Asymmetrical rolling and self-excited vibration in a hot roughing mill

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ASYMMETRICAL ROLLING AND SELF-EXCITED VIBRATION IN A HOT ROUGHING MILL

A thesis submitted in fulfilment of the requirements for the award of the degree of

DOCTOR OF PHILOSOPHY

from

UNIVERSITY OF WOLLONGONG

by

ALI ASGHAR JAFARI

BSc, MSc

Department of Mechanical Engineering

1994
This thesis is dedicated to my family
DECLARATION

I, ALI ASGHAR JAFARI certify that the work in this thesis was carried out by me in the Department of Mechanical Engineering at The University of Wollongong and has not been submitted for a degree to any other university or institution.

__________________________

ALI ASGHAR JAFARI
Torsional vibration of drive shafts, rolls, and vertical vibration of rolling mill housings due to impact rolling loads should be considered because they can shorten the lives of rolling mill components. Moreover these dynamic loads can affect the quality and physical properties of the strip.

This thesis contains the simulation of different vibration modes in a hot Roughing mill which consists of a Horizontal mill with an attached Edger (Fig. 5.1). Four vibrations of the Roughing Mill system have been studied:

(i) torsional vibration of Horizontal mill
(ii) vertical vibration of Horizontal mill
(iii) torsional vibration of Vertical Edger
(iv) horizontal vibration of slab between the two mills.

There are many published works on torsional and vertical vibration of rolling mills especially for cold rolling. In their simulations, the vibrations were calculated with the excitation of a step or ramp-step function, but they did not consider the oscillation of rolling force and torques caused by the rolls having different velocities due to torsional vibration under asymmetrical rolling conditions. However, the following parameters have been taken into account when calculating the vibration in this thesis:

(i) asymmetry in horizontal mill (different roll speed)
(ii) variation of rolling force and torques (top and bottom rolls) in horizontal mill
(iii) backlash on the drives couplings
(iv) non-linear stiffness and damping of hot slab
(v) push-pull between the two mills during interaction conditions
The study showed that the asymmetry and slab vertical damping introduced the greatest damping on the torsional and vertical vibration of horizontal mill respectively. Moreover, higher vibrations amplitudes achieved in both mills as a result of backlash on shafts or push-pull between the two mills.

Stability and torsional vibration of horizontal mill for different slippage conditions has also been investigated. The self-excited vibration (continual slippage when the torque is replaced by a friction torque) introduced the highest amplitude compared with normal slippage (no torque for a short time).
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NOMENCLATURE

\( A_0, a \)  = material constants

\( b, c \)  = material constants

\( c_v, c_t \)  = damping of slab for vertical and torsional vibrations, N.s/m

\( [C] \)  = damping matrix, N.s/m

\( d \)  = Moment arm, m

\( h \)  = slab thickness, m

\( h_A, h_B \)  = neutral points thickness, m

\( h_1 \)  = the exit thickness, m

\( \dot{h}_1 \)  = rate of the exit thickness, m

\( h_0 \)  = the entry thickness, m

\( J \)  = inertia, N.m.s^2

\( [J] \)  = inertia matrix, N.m.s^2

\( k_v, k_t \)  = stiffness of slab for vertical and torsional vibrations, N/m

\( [K] \)  = stiffness matrix, N/m

\( L \)  = the distance between exit and entry plan, m

\( m \)  = mass, Kg

\( [M] \)  = mass matrix, Kg

\( P \)  = rolling force, N

\( R \)  = roll radius, m

\( S \)  = pressure distribution, N/m^2

\( t \)  = time, s

\( t' \)  = temperature, °C

\( T \)  = rolling torque, N.m

\( v_1 \)  = the strip exit velocity, m/s

\( w \)  = slab width, m

\( y \)  = linear displacement, m

\( y' \)  = linear velocity, m/s

\( y'' \)  = linear acceleration, m/s^2
\( \alpha \) = rolling bite angle, Rad
\( \varepsilon \) = strain
\( \dot{\varepsilon} \) = rate of strain, 1/s
\( \theta \) = angular displacement, Rad
\( \dot{\theta} \) = angular velocity, Rad/s
\( \ddot{\theta} \) = angular acceleration, Rad/s\(^2\)
\( \mu \) = coefficient of friction
\( \sigma \) = flow stress, N/m\(^2\)
\( k \) = flow stress in shear, N/m\(^2\)
INTRODUCTION

This thesis simulates vibration of a hot Roughing mill taking into account the influence of slab damping and stiffness, spindle coupling backlash, push-pull between Horizontal mill and Edger and asymmetrical rolling force and torque due to different work-roll speed.

The calculation of rolling force and torques, stiffness and damping of slab for asymmetrical and symmetrical rolling needs a complete study of rolling theory. On the other hand, the rolling mill vibration analysis is the main aim of this thesis and demands a study of both modal analysis and transient response vibration theory. Thus, two separate chapters (1,2) have been necessary for the literature review of rolling mill vibration and rolling theory areas respectively.

The main aim of the thesis is to describe an improved method of hot flat rolling vibration analysis by the following aspects:

(i) chapter 3: Presentation of two sets of experiments results carried on the mechanical properties of hot steel which are flow stress and damping. These results are necessary for the calculations in the next chapters, because the energy parameters (rolling force and torque) and the vibration are dependent on flow stress and damping of the material respectively. This damping will be applied for the elastic regime between the two mills.

(ii) chapter 4: Application of a number of different analytical and numerical techniques such as slab method, FEM and upper-bound method to evaluate the symmetrical rolling force and torque of the industrial rolling mill model of this thesis (Roughing mill). The friction coefficient between the roll-slab interface is also estimated to give the best results for rolling force and torque when compared with practical mill data.
(iii) chapter 5: Presentation of the different vibrational models of Rouging mill and also calculation of the slab stiffness and damping in both elastic and plastic regimes to find the natural frequencies and mode shapes of the models. These data are essential when solving vibrational problems in rolling mill. Because, in practice, the vibration frequency of rolling mill normally correlates with one of the natural frequencies.

(iv) chapter 6: Vibration analysis of the Roughing mill based on symmetrical rolling assumption at Horizontal mill. Four vibrations of the Roughing Mill system are studied:

(a) torsional vibration of Horizontal mill
(b) vertical vibration of Horizontal mill
(c) torsional vibration of Vertical Edger
(d) horizontal vibration of slab between the two mills.

(v) chapter 7: Study of asymmetrical rolling based on upper-bound and slab methods. The influence of asymmetry on rolling force, torques and vibrations are investigated to compare with the results of chapter 6 (symmetrical rolling) and measurement.

(vi) chapter 8: Investigation of stability for both symmetrical and asymmetrical rolling in steady-state or transient conditions of the Roughing mill. The influence of slippage and variation of friction coefficient on the torsional vibration (self-excited vibration) are studied.

(vii) chapter 9: Conclusions and recommendation for future works.

The contributions of the work presented in this thesis are:
(1) A mathematical torque model has been developed to calculate the **slab stiffness** and **damping** in deformation zone (roll-bite) for torsional vibration.

(2) A new vibration model of tandem roughing mill has been developed to calculate the **interaction** between torsional vibrations of horizontal and vertical mills and linear vibration of slab acting as an independent mass between the two mills. The rigidity and damping of the slab in both plastic and elastic regions have been considered in this simulation. Moreover, variation of the **push-pull** forces between the two mills as a result of mismatch between the surface speed of the horizontal mill and vertical edger have been calculated which are essential data for roughing mill control.

(3) Investigation about the influence of **asymmetry** (rolling with different roll speeds) on the Roughing mill vibrations. The results showed that for asymmetrical rolling, the torque difference introduced the main source of damping for torsional vibration. It showed that the other damping such as internal damping of spindles and the damping from the variation of roll bite angle are negligible.

(4) Introducing a criterion for **stability** analysis of symmetrical and asymmetrical rolling which is an important aspect in the field of rolling control.
CHAPTER 1
LITERATURE REVIEW
ROLLING MILL VIBRATIONS
1.1 Introduction

Different vibration modes of rolling mill have been determined by Roberts [1,3]. Two common vertical chatter frequency ranges were categorised by their musical tones. The third octave (128 to 256-Hz) and fifth octave (500 to 700-Hz) chatter are mainly experienced by cold rolling mills. Furthermore, torsional chatter of rolling mill drive trains with frequency in the range of 5 to 15 Hz has been classified into two kinds, impulsive and self-excited torsional vibration [42].

Third octave vibration was characterised by the backup-roll bouncing on the entire roll stack. It results in gauge variation, interstand tension fluctuation, low frequency rumble, markings on the strip and strip breakage. Many cold mills (when reducing strip with thickness less than 0.2 mm) have reported in their last stands vibration at frequencies inside the third octave range [6, 16, 25], while a hot strip mill is reported as experiencing low frequency chatter between 40 to 50 Hz in the early mill stands (F2 and F3) [5]. In addition, some cold mills and temper mills have experienced fifth octave chatter outside the frequency range specified by Roberts.

As noted by Tamiya [13], the basic vibration mechanism of third octave chatter is believed to be self-excited due to speed changes of the preceding stand. Studies have shown that periodic speed changes produce negative damping caused by tension variation between the mill stands. A detailed discussion of this self-excitation phenomenon can be found in reference [4].

For hot rolling, the main factors causing self-excited vibration could be a sudden change in tension or push-pull force, unstable lubrication, large reduction, oxide layer formation on the roll surface, etc. Chen et al. [30] have attributed the formation of ferric oxide on the surface of steel as the main source of hot rolling self-excited torsional vibration. Gue et al. [31] have indicated the following sources and conditions in cold rolling which caused self-excited vibration:
- Thin gauge strip.
- Material with high carbon content or high strength.
- High interstand tension.
- Unstable oil emulsion or excessive lubricant particle size.
- High mill speed.
- Low friction coefficient or excessive lubrication.
- Inadequate lubrication.
- Extremely large or small separation force.
- Abrupt changes in the rolling parameters, such as the passage of welds.

1.2 Torsional Chatter

1.2.1 Introduction

Torsional chatter is the resonance vibration of the mill drive train with the frequency usually between 5 to 20 Hz [1]. At BHP Steel SPPD, the rolling mills have different types of drive systems. The Five Stand Cold Mill drive train comprises of motors, gearbox coupled to work-rolls by spindles, whilst the four-high reversing hot Roughing mill is driven by two independent drive trains as shown in Fig. 1.1.

The torsional chatter vibration mode consists of rolls acting as rotational inertia and the spindles acting as torsional springs, Gallenstein [10]. The motor inertia is large and its torsional vibration is small compared to the rolls. Thus, Roberts [1] considered the motor to be rigid and neglected its vibration. The one-degree of freedom torsional vibration system is represented by the roll inertia and spindle stiffness respectively.
1.2.2 Torque Amplification Factor (TAF)

During rolling a dynamic torque is superimposed on the steady state torque and the peak torsional strain on the mill occurs only milliseconds after an impact disturbance. The ratio of the peak dynamic torque value to steady state value is called torque amplification factor and is abbreviated as TAF.

If a load is applied to a drive system very slowly, the inertia would have very little effect on shaft torque. The torque is applied and the system reacts so slowly that the peak torque is equal to the maximum rolling torque and the TAF is one.

In practice, when the strip is fed into the mill roll-bite quickly or a weld joint is encountered at the first stand, then the system goes from idling or steady state rolling torque to a higher dynamic torque level, that is a torque amplification factor greater than one is observed.
1.2.3 Backlash

The dynamic analysis of rolling mill drive trains which have included the effect of backlash have been the source of much confusion. Backlash can be described in the same manner as hysteresis or dead zones, as inherent nonlinearities. Backlash is generally considered to cause instability in the system and as such becomes very important in determining the dynamic behaviour of a drive train as shown schematically in Fig. 1.2a [37].

![Fig. 1.2a Torsional system with backlash](image)

The treatment of backlash may be illustrated by considering the transmitted torque $T_{si}$ across a given span $i$ as follows [37]

\[
T_{si} = k_i (\theta_i - \theta_{i+1} - G_{ip}) + c_i (\dot{\theta}_i - \dot{\theta}_{i+1})
\]

for $\theta_i - \theta_{i+1} \geq G_{ip}$ and $T_{si} \geq 0$  \hspace{1cm} (1.1)

\[
T_{si} = k_i (\theta_i - \theta_{i+1} - G_{in}) + c_i (\dot{\theta}_i - \dot{\theta}_{i+1})
\]

for $\theta_i - \theta_{i+1} \leq G_{in}$ and $T_{si} \leq 0$  \hspace{1cm} (1.2)

\[
T_{si} = 0
\]

for $G_{in} (\theta_i - \theta_{i+1}) \geq G_{ip}$  \hspace{1cm} (1.3)
where

\[ J_i = \text{rotational element inertia} \]
\[ k_i = \text{rotational element stiffness} \]
\[ c_i = \text{rotational element damping} \]
\[ G_{in}, G_{ip} = \text{rotational element backlash for two different directions} \]

The above equation is shown graphically in Fig. 1.2b.

### 1.2.4 Torsional Vibration in Hot Rolling

A study to determine the TAF of a hot strip mill was carried out by Herman et al. [8]. They reduced both TAF and the first natural frequency by using a softer coupling.

Wright [39] reviewed the history of torsional vibration in industry especially in hot rolling mill drives (before 1976). A number of articles presented the theory (analytical and numerical solutions), measurement of frequencies and TAF of rolling drives.

Some of the useful analytical equations introduced by Wright [39] to calculate the TAF are as follows:

#### two-degrees of freedom system

\[
\text{TAF} = \frac{2J_2}{J_1+J_2} \tag{1.4}
\]

where

\[ J_1, J_2 = \text{moment of inertia of roll and motor respectively} \]

#### three-degrees of freedom system

\[
\text{TAF} = \frac{2J_3}{J_1+J_2+J_3} \left( \frac{\omega_2^2 - \omega_1^2}{\omega_2^2} \right) \tag{1.5}
\]

where

\[ J_1, J_2, J_3 = \text{moment of inertia of roll, pinion stand and motor respectively} \]

\[ \omega_1, \omega_2 = \text{the first and second natural frequencies respectively} \]
On the other hand, the methods of minimising TAF recommended in [39] are as follows:

(i) Experience indicates that separation of the first and second-mode natural torsional frequencies is the prime objective. This can best be achieved by lowering the first natural frequency. The reduction of stiffness is the most practical method. For example, the use of a resilient spacer coupling with a lead spindle was designed to reduce the stiffness.

(ii) It is apparent that the system with more masses and shafts, will have a lower second mode natural frequency and a greater TAF compared with a similar system with less masses and shafts.

(iii) It was found that the fundamental natural frequency should be 10 Hz or less to keep the TAF at manageable levels. Also, the fundamental natural frequency should not exceed 50% of the second mode natural frequency.

(iv) The investigators also explained the importance of couplings, pinion stand and gear backlash. The backlash will only have an effect if opened up and this will happen as a result of the impact torque at the rolls. If the location of backlash is close to a vibration node, the impulse load will significantly increase the TAF [39]. Also, the vibration will be excited with the location of node at the gears [45].

Monaco [11] developed a mathematical model of the mill drive train. The electrical and mechanical drive systems are combined to simulate transient and steady-state torque. One advantage of the above model is that the influence of both top and bottom drives on each other has been taken into account since both drive have been connected by a slab. Wright [39], had also mentioned the above point as being significant.

Simulated data was based on hot rolling, but the model can be applied to cold rolling.
Results of the computer simulation included the effects of bar ramming on torque and speed. If the bar speed is exactly equal to the entry rolling speed, there is no ramming and the torque disturbance can be adequately represented by a ramp type.

The entry time is a function of the rolling speed and bar reduction and the head end spread of the bar (see Fig. 1.3). The torque disturbance build-up time (entry time) for the bar having head end spread is considerably longer than for a square ended bar. Entry time can be calculated using the rolling parameters by the following equation [11].

\[
T = \frac{30}{\pi N} \left[ \cos^{-1} \left( 1 - \frac{\Delta h}{2R} \right) + \frac{S}{R} \right]
\]

where

- \( N \) = work-roll speed, rpm
- \( \Delta h \) = reduction
- \( S \) = head end spread
- \( R \) = work-roll radius
- \( T \) = entry time, s

![Fig. 1.3 Typical head end](image)

Moreover, the influence of backlash on drive train elements was studied using both simulation and measurement. Three cases were considered:
(i) backlash closed  
(ii) backlash half open  
(iii) backlash fully open  

The results showed that the TAF in the third case was the highest value.

It is assumed that strong damping occurs at the motors and at the roll bite. The damping coefficients were determined from the measured rate of vibration amplitude decay at the rolling mill. Monaco [11] also discussed the push-pull caused by the mismatch of speed between the horizontal and vertical mills in blooming and slabbing mills.

Furthermore, Monaco [46] later applied the stress analysis techniques to calculate the dynamic behaviour of a hot roughing mill. Also, a FM telemetry strain gauge system was used for measuring strain of the rotating components. The finite element method analysis for a typical roll coupling showed that the maximum stress occurs at the corners. On the other hand, the stress analysis of a spindle showed that for spindle design the location of a lubricating hole should not be in the maximum stress area due to high stress concentration factor of the hole.

Dobrucki et al. [38] introduced a model to calculate the torsional vibrations interaction between tandem slabbing mills. The nine-degrees of freedom model consists of inertia, stiffness, damping and electrical motor components of both horizontal mill and vertical edger. Thus, both mills are connected by the slab with its modulus of elasticity and damping. It seems that the above model was the first study about the interaction between the two stands. Torsional vibrations of rolls cause periodic variation in the velocities of the rolls and also in the longitudinal tension and compression stresses in the section of the slab between the stands. On the other hand, the variation of the push-pull between the stands influences the rolling processes and vibrations in the stands. The vibration of both mills spindles have been calculated for the following two cases:
(i) when the slab comes out from the edger and enters into the horizontal mill
(ii) when the slab exits from the horizontal mill and enters into the edger

The influence of different backlash values on the spindles was also investigated and it was found that backlash has a great effect on the drive dynamic load.

Swiatoniowski et al. [35] and Dobrucki et al. [36] reviewed the previous works regarding the dynamic load in rolling mills. They mentioned that the function representing the system rolling force and torque was usually assumed by other investigators as a trapezoidal (ramp-step function) or rectangular (step function) impulse.

However, the influence of mill vibration on rolling force is not negligible and should be considered. Any change in roll speed causes the variation of the inertial forces of rolled metal at the entry or exit side of roll-bite. Inertial forces act as back or front tension (push-pull) stress as shown in the following:

\[
\sigma_0 = -\frac{m_0 d^2 x_0}{s_0 dt^2} \quad \text{(entry side)} \tag{1.7}
\]

\[
\sigma_1 = -\frac{m_1 d^2 x_1}{s_1 dt^2} \quad \text{(exit side)} \tag{1.8}
\]

\[m_0 = \text{strip mass on entry side}\]
\[m_1 = \text{strip mass on exit side}\]
\[s_0 = \text{strip cross-section on entry side}\]
\[s_1 = \text{strip cross-section on exit side}\]

\[\frac{d^2 x_0}{dt^2} = \text{strip acceleration on entry side}\]
\[\frac{d^2 x_1}{dt^2} = \text{strip acceleration on exit side}\]

The above push-pull forces change the position of neutral point during torsional vibration:
\[ h_a = h_0 \frac{dx_0}{dt} (R \hat{\theta})^{-1} \]  
(1.9)

\[ h_0 \frac{dx_0}{dt} = h_i \frac{dx_i}{dt} \]  
(1.10)

Therefore, by applying these formulae, it is possible to derive the pressure distribution, rolling force and torque for each time step. The torsional vibration of the blooming mill top spindle was calculated from three different methods:

(i) from quasi-external trapezoidal rolling torque
(ii) from measurement
(iii) from the above mentioned theory

It showed that the result from the above theory is in agreement with measurements. Although no damping coefficient was introduced to the model, strong damping was observed using the third method which is realistic. Some investigators considered the damping during the rolling process by introducing a damping coefficient derived from measurement of the torsional vibration of rolling mill. Furthermore, it was shown that the torsional vibration in a system with gears is higher than without gears.

Swiatoniowski et al. [9] described the importance of asymmetry in the vibration of rolling mills and introduced a mathematical model to determine vertical and torsional vibrations, but they did not publish any results to show the influence of asymmetry on vibrations. In their mathematical model, the upper-bound method was applied to calculate the neutral points. The model of rolling force and torque was based on the slab method. The asymmetries can be caused by:

(i) a difference in friction conditions on the upper and lower rolls.
(ii) a difference in the upper and lower roll velocity.

Gallob et al. [43] have investigated the torsional vibration of a plate mill similar to that in Fig 1.1. Due to the different torsional stiffness of the top and bottom mill drive, an asymmetrical rolling torque distribution occurs. However, in their numerical
calculations the influence of asymmetry were neglected and their study concerning asymmetry was based on measurements of the variation of rolling force and rolling torque. The results demonstrated the difference between variations of the work-rolls torques and also between the torsional vibration of top and bottom drives lines respectively.

Chen et al. [42] reviewed previous torsional vibration studies (before 1985) of hot blooming mill. The torsional vibration was classified into impulsive and torsional self-excited vibration. A few impulsive torques models, a slippage model for impulsive torque at roll bite and a slippage model during rolling were introduced in the simulations. The self-excited vibration occurs when the slippage continues. Then the following relation has been applied as a friction torque on the rolls during slippage rather than rolling torque:

\[ T = \mu PR \]  
\[ \mu = \text{friction coefficient} \]  
\[ P = \text{rolling force} \]  
\[ R = \text{roll radius} \]

Their assumption was that the friction coefficient is dependent on the relative velocity of slab and rolls during slippage. The following empirical equation was used in the above investigation:

\[ \mu = a + bv + cv^3 \]  

where a, b, c are constants and v relative velocity.

Chen et al. [30] improved the above analysis of self-excited oscillation of a rolling mill with n-degrees of freedom. They attributed the formation of ferric oxide on the surface of steel as the main source of friction coefficient change and self-excited
vibration of hot rolling. Therefore when there are too many ferric oxides attached to
the surface of steel when rolling, they will eventually peel off resulting in a shock
disturbance which then starts the vibration and slippage. The coefficient of friction
between the roll and steel strip vary sharply and the self-excited vibration may occur
in the drive system.

In both references [30] and [42] on self excited vibrations, the rolling force $P$ in
equation (1.11) is assumed constant and is set equal to steady state rolling force
which is not correct. Moreover, the friction torque should be integrated throughout
the roll-bite and the value of rolling torque using equation (1.11) will be different
from the integrated value.

Tieu et al. [22] and Tieu [41] carried out a simulation to determine the TAF on the
BHP Roughing mill drive system. The BHP Rolling mill consists of a Vertical Edger
and a Horizontal mill. The Horizontal mill can be seen in Fig 1.1. The dynamic loads
of Horizontal mill drives have been calculated for the following conditions:

(i) slippage between Horizontal mill rolls and slab for both conditions with and
without backlash on spindles. Two cases were considered: the first case was when
the slab enters roll bite and the second case was during rolling.

(ii) push-pull between the two mills as a result of mismatch of speed between the two
mills.

The result showed that in the case of slippage and also when the edger is pushing the
Horizontal mill, the drive system of the Horizontal mill has a higher TAF than that of
the normal rolling or the case of pull. It also showed that introducing backlash to
spindles causes higher dynamic loads. Moreover, the study on vertical Edger gear
drive showed that very high dynamic loads on the bevel gears were caused by tooth
chattering when the gear backlash was opened.
Butler et al. [40] reviewed the following techniques to reduce TAF:

- use of compliant coupling on drive system
- addition of parasitically tuned secondary inertia
- tuning the drive train for increased or decreased resonant frequency
- using a cyclo-converter-drive induction motor to increase resonant frequency
- avoiding opening of backlash under load
- filtering of speed sensor signal
- changing lubrication and roll surface-finish
- scheduling the mill to avoid resonance

Then Butler et al. [40] introduced a new torque feedback control loop (TFB) on the Horizontal mill of BHP rolling mill. It showed that TFB is able to increase the damping by 50%.

1.2.5 Torsional Vibration in Cold Rolling

It seems Moller et al. [7] were the first to carry out documented research into torsional chatter associated with cold rolling. They observed that during the rolling operation, "bars" formed on the strip perpendicular to the direction of rolling due to the torsional vibration. These bars were light and dark bands and the bar spacing increased by increasing speed. Moller et al. [7], proposed that torsional chatter is explained by the coefficient of friction versus velocity curve. When the coefficient of friction decreases with increased speed, damping is negative and consequently the vibration will be amplified. Therefore self-excited vibration is likely to occur after a variation in torque. When the coefficient of friction increases with increased speed, damping is positive and vibration is likely to be dampened out after the initial transient.

Further laboratory investigations were carried out by Gallenstein [10], on a 4-high reversing mill, to determine conditions in which torsional chatter would be excited. The mill speed, rolling force, lubricant, strip thickness and hardness were varied
during the tests. The torsional chatter was excited when the rolling force exceeded 1780 kN and the rolling speed was between 0.25 and 0.75 m/s (50 and 150 fpm). Torque variation and gauge variation were plotted as a function of rolling speed. Both torque and gauge variation increased rapidly, then remained relatively constant and tapered off rapidly over the critical speed range. The maximum measured gauge variation was 4 μm. The peak-to-peak velocity oscillation was approximately 29 percent of the running speed. Investigations into lubrication conditions found that torsional chatter occurred more readily with low-viscosity lubricants. This agreed with the findings of negative damping by Moller et al. [7].

Roberts [1] states that torsional chatter can be excited by cyclic variation of lubricity in the roll-bite when the mill is heavily loaded and a positive incremental change in displacement of the work-roll coincides with a decrease in rolling torque

\[
\frac{dT}{d\theta} < 0 \tag{1.13}
\]

This condition occurs over a critical speed range,

\[
\frac{20 n L}{3 (1-r/4) (1+s)} \cdot V \cdot \frac{20 n L}{(1-r/4) (1+s)} \tag{1.14}
\]

\[
r = \text{reduction} \\
 s = \text{forward slip (})
\]

\[
n = \text{integer} = 1, 2, 3, 4, \\
 L = \text{arc of contact (m)}
\]

It was concluded that operation outside the critical speed range and minimising back up roll eccentricity leads to a better stability of the mill drive train.

Misonoh et al. [44] performed some experiments on chattering in asymmetrical cold rolling with different roll speeds and found that rolling with different speeds has a tendency to produce slip condition at the start of a coil or at low speed. When slip conditions occurs, the drive system experiences a self-excited vibration depending on the characteristic of the coefficient of friction to rolling speed. The torsional vibration
of the top (slower side) and the bottom spindles and vertical vibration of the top work-roll were measured. The torsional vibration of the higher speed side was higher than the slower side with the vibration frequency of the higher speed spindle and vertical vibration of the housing elements being 16 Hz which is lower than the frequency of the slower spindle, 32 Hz.

1.3 Vertical Vibration

1.3.1 Introduction

There are many papers documenting the vertical vibration of cold rolling. However, there are only a few contributions that have been made by hot rolling researchers. Therefore, the literature survey includes more cold rolling articles than hot rolling in this chapter, although the main aim of this thesis is the study of hot rolling.

1.3.2 Strip Stiffness and Damping

1.3.2.1 Strip Stiffness

According to the non-linear relation between rolling force and reduction, the following relation has been applied by the investigators to determine the strip stiffness for vertical vibration simulation of both hot and cold rolling[16], [34].

\[ K_v = -\frac{\partial p}{\partial h_1} \]  \hspace{1cm} (1.15)

where

- \( h_1 \) = exit strip thickness
- \( p \) = rolling force as a function of \( h_1 \)
Yarita et al. [16] applied the Hill's rolling force to study the influence of geometry, friction coefficient and yield stress on strip stiffness. This study demonstrated that for cold rolling, the strip stiffness is very sensitive to friction coefficient and strip thickness.

1.3.2.2 Strip Damping

When the roll is shifted up and down during vertical vibration, the rolling force changes due to variation of strip exit thickness. The vertical strip stiffness and damping are defined as the partial derivative of rolling force with respect to exit thickness and its rate of change respectively.

For cold rolling, Tamiya et al. [13] has proved the following relationship for equivalent viscous damping coefficient of strip during vertical vibration:

\[ C_{\text{cold}} = \sigma_1 \cdot W \cdot \frac{R}{V_r} \] (1.16)

- \( \sigma_1 \) = flow stress at exit side
- \( W \) = strip width
- \( R \) = roll radius
- \( V_r \) = peripheral speed of roll

Dukmasov [34] established a force model for vertical vibration of hot strip rolling and derived the hot strip damping coefficient:

\[ C_{\text{hot}} = \frac{p \cdot L \cdot a}{v_1 \cdot \Delta h} \] (1.17)

where
- \( p \) = rolling force
- \( L \) = roll-bite length
\begin{align*}
v_i &= \text{strip exit velocity} \\
\Delta h &= \text{draft} \\
a &= \text{material constant in the flow stress model of} \\
\sigma &= A_0 \dot{\varepsilon}^a \varepsilon^b e^{-ct} \\
\tag{1.18} \\
\end{align*}

(see part 3.1 for more details)

Then, he applied the dynamic properties of hot strip in a two high stand mill with a five-degrees of freedom model. He assumed that dry friction exists between top-roll chock and housing and studied the considerable influence of the dry friction on the amplitude-phase frequency domain.

Although the hot rolling strip damping coefficient (equation 1.17) introduced by [34] was the first model for hot rolling, it is applicable only if the flow stress model given in equation (1.18) is used. This flow stress equation does not include the dynamic recrystalisation which can occur in the low strain rate range.

1.3.3 Vertical Vibration in Hot Rolling

Keller et al. [20] detected the source of a low pitch loud rumble and the marking of strip with a series of lines parallel to the work-roll in a hot strip mill. To establish the natural frequencies and mode shapes of stand F2, the mill stand and roll system was analysed using a finite element computer program. Although many of the vibration modes are not symmetrical [25], only one half of the stand was modelled. Two types of elements were used: two dimensional plane stress elements to construct the housing and three dimensional beam elements to simulate the rolls, roll contact interface, chocks, nut, screw and hot strip. The model was used to analyse the lowest eight natural frequencies and mode shapes. Only two modes, mode 2 (61.9 Hz) and 7 (83.9 Hz), respectively, are likely to produce the marks observed on the strip. In both cases, the housing is ringing (bouncing) up and down, as a result of fluctuation in rolling force.
Wei-Jian et al. [33] carried out a three dimensional finite element to analyse the natural frequencies and mode shapes of a hot mill housing. Their 3D mesh included 244 elements. Moreover, Wilson method [78] has been applied to solve the differential equation of motion:

\[
[M] [x''(t)] + [K] [x (t)] = [0]
\]  

(1.19)

where

\[
[x (t)] = \text{displacement matrix}
\]

\[
[M] = \text{mass matrix}
\]

\[
[K] = \text{stiffness matrix}
\]

\[
[x''(t)] = [x_0] \cos \omega t
\]

The first three modes for xoz plane (19, 79.8 and 91.3 Hz) are shown in Fig. 1.4 a, b and c respectively. Also, the first three modes for yoz plane were 12, 51.4 and 70 Hz. Furthermore, they made up an experimental model for the housing to observe the mode shapes by means of holographic interferometry. The comparison of the results of both numerical and experimental methods showed a good agreement.
1.3.3 Vertical Vibration in Cold Rolling

1.3.3.1 Third Octave Mode Chatter

Third octave mode chatter is the resonance vibration of the top backup-roll on the roll stack, Roberts [1]. The frequency of vibration is usually in the third musical octave 128 - 256 Hz, and hence its name. The roll stack masses, with the exception of the top backup-roll mass, can be neglected due to their vibration being relatively small compared to the top backup chuck.

A study of cold rolling by Yarita et al. [16] stated that before chattering, more scratches appeared on the strip surface and the size of oil pits decreased. This implied that the lubrication between the strip and rolls was inadequate. The lubricating film is smaller than the roll and strip surface roughness because the strip surface is scratched when metal-to-metal contact takes place. It was observed that strip with tensile strength higher than 930 MPa tended to chatter, whereas strip with tensile strength of 830 MPa or lower did not. Strip that chattered had a gauge variation from 0.21 mm to 0.12 mm when the set up thickness was 0.16 mm. The fluctuation in gauge varies within the frequency range of 180 to 200 Hz. They showed that where the emulsion stability index (ESI) of the lubricant decreases after 40 hours, the poor lubricity and chatter occurred and the coefficient of friction was high between 0.02 - 0.03. The coefficient of friction was 0.01 to 0.02 before emulsion stability index decreased. During chatter the tension was recorded to vary at 200 Hz, with the entry and exit tension having a phase difference of 180 degrees. A four-degrees of freedom spring-mass model was developed to simulate roll stack vibration. This spring-mass model assumes that the effect of the mill housing masses are negligible on the roll stack vibration. This assumption has been made because the mill housings are rigidly secured to the foundations. The contact stiffness between the mill and the roll stack is
assumed symmetric. The natural frequencies were dependent on the coefficient of friction. It was determined from this model that the vibration of 200 Hz measured on the backup chuck and gauge variation was due to the top backup-roll and work-roll vibrating against the bottom backup-roll and work-roll. Yarita et al. [16] concluded that to prevent chatter in practice, improvements in lubrication are needed. At reductions close to rolling limit, chatter occurs due to degeneration of lubricant.

Roberts [14] stated that the third octave mode chatter can be excited by the rolling process being inherently unstable. The rolling process becomes unstable when the force decreases with increased reduction. Therefore when the rolling process becomes unstable it can be mathematically represented by:

\[
\frac{dF}{dr} < 0 \quad (1.20)
\]

\[
F = \text{roll force (N)}
\]

\[
r = \text{reduction}
\]

This was found to occur with excessive entry tension, or excessive roll flattening [14].

Tamiya et al. [13] found that chattering in tandem cold mill results in mill housing and the vibration of mill floor and causes a fluctuation of the strip thickness of more than 10% against the aimed thickness, with a constant frequency of fluctuation in the range of 160-170 Hz. Forward slip was measured during chatter by putting scratch marks on a work-roll. It was found that forward slip did not change significantly between normal rolling and during chatter. The self-excitation of vibration in the cold rolling process originated from a phase difference of 90 degrees between the vertical vibration of the roll and the fluctuation of entry tension. This mathematical model incorporates the same spring-mass model as the one developed by Yarita et al. [16], integrated with a mill control system. The mill control system includes tension, screw down and mill drive models. Vibration, gauge variation and tension fluctuation increases with increasing rolling speed. Rolling lighter gauge or changes in
lubrication condition may excite chatter. It was concluded that to suppress chatter, an improvement in roll bite lubrication was the most realistic and cheapest solution.

Misonoh [15], developed a five-degrees of freedom spring-mass model of a four high mill stand as shown in Figure 1.5. This spring-mass model includes the roll stack and mill housings. A backup-roll and associated chucks were assumed to vibrate in unison, and were therefore combined into one mass. The frequency of vibration was found to be between 100 to 120 Hz. The fluctuation of gauge and tension occurred at this frequency but an amplitude modulation or beating effect was exhibited.

Tlusty [4] developed a cold rolling model consisting of two stands, stand 2 has work- roll vibration and stand 1 was included to establish a back tension. Roll vibration causes the entry tension to fluctuate. The model was unstable and self-excitation occurred when:
\[
\frac{2E V_1^2 (r_m/R)^{0.5}}{(S-\sigma)L h_0 \omega^2} > 1
\]

(1.21)

**E** = elastic modulus of the strip (Pa)

**V_1** = exit velocity (m/s)

**r_m** = draft (m)

**S** = yield stress (Pa)

**\sigma** = interstand tension (Pa)

**L** = mill stand spacing (m)

**h_0** = entry strip thickness (m)

**\omega** = chatter frequency (rad/s)

**R** = work-roll radius (m)

Chefneux et al. [6] developed a computer model of the cold rolling process with mill control. Incorporated into this mathematical model was a two-degrees of freedom spring-mass system. The model assumed symmetry of vibration about the roll bite and the mill housings were neglected. It was found that entry tension plays a prominent role in self-excitation of vibration. Using the model, the time for strip to rupture under chatter vibration conditions was calculated for different rolling conditions. The chatter tendency is reduced or the time of rupture increased by decreasing speed, increasing friction or decreasing tension. It was noted that any sudden variation of mill setting, even if beneficial, could cause the excitation of chatter. It was observed at the Ferblatil plant [6] that chatter occurred when the roll forces were either high or low. The low roll forces were caused by excessive lubrication or polished rolls whereas high forces were caused by inadequate lubrication.

Paton and Critchley [25] stated that mill vibration can be used as an alarm. It was apparent that the tendency of the mill to chatter decreased when background mechanical vibration was low. Vibration increased when lighter gauge strip was
rolled. The back tension, rolling force and chuck acceleration were measured during chatter. Both signals of rolling force and back tension fluctuated at the same frequency as the backup chuck vibration. A tuned damper was developed to suppress mill vibration. The natural frequency of the tuned dampener was close to the natural frequency of the mill stand, 125 Hz. When the mill natural frequency is excited and the backup chuck vibrates, this excites the dampener into vibration with a larger amplitude and opposing the backup-roll chuck vibration. The energy of the vibration is dissipated by the rubber. Using the tuned dampener enabled an average increase of 15 percent of rolling speed.

Pawelski et al. [12], [18], [19] developed a mathematical model which predicted the stability of the rolling process. The spring-mass model has five-degrees of freedom. The backup-roll and associated chucks form one mass. Part of the mill housings below the housing feet were considered rigid. It was found that the vibration increased with rolling speed. Another parameter found to influence chatter was the coefficient of friction.

Swiatoniowski [28] developed a seven-degrees of freedom spring-mass model to simulate dynamic effects. The spring-mass model has the backup-roll and associated chucks forming one mass. The mill housings were divided into upper and lower masses. The dynamic effects were dependent on mill rigidity. The force variation on the upper-bearing of the backup-rolls at the moment of roll bite were recorded, and was found dependent on mill mechanical conditions. The vibration increases with bearing and screw down clearances. This is in agreement with Gasparic [5] and, Nieb and Nicolas [17], who stated that the tendency to chatter increases with the deterioration of mill mechanical condition or increased background noise. Poor mill mechanical condition can be attributed to mill components being outside of tolerances, improperly ground rolls, defective bearings, roll eccentricity, and poor ineffective maintenance causing unbalance and misalignment.
Furthermore, Guo et al. [31] mentioned the following recommendations about practical detection and suppression of third octave chatter: it is routinely detected by the low pitch rumble produced from vibrations in the roll stack. This type of chatter can further be detected by strip markings through visual inspection, tension variation measured from a tensiometer and gauge variation measured from an x-ray gauge sensor. Excitation can be due to work-roll surface undulations, defective roll bearings, tensions, improper roll bite lubricity, incoming strip chatter marks and an incoming weld segment. Third octave chatter can be eliminated or its effects lessened by improved grinding practices which detect and remove work-roll surface undulations, inspection procedures that detect and replace worn roll bearings or improved tension loop control which incorporates filters to offset the phase angle of the tension loop response. In addition, reduction of chatter can be accomplished through proper adjustments to the lubricant concentration, application or formulation to provide adequate lubrication to the roll bite.

1.3.3.2 Fifth Octave Mode Chatter

Fifth octave mode chatter is the resonance vibration of the work-rolls between the backup-rolls. The frequency of this vibration usually lies within the fifth musical octave of 512 to 1024 Hz and this type of chatter occurs in most types of high-speed mills.

Roberts [9] stated fifth octave mode chatter vibration consisted of the work-rolls acting as vibrating masses between the backup-rolls which appear fixed. The backup-roll, work-roll contact acts as a spring. The roll stack masses are symmetric about the roll-bite.

The frequency of vibration may be approximated by the following equation:

\[ f = \frac{360.68}{D_w} \] (1.22)
f = fifth-octave-mode vibration frequency (Hz)

\[ D_w = \text{work-roll diameter (m)} \]

Roberts [2] stated that fifth octave mode chatter caused chatter marks on backup-rolls. These marks may subsequently be printed on the strip. They are striation of alternating light and dark regions, perpendicular to the rolling direction, but no measurable gauge variation is associated with this chatter vibration. The work-rolls are rarely affected by chatter marks, but the backup-rolls may form flats on the roll surface, which is called "fluting".

This type of chatter may be excited due to malfunction of drive motors, gears, pinions and pinion stands. Fifth octave-mode vibration chatter can be excited by any mill vibration. Rolling for extended periods of time at critical speeds should be avoided.

The critical velocity for backup-rolls of equal diameters can be determined by

\[ V_c = \frac{f \pi D_h}{n} = \frac{(360.68 \pi) D_h}{n D_w} \]

\[ V_c = \text{critical velocity of work-roll (m/s)} \]

\[ D_h = \text{backup-roll diameter (m)} \]

\[ D_w = \text{work-roll diameter (m)} \]

\[ n = \text{integer 1,2,3,...} \]

It can be seen that the (backup-roll diameter / work-roll diameter) ratio is of significant importance in determining the critical speed.

Nessler et al. [26] carried out impact tests on a backup-roll. The backup-roll had a diameter of 1.52 m and was 2.13 m wide. The excitation of the roll stack bending frequencies imprints chatter marks on the top and bottom backup-rolls with the same integer wavelength. If the top and bottom backup-rolls are of different diameters, there will be a different number of chatter marks printed on the roll surfaces. The critical speed \( V_c \) is then determined by:
For a given speed range, the greater the difference between the top and bottom backup-roll diameter, the higher the critical speeds in that speed range. It was concluded that to have no critical speeds in the operating speed range of most mills, the difference of top and bottom backup-roll diameters would have to be less than 5 mm.

Nieb et al. [17] stated that neither Roberts [2], or Nessler et al. [26], explained why different wavelengths build up on top and bottom backup-rolls in the same stand, and why chatter marks do not appear on works rolls. Roberts [9], and Nieb et al. [17] stated that the cause of fifth octave mode chatter is the mill background vibration. Eccentricity of rolls, unbalance, defective bearing, misalignment or improperly ground rolls are possible sources that may excite chatter.

Recently, Tieu et al. [32] have investigated the relationship between the chatter and the backup-roll fluting of a temper mill at BHP steel. They made a seven-degrees of freedom vertical vibration model with 7 masses being work-rolls, backup-rolls, chucks and housing. The strip stiffness was also considered. The modal analysis showed that at 675 Hz, the work-rolls move against backup-rolls while the backup-rolls are fairly rigid. Furthermore, the experimental observation showed that the 675 Hz was the highest peak in the spectrum of top work-roll vibration, while the 675 Hz vibration of backup-roll was relatively small. This fifth octave mode chatter has caused backup-roll fluting which has been a problem over the last ten years at BHP. Also the influence of varying strip thickness, friction coefficient and strip force and their influence on vibration were studied using the model.

Furthermore, Guo et al. [31] mentioned the following recommendations about a practical detection and suppression of fifth octave chatter:
Fifth octave chatter is usually not detected immediately upon occurrence, since the higher frequency noise may not be heard by the operator. Without a vibration monitoring system, in which the operator would be notified of chatter and reduce operating speed to minimise its effect, fifth octave chatter is typically first recognised visually by the markings it produces on the rolls or strip several hours after the mill has been vibrating. In some cases, fifth octave chatter has occurred after rolling only one coil. It could be caused from undulations on the surface of the work-roll, defective roll bearing wear patterns on the backup-roll and chatter marks on incoming strip. Its effect on the rolls and strip can be lessened by the utilisation of the work and backup-roll pairs with certain diameters. If the circumferential length difference of the backup-rolls is a multiple of the chatter mark wavelength (length of mark separation), the vibration will cause the rolls to impact each other periodically in exactly the same locations which causes the development of wear marks. By pairing backup-rolls of different diameters, the impact locations are scattered and the formation of wear marks minimised. In addition, under normal mill operating condition, if the wavelength of the wear marks on the work-roll surface is close to the chatter wavelength, the work-roll will tend to excite the mill into a chatter condition. Vibration causes repeated impacts in the same locations on the backup-roll if both work and backup-roll diameters match a specific ratio. These repetitious impacts promote wear mark development and can be avoided by pairing work and backup-rolls with certain diameter ratios.
CHAPTER 2
LITERATURE REVIEW
ROLLING THEORY
2.1 Introduction

The theory of hot strip rolling is based on the theory of plasticity, continuum mechanics, heat transfer and strength of materials. The first step for designers and technologists is to calculate rolling force, torque and power based on steady state rolling conditions.

Mathematical models of flat rolling process are numerous. In each model, the equation of motion, thermal balance, material properties and roll deformation are used to calculate the stress, strain, strain rate, velocity and temperature fields, roll pressure distribution, roll separating force and torque. The consistency and accuracy of the predictive capabilities of these computational methods depend exclusively on the quality of assumptions made either during the derivation or solution of the basic equations. In conventional models, researchers assume an isotropic and homogeneous material that is incompressible in the plastic state [59].

In most rolling theories symmetrical rolling conditions have been considered. However, asymmetrical rolling process are considered in the manufacture of plain, cladding or composite plates and sheets.

The theory of flat rolling, both hot and cold, have been discussed comprehensively by Tselikov [51], Alexander [52], Roberts [41] and Pietrzyk [23], who have made notable contributions to this field, by both theoretical and experimental research. Hot rolling is more complex than cold rolling, it is for this reason that the theoretical study is less complete. The yield stress of the material is dependent on both temperature and rate of straining, and frictional condition are not completely defined. The theories which have been developed give only an estimate of the roll force and torque [52].
2.2 Symmetrical Rolling

There are a number of rolling models and methods in this area which will be reviewed in this chapter: slab method, upper bound method, finite element method, boundary element method, slip line field, shear plane theory and hydrodynamic theory.

2.2.1 Slab Method

The slab method has been used in both hot and cold rolling. The first to work on this method were Siebel [55] and Von Karman [53]. One of the most comprehensive of the rolling theories was that developed by Orowan [59]. He considered most of the factors such as different frictional conditions and inhomogeneity of the deformation during rolling. Also the fact that the yield stress of the material would vary during its passage through the arc of contact due to work-hardening, temperature and strain rate variation, was also considered.

Later research workers such as Bland & Ford [60] and Sims [58] developed solutions based on some assumptions to simplify the Orowan's theory.

2.2.1.1 Simple Model of Flat Rolling

The following assumptions are made to achieve a simple model of flat rolling:

(1) arc of contact remains circular with original roll radius.
(2) elastic compression of strip and roll is negligible.
(3) deformation is under plane strain conditions.
(4) the Maxwell (Von Mises) criterion of yielding holds. If $\sigma_x$ and $\sigma_y$ are the principal stresses, this reduces to $\sigma_x - \sigma_y = \sigma_0 = 2k = 2\sigma / \sqrt{3}$
(5) plane sections remain plane (homogeneous deformation).
(6) coefficient of friction between the roll and slab is constant
(7) normal stress $s$ is assumed to be equal to principal stress $\sigma_y$.

Fig. 2.1 illustrates the stresses acting on a slab element at an angle $\phi$ with respect to the centre line. The stress $\sigma_x$ is assumed to be uniformly distributed across the element. The stress $s$ is the pressure normal to the roll surface and the shear stress transmitted between the roll and slab is $\tau$. The constancy of volume requires:

$$v_0 h_0 = v h = v_1 h_1$$

(2.1)

where $v_0$, $v$ and $v_1$ are the velocities before entry plane, at the roll gap and after exit plane respectively. Also, $h_0$, $h$, and $h_1$ are the corresponding thicknesses. It can be seen from Equation 2.1 that, since $h_1$ is less than $h_0$, $v_1$ must be greater than $v_0$.

The peripheral velocity of the roll $v_R$ will have some intermediate value between $v_0$ and $v_1$, such that the roll surface will be moving faster than the strip at the plane of entry, and slower than the strip at the plane of exit. Thus the frictional forces acting
on the strip will be in opposite directions at entry and exit, and as the strip speed increases in the roll gap there must be a point at which \( v = v_R \) and there is no relative velocity between strip and roll. This is called the neutral point, and is a result of the assumption of plane sections remaining plane.

To derive an equation for the stresses in the roll bite interface, we can consider an elemental slab of material in the arc of contact. Due to the rolling pressure, the roll surface will be flattened in the arc of contact from its original radius \( R \) to a new larger radius \( R' \). In hot rolling, roll pressures are less than those in cold rolling and it is reasonable to neglect roll flattening. The horizontal stress \( \sigma_x \) is assumed to be distributed uniformly over the vertical section. Then, the horizontal force \( f \) per unit width, is equal to \( \sigma_x h \) and the horizontal forces acting on the element shown will be in equilibrium if:-

\[
d(\sigma_x h)/d\phi = 2R (s \sin \phi \pm \tau \cos \phi)
\]  

(2.2)

where the negative sign refers to the conditions on the entry side of the neutral point, and the positive sign refers to the conditions on the exit side. This equation can not be solved analytically, but it has been solved by a step-by-step numerical method.

The differential Equation (2.2) expressed as above, or in one of its alternative forms, represents the starting point in the analysis known as the theory of homogeneous deformation which was first introduced by Von Karman [53].

2.2.1.2 Friction Models

A number of solutions for this equation have been proposed and some of them will be reviewed below. These solutions differ mainly in the assumed nature of the frictional forces in the arc of contact between the roll and slab.
Von Karman's solution [53] is based on the assumption that the dry slipping occurs over the whole arc of the contact between the rolls and the rolled material. Also the frictional force is directly proportional to the value of local normal pressure, i.e.,

\[ \tau = \mu s \]  
\[ \mu = \text{friction coefficient}. \]  

Ekelund's solution [54] assumed that the dry slipping occurs over the whole entry side, and sticking over the whole exit side of the arc of the contact.

Siebel's solution [55] is obtained for the case when the dry slipping occurs over the whole arc of contact between the rolls and the rolled material. Moreover, it is assumed that the frictional force is constant along the arc of contact.

Nadai's solution [56] is based on the assumption that viscous slipping exists in the roll contact zones. Also that the frictional force is proportional to the relative velocity of the slip. Thus,

\[ \tau_x = \mu (v_x - v)/h \]  
\[ \mu = \text{friction coefficient}. \]  

Orowan and Pascoe's solution [57] is for the case when sticking occurs over the whole arc of the contact. Similar assumption was made by Sims [58] and Alexander [67].
Tselikov's solution [51] is given for the case which includes a zone of restricted plastic deformation (called adhesion zone) in the middle of the sticking zone. It is also assumed that dry slipping occurs in the arc of contact close to entry and exit sides. It seems Tselikov's friction model is more realistic than the other models introduced for flat rolling. Most of the FEM solutions showed that a neutral zone (similar to the above adhesion zone) rather than a neutral point can occur in the roll-bite [75].

2.2.1.3 Pressure Distribution along the Arc of Contact

2.2.1.3.1 Sticking Friction

This model has been used by several investigators such as Orowan and Pascoe [57], Sims [58] and Alexander [68]. In this model, frictional resistance between strip and roll is sufficient to raise the shear stress to the yield shear strength \( k \) of the strip, which it cannot exceed.

The simple theory of Orowan and Pascoe [57] has been presented for hot rolling. They replaced the curved surface of the rolls with flat parallel platens, so that the equilibrium Equation 2.2 becomes modified as follows:

\[
\frac{h_1 \, d\sigma_x}{dx} = \pm 2k
\]  

(+) exit side

(-) entry side

According to the 4th and 7th assumptions

\[
s - \sigma_x = 2k
\]  

(2.6)

Integrating this equation, the roll pressure \( s \) is seen to be
\[ s = s_1 + \left( \frac{2k}{h_1} \right) x \quad \text{(on exit side)} \]  

(2.7)

where \( s_1 \) is the value of \( s \) at the exit plane \((x=0)\). At the exit plane where \( \sigma_x \) is zero (i.e. no longitudinal force applied to the slab from the rolls) and \( s_1 = 2k \). Orowan actually used a value of \( s_1 = 2k(\pi/4) \approx 0.8(2k) \) to take into account the influence of inhomogenous deformation from Nadai's solution for flow through a tapered channel. The pressure distribution is as follows:

\[ s = 2k \left( 0.8 + \frac{x}{h_1} \right) \quad \text{(on exit side)} \]  

(2.8)

### 2.2.1.3.2 Slipping Friction

In order to solve Von Karman's differential equation (2.2), another simplified solutions of this equation was proposed by Tselikov [51]. For the case of dry slipping friction, the simplification was achieved by replacing the variable parameter \( \phi \) with a constant value, such as

\[ \phi = \alpha/2 \]

where \( \alpha \) = roll bite angle (Fig. 2.1).

This assumption allows one to reduce Equation (2.2) to a form which can be integrated. However, the obtained results are valid only when the bite angle is small.

If the material is rolled with entry tension \( \sigma_0 \) and exit tension \( \sigma_1 \), the solution for normal pressure distribution in the arc of contact is given by

\[ s = 2k \frac{h}{\delta} \left( \frac{h}{h_1} \right)^{\delta} \left( E_0 \delta + 1 \right) - 1 \]  

(2.9)  

(on exit side)
\[ s = 2 \frac{k}{\delta} \left( \frac{h_0}{h} \right)^{\delta} (\xi_0 \delta - 1) + 1 \]  
(on entry side)  \hspace{1cm} (2.10)

where,

\[ \delta = \frac{2\mu L}{h_0 \varepsilon} \]

\[ L = \sqrt{R \Delta h} \]

\[ \xi_0 = \frac{2k_0 - \sigma_0}{2k_0} \]

\[ \xi_1 = \frac{2k_1 - \sigma_1}{2k_1} \]

According to the above pressure distributions, theoretically, as the coefficient of friction \( \mu \) increases, both the peak pressure and the average pressure increase. The position of neutral point, which lies at the normal pressure peak, shifts toward the entry as \( \mu \) increases. Also, as reduction increases, the peak normal pressure and average pressure increase. Similar effect is produced when the ratio of the roll radius to the exit thickness increases. Moreover, Equations (2.9) and (2.10) show that the higher the entry or exit side tensions, the lower the normal pressure in the arc of contact.

Bland and Ford [60], Sims [58], Ford and Alexander [67] have introduced approximate analytical solutions for the roll pressure. Alexander [61] reviewed and evaluated some of these solutions. Also, he presented a complete numerical solution of Orowan’s equations considering both sticking and slipping friction. This model predicts a sharp peak in pressure distribution and instantaneous change in the magnitude of the shear stress. Later, he [52] introduced the adhesion zone similar to that proposed by Tselikov [51] which resulted in a curved maximum pressure and a more gradual change in the magnitude of the shear stress in the vicinity of maximum pressure. The Alexander’s numerical method and some of the approximate solutions
will be reviewed in details in chapter 4, where the rolling force and torque are calculated by the finite element method, slab method (Orowan's theory) and upper-bound method.

Another numerical solution based on slab method was introduced by Lenard et al. [77]. They discussed the limitation of Orowan's model. The Hitchcock [51] model used in Orowan's theory to calculate roll flattening assumes the deformed roll remains circular which is not realistic in certain circumstances. However, Lenard et al. [77] described the deformed roll function as a curve by a function

\[ y = f(x) \] (2.11)

which is determined as a part of the calculations. Moreover, the following equilibrium equation was derived:

\[ \frac{d}{dx} \left[ h_x (s - 2k + \tau \frac{dy}{dx}) \right] = 2 (s \frac{dy}{dx} + \tau) \] (2.12)

2.2.2 Slip-Line Field

Slip-line fields always refer to plane strain and the material is assumed to be incompressible and ideally plastic. Slip-line fields consist of two sets of curves which cut each other orthogonally to show the direction of maximum shear at every point in the deformation zone. The stresses at any point can be determined by the shear stress and a hydrostatic pressure (mean compressive stress) \( s \).

The following equations have been applied by the investigators to determine the variation of the normal pressure \( s \) along the slip - lines [3]:

\[ s + 2k \theta = c_1 \] (2.13)
\[ s - 2k \theta = c_2 \] (2.14)
where $\theta$ is the angle between $s$ and the vertical direction at any point, $c_1$ and $c_2$ are constant.

Hill [65] described a method for the numerical calculation of slip-line fields. However, in the steady-state case, the use of Hill's method to obtain an acceptable slip-line field which satisfies the velocity boundary condition is difficult.

Prager [66] suggested a geometrical construction to solve the slip-line field equations. The geometrical representation takes the form of three diagrams; the physical plane, showing the slip-line field, the stress plane, representing the stress variation along slip-line, and the hodograph, representing the velocity of the points in the physical plane.

Alexander [67] improved this method assuming that such a solution gave a possible slip-line field for a single geometry of the hot rolling process. Then, Ford and Alexander [68] proposed an approximate slip-line method to obtain solutions for various rolling geometries of hot rolling. They assumed that the slip line at the entrance to the roll gap is straight and that the slip line at the exit is a circular arc. Their relationship with rolling force and torque will be introduced in chapter 4.

### 2.2.3 Shear Plane Theory

Green et al. [69] introduced a simplified solution for hot rolling of ingots. The material was considered to be a rigid triangular area in the roll gap with its base representing the arc of contact. All deformations occurred along the sides of the triangle by pure shear.
2.2.4 Hydrodynamic Theory

Weber [70] developed another theory of rolling by application of hydrodynamics of viscous fluid. Thus, the differential equation of the velocity component of the material in the roll gap described the flow of the material during the rolling process. He assumed the sticking friction condition between the deforming material and the roll surface, and the results were in agreement with experimental results. However, the application of hydrodynamic theory to predict roll force and torque requires a knowledge of material dynamic viscosity which is not yet available [70]. Thus, a quantitative solution is not possible.

Furthermore, Ichinoi et al. [71] assumed a fluid film exists on the strip-roll interface of cold rolling and they applied a theory of hydrodynamic lubrication. Also, Von Karman's equation was used to describe the plastic deformation of the material. The rolling pressure distribution obtained was similar to that derived by Nadai [56] for the case when the shearing stress was assumed proportional to the relative velocity between the material and the rolls.

2.2.5 Upper-Bound Method

A lower-bound solution will give a load prediction which is less than or equal to the load required to cause a body to experience full plastic deformation.

An upper-bound analysis predicts a load that is at least equal to or greater than the exact load needed to cause plastic deformation with no attention being paid to stress equilibrium. This kind of analysis results in a kinematically admissible solution. The upper-bound theorem may be stated as follows:
Any estimate of the collapse load of a structure made by equating the internal rate of energy dissipation to the rate at which external forces do work in some assumed pattern of deformation will be greater than or equal to the correct load.

The basis of an upper-bound analysis is:

(i) an internal flow field is assumed and must account for the required deformation. As such, the field must be geometrically self-consistent.

(ii) the energy consumed internally in this deformation field is calculated using the appropriate work material properties.

(iii) external forces (or stresses) are calculated by equating the external work with the internal energy consumption.

In recent years less attention has been paid to upper-bound analysis. Some investigators have used this method of analysis for the asymmetrically rolling of plain or bi-metallic sheets. More discussion regarding this method will be given in chapter 7.

2.2.6 Finite Element Method

Several finite element models of rolling have been established. Zienkiewicz et al. [72] and Dawson [73] considered the rolling problem with an assumption of viscoplastic behaviour for the deforming strip. Dawson incorporated the effect of temperature into his solution.

The plane strain rolling problem was solved by the rigid-plastic finite element method (FEM) by Li and Kobayashi [74], on the basis of the infinitesimal theory of plastic deformation. They simulated the rolling with different geometry conditions and material properties, numerically and compared them with the experimental data found in the literature. Both steady-state and non steady-state rolling were analysed and the frictional stress was assumed to be velocity dependent. For each case, they evaluated
the velocity field, grid distortion pattern, distributions of stresses, strain rates and total effective strain, normal pressure variation along the roll-work-piece interface, roll separating force and torque. The results were compared with the experimental data on contact pressure, roll separating force, and torque obtained by Al-Salehi et al. [163]. Al-Salehi et al. not only measured the roll separating force and torque, but also measured the contact pressure distribution and the coefficient of friction using the pressure-pin technique for cold rolling.

In some cases, the pressure distribution calculated by FEM had two curves peaks. However, only one peak was obtained in the slab method and the slip line method which indicates an error in these methods. Some of the most important hypothesis for rheological behaviour of strip during the rolling process are:

(i) rigid plastic behaviour
(ii) viscoplasticity
(iii) elasto-plasticity

A brief description of the method used by Pietrzyk et al. [23] to analyse hot rolling is given here.

The approach is based on an extremum principle. The principle states that for a plastically deforming body of volume $V$ under traction $F$ prescribed on a part of the surface $S$ and the velocity $v$ prescribed on the remainig part of the surface, the actual solution minimises the following functional:

$$ J = \int_{V} \sigma_{i} \dot{\varepsilon}_{i} \, dV - \int_{S} F \cdot v \, dS $$

(2.15)

under the constraint

$$ \dot{\varepsilon}_{v} = \dot{\varepsilon}_{x} + \dot{\varepsilon}_{y} = 0 $$

(2.16)
where

\[
\dot{\varepsilon}_i = \sqrt{\frac{2}{3}} \dot{\varepsilon}_i^T \mathbf{E} \dot{\varepsilon}_i = \text{effective strain rate}
\]

\[
\dot{\varepsilon} = [\dot{\varepsilon}_x, \dot{\varepsilon}_y, \dot{\varepsilon}_{xy}]^T = \text{vector of the strain rate components}
\]

\[
\sigma_i = \text{effective stress}
\]

\[
\mathbf{E} = \text{identity matrix.}
\]

They assumed a rigid-plastic material which obeys the Huber-Mises yield criterion with a flow rule of:

\[
\sigma = \frac{2}{3} \mathbf{E} \dot{\varepsilon}_i \frac{\sigma_i}{\dot{\varepsilon}_i}
\]

(2.18)

The incompressibility constraint can be removed by introducing a Lagrange multiplier \( \lambda \) and the stationary problem is then achieved for the finite element problem.

\[
J = \int \sigma_i \dot{\varepsilon}_i \, dV + \lambda \int \dot{\varepsilon}_v \, dV - \int \mathbf{F} \cdot \mathbf{v} \, dS
\]

(2.19)

Minimisation of the above functional gives two non linear equations that should be solved simultaneously and Taylor's expansion can be used to form a set of linear equations with the increments of nodal velocities \( \Delta \mathbf{v} \) and Lagrange multiplier \( \lambda \) as the unknowns. Thermal phenomena are considered by introducing the temperature-dependent properties of the rolled metal. For the thermal-mechanical problem, the most important combined phenomena includes: heat generation due to plastic work, heat generation due to friction forces, accumulation of the deformation work connected with an increase of dislocation density, thermal events connected with metallurgical transformations, cooling by air or water spray on free surfaces, and cooling due to contact with the roll.
2.2.7 Boundary Element Method

The boundary element method (BEM) is one of the most useful general numerical methods, like the FEM. The BEM can be applied to all fields of analysis of the boundary value problems.

The application of either elastoplastic or rigid-plastic FEM to calculate the displacement field has demonstrated the power and usefulness of FEM in rolling technology considerably over the last decade [47]. However, FEM is quite sensitive to the aspect ratios of individual elements. Moreover, the secondary variables (stresses in displacement formulations) obtained through numerical differentiation are inherently less accurate than the primary ones [47]. A more accurate numerical method than FEM would advance significantly rolling technology. The BEM is another powerful numerical technique [48,49].

The BEM is based on both the reciprocal theorem and the principle of virtual work. It involves solving linear simultaneous equations of continuum mechanics. However, there are few examples of BEM in the rolling process.

Chandra [47] was the first to apply BEM to analyse the stress and deformation in rolling incorporating a rate-dependent inelastic constitutive model. The normal pressure distribution obtained from his work compared quite well to those obtained from a rigid-plastic finite element analysis. Moreover, the calculation of residual longitudinal stress distribution across the cross section of the work-piece showed that the residual longitudinal stress is tensile at the top and bottom faces and compressive at the centre. As the roll velocity increases, the residual stress peaks change.

Also, Kihara [50] investigated the residual stress analysis of rolled sheet, estimation of the roll deformation, the thermal property inverse analysis of a cooled billet and the estimation of work-roll temperature distribution in cold rolling.
2.3 Asymmetrical Rolling

2.3.1 Cold Rolling

Asymmetrical rolling is becoming popular for cold rolling of thin strip, mainly because the rolling force is less than that for symmetrical rolling.

It seems, for the first time the advantages of asymmetry were described by Vydrin et al. [80]. They also designed a model of strip rolling with different roll speed and his study showed that the neutral point of the higher speed roll moves to the exit side of the arc of contact and the neutral point of slower speed roll to the entry of it. Consequently, the model exhibits a maximum reduction of rolling force by increasing the roll speed difference.

IHI (Ishikawajima-Harima Heavy Industries Co., Ltd, Japan) has developed another cold rolling model which uses a different optimum roll speed ratio to satisfy each rolling operation conditions and mill capacity. It has the advantages of a 4-high mill and has much the same effect as Vydrin's model on the reduction of rolling force [81].

An experimental study has been carried out by Shiozaki et al. [82] at IHI on cold rolling with different roll speed with the following conditions: By measuring the forward slip of top and bottom work-rolls and using the following relations, the influence of speed mismatch between the work-rolls on the neutral point positions were studied.

\[ h = h_1 + \frac{x^2}{2R'} \]  
(2.21)

\[ h_{NH} = h_1 \frac{v_1}{v_H}, h_{NL} = h_1 \frac{v_1}{v_L} \]  
(2.22)
\[ x_{NH} = \sqrt{R' h_1 \left( \frac{v_1}{v_H} - 1 \right)} \quad , \quad x_{NL} = \sqrt{R' h_1 \left( \frac{v_1}{v_L} - 1 \right)} \]  

(2.23)

where,

- \( h \) = strip thickness at the distance \( x \) from the exit side
- \( h_1 \) = exit side strip thickness
- \( R' \) = flattened roll radius from Hitchcock's formula
- \( x_{NH} \) = neutral point of higher speed roll
- \( x_{NL} \) = neutral point of lower speed roll
- \( v_1 \) = strip exit side speed
- \( v_H \) = speed of higher speed work-roll (bottom roll)
- \( v_L \) = speed of lower speed work-roll (top roll)

Also, they showed the variation of rolling force, forward slip and rolling torques with different roll speed ratio. When the roll speed ratio is 1.43, the stability conditions of rolling are satisfied so that the forward slip of higher speed side is zero and it increases to the positive side with decreased roll speed ratio and reduces negative side (slippage condition) with increased roll speed ratio. However, the forward slip of lower speed side increases conversely with an increase of roll speed ratio. The rolling force decreases linearly with an increase of roll speed ratio and this fact is conspicuous with the thin strip. The rolling torque of the higher speed roll is higher than the symmetrical case and that of lower speed roll is often lower. On the rolling of thick strip, the absolute values of rolling torques are approximately proportional to the roll speed ratio, but on the rolling of thinner strip, it increases steeply to peak value, after which it decreases gradually. The latter phenomenon is due to the decrease in the rolling force with increased roll speed ratio. It is clear that for the same roll speed ratio, the rolling force and torque increase with an increase of work-roll diameter (by neglecting the difference between the front and back tensions). The effect of front and back tension (push-pull) on forward slip showed that the forward slip increased with an increase of forward tension or a decrease of the backward tension. Moreover, the higher speed roll has a negative forward slip when the bottom neutral point is not inside the roll bite.
Wusatowski [83] introduced an equivalent diameter for the rolling with different roll diameters using the symmetrical rolling relations:

\[ D_{eq} = \frac{2D_1D_2}{D_1 + D_2} \]  

(2.24)

T. Kawanami et al. [84] have studied the influence of asymmetry on rolling parameters such as rolling force, using different work-roll diameters at the same speeds. In their experiments, an asymmetric cluster mill was developed to attain a higher reduction in thickness than that of a four-high mill or a six-high mill. The back and front tensions were in the range of 4.6-5.4 Kg/mm\(^2\) and 9.7-10.1 Kg/mm\(^2\) respectively and the experiments showed that the rolling force decreases if the equivalent diameter or the draft decreases. Moreover, it showed close results for both cases of asymmetry and symmetry with the same equivalent diameter. Thus, the above equivalent diameter relation is a good approximation for industrial applications.

On the other hand, when the equivalent diameter was less than a special value (here 50 mm), the forward slip is negative and the rolling condition can be unstable, because the neutral point is not in the rolling bite and slippage occurs. Moreover, the forward slip drops as reduction increases.

Another study on asymmetrical cold rolling with two different diameter (same speed) was carried out by Wang et al. [85]. By assuming the same pressure distribution in both rolls interfaces, the slab method was applied to calculate the roll pressure distribution, rolling force and torques. The variation of rolling force calculations with respect to reduction was compared to experimental data. As the work-roll diameter ratio increases, the rolling force decreases significantly and rapidly. The rolling torque for a larger diameter work-roll was always 20-30% less than that for the other driven roll. As the ratio of the diameters increases, the total rolling torque decreases except for cases with work-roll diameter ratio greater than 1.3.
The strain analysis for asymmetry was studied by Yamamoto et al. [86]. The distribution of strain across the thickness of rolled strips has been calculated for different friction coefficient.

The most comprehensive study based on the slab method was carried out by Heurtault [87]. The model can be used to calculate the different rolling parameters (pressure distribution, rolling force and torque) for asymmetries caused by:

(i) two work-rolls of different diameters  
(ii) two work-rolls with different velocities  
(iii) different friction coefficient in top and bottom layer using Coulomb friction model

The work hardening effect in both the thickness and rolling directions has been taken into account. Three differential equations were written for the three regions of forward slip of higher speed side, backward slip of slower speed side and the region between the neutral points. The Runge-Kutta method was used to solve the equations.

2.3.2 Hot Rolling

2.3.2.1 Plain Strip Rolling

The sensitivity of rolling parameters to the asymmetry in hot rolling is also important due to the higher values of friction coefficient than those of cold rolling. The selection of asymmetric rolling regimes in hot rolling of thick sheet and wide strip requires serious theoretical and experimental investigations. It is well known that such experimental investigation would be very labor-intensive, and limited by the working condition and production range of the rolling mill used.
Moreover, hot rolling is a very complex process. Most of the rolling process on hot rolling mills are assumed to be symmetric but in practice they are really asymmetric. For example, Kun et al. [90] studied the processes occurring during rolling on a four-high skin pass mill with single-spindle drive which had not been studied in sufficient detail before. A series of experiments employing the punch mark method was carried out to study the characteristics of sheet skin pass on a 1700 mm four-high mill with a single spindle drive. During the experiment, they measured the forward slip on the upper surface of the deformation zone \( \left( S_u \right) \) and the forward slip on the lower surface of the deformation zone \( \left( S_l \right) \). The measurements for \( \epsilon \) (strain), \( S_u \) and \( S_l \) were processed using a Nairi-K computer and the following linear regression equations were obtained for the skin pass

\[
S_u = 0.308 \epsilon - 0.950 \quad (2.25)
\]

\[
S_l = 0.189 \epsilon + 0.0475 \quad (2.26)
\]

When \( S_u \neq S_l \), the system was asymmetric.

It seems Collins et al. [88] were the first to present an analysis of hot asymmetrical rolling based on the slip line field method. Their theoretical results were compared with the experimental data and the correlation was extremely good. The first effect studied by Collins et al. was the influence of unequal roll diameters on exit side strip curvature. It had previously been expected that this would produce strip that curved away from the larger roll, which inherently had a higher surface speed. The strip actually curved either way, depending on the ratio of roll diameters. The maximum strip curvature occurred with only 5% difference in roll diameters, and the strip came out straight with a difference of about 9%. The torques on the rolls varied in a similar manner with the roll diameter ratio. Note that in two cases, one roll actually drove the other. The second and third effects studied were the variations of strip curvature when changing the angular velocity of one roll and changing the reduction (for a given asymmetry) respectively. The interesting result that came out of this work was
that the material can curve either way as a result of different roll angular velocities or by changing the reduction in asymmetry.

Recently the application of asymmetrical rolling was studied by some investigators at a few hot rolling mills around the world. For instance, the 1700 mm continuous hot-strip mill at the Karaganda Metallurgical Combine (Central Scientific-Research Institute of Ferrous Metallurgy, USSR) implemented asymmetrical rolling which will be reviewed here.

The work-rolls on the 1700 mm mill are driven through a pinion stand and the process can be made asymmetrical by using rolls of different diameters. To determine the main parameters of this process, Soskovents et al. [89] modelled the asymmetrical rolling on an experimental 450 mm hot rolling mill. They rolled specimens 1.2-7.4 mm thick made of steel. The strip dimensions and the reheating temperature corresponded to the finishing temperature on the 1700 mm mill. They used the experimental mill to study the effect of the difference in the roll diameters on the bending of the strip, the power-force parameters, and the uniformity of the load on the spindles of the main drive. The results showed that torque on the slower roll \( T_L \) approaches zero by increasing the roll-speed difference parameter \( a = \frac{v_H}{v_L} \). For this case, the entire rolling torque is transmitted by the driving roll (higher speed roll) as shown in Fig. 2.2. With a further increase in the speed difference, the torque on the lower speed roll has a negative value. When the speed difference is 4-5%, the roll torque \( T_H \) on the driving roll is 4-5 times greater than the torque during symmetrical rolling. It was noted that bending of the strip occurred on the 450 mm rolling mill with different roll speeds. The end of the strip is bent either on the driven (slower roll) or the driving roll (faster roll), depending on the degree of asymmetry and reduction. When the strain was 18%, the curvature was close to zero and the strip left the rolls without bending for asymmetry with 1.12 diameter ratio. It was shown that by moving the roll with the lower peripheral velocity forward in the rolling direction, the strip bending was eliminated. The bending of strip was also affected by roll
hardness difference. An increase in the body hardness of one roll reduces the friction and forward-slip on that roll. In that case, the strip is curved towards the roll with the greater hardness. The asymmetrical rolling force was slightly lower (by 10-15%) than that of symmetrical rolling. Furthermore, their study showed that the quality and accuracy of rolling improves by asymmetry. Because the asymmetry not only reduced the rolling force, but also reduced the roll flattening and fluctuation of this force along the product. It showed that the variation of the strip thickness and flattening of the strip in the asymmetrical rolling is less than the symmetrical case.

The asymmetrical rolling model developed on the experimental mill was tested and introduced on the 1700 mm mill. The difference between the diameters of the rolls reached 3-4%, which was permissible with regard to the strength of the main drive of the mill. They also studied the effect of a difference in roll speed (to 4%) on the roll flattening and accuracy of the rolled product of 1700 mm mill. The rolling torque was monitored from the current on the main drive.

![Asymmetrical rolling torques at the rolls with different velocity](image)
In all cases of asymmetrical rolling, the total torque increased but it did not exceed the permissible value. The curvature decreased by an average of 25%. However, a difference in roll speed increased contact slip in the deformation zone, and thus wear of the rolls. Measurements made in the roll-dressing section showed that work-roll wear increased with the use of asymmetrical rolling. The study conducted here established that the optimum range of roll-speed differences for the 1700 mm mill at the Karaganda works is 1-3%.

Unfortunately, the above investigators did not provide the specific data used (such as mechanical properties of hot strip) in their paper. Thus, the results in this thesis can only be used for a qualitative comparison with other investigations and the quantitative comparison is not possible.

The main factors causing asymmetry in hot rolling were analysed in [91,92] are as following:

(i) geometric asymmetry (different roll diameter)
(ii) kinematic asymmetry (different roll velocity)
(iii) surface asymmetry (different friction coefficient on the top and bottom surface of strip)
(iv) physical or mechanical properties asymmetry (for example, the difference between the flow stress of the top and bottom surface of slab or strip in the cladding sheet)
(v) temperature asymmetry.

Each of the above asymmetrical condition influences the rolling characteristics such as asymmetry of contact stresses, stressed state, and metal flow in the deformation zone. Most of the theoretical and experimental investigations were carried on the geometric and kinematic asymmetries [92-97]. Surface asymmetry was analysed in [98-100].
Asymmetry was introduced in a rolling mill in order to compensate for the nonuniform distribution of mechanical properties caused by the cooling of the bottom slab surface which took place during its contact with the skids in the furnace. The asymmetry was created by rolls with different diameters or additional lubrication on the bottom roll in order to prevent bending of the slab [108].

A theoretical analysis of asymmetric rolling (measurement of contact stresses for instance) is usually based on symmetrical rolling theory. In order to apply that theory, asymmetric rolling is regarded as a combination of two separate symmetrical processes for the driving and driven rolls (quasi-symmetrical) [93-97]. However, that approach is not applicable for velocity asymmetry, because the latter would require integration of the contact stress equation, with consideration of boundary conditions for each roll which is not known, being dependent on the velocity mismatch [102].

An application of slip-line method to investigate asymmetric rolling has been carried out by Brovman [94,95] and Mazutani et al. [96]. The slip-line method is based on an assumption that plastic regions in the deformation zone merge at one point on the symmetry axis. This assumption means that tangential stresses along the contact surface should be of the same sign which in reality is wrong. It is also assumed that the ratio of the roll torque is known. These and other assumptions of the slip-line method are different from the real conditions of asymmetric rolling, making it difficult to apply the results [102].

An attempt was made in Too et al. [101], Kiumi et al. [103] to solve the problem of asymmetrical rolling using the finite difference method, which presents a better model than the above method.

Application of the finite element method for calculation of the stress-strain state of metal in rolling provides more reliable and more complete information on energy parameters and shaping than the other methods in the theory of rolling. There are
many publications on FEM analysis application for calculating symmetrical flat rolling process, [see review on symmetry]. However, all the existing FEM calculation algorithms and programs are not suitable for calculation of asymmetric regimes because in such regimes the boundary condition on the contact surface between metal and rolls are not known, and they are quite different for each roll. There is no complete adhesion contact surface in asymmetric rolling of even thick strips.

A comprehensive paper based on FEM analysis has been introduced by Kalmykov et al. [102] for complete solution of hot rolling of plain strip. Mixed boundary conditions on the top or bottom strip surface were used to account simultaneously for the formation of adhesion and sliding zones. His results will be reviewed here:

(i) Influence of velocity mismatch:

The paper presents the variation of rolling force and torque on work-rolls as a function of velocity mismatch. When the mismatch is up to 4%, the rolling force decreases in comparison with symmetrical rolling by 2-4%. As the velocity mismatch increases to 8%, the rolling force decreases by 8-12%. Further increases of the mismatch decreases that effect. The relative change of force in asymmetric rolling also depends on \( \frac{L}{h_{av}} \)

where

\[
h_{av} = \frac{h_0 + h_1}{2}
\]  

(2.27)

When the \( \frac{L}{h_{av}} \) ratio increases the contribution of the friction force on the contact surface increases as well. At a velocity mismatch of 5-6% , the driving roll was overloaded 4-5 times in comparison with symmetrical rolling. Transition of the driven roll into generator regime occurred at a mismatch of 1.5-2.0% which was well supported by experimental data [89]. If the velocity mismatch exceeds 10%, the slippage may occur on the driving roll (faster roll), which is unstable rolling.
(ii) influence of velocity mismatch and geometric asymmetry simultaneously:

As the diameter of one of the rolls is reduced, the rolling force decreased even in a symmetrical regime. For instance, for a diameter difference of 20%, the rolling force decreases by 8-12% which agreed with experimental data [84]. The simultaneous velocity mismatch and geometrical asymmetry reduces the rolling force up to 22% ($\Delta V = 12\%, \Delta D = 20\%$) in comparison with symmetrical regimes. In that case, the decrease of rolling force due to the geometric asymmetry alone was 7-8%. Therefore, the simultaneous effect of the velocity and roll geometry asymmetry is 2-3 times more effective than the roll velocity mismatch alone. This effect depends on which roll has a higher velocity, because, by increasing the velocity of the smaller roll, one can reduce the rolling force to a much greater degree than increasing the velocity of the larger roll. Variation of the rolling torque showed that a decrease of roll diameters in the general case decreases the torque. If the rolls have the same velocity, the difference of their torque depends on the difference of their diameters. For instance, if the diameter difference is 20%, the torque on the larger roll is greater by 20-25% than that of the smaller roll.

(iii) The influence of surface asymmetry:

If one of the rolls has a lower friction coefficient than the average value, while the other roll has a higher value, the rolling force is lower in comparison with the case when both rolls have the same friction coefficient. For instance, if $\mu_1 = 0.15$ and $\mu_2 = 0.3$ ($\mu_{av} = 0.225$), the rolling force decreases by 3-5%. This effect is stronger if the driving roll has a higher friction coefficient than the driven one. The reduction of rolling force in comparison with the case of equal friction on the rolls, decreases by 8-14% at a velocity mismatch of 12%. If the friction coefficient of a roll is increased, the roll torques become redistributed. Even if there is no velocity mismatch that difference might reach 20-30%. When the driving roll has a higher friction coefficient, the rolling torque depends more strongly on velocity mismatch than when
the driving roll has a lower friction coefficient. Although the above explained paper is a very worthwhile contribution in asymmetrical rolling, the specific data used were not shown in the paper.

2.3.2.2 Cladding Sheet Rolling

The most complicated asymmetrical rolling conditions occurred during hot rolling of bimetal (cladding) plates. The slab method is not able to handle the asymmetries caused by mechanical property variations in the slab thickness in the roll gap. Thus, the other methods such as upper-bound, FEM or BEM should be applied to these kinds of asymmetries.

Kiuchi et al. [105] has a solution to the kinematic problem of cladding sheet using upper-bound method including all mentioned asymmetry conditions and considering the difference between the initial thickness of the two metals and the inlet angle of the sheet at the entrance of roll-gap. The flattening and bending of the rolls and spreading of the sheet were not considered. Furthermore, the linear velocity distribution was applied.

Pietrzyk et al. [106-108] have offered excellent contributions to the field of bimetal rolling. At first, they used the numerical method based on the upper-bound method by simplifying the geometry of the system and assuming linear contact surfaces between the plates and rolls [106]. Then, finite element simulation was applied to analyse the distribution of velocities, strain rates, strain, roll pressure and temperatures in the roll-gap [107,108].

Analysis of data given by Pietrzyk et al. [108] showed reasonable accuracy of both the upper bound and finite element methods. It leads to the conclusion that the methods based on a simplified velocity field can be used successfully to simulate the overall parameters of the rolling process such as rolling force or torque. Given
satisfactory accuracy of results, these methods require less computer memory and shorter computing time when compared with the finite element approach. However, the finite element method is irreplaceable when simulating the pattern of the metal flow through the gap. Their method was based on thermo-mechanical model and rigid-plastic rheological behaviour. Thus, both metal flow and temperature distribution were calculated for various reductions and dimensions of bimetal plates. Two possible cases for boundary conditions were considered. One assumed the presence of the special guides (tensions) to prevent bending of the plate, the second allowed free bending of the plate on both sides of the mill caused by asymmetry. Their main results are the following:

(i) The influence of asymmetry caused by cladding on the distribution of velocities, strain rates and strains in the case of rolling with tension is negligible.

(ii) Strains in the soft layer exceed those in the hard one. The neutral point in contact with the soft layer moves towards the entry side and the hard layer towards the exit side.

(iii) The effect of cladding on the temperature distribution (the difference between the top and bottom layers) is negligible.

(iv) It is possible to find an optimum value for the ratio of top to bottom roll diameter to prevent bending in rolling of cladding material.
CHAPTER 3

MECHANICAL PROPERTIES
3.1 Flow Stress

3.1.1 Introduction:

The resistance of material deformation is required in the prediction of parameters in hot strip rolling. For example, the accuracy of rolling force and torque calculations are directly dependent on the results of the resistance to deformation measurement. The resistance to deformation at high temperatures is affected by some metallurgical processes such as hardening, softening and recrystallization. In hot working at high strain rates, the flow stress rises to a maximum. Then, as a result of dynamic recrystallization, it reduces to a value intermediate between the yield stress and the maximum or peak stress (Fig. 3.1). At lower strain rates, the softening produced by dynamic recrystallization is followed by renewed hardening and a cyclic flow curve of approximately constant period with damped amplitude height [119].

![Stress-strain curves](image)

**Fig. 3.1** Stress-strain curves
3.1.2 Literature Review

3.1.2.1 Experimental Methods

Pietrzyk et al. [23] and Ginzburg [126] have reviewed the experimental methods and the empirical constitutive relations for carbon steels. The four different test methods to measure the flow strength are as follows:

3.1.2.1.1 Tension Test

It is the simplest method to perform, but there are some disadvantages and limitations with it. The magnitude of strain is significantly lower than that of the other methods, and also necking occurs during tension test.

3.1.2.1.2 Torsion Test

The strains with higher magnitude than tension test can be achieved by a torsion test. However, the radial variation of the strain rates is one disadvantage of this method.

3.1.2.1.3 Compression Test

Solid or hollow cylindrical, conical or flat specimens are used in a compression test. The strains are higher than those in a tension test. Usually, the researchers use the cam plastometer to achieve a constant rate of strain in compression testing of solid cylindrical specimens. The friction of the contact surfaces between the specimen and the compression test machine can affect the results of the compression test. One of the methods to reduce the effects of friction is by using a slender cylinder [23]. Another way is by using lubricant on the contact surfaces.
3.1.2.1.4 Rolling Mill Test:

In the previous experimental methods, the deformation resistance was available directly from stress-strain data file. However, the mean flow stress is measured indirectly by the mill data method. An analytical or numerical rolling force or torque model is applied to evaluate the unknown flow stress by measuring the rolling force or torque on a rolling mill.

Sims' model [58] has been applied by some investigators' to calculate the strength of steels under hot rolling conditions [110-113]. The reduced Sims' model for a known rolling force $p$ is in the form of

$$P = \sigma_m L Q_p$$

(3.1)

where $Q_p$ is geometric term and $\sigma_m$ is the mean flow stress which can be found from the above relation.

The flow stress determined by this method seems to be more realistic to apply for rolling process than those from tension, torsion or compression tests [126]. However, there are some approximations or simplifications in every rolling force or torque model and these approximations have an influence on the calculation of flow stress obtained from rolling mill data. Ginzburg [126] has given a comprehensive review of several investigators results in the field of resistance to deformation using rolling test method.

3.1.2.2 Empirical Constitutive Relations For Carbon Steels

The accurate, reliable and repeatable flow strength data, by themselves, are not sufficient for the calculation of force, torque, pressure and power during bulk or sheet metal forming processes. The material's behaviour subjected to loading must
also be represented by an appropriate constitutive equation, giving the strength as a function of other process parameters, such as strain, strain rate and temperature. This involves selecting the form of the equation as well as using careful nonlinear regression analysis in determining the parameters of that relation [23].

Nadai et al. [56] determined such relationships for different steels and examined the effect of temperature and strain rate on tensile strength. In all tests of this kind the strain rate varied during the deformation process, so for different temperatures, different mean strain rates were employed. Some works have been done on the cam plastometer, which can carry out a compression test at a constant strain rate within the range of 0.2-100 per sec. Adler et al. [117] used such an instrument to measure the flow stress of aluminium, copper and 0.17 carbon steel in the strain rate range of 1 - 40 1/s and up to a strain of 50%, while Cook [115] obtained data for steel at high strain rates and with temperatures between $900^\circ$ C and $1200^\circ$ C.

The most comprehensive series of results on yield stress, using both cam plastometer and a drop hammer, were carried out by Suzuki et al. [114], on a wide range of ferrous and nonferrous metals. These results were presented in the form of about 100 data sheets.

The yield stress curve obtained at the relevant temperature and a constant strain rate equivalent to the mean strain rate during rolling, enables a mean yield stress to be determined by integration over the strain of interest. Sims [58] suggested the following expressions:

\[
\text{Mean Flow Stress for Rolling Force} = \frac{1}{\alpha} \int_{0}^{\alpha} \sigma \, d\phi
\]

\[
\text{Mean Flow Stress for Rolling Torque} = \frac{1}{\varepsilon} \int_{0}^{\varepsilon} \sigma \, de
\]
where \( e \) is strain and \( \phi \) the angle of a point at roll bite from the exit plane. Unfortunately, the above equations did not include any variation of strain rate during deformation.

During hot deformation ferritic and austenitic steels undergo work-hardening until a critical strain is attained beyond which a recovery process and a dynamic recrystallization are initiated [119], causing a drop in yield stress as deformation proceeds (Fig. 3.1). The Zener-Holman parameter \( Z \), which could be associated with the dynamic recrystallization process during hot deformation in low carbon steels remains constant over a wide range of strain rates [118]

\[
Z = \dot{e} \exp \left( \frac{Q}{RT} \right) \quad (3.4)
\]

where,

\[
T = \text{absolute temperature},
\]

\[
R = \text{the gas constant},
\]

\[
Q = \text{the activation energy},
\]

which are obtained experimentally for various metals [119].

Many empirical constitutive equations have been presented in the technical literature. Those specially designed for high temperature, and high strain rate applications include the work of Altan et al. [120], Shida [121], Gittins et al.[122] and Hajduk et al. [123]. The equation of Ekelund presented by Wusatowski [124] should also be mentioned partly for historical reasons.

Altan et al. [120] have compiled a considerable amount of data on the behaviour of engineering materials. For high temperature flow they used the well-known power law

\[
\sigma = c \dot{\varepsilon}^n \quad (3.5)
\]
where \( c \) and \( m \) are strength and strain rate hardening coefficients respectively, for specific temperatures and strains. The chemical compositions of the materials were also given in the above reference.

Tarokh et al. [125] developed an expression

\[
\sigma = a_0 + \frac{a_1}{t^{1/2}} + \sqrt{\varepsilon} \left( a_2 + \frac{a_3}{t'} + a_4 \ln \dot{e} \right)
\]  

(3.6)

where \( a_0 \) to \( a_4 \) are constants. For steel with 0.2% C deformed at a strain rate of 2 (1/s), and using the above expression, Moller [110] established the following empirical relationship

\[
\sigma = -1.429 + 4.985 \left( \frac{t'}{1000} \right)^{-2} - 2.13 \sqrt{\varepsilon} + 33.08 \sqrt{e} \left( \frac{t'}{1000} \right)^{-1}
\]  

(3.7)

One of the most comprehensive set of empirical equations for high temperature behaviour of carbon steels were developed by Shida [121]. The results were obtained using a constant strain rate cam plastometer. The flow strength was given in terms of strain, strain rate, temperature and carbon content. Shida also compared experimental results by performing at least ten different tests for the same experiment. Shida [121] expressed the flow strength in the form

\[
\sigma = \sigma_f f \left( \frac{\dot{e}}{10} \right)^m
\]  

(3.8)

where \( \sigma_f \) is a function of temperature and carbon content. Also, \( f \) is a function of strain and carbon content.

Gittins et al. [122] derived the coefficients of an equation giving the mean yield strength as
\[ \sigma = a_0 + \varepsilon^{0.2} \left( a_1 + a_2 \ln \dot{\varepsilon} + a_3 - \frac{t'}{1000} \right) \]  

(3.9)

In this equation, the symbol \( \varepsilon \) represents the engineering strain, \( \dot{\varepsilon} \) is the mean strain rate and \( t' \) is the temperature.

An equation originally proposed by Ekelund and used by Wusatowski [124] defines the yield strength as a function of temperature, carbon, manganese and chromium contents as

\[ \sigma = 9.81 \left( 14 - 0.011 t' \right) \left( 1.4 + C + Mn + 0.3 Cr \right) \]  

(3.10)

where \( C \), \( Mn \) and \( Cr \) are the carbon, manganese and chromium contents, respectively, in weight percentage. The above equation indicates ideally plastic, rate-independent behaviour. Wusatowski [124] includes this as an empirical relationship which attempts to predict roll separating forces in hot flat rolling. The formula presented by Wusatowski [124] contains corrections for the rate effects but those corrections do not appear to contribute significantly to the yield strength.

The equation, given by Gittins et al. [122] for mild steel, which has the chemical composition closest to AISI 1015 with a carbon content of 0.28% and a manganese content of 1.0%, is

\[ \sigma = 0.49 + \left[ 1 - \exp(-\dot{\varepsilon})^{0.2} \right] \left( 516.56 + 21.3 \ln \dot{\varepsilon} - \frac{357230}{t'} \right) \]  

(3.11)

The equation of Hajduk et al. [123] is given in the form

\[ \sigma = \sigma_0 K_T K_\varepsilon K_u \]  

(3.12)

where \( \sigma_0 \), \( K_T \), \( K_\varepsilon \) and \( K_u \) are the functions of the material, temperature, strain and strain rate, respectively. The values of constants for 25 different low and medium
carbon steels are calculated. For example, the yield stress equation given for the carbon steel containing 0.13% C and 0.4% Mn, is

\[ \sigma = 98.1[17.8 \exp(-0.0029t')] (1.79 \epsilon^{0.252}) (0.72 \epsilon^{0.143}) \]  

(3.13)

Also, the flow stresses of steels have been measured in the past decade [111-113, 77,128,129]. Lenard et al. [77] carried out some experiments on low carbon steel and Vanadium steel.

The torsion test and rolling mill test have been performed by Hodgson [111] with 0.1%C at BHP steel. The average flow stresses based on torsion data (model) have been determined using a constitutive relation to compare with the results of rolling test method (measured curve) based on Sims' model, using BHP hot rolling mill data.

3.1.3 Experiments and Results

3.1.3.1 Experimental Material

The tested low-carbon steel from BHP Steel has the following chemical composition by weight:

- C=0.08, Al=0.041, Cr=0.041, Cu=0.007, Mn=0.25,
- Mo=0.002, N=0.002, Ni=0.028, P=0.016, S=0.015,
- Si=0.005, Sn=0.002, V=0.003

3.1.3.2 Experimental Equipment

The uniaxial hot compression test was carried on a microprocessor-controlled servohydraulic test machine INSTRON MODEL 1343 from Material Engineering Department with a movable furnace as shown in Fig 3.2.
3.1.3.3 Experimental Procedure

Cylindrical samples of 8 mm diameter and 12 mm length were machined from the above material and they were compressed to a strain of 0.9 at constant strain rates 0.4 to 5 (1/s). The temperatures used in this test were 900, 1000 and 1100 °C. Glass powder was used as a lubricant to reduce friction during the compression tests. Testing temperature, strain and constant strain rates were set and controlled by a PC. Load and extension were recorded by a load cell and an extensometer fixed on the machine. The data was collected by the A/D converter of the computer and the resulting true stress-true strain curves were obtained directly on the attached plotter.

![Flow Stress Experimental Equipment](image)

Fig3.2 Flow Stress Experimental Equipment

3.1.3.4 Stress Strain Curves

The true stress-true strain curves for different temperatures and various strain rates are given in Fig.3.3 (a,b,c). For the lowest rate of strain 0.4 (1/s), there are a few dynamic recrystallization cycles, while at the higher strain rates the initiation of dynamic recrystallization is followed by softening. Also, the number of dynamic
recrystallization cycles at a higher temperature (1100°C) is more than that at the lower temperature (900°C) for the lowest strain rate.

Fig. 3.3a Flow stress-strain at $t' = 900^\circ$C

($\dot{\varepsilon}$ = rate of strain, 1/s)
Fig. 3.3b  Flow stress-strain at $t' = 1000^\circ C$

($\dot{\varepsilon}$ = rate of strain, 1/s)

Fig. 3.3c  Flow stress-strain at $t' = 1100^\circ C$

($\dot{\varepsilon}$ = rate of strain, 1/s)
3.1.3.5 Empirical Constitutive Relation

The flow curves of hot steel may be represented by the following equation

\[ \sigma = f(\varepsilon, \dot{\varepsilon}, t') \]  

(3.14)

where \( \varepsilon \) is the true strain, \( \dot{\varepsilon} \) the strain rate and \( t' \) the temperature. Several mathematical models for the above generalised equation have been introduced by many investigators. The simplest model has been given by Pietrzyk et al. [23] as

\[ \sigma = A_0 \dot{\varepsilon}^a \varepsilon^b e^{-ct'} \]  

(3.15)

where \( A_0, a, b, c \) are the coefficients which are dependent on the steel type and the stress-strain curve region. These regions include the initial hardening part followed by softening and then cyclic hardening-softening. The dynamic recrystallisation appears in the experimental results especially at low strain rates, but we have not considered this phenomenon in the above formula. The results of this experimental work will be used for steady state and transient load calculations on the Hot Roughing Mill of BHP Steel (Port Kembla). The strain rates in the Roughing Mill of BHP are higher than experimental conditions considered here and the recrystallisation is negligible. Moreover, equation 5.6 (strip damping) used in this thesis can only be applied if equation 3.15 is used.

The strains and strain rates at the roughing mill are 0.3-0.5 and 3-18.5 (s\(^{-1}\)) respectively, but the coefficients of the above empirical constitutive relation have been calculated based on the 0.15-0.5 strains domain and 0.4-5 (s\(^{-1}\)) strain rates domain because of the limitations of the test rig speed. Thus, extrapolation of the measured data is required for the strain rate range 5-18.5 (s\(^{-1}\)), but we have not considered this matter in the analysis.
From equation 3.15 we can write

\[ z = A + a B + b C + c T \]  \hspace{1cm} (3.15a)

where

\[ Z = \ln \sigma \]

\[ A = \ln A_0 \]

\[ B = \ln \dot{\varepsilon} \]

\[ C = \ln \varepsilon \]

\[ T = - t' \]

Thus we need four equations to find the four unknowns \( A_0, a, b \) and \( c \). Using non-linear regression the following results are obtained:

\( A_0 = 1896 \text{ MPa} \), \( a = 0.1608 \), \( b = -0.0538 \), \( c = 0.003 \)

where the temperature \( t' \) is in °C in the above model.

The results are in agreement with the results obtained by Lenard et al. [77] and Suzuki [114].
3.2 Damping

The Relationship amongst various measures of damping has been given by [159] (valid for small values of damping: \( \tan \phi < 0.1 \))

\[ \eta = \frac{\delta}{\pi} = \tan \phi = \phi = 2\zeta \]  

(3.16)

where

- \( \eta \) = Loss Factor
- \( \delta \) = Logarithmic Decrement
- \( \phi \) = Phase Angle by Which Stress Leads Strain
- \( \zeta \) = Damping Ratio or Damping Factor

3.2.1 Introduction

Damping is the dissipation of vibrational energy and the consequent reduction or decay of motion. The development of lighter and more flexible aerospace structures has required a more complete understanding of the damping properties of candidate materials.

There are two primary forms of damping, internal damping and external damping.

- Internal damping refers to energy dissipation within a material.
- External damping is the energy dissipated by mechanisms external to a material, such as joint friction or air damping and is beyond the scope of this thesis.

Even at stress levels well below the elastic limit, no real material behaves as a perfect elastic solid. During cyclic loading, time-dependent mechanisms within the material always cause the strain to lag behind the applied stress [132]. Examples of such mechanisms are the motion of a dislocation through a crystal lattice or the rearrangement of a long-chain molecule in a polymer. Therefore, the cyclic stress-strain curve is not absolute but forms hysteresis loop. The area enclosed by the loop is proportional to the
energy dissipated and internal damping is defined herein as the energy dissipated during the cycle.

The study of solid mechanics and the dynamic response of solids has developed from sound mathematics, established by such men as Euler [1707-1789], D' Alembert [1717-1883], Lagrange [1736-1813], Laplace [1749-1827] and Cauchy [1789-1857]. However, in 1267, Bacon in his Opus Majus and Opus Tertium [133] wrote, "Without experiment nothing can be adequately known " [134]. Beginning around the late 17th century, a number of investigators conducted theory-related experiments on the dynamic response of solids. Among these were Bernoulli in 1751 [135], Riccati in 1782 [136], Coulomb in 1784 [137], and Kohlrausch in 1863 [138]. In his paper, Riccati introduced a dynamically determined measure of the modulus of elasticity 25 years before Young. Among other studies, Kohlrausch measured the decay rate of amplitude as a function of different initial amplitudes in glass and brass. However, it was not until the late 1930s and 1940s that researchers such as Wegel et al. [139], Kimball [140], Hatfield et al. [141], Robinson et al. [142], and Hanstock et al. [143] began to establish a collection of data suitable for use in scientific studies of the mechanisms responsible for energy dissipation in materials. With this accumulation of damping data came further development of theories for the causes of the observed trends and behaviour. Notable contributions were the studies of atomic relaxation and thermoelastic damping by Zener [144,145] and the theory of dislocation damping developed by Granato et al. [146,147]. In recent years, there has been a significant advance in the theoretical modelling of damping due to the development of composite materials [148], amongst others, who have extended these studies to the viscoelastic moduli using the principle of linear viscoelasticity [149]. Nowick et al. [132], Zener [145], Mason [150], and Lazan [151] have discussed many mechanisms of damping. Wood et al. [152], Ganc [153], and Ritchie [154], present comprehensive bibliographies of studies in material damping.
3.2.2 Literature Review

The history of damping has been divided by Plunkett [130] into three epochs: first from 1784 to 1920 in which a natural philosophy approach was used; second from 1920 to 1940 in which industrial applications were made and third from about 1940 to the present in which the principles of physics, applied mechanics, and materials science has been used to characterise and analyse the influence of damping on dynamic systems. The literature regarding the third period is given here and the other parts are available in appendix 1.

-Physics and Metallurgy (1940-)

Micromechanical Damping

Damping measurements have long been used by metallurgists and physicists to study the details of single crystal and polycrystalline materials [145,79]. Among the phenomena studied are solute atoms, atomic diffusion, phase changes, precipitation, location of interstitial and substitution atoms, and twinning. With the exception of phase changes, none of these mechanisms causes very high damping. The damping factor changes with temperature and frequency, peaking at a critical temperature for each frequency and at a critical frequency for each temperature [145]. Polymers have a combined temperature-frequency relationship which can usually be closely approximated by an Arrhenius relationship [158].

3.2.3 External Damping

3.2.3.1 Acoustic Radiation Damping

The vibrational response of a structure will always couple with the surrounding fluid medium, which may commonly be air, water, oil, or other gases or liquids.
As the foregoing discussion shows, acoustic damping can sometimes be a very important factor in controlling structural response, but its order of magnitude is often too small to be useful. This occurs when the density of the fluid medium is too low in relation to the massiveness of the structure or if acoustic pressures from some parts of the vibrating structure cancel out those from the other parts, as would happen for modes of vibration in which adjacent areas vibrate in antiphase with each other. For spacecraft, acoustic damping does not exist. For some thin, lightweight, stiffened structures such as aircraft panels, the acoustic damping can on occasion be important.

3.2.3.2 Linear Air Pumping

The fluid in which a structure is immersed can provide other damping mechanisms besides the radiation of energy away as sound waves. If the vibrating structure is backed by a nearly airtight volume, as often happens in the construction of complex structures, the entrapped air is alternately compressed and rarefied by the motion of the panel, leading to a pressure increment proportional to the panel motion. If the enclosed air is totally encapsulated, no energy dissipation can occur. If, however, there are any small leaks, the energy dissipation occurs depending on the flow rate of the air leak.

3.2.3.3 Coulomb Friction Damping

Frictional forces arising from the relative motion of two contacting surfaces are generally modelled by a constant force proportional to the normal load between the surfaces and directed against the velocity vector at each instant.

3.2.4 Internal Damping

After all external sources of damping have been accounted for, there still remain a large number of mechanisms whereby vibrational energy can be dissipated within the volume of a material element as it is cyclically deformed. We shall not in any way endeavour to
review all of these mechanisms, only a few of which are dominant at any one time. Included are magnetic effects (magneto-elastic and magneto-mechanical hysteresis), thermal effects (thermo-elastic phenomena, thermal conduction, and thermal diffusion, and thermal flow), and atomic reconstruction (dislocation, concentrated defect of crystal lattices, phono-electronic effects, stress relaxation at grain boundaries, phase processes in solid solutions, blocks in polycrystalline materials, etc.)

An ideal elastic material is defined by

\[ \int \sigma \, d\varepsilon = 0 \quad (3.17) \]

where \( \sigma \) and \( \varepsilon \) are stress and strain respectively.

From this postulate follow the properties of complete recoverability and an instantaneous equilibrium response. Inelasticity refers to any deviation from ideal elasticity which implies the existence of internal damping.

This approach toward quantifying the internal damping behaviour of materials is through the hysteresis loop. The hysteresis loop of typical constructional metal alloys is extremely thin, unless the metal is strained into its plastic range, and is not easily observed directly. Many nonlinear analyses of damping response of structures have been carried out using analytical representations of such a hysteresis loop having a different functional form. One representation is

\[ \sigma_1 = E\left\{\varepsilon - \frac{V}{n}[(\varepsilon_0 + \varepsilon)^n - 2^{n-1}\varepsilon_0^n]\right\} \quad (3.18) \]

\[ \sigma_2 = E\left\{\varepsilon + \frac{V}{n}[(\varepsilon_0 - \varepsilon)^n - 2^{n-1}\varepsilon_0^n]\right\} \quad (3.19) \]

Where

\( \sigma_1 \) = the stress during the loading part of the cycle

\( \sigma_2 \) = the stress during the unloading

\( E \) = Young's modulus

\( v \) = Poisson's ratio

\( n \) and \( \varepsilon_0 \) = constant
The identification of the parameters in these equations is not a simple task, it requires a very accurate set of measured hysteresis loop at various strain levels, and at various frequencies and temperatures. For most structural metals, the deviation of the hysteresis loop from a single line is extremely small, and hence the material damping is insignificant in comparison with other commonly operating damping mechanisms.

3.2.4.1 Models and Criteria of Internal Damping

In general, internal damping is a function of temperature, strain and strain rate. Three different damping models for one-degree of freedom system are viscous, viscoelastic and hysteresis models.

3.2.4.1.1 Viscous Model

The equation of motion is given by:

\[ m \ddot{x} + c \dot{x} + k x = f(t) \] (3.20)

\[ c = 2\zeta \sqrt{km} = \eta \sqrt{km} \]

Where the \( c \) and \( k \) (damping and stiffness respectively) are constant and are independent of frequency of oscillation.

3.2.4.1.2 Viscoelastic Model

Here, both damping and stiffness of the system are proportional to the displacement by complex stiffness of \( k \) (1+\( j \eta \)):

\[ m \ddot{x} + k (1 + j \eta) x = f(t) \] (3.21)
\[ x = X_0 e^{j\omega t} \]
\[ j = \sqrt{-1} \]

Where, both \( k \) and \( \eta \) are dependent on the frequency and temperature. Temperature is usually considered to be the single most important environmental factor affecting the properties of damping materials [159]. Four distinct regions can be observed as a result of this effect. The first is the so-called glassy region, where the material takes on its maximum value of the storage modulus while having extremely low values for the loss factor. The modulus in the glassy region changes slowly with temperature, while the loss factor increases significantly with increasing temperature. The second region is characterised by having a modulus that decreases rapidly with increasing temperature, while the loss factor takes on its maximum value. The third is the rubbery region where both the modulus and the loss factor usually take on somewhat low values and change very slowly with temperature. The fourth region is typical of a few damping materials such as vitreous enamels and thermoplastics. In this region the material continues to soften with increasing temperature, as it melts, the loss factor takes on a very high value.

Although the fourth region is important for complete characterisation of the damping properties, it is usually of little use in the design of damped systems because of instability and other unwanted physical properties. It should also be noted that, for most rubber like materials, such as cross-lined polymers, this fourth region does not exist. For the remainder of this chapter, the damping properties will be discussed with regard to the first three regions: glassy, transition, and rubbery.

Although this behaviour is typical of all rubber like materials, different materials have different specific properties, characterised mainly by the various levels of the storage modulus and loss factor within each temperature region and the location of each region with respect to temperature.

The variation of the damping properties with frequency at a fixed temperature is expected to be qualitatively the inverse of the temperature but to a lesser degree: that is, it takes several decades of frequency to reflect the same change as a few degrees of temperature.
This phenomenon is one of the most important aspects of viscoelasticity theory, especially in regard to the characterisation of viscoelastic materials. This behaviour provides the basis for the temperature-frequency superposition principle which is used to transform material properties from the frequency domain to the temperature domain, and vice versa.

3.2.4.1.3 Hysteresis Model

Here, the assumption is that the stiffness $k$ and damping $c$ of the linear viscoelastic material are constant. If the material has been excited by a harmonic force of $f(t) = F_0 \cos t$, the response $x(t) = X_0 \cos (t-\Delta)$ is also harmonic. The coefficient of complex stiffness can be replaced by $\frac{k\eta}{\omega}$ which is equivalent viscous damping.

3.2.4.1.4 Comparison of Damping Models

The comparison of results based on the three damping models, viscous, viscoelastic and hysteresis was carried out by Nashif et al. [159] using the response of a single degree of freedom system to an impulsive excitation. The results showed that for low damping ($\eta<0.2$), the three models agree well, but at high damping the differences are considerable.

3.2.4.2 Damping Measurement Methods

(1) The harmonic exciter force $f(t) = F_0 \cos t$ is applied to the system and the phase difference ($\phi$) between the exciter and response can be measured. Then, $\eta = \tan \phi = \phi$ for low damping.

(2) Logarithmic decrement method:
(2-a) When a damped system is struck by an impulsive load or is released from a displaced position to its equilibrium state, a decaying oscillation usually takes place. A measure of damping called logarithmic decrement $\delta$ is defined as the natural logarithm of the ratio of amplitudes of successive peaks (Fig. 3.4)

$$\delta = \ln \frac{x_1}{x_2} = \ln \frac{x_n}{x_{n+1}}, \quad \eta = 2\zeta = 2\delta/\pi$$

(3.22)

![Logarithmic Decrement Method](image)

The damping ratio $\zeta$ can be calculated by measuring logarithmic decrement. These relations are applicable only for low damping conditions which means for $\zeta$ less than one.

(2-b) The logarithmic decrement can be calculated by an alternative technique. A steady-state vibration is induced at resonant frequency by a harmonic exciter and then is allowed to decay freely [132-133].

(3) Half power bandwidth method: The equation of motion is transformed to frequency domain using the Fourier Transform in this method

$$2\zeta = \frac{\omega_2 - \omega_1}{\omega_d}, \quad \omega_d = \omega_n \sqrt{1 - \zeta^2}$$

(3.23)
where \( \omega_1 \) and \( \omega_2 \) are the special frequencies corresponding to 0.69 times of maximum amplitude in the frequency domain. By using impulse or harmonic exciter and FFT equipment, it is possible to find the amplitude and phase curves to calculate \( \zeta \) from the above formulae.

### 3.2.4.3 Hot Steel Damping

The experiments by other investigators for low temperature showed that the internal damping of steel is low. For example, Wren et al. [131] has given a table to compare the damping capacity of different materials as shown in appendix 3. The loss factor of stainless steel is about 0.0018 and for Cast iron is about 0.045. Also, the loss factor for steel has been found by Gibson et al. [116] to be \( \eta = 0.001 \). So, the value of the loss factor for different compositions of iron should be in the range of 0.001-0.045. On the other hand, as we discussed before, the most important factor affecting on damping is temperature. The influence of frequency during experiments to measure hot steel damping in this thesis has been neglected.

### 3.2.4.4 Experimental Procedure and Results

Nashif et al. [159] compared the results based on three different models, viscous, hysteresis and viscoelastic damping for response to impulsive excitation. For low damping, the three models agree well, but at high damping the differences are considerable. As the experiments by the other investigators for low temperature showed that the structural damping of steel is low, the model of viscous damping has been used in this experiment. Specimens with dimensions 19 mm width x 4 mm thickness x 150 mm long were made from the following two types of steel:

- case a: mild steel
- case b: low carbon steel (the composition described in section 3.1.3.1)
The electrical heater was designed for high temperature testing of the above specimen and a controller was used to control the temperature. A B&K accelerometer model 4370 (charge sensitivity 10 pc/ms\(^2\)) was used to measure the acceleration of the beam, the output of which was captured by a Mac Adios A/D card on a Mac II CX computer. As the maximum working temperature of the accelerometer was 250 °C, a movable rod made from stainless steel with 15 mm diameter x 120 mm length was used as a spacer between the accelerometer and the high temperature test beam. The Mac Instrument software controlled the collection of data and output presentation (see Fig. 3.5). Then the logarithmic decrement was calculated from the data. The experiments were repeated five times and the results are based on the average.

![Fig. 3.5 Hot Steel Damping Experimental Procedure](image)

The results for both types of steels are shown in Fig. 3.6 a,b where it can be seen that the logarithmic decrement \( \delta \) increases nonlinearly with temperature. The standard deviation were in the range of 0.0158 - 0.0164. On the other hand, Fig 3.7 shows the variation of Young's modulus (\( E \)) from a research report [127] for two different steel composition. It can be seen that \( E \) decreases with temperature and there is a significant
change in the slope of the curve at about 700 °C. Similar phenomenon has been observed for the variation of damping with temperature in Fig. 3.6 at about 600 °C. Also, the damping values for room temperature in Fig. 3.7 are in agreement with results obtained by Wren et al. [131] as given in appendix 2. However, in these measurements, difference up to 15% with the results of Wren et al. [131] can be attributed to external damping of the cantilever beam. This external damping is caused by clamp edge friction and the stiffening of the accelerometer support spacer.

Fig. 3.6a  Logarithmic decrement of mild steel (case a)
Fig. 3.6b  Logarithmic decrement of BHP steel (case b)

Fig. 3.7  The variation of Young's modulus with temperature [127]
CHAPTER 4

ROLLING FORCE AND TORQUE
4.1 Introduction

As mentioned in chapter 2, one of the most comprehensive plane strain slab model for hot and cold rolling is that by Orowan [59]. Bland and Ford [60], Sims [58], Ford and Alexander [67] have introduced approximate analytical solutions for the roll pressure. Alexander [61] reviewed and evaluated some of these solutions. Also, he presented a complete numerical solution of Orowan's equations. In this chapter Alexander's numerical method and some of the approximate solutions will be reviewed. Moreover, the rolling force and torque are calculated by various numerical methods and the analytical slab method (Orowan's theory) will be compared with the finite element and upper-bound methods.

Test data from BHP Steel Hot Roughing Mill have been used in this investigation. The Roughing Mill consists of a Horizontal mill coupled with an attached Edger on its entry side as shown in chapter 5. The velocity and temperature of the Roughing Mill during seven passes were in the range of 3-5.5 m/s and 1100-1200°C respectively.

4.2 Alexander's Force and Torque Model

4.2.1 Roll Deformation

Due to high pressure at the deformation zone (roll bite), the work-rolls flatten elastically. This elastic distortion increases the length of the deformation zone. An expression for the roll bite curvature was derived by Hitchcock [62] who assumed that the pressure distribution was elliptical (see Fig. 2.1).

\[ \frac{R'}{R} = 1 + \frac{C_p}{\Delta h} \]

\[ \Delta h = h_0 - h_1 \]
\[ C = \frac{16(1 - \nu^2)}{(\pi E)} \]  \hspace{1cm} (4.2)\\

where

\[ R' = \text{deformed work-roll radius (m)} \]
\[ R = \text{work-roll radius (m)} \]
\[ p = \text{force per unit width (N/m)} \]
\[ \nu = \text{Poisson's ratio} \]
\[ h_0 = \text{entry strip thickness (m)} \]
\[ h_x = \text{exit strip thickness (m)} \]
\[ E = \text{Young's modulus (Pa)} \]

This equation has been proven as being incorrect by Lenard et al. [77] (see also chapter 2) but this error does not seem to affect significantly the roll force and torque. Ford et al. [60] modified the equation derived by Hitchcock to include the effects of both entry and exit elastic zones.

\[ R'/R = 1 + \left( \frac{Cp}{g} \right) \]  \hspace{1cm} (4.3)\\

\[ g = \left\{ \left[ \Delta h + \delta_{el} + \delta_1 \right]^{0.5} + \left[ \delta_{el} \right]^{0.5} \right\}^2 \]
\[ \delta_{el} = (1 - \nu^2)(2k_1 - t_{el}) h_1 / E \]
\[ t_{el} = \sigma_1 - \frac{2\mu p_{el}}{h_1} \]  \hspace{1cm} (4.4)
\[ \delta_1 = \nu(\nu + 1)(h_1 \sigma_1 - \sigma_0 h_0) / E \]
\[ 2k_1 = \text{strip plane strain yield stress at exit (Pa)} \]
\[ \sigma_0 = \text{back tension (Pa)} \]
\[ \sigma_1 = \text{forward tension (Pa)} \]
\[ t_{el} = \text{forward tension at the end of the exit plastic zone (Pa)} \]
\[ p_{el} = \text{roll force due to exit elastic zone (N/m)} \]
4.2.2 Flow Stress

The plane strain yield stress of the strip using Von Mises yield criterion applied to the state of uniaxial tension or compression gives:

\[ 2k = 2 \frac{\sigma}{\sqrt{3}} \]  \hspace{1cm} (4.5)

where

- $2k$ = plane strain yield stress (Pa)
- $\sigma$ = uniaxial yield stress (Pa)

A constitutive equation for flow stress has been introduced by Alexander [61],

\[ \sigma = Y_0 (1+B\bar{e})^n (1+D\bar{\varepsilon})^m \exp(-ct') \]  \hspace{1cm} (4.6)

\[ \bar{e} = \frac{2}{\sqrt{3}} \ln \left( \frac{h_0}{h} \right) \]  \hspace{1cm} (4.7)

\[ \bar{\varepsilon} = \frac{2}{\sqrt{3}} v_1 h_1 \tan \phi \]  \hspace{1cm} (4.8)

where

- $\sigma$ = uniaxial yield stress (Pa)
- $\bar{e}$ = effective strain
- $\bar{\varepsilon}$ = equivalent strain rate
- $t'$ = temperature (°C)
- $h_0$ = strip thickness at entry plane (m)
- $h$ = strip thickness of an elemental slice of roll bite (m)
- $h_1$ = strip thickness at exit plane (m)
- $v_1$ = velocity of strip at exit plane (m/s)
- $\phi$ = angle of an elemental slice in the roll-bite (radian)

and $Y_0$, $B$, $D$, $n$, $m$ and $c$ are constants.

The above constants ($Y_0$, $B$, $D$, $n$, $m$ and $c$) were found by the regression of constitutive equation introduced in chapter 3 for a grade of BHP steel.
4.2.3 Equation for Normal Pressure

Equilibrium of an element in the deformation zone as shown in Figure 2.1 gives a first-order differential equation. The equilibrium equation for an elemental slice in the deformation zone on the exit side of the neutral plane is given by:

\[ \frac{d[h(s - 2k - \tau \tan \phi)]}{d\phi} = 2R'(s \sin \phi + \tau \cos \phi) \]  \hspace{1cm} (4.9)

The equilibrium equation for an elemental slice in the deformation zone on the entry side of the neutral plane is given by

\[ \frac{d[h(s - 2k + \tau \tan \phi)]}{d\phi} = 2R'(s \sin \phi - \tau \cos \phi) \]  \hspace{1cm} (4.10)

where

- \( s \) = normal pressure (Pa)
- \( \tau \) = surface shear stress (Pa)

The elemental thickness can be derived from geometry

\[ h = h_1 + 2R'(1 - \cos \phi) \]  \hspace{1cm} (4.11)

4.2.4 Boundary Conditions

The entry and exit boundary conditions for the differential equation are obtained from the equilibrium of an elemental slice on the exit and entry side of the neutral plane:

\[ \sigma_y R' \delta \phi \cos \phi = s R' \delta \phi \cos \phi \pm \tau R' \delta \phi \sin \phi \]  \hspace{1cm} (4.12)

If the vertical compressive stress \( \sigma_y \) is numerically larger than the horizontal compressive stress \( \sigma_x \), then

\[ \sigma_y - \sigma_x = 2k \]  \hspace{1cm} (4.13)

where

- \( \sigma_y \) = vertical stress
- \( \sigma_x \) = horizontal stress
Substitution of equation (2.27) into equation (2.25) gives the exit and entry side plane boundary conditions

at entry

\[ \phi = \alpha \]
\[ s_0 = 2k_0 - t_{e_0} - \tau \tan \alpha \]  \hspace{1cm} (4.14)
\[ t_{e_0} = \sigma_0 - \frac{2 \mu \pi_{e_0}}{h_0} \]

at exit

\[ \phi = 0 \]
\[ s_1 = 2k_1 - t_{e_1} \]  \hspace{1cm} (4.15)

4.2.5 Friction Condition

The friction condition determines the shear stress acting on the strip. Two conditions are considered:

(i) slipping

(ii) sticking

4.2.5.1 Slipping Condition

For slipping condition, substitute \( \tau = \mu s \) into equations (4.9) & (4.10) and differentiate with respect to \( \phi \). The equations for normal pressure \( s \), as a differential of \( \phi \) is given by

\[ \frac{ds}{d\phi} = g_1(\phi) s + g_2(\phi) \]  \hspace{1cm} (4.16)

\[ g_1(\phi) = \pm \mu \sec \phi \frac{2R' + \sec \phi}{h} \]  \hspace{1cm} (4.17)
\[ g_2(\phi) = \frac{2R'}{h} \frac{2k \sin\phi + \frac{d(2k)}{d\phi}}{1 + \mu \tan\phi} \]  \hspace{1cm} (4.18)

where the upper algebraic sign refers to the region between the neutral plane and exit and the lower sign denotes the region between the neutral plane and entry.

### 4.2.5.2 Sticking Condition

For sticking condition, \( \tau = k \) is substituted into equations (4.9) & (4.10) which are then differentiated with respect to \( \phi \). The equations for normal pressure \( s \), as a differential of \( \phi \) are

\[ \frac{ds}{d\phi} = g_3(\phi) \]  \hspace{1cm} (4.19)

where

\[ g_3(\phi) = 2k \left[ \frac{R'}{h} \sin\phi (2 \pm \tan\phi) \pm \left( \frac{R'}{h} \cos\phi + 0.5 \sec^2\phi \right) \right] \]

\[ + \frac{dk}{d\phi} (2 \pm \tan\phi) \]

The two values for shear stress are determined for both sticking and slipping models in each position of \( \phi \). Then the calculation will be continued based on the smaller value.

The pressure distributions determined by equations 4.16 and 4.19 introduce a sharp peak which is not realistic. Later, Alexander [52] introduced the adhesion zone based on Tselikov's model [51] around the neutral point. In fact, the roll-bite has been divided to three different zones which are entry side, adhesion zone and the exit side. The rolling force and torque in the modified program showed better agreement with measurement. Moreover, most of the FEM solutions showed that a neutral zone (similar to the above adhesion zone) rather than a neutral point can occur in the roll-bite [75].
4.2.6 Rolling Force

The rolling force per unit width is determined by:

\[
p = R' \int_0^\alpha s \cos(\phi - 0.5\phi_1) d\phi + R' \left[ \int_\phi^\alpha \tau \sin(\phi - 0.5\phi_1) d\phi - \int_0^\phi \tau \sin(\phi - 0.5\phi_1) d\phi \right]
\]

(4.20)

where \( \phi_1 \) is the roll bite angle.

Ford et al. [15] developed the following equations to calculate the entry and exit elastic zone forces:

\[
p_{e0} = \frac{(1-v^2)h_0}{4} \sqrt{\frac{R'}{\Delta h}} \frac{(2k_0 - t_{e0})^2}{E}
\]

(4.21)

\[
p_{el} = \frac{2}{3} \sqrt{\frac{R' h_1 (1-v)^2}{E}} (2k_1 - t_{el})^{1.5}
\]

(4.22)

where

\[
t_{e0} = \sigma_0 - \frac{2 \mu p_{e0}}{h_0}
\]

\[
t_{el} = \sigma_1 - \frac{2 \mu p_{el}}{h_1}
\]

The elastic zone forces \( p_{e0} \) and \( p_{el} \) are usually between 0.1 to 2 percent of the plastic zone force. The total force per unit width is

\[
p_t = p + p_{el} + p_{e0}
\]

(4.23)

4.2.7 Rolling Torque

The roll torque for one roll in the plastic zone is determined by integrating the moments over the roll-bite caused by normal pressure \( s \), and the shear stress \( \tau \) about the work-roll centre:
\[ T = R' (R' - R) \int_{0}^{\pi} \sin(\phi - 0.5\phi_1) d\phi + R' \int_{\phi_s}^{\phi_e} \left[ R' \tau - (R' - R) \tau \cos(\phi - 0.5\phi_1) \right] d\phi \]

- \[ R' \int_{0}^{\phi_s} \left[ R' \tau - (R' - R) \tau \cos(\phi - 0.5\phi_1) \right] d\phi \quad (4.24) \]

The elastic zone contributes to the torque as follows:

\[ T_{e0} = +\mu R \rho_{e0} \quad (4.25) \]
\[ T_{el} = -\mu R \rho_{el} \quad (4.26) \]

The total roll torque per unit width for one roll is:

\[ T_t = T + T_{e0} + T_{el} \quad (4.27) \]

### 4.3 Sims' Method

The assumptions made by Sims [58] in his paper are as follow:

(i) the roll gap angle \( \alpha \) would in most cases be a small angle,

(ii) \[ \tan \phi = \sin \phi = \phi \quad ; \quad \cos \phi \approx 1 \quad ; \quad 1 - \cos \phi = 0 \quad (4.28) \]

(iii) introduction of the correction factor \( \frac{\pi}{4} \) for non-uniform deformation

(iv) the strip thickness in the roll gap may be approximated by

\[ h = h_1 + R' \phi^2 \quad (4.29) \]

(v) Sims further assumes that the term \( \tau \phi \) is negligibly small in comparison to the other terms.

(vi) the friction stress \( \tau \) remains constant and is equal to the shear yield strength \( k \) of the material.

thus,

\[ \frac{d}{d\phi} \left[ h (s - \frac{\pi}{2}k) \right] = 2R' (\phi s \pm k) \quad (4.30) \]
which forms the basis of his rolling theory. Using the present notation, Sims' solution for the roll pressure from the exit to the neutral plane is given as

$$\frac{s}{2k} = \frac{\pi}{4} \ln \frac{h}{h_0} + \frac{\pi}{4} + \frac{R'}{h} \tan^{-1}\left(\frac{R'}{h} \alpha \right)$$  \hspace{1cm} (4.31)

and for the portion of the roll gap from entry to the neutral plane

$$\frac{s}{2k} = \frac{\pi}{4} \ln \frac{h}{h_0} + \frac{\pi}{4} + \frac{R'}{h} \tan^{-1}\left(\frac{R'}{h} \alpha \right) - \frac{R'}{h} \tan^{-1}\left(\frac{R' \phi}{h} \right)$$  \hspace{1cm} (4.32)

The position of the neutral point \( \phi_n \) may be calculated from:

$$\frac{\pi}{4} \ln(1-\epsilon) = 2 \frac{R'}{h} \tan^{-1}\left(\frac{R' \phi}{h} \right) - \frac{R'}{h} \tan^{-1}\left(\frac{\epsilon}{1-\epsilon} \right)$$  \hspace{1cm} (4.33)

The roll separating force per unit width is given as

$$p = 2k R' \left[ \frac{\pi}{2} R' \tan^{-1}\left(\frac{\epsilon}{\sqrt{1-\epsilon}} \right) - \frac{\pi \alpha}{4} - \ln \frac{h(\phi)}{h_1} + \frac{1}{2} \ln \frac{h_0}{h_1} \right]$$  \hspace{1cm} (4.34)

and the roll torque per unit width for one roll as

$$T = 2k R R' \left( 0.5 \alpha - \phi_n \right)$$  \hspace{1cm} (4.35)

### 4.4 Ford and Alexander's Method

Ford and Alexander [68] presented the following formulae for the calculation of roll separating force and roll torques in hot, flat rolling based on sticking friction assumption:

$$\frac{p}{kh_0} = 1.571 \left( \frac{R \epsilon}{h_0} + \frac{R}{h_0} \frac{\epsilon}{2-\epsilon} \right)$$  \hspace{1cm} (4.36)
and

\[
\frac{T}{k h_0^2} = \frac{R e}{h_0} \left[ 1.571 + \frac{(2/3) \left( 3 - 2\varepsilon \right)}{(2 - \varepsilon)^2} \sqrt{\frac{R e}{h_0}} \right]
\]

where

\[ p \] = separating force per unit width
\[ T \] = roll torque per unit width for both rolls
\[ k \] = average shear yield strength in the pass
\[ R \] = roll radius
\[ h_0 \] = strip thickness at entry
\[ \varepsilon \] = reduction

### 4.5 Tselikov's Solution

Another solution of the equilibrium equation was developed by Tselikov [51]. His assumptions are:

(i) substitution of the arc of contact by its chord
(ii) sliding model of friction (\( \tau = \mu s \))
(iii) constant value of the yield strength \( k \) in the roll gap
(iv) in the hot rolling process both entry and exit tensions are equal to zero

With these assumptions, the equilibrium equation written in Cartesian coordinates is:

\[
\frac{ds}{dx} - \frac{2k}{y} \frac{dy}{dx} + \frac{\tau}{y} = 0
\]

(4.38)

and the pressure distributions (at entry and exit side) are given by the following:

\[ s = 2 \frac{k}{\delta} \left[ \left( \frac{h}{h_1} \right)^\delta (\delta + 1) - 1 \right] \] (on exit side)

(4.39)

\[ s = 2 \frac{k}{\delta} \left[ \left( \frac{h_0}{h} \right)^\delta (\delta - 1) + 1 \right] \] (on entry side)

(4.40)

where,
\[ \delta = \frac{2\mu L}{h_0 \varepsilon} \]
\[ L = \sqrt{R \Delta h} \]

also, the final formula for the roll separating force per unit width in the hot rolling process is in the form of

\[ p = \frac{2kLh_n}{h_1 \varepsilon (\delta - 1)} \left( \left( \frac{h_n}{h_1} \right)^{\delta} - 1 \right) \]  

(4.41)

where \( h_n \) represents the strip thickness at the neutral point:

\[ h_a = h_1 \left[ \frac{1 + \sqrt{1 + (\delta^2 + 1)\left( \frac{h_0}{h_1} \right)^{\delta}}}{\delta + 1} \right]^{\frac{1}{\delta}} \]  

(4.42)

Two different methods have been discussed by Tselikov for rolling torque \( T \) per unit width:

(i) rolling torque can be determined by the integration of the moment of shear stress with respect to roll centre:

\[ T = R \int_0^L \tau \, dx \]  

(4.43)

(ii) moment arm (lever arm) \( d \) is defined as the ratio of rolling torque to rolling force:

\[ d = \frac{T}{p} \]  

(4.44)

an alternative parameter is lever arm ratio defined as:

\[ \psi = \frac{d}{L} \]  

(4.45)

thus

\[ T = \psi \, L \, p \]  

(4.46)
where $\psi$ is the lever arm ratio (coefficient of lever arm).

Pietrzyk et al. [23] has given an approximation for $\psi$:

$$\psi = 0.5494 \Delta^{0.3146} \quad (4.47)$$

where $\Delta$ the shape coefficient is an important parameter in rolling

$$\Delta = \frac{h_{av}}{L}$$

$$h_{av} = \frac{h_0 + h_1}{2}$$

Analysing the cold rolling process, Tselikov introduces the tensions and assumes that the yield strength in shear is a constant, equal to $k_0$ in the backward slip zone and to $k_1$ in the forward slip zone. From the above assumptions, the rolling force can be determined from the following equation:

$$p = \frac{4k_0 \xi_0}{\mu h_0 (2-\tau)} \left[ \left( \frac{k_0 \xi_0}{k_1 \xi_1} \right)^{a_0} \exp \left( -\frac{2\mu L}{h_0 (2-\tau)} \right) - \frac{k_0 \xi_0}{k_1 \xi_1} a_0 - \frac{h_1}{h_0 + h_2} \right] \quad (4.48)$$

where

$$a_0 = \frac{h_0}{h_0 + h_1}$$

$$\xi_0 = \frac{2k_0 - \sigma_0}{2k_0}$$

$$\xi_1 = \frac{2k_1 - \sigma_1}{2k_1}$$

$\sigma_0$ = tension stress at entry side

$\sigma_1$ = tension stress at exit side
4.6 Finite Element Method

A two-dimensional FEM program written by Khoddam [75] is used here to evaluate rolling force and torque in roll gap. This program is based on the work by Kobayashi [76]:

The assumption applied in this program are as follows:
(i) plane strain iso-thermal rolling simulation without heat transfer.
(ii) symmetrical rigid-viscoplastic rolling condition with rigid roll.
(iii) rectangular isoparametric elements.

4.7 Upper-Bound Method

The roll torque can be also calculated by the upper bound approach. The basic equation which describes the power balance in the deformation zone is [23]

\[ T\omega = \int_V \sigma_i \dot{\epsilon}_i \, dV + \int_{S_i} \frac{\sigma}{\sqrt{3}} |v_{y0}| \, dS + \int_S \tau |v_s| \, dS \]  

\[ (4.49) \]

where

\[ V = \text{volume of the deformation zone} \]
\[ S_i = \text{represents the surface of the velocity discontinuity} \]
\[ S = \text{contact surface} \]
\[ T = \text{roll torque} \]
\[ \omega = \text{roll angular velocity} \]
\[ \sigma_i = \text{effective stress} \]
\[ \dot{\epsilon}_i = \text{effective strain rate} \]
\[ \sigma = \text{yield strength} \]
\[ v_{y0} = \text{the vertical component of slab velocity at the surface of velocity discontinuity} \]
\[ \tau = \text{shear stress at the contact surface} \]
\[ v_s = \text{slip velocity} \]
Moreover, the following approximation is applied by the investigators to calculate rolling force from rolling torque:

$$p = \frac{2T}{L}$$

$$L = \sqrt{R \Delta h}$$

(4.50)

More details about the numerical calculation of upper-bound method are given in chapter 6.

### 4.8 Rolling Force and Torque Methods Application

Rolling Force (from load cell) and Torque (from motor current) for the 7th pass of BHP Horizontal Roughing Mill has been compared with the computed data shown in Table 4.1.

The rolling condition for the above investigation are as follow:

- \(h_0 = 50 \text{ mm}\)
- \(h_1 = 34 \text{ mm}\)
- \(R = 577 \text{ mm}\)
- \(w = 769 \text{ mm}\)
- \(\omega = \text{ roll angular velocity} = 9.53 \text{ rad/sec}\)
- \(t' = \text{ temperature} = 1100\degree \text{C}\)

For these calculations a friction coefficient of 0.3 was used. This value has been applied by many researchers in hot rolling. Moreover, the flow stress model presented in chapter 3 for a grade of BHP Carbon Steel have been introduced in the above methods and formulae. The average flow stress was 140 MPa and the roll flattening (except in Alexander program) has been neglected.
<table>
<thead>
<tr>
<th>Method</th>
<th>Force [MN]</th>
<th>Torque [KN-m]</th>
<th>Friction model</th>
</tr>
</thead>
<tbody>
<tr>
<td>BHP measurement</td>
<td>15.6</td>
<td>501.3</td>
<td>-</td>
</tr>
<tr>
<td>Sims</td>
<td>13.8</td>
<td>681.0</td>
<td>sticking</td>
</tr>
<tr>
<td>Ford &amp; Alexander</td>
<td>15.0</td>
<td>565.8</td>
<td>sticking</td>
</tr>
<tr>
<td>Tselikov</td>
<td>14.5</td>
<td>575.0</td>
<td>sliding</td>
</tr>
<tr>
<td>Alexander program</td>
<td>14.6</td>
<td>529.9</td>
<td>mix model</td>
</tr>
<tr>
<td>Finite element</td>
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<td>667.7</td>
<td>sliding</td>
</tr>
<tr>
<td>Upper-bound</td>
<td>13.9</td>
<td>660.7</td>
<td>sliding</td>
</tr>
</tbody>
</table>

Table 4.1 BHP Rolling Force & Torque

for the 7th pass

![Pressure distribution](image)

**Fig. 4.1 Pressure distribution s in deformation zone**

The unit of the angular distance from the exit side is rad in the figures.
**Fig. 4.2** Shear stress $\tau$ in roll bite

**Fig. 4.3** Flow stress variation in deformation zone

<table>
<thead>
<tr>
<th>pass Number</th>
<th>$h_0$ [mm]</th>
<th>$h_1$ [mm]</th>
<th>$R$ [mm]</th>
<th>$w$ [mm]</th>
<th>$\omega$ [rad/sec]</th>
<th>$t'$ [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>233</td>
<td>201</td>
<td>577</td>
<td>768</td>
<td>5.2</td>
<td>1280</td>
</tr>
<tr>
<td>2</td>
<td>201</td>
<td>167</td>
<td>&quot;</td>
<td>775</td>
<td>5.7</td>
<td>1250</td>
</tr>
<tr>
<td>3</td>
<td>167</td>
<td>133</td>
<td>&quot;</td>
<td>752</td>
<td>4.2</td>
<td>1220</td>
</tr>
<tr>
<td>4</td>
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<td>762</td>
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<td>1190</td>
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<td>74</td>
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<td>1160</td>
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<td>50</td>
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<td>767</td>
<td>7.45</td>
<td>1130</td>
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<tr>
<td>7</td>
<td>50</td>
<td>34</td>
<td>&quot;</td>
<td>770</td>
<td>9.53</td>
<td>1100</td>
</tr>
</tbody>
</table>

Table 4.2 BHP 7 Passes Data
Table 4.3 BHP Rolling Force & Torque For 7 Passes

<table>
<thead>
<tr>
<th>pass Number</th>
<th>Tselikov measured Force [MN]</th>
<th>BHP measured Force [MN]</th>
<th>Tselikov measured Torque [KN-m]</th>
<th>BHP measured Torque [KN-m]</th>
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<td>9</td>
<td>13</td>
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<td>4</td>
<td>11</td>
<td>13.6</td>
<td>740</td>
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<td>574</td>
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</tr>
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<td>14.2</td>
<td>14.7</td>
<td>570</td>
<td>584</td>
</tr>
<tr>
<td>7</td>
<td>14.5</td>
<td>15.6</td>
<td>575</td>
<td>501</td>
</tr>
</tbody>
</table>

4.9 Results and Discussion

The calculated results for rolling force and torque based on six different analytical and numerical methods are given in Table 4.1 for the 7th pass of BHP Roughing Mill.

Fig. 4.1, 4.2, 4.3 show respectively the variations of pressure, shear stress and flow stress in the deformation zone produced by the Alexander program. The pressure distribution is a maximum at the neutral plane where the shear stress direction changes. The Alexander program considers the possible existence of both sliding and sticking friction conditions (according to Orowan's theory). Therefore, this program is a good criterion to justify the friction condition of the roll slab interface. By comparing of the pressure and shear stress distribution (Fig. 4.1 and Fig. 4.2), it is clear that the sliding friction occurs in the 7th pass in the Horizontal Roughing Mill, because the ratio of $\frac{\tau}{s}$ is constant in the roll gap (from Fig. 4.1 and Fig. 4.2) and is equal to $\mu = 0.3$.

Furthermore, the flow stress on the exit side is about 40% greater than that on the entry side in the 7th pass. This is the influence of strain hardening on the resistance to deformation of hot steel.
Ford and Alexander method has given better predictions for rolling force and torque of the 7th pass compared with the other simplified methods. Furthermore, there is not much difference between the calculation of rolling torque using finite element or upper-bound method. Pietrzyk et al. [108] mentioned that the upper-bound method with shorter computing times gave satisfactory accurate results when compared with the finite element approach. The finite element method is required for the simulation of the metal flow pattern through the roll gap.

Table 4.2 shows the input data of BHP for all seven passes and table 4.3 shows the rolling force and torque of all seven passes by from the Tselikov's method and they are compared with BHP measurements. Table 4.3 demonstrates that the Tselikov's analytical solution has acceptable accuracy and it can be applied in chapter 6 and 7. Moreover, this method is based on the slipping friction model which has been demonstrated to be reasonable in tables 4.1 and 4.3. On the other hand, we believe that the slipping friction model is a realistic model for all passes of Roughing Mill during vibrations conditions. This is because the possibility of slipping friction is more significant than sticking friction when the Roughing Mill Stands experience vibrations. For steady-state condition, the assumption of slipping friction may not be appropriate.

In practice, the rolling mill data exhibit a lot of scatter owing to the widely varied mill condition. Therefore, comparison with mill data should be carried out on many slabs (instead of only one as in this thesis). However, this is beyond the scope of this thesis.
CHAPTER 5

MODAL ANALYSIS
5.1 Introduction

The natural frequency and mode shapes are essential data when solving vibrational problems in rolling mills. The torsional vibration occurs normally at the first natural frequency. Moreover, the vertical resonance vibration of hot rolling mills correlates well with one of the natural frequencies, normally with the first or second one. Third octave or fifth octave mode chatter are the frequencies of the oscillation of most cold rolling mills. These frequencies are also close to natural frequencies for linear vibration. The third octave mode chatter normally exists in the second natural frequency and the fifth octave mode is usually obtained in the 4th or 5th natural frequency of the system.

An industrial rolling mill has been analysed in this thesis. The study has been carried out on the Hot Rolling Mill at BHP Steel at Port Kembla, NSW, Australia which consists of a Horizontal mill coupled with an attached Edger on its entry side as shown in Fig 5.1.

![Fig. 5.1 BHP Roughing Mill](image-url)
There are four main kinds of vibration in this system. Torsional and linear vibration of the Horizontal mill, torsional vibration of Vertical Edger and linear vibration of hot steel slab between the two mills. As discussed before, the natural frequencies and mode shapes of the system elements are important information for the transient response of the system when it is excited as a result of impact loads (rolling force and torque). In this chapter the torsional vibration modes and natural frequencies of the Horizontal mill and Vertical Edger will be studied.

5.2 Slab Stiffness and Damping

5.2.1 Slab Stiffness and damping in elastic regime

The modulus of elasticity of hot steel slab at 1100°C is about half of the value at room temperature (see Fig. 3.8 in chapter 3). Thus, the stiffness of the hot slab is more important than that in cold rolling and should be considered in the vibration calculations. Also, according to the experimental work described in part 3.2, the logarithmic decrement of hot steel (at 1100°C) measured was about $\delta = 0.23$. Then, we can determine the stiffness $k_x$ and damping $c_x$ of slab for the linear vibration of slab between the two mills (see Fig 5.1):

$$k_x = \frac{E A}{l}$$  \hspace{1cm} (5.1)

$$c_x = 2\xi \sqrt{k_x M}$$  \hspace{1cm} (5.2)

$$\xi = \frac{\delta}{2\pi}$$  \hspace{1cm} (5.3)

where

$E = $ Young's modulus of hot steel

$l = $ distance between two mills

$A = $ cross section of slab
\( \xi = \text{damping ratio of hot steel} \)
\( M = \text{mass of slab between the two mills} \)

### 5.2.2 Slab Stiffness and Damping in Plastic Regime

#### 5.2.2.1 Slab Stiffness for Vertical Vibration

According to Dukmasov [34] who established a force model for vertical vibration of hot strip rolling the dynamic change of the rolling force \( \delta p(t) \) is given by:

\[
\delta p = \delta p_1 - k_v \delta h_1 - c_v \delta h_1
\]  
(5.4)

\[
k_v = -\frac{\partial p}{\partial h_1}
\]  
(5.5)

\[
c_v = \frac{pL}{v_1 \Delta h}
\]  
(5.6)

where

- \( \delta p_1 = \text{change in rolling force caused by changes in properties such as roll radius and coefficient of friction} \)
- \( \delta h_1 = \text{variation of exit thickness} \)
- \( \delta h_1 = \text{rate of the variation of exit thickness} \)
- \( k_v = \text{strip stiffness for vertical vibration} \)
- \( c_v = \text{strip damping for vertical vibration} \)
- \( \Delta h = h_0 - h_1 \)
- \( L = \text{distance between exit and entry plan} \)
- \( v_1 = \text{strip exit velocity} \)
- \( a = \text{material flow stress constant (see part 3.1)} \)
The rolling force model that we apply for vibration is based on sliding friction model. It has been shown in section 4.9 that the assumption of sliding friction prevailing in the 7 passes of the Roughing Mill is reasonable. Thus, the Tselikov method for hot rolling is used in this thesis to calculate the rolling force and torque because its friction model is sliding. The details of the method are given in part 4.5.

Thus, we can calculate the partial derivative of rolling force with respect to $h_1$. Analytical solution of $k_v$ has been carried out by the Mathematica package. Also, numerical solution can be obtained using the following expression and central difference method:

$$k_v = -\frac{\Delta p}{\Delta h_1} \quad (5.7)$$

### 5.2.2.2 Slab Stiffness for Torsional Vibration

Here, the equivalent values of stiffness and damping for torsional vibration are calculated from the following calculations:

According to the relation $T=\psi L p$ (equation 4.46), and by multiplication of equation (5.4) by moment arm $d=\psi L$,

$$d \delta p = d \delta p_1 - d k_v \delta h_1 - d c_v \delta h_1 \quad (5.8)$$

By the assumption of small value for roll bite angle $\alpha$

$$\alpha = \sin \alpha = \sqrt{\frac{h_0 - h_1}{R}} \quad (5.9)$$

and

$$h_1 = -R\alpha^2 + h_0 \quad (5.10)$$
the entry side thickness is constant. Thus, the variation in exit thickness and its rate of variation rate are

\[ \delta h_1 = -2R \alpha \delta \alpha \] (5.11)

\[ \delta h_1 = -2R (\dot{\alpha} \delta \alpha + \alpha \ddot{\alpha}) \] (5.12)

by replacing equation (5.12) and (5.11) into equation (5.8), then the dynamic torque model will appear. Then

\[ \delta T = \delta T_1 + k_1 \delta \alpha + c_1 \delta \dot{\alpha} \] (5.13)

where

\[ \delta \alpha = \text{change in roll bite angle} \]
\[ \delta \dot{\alpha} = \text{rate of change in roll-bite angle} \]

According to Fig. 2.1

\[ \alpha = \int d\phi \] (5.14a)

where \( \phi \) is the angular co-ordinate.

Sims [58] has replaced the derivative \( \frac{d\phi}{dt} \) with angular velocity \( \dot{\theta} \) of the roll. Thus:

\[ \frac{d\phi}{dt} = \frac{d\theta}{dt} \]
\[ \delta \alpha = \delta \theta \] (5.14)
\[ \delta \dot{\alpha} = \delta \dot{\theta} \] (5.15)

where

\[ \delta \theta = \text{change in roll torsional displacement} \]
\[ \delta \dot{\theta} = \text{variation of roll angular velocity} \]
\[ \delta T = \delta T_1 + k_t \delta \theta + c_t \dot{\delta \theta} \]  
(5.16)  

dslab rotation stiffness for torsional vibration

\[ k_t = 2 d (k_v L + R c_v \dot{\theta}) \]  
(5.17)  

slab damping for torsional vibration

\[ c_t = 2 d L c_v \]  
(5.18)  

where \( \dot{\theta} \) is the angular velocity of work-roll and \( c_t \) is the damping caused by a change in the roll bite angle.

### 5.3 Torsional Vibration Modes of Horizontal Mill

A 4-high horizontal mill has been shown in Fig. 1.1 and Fig. 5.1 together with the Vertical Edger on its entry side. The torsional vibration model of the Horizontal mill is not symmetrical. This is because the stiffness and inertia of the top drive system are not the same as the bottom drive line, and the nature of vibration should be different in both.

Furthermore, the torsional vibration of both drive lines can have an influence on each other. Thus, the top and bottom drives should be represented by two different models connected together by the slab [39].

Tieu [41] has calculated the natural frequencies of the above system based on 20-degrees of freedom. However, as discussed before, in practice the frequency of torsional vibration is close to the first or second natural frequencies of the system and the higher natural frequencies are not involved in this oscillation.
Therefore, a system with four and three-degrees of freedom has been considered for the top and bottom drive lines respectively as shown in Fig 5.2. This rolling mill, has work-roolls that are driven by motors and the backup-roolls rotate as a result of friction between the work-roolls and backup-roolls. The backup-roll and corresponding work-roll are combined into one inertia [1]. In practice, the inertia of the slab with respect to roll centres is not negligible and should be combined to top and bottom work-roolls. Thus, neglecting the thickness of slab, the combined inertia of backup-roll, work-roll and slab can be obtained from:

\[ J_4 = J_w + \left( \frac{d}{D} \right)^2 J_b + \frac{1}{2} M R^2 \]  

(5.19)

where,

- \( J_b \) = backup-roll inertia,
- \( J_w \) = work-roll inertia,
- \( M \) = mass of slab
- \( R \) = work-roll radius

The model with 6-degrees of freedom is shown in Fig 5.3. This system is similar to the Monaco model [11].

5.3.1 Top and Bottom Drive Torsional Models:

The free vibration of the two drives should be considered. Usually, the damping is not considered in the calculation of natural frequencies and modal analysis. Moreover, we discussed in part 3.2.4.4 that the internal damping of steel in room temperature is low. Thus, the influence of damping has been neglected in this chapter. The data of the stiffness and inertia in Fig. 5.2 is given in Table 5.1
Fig. 5.2 Torsional model of Horizontal mill
(Separate drive model)

Table 5.1 Dynamic characteristic of Horizontal Mill
The free vibration equations of the motions have been written for all 7 inertia. For example, for top work-roll $J_4$ we can write:

$$J_4 \ddot{\theta}_4 + k_3 (\dot{\theta}_4 - \dot{\theta}_3) + c_3 (\dot{\theta}_4 - \dot{\theta}_3) = 0$$  \hspace{1cm} (5.20)$$

or in matrix form the equations of motions of top drive inertia will be:

$$[J]\{\ddot{\theta}\} + [k]\{\theta\} = 0$$  \hspace{1cm} (5.21)$$

where

$$[J] = \begin{bmatrix} J_1 & 0 & 0 & 0 \\ 0 & J_2 & 0 & 0 \\ 0 & 0 & J_3 & 0 \\ 0 & 0 & 0 & J_4 \end{bmatrix}$$  \hspace{1cm} (5.22)$$

and

$$[k] = \begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & k_1 + k_2 & -k_2 & 0 \\ 0 & -k_2 & k_2 + k_3 & -k_3 \\ 0 & 0 & -k_3 & +k_3 \end{bmatrix}$$  \hspace{1cm} (5.23)$$
the stiffness and inertia values for top drive line of Horizontal mill of BHP are as follows:

\[ [J] = \begin{bmatrix}
0.11 \times 10^6 & 0 & 0 & 0 \\
0 & 0.11 \times 10^5 & 0 & 0 \\
0 & 0 & 4500 & 0 \\
0 & 0 & 0 & 9500 \\
\end{bmatrix} \]  

(5.24)

\[ [k] = \begin{bmatrix}
1.27 & -1.27 & 0 & 0 \\
-1.27 & 6.67 & -5.4 & 0 \\
0 & -5.4 & 7.23 & -1.8 \\
0 & 0 & -1.8 & 1.83 \\
\end{bmatrix} \times 10^8 \]  

(5.25)

and for bottom drive system:

\[ [k] = \begin{bmatrix}
1.4 & -1.4 & 0 \\
-1.4 & 6.9 & -5.5 \\
0 & -5.5 & 5.5 \\
\end{bmatrix} \times 10^8 \]  

(5.26)

\[ [J] = \begin{bmatrix}
9500 & 0 & 0 \\
0 & 10500 & 0 \\
0 & 0 & 0.11 \times 10^6 \\
\end{bmatrix} \]  

(5.27)

Similarly, the stiffness matrix and inertia matrix of the connected drives system with 6-degree of freedom (see Fig. 5.3) can also be derived.

All the above free torsional vibration systems are governed by the following sets of equations:

\[ [J]\ddot{\theta} + [k]\theta = 0 \]  

(5.28)

\[ \theta_i = \Theta_i \sin(\omega t + \Phi_i) \]  

(5.29)
Equation (5.28) is of the form:

\[\begin{bmatrix} -\omega^2 [J] + [k] \end{bmatrix} \{\theta\} = 0 \quad (5.30)\]

Equation (5.30) has a solution if the characteristic determinant is equal to zero.

\[| [J]^{-1} [k] - \omega^2 [I] | = 0 \quad (5.31)\]

The modal vector \(\{u\}\), gives the relative vibration amplitude of the mass at a given natural frequency. The modal vector is determined by

\[\begin{bmatrix} -\omega^2 [J] + [k] \end{bmatrix} \{u\} = 0 \quad (5.32)\]

Each natural frequency \(\omega\) has a modal vector \(\{u\}\). A modal matrix \([u]\) is the combination of modal vectors.

The program "Modal Analysis" has been written in Fortran 77 to solve the equation (5.32) for all three torsional vibration models.

5.3.2 Discussion and Result of Torsional models

5.3.2.1 Top Drive Line

The natural frequencies and the modal matrix of top drive line are as follows:

Natural Frequencies (Hz)

<table>
<thead>
<tr>
<th>Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.</td>
</tr>
<tr>
<td>11.4082</td>
</tr>
<tr>
<td>28.0560</td>
</tr>
<tr>
<td>72.1914</td>
</tr>
</tbody>
</table>
Modal Matrix

\[
\begin{pmatrix}
1.00000 & 1.00000 & 1.00000 & 1.00000 \\
1.00000 & -3.44573 & -25.8880 & -177.024 \\
1.00000 & -4.13102 & -15.8410 & 522.283 \\
1.00000 & -5.63180 & 25.8984 & -53.9061
\end{pmatrix}
\]

The relative modal vector is based on unit displacement of the top motor. The first natural frequency is zero. It means, there is no relative motion in the system. This system can move as a rigid body and is called semi-definite. The first modal vector confirms this point. The second (or the first non-zero) natural frequency is close to the main frequency of the torsional vibration of the top drive components when the calculations are based on the symmetrical rolling (see chapter 6). Fig. 5.4 shows the second mode shape of the top drive line components.

Fig. 5.4 The Second Mode Shape of Top Drive System

5.3.2.2 Bottom Drive Line and Connected Drive Lines System

The natural frequencies and the modal matrix of bottom drive line are as follows
The first non-zero natural frequency is close to the main frequency of the torsional vibration of the bottom drive components when the calculation is based on symmetrical rolling theory (see chapter 6). Also, the results for the connected drive lines system are
The second non-zero natural frequency (14 Hz) is close to the torsional vibration frequency of both top and bottom drive components when the calculation is based on asymmetrical rolling (see chapter 7). Furthermore, this frequency is also close to the measured frequency of the spindles which is given in Fig. 5.5.

Fig. 5.6 and Fig. 5.7 show the torsional mode shapes of bottom drive system and connected drive system respectively.
Fig. 5.6 The second mode shape of bottom drive system

Fig. 5.7 The third mode shape of the connected system (14.03 Hz)
5.4 Vertical Vibration Modes of Horizontal Mill

5.4.1 Vertical Vibration Model

The 4-high Horizontal Roughing mill housing has been shown in Fig. 5.8a. A six-degrees of freedom model has been designed for vertical vibration of the housing elements as shown in Fig. 5.8b, where

\[ \begin{align*}
    k_0 &= \text{housing stiffness} \\
    k_1 &= \text{stiffness of screw} \\
    k_2 &= k_6 = \text{equivalent bending stiffness of the backup-rolls and bearing oil} \\
    k_3 &= k_5 = \text{effective stiffness for compressive contact of backup-rolls and work-rolls} \\
    k_4 &= \text{the non-linear stiffness of slab in the vertical vibration} \\
    c_4 &= \text{damping of slab in the vertical vibration} \\
    m_1 &= \text{housing mass} \\
    m_2 &= \text{mass of backup-roll bearings} \\
    m_3 &= m_6 = \text{mass of backup-rolls} \\
    m_4 &= \text{mass of top work-roll} \\
    m_5 &= \text{mass of the bottom work-roll and equivalent mass of slab}
\end{align*} \]

Table 5.2 shows the values of the mass and stiffness. The advantage of this model is that the bending stiffness of the top and bottom back-up roll have been taken into account compared with 5-degrees of freedom model shown in Fig. 5.8c (Misonoh model [44]).
Fig. 5.8a  Housing Elements

Fig. 5.8b  6-Degrees of freedom vertical vibration model
Fig. 5.8c  5-Degrees of freedom
vertical vibration model [15]

The spring constant due to the backup-roll, work-roll contact is given by Yarita et al. [16] as

\[ k_3 = k_5 = \frac{\pi L_r}{B \left[ 0.1182 + \ln (d_w + d_b) - \ln B - \ln \frac{p}{L_r} \right]} \]  (5.33)

where

\[ B = \frac{(1 - \nu_w^2)}{E_w} + \frac{(1 - \nu_b^2)}{E_b} \]  (5.34)

\[ L_r = \text{roll width (m)} \]
\[ p = \text{rolling force} \]
\[ d_b \text{ and } d_w = \text{diameter of backup-roll and work-roll (m)} \]
\[ \nu_b \text{ and } \nu_w = \text{Poisson's ratio for backup-roll and work-roll} \]
\[ E_w \text{ and } E_b = \text{Modulus of elasticity for backup-roll and work-roll (Pa)} \]
The screw down is considered to be a bar under axial load. The spring constant can be determined by

\[ k = \frac{EA}{l} \quad (5.35) \]

- \( E \) = Young's modulus of elasticity (Pa)
- \( A \) = cross sectional area of bar (m²)
- \( l \) = length of bar (m)

The housing mill columns above the floor are considered as bars under axial load. The 4 mill columns are treated as 4 springs in parallel. Thus, \( k_0 \) is the equivalent stiffness of two springs.

The backup-roll bearing stiffness was taken from Reference [109] which has been shown in Fig. 5.9. The values of oil film stiffness with respect to bearing load are presented in a group of curves for different mill speed. The curves are given only for a 55 in (1.4 m) diameter back-up roll which is close to the Horizontal mill back-up roll diameter.

The bending stiffness of backup-rolls is based on the beam theory and the relation introduced by Tselikov[51]. The forces acting on the back-up rolls (neglecting the counter bending force) are \( q \) the rolling force per unit slab width and the supports reactions \( F = \frac{P}{2} \) presented in Fig. 5.10. The bending stiffness is defined as the ratio of rolling force to maximum deflection which is given in the following:

\[ k_{\text{bending}} = \frac{P}{y_{\text{max}}} = \frac{18.8ED^4}{8a^3 - 4ab^2 + b^3 + 64c^3 \left( \frac{D}{d} \right)^4 - 1} \quad (5.36) \]

where, \( E \) is the modulus of elasticity.

Yuen et al.[164] have shown the influence of shear force on the deflection of rolls.
can be important because of high rolling force. However, the shear stiffness has been neglected in this thesis as did most of the investigators in the analysis of the practical dynamic systems such as that in [161].

![Graph showing the Backup-roll oil bearing stiffness](image)

**Fig. 5.9** The Backup-roll oil bearing stiffness [109]

![Diagram showing forces acting on a roll](image)

**Fig. 5.10** Forces acting on a roll [51]

The stiffness and damping of slab $k_4$ and $c_4$ were calculated in part 5.2. Moreover, one third of the slab mass was combined to the bottom work-roll because this weight is not negligible. The above approximation has been derived as shown in appendix 5.
The equation of motion for free undamped vibration as follows
\[ [m] \ddot{y} + [k]y = 0 \quad (5.44) \]

The method of solution is similar to the method applied for torsional vibration. The natural frequencies and modal matrix are as follows

**Natural Frequencies (Hz)**

- 71.7038
- 122.394
- 130.037
- 304.496
- 355.123
- 392.421

**Modal Matrix**

\[
\begin{array}{cccccc}
26.3847 & -0.306359 & 0.750378 & 43.1437 & 6.86141E-04 & -0.159114 \\
41.3280 & 3.97511E-02 & -0.216233 & -357.802 & -7.69951E-03 & 3.68351 \\
54.7388 & 0.446166 & -1.31631 & -418.411 & -6.25916E-03 & -1.98836 \\
63.9892 & 0.845693 & -2.27654 & 572.095 & 1.69510E-02 & 0.643052 \\
3.00132 & 2.04294 & 2.02183 & -15.2813 & -0.613697 & -0.983453 \\
1.00000 & 1.00000 & 1.00000 & 1.00000 & 1.00000 & 1.00000 \\
\end{array}
\]

### 5.4.2 The Influence of Slab Stiffness

The effect of the slab stiffness can be demonstrated by comparing the natural frequencies and modal matrix of the Horizontal mill with and without strip in bite. In this section, the mill without strip will be analysed. The spring constant \( k_4 \) is replaced by the contact stiffness between the work-rolls. Equation (5.33) is used and the backup-roll diameter is replaced by the work-roll diameter. The modal analysis
results of the above study is given here:

Natural Frequencies (Hz)

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>78.5300</td>
</tr>
<tr>
<td>116.248</td>
</tr>
<tr>
<td>181.852</td>
</tr>
<tr>
<td>250.064</td>
</tr>
<tr>
<td>308.554</td>
</tr>
<tr>
<td>357.437</td>
</tr>
</tbody>
</table>

Modal Matrix

<table>
<thead>
<tr>
<th></th>
<th>2.13221</th>
<th>-0.713808</th>
<th>0.178395</th>
<th>-3.91365E-02</th>
<th>6.18791E-02</th>
<th>-0.779701</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.86141</td>
<td>0.111842</td>
<td>-0.739629</td>
<td>0.397470</td>
<td>-1.04088</td>
<td>18.2935</td>
<td></td>
</tr>
<tr>
<td>3.35106</td>
<td>1.07597</td>
<td>-1.30630</td>
<td>0.373660</td>
<td>-0.174988</td>
<td>-10.50842</td>
<td></td>
</tr>
<tr>
<td>3.17294</td>
<td>2.00532</td>
<td>-0.156302</td>
<td>-0.810578</td>
<td>2.11434</td>
<td>6.18378</td>
<td></td>
</tr>
<tr>
<td>2.40103</td>
<td>2.04171</td>
<td>1.08438</td>
<td>-0.357870</td>
<td>-1.95245</td>
<td>-2.53056</td>
<td></td>
</tr>
<tr>
<td>1.00000</td>
<td>1.00000</td>
<td>1.00000</td>
<td>1.00000</td>
<td>1.00000</td>
<td>1.00000</td>
<td></td>
</tr>
</tbody>
</table>

5.4.3 Five-Degrees of Freedom Model

Also, a 5-degrees of freedom model introduced by Misonoh [15] (see Fig. 5.8c) was used to compare with the above 6-degrees of freedom model of roughing mill. The backup-rolls bending and the equivalent mass of slab have been neglected in the 5-degrees of freedom model. The data applied in 5-degrees of freedom model are given in Table 5.3 where \( m_2 \) is the mass of backup-roll and backup-roll bearing. The program output regarding this model is
5.4.4 Discussion on Vertical Vibration Analysis

The first natural frequency is close to the vertical vibration frequency of housing elements excited by rolling force (see chapter 6). The study showed that the influence of slab stiffness on the values of the first and the second natural frequencies of the system is not as great as at higher natural frequencies. Moreover, slab stiffness increases the amplitudes specially at the first natural frequency. In hot rolling the slab stiffness is lower than the other components and the system is softer with the hot slab in the roll bite than without it.

There is a difference between the natural frequencies and modal amplitudes of the 6-degrees of freedom model and 5-degrees of freedom model. The difference is due to the simplifications in 5-degrees of freedom model compared with 6-degrees of freedom model. Using the same mass for work-rolls, the neglect of the back-up rolls
reasons for this difference. Thus the 5-degrees of freedom model is stiffer than the 6-
degrees of freedom model as demonstrated by the first and the second natural
frequencies of the 6-degree of freedom model which are 18% and 30% respectively
less than those of the 5-degrees of freedom model.

5.5 Torsional Model of Edger and
Interaction Vibration Model

A simplified two-degrees of freedom model of Vertical Edger for torsional vibration
has been designed as can be seen in Fig. 5.11. The Edger torsional model is
symmetrical because the drives are identical. Thus, calculation of one drive line is
adequate and the fundamental natural frequency of Edger can be determined by the
analytical relation of:

\[ \omega = \sqrt{\frac{k_7}{J_8 + J_9}} = 31.81 \text{ Hz} \]

where,

- \( J_8 \) = Edger motor and gears inertia = 54921. Kg.m²
- \( J_9 \) = Work-roll inertia = 3180 Kg.m²
- \( k_7 \) = Spindle Stiffness = 1.2 \times 10^8 N/m

Fig. 5.11 Torsional Model of Edger

The model to simulate the interaction between torsional vibration of Edger and
Horizontal mill is shown in Fig. 5.12. In the study of vibration in tandem mills, the
Horizontal mill is shown in Fig. 5.12. In the study of vibration in tandem mills, the following factors will be considered in chapter 6 which have not been discussed previously:

(i) the slab acting as an independent mass between the two mills.

(ii) the rigidity and damping of the slab in both plastic and elastic regions in the simulation.

This model of the horizontal mill includes the top and bottom spindles and motors which have been connected by the damping and stiffness of slab in the plastic region.

(iii) calculation of the push-pull forces between the two mills as a result of mismatch between the surface speed of the horizontal mill and Vertical Edger.

Fig. 5.12 Interaction Vibration Model
The influence of asymmetries on vibrations will also be included in Chapter 7. The asymmetries are caused by:

(i) difference in friction conditions on the upper and lower rolls.
(ii) difference in angular velocity of the rolls due to torsional vibration or mismatching speed.

Another important factor during rolling mill vibration is the oscillation of kinetic coefficient of friction which will be investigated in Chapter 8.
CHAPTER 6

VIBRATION ANALYSIS IN
SYMMETRICAL ROLLING
6-1 Introduction

Torsional vibration of drive shafts, rolls, and vertical vibration of rolling housings due to impact rolling loads should be considered because they can shorten the lives of rolling mill components. Moreover these dynamic loads can affect the quality and physical properties of the strip.

There are many published works on torsional vibration of rolling mills. Some of them are mathematical models and computer simulations that calculate the natural frequencies of torsional and linear vibration (refer to chapter 5). Other work has correlated the dynamic response of rolling mill elements due to impact rolling force or torque against experimental studies. Dukmasov [34] established a force model for vertical vibration of hot strip rolling taking into account the stiffness and damping of hot steel. In chapter 5, this technique was applied to a torque model to find the torsional stiffness and damping of the slab between the work-rolls. Dobrucki et al. [38] introduced a model to calculate the interaction between the torsional vibrations of a tandem slabbing mill.

In this chapter, four different types of vibrations of the Roughing Mill at BHP Steel SPPD (see Fig 5.1) will be studied:

(i) torsional vibration of the Horizontal mill
(ii) vertical vibration of the Horizontal mill
(iii) torsional vibration of Vertical Edger
(iv) horizontal vibration of slab between the two mills.

This simulation is based on symmetrical rolling force and torque represented as a step or a ramp-step function. The influence of asymmetrical rolling on the above vibrations will be studied in Chapters 7 and 8.
6.2 Vertical Vibration of Horizontal Mill

6.2.1 Step Function Simulation

6.2.1.1 Dynamic Model and Formulation

A six-degrees of freedom model for vertical vibration shown in Fig 5.9 (Chapter 5) is reproduced here. To calculate the transient response of housing elements, two forces equal to symmetrical rolling force \( P \) were applied on the top and bottom work-rolls. The above forces are constant for the step function simulation and the value has been determined in chapter 4. The slab stiffness and damping \( (k_4, c_4) \) have been determined from equations (5.5) and (5.6) respectively. They will be constant during the course of step function simulation. The slab damping applied as the only source of energy dissipation. The internal damping of housing elements (cold steel) have been neglected [131].

Fig. 6.1a  Vertical vibration model
Six differential equations of vertical vibration, one for each mass has been derived. For example, the equations of motion of top work-roll $m_4$ and housing $m_1$ are as follows

\[ m_4 \ddot{y}_4 + k_3 (y_4 - y_5) + k_4 (y_4 - y_5) + c_4 (\dot{y}_4 - \dot{y}_5) = -P(t) \]  \hspace{1cm} (6.1)

\[ m_1 \ddot{y}_1 + k_1 (y_1 - y_2) + k_0 y_1 = 0 \]  \hspace{1cm} (6.2)

Subroutine Dverk was introduced by Hull et al. [64] based on the sixth order Runge Kutta method to find the approximate solution to the first order differential equations with initial values. The above subroutine was applied to some differential equations having analytical solutions. The numerical results from the Dverk subroutine were accurate when compared with the analytical solution. Therefore, this method was used to solve the differential equations in this thesis.

### 6.2.1.2 Discussion and Results

Fig 6.1b illustrates the damped dynamic deflection of four elements whereas the damped and undamped vertical vibration of two other mass are shown in Fig. 6.2 and Fig. 6.3. The top rolls have the greatest deflections $y_3$ and $y_4$ and their overshoots i.e. the difference between dynamic and static deflections, is higher than the others. On the other hand, the mill housing has the smallest deflection. It should be pointed out that these elongations are in the range of millimetre.

The important influence of slab damping has been demonstrated in Fig 6.1b and 6.2 where it significantly reduces the vibration. Thus, the slab vertical chatter can be affected partly by this damping. The vibration frequency of the elements above the slab is lower than that for those under the slab, because of the difference between the equivalent stiffness.
The FFT (Fast Fourier Transform) for the top work-roll, housing and bottom backup-roll are given in Fig 6.4, 6.6 and 6.7 respectively. The main vibration frequency of the elements above the slab is 72.2 Hz while under the slab is 123 Hz. It was shown in chapter 5 that the above values were the first and second natural frequencies of vertical vibration. Moreover, the nature of vibration of top work-roll velocity shown in Fig. 6.5 is similar to the vibration of top work-roll displacement.

![Housing, backup-rolls and bottom work-roll displacements](image)

**Fig. 6.1b**  Housing, backup-rolls and bottom work-roll displacements
Fig. 6.2 Back-up roll chuck displacement

Fig. 6.3 Vertical vibration of top work-roll
Fig. 6.4 FFT of top work-roll vertical vibration

Fig. 6.5 Top Work-roll Velocity
6.2.2 Ramp-Step Function Result

The entry time is defined as the time that the slab needs to contact completely the work-rolls at the roll bite. If this time is not near zero (slow rolling), the rolling force acts as a ramp-step function and the vibration amplitude is less than that of the step function. Using 0.04 sec for entry time, the vertical vibration of the top work-roll is shown in Fig. 6.8.
6.3 Torsional Vibration of Horizontal Mill

6.3.1 Dynamic Model

A 7-degrees of freedom model is given in Fig 5.2 (Chatter 5). If we connect the top work-roll inertia $J_4$ and bottom work-roll $J_5$ by torsional damping $c_4$ (determined in chapter 5), then the complete dynamic torsional model of Horizontal mill will be available.

6.3.2 Discussion and Results

The input rolling torque on the two work-rolls is as a step function and there is a delay time in the response of the electrical motors. Torque amplification factor, TAF, is defined as the ratio of maximum dynamic torque to the steady state rolling torque.
The step function rolling torque $T$ was applied on both the top and bottom work-rolls. The value of symmetrical rolling torque $T$ was calculated in chapter 4. The seven differential equations of motion for torsional vibration of all mass inertia were derived. For example, for torsional vibration of the top work-roll:

$$J_4 \ddot{\theta}_4 + c_4 (\dot{\theta}_4 - \dot{\theta}_5) + k_3 (\theta_4 - \theta_3) + c_3 (\dot{\theta}_4 - \dot{\theta}_3) = -T \quad (6.3)$$

After solving the equations numerically, then the dynamic torque $T_{si}$ and $TAF_i$ of each shaft was calculated using the following relations:

$$T_{si} = k_i \left( \theta_i - \theta_{i+1} \right) + c_i \left( \dot{\theta}_i - \dot{\theta}_{i+1} \right) \quad (6.4)$$

$$TAF_i = \frac{T_{si}}{T} \quad (6.5)$$

If the couplings include backlash, then the equations of motions should take it into account.

The backlash was discussed in part 1.2.3. Here, the absolute value of the backlash of each shaft is:

$$G_i = G_{ip} = |G_{in}| \quad (6.6)$$

For example, if the top spindle (shaft between $J_3$ and $J_4$) includes backlash $G_3$, then the equation (6.4) will be modified in the following way:

$$J_4 \ddot{\theta}_4 - T_{s3} + c_4 (\dot{\theta}_4 - \dot{\theta}_5) = -T \quad (6.7)$$

where,

$$T_{s3} = k_3 (\theta_4 - \theta_3 - G_3) + c_3 (\dot{\theta}_3 - \dot{\theta}_4) \quad (6.8)$$
for \( \theta_3 - \theta_4 \geq G_i \) \( , \theta_3 - \theta_4 \leq -G_i \)

and \( T_{s3} = 0 \) \hspace{1cm} (6.9)

\( \text{for } -G_i \leq \theta_3 - \theta_4 \leq G_i \)

6.3.2.1 Torsional Vibration without Backlash

Fig 6.9 and Fig 6.11 show the torque amplification factor variation of top and bottom drive shafts against time respectively. The maximum TAF on the top and bottom drive shafts occurs on the motor shafts.

The frequency of torsional vibration of top-drive line is 11.7 Hz (see Fig. 6-10) while that of bottom drive is 18 Hz. Moreover, the variation of top work-roll velocity is given in Fig 6.12.
Fig. 6.9  Torsional Vibration of Top Drive Line

Fig. 6.10  FFT of top motor shaft
Fig. 6.11  Torsional Vibration of Bottom Drive Line

Fig. 6.12  Variation of top Roll Speed
6.3.2.2 Torsional Vibration with Backlash

Here, backlash was introduced to the dynamic system. For example, Fig. 6.13 illustrates the torsional vibration of top spindle without backlash and also with backlash $G_3 = 0.001 \text{ rad}$. The result showed that the backlash increases the torsional vibration significantly by up to 13.6%.

![Torsional Vibration of top Spindle](image)

Fig. 6.13  Torsional Vibration of top Spindle

6.4 Interaction Between the Vibration of Coupled Tandem Mills

6.4.1 Dynamic Model and Formulation

A ten-degrees of freedom model was designed to calculate the torsional vibration of the Horizontal mill and Edger and horizontal vibration of the slab between two mills. The model was shown in Fig. 5.12 in chapter 5. There are some new factors in this study which have not been discussed previously:

(i) the slab acting as an independent mass between the two mills.
(ii) the damping of the slab in both plastic (slab vertical and torsional damping) and elastic zone (slab horizontal damping) and also the stiffness of slab in elastic region (horizontal stiffness between the two mills) in the simulation. In this model, the Horizontal mill includes the top and bottom spindles and motors which have been connected by the damping of slab in the plastic region. Then, it is possible to investigate the influence of asymmetry caused by the difference between the speed of the top and bottom drive lines during torsional vibration.

(iii) calculation of the push-pull forces between the two mills as a result of mismatch between the surface speed of the Horizontal mill and Vertical Edger.

In the forward pass of the Roughing Mill at BHP, the Vertical Edger first experiences impact of slab before it enters the Horizontal mill roll bite. Thus there are two kinds of torsional impacts for the Edger. The first vibration is before interaction with the Horizontal mill. The second is during the interaction with Horizontal mill. The calculation of Horizontal mill torsional vibration has been discussed in 6.2. The ramp-step function is used for this study and the result is given in Fig. 6.16 (before interaction). The amplitude of this vibration is small and the frequency of vibration of is 31.2 Hz (see Fig 6.17) which is close to natural frequency of Edger (see chapter 5).

The values of displacement and velocity of Edger at time t=0.17 second from the first vibration was applied as initial values for the interaction condition. Ten differential equations were written for linear vibration of slab and torsional vibrations of both the Horizontal and Vertical mills. For example, the equation of motion for Edger roll is as follows:

\[ J_9 \ddot{\theta}_9 + c_8 (\dot{\theta}_9 - \dot{\theta}_8) + k_8 (\theta_9 - \theta_8) + f_9 R_9 = -T_v \quad (6.10) \]

where

\[ f_9 = c_x (R_9 \dot{\theta}_9 - \dot{x}) + k_x (R_9 \theta_9 - x) \quad (6.11) \]
\[ R_9 = \text{Edger roll radius} \]
\[ x, \dot{x} = \text{slab displacement and velocity} \]
\[ T_v = \text{Edger rolling torque} \]
\[ c_x = \text{slab damping in elastic zone} \]

Fig. 6.14 Torsional vibration of top spindle

Fig. 6.15 FFT of top spindle torsional vibration
Fig. 6.16  Torsional Vibration of Edger

Fig. 6.17  FFT of Edger torsional vibration
Fig. 6.18  Push-pull between the two mills

Fig. 6.19  FFT of Push-pull
6.4.2 Discussion and Results

The Horizontal mill torsional vibration and FFT of top spindle is given in Fig 6.14 and Fig. 6.15 respectively. The main vibration frequency is 11.7 Hz, similar to the condition of no interaction.

The TAF of interaction condition for both mills is higher than without interaction which agrees with the results by Tieu [41]. However, the influence of interaction increased the amplitude of Edger vibration more significantly than that of Horizontal mill (see Fig 6.16). The main frequency of Edger vibration is 31.2 Hz, which is the
natural frequency of the Edger. The other frequencies are 11.7 Hz and 100 Hz, which are the frequencies of the Horizontal mill torsional vibration and the horizontal vibration of the slab (or push-pull) respectively. Also, the variation of push-pull force and slab velocity are shown in Fig 6.18 and 6.20 respectively. Furthermore, the FFT in Fig 6.19 shows that the vibration frequency of slab (or push-pull) is 100 Hz.
CHAPTER 7

VIBRATION ANALYSIS IN ASYMMETRICAL ROLLING
7.1 Introduction

Some analytical and experimental studies have been carried out on asymmetrical rolling. Analytical methods were based on the slab method which were not able to predict the radius of curvature of the rolled products. Kiuchi et al.[105] used upper-bound method and introduced a numerical solution for the rolling of both plain and cladding sheets by optimisation of the velocity field. Pietrzyk et al. [108] also investigated the asymmetrical hot rolling of bimetallic sheet and their results show the accuracy of both the upper bound and finite element methods to be reasonable. They concluded that the upper-bound method based on a simplified velocity field can be used successfully to calculate the rolling process parameters such as rolling force or torque. This method requires less computer memory and shorter computing times when compared with the finite element approach but it still produces satisfactory accurate results. Thus, a numerical solution based on the upper-bound method will be applied in this chapter to evaluate the velocity field and also rolling force and torque of asymmetrical hot slab rolling with work-rolls having two different diameters. We recognize that the upper-bound method is not unique, but only one among the many solutions can provide results which are reasonable and practical (see section 7.2.3).

On the other hand, the complete numerical upper-bound method solution for transient condition study (vibration) requires large computation time, because the rolling force and rolling torque are calculated for each time step. Therefore, the simplified analytical formulae based on the upper-bound method has been used in this chapter for vibration analysis of asymmetrical rolling.
7.2 Upper-Bound Method

7.2.1 Introduction

The upper-bound method is based on a determination of the external power required for the metal forming process and an evaluation of the velocity field in the metal forming process.

Fig 7.1.a shows the geometrical configuration of the slab and two rolls with two different diameter.

Fig. 7.1.a Neutral point positions on the rolls with different diameters and velocities
The subscript 1 is used for top-roll and subscript 2 for the bottom roll. The \( xy \) co-ordinate axis origin is positioned on the exit plane at mid-thickness between the two rolls. Also, the angular co-ordinate \( \varphi \) is zero at the exit plane. The other assumptions in Fig. 7.1.a are as follows:

(i) \( R_2 > R_1 \)

\( \alpha_1 > \alpha_2 \)

bottom roll velocity \( V_2 \) > top roll velocity \( V_1 \)

neutral point thickness \( h_A \) > neutral point thickness \( h_B \)

on top surface on bottom surface

(ii) plane strain deformation exists

(iii) distribution of slab velocity horizontal component between the top and bottom work-roll is linear.

The simplified horizontal and vertical components of the slab velocity based on linear horizontal velocity assumption in the roll bite are determined as follow [92]:

---

**Fig. 7.1.b** Neutral points positions on the rolls with different angular velocities \( \dot{\theta}_2 > \dot{\theta}_1 \)
\[ v_x = E \left\{ \frac{1}{h_x} + C_1 \left[ 2y - (h_{x1} - h_{x2}) \right] \right\} \]  
\hspace{1cm} (7.1)

\[ v_y = E \left\{ \frac{\dot{h}_{x1}}{h_x} + \frac{\dot{h}_x}{h_x^2} (y - h_{x1}) + \frac{\partial C_1}{\partial x} (h_{x1} - y)(h_{x2} + y) + C_1 D \right\} \]  
\hspace{1cm} (7.2)

where

\[ \dot{h}_x = \frac{dh_x}{dx}, \quad \dot{h}_{x1} = \frac{dh_{x1}}{dx}, \quad \dot{h}_{x2} = \frac{dx_2}{dx} \]  
\hspace{1cm} (7.3)

\[ C_1 = -\frac{\beta_1}{h_x h_1} \left( 1 - \beta_2 \frac{h_x}{h_0} \right) \]  
\hspace{1cm} (7.4)

\[ D = h_x \dot{h}_{x1} - (\dot{h}_{x1} - \dot{h}_{x2})(h_{x1} - y) \]  
\hspace{1cm} (7.5)

\[ E = -v_0 h_0 \]  
\hspace{1cm} (7.6)

\[ \beta_1 \text{ and } \beta_2 = \text{constant} \]  
\hspace{1cm} (7.7)

\[ v_0 = \text{entry slab velocity} \]

Detailed derivation of the slab velocity components \( v_x \) and \( v_y \) in equations (7.1) and (7.2) are given in Appendix 3.

The externally supplied power per unit width in the metal forming process is defined as:
\[ N = N_p + N_t + N_s = \int_{v} \bar{\sigma} \bar{\epsilon} \, dv + \int_{s} \tau |v_s| \, ds + \sigma \int_{s_t} |v_{y_L}| \, ds \quad (7.8) \]

where

- \( N_p \) = power for internal deformation
- \( N_t \) = friction losses in the roll-slab interface
- \( N_s \) = shear power over the surface of velocity discontinuity
- \( \bar{\sigma} \) = effective stress
- \( \sigma \) = yield strength
- \( \bar{\epsilon} \) = effective strain rate
- \( V \) = volume of deformation zone
- \( s_t \) = contact surface
- \( s \) = surface of velocity discontinuity

(cross section of slab at point C in Fig. 7.1a)

- \( \tau \) = shear stress at the contact surface
- \( v_{y_L} \) = \( v_y \) at \( x = L \)
- \( v_s \) = slip velocity

After substitution of equations (7.1) and (7.2) into equation (7.8), there are five unknowns (\( \beta_1 \), \( \beta_2 \), \( v_0 \), \( h_A \) and \( h_B \)) in the calculation of total power \( N \).

If the horizontal component of velocities across the slab section at point C (entry plane) is the same as the entry slab velocity, then:

\[ v_x = -v_0 \quad (7.9) \]

at this point and with substitution of this value into equation (7.1) gives

\[ \beta_2 = 1 \quad (7.10) \]

The effective stress and effective strain rate have been defined as
\[ \sigma = \sigma = \sqrt{3} k \]  
(7.11a)

\[ \bar{\varepsilon} = \sqrt{\frac{2}{3} \left( \dot{\varepsilon}_x \dot{\varepsilon}_y \right)} = \sqrt{\frac{2}{3} \left( \dot{\varepsilon}_1^2 + \dot{\varepsilon}_2^2 + \dot{\varepsilon}_3^2 \right)} \]  
(7.11b)

where,

\[ \sigma \] = flow stress from uniaxial test (see chapter 3)

\[ k \] = shear yield stress

\[ \dot{\varepsilon}_1, \dot{\varepsilon}_2, \dot{\varepsilon}_3 \] = principal strain rates

Also, for principal strain rates we have

\[ \dot{\varepsilon}_1^2 + \dot{\varepsilon}_2^2 + \dot{\varepsilon}_3^2 = \]

\[ \left[ \frac{1}{2} \left\{ (\dot{\varepsilon}_x - \dot{\varepsilon}_y)^2 + (\dot{\varepsilon}_y - \dot{\varepsilon}_z)^2 + (\dot{\varepsilon}_z - \dot{\varepsilon}_x)^2 \right\} + \frac{3}{4} \left( \dot{\gamma}_{xy}^2 + \dot{\gamma}_{yz}^2 + \dot{\gamma}_{zx}^2 \right) \right]^{\frac{1}{2}} \]  
(7.11c)

where

\[ \dot{\varepsilon}_x, \dot{\varepsilon}_y, \dot{\varepsilon}_z \] = strain rates

\[ \dot{\gamma}_{xy}, \dot{\gamma}_{yz}, \dot{\gamma}_{zx} \] = shear strain rates

The plastic deformation is assumed to be incompressible, thus

\[ \varepsilon_x + \varepsilon_y + \varepsilon_z = \dot{\varepsilon}_x + \dot{\varepsilon}_y + \dot{\varepsilon}_z = 0 \]

and for the plane strain conditions

\[ \dot{\varepsilon}_z = 0, \quad \dot{\gamma}_{zx} = \dot{\gamma}_{yz} = 0, \quad \dot{\varepsilon}_y = -\dot{\varepsilon}_x \]  
(7.11d)

Then, the following equation is derived from equations 7.11b, c, d

\[ \bar{\varepsilon} = \frac{2}{\sqrt{3}} \sqrt{\varepsilon_x^2 + \frac{1}{4} \dot{\gamma}_{xy}^2} \]  
(7.12)
where
\[ \dot{\gamma}_{xy} = \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \]

\[ \dot{\varepsilon}_x = \frac{\partial v_x}{\partial x} \]

then

\[ N_p = k \int_{0}^{L} \int_{-b x_2}^{b x_1} \sqrt{4 \dot{\varepsilon}_x^2 + \dot{\gamma}_{xy}^2} \, dx \, dy \quad (7.13) \]

From equation 7.8 the second term of the right hand side can be written as:

\[ N_t = \int_{s} \tau \, |v_s| \, ds \quad (7.13a) \]

where the shear stress \( \tau \) is equal to \( \mu S \) because of the slipping friction model assumption in this thesis (see section 4.9). Let us estimate the average shear stress to simplify the above equation.

According to the 4th and 7th assumption in the simple model of flat rolling (part 2.2.1.1):

\[ S - \sigma_x = 2k \quad \text{(for exit side)} \quad (7.13b) \]

Also by replacing the curved surface of the rolls with flat parallel platens [57], then equation 2.2 becomes:

\[ h_1 \frac{d \sigma_x}{dx} = 2 \tau \quad \text{(for exit side)} \quad (7.13c) \]

\[ \tau = \mu S \quad (7.13d) \]

The pressure distribution \( S \) can be determined by integrating equation 7.13c and by using equations 7.13b and 7.13d:

\[ S = S_1 e^{\frac{2 \mu x}{h_1}} \quad (7.13e) \]
Where \( S_1 = 2k \) is the value of \( S \) at the exit plane \((x = 0)\).

Stone et al. [165] calculated the average pressure by integrating equation 7.13e at exit side. Thus

\[
\tau_{av} = \mu S_{av} = (2\mu k) \frac{e^A - 1}{A}
\]  
(7.13f)

where

\[
A = \frac{\mu L}{h_1}
\]

Normally the value of \( A \) is much less than one in the rolling of thick plates and slabs and \( e^A \) series is estimated as:

\[
e^A = 1 + A
\]

then

\[
\tau_{av} = 2\mu k
\]  
(7.13g)

\[
N_1 = 2k\mu \left[ \int_0^L |v_{top} - V_1| \, ds_1 + \int_0^L |v_{bottom} - V_2| \, ds_2 \right]
\]  
(7.14)

\[
N_s = k \int_{h_{es1}}^{h_{es2}} |v_{yl}| \, dy
\]  
(7.15)

where

\[
ds_1 = \frac{dx}{\cos \frac{\alpha_1}{2}}
\]  
(7.16)

\[
ds_2 = \frac{dx}{\cos \frac{\alpha_2}{2}}
\]  
(7.17)
\[ k = \text{shear flow stress for plane strain } = \frac{\sigma}{\sqrt{3}} \]

\[ v_{\text{top}} = \text{slab velocity at } y = h_1 \]

\[ v_{\text{bottom}} = \text{slab velocity at } y = -h_2 \]

\[ v_{yL} = v_y \text{ at } x = L \]

\[ \mu = \text{friction coefficient} \]

\[ h_{o1} = h_{x1} \text{ at } x = L \]

\[ h_{o2} = h_{x2} \text{ at } x = L \]

For more simplification, let us assume that the angles \( \alpha_1 \) and \( \alpha_2 \) are small. Thus,

\[ \cos \frac{\alpha_1}{2} = 1 \]

\[ \cos \frac{\alpha_2}{2} = 1 \]

And

\[ v_{xt} = v_{\text{top}} \]

\[ v_{xb} = v_{\text{bottom}} \]

Where \( v_{xt} \) and \( v_{xb} \) are the slab velocity horizontal components at top and bottom interface respectively.

Another assumption is that the thickness variation has a parabolic relationship with respect to \( x \):
\[ h_{x1} = \frac{h}{2} + \frac{1}{2R_1}x^2 \]  
\[ h_{x2} = \frac{h}{2} + \frac{1}{2R_2}x^2 \]  

then

\[ N_t = 2k\mu \left[ \int_0^{x_A} (v_{xt} + V_1) \, dx + \int_{x_A}^L (v_{xt} + V_1) \, dx \right. 
\left. - \int_0^{x_B} (v_{xb} + V_2) \, dx + \int_{x_B}^L (v_{xb} + V_2) \, dx \right] \]  

\[ x_A = \text{top layer interface neutral point distance from the exit side and it can be determined from the equation:} \]

\[ v_{xt} = V_1 \]  
\[ (7.21) \]

\[ x_B = \text{bottom layer interface neutral point distance from exit side and it can be calculated from the relation:} \]

\[ v_{xb} = V_2 \]  
\[ (7.22) \]

Also,

\[ N_s = k \int_{-h_m}^{y_1} v_{yL} \, dy - k \int_{y_1}^{h_{ol}} v_{yL} \, dy \]  
\[ (7.23) \]

the value of \( y_1 \) is related to the point with \( v_y = 0 \) at the entry cross section. Thus, \( y_1 \) can be found from the following equation:
\[ v_{yL} = 0 \] (7.24)

In the upper-bound method, the power \( N \) in equation (7.8) should be minimised with respect to the four unknowns \( x_A, x_B, \beta_1 \) and \( v_0 \).

### 7.2.2 Optimisation

#### 7.2.2.1 Random Search Methods

Rao [166] has explained different methods of optimisations. In this section, we explain the Random Search Methods which are branches of Direct Search Methods of Unconstrained Optimisation Techniques of Non linear Programming. This is because the Random Walk Method with Direction Exploitation has been applied for the upper-bound method in this chapter. Moreover, since most of the computers softwares have random number generators, the Random Search Methods can be used conveniently.

#### 7.2.2.2 Random Jumping Method

The problem is to find the minimum of \( f(x) \) in the \( n \)-dimensional co-ordinated defined by

\[ l_i \leq x_i \leq u_i , \quad i = 1,2, \ldots, n \] (7.24a)

where \( l_i \) and \( u_i \) are the lower and upper bounds on the variable \( x_i \). In the Random Jumping Method, sets of \( n \) random numbers \((r_1, r_2, \ldots, r_n)\) between 0 and 1 are generated. Each set of these numbers, is used to find a point \( X \), inside the domain defined by Equations 7.24a as
and the value of the objective function \( f(x) \) is evaluated at this point \( x \). By generating a large number of points and evaluating the value of the objective function at each of these points, we take the smallest value of \( f(X) \) as minimum point.

Although this random jumping method is simple, it is not very efficient for problems with many variables. Random Walk Method is more efficient than previous method as described below.

**7.2.2.3 Random Walk Method**

This method is based on generating a sequence of improved approximations to the minimum, each derived from the preceding approximation. Thus, if \( X_i \) is the approximation to the minimum obtained in the \((i-1)\)th iteration, the new or improved approximation in the \(i\)th stage is found from the relation

\[
X_{i+1} = X_i + \lambda u_i
\]  

(7.24c)

where \( \lambda \) is called scalar step length, and \( u_i \) is a unit random vector generated in the \(i\)th iteration. The steps of this method are given in the following:

1- Start with an initial point \( X_1 \) and a \( \lambda \) that is sufficiently large in relation to the desirable final accuracy. Determine the objective function \( f_1 = f(X_1) \).

2- Set the iteration number, \( i = 1 \).

3- Generate a set of \( n \) random numbers \( r_1, r_2, \ldots, r_n \) and calculate the unit random
vector \( u_i \):

\[
 u = \frac{1}{(r_1^2 + r_2^2 + \ldots + r_n^2)^{1/2}} \begin{pmatrix}
 r_1 \\
 r_2 \\
 \vdots \\
 r_n
\end{pmatrix}
\]  

(7.24d)

4- Determine the new objective function

\[
 f = f( X_1 + \lambda \ u )
\]

5- If \( f < f_i \), set \( X_i = X_1 + \lambda \ u \), and \( f_1 = f \), and repeat steps 3 - 5. If \( f \geq f_1 \), only repeat steps 3 - 5.

6- If a sufficiently large number of iteration (M) can not generate a better point, \( X_{i+1} \) reduce \( \lambda \) to half and go back to step 3.

7- If a better point could not be generated and the value of \( \lambda \) is less than the prescribed tolerance (\( \varepsilon \)), take the current point \( X_1 \) as the optimum point, and stop.

7.2.2.4 Random Walk Method With Direction Exploitation

In the previous random walk method a new unit random vector \( u_{i+1} \) is generated as soon as we find that \( u_i \) is successful in reducing the objective function value for a fixed step length \( \lambda \). However, we can expect to achieve a further decrease in the function value by taking a longer step length along the direction \( u_i \). Thus, the random walk method can be improved if each successful direction is applied. This can be achieved by using one-dimensional minimisation methods. Thus, the new point \( X_{i+1} \) is found as

\[
 X_{i+1} = X_i + \lambda^* u_i
\]  

(7.24e)
where $\lambda_i^*$ is the optimal $\lambda$ found along the direction $u_i$ so that

$$ f_{i+1} = f( X_i + \lambda_i^* u_i) = \min_{\lambda_i} f( X_i + \lambda_i u_i) \quad (7.24f) $$

The above search method is called the random walk method with direction exploitation. The flow chart of this method is shown in Fig. 7.2a.

![Flow Chart of the Random Walk Method with Direction Exploitation](image-url)
7.2.3 Asymmetrical Rolling Optimisation

The random walk method with direction exploitation was used to minimise the total of rolling power $N$ with respect to the two main variable $\beta_1$ and $v_0$. But, the upper-bound method is not unique. Because equations 7.21 and 7.22 (after substitution of equations 7.18 and 7.19 in equation 7.1) are 4th-orders equations. Therefore, many solutions are available, but only one is of a practical value. We have introduced two new loops in the flow chart of Fig. 7.2a to calculate the values of $x_A$, $x_B$ and $y_1$ and to select the practical values. The values of $x_A$ and $x_B$ must be inside the interval $0 < x < L$ for a stable rolling condition and also the condition of $-h_0 < y_1 < h_0$ must be satisfied (in equation 7.24). Fortunately, we found that for every calculation only one real answer was obtained.

The asymmetrical rolling optimisation flow chart is given in Fig. 7.2b. This program considers the above limitation of upper-bound method as explained in the following:

(a) The program starts by assuming $M$ and small values for $\varepsilon_v$ and $\varepsilon_\beta$ and the initial values for $v_0$, $\beta_1$, $\lambda_v$ and $\lambda_\beta$.

(b) Then the power $N_1$ as the objective function will be estimated.

(c) Two random numbers $r_v$ and $r_\beta$ will be generated using Mathematica Package:

$$\begin{bmatrix} u_v \\ u_\beta \end{bmatrix} = \frac{1}{(r_v^2 + r_\beta^2)^{1/2}} \begin{bmatrix} r_v \\ r_\beta \end{bmatrix}$$

and

$$\text{new } v_0 = \text{last } v_0 + \lambda_v u_v$$
$$\text{new } \beta_1 = \text{last } \beta_1 + \lambda_\beta u_\beta$$

(d) $x_A$, $x_B$ and $y_1$ will be calculated. If there is a real value for each $x_A$, $x_B$ and $y_1$ the new $N$ will be estimated. Otherwise, it returns to generate the new random numbers.
(e) If the new $N$ is less than $N_1$, then $N_1$ will be replaced by the new $N$ and the newer $v_0, \beta_1, x_A, x_B$ and $y_1$ will be calculated to find newer $N$.

(f) If the above newer $N$ is less than $N_1$, then the calculation is similar to step e.

The optimisation procedure will be stopped when $N$ is greater than $N_1$ and the values of $\lambda_v$ and $\lambda_{\beta}$ are less than small values of $\varepsilon_v$ and $\varepsilon_{\beta}$ respectively.

The list of the program is given in appendix 4.

The following data were used in the calculations:

<table>
<thead>
<tr>
<th>$R_1$</th>
<th>$R_2$</th>
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<tbody>
<tr>
<td>512.5 mm</td>
<td>639.8 mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$V_1$</th>
<th>$V_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5 m/s</td>
<td>5.7 m/s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$h_0$</th>
<th>$h_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 mm</td>
<td>34 mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>70 MPa</td>
</tr>
</tbody>
</table>

**Table 7.1 Data used for rolls with different diameters and velocities**

The radii $R_1$ and $R_2$ were arbitrarily chosen to be $\pm 10\%$ deviation from the nominal roll diameter to demonstrate the effect of asymmetry. This is because symmetrical rolling under steady state rolling condition is prevailing at the roughing mill.

The optimum power per unit width is 17.6 kw/m which corresponds to the optimum values of $v_0 = 3.95$ m/s and $\beta_1 = 0.03$. 
Start with $v_0$, $\beta_1$, $\lambda_v$, $\lambda_\beta$, $\epsilon_v$, $\epsilon_\beta$ and $M$

Find $N_1 = N(v_0, \beta_1)$

Set $i = 1$

Generate a set of 2 random numbers and calculate $u_v$ and $u_\beta$

$x_A = x_A (new \, v_0 \, and \, new \, \beta_1)$

$x_B = x_B (new \, v_0 \, and \, new \, \beta_1)$

Are $x_A$ and $x_B$ inside $0 < x < L$?

Set $N = N (new \, v_0 \, and \, new \, \beta_1)$

Are $x_A$ and $x_B$ inside $0 < x < L$?

Is $N < N_i$?

Set $i = i + 1$

Is $i > M$?

Set $\lambda_v = \lambda_v / 2$

Set $\lambda_\beta = \lambda_\beta / 2$

Is $\lambda_v < \epsilon_v$?

Is $\lambda_\beta < \epsilon_\beta$?

Yes

Set $v_0(\text{optimised}) = v_0$, $\lambda_v(\text{optimised}) = \lambda_v$

$N_{\text{optimised}} = N_1$

No

Is $x_A$ and $x_B$ inside $0 < x < L$?

Set $N = N (new \, v_0 \, and \, new \, \beta_1)$

Is $N < N_i$?

$N = N (new \, v_0 \, and \, new \, \beta_1)$

Yes

No

Fig. 7.2b Flow Chart of Asymmetrical Rolling Optimisation
A second method of optimising equation 7.8 is shown in details in appendix 6. In this method $\beta_1$ is assumed to be constant and $v_0$ is determined to achieve the local minimum power $N$. Then, this $v_0$ value is kept constant and $\beta_1$ is calculated to find the minimum $N$ which may or may not be the global minimum. This method is not a standard method to give a global minimum of $N$ but it provides a specific testing of the random walk method. The results are not significantly different from those determined by the random walk method with direction exploitation. Thus, it confirms the correctness of the random walk method in the minimisation process required in the upper-bound method.

The above program (Fig. 7.2b) is useful to evaluate the influence of rolling parameters such as thickness, flow stress, roll radius on the rolling power or slab velocity. For example, the influence of the entry thickness has been demonstrated in Fig. 7.2c while $h_1$, $R_1$, $R_2$, $V_1$ and $V_2$ are similar to table 7.1. The minimum value corresponds with the data used before. The power increases as a result of increasing the entry thickness of the slab.
Moreover, the above optimisation program was applied to the case of symmetrical rolling (introduced in section 4.8) and the result for optimised entry side slab velocity shows good agreement with actual measurement (see table 7.2). The results for rolling force and torque however show fair agreement. The following relations was used to calculate rolling torque $T$ and force $p$ from the calculated total power $N$ per unit width

$$T = \frac{N}{2\dot{\theta}}$$

$$p = \frac{N}{L\dot{\theta}} \quad (7.25a)$$

where, $\dot{\theta}$ is the angular velocity of roll in symmetry.

<table>
<thead>
<tr>
<th>Method</th>
<th>Force [MN]</th>
<th>Torque [KN-m]</th>
<th>Slab Entry Velocity [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>BHP measurement</td>
<td>15.6</td>
<td>501.3</td>
<td>3.9</td>
</tr>
<tr>
<td>Upper-bound method</td>
<td>13.9</td>
<td>660.68</td>
<td>3.9</td>
</tr>
</tbody>
</table>

Table 7.2 Symmetrical rolling results

### 7.3 Simplified Method for Asymmetrical Rolling with the Same Diameter

At BHP Steel SPPD, both top and bottom work-roll of the Roughing Mill are of the same nominal diameter and the asymmetry is caused by the difference between angular velocities of the rolls due to torsional vibration (see Fig. 7.1.b).
Sinicyn [92] introduced an analytical solution for the case of asymmetry caused by two rolls having different velocity with the same diameters. Thus, the geometry of rolling is symmetrical and for further simplification, the arc of contact surface was replaced by a chord and the roll bite angle was assumed to be small. Then,

\[ h_{01} = h_{02} = \frac{h_0}{2} \]

\[ h_{x1} = h_{x2} = \frac{h_x}{2} \]

Thus, equations (7.1) and (7.2) were simplified to the following:

\[ v_x = \frac{E}{h_x} \left[ 1 - \frac{2\beta_1 y}{h} \left( 1 - \frac{h_x \beta_2}{h_0} \right) \right] \]

\[ (7.25) \]

\[ v_y = \frac{E}{h_x^2} \left[ \frac{y}{h_x} \left( 1 - \frac{\beta_1 y}{h} \right) - \frac{\beta_1}{2h} \left( \frac{1}{2} - \frac{h_x \beta_2}{h_0} \right) \right] h_x \]

\[ (7.26) \]

and

\[ h_x = h_1 + \alpha x \]

\[ (7.27) \]

then by substitution of equations (7.25) and (7.27) in equation (7.8) and replacing the variable \( h_x \) using equation (7.27) the total power per unit width can be calculated with five unknowns \( h_A, h_B, \beta_1, \beta_2 \) and \( v_0 \). Then, the total power \( N \) can be minimised from the following set of equations:

\[ \frac{\partial N}{\partial v_0} = 0 , \quad \frac{\partial N}{\partial \beta_1} = 0 , \quad \frac{\partial N}{\partial \beta_2} = 0 \]

\[ (7.28) \]
\begin{equation}
\nu_{xt} = V_1 = R \dot{\theta}_1 \quad \text{at} \quad h_x = h_B 
\end{equation}
(7.29)

\begin{equation}
\nu_{xb} = V_2 = R \dot{\theta}_2 \quad \text{at} \quad h_x = h_B 
\end{equation}
(7.30)

Sinicyn [92] introduced the following simplified formulae for the above set of equations by neglecting the small terms.

\begin{equation}
h_A = \left[ \frac{h_0 h_1}{\exp \left( \frac{\alpha^2}{\delta \mu} + \frac{\alpha}{\mu} \ln \frac{h_0}{h_1} \right) \dot{\theta}_2} \right] \dot{\theta}_1 
\end{equation}
(7.31)

\begin{equation}
h_B = h_A \frac{\dot{\theta}_1}{\dot{\theta}_2} 
\end{equation}
(7.32)

Let us compare the value of \( h_A \) and \( h_B \) (neutral points thicknesses) using the above Sinicyn equations (7.31) and (7.32) and the results obtained from the Random Walk optimisation program (appendix 4).

Fig. 7.2d shows the variation of \( h_A \) respect to \( \frac{h_0}{h_1} \) ratio for both methods while \( h_1=34\,\text{mm}, R_1 = R_2 = R = 512.5\,\text{mm}, V_1 = 5.5\,\text{m/s} \) and \( V_2 = 6\,\text{m/s} \). The maximum difference in \( h_A \) from Sinicyn's value is about 4\%. Moreover, Fig. 7.2e shows the maximum difference \( h_B \) of Sinicyn is only about 2\%. Thus, the simplified method of Sinicyn has a good accuracy in the calculation of asymmetrical rolling vibrations. Furthermore, the vibrations analysis using the simplified formulae requires less computer time than that of a more accurate method.
Fig. 7.2d  The influence of entry thickness on $h_A$

Fig. 7.2e  The influence of entry thickness on $h_B$
7.4 Model of Rolling Force and Torque

An approximation of rolling force is given by Equation (7.25a) but it is not accurate. The calculation of rolling force and torque by the upper-bound method is based on the evaluation of total power in equation (7.8):

\[ N = T_1 \dot{\theta}_1 + T_2 \dot{\theta}_2 \]  

(7.33)

The upper-bound method is unable to divide the power and torque between the rolls which is given by equation (7.33). Normally investigators assume power and torque to be symmetrical as considered by Kiuchi et al. [105] and Dyja et al. [106]. However, as discussed in chapter 2, the rolling torques are not equal in asymmetrical rolling. To overcome this limitation of the upper bound method, Tselikov's slab method is used in this thesis to find rolling force and torque after the neutral points are evaluated by the upper-bound method. Tselikov's method [51] was applied for the symmetry case in chapter 6. The pressure distribution in the 'asymmetry' case is different to that of the 'symmetry' case because there are three different regimes in asymmetry rolling as can be seen in Fig 7.1.b. For example, B'D, B'A and AC on the top roll-slab interface.

The pressure distribution for the above regimes introduced by Tselikov for hot rolling are as following:

regime B'D : \[ s = 2 \frac{k}{\delta} \left[ \left( \frac{h_I}{h_X} \right)^6 (\delta + 1) - 1 \right] \]  

(7.34)

regime B'A : \[ s = s_A - 2 k \ln \left( \frac{h_A}{h_X} \right) \]  

(7.35)
regime \: AC \: \: s = 2 \frac{k}{\delta} \left[ \left( \frac{h_1}{h_x} \right)^\delta (\delta - 1) + 1 \right] \tag{7.36}

\text{where} \quad \delta = \frac{2 \mu L}{h_0 \varepsilon} \tag{7.37}

Where \( s_A \) is the pressure at the neutral point \( A \) and in the slab method, the pressure distribution for the top roll-slab interface has been assumed to be equal to the bottom roll-slab interface. Rolling force has been calculated by integrating the vertical component of pressure distribution in the three regions. Also, the rolls torques have been determined by integrating the moment of shear stress with respect to the work-roll centre.

Tselikov also assumed a small value for angle \( \alpha \) and replaced the arc CD with a chord. Thus

\[ \cos \alpha = 1 \quad \text{and} \quad dx = \frac{dh_x}{\alpha} \tag{7.38} \]

Thus, for rolling force

\[ p = \int_0^L s \cos \varphi \, ds = \int_0^L s \, dx = \int_{s_B}^{s_B} s_B \, dx + \int_{s_A}^{s_B} s_A \, dx + \int_{s_A}^{s_A} s_C \, dx. \tag{7.39} \]

After integration and simplification, we have

\[ p = \frac{k}{\mu} \left( \frac{h_0}{h_A} \right)^\delta \left[ \delta h_A - (\delta - 1) h_B \right] + h_B \left[ \left( \frac{h_B}{h_1} \right)^\delta - 2 \right] - \delta \left[ (h_A - h_B) - h_B \ln \frac{h_A}{h_B} \right] \]
The formulae for top and bottom rolling torque are given by:

\[ T_{\text{top}} = R \left\{ 2k \left[ h_A \left( \frac{h_0}{h_A} \right)^6 - h_A \right] - \mu p \right\} \] 

(7.41)

\[ T_{\text{bottom}} = R \left\{ \mu p - 2k h_B \left[ \left( \frac{h_B}{h_1} \right)^6 - 1 \right] \right\} \] 

(7.42)

The above three formulae and also the simplified equations (7.31) and (7.32) were applied to the case of asymmetrical rolling with different roll speeds (same diameter). The rolling condition for this investigation are as follow (seventh pass in the BHP Roughing mill):

- \( h_0 = 50 \text{ mm} \)
- \( h_1 = 34 \text{ mm} \)
- \( R = 577 \text{ mm} \)
- \( w = 769 \text{ mm} \)
- \( \omega = \text{roll angular velocity} = 9.53 \text{ rad/sec} \)
- \( t' = \text{temperature} = 1100^\circ\text{C} \)

For these calculations a friction coefficient of 0.3 is used (has been shown accurate in part 4.8). The variation of rolling force, top and bottom work-roll torque with respect to speed ratio (the ratio of the bottom roll to top roll speed) are given in Fig. 7.3.a and Fig. 7.3.b respectively. At first the speed ratio is one and the values of rolling force and torque are equal for symmetrical rolling conditions (see chapter 4). Then, the rolling force decreases with an increase in speed ratio. At the same time, the bottom roll (faster roll) torque increases and the top roll (slower roll) torque decreases. These results agree with those given by Shiozaki et al. [82] and Soskovets et al. [89] (see Fig. 2.2) for cold and hot rolling experiments and those obtained from the finite element method by Kalmykov [102] for hot rolling. However, the quantitative
comparison between our results and the above investigators is not possible, because they have not provided the specific data used in their papers.

Furthermore, the slab method is based on the assumption of the same pressure distribution on the top and bottom interface which is not true in asymmetrical rolling and this is the main source of error in our calculation. However, as the main aim of this thesis is the investigation of the vibrational problems in asymmetrical hot rolling (which no body has investigated before), the above approximation is acceptable. The simulation can be improved by using FEM or BEM in the calculation.

![Graph showing the variation of rolling force with respect to speed ratio in asymmetrical rolling](image-url)
Fig. 7.3.b Variation of top and bottom work-roll torque with respect to speed ratio in asymmetrical rolling
7.5 Vibration Analysis of Horizontal Mill

7.5.1 Dynamic Model and Formulation

A seven-degrees of freedom model of the Horizontal mill has been introduced in Fig. 5.2. The dynamic characteristics of the top drive line (mass inertia and stiffness) are not equal to those for bottom drive and the torsional model of the Horizontal mill is asymmetric. As the work-rolls of the same diameter are used in the rolling mill, the asymmetry is the result of different angular velocities. The simplified equations (7.31) and (7.32) were used to determine the neutral point thicknesses (or positions) for each speed ratio value of the bottom-roll to the top-roll at each time step.

The next step is the calculation of average angular velocity of top and bottom rolls to determine the average strain rate and the average flow stress (equation 3.15) of steel at a given temperature. Tselikov [51] has introduced the following formula to estimate the average strain rate.

\[
\dot{\varepsilon}_{av} = \frac{v_1}{L} \varepsilon_{av} \tag{7.42a}
\]

where

\[
\varepsilon_{av} = \frac{h_0 - h_1}{h_0}
\]

\[
v_1 = \text{exit side slab velocity} \approx R \dot{\theta}_{av}
\]

Then, rolling force and torques are estimated from equations (7.41), (7.42) and (7.43). The last step is the substitution of rolling force and torque into the vibration equations for vibration analysis as shown by the flow chart in Fig 7.3c.

In chapter 6, the rolling force and torque were a step (constant value) or ramp-step function. However, here in this chapter, the rolling force and torques are not constant and they are time dependent. Also, the slab stiffness and damping for vertical and torsional vibrations are recalculated from equations (5.5), (5.6), (5.17) and (5.18) respectively for every time step. Furthermore, rolling force, asymmetrical rolling
torques, vertical vibration and torsional vibration are determined simultaneously.

7.5.2 Positive and Negative Damping

Let us consider the following one-degree of freedom equation of motion for forced vibration analysis.
\[ m\ddot{y} + c\dot{y} + ky = F(\dot{y}) \quad (7.43) \]

with the assumption that the external force in the right hand side of the equation is a function of velocity. The above equation is not homogeneous and if we transfer the external force to the left hand side of equation, then

\[ m\ddot{y} + c\dot{y} - F(\dot{y}) + ky = 0 \quad (7.44a) \]

and it can be rewritten as:

\[ \ddot{y} + f(y,\dot{y}) = 0 \quad (7.44b) \]

Equation (7.44b) is in a homogeneous form and in vibration the nature of the solution has two possibilities:

(i) damped vibration: it means, the dynamic characteristic of the external force is similar to viscous damping (positive damping) and the amplitude decreases with respect to time and the dynamic system is stable

(ii) self-excited vibration: here, the dynamic characteristic of the external force is similar to negative damping and the amplitude increases with respect to time. Thus, the system can be a self-excited vibration which is unstable.

7.5.3 Results and Discussion

Two cases are compared with each other to find out the influence of time dependent asymmetrical rolling force and torques on vibrations:

Case a: "step function simulation" based on symmetrical rolling which was studied in
Case b: "asymmetrical rolling simulation" based on the flow chart given in Fig 7.3.

Here, the speed ratio is defined as a ratio of bottom work-roll rotational speed and top work-roll speed. Thus, it is a measure for the degree of asymmetry in rolling.

**Fig. 7.4** Speed ratio of bottom work-roll and top work-roll

**Fig. 7.5** Variation of Neutral Points Thickness
Fig. 7.6 Variation of Asymmetrical Rolling Torque

Fig. 7.7 Variation of Rolling Force
Fig. 7.8 Measurement of torsional vibration of Horizontal mill bottom spindle

Fig. 7.9 FFT of bottom work-roll torsional vibration

Fig 7.4 shows the variation of speed ratio for both cases a, b. The symmetry simulation introduces about 11% difference between the speeds of two rolls. However, 4% difference has been achieved using the asymmetry theory. It means the influence of damping which is energy dissipation during metal forming has been active in the asymmetry simulation.

This strong damping is caused by the rolling process and is a result of change in
neutral points. In practice, the three main sources of damping are:

(i) Internal damping of shafts and housing elements which have been neglected. Because, according to the data given in appendix 2, the loss factor value for cold steel is very small, in the order of 0.0001.

(ii) The damping caused by the variation of roll bite angle $\alpha$ as a result of vertical vibration which we have considered for both vertical and torsional vibrations using the formulation given in chapter 5.

(iii) The damping caused by the dynamic loads as a result of variation of neutral points which is the most important source of damping for torsional vibration.

Let us consider the following:

The speed ratio is not unity in the case of asymmetry. According to the relations (7.31) and (7.32):

$$h_A = C \sqrt{\frac{\theta_2}{\theta_1}}$$  \hspace{1cm} (7.45)

$$h_B = \frac{C}{\sqrt{\frac{\theta_2}{\theta_1}}}$$  \hspace{1cm} (7.46)

where

$$C = \sqrt{\frac{h_0 h_1}{\exp \left( \frac{\alpha^2}{8\mu} + \frac{\alpha}{\mu} \ln \frac{h_0}{h_1} \right)}}$$

Therefore, in theory, the neutral points thicknesses $h_A$ and $h_B$ are respectively proportional and inversely proportional to the root of speed ratio value with a constant
C. It means, if the speed ratio increases, the thickness $h_A$ increases and $h_B$ decreases. The variations of $h_A$ and $h_B$ given in Fig 7.5 show that the neutral points variations are similar to speed ratio variation (see Fig. 7.4) and there is a good agreement with the above theory.

Also, in theory, the rolling force and torques will vary as a function of speed ratio, because they are dependent on the neutral point positions (see equations 7.40, 7.41, 7.42).

Thus, the rolling force $p(t)$ on the right hand side of equation (6.1) and rolling torque $T(t)$ on the right hand side of equation (6.3) are a function of speed ratio.

By comparing both the top and bottom rolling torques variations as shown in Fig 7.6 and the rolling force variation as shown in Fig. 7.7 with the speed ratio variation in Fig 7.4, the following conclusions can be reached:

(i) The influence of speed ratio on rolling force is much less than that on rolling torques of top and bottom-rolls.

(ii) The bottom-roll torque increases when the speed ratio increases (the bottom roll speed is higher than top roll speed in this case due to definition of speed ratio). At the same time, top-roll torque decreases. Similar results were shown for hot rolling experimentally by Soskovets et al. [89] and theoretically by Kalmykov et al. [102] for steady state asymmetrical rolling which have been discussed in chapter 2. They showed that the rolling torque on the higher speed roll increased with increasing speed ratio while the rolling torque on the lower speed roll decreased (see also part 7.4).

(iii) The oscillations of bottom-roll torque and top-roll torque are similar to the speed ratio. This relation was also predicted by Shiozaki et al. [81] for thick strip rolling. Moreover, a strong damping is observed in these oscillations.
The value of the damping is very high because of the high sensitivity of roll torques to the speed ratio and this damping is the main source of damping in the rolling process. Some investigators such as Monaco [11] have tried to improve the results of symmetrical rolling vibration simulation to agree with measurements. They introduce some artificial damping factors as an internal damping of shafts. However, their method was not realistic.

Fig. 7.10   Torsional vibration of top spindle
Fig. 7.11 FFT of torsional vibration of top spindle for asymmetry

Fig. 7.12 Torsional vibration of bottom spindle

Fig 7.9 and Fig 7.11 show that the torsional vibration fundamental frequency of the Horizontal mill drive elements is 13.7 Hz which is one of the torsional vibration natural frequencies for the combined roll system (see chapter 5). Moreover, it is in agreement with the measured frequency of bottom work-roll shown in Fig 7.8 (5.5
cycles for 0.4 Sec or 13.75 Hz). Also the torsional oscillation of the bottom roll from asymmetrical simulation in Fig 7.6 also agrees with the measurement shown in Fig 7.8. A similar study on symmetrical rolling was carried out by Dobrucki et al. [36] taking into account the inertia force of slab on entry and exit side of rolling. The torsional vibration of top and bottom spindles are given in Fig 7.10 and Fig 7.12 respectively, which show the influence of asymmetry on the amplitude and frequency of vibration. The $TAF_{\text{max}}$ value of bottom spindle is about 2 as can be seen in Fig. 7.10 which also agrees with the measured $TAF$ ($TAF_{\text{max}} = 43/22.5 = 1.9$ from Fig. 7.8). The FFT of torsional vibration of top spindle illustrates that the second frequency of this oscillation is 27.3 Hz which is close to the second natural frequency of the top drive line.

Fig. 7.13 Vertical vibration of top work-roll
Vertical vibration of top work-roll and its FFT are given in Fig. 7.13 and Fig. 7.14 respectively. They show that the influence of asymmetry caused by the difference between roll speeds is negligible (compared with Fig. 6.3 and Fig. 6.4). The variation of rolling force in Fig. 7.7 is not as high as roll torques which is in agreement with the results of Soskovets et al. [89] and Kalmykov et al. [102]. The main source of damping in vertical vibration is the damping caused by the change in the roll-bite angle (slab damping for vertical vibration). This damping has been calculated from equation (5.6) for every time step as shown in Fig. 7.15b. The variation of non-linear slab stiffness for vertical vibration ($k_v$) is also given in Fig. 7.15a.
7.5.4 The Influence Of Rolling Parameters on Vibrations

Here, we investigate the influence of some important parameters on horizontal mill torsional vibrations by using the flow chart shown in Fig. 7.3c.
Fig. 7.15c demonstrates that smaller rolling torque with lower amplitude of vibrations have been achieved with increasing slab temperature.

On the other hand, Fig. 7.15d shows that greater rolling torque with the higher amplitude of vibrations have been obtained with increasing slab entry side thickness.

The above results are in agreement with equation 7.14, which shows that the rolling torque is proportional to the flow stress. The flow stress given in equation 3.15 decreases with temperature. However the flow stress increases when the strain increases with entry slab thickness.

![Graph showing variation of asymmetrical rolling torque for different temperatures](image)

**Fig. 7.15c Variation of Asymmetrical Rolling Torque for Different Temperature (ho = 50, h1 = 34, R = 577 mm)**
7.6 Interaction Between Different Modes of Vibration in Coupled Tandem Mills

7.6.1 Dynamic Model and Formulation

A ten-degree of freedom model was introduced in chapter 5 and it is reproduced here in Fig. 7.16 to calculate the transient vibration of Horizontal mill and Edger and horizontal vibration of slab coupled between two mills. The vibration analysis based on symmetrical rolling conditions of Horizontal mill was studied in chapter 6. Here, the study is about the influence of asymmetry on the vibrations of the Horizontal mill, Vertical Edger and slab in coupled system. The formulation is similar to that discussed in 6.4.1 except that the values of rolling force and roll torques are calculated from the flow chart shown in Fig. 7.3.
7.6.2 Discussion and Results

Torsional vibration of horizontal mill in the case of asymmetrical rolling is given in Fig. 7.17a for the two different cases of 'with interaction' and 'without interaction' conditions. It shows that during interaction, the oscillation amplitude is higher and has multiple frequencies. However, the main frequency of vibration is similar to the case 'without interaction'.

Fig. 7.17b and Fig. 18 show the influence of backlash (backlash = 0.001 radian on the top spindle) on increasing the torsional vibration amplitude of the top spindle and the top work-roll.
Fig. 7.19 illustrates the comparison between horizontal vibration of slab in two different cases of symmetry (Fig. 6.20) and asymmetry. It shows that the damping caused by asymmetry in Horizontal mill does not affect the slab vibration significantly. However, the damping caused by asymmetry can affect significantly the Edger torsional vibration during interaction conditions (see Fig. 7.20). The influence of asymmetry on slab and Edger vibrations starts with a time delay of about 0.05 second.

The rolling force is shown in Fig. 7.21, it indicates that the push-pull force caused by the mismatch of the speed between the two mills introduces multiple frequency oscillation compared with that of 'without interaction' shown in Fig. 7.7. The influence of push-pull on the amplitude of vibration is negligible.

![Torsional vibration of top spindle for asymmetry](image)
Fig. 7.17b  Torsional vibration of top spindle for asymmetry

Fig. 7.18  Torsional vibration of top work-roll for asymmetry
Fig. 7.19  Linear vibration of slab

Fig. 7.20  Influence of asymmetry on Edger torsional vibration
Fig. 7.21 Variation of rolling force for asymmetry and during interaction
CHAPTER 8

SELF-EXCITED TORSIONAL VIBRATION AND STABILITY
8.1 Introduction

Chen [42] classified the blooming mill torsional vibration into impulsive torsional and self-excited vibration. The first kind was similar to the simulations in chapter 6. The impulsive simulation takes into account the influence of asymmetry in chapter 7. Here, not only the influence of asymmetry is considered, but the variation of friction coefficient during oscillation is also accounted for.

Moreover, Tieu [41] has studied Roughing mill impulsive torsional vibration caused by slippage between roll and slab using ramp-step function simulation (symmetrical rolling). His results would be improved by a consideration of asymmetrical rolling.

Self-excited vibration was studied by a few investigators on blooming mills in China during the last decade. This phenomenon occurs when the slippage continues and the rolling process becomes unstable. During slippage, the rolling torque decreases close to zero for a short time. However, the friction torque (see chapter 1) is active on rolls during self-excited vibration.

8.2 Stability Analysis

The neutral point is defined as a point on the contact surface between the roll and slab where the linear velocity of the roll is equal to the slab velocity. At exit side, the slab speed \( v_1 \) is higher than peripheral speed \( V_R \). Forward slip is defined as:

\[
s_f = \frac{v_1 - V_R}{V_R} \tag{8.1}
\]
8.2.1 Symmetrical Rolling

The neutral point should be inside the roll bite angle for a stable condition of rolling. Otherwise, slippage occurs which is an unstable condition. In other words, rolling is stable if the forward slip is positive.

There are a few analytical formulae to find the neutral point:

Ekelund [54] derived the following formula to determine the hot rolling neutral angle for sliding friction model assuming the pressure distribution between the roll and slab is constant in the roll gap

\[ \phi_n = \frac{\alpha}{2} \left( 1 - \frac{\alpha}{2\mu} \right) \] (8.2)

Thus the neutral point is dependent on the geometry and friction coefficient. Sims [58] method (see chapter 4) was based on sticking friction condition and a formula was introduced for hot rolling neutral point. That relation was only dependent to geometry.

Bland and Ford [60] introduced a solution for cold rolling. The neutral point can be found from the following equation:

\[ e^{\mu H_*} = \frac{h_1}{h_0} e^{\mu H} \] (8.3)

where

\[ H_* = 2 \sqrt{\frac{R'}{h_1}} \tan^{-1} \left( \phi_n \sqrt{\frac{R'}{h_1}} \right) \] (8.4)

\[ H_1 = 2 \sqrt{\frac{R'}{h_1}} \tan^{-1} \left( \alpha \sqrt{\frac{R'}{h_1}} \right) \] (8.5)
Ekelund presented the following simple formula for forward slip:

$$s_f = \frac{R' \alpha}{2h_1} \left(1 - \frac{\alpha}{2\mu}\right)^2$$  \hspace{1cm} (8.6)

Thus, for stability we should have

$$\mu \geq \frac{\alpha}{2}$$  \hspace{1cm} (8.7)

Also, there are some different empirical relations for friction coefficient in hot rolling. Good reviews were given by Roberts [1] and Pietrzyk et al. [23].

In some cases, the friction coefficient depends on geometry, and in some others it depends on the temperature. Moreover, $\mu$ is related on the rolling speed for some empirical formulae, and it depends to both temperature and velocity in a few others. For example, Geleji [63] introduced the following relation for steel rolls

$$\mu = 1.05 - 0.0005 t' - 0.056 V$$  \hspace{1cm} (8.8)

where $t'$ is temperature and $V$ rolling speed m/s.

One of the most comprehensive relations for $\mu$ is given by Sato et al. [101]:

$$\mu = e_0 + e_1 S_i^2 + e_3 t'^2 + e_4 t' + e_5 \Delta v^3 + e_6 \Delta v^2 + e_7 \Delta v$$  \hspace{1cm} (8.9)

where,

$\Delta v = \text{relative speed}$

$S_i = \text{Silicon content (\%)}$

$e_0, e_1, \ldots, e_7 = \text{constants}$
Here, we analyse the stability of the 7th pass (the highest speed pass) of Roughing Mill at B.H.P. for symmetry and steady condition by replacing equation (8.8) in (8.7) and using $\alpha = 0.16$ rad and $t' = 1100 ^\circ C$. Thus, the calculations show that we should operate $V \leq 7.5 m / s$ for stability. As the rolling speed used in B.H.P is $5.5 m / s$, the system is stable.

For vibration condition of hot rolling, Butler et al. [40] reviewed the methods which decrease the torsional chatter for stability [see part 1.2.4]. Also, a review for stability criteria of cold rolling was given by Robert [1][see chapter 1].

8.2.2 Asymmetry

The stability analysis of asymmetry is more complex than symmetry. There are less analytical methods to find the neutral points in asymmetrical rolling than symmetrical rolling. For the asymmetry rolling with two different velocities and the same diameter, equations (7.31), (7.32) were used for vibration analysis. These equations are also used to study stability on steady-state condition. For stable condition, the neutral points $h_A$ and $h_B$ must be inside the roll-bite. It means

\[
h_A \leq h_1 \quad \text{and} \quad h_B \geq h_0
\]

\[
h_A = \sqrt[\hat{\theta}_2]{\frac{h_0}{h_1} \left( \frac{\hat{\theta}_2}{\hat{\theta}_1} \right) \exp \left( \frac{\alpha^2 - \frac{\alpha}{\ln h_0}}{8\mu} \right) \ln h_1}
\]

(8.10)

\[
h_A \leq h_1 \quad \text{(8.11)}
\]

\[
h_{A\hat{\theta}_2} \geq h_0 \quad \text{(8.12)}
\]
where
\[ \alpha = \cos^{-1}\left[1 - \frac{h_2 - h_1}{2R}\right] \quad \text{and} \quad \dot{\theta}_2 \geq \dot{\theta}_1 \] (8.13)

for a given speed ratio, the friction coefficient and the geometry must satisfy the above equations. For example, let us study the stability of asymmetry using the geometry and the other properties introduced in chapter 4 regarding the 7th pass of BHP Roughing mill. It showed that if the speed ratio is 1.18 (15% difference between speed of the rolls) then
\[ h_A = 33.8\text{mm} \quad \left< \quad h_1 = 34\text{mm} \]
and slippage occurs on the roll with the higher speed.
Fig. 8.1 The influence of friction coefficient variation on torsional vibration of top work-roll

Fig. 8.2 Variation of friction coefficient
8.3 Discussion and Result

8.3.1 Impulsive Simulation

(i) Friction variation in asymmetry:

As we mentioned in the previous part, there are very few empirical relations for friction coefficient which define it as a function of rolling speed. According to the above limitation, equation (8.8) has been applied to the asymmetry model of chapter 7. In other words, the variation of $\mu$ as a result of the variation of the rolls velocities has been taken into account here. Fig 8.1 shows the influence of the variation of $\mu$ on torsional vibration of top work-roll is similar to positive damping. The linear relation of $\mu$ to velocity in equation (8.8) introduces the viscous damping characteristic of $\mu$.

The variation of friction coefficient and its FFT are give by Fig 8.2 and Fig 8.3 respectively. The frequency of oscillation is the fundamental frequency of torsional vibration of drives.

(ii) Torsional vibration in slippage condition and asymmetrical rolling
Here, slippage means that the rolling torques suddenly become zero for a short time. For example, in Fig 8.4 rolling torques have been assumed to be zero after 0.17 sec from the start time of torsional vibrations. Also the torsional vibration of both top and bottom spindles are given in Fig 8.5. These results show that in slippage condition, the TAF is about 25% higher than the case of without slippage. Tieu [41] had shown similar result for symmetrical rolling.

![Diagram of Torsional Vibration](image)

**Fig. 8.4** Torsional vibration of top and bottom work-rolls for slippage case
8.3.2 Self-Excited Simulation

The assumption of slippage rolling condition is considered for self-excited simulation. When slippage occurs, the friction torque exists between the top and bottom interfaces. The friction torque applied by several investigators such as Chen et al. [30] as follows:

\[ T = \mu R P \]  \hspace{1cm} (8.14)

If a constant value for \( \mu \) is applied in (8.14) the simulation will be similar to step function (impulsive) simulation. It means the rolling torque will be replaced by friction torque during slippage in the equations of motions, of the rolls. However, for self excited simulation, the friction torque should be defined as a function of velocity (see 7.5.2).
The following empirical equation has been applied by some investigators such as Chen et al.[30] regarding self excited vibrations.

\[ H = P_0 - e v_r + f v_r^2 \]  
(8.15)

where,

\[ v_r = \text{relative velocity of roll and slab} \]
\[ \mu_0 = 0.2-0.49 \]
\[ e = 0.03 - 0.09 \]
\[ f = 0.0015 - 0.0033 \]

for this study, the values of \( \mu_0 = 0.3 \), \( e = 0.06 \) and \( f = 0.0024 \) have been chosen.

For self excited simulation, the method of part 7.5 (asymmetry and impulsive simulation with constant friction coefficient) was applied to the time between 0. and 0.2 sec. Then three cases were compared with each other:

case (i) a constant value \( \mu = 0.3 \) was substituted into equation (8.14 ) to determine the vibration.

case (ii) a value of \( \mu \) from the equation (8.15) was substituted into (8.14) for vibration simulation after 0.2 sec. It is assumed that the slab has the constant steady-state velocity of the work-rolls.

(iii) the same as case (ii) except that the slab velocity is assumed to be zero, but this is not realistic due to the great inertia of the slab.

The torsional stiffness and damping of slab are zero in all the above three cases due to slippage conditions.

Fig 8.6 shows that the vibration with higher amplitude exists for all three cases of self-excited vibrations:
(i) in the case of self-excited vibration using a constant value of 0.3 for \( \mu \), TAF is about 3 times higher than that of normal slippage (compared with Fig. 8.5)

(ii) in this case of self excited vibration, the TAF is 2.5 times higher than normal slippage. The value of parameter \( f \) in equation (8.15) is much smaller than \( \mu_0 \) and \( \epsilon \), and the value of \( v_r^3 \) is small due to small relative speed between the roll and slab. Thus the last term in the right hand of equation (8.15) is negligible compared with the two others and \( \mu \) is a linear function of relative velocity \( (\mu = \mu_0 - \epsilon v_r) \) which contributes little viscous damping to the system.

(iii) in the case of self excited vibration using relation (8.15), TAF_{max} is approximately 2 times higher than that of normal slippage. This vibration has a significant viscous damping due to equation (8.15) where the term \( f v_r^3 \) is negligible and \( \mu \) is approximately a linear function of work-roll velocity.
Fig. 8.6  Self-excited torsional Vibration of top spindle

The fundamental frequency of self-excited vibration of top spindle for case(iii) is 10.7 Hz (see Fig. 8.7) which is different with 13.6 Hz for impulsive simulation without slippage (Fig. 8.3). The difference is due to independent vibrational behaviour of top and bottom drives of Horizontal mill during slippage condition. However, the frequency of self-excited vibration of top spindle is close to the first non-zero natural frequency of top drive which agrees with the results given by Chen et al. [30].
Fig. 8.8 shows the friction coefficient variation of top and bottom rolls are different due to the great difference between the velocities of rolls during self-excited torsional vibration as shown in Fig. 8.9.

![Fig. 8.7 FFT of Torsional Vibration of Top Spindle when slab stops case (iii)](image1)

![Fig. 8.8 Friction coefficient variation of top and bottom work-rolls when the slab stops case (iii)](image2)
Fig. 8.9  Speed Ratio During Self Excited Vibration
CHAPTER 9

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK
The main aim of the thesis was to simulate hot rolling mill vibrations. The main source of excitation was found to be the rolling force and torque. In asymmetrical rolling, the dynamic rolling force and the two dynamic roll torques (top and bottom work-roll) were evaluated to achieve more accurate results for vibration compared with the previous works by other investigators.

The energy parameters and the vibration are dependent on flow stress and damping of the material respectively. Thus, in chapter 3 two sets of experiments were performed to determine the strength and damping properties of hot steel.

In chapter 4, a number of different analytical and numerical techniques were used to evaluate the symmetrical rolling force and torque. Using a value of 0.3 for the friction coefficient gave the best results for rolling force and torque when compared with mill data. Also, it was found that the slippage friction model is suitable for the roughing mill during vibration conditions.

A new vibration model of tandem roughing mill was developed in chapter 5 to calculate the interaction between torsional vibrations of horizontal and vertical mills and linear vibration of slab acting as an independent mass between the two mills. The rigidity and damping of the slab in both plastic and elastic regions were considered in this simulation.

Four modes of vibrations of the Roughing Mill system were studied in chapters 6 and 7:

(a) torsional vibration of Horizontal mill
(b) vertical vibration of Horizontal mill
(c) torsional vibration of Vertical Edger
(d) horizontal vibration of slab coupled between the two mills.
Moreover, the variation of the **push-pull** forces between the two mills as a result of mismatch between the surface speed of the Horizontal mill and Vertical Edger were calculated which are essential data for roughing mill control. The vibration analysis based on symmetrical rolling force and torque in chapter 6 showed the importance of TAF which is representative of the dynamic load caused by torsional vibration. Introducing coupling backlash and taking into account the push-pull between the two mills (Horizontal mill and Vertical Edger), showed that these parameters increase the TAF. The influence of push-pull on the Vertical Edger torsional vibration was greater than the Horizontal mill. The torsional and vertical vibration of the Horizontal mill and torsional vibration of the Edger were close to their first natural frequencies as calculated in chapter 5.

Furthermore, the following parameters were taken into account when calculating the vibration in chapter 7:

(i) asymmetry in horizontal mill (different roll speed)
(ii) variation of rolling force and torques (top and bottom rolls) in horizontal mill
(iii) backlash on the drives couplings
(iv) non-linear stiffness and damping of hot slab
(v) push-pull between the two mills during interaction conditions

Thus, the influence of asymmetry on rolling force, rolls torques and vibration were investigated in chapter 7. It shows that by using the rolls with different speeds decreases the rolling force and increases the faster roll torque and decreases the slower roll torque. During vibration conditions, the results showed that for asymmetrical rolling, the torque difference introduces the main source of damping for torsional vibration. It showed that other damping such as internal damping of spindles ([131]) and the damping from the variation of roll bite angle are negligible in comparison with the damping caused by asymmetry. However, the damping caused by the variation of roll bite angle is the main source of damping for vertical vibration.
In chapter 8, for both symmetrical and asymmetrical rolling when the position (or positions) of neutral points are inside the roll bite, rolling is stable. When the neutral point or points are outside the roll bite, rolling becomes unstable and slippage occurs. The slippage introduces higher values of TAF and the self-excited vibration with high TAF values occurs when a constant friction coefficient was used in the friction torque equation.

Although the above mentioned results have been obtained from a special roughing mill model (BHP Roughing Mill as an example), the results and methodology can be generalised for the other roughing mills. The results can be used for both qualitative and quantitative investigations if the temperature, strain and rate of strain of a roughing mill are in the ranges 1000 - 1200 °C, 0.3 - 5 (strain) and 3 - 20 s⁻¹ (strain rate) respectively.

Furthermore, the results are applicable for qualitative vibration analysis of a roughing mill when the rolling mill parameters are not in the above mentioned ranges.

The outcome of this thesis is not an end to itself. The following research areas are recommended for future work:

(i) finite element or boundary element methods should be used to achieve a more accurate rolling force and roll torques in asymmetrical rolling vibration calculations.

(ii) more experiments should be carried out to find a better empirical relation between the friction coefficient, temperature, and rolling speed for stable and unstable (slippage) conditions of rolling to apply in self-excited vibration simulations.

(iii) more correlation should be carried out between the results in this thesis and actual rolling data.
(iv) Vibration simulation for asymmetrical rolling with two different diameters.

(v) Model of electrical control system should be incorporated together with the mechanical model.

(vi) Shear stiffness can be considered in the vibration model of the rolling mill.

(vii) The variation of equivalent mass of slab on the bottom roll respect to time can also be considered.
REFERENCES


Those marked with * have not been read by the author.


APPENDICES
APPENDIX 1

LITERATURE REVIEW ABOUT DAMPING
The history of subject damping has been divided by Phunkett [130] into three epochs: the first from 1784 to 1920 in which a natural philosophy approach was used; the second from 1920 to 1940 in which industrial applications were made and the third from about 1940 to the present in which the principles of physics, applied mechanics, and materials science has been used to characterise and analyse the influence of damping on dynamic systems and the material structure on damping.

**Natural Philosophy (1784 to 1920)**

Lazan [151] states that Coulomb, in his Memoir on Torsion (1784), deduced from his experiments that metals dissipated energy when cyclically strained; he did not have any reasonable model to describe the physical behaviour because the crystalline structure of metals was not known at that time. Coulomb did, however, conclude that the stress-strain curve would trace a hysteresis loop. Zener [145] attributes the first measurements of internal friction to Webre in 1837 using free decay of torsional vibration. Todhunter et al. [155] in their discussion of Coulomb's memoir do not discuss his observation of decay rates. They do refer to a paper of Weber's in Latin (1835) which describes decay of torsional pendulums. Rayleigh [156] gave the differential equations for the vibration of linear, viscously damped discrete and coetaneous mechanical, acoustic, and electrical systems and solution for some of them.

As reported by Todhunter et al. [155], mathematical physicist with the problem of formulating the continuum equations as a limit from the forces of molecular attraction and the details of crystalline during the nineteenth century. In 1821 Navier started which central forces and simple forms and derived equations containing only one elastic constant and a Poisson's ratio of 0.25. This was extended by Poisson, Voigt, and Kelvin to multi-point molecular forces. However, it is not possible to calculate damping factors from models based on atomic or molecular forces alone. Qualitative results for the frequency and temperature
dependence of damping due to dislocation motion have been obtained from measurements of stress dependent dislocation velocity. Thermoelastic damping factors [145] can be calculated with remarkable accuracy using a continuum mechanics approach and independent measurements of other thermoelastic constants such as the coefficient of thermal expansion and diffusivity. These in turn can be determined with varying degrees of accuracy from atomic and molecular models. There have been some recent progress in calculating energy dissipation caused by stress-induced phase changes.

Using the crude instrumentation available during the nineteenth century, physicists made calibration measurements which agreed reasonably well with the prediction of the linear viscoelastic model. In particular, they were able to predict the natural frequencies of uniform beams and of square and rectangular flat plates. Within their ability to measure, they found that the decay rates for free vibration were independent of amplitude. They did find, however, that the frequency dependence of decay and damping did not follow the predictions using simple viscous dampers and there were many attempts to construct models that would behave properly.

Starting with Maxwell, Voigt, and Kelvin, mathematical physicists tried to construct dissipation models using combinations of linear springs and dashpots to give better prediction of the behaviour at very high or low frequencies but they were not successful [157]. Linear viscosity is not a good model for the damping at engineering strain levels in any material at temperatures well below the glass transition temperature. Contrary to the prediction of any linear viscoelasticity model, there is a measurable hysteresis loop even under quasi-static loading (Weber's elastische Nachwirkung [155]) and this value is approached in the limit as frequency approaches zero. There are several different mechanisms for energy dissipation in polystalline metals and the effect of each peaks at a different critical frequency [145]. As a result, the curve of damping factor as a function of frequency is quite irregular. It is possible,
but not very useful, to construct this function by superimposing a simpler function representing the effect of each mechanism separately.

Over appropriate frequency ranges, certain relationships between creep and frequency response can be used to calculate damping properties [145,158]. In addition, the generalised Arrhenius times temperature relationships may be used for damping and elasticity modulus prediction for polymeric and porcelain enamels[157].

**Industrial Applications 1920 to 1940**

The increase in size and speed of machinery in the 1920s and 1930s made vibration control a subject of great interest to industry especially in rotating machinery, aircraft, and large structures like bridges and buildings.

**-Rotating Machinery**

The first really high-speed machinery included electrical generators and motors and their associated steam turbines. It was soon found that vibrations could not be avoided, and there was great concern for finding ways to introduce damping into the system. Major manufacturers like Westinghouse, GE, AEG, and Brown-Boveri conducted research into better material and measurement techniques, and texts on the subject began to increase in numbers [160-162]. Two of the most serious problems were the behaviour of rotating shafts at their critical speed and the fatigue caused vibrations of turbine blades.

The use of geared turbines in ships, starting in the early part of the twentieth century quickly uncovered problems of torsional vibrations and fatigue in propeller shafts in addition to the vibration problems in the turbines and blade vibration in the propellers themselves [161].
The increased use of internal combustion engines soon uncovered torsional problems on their crankshafts. There were added complications from the coupling between torsional and lateral vibrations caused by the crank throws [161].

In the 1920s and 1930s, engineers tried to deduce system damping by rather simple calculations from material damping measurements made using torsional pendulums. As instrumentation was improved, it was found that the measured system damping was many times what could be ascribed to the material alone. For want of a better method, empirical factors were used; for example, Den Hartog [161] recommends multiplying the calculated energy dissipation by a factor of 10. It was not realised that most of the energy loss was due to lateral motion and rotation of crankshafts in bearings or to fluid forces on marine propellers and so was almost independent of hysteresis in the material.

-Aircraft

Aircraft had structural vibration in addition to those in their engines. The primary problem was of the aircraft wings and panels. The aero-elastic analysis showed that such behaviour was caused by fluid-solid interaction that behaved like negative damping. Since the addition of inherent positive damping could increase the flutter velocity, there was great interest in developing ways of increasing passive damping of the materials, but it was largely unsuccessful. Since it was found that the damping factor for aircraft structures was almost independent of frequency in the structural vibration range from 1 to 100 Hz, the concept of complex modulus or stiffness became very popular. Measurements of prototype aircraft showed that the damping factor, the ratio of dissipative to storage modulus, lies between 0.01 and 0.03 [151].
Large structures like bridges, buildings, and ships were thought to be largely immune from vibration problems, since measurements indicated damping factors of 10 to 20%. Welded construction became more prevalent in the 1940s, effective damping dropped, and vibration problems appeared. Ships developed fatigue cracks at the corners of hatch openings. Aircraft displayed fatigue cracks in the fuselage and wings; the first commercial jet, the Comet, crashed from fatigue failure. The Tacoma Narrows bridge failed from flow induced vibration. Tall building had large amplitude that were wind driven. Measurements showed that much of the damping in previous construction had been in the connection because the rivets and bolts permitted slip and coulomb friction-dissipated energy. Welded and other monolithic construction methods eliminated this source of slip damping. There was a pronged attack on the problems. Where possible the excitation was reduced, the structure was stiffened, and stress concentration were reduced. In other cases, additional damping was deliberately introduced. A few examples may illustrate this activity; external coatings, such as undercoat were sprayed on the panels of automobiles to reduce vibration and noise; torsional absorbers were added to internal combustion engines; 20000 viscous damping layers were added to panels in aircraft [159].
APPENDIX 2
DAMPING CAPACITY OF DIFFERENT MATERIAL FOR ROOM TEMPERATURE
The experiments by the other investigators for low temperature showed that the internal damping of steel is low. For example, Wren et al. [131] has given a table to compare the damping capacity of different materials which had been collected before by the others and it has been shown here:

![Damping capacity of different material at room temperature][131]

The loss factor of stainless steel is about 0.0018 and for Cast iron is about 0.045. Also, the loss factor for steel has been found by Gibson et al. [116] to be $\eta = 0.001$.

Thus, the value of the loss factor for different compositions of iron should be in the range of 0.001-0.045.
APPENDIX 3
THE VELOCITY FIELD IN
UPPER-BOUND METHOD
The material flow and continuity equations can be derived from two dimensional problems fluid mechanics principles as (see Fig. 7.1.a) [92]

\[ \int_{-h_x}^{h_x} v_x \, dy = v_0 h_0 = E = \text{constant} \]  

(1)

\[ \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \]  

(2)

where \( v_x \) and \( v_y \) are the slab velocity components.

The horizontal velocity component is assumed to be a polynomial as

\[ v_x = \frac{E}{h_x} + E \sum_{k=1}^{n} C_k(x) \left[ (1 + k) y^k - \frac{\varphi_{k1} - \varphi_{k2}}{h_x} \right] \]  

(3)

where,

\[ \varphi_{k1} = h_x^{k+1}, \quad \varphi_{k2} = (-h_x^2)^{k+1} \]  

(4)

The partial derivative of \( v_x \) respect to \( x \), \( \frac{\partial v_x}{\partial x} \) can be derived from the equation (3).

Then, by substitution of \( \frac{\partial v_x}{\partial x} \) into equation (2), the velocity component \( v_y \) can be determined by integrating equation (2) with respect to \( y \):

\[ v_y = E \left\{ \frac{h_{x1}}{h_x} + \frac{h_x}{h_{x1}^2} (y - h_{x1}) + \sum_{k=1}^{n} \frac{\partial C_k(x)}{\partial x} B + \sum_{k=1}^{n} C_k(x) . A \right\} \]  

(5)

where

\[ A = \varphi_k (h_{x1} - y) + \left( \frac{1+k}{h_{x1}} \varphi_{k1} - \psi \right) h_{x1} \]

\[ B = \left[ \varphi_{k1} - y^{k+1} - \psi_k (h_{x1} - y) \right] \]
The assumption is the linear distribution of \( v_x \) with respect to variable \( y \). Thus

\[ k = 1 \]

Then, by substitution of this value in the equations (3) and (5), the simplified horizontal and vertical components of the slab velocity are as follows:

\[
v_x = E \left\{ \frac{1}{h_x} + C_1(x) \left[ 2y - (h_{x1} - h_{x2}) \right] \right\}
\]

(6)

\[
v_y = E \left\{ \frac{h_{x1}}{h_x} + \frac{h_x}{h_x^2} (y - h_{x1}) + \frac{\partial C_1(x)}{\partial x} (h_{x1} - y)(h_{x2} + y) + C_1(x) D \right\}
\]

(7)

where

\[ D = h_x \dot{h}_{x1} - (\dot{h}_{x1} - \dot{h}_{x2})(h_{x1} - y) \]

The next step is an approximation for the function of \( C_1(x) \) which depends on variable \( x \). The \( C_1(x) \) is assumed to be linear function of \( h_x \) as

\[ C_1 = -\frac{\beta_1}{h_x} h_1 \left( 1 - \beta_2 \frac{h_x}{h_0} \right) \]

\( \beta_1 \) and \( \beta_2 = \text{constant} \)
APPENDIX 4

ASYMMETRICAL ROLLING

OPTIMISATION PROGRAM
The following program was run using the **Mathematica Package**.

**Nomenclature:**

- \( a \) = initial value of \( \beta_1 \)
- \( V \) = initial value of \( v_0 \) (m/s)
- \( l \) = roll-bite distance (mm)
- \( k \) = shear flow stress (MPa)
- \( V_1 \) = top work-roll speed (m/s)
- \( V_2 \) = bottom work-roll speed (m/s)
- \( r_m \) = top work-roll radius (mm)
- \( r_s \) = bottom work-roll radius (mm)
- \( h \) = exit side thickness (mm)
- \( S_I \) = friction coefficient
- \( b_e = \beta_2 = 1 \)
- \( \text{num} = M = \) iteration number
- \( e_{pv} = \epsilon_v \)
- \( e_{pa} = \epsilon_\lambda \)
- \( l_{ana} = \lambda_\beta \)
- \( l_{anv} = \lambda_v \)

Also, see section 7.2.3.

**MAIN PROGRAM**

```
(((
  a=0.035;
  V=4030;
  l=95.34;
  k=73;
  V1=5500;
  V2=5775;
```
rm=512.2;
rs=639.86;

h=34;
SI=0.31;
be=1;

num=5;

epv=15;
epa=0.013;

lana=0.2;
lanv=200;
(*lana=0.2;*)
(*lanv=200;*)

<<sam2.ma;
(*N111=10^20;*)

Label[ij];
lana=lana/2;
lanv=lanv/2;
Print["lana=",lana];
Label[w];
i=1;
Label[qw];
<<sam1.ma;
(*N11=10^19;*)
If[N11>=N111,Goto[es]]; Label[ss];
Clear[N111];
N111=N11;
Print["N111=",N111];
a=a+lana*ua;
V=V+lanv*uv;
(*N11=N111/2;*)
<<sam.ma;
Print["EEE"]; If[N11<N111, Goto[ss], Goto[w]]; Label[es];

i=i+1;
If[i>num,, Goto[qw]]; If[lana<epa, Print["amin=",a]; Print["Vmin=",V]; Print["Nmin=",N111], Goto[ij]]
)))));
SUBROUTINE SAM.MA

ClearAll[NT1,NT2,NT3,NT4,NT,NP,N11,
NS1,NS2,NS,j1,j2,j3,j4,k1,k2,k3,
k4,e1,e2,VT,A,VTB,VXA,VXB,HA,HB,C1,C2,m,n,t,VX,VY,VYY,XA, XB,y1,
C1A,C1B,ETAxy,
,KISIX,FXY1];

Print["begin sam.ma"]; 

kaa=a+lana*ua;
kvv=V+lanv*uv;
sa=a;
sv=V;
ClearAll[a,V];
HX1=h/2+(0.5/rm)*x^2;
HX2=h/2+(0.5/rs)*x^2;
H01=HX1 /. \{ \{ x->l \}\};
H02=HX2 /. \{ \{ x->l \}\};
H0=H01+H02;
E1=-H0*V/.V->kvv;
Print["i"]; 

Print["H0= ",H0];
HP1=D[HX1,x];
HP2=D[HX2,x];
HPX=HP1+HP2;
HX=HX1+HX2;
Print["HX= ",HX];
Cl=-a(l-be*HX/H0)/(HX*h)/.a->kaa;
C2=D[Cl,x];
VX=El*(1/HX+C1*HX)/(HX*h)/.a->kaa;
VY=El*(1/HX+C1*HX)/(HX*h)/.a->kaa;
C2*(HX1-y)*(HX2+y)+C1*(HX*HP1-(HP1-HP2)*(HX1-y));
VXT=El*(1/HX+C1*HX);
VXB=El*(1/HX-C1*HX);
HA=HX/.x->XA;
HB=HX/.x->XB;
C1A=C1/.HX->HA;
C1B=C1/.HX->HB;
VTA=El*(1/HA+C1A*HA);
VTB=El*(1/HB-C1B*HB);
N11=10^20;

V1=Solve[VTA == -V1,XA]/.XA_Complex->10000;
j1=t[[1,1,2]]; 
j2=t[[2,1,2]]; 
j3=t[[3,1,2]]; 
j4=t[[4,1,2]]; 
Print["a"]; 
If[0<j1<l,XA=j1, 
If[0<j2<l,XA=j2, 
If[0<j3<l,XA=j3, 
If[0<j4<l,XA=j4,Goto[v]]]]]]);
V2=Solve[VTB==V2,XB]/.XB_Complex->10000;
k1=n[[1,1,2]]; 
k2=n[[2,1,2]]; 
k3=n[[3,1,2]];
k4=n[[4,1,2]]; Print["b"]; If[0<k1<l, XB=k1, If[0<k2<l, XB=k2, If[0<k3<l, XB=k3, If[0<k4<l, XB=k4, Goto[v] ]]]]; If[XA<XB, Goto[vaa]]; VYY=VY /. {x->l, y->y1}; m=Solve[VYY==0,y1]/.y1_Complex->10000; e1=m[[1,1,2]]; e2=m[[2,1,2]]; If[-H02<e1<H01,y1:=e1, If[-H02<e2<H01,y1:=e2, Goto[vaaa]]]; Print["c"]; Clear["c"]; NT1=N[Integrate[VXT+V1,{x,XA,l}]]; NT2=N[Integrate[-VXT-V1,{x,0,XA}]]; NT3=N[Integrate[VXB+V2,{x,XB,1}]]; NT4=N[Integrate[-VXB-V2,{x,0,XB}]]; NT=(NT1+NT2+NT3+NT4)*2*SI; VYY=VY /. {x->l}; NS1=N[Integrate[VYY,{y,-H02,y1}]]; NS2=N[Integrate[VYY,{y,y1,H01}]]; NS=NS1-NS2; KISIX=D[VX,x]; ETAXY=D[VX,y]+D[VY,x]; FXY1=Sqrt[4*(KISIX^2)+(ETAXY^2)]; NP=NIntegrate[NIntegrate[FXY1,{y,-HX2,HX1},{x,0,l}]; (*NP:=2*V*H0*Log[H0/h]/.V->kvv;*) N11=k*(NP+NT+NS); sou=N11; N11=sou; Clear["Nll"]; Nll=sou; Print["Nll=", Nll]; Print["end sam.ma"])))
SUBROUTINE SAM1.MA

Print["begin sam1.ma"]; ClearAll[XA, XB, y1, NT1, NT2, NT3, NT4, NT, NP, NS1, NS2, NS, j1, j2, j3, j4, k1, k2, k3, k4, e1, e2, ETA, FX, FXY, KISIX, C1A, C1B, VTA, VTB, HA, HB, m, n, t, VYY, ua, uv, a0, V0];

ka = a + lana*ua; kv = V + lanv*uv;

a10 = a; V10 = V;

(Label[aaa]; Label[ab]; ClearAll[XB]; Label[aa]; ClearAll[XA]; Label[a]; ClearAll[a0, V0, ua, uv, a, V]; a0 = Random[Real, {-1, 1}]; V0 = Random[Real, {-1, 1}];

ua = a0/(Sqrt[a0^2 + V0^2]);

uv = V0/(Sqrt[a0^2 + V0^2]);

Print["ua = ", ua]; Print["uv = ", uv];

HX1 = h/2 + (0.5/rm)*x^2;

HX2 = h/2 + (0.5/rs)*x^2;

H01 = HX1 /. {x -> 1};

H02 = HX2 /. {x -> 1};

H0 = H01 + H02;

E1 = -H0*V/V -> kv;

HP1 = D[HX1, x];

HP2 = D[HX2, x];

HPX = HP1 + HP2;

HX = HX1 + HX2;

C1 = a(1 - be*HX/H0)/(HX*h)/.a -> ka;

C2 = D[C1, x];

VX = E1*(1/HX + C1*(2*y - (HX1 - HX2)));

VY = E1*(HP1/(HX) + (HPX)*(y - HX1)/(HX^2) +

C2*(HX1 - y)*(HX2 + y) + C1*(HX*HP1 - (HP1 - HP2)*(HX1 - y)));

VXT = E1*(1/HX + C1*HX);

VXB = E1*(1/HX - C1*HX);

HA = HX/. x -> XA;

HB = HX/. x -> XB;

C1A = C1/. HX -> HA;

C1B = C1/. HX -> HB;

VTA = E1*(1/HA + C1A*HA);

VTB = E1*(1/HB - C1B*HB);

t = Solve[VTA == -V1, XA]/.XA_Complex -> 1000;

j1 = Label[{1, 1, 2}];
j2=t[[2,1,2]]; j3=t[[3,1,2]]; j4=t[[4,1,2]]; Print["a"]; If[0<j1<l,XA=j1, If[0<j2<l,XA=j2, If[0<j3<l,XA=j3, If[0<j4<l,XA=j4,Goto[a]]])); n=Solye[VTB==-V2,XB]/.XB_Complex->1000; k1=n[[1,1,2]]; k2=n[[2,1,2]]; k3=n[[3,1,2]]; k4=n[[4,1,2]]; Print["b"]; If[0<k1<l,XB=k1, If[0<k2<l,XB=k2, If[0<k3<l,XB=k3, If[0<k4<l,XB=k4,Goto[aa]]])); If[XA<XB,Goto[ab],]]; VYY=VY/. {x->l, y->yl}; m=Solve[VYY==0,yl]/.yl_Complex->10000; e1=m[[1,1,2]]; e2=m[[2,1,2]]; If[-H02<el<H01,yl=el, If[-H02<e2<H01,yl=e2, Goto[aaa]]]); Print["c"]; Print["aa=",kv]; Print["vv=",ka]; NT1=N[Integrate[VXT+V1,{x,XA,l}]]; NT2=N[Integrate[-VXT-V1,{x,0,XA}]]; NT3=N[Integrate[VXB+V2,{x,XB,l}]]; NT4=N[Integrate[-VXB-V2,{x,0,XB}]]; NT=(NT1+NT2+NT3+NT4)*2*SI; VYY=VY/. {x->l}; NS1=N[Integrate[VYY,{y,-H02,yl}]]; NS2=N[Integrate[VYY,{y,y1,H01}]]; NS=NS1-NS2; KISIX=D[VX,x]; ETAXY=D[VX,y]+D[VY,x]; FXY1=Sqrt[4*(KISIX^2)+(ETAXY^2)]; NP=NIntegrate[NIntegrate[FXY1,{y,-HX2,HX1}],{x,0,l}]; (*NP=2*V*H0*Log[H0/h]/.V->kv;*) N11=k*(NP+NT+NS); a=a10//N; V=V10//N; Print["V=",V]; Print["a=",a]; Print["XA=",XA]; Print["XB=",XB]; Print["yl=",yl]; Print["N11=",N11]; ClearAll[XA, XB, yl]; Print["end sam1.ma"];
APPENDIX 5
EQUIVALENT SLAB MASS ON BOTTOM ROLL
At first, when the slab reaches the rolls, there is no reaction force on the bottom roll caused by the slab weight. At the same time, the rolls vibrations start.

The vibrations will finish when about half of the slab pass the rolls and the bottom roll reaction \( F' \) can be determined from the following figure and by using a basic formula of strength of materials:

\[
F' = \frac{5}{8} ql = 0.625 w
\]

where

\[
\begin{align*}
w &= \text{slab weight} \\
q &= \text{slab weight per unit length} \\
l &= \text{slab length}
\end{align*}
\]

Then we can calculate the equivalent weight of slab by determining the average value of the two above conditions.

\[
F'_{av} = \frac{(0 + 0.625 w)}{2} = \frac{w}{3}
\]

Thus, the equivalent slab weight on the bottom roll is approximated by one third of the total slab weight.
APPENDIX 6

SECOND METHOD OF OPTIMISATION
The Mathematica Package has been used to gain a numerical solution to find the minimum of power $N$ (equation 7.8) using the data given in table 7.1 in the following way: the slab entry side velocity $v_0$ is the first value to be calculated by assuming a small initial value for $v_0$, we can calculate $x_A$, $x_B$ and $N$ using equations (7.8), (7.21) and (7.22). The $v_0$ value is incrementally increased to calculate the new value for $N$ (see flow chart given in the following).

The procedure for optimising $\beta_1$ is similar to Fig. A-6.1

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**Fig A-6.1** Flow chart for optimisation of slab entry velocity for steady-state case.
Thus, it is possible to draw a curve for $N$ with respect to $v_0$ to find the optimum value of $v_0$ (where $N$ has been minimised) as shown in Fig. A-6.2. From Fig. A-6.3, the optimum value of $v_0$ is 4.03 m/s. It can be seen from Fig. A-6.3 that at this velocity, the optimum power per unit width is 17.635 kW/m which corresponds to the optimum value $\beta_i = 0.035$.

![Graph](image-url)
Second Optimisation Program

l=95.34
a1=0.186
a2=0.149
SI=0.31
rm=512.5
rs=639.86
h=34
V1=5500
V2=5775
k=73
be=1

HX1=h/2+(0.5/rm)*x^2
HX2=h/2+(0.5/rs)*x^2
H01=HX1 /. {x->1}
H02=HX2 /. {x->1}
H0=H01+H02
E1=-H0*V

Print["H0= ",H0]
HP1=D[HX1,x]
HP2=D[HX2,x]
HPX=HP1+HP2

HX=HX1+HX2
Print["HX= ",HX]
ii = 0
jj = 31
NMIN = 1000000000000000000

For[j = 1, j < 100, j++,
jj = ii + j;
ClearAll[a, XA, XB, y1, yl1, N1];
a = jj/1000;
V = 4030;

C1 = -a(1-be*HX/H0)/(HX*h);
C2 = D[C1, x];

VX = E1*(1/HX+C1*2*y-(HX1-HX2));
VY = E1*(HP1/(HX)+(HPX)*(y-HX1)/(HX^2)+
    C2*(HX1-y)*(HX2+y)+C1*(HX*HP1-(HP1-HP2)*(HX1-y));

VXT = E1*(1/HX+C1*HX);
VXB = E1*(1/HX-C1*HX);
HA = HX /. x -> XA;
HB = HX /. x -> XB;
C1A = C1 /. HX -> HA;
C1B = C1 /. HX -> HB;
VXTA = E1*(1/HA+C1A*HA);
VXTB = E1*(1/HB-C1B*HB);
n = Solve[VXTA == -V1, XA];
t = Solve[VXTB == -V2, XB];

XA = n[[3, 1, 2]];
XB = t[[3, 1, 2]];
Print["XA = ", XA];
Print["XB = ", XB];

NT1 = Integrate[VXT + V1, {x, XA, 1}];
NT2 = Integrate[-VXT - V1, {x, 0, XA}];
NT3 = Integrate[VXB + V2, {x, XB, 1}];
NT4 = Integrate[-VXB - V2, {x, 0, XB}];
NT = (NT1 + NT2 + NT3 + NT4)*2*SI;

VYY = VY /. {x -> 1, y -> y1};
m = Solve[VYY == 0, y1];
y11 = m[[2, 1, 2]];

Print["y1 = ", y11];

Print["y1 = ", y1];
NS1 = Integrate[VYY, {y, -H02, y1}];

NS2 = Integrate[VYY, {y, y1, H01}];

NS = NS1 - NS2;

KISIX = D[VX, x];

ETAXY = D[VX, y] + D[VY, x];

FXY1 = (4*(KISIX^2) + (ETAXY^2))^0.5;
FXY = Simplify[FXY1];
ss = Integrate[FXY, {y, -HX2, HX1}];
NP = NIntegrate[ss, {x, 0, 1}];

N11 = k*(NP + NT + NS);
N12 = N11;
N1 = N[N12];
Print["N1 = ", N1];
If[NMIN > N1, NMIN = N1; AMIN = a; VMIN = V, Print["N1 = ", N1]; Print["NMIN = ", NMIN];
Print["================================== jj = " jj ];
Pause[1];
Print["NMIN = ", NMIN]
Print["AMIN = ", AMIN]
Print["VMIN = ", VMIN]