measurements, so average values of the volume withdrawn per revolution are presented.

6.3.2 Experimental Results for Draw-Down Performance

The volume withdrawn was different between the rising side and the descending side of the flight. A detailed trial was undertaken to show the variable nature of the withdrawn volume on each side of the screw during 360-degree of rotation. It was clearly observed that in the first pitch the volume withdrawn on the rising side was larger than that withdrawn on the descending side. This observation was previously described by Bates [8]. But for the test materials the situations were different in subsequent pitches. In the downstream pitches the volume withdrawn on the
descending side was larger than that withdrawn on the rising side for all four screws, with the exception of the last or exit pitch. The reason for this case may be that the bulk solid in the last pitch (including the bulk solid in the clearance between the screw flight and trough surface) is pushed into the choke section rather than sheared off on the upper shear surface in the next pitch.

Experimental results for white plastic pellets and semolina with the four screws are shown in Figs 6.14 to 6.29. For each screw and each bulk material both the drawdown performance and the volume withdrawn per revolution (average values of both side of the screw) along the screw are given in the figures.

The theoretical predictions represented by lines in the figures for the volume withdrawn per revolution along the screw are calculated from Eq. (3.35) for the last pitch in the feed section and Eq. (3.31) for the other pitches.

\[
V_{con} = \pi \eta_v P \left(R_o^2 - R_c^2\right) \quad \text{(3.31 repeated)}
\]

\[
V_o = \pi \eta_v P \left(R_o^2 - R_c^2\right) + (1 - k) \left(2cR_o + c^2\right) \quad \text{(3.35 repeated)}
\]

In Eqs. (3.31) and (3.35) the volumetric efficiency \( \eta_v \) is calculated based on the theoretical model described in Chapter 3.

Figs. 6.14 to 6.17 show the draw-down performance and the volume withdrawn per revolution along No. 1 screw feeder. No. 1 screw had pitches stepped in groups of three equal pitches (Fig. 4.7). It can be seen that there are large fluctuations in the volume withdrawn in this screw. It can also be clearly seen in these figures that the deviations of the volume withdrawn between measurements and theoretical predictions are greater. If the length of a pitch is equal to the length of the downstream pitch, theoretically speaking, there is no change in the transported volume in the downstream pitch. However, the evidence from experiments indicates some increment. One main reason for this is that some cross-flow occurred between the dividing plates because there was a gap between the bottom of the grid and the top of the screw flights.
Another reason is that higher pressures tends to form in the trough in the downstream pitches of equal length. These higher pressures may cause some bulk solid in the clearance volume of the feed section to be withdrawn. Probably this effect is amplified by the relatively large particle size of the plastic pellets. The higher pressure may also result in an increase in the bulk density of the withdrawn solids.

![Fig. 6.14 Draw-down performance with No. 1 screw (white plastic pellets)](image)

![Fig. 6.15 Volume withdrawn per revolution along No. 1 screw (white plastic pellets)](image)
Fig. 6.16 Draw-down performance with No. 1 screw (semolina)

Fig. 6.17 Volume withdrawn per revolution along No. 1 screw (semolina)
The draw-down performance and the volume withdrawn per revolution by No. 2 screw feeder are shown in Figs. 6.18 to 6.21. No. 2 screw had both a tapered core shaft and stepped pitches (Fig. 4.7). It can be seen from Figs. 6.18 to 6.21 that No. 2 screw gave better draw-down performance because of the ratios \( \frac{d_1}{D} = 0.87 \) and \( \frac{P_1}{D} = 0.33 \) which limited the volume withdrawn in the first pitch and increased the volume withdrawn in downstream pitches.

![Graph showing draw-down performance with No. 2 screw](white plastic pellets)
Fig. 6.19 Volume withdrawn per revolution along No. 2 screw (white plastic pellets)

Fig. 6.20 Draw-down performance with No. 2 screw (semolina)
Fig. 6.21 Volume withdrawn per revolution along No. 2 screw
(semolina)

Although No. 3 screw had variable width ribbon flights and stepped pitches, for the
tested materials, the ribbon flights seem not to have had the desired effect on
increasing the withdrawn volume in the feed direction. Figs. 6.22 to 6.25 show the
draw-down performance and the volume withdrawn per revolution along the No. 3
screw. For the volume withdrawn per revolution along the screw, the experimental
results are compared with the calculated values based on a screw with full flights and
stepped pitch. It can be observed that even for the relatively fine semolina the flow
pattern produced by a screw with ribbon flights was close to that with full flights.
Chapter 6  Experimental Investigation into Single Screw Feeders

Fig. 6.22 Draw-down performance with No. 3 screw
(white plastic pellets)

Fig. 6.23 Volume withdrawn per revolution along No. 3 screw
(white plastic pellets)
Fig. 6.24 Draw-down performance with No. 3 screw (semolina)

Fig. 6.25 Volume withdrawn per revolution along No. 3 screw (semolina)
No. 4 screw gave a good comparison of the effect on flow pattern between a screw with stepped pitch and a screw with stepped shaft diameter. Figs. 6.26 to 6.29 show the draw-down performance and the volume withdrawn per revolution by the No. 4 screw feeder. No. 4 screw had only stepped pitches in the first four pitches: 50, 75, 100 and 125 mm respectively (Fig. 4.7). The pitch ratios (ratio of length of downstream pitch to upstream pitch) are 1.5, 1.33 and 1.25. For the two test materials, it can be observed from Figs. 6.26 to 6.29 that the stepped pitch, as a method of increasing screw capacity, is less effective for obtaining an even flow pattern. This is because the possible range of the pitch length is limited. In general, the ratio of pitch to screw diameter is limited from about 1/3 to 1 for most bulk solids. Due to the limitation of the pitch length, it is impossible to obtain large pitch ratios along the whole feed length, especially for long screw feeders. For small pitch ratios, the effectiveness of increasing screw capacity is very limited. Thus, it is quite evident that if a screw with only stepped pitches is to provide even draw-down performance for some bulk solids, then other factors, such as cross flow of bulk solids in the bin or hopper, must play very important roles. It can also be seen from Figs. 6.26 to 6.29 that, the volume withdrawn in the fifth pitch, in which the core shaft diameter was 89 mm (stepped down from 114 mm in the first four pitches), increased significantly when compared with the upstream pitch. This indicates that, compared with stepped pitches, stepping the shaft can provide a more significant increment in withdrawn volume.
Chapter 6  
Experimental Investigation into Single Screw Feeders

Fig. 6.26 Draw-down performance with No. 4 screw  
(white plastic pellets)

Fig. 6.27 Volume withdrawn per revolution along No. 4 screw  
(white plastic pellets)
Fig. 6.28 Draw-down performance with No. 4 screw (semolina)

Fig. 6.29 Volume withdrawn per revolution along No. 4 screw (semolina)
6.3.3 Comparisons of Profile Coefficient

In Section 3.4.2 a profile coefficient was introduced to allow the withdrawal performance of a particular screw to be compared to the ideal

\[ f_{pi} = \frac{(A_{ai} - A_{ai-1})}{P_{i}} / A_{a1} \]  

(3.124 repeated)

\( f_p = 1 \) for ideal uniform withdrawal performance and \( f_p = 0 \) for no volumetric increment in the pitch.

Figs. 6.30 to 6.33 show comparisons of the profile coefficients for the four screws between the results from experiments (plots) and theoretical predictions (lines). Experimental results for the profile coefficients were obtained by taking measurements of the falling level of the individual pitches. In actual, the measured value of the falling level multiplied by the outlet width represented the net increment in the average effective area in this pitch \( (A_{ai} - A_{ai-1} P_{i-1}) / P_{i} \). Thus, the ratio of the value of the falling level in a pitch to that in the first pitch is the profile coefficient for this pitch. In Figs 6.30 to 6.33 the profile coefficients for the individual pitches were obtained from the average values for all measurements.

For No. 3 screw the comparison was made with a screw having full flights and the same stepped pitches, as the experiments suggested that the variable width ribbon flight did not have a strong influence on decreasing the effective cross sectional area of the screw.

Although cross-flow may have occurred between the dividing plates and this could modify the hopper flow pattern, the theoretical predictions still attained reasonable consistency with the measurements.
Fig. 6.30 Comparison of profile coefficient for No. 1 screw

Fig. 6.31 Comparison of profile coefficient for No. 2 screw

Fig. 6.32 Comparison of profile coefficient for No. 3 screw
6.3.4 Influence of Cross-Flow

Experiments without the grid in the hopper were also performed. These experiments were conducted under the same conditions as those using a dividing grid, i.e., the screw was rotating at the speed of 20 rpm and the trough had a 5 mm radial clearance between the screw flights and trough surface. The bulk solid was loosely poured into the hopper. After several revolutions of the screw, the hopper was refilled and trimmed level to secure consistent conditions in the bulk solid. Measurements of fall of level were then taken after each two successive rotations of the screw at the same positions as those taken with the grid.

Flow patterns developed with four screws and two different of bulk solids are shown in Figs. 6.34 to 6.41. Compared to the draw-down performance with the grid over the screws, it can be seen that the profiles of bulk material in the hopper were modified significantly. For No. 1 screw, the fluctuations in profile indicated in Figs. 6.14 and 6.16 have been eliminated. Almost completely uniform draw-down performances were achieved for No. 2 and No. 4 screws. No. 3 screw also gave much better flow patterns in the hopper.
Without a grid the flow of the bulk solid in the hopper is not limited by the dividing plates. Due to the differences in the capacity of various pitches along the whole feed section, pitches with higher capacity (or higher increase in the capacity in the feed direction) will withdraw mainly from the bulk solid directly above them and partly from the bulk solid above adjacent pitches. Hence, some cross-flow of the bulk solid in the hopper occurs. Results from experiments without a dividing grid indicate that the cross-flow of the bulk solid in a bin or hopper can substantially modify the actual flow patterns obtained when using a dividing grid. This is considered an interesting result which is worth further investigation, but has been regarded as being beyond the scope of this study.

Fig. 6.34 Flow pattern without grid for No. 1 screw
(white plastic pellets)
Fig. 6.35  Flow pattern without grid for No. 1 screw
(semolina)

Fig. 6.36  Flow pattern without grid for No. 2 screw
(white plastic pellets)
Fig. 6.37 Flow pattern without grid for No. 2 screw
(semolina)

Fig. 6.38 Flow pattern without grid for No. 3 screw
(white plastic pellets)
Fig. 6.39 Flow pattern without grid for No. 3 screw
(semolina)

Fig. 6.40 Flow pattern without grid for No. 4 screw
(white plastic pellets)
6.4 Torque Requirements

6.4.1 Influence of Operating Conditions

Detailed experiments under two initial filling levels in the hopper and for No 1 screw with a speed range from 20 to 80 rpm, were carried out using a trough with clearance $c = 5\, \text{mm}$. Experimental results showed only small differences due to the filling levels and screw speeds for the test materials. Torque requirements versus the speed of rotation for cement and semolina are shown in Figs. 6.42 and 6.43. In the legends L.I. is short for low initial fill and H.I. for high initial fill. For cement the average values of the torque for the high initial fill state was 3.3\% higher than that for the low initial fill state; for semolina the equivalent torque was 3.9\% higher. Compared to the increase in the filling level, the increases in the torque requirement were very low.
Fig. 6.42 Torque versus speed of rotation for No. 1 single screw feeder with cement

Fig. 6.43 Torque versus speed of rotation for No. 1 single screw feeder with semolina
6.4.2 Comparison Between Theoretical Predictions and Experimental Results

Figs. 6.44 and 6.45 give a comparison between the theoretical predictions of the torque requirements and the experimental results for No. 2 and No. 4 screws with three troughs. In the legends C is short for cement and S for semolina, while 5, 10 and 20 are the three radial clearances. It can be seen that the theoretical predictions are reasonably consistent with the experimental data, except for the screw operating in the largest trough \((R_t = 95 \text{ mm})\) with semolina, where the calculated values are lower than the observed results.

Semolina is a very free flowing material. The output of the screw feeder with semolina increases with increasing radial clearance (as indicated previously in Fig. 6.10). However, the torque requirement does not show the same pattern. It can be observed from the experimental results that for both single screw feeders the torque required for the middle trough \((R_t = 85 \text{ mm, } c = 10 \text{ mm})\) is less than that for the other two troughs. There are two reasons for this situation. Due to the bigger shaft diameters of the screws, the small clearance \((c = 5 \text{ mm})\) would result in an increase in the pressure in the lower region (as defined in Fig. 3.6), thereby increasing the torque requirement. On the other hand, the large radial clearance \((c = 20 \text{ mm})\) may result in an additional resisting force due to the difficulty of the larger mass of bulk solids (assumed to be sliding on the trough surface in the lower region) trying to "enter" the region of the shear surface above the screw.
Fig. 6.44 Comparison of torques for No. 2 single screw feeder

Fig. 6.45 Comparison of torques for No. 4 single screw feeder
6.4.3 Power Efficiency

As mentioned in Section 3.3.8, the power efficiency of a screw feeder is defined as the ratio of the power required to transport the bulk solid in the axial direction to the power needed for turning the screw in the feed section. Figs. 6.46 and 6.47 give the comparison between the calculated and the observed power efficiency for No. 2 and No. 4 screw feeders. It can be seen that the power efficiency for both screw feeders is very low. The range of values of the power efficiency for No. 2 screw feeder (using the screw with a tapered shaft and stepped pitches) was from 2 to 3.5 % with the two test bulk solids, while the power efficiency was 1.5 to 3% for No. 4 screw feeder (using the screw with a stepped shaft and stepped pitches).

Fig. 6.46 Comparison of power efficiency for No. 2 single screw feeder
6.4.4 Power Calculation

A simplified method for the calculation of the power requirement of a screw feeder is introduced in Section 3.3.9. This approximation is based on the assumption that the power efficiency for a screw feeder is proportional to the ratio of the last pitch length to the sum of the length of the feed section and the length of the choke section, i.e.

\[ \eta_{ps} = \eta_{ps} \frac{P_L}{(L + L_c)} \]  
(3.113 repeated)

The power requirement for a screw feeder can be obtained from

\[ P_f = \eta_{ps} \frac{n\gamma V_o P_L}{\eta_{ps}} = \eta_{ps} \frac{n\gamma V_o (L + L_c)}{\eta_{ps}} \]  
(3.114 repeated)

As previous experimental results did not show any obvious differences in torque requirement due to the filling states and screw speeds for the test materials, the tests for the power requirement with two different bulk solids and the three troughs were carried out at 60 rpm, giving one revolution per second.
The comparisons of the power requirements between the values calculated by this simplified method and the observed values for No. 2 and No. 4 screw feeders using the three troughs are shown in Figs. 6.48 and 6.49. It can be seen that the calculated values based on the simplified method are higher than the experimental values for cement and for semolina using troughs with the clearance $c = 5\,\text{mm}$ and $c = 10\,\text{mm}$.

In the choke section the casing is cylindrical and has the same radial clearance as the lower part of the trough in the feed section. The shear surface which occurs in the feed section does not exist but is replaced by a cylindrical sliding surface. From the analyses in Section 3.3.7.1 the torque required for a screw feeder is determined mainly by the resisting torque acting on the shear surface, which contributes 50% of the total torque. Using the power efficiency for the last pitch in the feed section to approximate the power efficiency for both the feed section and the choke section will result in a higher estimate of the total power requirement.

For semolina, a very free flowing bulk solid, the large radial clearance ($c = 20\,\text{mm}$) may result in an additional resisting force due to the difficulty of transporting the larger mass of bulk solids. This case is explained in Section 6.4.2. Even for the screw feeders using the trough with the clearance $c = 20\,\text{mm}$, the calculated values are very close to the observed values. It is expected that the simplified approach to the calculation of the power requirement will provide an estimate that is reasonably safe. This method also leads to a much easier procedure for the power calculation.
Fig. 6.48 Comparison of power requirements for No. 2 single screw feeder

\[ (n_m = 60 \text{ rpm}) \]

Fig. 6.49 Comparison of power requirements for No. 4 single screw feeder

\[ (n_m = 60 \text{ rpm}) \]
7.1 Introduction

Compared with other mechanical feeding devices, one obvious advantage of the screw feeder is its great flexibility of design. Bates [9, 10] gave a very detailed description of screw feeders in a variety of applications: increasing conveying section to extend feeder discharge location, alternating outlet positions, inclination upwards or downwards, etc. The design flexibility of screw feeders allows them to be used in situations where no other feeder can be considered. One aspect of this flexibility is multiple screw applications, by which two or more screws arranged side by side in a feeder, can service a larger slot width to meet the design requirement of a hopper.

Although some experimental work on screw feeders is found in the literature [8, 14, 15, 28, 29, 40, 44, 55, 63-65, 77, 81], these studies have focused attention on the performance of single screw feeders. The performance of multi screw feeders does not appear to have received extensive investigation, and no papers on this topic have been found. Therefore, it was considered important that some data be obtained on the influence of the properties of the bulk solid, direction of rotation of screws and screw geometry on the performance characteristics. Such data would allow strategies to improve performance characteristics of multi-screw feeders to be determined.

In this study experimental observations of the performance of counter rotating twin screw feeders have been made for different screw configurations, bulk solids and operating conditions. Based on these results, the influences of the constructional features of the
screw and the properties of the bulk solid on the volumetric output and the torque requirement are reported. Theoretical predictions based on single screw feeders are compared with the experimental results.

7.2 Volumetric Output

7.2.1 Volumetric Efficiency and Output

The volumetric efficiency for a twin screw feeder can be chosen in the same manner as that for a single screw feeder. However, for the volumetric output of a twin screw feeder, the geometry of the trough at the choke section should be taken into consideration. Similar to single screw feeders, the clearance between the trough and the tips of the flight in the choke section is also an important constructional feature affecting the output of a twin screw feeder. In the choke section the predicted feeder capacity increases with an increase in the "effective" area. For a twin screw feeder, the effective area is defined as:

$$A_e = 2\pi(R_o^2 - R_c^2) + (1-k)[\pi(4c + 4cR_o - R_o^2) + 2(2c + R_o)L]$$  \hspace{1cm} (7.1)$$

The coefficient $k$ ($k = 0 \sim 1$) is introduced to account for the possible dead layer of the bulk material between the screw flights and the trough wall. The output per revolution of a twin screw feeder can be expressed as

$$V_o = \eta_n P_L \left[2\pi(R_o^2 - R_c^2) + (1-k)[\pi(4c + 4cR_o - R_o^2) + 2(2c + R_o)L]\right]$$ \hspace{1cm} (7.2)$$

Experiments were conducted on the test rig for the twin screw feeder depicted in Fig. 4.2. Two sets of screws (one set of screws is the No. 2 screw with a tapered core shaft and stepped pitches and the other is the No. 4 screw with a stepped core shaft and stepped pitches, as shown in Fig. 4.9), two troughs with the same inside radius but different centre distances (as shown in Fig. 4.4) and three different bulk solids were employed in the experimental program. Experiments were conducted to examine the
influence of the rotational directions of the screws on the volumetric output of the twin screw feeder. Experiments were carried out at a speed of 20 rpm.

Figs. 7.1 and 7.2 give comparisons of the volumetric output between the theoretical calculations and the experimental results. In the legends W is short for white plastic pellets, S for semolina and C for cement; Up is short for the screws rotating upward near the trough wall and Down for the screws rotating downward near the trough wall. In the figures the smaller symbols represent the values for the operating condition using the smaller screw centre distance \( (l_t = 10 \text{ mm}) \) and the bigger symbols are for the larger screw centre distance \( (l_t = 25 \text{ mm}) \).

It can be seen from both figures that the theoretical predictions are reasonably consistent with the experimental results. Thus, for a twin screw feeder it appears reasonable to choose the volumetric efficiency calculated on the basis of a single screw feeder with the same geometry. Experimental results indicate that for white plastic pellets and semolina the volumetric output is dependent on both the last pitch in the feed section and the geometry of the trough in the choke section; \( k = 0 \) again has been chosen. This situation is consistent with that for the single screw feeders discussed in Chapter 6. Thus, the output of a twin screw feeder with a larger screw centre distance is higher, as indicated by the bigger symbols in both figures. For cement the coefficient \( k \), accounting for the dead layer between the screw flight and the trough, is chosen to be the same as that for single screw feeders with the same clearance, \( k = 0.45 \) for \( c = 10 \text{ mm} \).

The experimental results for the twin screw feeder indicate that the volumetric output is influenced by the rotating directions of the screws. For the twin screw feeder with No. 2 screws the volumetric output when the screws were rotating in the downward direction was, on average, 5.6% higher than when the screws were rotating in the upward direction; for the twin screw feeder with No. 4 screws an average 5.4% increment was observed for the same conditions.
Fig. 7.1 Comparison of volumetric output per revolution
for No. 2 twin screw feeder

Fig. 7.2 Comparison of volumetric output per revolution
for No. 4 twin screw feeder
7.2.2 Volume Withdrawn by Each Screw

Experiments on the volume withdrawn by each screw (left or right screw), when the screws were rotating in the upward or downward direction, were also carried out at a speed of 20 rpm on the test rig for the twin screw feeder. The trough with a smaller centre distance ($l_r = 160$ mm) was used in this experimental program. To obtain the values of the volume withdrawn by each pitch of the screws in the feed section, a dividing grid (as shown in Fig. 4.6) was put into the hopper. This grid with a centre dividing plate between the two screws and radial dividing plates matched the different pitch lengths. Only white plastic pellets were used in the test. White plastic pellets were loosely poured into the hopper. After several revolutions of the screws the hopper was refilled and trimmed level to secure consistent conditions in the bulk solid. Similar to the experimental program for the single screw feeder, measurements of fall of level were taken at each division after each two successive rotations of the screws.

Experimental results for the volume withdrawn per revolution by each screw (average of 10 repeat runs) are presented in Figs. 7.3 to 7.6. For both twin screw feeders the differences in the withdrawn volume between the left screw and the right screw in the upward direction were larger than they were when rotating in the downward direction. It can be said that the bulk solid was withdrawn more evenly when the screws were rotating in the downward direction than in the upward direction. When the screws were rotating in the downward direction, the bulk solid was advancing mainly on the trough surface and not interacting on the shear surface formed between the two screws. This could be the reason why the bulk solid was withdrawn more evenly when the screws were rotating in the downward direction.
Fig. 7.3 Volume withdrawn per revolution by each pitch of No. 2 twin screw (Upward)

Fig. 7.4 Volume withdrawn per revolution by each pitch of No. 2 twin screw (Downward)
Fig. 7.5 Volume withdrawn per revolution by each pitch of No. 4 twin screw (Upward)

Fig. 7.6 Volume withdrawn per revolution by each pitch of No. 4 twin screw (Downward)
7.2.3 Metering Characteristics

The tests for the metering characteristics of a twin screw feeder were conducted in a speed range from 10 to 80 rpm. Two sets of twin screw feeders, one used the No. 2 screw and the other used the No. 4 screw, were used to examine the influence of the rotating speed and direction of the screws on the metering characteristics of a twin screw feeder. Two bulk solids (white plastic pellets and semolina) and the trough with the smaller centre distance \( l_t = 160 \text{ mm}, c = 10 \text{ mm} \) were used in the experiments. The length of the feed section was 700 mm, providing an exit pitch with a length of 125 mm.

Figs. 7.7 and 7.8 show the mass output versus rotating speed for No. 2 twin screw feeder with semolina and for No. 4 twin screw feeder with white plastic pellets. The mass output was directly proportional to the speed of rotation and a good linear relationship was obtained. That means that in the range of the test speeds an excellent metering characteristic was observed. Also it can be seen that for both twin screw feeders and both bulk solids, the mass output when the screws were rotating in the downward direction was higher than when rotating in the upward direction. The increase in the output rates over the range of the test speeds and the average increase in the output rates are presented in Fig. 7.9. The average value of the increase rate is 5.4% for No. 2 twin screw feeder and 6.1% for No. 4 twin screw feeder, respectively.

When the screws were rotating in the upward direction, the bulk solid was pushed toward the middle between the two screws, where the shear surface between the bulk solid and the screws developed. Part of the bulk solid between the two screws would be sheared off or delayed behind the screw flights. On the other hand, when the screws were rotating in the downward direction, the bulk solid was pushed toward the trough walls, where the bulk solid was sliding on the trough wall surface. Thus, the output of the twin screw feeder when the screws were rotating in the upward direction was lower than when rotating in the downward direction.
Fig. 7.9 also shows the different trends for two bulk solids over the range of the test speeds. It is known from Table 5.1 that the mean equivalent particle diameter is 3.66 mm for white plastic pellets and 0.39 mm for semolina. Thus, over the lower range of the test speeds (10~20 rpm) the larger particle size of white plastic pellets had a stronger influence on the output of the twin screw feeder when the screws were rotating in the upward direction, because of the shear occurring between the bulk solid and the two screws. Semolina is a relatively fine bulk solid. The particle size had no obvious influence in the range of test speeds. The differences in the increased output rates over the range of test speeds were also relatively smaller.

![Graph showing mass output versus rotating speed with No. 2 twin screw](Semolina)
Fig. 7.8 Mass output versus rotating speed with No. 4 twin screw
(White plastic pellets)

Fig. 7.9 Increase in output rate of screws rotating in downward direction versus rotating speed
7.3 Draw-Down Performance

Only white plastic pellets were used in the experimental program to assess the draw-down performance of the two twin screw feeders. For each twin screw feeder experiments were carried out for both directions of rotation to monitor the influence of the direction of rotation on the performance. The trough with a smaller centre distance \((l_t = 160 \text{ mm})\) was employed in the tests. A dividing grid (as shown in Fig. 4.6) was put into the hopper to measure the fall level of the bulk solid in each division after each two successive rotations of the screws. As mentioned in Section 7.2.2, experimental results indicated that the volume withdrawn by the screws (left or right screw) was different. The results presented for the draw-down performance of a twin screw feeder were the average values for the flights on the left and the right screw in each division. All experiments were undertaken at a speed of 20 rpm.

The experimental results for the draw-down performance are shown in Figs. 7.10 to 7.17. The volume withdrawn per revolution along the flow direction and comparisons between the theoretical predictions and the observed values are also given in the figures. The theoretical predictions for the volume withdrawn per revolution along the screw are represented by lines in the figures. Eq. (7.2) has been used to calculate the volume withdrawn in the last pitch in the feed section, i.e., the volumetric output of the twin screw feeder; for white plastic pellets the coefficient \(k = 0\). For the other pitches in the feed section the volume withdrawn by the screws are calculated based on two single screws, which leads

\[
V_{\text{con}} = 2\pi \eta_v P_L \left( R_o^2 - R_c^2 \right) \quad (7.3)
\]

where \(P_L\) is the length of the last or exit pitch in the feed section.

In Eqs. (7.2) and (7.3) the volumetric efficiency \(\eta_v\) is calculated based on a single screw with the same geometry.
The experimental results for the volume withdrawn per revolution along the flow direction presented in the figures are the average values obtained from the five successive measurements (ten revolutions). It can be seen that a good consistency was obtained between the theoretical predictions and experimental results.

There was no obvious difference in the draw-down performance between the two directions of rotation. But significant differences existed in the volume withdrawn in each pitch along the screws. For both twin screw feeders, the volume withdrawn when the screws were rotating in the downward direction was larger than when the screws were rotating in the upward direction. A comparison of the percentage increase in the volume withdrawn in each pitch for the different rotating directions is presented in Fig. 7.18. Although large deviations can be seen between the different pitches, the average values of the percentage increase in the volume withdrawn were close to those obtained for the last pitch (ie the output of a twin screw feeder). The average value of the increase in rate was 5.2% for No. 2 twin screw feeder and 5.6% for No. 4 twin screw feeder. These values agree well with those discussed in Section 7.2.1 and Section 7.2.3.

It can be seen from Fig. 7.18 that the largest increase in the volume withdrawn was in the first pitch, which had the shortest pitch length. For both twin screw feeders the diameter of the core shaft in the first pitch was largest (as shown in Fig. 4.7). This means that less room exists between the two screws. As discussed in the previous section in this chapter, when the screws were rotating in the upward direction, the bulk solid was pushed toward the middle between the two screws, where the shear surface between the bulk solid and the screws developed. It is suspected that a proportion of the bulk solid between the two screws may be sheared off and its transport delayed. Where there was less room between the screw shafts and for smaller pitches, the differences observed between the upward and downward directions were more significant.
Fig. 7.10 Draw-down performance with No. 2 twin screw
(Upward)

Fig. 7.11 Volume withdrawn per revolution along No. 2 twin screw
(Upward)
Fig. 7.12 Draw-down performance with No. 2 twin screw (Downward)

Fig. 7.13 Volume withdrawn per revolution along No. 2 twin screw (Downward)
Fig. 7.14 Draw-down performance with No. 4 twin screw (Upward)

Fig. 7.15 Volume withdrawn per revolution along No. 4 twin screw (Upward)
Fig. 7.16 Draw-down performance with No. 4 twin screw (Downward)

Fig. 7.17 Volume withdrawn per revolution along No. 4 twin screw (Downward)
7.4 Torque Requirements

Experiments on the torque requirement of a twin screw feeder were carried out with two sets of screws (as shown in Fig. 4.9), two troughs with different centre distances ($l_l = 160$ mm and $l_l = 175$ mm) and two bulk solids (white plastic pellets and semolina). The length of the feed section was 700 mm. Only the high initial filling state (hopper filled from an empty state to a level of 600 mm) was used for this test program. To assess the influence of the rotating speeds of the screws on the torque requirements of the twin screw feeder, experiments were undertaken over a range of the speeds from 20 to 80 rpm using the trough with a smaller centre distance ($l_l = 160$ mm) and two bulk solids.

Fig. 7.18 Increase in volume withdrawn by screws rotating in downward direction

(White plastic pellets)
7.4.1 Comparison between Theoretical Predictions and Experimental Results

According to Eq. (3.42), the stress exerted by bulk solids in the hopper on the twin screw feeder should be approximately twice the stress exerted on the single screw feeder. This was found to be basically true for semolina. However, in the twin screw feeder experiments with cement it was found that a cement "wall" built up between the two screws. This meant that two separate arches tended to form and reduce the stress acting on the screws. These observations suggest that the calculated torque should be distinguished between cement and semolina. For semolina, estimates of the torque requirements have been calculated on the basis of Eq. (3.42) while for cement the estimated torque requirements are based on two single screw feeders. Comparisons between calculations and experimental results for twin screw feeders are shown in Figs. 7.19 and 7.20. In the legends C is short for cement and S for semolina. 10 is for the trough with a smaller centre distance ($l_s = 160$ mm), giving a gap of 10 mm between the two screws and 25 is for the trough with a larger centre distance ($l_s = 175$ mm), giving a gap of 25 mm. "up" is short for the operating condition when the screws were rotating in the upward direction and "down" is short for the operating condition when the screws were rotating in the downward direction.

The theoretical predictions for cement are smaller than the experimental results. The calculated values for semolina show good consistency with the torque requirements observed when using the smaller screw centre distance and when the screws were rotating in the upward direction. For the other conditions the calculated values are higher than the observed values.

An unexpected outcome from the experiments is that providing a greater separation of the screws may be a useful method to reduce the load on the screws exerted by the bulk solid in the hopper, thereby reducing the required torque. The experiments indicate that this
may be effective for some relatively free-flowing bulk materials like semolina. However, care would need to be taken with cohesive bulk solids where an excessive gap between the screws may result in stable arches forming around each individual screw.

Fig. 7.19 Comparison of torque requirements for No. 2 twin screw feeder

Fig. 7.20 Comparison of torque requirements for No. 4 twin screw feeder
7.4.2 Influence of Rotating Direction of Screws

An interesting phenomenon observed was that for both twin screw feeders, the torque required for semolina when screws were rotating in the upward direction was significantly higher than when the screws were rotating in the downward direction. Fig. 7.21 shows the observed results with No. 4 twin screw feeder for semolina in the test speed range from 20 to 80 rpm. The required torque with the screws rotating in the upward direction was up to 40% higher than the torque required for the screws rotating in the downward direction. For upward rotation bulk solids between the two screws will be pushed down into the lower region of the screws, which consists of rigid surfaces. Narrow gaps will result in higher stress on the bulk solids moving forward and higher shear forces will occur; a larger centre distance seems to reduce this problem. For cement, however, due to the "material wall" built up between the two screws, the screws actually operate independently and the centre distance between the two screws had no obvious effect on the torque requirements.

![Graph showing torque versus screw rotating speed for No. 2 twin screw feeder with semolina (l₁ = 160 mm)](image)

Fig. 7.21 Torque versus screw rotating speed for No. 2 twin screw feeder with semolina (l₁ = 160 mm)
CHAPTER 8

CASE STUDIES

8.1 Introduction

Models for predicting the performance of screw feeders were developed in Chapter 3. Experimental programs reported in Chapter 6 and Chapter 7 examined the reliability of the theoretical analyses. It was found that the theoretical predictions have a good consistency with the experimental results determined from the laboratory scale test rigs.

As mentioned in Chapter 1, most screw feeders are custom-designed. The design of this type of equipment still relies heavily on experience and trial-and-error. As a result, unnecessary problems can occur. During the latter part of this study the opportunity was provided to apply the models developed in this study to some industrial problems. In this Chapter four upgrade projects are chosen for case studies.

8.2 Case 1

There are two screw feeders in Case 1. One screw feeder was used with a 300T bin containing crushed bath, as shown in Fig. 8.1, and the screw geometry is shown in Fig. 8.2. The other screw feeder was used with a 75T product bin. Both screw feeders were employed for symmetrical funnel-flow bins, so the flow pattern was not a particular concern. However, for both screw feeders effective control over feeding rate could not be obtained. The reason for this problem was that the length of the screw for feeding control was only that length under the bin outlet. The casing was a U-trough with the attendant large gap between the trough and flights in the upper half
of the casing. There was no effective choke section so that both screw feeders worked as “flood fed screw conveyors”.

The problem was solved by forming a choke section to allow the feed to be controlled at the desired rate. It was required that there be no big change in the configurations of both screw feeders. For the screw feeder used with the 300T bath bin the geometry of the screw in the choke section was redesigned and extended. The diameter of the screw in the choke section was reduced from the original 500 mm to 400 mm. A modified trough and shroud were put into the original trough to form an effective choke section. Fig. 8.4 shows a sketch of the modified screw for the feeder with the 300T bath bin.

The original screw used with the feeder for the 75T product bin had two sections: one comprised a 300 mm diameter screw with a 200 mm pitch length, used for feeding the bulk material from the bin; the other comprised a 500 mm diameter screw with a 500 mm pitch length, used for conveying the bulk material. This screw was revised with the same screw diameter (300 mm) for both the feeding section and the conveying section. The choke section was extended with a cylindrical trough. The “fullness” was 42% in the conveying section. Fig. 8.5 shows a sketch of the modified screw for the feeder with the 75T product bin.

The parameters assumed for both screws and the calculated results from the models in this study and the equations in CEMA [22] are listed in Table 8.1.
Table 8.1 Assumed parameters and calculated results for Case 1

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Feeding section</th>
<th>Conveying section</th>
<th>Screw 1</th>
<th>Screw 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk density (kg/m$^3$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wall friction angle ($^\circ$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effective angle of internal friction ($^\circ$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capacity (t/h)</td>
<td></td>
<td></td>
<td>40</td>
<td>18</td>
</tr>
<tr>
<td>Speed of rotation (rpm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total power requirement (kW)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volumetric efficiency</td>
<td></td>
<td></td>
<td>0.82</td>
<td>0.85</td>
</tr>
<tr>
<td>Output per revolution (m$^3$)</td>
<td></td>
<td></td>
<td>0.018</td>
<td>0.0077</td>
</tr>
<tr>
<td>Pitch length (m)</td>
<td></td>
<td></td>
<td>0.25</td>
<td>0.15</td>
</tr>
<tr>
<td>Screw diameter (m)</td>
<td>0.4</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shaft diameter (m)</td>
<td>0.219</td>
<td>0.1143</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conveying section</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Screw diameter (m)</td>
<td>0.5</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shaft diameter (m)</td>
<td>0.114</td>
<td>0.1143</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pitch length (m)</td>
<td>0.5</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volumetric efficiency</td>
<td></td>
<td></td>
<td>0.82</td>
<td>0.85</td>
</tr>
<tr>
<td>Output per revolution (m$^3$)</td>
<td></td>
<td></td>
<td>0.018</td>
<td>0.0077</td>
</tr>
<tr>
<td>Speed of rotation (rpm)</td>
<td></td>
<td></td>
<td>24.6</td>
<td>26.0</td>
</tr>
<tr>
<td>Total power requirement (kW)</td>
<td></td>
<td></td>
<td>13.5</td>
<td>4.1</td>
</tr>
<tr>
<td>Power requirement from CEMA (kW)</td>
<td></td>
<td></td>
<td>8.7</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Note: Screw 1 is for 300T bath bin and Screw 2 is for 75T product bin.
Fig. 8.1 Original screw feeder for 300T bath bin
Fig. 8.2 Original screw used in feeder for 300T bath bin
Fig. 8.3 Original screw feeder for 75T product bin
Fig. 8.4 Proposed screw for 300T bath bin
Chapter 8

Case Studies

Fig. 8.5 Proposed screw for 75T product bin

FLIGHTS 300 O.D.
300 PITCH
1,500 LONG

PIPE 114 O.D.

MATERIAL FLOW

150 * 10 = 1,500

300 + 5 = 1,500
8.3 Case 2

The screw feeder in Case 2 was used with a bath crusher feed silo. The general arrangement of this screw feeder is shown in Fig. 8.6. The original screw had a 400 mm diameter in the feeding section and a 500 mm diameter in the conveying section. It was found that there were two main problems with this design. First, the screw geometry could not meet the requirement of the high output (up to 150 t/h) at a reasonable speed of screw rotation. Secondly, the distance between the inlet of the bin and the outlet of the screw feeder was too short to provide effective control over the flow rate. Thus, the screw diameter was changed to 600 mm and the length of the screw was extended to provide an effective choke section. Fig. 8.7 shows a sketch of the modified screw while the assumed parameters and calculated results are listed in Table 8.2.
Table 8.2 Assumed parameters and calculated results for Case 2

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk density</td>
<td>(kg/m³) 600-1100</td>
</tr>
<tr>
<td>Wall friction angle</td>
<td>(°) 28-35</td>
</tr>
<tr>
<td>Effective angle of internal friction</td>
<td>(°) 49</td>
</tr>
<tr>
<td>Capacity</td>
<td>(t/h) 150</td>
</tr>
<tr>
<td>Screw</td>
<td></td>
</tr>
<tr>
<td>First pitch</td>
<td></td>
</tr>
<tr>
<td>Screw diameter</td>
<td>(m) 0.6</td>
</tr>
<tr>
<td>Shaft diameter</td>
<td>(m) 0.324</td>
</tr>
<tr>
<td>Pitch length</td>
<td>(m) 0.4</td>
</tr>
<tr>
<td>Other pitches</td>
<td></td>
</tr>
<tr>
<td>Screw diameter</td>
<td>(m) 0.6</td>
</tr>
<tr>
<td>Shaft diameter</td>
<td>(m) 0.168</td>
</tr>
<tr>
<td>Pitch length</td>
<td>(m) 0.6</td>
</tr>
<tr>
<td>Volumetric efficiency</td>
<td>0.72</td>
</tr>
<tr>
<td>Output per revolution</td>
<td>(m³) 0.113</td>
</tr>
<tr>
<td>Speed of rotation</td>
<td>(rpm) 37.0</td>
</tr>
<tr>
<td>Total power requirement</td>
<td>(kW) 32.2</td>
</tr>
<tr>
<td>Power requirement from CEMA</td>
<td>(kW) 22.8</td>
</tr>
</tbody>
</table>
Fig. 8.6 Original screw feeder for bath crusher feed silo
Fig. 8.7 Proposed screw for bath crusher feed silo
8.4 Case 3

Two screw devices were involved in Case 3: a rotary breaker discharger, as shown in Fig. 8.8, and a transfer screw conveyor shown in Fig. 8.9.

The rotary breaker discharger resembled a typical screw feeder, but, because it was placed under the rotary breaker and its capacity was higher than the output of the rotary breaker, this screw actually worked as a conveyor. Uniform screw and shaft diameters were chosen with constant pitch length of 350 mm in the feeding section and 400 mm in the conveying section. The original cylindrical casing was used to provide effective flow rate control.

For the transfer screw conveyor a diameter of 500 mm and a pitch length of 300 mm were chosen. Because of a strict requirement on screw rigidity (maximum permissible shaft deflection under full load should be less than 1 mm per 1 m length) and due to the longer span of this screw, the diameter of the core shaft was made 219 mm. This screw had an inclination of about 25°, so a short pitch length (300 mm) was chosen for all screw flights.

Details of the proposed screws are given in Fig. 8.10 and 8.11 while the assumed parameters and calculated results are listed in Table 8.3.
### Table 8.3 Assumed parameters and calculated results for Case 3

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk density (kg/m$^3$)</td>
<td>1560–1950</td>
</tr>
<tr>
<td>Wall friction angle (°)</td>
<td>22–23</td>
</tr>
<tr>
<td>Effective angle of internal friction (°)</td>
<td>42–43</td>
</tr>
<tr>
<td>Capacity (t/h)</td>
<td>35/42</td>
</tr>
<tr>
<td>Screw 1</td>
<td>0.4/0.5</td>
</tr>
<tr>
<td>Screw 2</td>
<td>0.4/0.219</td>
</tr>
<tr>
<td>Volumetric efficiency</td>
<td>0.6/0.21</td>
</tr>
<tr>
<td>Output per revolution (m$^3$/rev)</td>
<td>0.022/0.01</td>
</tr>
<tr>
<td>Speed of rotation (rpm)</td>
<td>17.0/45.0</td>
</tr>
<tr>
<td>Total power requirement (kW)</td>
<td>21.5/15.2</td>
</tr>
<tr>
<td>Power requirement from CEMA (kW)</td>
<td>10.4/7.9</td>
</tr>
</tbody>
</table>

Note: Screw 1 is for the rotary breaker discharger and Screw 2 is for the transfer conveyor.
Fig. 8.8 Rotary breaker discharger
Fig. 8.9 Transfer screw conveyor
Fig. 8.10 Proposed screw for rotary breaker discharger
Fig. 8.11 Proposed screw for transfer conveyor
8.5 Case 4

The screw feeder in Case 4 was used to feed ground spent potlinings to the calcining furnaces. The position of the screw feeder in the system and the configuration of the original screw are shown in Figs. 8.12 and 8.13 respectively. The problems experienced by operators included uneven draw-down and bridging of the bulk material in the feed hopper, variations in the feed rate from the screw that reduced with time and the generation of local hot spots in the trough combined with an increasing power consumption.

Observation of the operation indicated that the bulk material at the centre of the hopper outlet remained undisturbed and formed an inverted wedge, eventually becoming quite compacted. The compacted material required breaking down with a paddle and feeding to either the front or rear end of the slot. A second problem was the inability of the screw to maintain the required feed rate. In conjunction with this occurrence, the current drawn by the screw feeder motor increased and the trough was observed to heat up, in an area at the transition between the feeding section and the conveying section, to a point that it became too hot to touch. As a result, the outside diameters of some screw flights were reduced through wear by up to 8 mm and the trough had a spiral groove worn on its inside surface that was estimated to be in the order of 2 mm deep and 60 mm wide.

It was found from an inspection of the screw flights that at the transition between the feeding section and the conveying section the gap between corresponding flights in adjacent “leads” reduced by 10 mm. At the point where the gap was narrowest, the wear was most severe. This was the reason why the hot spot on the trough was generated and the severe wear on the flights and the trough surface resulted. It was considered that the double start flights over all the screw for this screw feeder was unnecessary. This configuration can easily result in uneven spacing between
subsequent flights, due to a lack of attention to the manufacturing and assembling tolerances. The bridging of bulk material in the feed hopper was related to the design of the screw in the feeding section. Although the core shaft was tapered over the feeding section, the conical shape of the shaft was too small and a constant pitch length in the feed section meant that the requirements for even draw-down performance were not satisfied.

In the proposed screw design the double start flights were changed to single start flights. The geometry of the core shaft was altered and the variable width of ribbon flights and stepped pitches were employed in the feed section. Details of the proposed screw are given in Fig. 8.14 while the assumed parameters and calculated results are listed in Table 8.4.
Table 8.4 Assumed parameters and calculated results for Case 4

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk density</td>
<td>900–1600 kg/m³</td>
</tr>
<tr>
<td>Wall friction angle</td>
<td>22 °</td>
</tr>
<tr>
<td>Effective angle of internal friction</td>
<td>43 °</td>
</tr>
<tr>
<td>Capacity</td>
<td>0.75 t/h</td>
</tr>
<tr>
<td>Screw</td>
<td>0.168 m</td>
</tr>
<tr>
<td>variable</td>
<td>variable</td>
</tr>
<tr>
<td>Speed of rotation</td>
<td>5.0 rpm</td>
</tr>
<tr>
<td>Total power requirement</td>
<td>2.1 kW</td>
</tr>
<tr>
<td>Power requirement from CEMA</td>
<td>1.1 kW</td>
</tr>
</tbody>
</table>
Chapter 8

Case Studies

Fig. 8.13 Original screw geometry in Case 4
Fig. 8.14 Proposed screw geometry in Case 4
(Br: width of ribbon flights)
9.1 Conclusions

This study was aimed at developing models for predicting the performance of screw feeders. For this purpose, theoretical analyses were carried out for the volumetric efficiency, draw-down performance and torque characteristics of screw feeders. A series of systematic experiments were undertaken to demonstrate the validity of the theoretical analyses and to investigate the influence of geometrical parameters, properties of the bulk solid and operating conditions, on the performance of screw feeders. Experimental data were compared with the results obtained from the theoretical predictions. The performance characteristics of screw feeders, such as the volumetric efficiency, draw-down performance and torque requirements, predicted by the theoretical models developed in this work were found to provide results which agreed well with the experimental data.

The performance prediction models were also applied to some industrial problems. The problems were analysed and new designs for screw feeders were given for four upgrade projects.

The following conclusions are based on the theoretical analyses, experimental investigations and findings of this study.
9.1.1 Volumetric Efficiency

- Based on the analysis, in which an element of the bulk solid sliding on the helical surface of a screw flight is assumed to move in a direction related to the angle of friction between the bulk solid and the flight surface, and helical angle of the flight surface, an analytical solution to the integral equation to express the volumetric efficiency of screw feeders has been obtained [Eqs. (3.21) and (3.22)].

- Equivalent helical angles [Eqs. (3.28) and (3.29)] are developed by applying the principle that the direction of the resultant force acting on the bulk material element should be consistent with the direction of movement of this element within a pitch. This leads to an approximate solution for the volumetric efficiency which is identical to that determined previously by Roberts et al. [65, 69].

- Comparison of the results calculated from the analytical and the approximate equations shows that they are very similar, however, the equation using equivalent helical angles is more concise.

- At the point where the volumetric efficiency is nearly equal to 0.5 and the specific volume begins to drop from the highest point, the corresponding ratio of P/D can be considered to be the upper boundary condition (Fig. 3.4).

- The radial clearance at the choke section can have a strong influence on the output of a screw feeder. For some free-flowing bulk solids it is both the last pitch in the hopper and the inside diameter of the trough at the choke section that determines the output of a screw feeder.

- The volume transported in the feed section and output of a screw feeder should be differentiated, especially for some free-flowing bulk solids which can easily result in full wall slip and for screw feeders with large radial clearances. This differentiation is also important when considering the flow pattern in the bin or hopper above the screw feeder.
Experimental results for the metering characteristics of single screw feeders indicate that a good linear relationship existed between the output and the speed of rotation under the three different filling states and for the screw speed range from 20 to 80 rpm.

### 9.1.2 Draw-Down Performance

- The first pitch has a great effect on the draw-down performance of screw feeders. Even if the ratio of pitch to the screw diameter is near to the low limit and the ratio of the shaft diameter to the screw diameter is close to the upper limit, it is still hard for the downstream pitches to provide a uniform draw-down pattern.

- Because the range of the pitch values is limited, (normally from $P/D=1/3$ to $P/D=1$), even draw-down performance with a screw which has only stepped pitches is unlikely, particularly for a long feed section.

- The condition that the increment of the volumetric flowrate is constant along the screw length does not guarantee uniform draw-down performance, unless the volumetric flowrate in the first pitch is equal to the increment in the downstream pitches.

- The effect of the clearance between the flight tips and the inside surface of the trough on the draw-down performance should not be neglected. For free-flowing bulk solids the volume withdrawn in the last pitch must take account of the clearance.

- In consideration of the draw-down performance, not only should the geometry of screw and the properties of bulk solids be considered, but hopper geometry (e.g. sloping end walls) which can improve the flow pattern should be given attention.
• Because of cross-flow of the bulk solid, the actual flow patterns in a bin or hopper (without the dividing grid used in the experiments) are modified. For some very free-flowing bulk solids such modification may be substantial.

9.1.3 Torque and Power Characteristics

• A theoretical model for the torque requirement of a single screw feeder was developed by applying principles of powder mechanics to a moving bulk material element within a pitch. The model indicates that the torque requirement is proportional to the stress exerted on the feeder by the bulk solid in the hopper and to the third power of the screw diameter.

• Consideration of the forces acting on the five confining surfaces surrounding the bulk solid contained within a pitch, and the pressure distribution in the lower region of the screw, leads to a model which provides a reasonable prediction of the torque requirement for a screw feeder.

• The analytical model for the calculation of the torque requirement gives an indication of the relationship among the screw geometry parameters, properties of the bulk solid and feeder load arising from the hopper.

• Based on the parameters used for this study, the torque required for a screw feeder is determined mainly by the resisting torque acting on the shear surface, which contributes approximately 50% of the total torque requirement.

• Experimental results indicate that the starting torque is close to the running torque for the test bulk solids and situations. The feeder loads exerted by bulk solids in the hopper were determined adequately by the flow loads on the basis of the major consolidation stress.
• Calculated and experimental results indicate that the power efficiency of a screw feeder is very low. For the test screw feeders and test bulk materials the power efficiency was in the range of 1.5~3.5%.

• A simplified approach to the calculation of the power requirement of a screw feeder was proposed, based on the power efficiency of the last pitch in the feed section and the ratio of the sum of the length of the feed section and the choke section to the last pitch length. Comparison of the power requirement between the calculated and experimental results indicates that the simplified approach is a reasonable and safe method to calculate the power requirement for a screw feeder.

9.1.4 Performance of Twin Screw Feeders

• Twin screw feeders, similar to single screw feeders, can provide a good metering characteristics. The output of the test twin screw feeders had a good linear relationship with the rotating speed in the test range.

• The output of the twin screw feeder when the screws were rotating in the downward direction was higher than when the screws were rotating in the upward direction. For the test bulk solids and the geometry of the screws and troughs, the increase in the output was about 5 ~ 6 %.

• The choke section for a twin screw feeder has the same effect on the output as it does for a single screw feeder. Thus, in calculating the output of a twin screw feeder, consideration should be given to the “effective” area.

• The physical properties of the conveyed bulk solid have a stronger influence on the performance of twin screw feeders than on the performance of single screw feeders, eg the bulk material “wall” that may be built between the screws will reduce the stress acting on the screws and reduce the torque requirements.
• The torque requirement of a twin screw feeder when the screws were rotating in the upward direction was higher than when the screws were rotating in the downward direction. For semolina the increase in the torque was up to 40%.

• A twin screw feeder with a larger centre distance can provide a higher output and yet need a lower power consumption.

9.2 Suggestions for Further Work

9.2.1 Pressure Exerted on a Screw Feeder

In this study the load which is exerted on a screw feeder by the bulk solid in a hopper is considered to be that generated under flow conditions. Based on the flow case, the stress is obtained by using Eq. (3.42), which is related to the hopper geometry and the properties of the bulk solid, but not to the filling level of the bulk solid in the hopper. Experimental evidence in this study also indicates that the torque requirements under the high initial fill state (600 mm) are 3-4% higher than that under the low initial fill state (300 mm) for the test bulk materials, as shown in Figs. 6.41 and 6.42. This suggests that the pressure or stress exerted by the bulk solid is influenced by the filling level, although such an influence is small compared to the actual changes in the filling level. Hence, investigations into much higher filling levels are necessary so that the effect of the filling level of the bulk solid in the hopper on the torque requirements can be predicted reliably. Such investigations should also seek to establish (by considering a wider range of conditions) the extent to which the stress states established under “initial” and “flow” conditions influence the performance of screw feeders.

9.2.2 Effect of Cross-Flow on Draw-Down Performance

From the theoretical analyses in this study it is very difficult for some screw geometries to provide uniform draw-down performance, because of the existence of the end effects. Results from experiments without a dividing grid indicate that the cross-flow of the bulk solid in a bin or hopper can substantially modify the actual flow
patterns obtained when using a dividing grid. Due to the differences in the capacity of various pitches along the whole feed section, pitches with higher capacity will withdraw mainly from the bulk solid directly above them and partly from the bulk solid above adjacent pitches. Hence, flow boundaries formed and the movement of the bulk solid in actual bins or hoppers are much more complex than the situations studied in this work where a dividing grid was used. The phenomenon of cross-flow is worthy of further investigation.

9.2.3 Effect of Trough Shape on Twin Screw Feeders

In this study only the one trough shape shown in Fig. 4.4 was employed for the experimental programs for twin screw feeders. Experimental results indicate that the operating conditions (eg the screws rotating in the downward or upward direction) and the physical properties of the bulk solid (eg the bulk material “wall” that may be built between the screws and reducing the stress acting on the screws and the torque requirements) have a strong influence on the performance of twin screw feeders. Hence, further investigations should be carried out on different types of trough shape (two of the different types of trough shape used in practice are shown in Fig. 9.1 and 9.2), in order to assess their influence on the performance of twin or multi-screw feeders.
9.2.4 Performance of Screw Feeders with Different Configurations and Arrangements

This research is concentrated only on the counter-rotating twin screw feeder. With the development of design techniques of screw feeders, different configurations and arrangements of multi-screw feeders are employed in industrial applications, e.g., the "self-cleaning" multi-screw feeder, "overlapping" multi-screw feeder and so on. The mechanism of interaction between the screws will be much different from that used in this study. Further work on these types of multi-screw feeders would be helpful in widening our understanding of their performance.

9.2.5 Investigation of Screws with Ribbon Flights

The screw with ribbon flights is often employed in screw feeders. Ribbon flights may be good for sticky or cohesive bulk solids where the gap between the screw and the core shaft prevents bulk solid build-up. But, from the experiments in this study using white plastic pellets and semolina (free flowing bulk solids), this type of screw is not effective in increasing the capacity of the screw in the feed direction. The influence of the physical properties of the bulk solid (e.g., cohesion, particle size) and the ribbon geometry on the draw-down performance and torque requirements needs further investigation.
REFERENCES


References


A Comparison of Torque Calculation between Roberts’ Method and Model in this Study

A method for the calculation of the torque requirement of a screw feeder proposed by Roberts et al. is outlined in Chapter 2. An example, which was used by Roberts et al. in References [65, 69], is presented to compare the results calculated by Roberts’ method and the model developed in this study.

The parameters for the calculation are the same as those used by Roberts et al. To avoid confusion, the parameters, their symbols and assumed values are outlined below:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>opening width of hopper outlet</td>
<td>$B$</td>
<td>m</td>
<td>0.165*</td>
</tr>
<tr>
<td>screw diameter</td>
<td>$D$</td>
<td>m</td>
<td>0.15</td>
</tr>
<tr>
<td>core shaft diameter</td>
<td>$d$</td>
<td>m</td>
<td>0.04</td>
</tr>
<tr>
<td>surcharge factor for flow condition</td>
<td>$q_{s}\sigma l$</td>
<td></td>
<td>1.15**</td>
</tr>
<tr>
<td>effective angle of internal friction</td>
<td>$\delta$</td>
<td>°</td>
<td>50</td>
</tr>
<tr>
<td>friction angle on flight surface</td>
<td>$\phi_f$</td>
<td>°</td>
<td>20</td>
</tr>
<tr>
<td>wall friction angle on trough surface</td>
<td>$\phi_w$</td>
<td>°</td>
<td>25</td>
</tr>
<tr>
<td>loose poured bulk density</td>
<td>$\rho_l$</td>
<td>kgm$^{-3}$</td>
<td>550</td>
</tr>
</tbody>
</table>

* The opening width of the hopper outlet is assumed to be equal to the screw diameter plus twice the normal radial clearance.

** The value of $q_{s}\sigma l$ is chosen from the calculated result in Reference [65, 69].
For convenience, the comparison of the torque calculation is based on a single pitch of the screw. According to Roberts et al., the vertical force acting on a pitch is

\[ F_V = 9.81 \rho g \rho_b PB^2 \]  \hspace{1cm} (A-1)

The force \( F_{AS} \) to shear the bulk solid along the shear surface is

\[ F_{AS} = \mu_e F_V \]  \hspace{1cm} (A-2)

where the equivalent friction coefficient \( \mu_e = 0.8 \sin \delta \).

The force \( F_{AC} \) to slide the bulk solid along the trough surface is

\[ F_{AC} = \mu_w F_V (K_1 + K_2 + K_3) \]  \hspace{1cm} (A-3)

For this example, \( K_1 = K_2 = 0.4 \) and \( K_3 = 0.6 \) were chosen by Roberts et al. Thus, the axial force acting on the driving side of the screw flight can be expressed as

\[ F_A = F_{AS} + F_{AC} = F_V (0.8 \sin \delta + 1.4 \mu_w) \]  \hspace{1cm} (A-4)

The torque requirement for a single pitch is

\[ T = \frac{2F_A}{R_o^2 - R_c^2} \int_{R_c}^{R_o} r^2 \left( \frac{1 + 2 \pi \mu f P}{2 \pi P - \mu f} \right) dr \]  \hspace{1cm} (A-5)

The comparisons of the axial force acting on the driving side and the torque requirement versus the ratio of pitch to screw diameter are shown in Fig. A-1 and A-2. It can be seen that for both the axial force and the torque requirement, the values calculated based on Roberts' method are higher than those based on the model developed in this study. The differences between the calculated results increase with an increase in the ratio of P/D. When the ratio of P/D is small, the calculated results are very close. When P/D reaches 1, the differences in both the axial force and the torque requirement are up to 35%. It should be noted that in this example the pressure ratios (\( K_1 \), \( K_2 \) and \( K_3 \)) are chosen at their
lowest suggested limit. If the pressure ratios are chosen as the middle values in the suggested ranges, the differences in the axial force and torque values will be up to 55%.

Fig. A-1 Comparison of axial force on driving side

Fig. A-2 Comparison of torque requirement
B Publications List as PhD Candidate


