Conveyor trajectory prediction methods - a review

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ABSTRACT
The accurate design of conveyor transfers for the efficient transfer of product from one conveyor to another is of utmost importance and companies cannot afford a hit and miss approach in their construction. Several key particle mechanisms occur within a transfer chute, discharge from a belt conveyor, trajectory, impact, free-fall and chute flow.

This paper will focus on the prediction of product discharge and product trajectory from conveyor belts, which are the defining elements determining the flow through a conveyor transfer. There are several approaches available in the literature which will be reviewed taking into account issues such as the parameters used in the trajectory determination, complexity of method and potential accuracy.

Following this review, trajectory curves for a range of belt velocities and pulley diameters will be generated to allow visual scrutiny of each method and to better compare one method to another.

1 INTRODUCTION
There are literally hundreds of applications for the use of belt conveyors for the transport of their products, including; coal mines both underground and open-cut, power stations, iron ore mines, gold mines and processing plants to name but a few. It would be no stretch to say that the majority of industries in these fields would rely on conveyor transfers to divert material from one conveyor to another.

With the reality that some companies will be driven by cost effectiveness for the short-term rather than planning for the long-term, bad decisions can sometimes be made resulting in the incorrect conveyor transfer being installed. This can result in substantial downtime in the future when modifications need to be instigated. Some of the specific issues which may arise from a poorly designed and/or installed conveyor transfer are;

- belt shift due to non-central loading of product
- belt wear resulting from turbulent flow through the transfer
- structural damage to the transfer chute
- product spillage resulting in product loss
- product degradation
- excessive noise
- dust emissions

The obvious solution to eliminating, or at least greatly minimising, these issues is to fully understand the behaviour of the material being conveyed and from this knowledge, apply it to the design of the conveyor transfer. The main particle mechanisms occurring within a conveyor transfer are product discharge, trajectory, impact, free-fall and chute flow (Burnett 2000). An illustration of these mechanisms is depicted in Figure 1.

This paper will focus on the determination of material trajectory as it is discharged from a belt conveyor. This process will also determine the point at which the material leaves the belt, referred to as the discharge angle. There are several methods available in the literature focusing on the modelling of material discharge and trajectory, including: C.E.M.A. (1966; 1979; 1994; 1997; 2005); M.H.E.A. (1977; 1986); Korzen (1989); Booth (1934); Golka (1992; 1993); Dunlop (1982); Goodyear (1975) and Colijn and Conners (1972). These methods will be reviewed and evaluated.
using several sets of parameters and compared to allow comment on their ease of use, completeness and potential accuracy.

![Figure 1 Key material mechanisms within a conveyor transfer (Burnett 2000)](image)

2 TRAJECTORY METHODS EXAMINED

2.1 C.E.M.A.
The Conveyor Equipment Manufacturers Association have released six editions of the C.E.M.A. guide, ‘Belt Conveyors for Bulk Materials’ since 1966. The first five editions follow the same procedure for determination of material trajectory with the only variation, slight adjustments to various values in reference tables.

Two reference tables are used, one is load height and centre of gravity for a range of surcharge and troughing for belt widths between 450 mm and 2400 mm. The other consists of fall intervals, to plot the vertical component of the trajectory path. The C.E.M.A. method addresses seven main belt conveyor cases;

- slow-speed belt where material wraps around pulley before discharge (horizontal, inclined and declined belts),
- fast-speed belt where material discharges at the tangent point of belt to pulley (horizontal, inclined and declined belts), and
- inclined belts where material discharges at the vertical point of contact.

To determine which case is applicable, a velocity check is made. For equation (1), if the speed condition is \( \geq 1 \), then high-speed conditions apply. If the speed condition is \(< 1\), then low-speed conditions apply. There is an additional special case for an inclined belt where if the speed condition = 1, then the material will leave the belt at the top of the pulley.

\[
\text{Speed condition } \Rightarrow \frac{V^2_s}{g R_c}
\]  

(1)

For the low-speed conditions, the angle at which the material leaves the belt, \( \alpha_d \), is determined by solving equation (2).
\[
\frac{V_s^2}{g R_c} = \cos \alpha_d
\]  

(2)

To plot the trajectory, a tangent line is drawn from the point at which the material leaves the pulley at radius \( R_c \). Time intervals of \( \frac{1}{20} \)th of a second are marked along this line, equating to 50 mm for each metre per second of tangential velocity, \( V_s \). Lines are then projected vertically down and the corresponding fall distances for each point are marked. A smooth curve is then drawn to produce the centroid trajectory curve.

C.E.M.A. also allows for the plotting of the upper and lower trajectory limits by marking a distance of \( (h - a_1) \) and a distance of \( a_1 \) perpendicular to each drop point for the upper and lower trajectories respectively. A smooth curve is then drawn through these points to produce the trajectory curves. It is evident from this procedure, that a constant width trajectory path results.

In the 6th Edition of C.E.M.A. (2005), there is a change to how the time interval is calculated for high-speed belts, now being calculated from the belt speed rather than the tangential velocity. For all other conditions the time intervals are calculated as previously explained.

As previously mentioned, the trajectory path has a constant width throughout, however C.E.M.A. does note that for light fluffy materials, a high belt speed will alter the upper and lower limits with both vertical and lateral spread resulting from air resistance for such materials.

2.2 M.H.E.A. 1977

The Mechanical Handling Engineer’s Association guide, ‘Recommended Practice for Troughed Belt Conveyors’ (M.H.E.A. 1977) addresses both low-speed and high-speed belts via centripetal acceleration. The speed condition of equation (1) is once again used, this time using pulley diameter rather than material centroid.

The calculations result in a plot of the lower trajectory. Time intervals along the tangent line are marked off at divisions of 50 mm for each metre per second of belt speed. A table is supplied with the vertical fall distances for each division, through which a smooth curve is drawn.

The M.H.E.A. method (1977) also provides an approximation for the outer trajectory of the material by first determining the angle, \( \alpha_d_2 \), at which the upper surface of material starts its trajectory, based on equation (3). Following the above method, a smooth curve can then be drawn.

\[
\frac{V_h^2 R_h}{g R_p} = \cos \alpha_{d_2}
\]  

(3)

2.3 M.H.E.A. 1986

The Mechanical Handling Engineer’s Association guide, ‘Recommended Practice for Troughed Belt Conveyors’ (M.H.E.A. 1986) is identical to the C.E.M.A. (1966; 1979; 1994; 1997) method up to and including the 5th edition, however there are some minor differences to the values in the supplied fall distance table. The M.H.E.A. (1986) method uses metric units rather than imperial units and the conversion results in small rounding differences, ultimately causing minor variations to the trajectory curves.

2.4 KORZEN

Of all the methods reviewed, Korzen (1989) is the most complex in its approach, see Figure 2, addressing the issue of adhesive materials and also combines inertia and slip into its calculations. This method is the only one incorporating air drag into its calculations. Further to this, there is also a distinction between static friction, \( \mu_s \), and kinematic friction, \( \mu_k \), used in the determination of the discharge velocity, \( V_d \), and discharge angle, \( \alpha_d \).
For high-speed belts, the following conditions apply,

\[ \alpha_d = \alpha_b \quad \text{and} \quad V_d = V_b \]

For low-speed belts the angle at which material begins to slip on the belt before discharge, \( \alpha_r \), is determined from equation (4).

\[
\alpha_r = \tan^{-1} \mu_s \pm \sin^{-1} \left[ \sin \left( \tan^{-1} \mu_s \right) \left( \frac{V_b^2}{R_c g} - 2 \frac{\sigma_w}{\gamma h} \right) \right]
\]  

(4)

The discharge angle, \( \alpha_d \), is found from equation (5) via substitution:

\[
V^2(\psi) = C e^{4\mu_k \psi} + \left( \frac{2R_c g}{1 + 16\mu_k^2} \right) \left[ (4\mu_k^2 - 1) \cos \psi - 5\mu_k \sin \psi \right]
\]  

(5)

The discharge velocity, \( V_d \), is then found using equation (6):

\[
V_d = \sqrt{R_c g \cos \alpha_d}
\]  

(6)

The discharge angle and discharge velocity calculated above are for the centre height of the material stream. Korzen also allows for calculation of the upper and lower trajectory limit discharge velocities. For high-speed belts the upper and lower discharge velocities are the same as that for the centre height trajectory but for a slow-speed belt, equations (7) and (8) are used:

\[
V_{d,\text{lower}} = V_d \cdot \frac{R_p}{R_p + 0.5 h_d}
\]  

(7)

\[
V_{d,\text{upper}} = V_d \cdot \frac{R_p + h_d}{R_p + 0.5 h_d}
\]  

(8)

Although the discharge velocity for the lower and upper trajectories can be determined, there is no method described for the determination of these trajectories. The assumption has been made that the
same approximation method is used, substituting the calculated discharge velocity for the appropriate trajectory.

As previously mentioned, Korzen (1989) incorporates air drag into the calculations for the determination of the trajectory profile. The detailed numerical analysis developed is explained by a series of successive approximations. The successive approximations incorporate ‘corrected’ air drag coefficients based on particle shape and a proportionality factor for air drag, $a_w$.

From the values presented above, the trajectory, $y(x)$, trajectory angle, $\xi(x)$, and resultant velocity, $v(x)$, can be determined for the free fall of a particle by equations (9), (10) and (11).

$$y(x) = x \tan \alpha_d - \frac{g}{2 V_d^2 \cos^2 \alpha_d} x^2 - \frac{a_w g}{3 \frac{dm}{V_d^2} \cos^2 \alpha_d} x^3$$  \hspace{1cm} (9)

$$\xi(x) = \tan^{-1} \left( \tan \alpha_d - \frac{g}{V_d^2 \cos^2 \alpha_d} x - \frac{a_w g}{\frac{dm}{V_d^2} \cos^2 \alpha_d} x^2 \right)$$  \hspace{1cm} (10)

$$v(x) = V_d e^{\left( \frac{a_w x}{dm} \right)} \cos \alpha_d \sqrt{1 + \left( \tan \alpha_d - \frac{g}{V_d^2 \cos^2 \alpha_d} x - \frac{a_w g}{\frac{dm}{V_d^2} \cos^2 \alpha_d} x^2 \right)^2}$$  \hspace{1cm} (11)

The first approximation for the equations is for a free falling particle in a vacuum, which is used as the initial estimate, hence $a_w = 0$, for all other approximations, $a_w$ is calculated. The successive approximations are continued until the differential error between successive approximations for $y(x)$, $\xi(x)$, and $v(x)$, have deviations no greater than 1% or 2%. Once the analysis has been completed for a suitable range of X values, the X and Y coordinates are plotted to produce the trajectory for the central path.

A statement made by Korzen (1989) is that for particles over 1g in mass, the effects of air drag can be dismissed. This being the case, only the first approximation is performed and the trajectory plotted from those values. This will also result in the trajectory curves having a closer match to trajectory curves generated by other methods.

Korzen (1989) does not include the belt thickness when determining the trajectory curves. This omission will result in a minor vertical offset of the plotted trajectories compared to methods which do incorporate belt thickness. An adjustment to the radii used in this method has been made to accommodate belt thickness which will allow a direct comparison between all the methods presented.

There seem to be several errors present in the worked examples presented by Korzen (1989), as a result of one group of errors additional approximations were required to achieve an adequate error level. Also, in several quoted equations there are what would most likely prove be typographical errors but as a result can raise questions as to whether the method is being applied correctly.

2.5 BOOTH

Booth (1934) found that while using available theory to determine material trajectory from a conveyor belt, a large discrepancy was present between the theory and that of the actual trajectory. After careful investigation and confirmation of these errors, Booth concluded that the existing theory was incomplete, for one, not taking into account the effects of the material slip as material discharged over the head pulley. This lead to an analytical analysis to develop a more representative theory, however Booth uses a single particle only in the determination of the trajectory.

The starting point of the Booth method is to determine the angle, $\alpha$, at which the particle will leave the head pulley, again based on the slow-speed and high-speed conditions described by the C.E.M.A. (1966; 1979; 1994; 1997; 2005) method.
If the belt is operating at slow-speed, the angle, $\alpha_r$, where the product begins to slip on the belt can be found by solving equation (12).

$$\cos \alpha_r - \frac{V_b^2}{g R_b} = \frac{1}{\mu} \sin \alpha_r$$  \hspace{1cm} (12)

The discharge angle, $\alpha_d$, is found from equation (13) via substitution:

$$\frac{V^2(\psi)}{2 g R_b} = \frac{(2 \mu^2 - 1) \cos \psi - 3 \mu \sin \psi}{(4 \mu^2 + 1)} + C e^{2 \mu \psi}$$  \hspace{1cm} (13)

The discharge velocity, $V_d$, is then found using equation (14):

$$V_d = \sqrt{R_b g \cos \alpha_d}$$  \hspace{1cm} (14)

Booth acknowledged that this method was both tedious and complicated and as such, developed a chart to minimise the time required to analyse a particular belt conveyor geometry with reasonable accuracy.

There is no mention of how the upper trajectory should be determined and no mention how to determine the material height at the discharge point. As an estimate, the method used by C.E.M.A. (1966; 1979; 1994; 1997; 2005) has been applied but only offsetting a perpendicular distance equal to the material height, followed by fitting a curve through these points. This results in an upper trajectory parallel to the lower trajectory, hence no divergence or convergence of the flow pattern.

An alternative method of producing the upper trajectory limit could be to use the same method as for the lower boundary with an increased radius used to account for the belt thickness and material height. This method would also result in a second discharge angle being produced for slow-speed belt conditions which may or may not generate a divergent or convergent flow pattern.

2.6 GOLKA

Golka’s method (1992; 1993) for determining material trajectory is based on the Cartesian coordinate system. This method is for materials without cohesion or adhesion.

For slow-speed belts, this method once again follows that of C.E.M.A. (1966; 1979; 1994; 1997; 2005) in determining the discharge angle of the lower trajectory using equation (2). It also calculates a separate discharge angle for the upper trajectory by substituting the radius of the upper trajectory into equation (2). An adjusted material height, $h_2$, is also calculated for the point at which the upper trajectory discharges from the pulley, see equation (15).

$$h_d = R_p \left( \left[ 1 + \frac{2h}{R_p} \right]^{0.5} - 1 \right)$$  \hspace{1cm} (15)

Two divergent coefficients have been introduced by Golka (1992; 1993), $\varepsilon_1$ for the lower and $\varepsilon_2$ for the upper trajectory, which takes into account variables such as air resistance, size distribution, permeability and particle segregation. Unfortunately there is no explanation how these divergent coefficients are determined. Without knowing with any certainty what divergent coefficients should be used for a given product the predicted trajectory curves could vary substantially as can be seen in Figure 3. Five distinct conditions have been presented, the case where divergent coefficients have been neglected and the values of 0.1, 0.2, 0.3 and 0.4, to indicate the extent these divergent coefficients will have on trajectory predictions.
Golka (1992; 1993) uses three cases to predict the material trajectory from the pulley depending on predefined conditions. The critical velocity, \( V_{cr} \), needs to be determined, indicating the velocity at which the transition from slow-speed to fast-speed occurs, see equation (16) where \( R \) is either the radius for the lower or upper trajectory.

\[
V_{cr} = \sqrt{g \cdot R}
\]  

(16)

Case 1: \( V_1 < V_{cr} \) and \( V_2 < V_{cr} \)

Case 2: \( V_1 > V_{cr} \)

Case 3: \( V_1 < V_{cr} \) and \( V_2 > V_{cr} \)

Belt thickness is neglected in this method also and again for direct comparison between all the methods, has been included by adjusting the relevant radii in all calculations. Material height is used in the Golka method (1992; 1993), however the value used in the worked examples is that of early C.E.M.A. (1966; 1979). As the newest version of C.E.M.A. (2005) has updated values for material heights, these have been used in the comparisons in section 3.

2.7    DUNLOP

The Dunlop Conveyor Manual (1982) uses a graphical method to determine the material trajectory leaving the conveyor for slow-speed belts and analytical method for fast-speed belts. For high-speed belts, material will leave the belt at the point where the belt is at a tangent to the pulley. To calculate the X coordinate, the distance travelled in line with belt along line A-A, equation (17) is used. The Y coordinate, the distance material falls below the line of discharge, equation (18) is used.

\[
X = V_b \cdot t
\]  

(17)
Setting the time interval to 0.143 seconds simplifies the calculations for X and Y, the coordinates are then used to plot the trajectory as represented in Figure 4.

\[ Y = \frac{gt^2}{2} \]  

(18)

Figure 4 Method of plotting trajectory for high-speed belts (Dunlop 1982)

Dunlop goes on to say that the mathematics behind the trajectory for slow-speed conveyors is complex and so a graphical method has been developed. By knowing the belt speed and pulley diameter, it is a simple process to determine \( \alpha_d \), the angle at which material will leave the belt, and X, the incremental distance along the project line A-A. At each of these incremental distances, Y is projected vertically down with the same values as for fast conveyors. If it is found that the desired belt speed does not intersect with the required pulley diameter then the method for fast belts should be used. For the slow-speed condition, pulley diameters between 312mm and 1600mm are presented. Outside of this range there is no way to estimate \( \alpha_d \) or X.

The resulting trajectory is a prediction of the lower boundary of the particle stream. There is reference to material depth in the worked examples in the Dunlop Conveyor Manual (1982) but no explanation of how it is determined, as a result, the material height obtained via the C.E.M.A. method (2005) has been used. With no method given to determine the upper trajectory path, the approach used by C.E.M.A. (1966; 1979; 1994; 1997; 2005) has been employed. This results in a parallel flow pattern throughout the trajectory with no divergence or convergence. This, goes against the indication that the flow pattern is convergent in the examples provided in the Dunlop Conveyor Manual (1982).

2.8 GOODYEAR

The ‘Handbook of Conveyor and Elevator Belting’ (Goodyear 1975) is again quite simplistic in its approach to determining material trajectory. Goodyear states that there are three relationships which determine the trajectory of material leaving the belt, they being;

1. the centrifugal force which determines where material will leave the belt
2. discharge velocity of the material as it leaves the belt
3. the force of gravity on the material after it leaves the belt

Using the principles of projectile motion, equation (17) and equation (18) provided by Dunlop (1982) are again used to determine the X and Y coordinates of the trajectory. The discharge angle is
determined for slow-speed belt conditions by again using equation (2), this time using the radius of the central height of the material.

The Goodyear Handbook defines six individual cases for where discharge begins depending on high-speed or low-speed belts for horizontal, inclined and declined belts. These six cases are identical to those presented by C.E.M.A. (1966; 1979; 1994; 1997; 2005) with the exception of the special inclined belt case. Once the X and Y coordinates and the discharge angle have been determined, producing the trajectory curve is straightforward.

Again, belt thickness is not used in the Goodyear method (1975) but has been incorporated for comparison purposes. As this method determines only the centre trajectory path, a material height must be known. The worked examples by Goodyear (1975) incorporate half the material height into the radius used to determine the central trajectory path without implicitly providing details of the material height. As a result, the material height obtained from C.E.M.A. (2005) has once again been used. The method of determining the lower and upper trajectory paths is not provided by the Goodyear method and as such, the C.E.M.A. method (1966; 1979; 1994; 1997; 2005) is once again applied.

Goodyear (1975) states that the actual trajectory may be different to that of the one calculated due to other forces acting on the particle stream which haven’t been used in these calculations.

2.9 COLIJN AND CONNERS
Colijn and Conners (1972) state that the belt speed should be at least 500 fpm (2.5 m/s) so as not to allow free flowing materials to spill over the sides of the head pulley as the belt flattens through the transition stage. Colijn and Conners’ method of determining material trajectory is a reproduction of the 1st edition of C.E.M.A. (1966).

Colijn and Conners also state that a different trajectory will result if the head pulley is positioned too high above the centre roll of the adjacent idler or if there is too much belt sag present. This is due to the humping effect that the flow stream takes for high speed belts.

2.10 TRAJECTORY METHOD COMPARISON CHART
As an overall summary of the parameters included or omitted for each of the trajectory methods reviewed above, a summary chart has been produced, Table 1.
3 TRAJECTORY METHOD COMPARISONS

The most obvious way to directly compare one model against another is to use a set of arbitrary conditions which are kept constant and used as input for each of the models previously discussed. Table 2 shows such a set of parameters and for the following graphical comparisons, three varying belt speeds have been used, 1.25m/s, 3m/s and 6m/s and for each belt speed, pulley diameters of 0.5m, 1.0m and 1.5m have been used, with all other parameters being constant throughout. It should also be noted that not all values are used in each model, as was summarised in Table 1.

Table 2 Conveyor parameters used for comparisons

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belt Width, ( w_b )</td>
<td>30 inch</td>
</tr>
<tr>
<td>Belt Thickness, ( b )</td>
<td>0.4375 inch</td>
</tr>
<tr>
<td>Belt Speed, ( V_b )</td>
<td>295.275 fpm</td>
</tr>
<tr>
<td>Pulley Diameter, ( D_p )</td>
<td>19.685 inch</td>
</tr>
<tr>
<td>Surcharge Angle</td>
<td>20 °</td>
</tr>
<tr>
<td>Troughing Angle</td>
<td>20 °</td>
</tr>
<tr>
<td>Belt Inclination Angle, ( \alpha_b )</td>
<td>0 °</td>
</tr>
<tr>
<td>Divergent Coefficient, Lower, ( \varepsilon_1 )</td>
<td>0.1</td>
</tr>
<tr>
<td>Divergent Coefficient, Upper, ( \varepsilon_2 )</td>
<td>-0.1</td>
</tr>
<tr>
<td>Coefficient of Friction, Static, ( \mu_s )</td>
<td>0.42</td>
</tr>
<tr>
<td>Coefficient of Friction, Kinematic, ( \mu_k )</td>
<td>0.5</td>
</tr>
<tr>
<td>Adhesion Stress, ( \sigma_a )</td>
<td>0 kPa</td>
</tr>
<tr>
<td>Equivalent Spherical Particle Diameter</td>
<td>0.001 m</td>
</tr>
<tr>
<td>Atmospheric Temperature, ( T_{atm} )</td>
<td>20 °C</td>
</tr>
<tr>
<td>Atmospheric Pressure, ( P_{atm} )</td>
<td>101 kPa</td>
</tr>
<tr>
<td>Air Viscosity, ( \eta_f )</td>
<td>1.80E-05 Ns/m²</td>
</tr>
<tr>
<td>Product Density, ( \rho_b )</td>
<td>2000 kg/m³</td>
</tr>
</tbody>
</table>

3.1 SLOW-SPEED TRAJECTORY COMPARISONS

For the slow speed belt condition, the material being conveyed will discharge after wrapping around the head pulley to the calculated or graphically determined discharge angle, \( \alpha_d \). The various trajectory techniques use varying methods of determining the discharge angle and discharge velocity, \( V_d \), and as a consequence a range of discharge angles results for any given belt speed condition with Korzen (1989) always having the lowest discharge angle and Goodyear (1975) having the largest discharge angle. These results are summarised in Table 3 below for the three pulley diameters.

Table 3 Range of discharge velocities obtained from the trajectory methods

<table>
<thead>
<tr>
<th>TRAJECTORY METHOD</th>
<th>( V_b = 1.25 \text{ m/s, } D_p = 0.5 \text{ m} )</th>
<th>( V_b = 1.25 \text{ m/s, } D_p = 1.0 \text{ m} )</th>
<th>( V_b = 1.25 \text{ m/s, } D_p = 1.5 \text{ m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C.E.M.A. 1, 2, 4, 5</td>
<td>45.18°</td>
<td>70.34°</td>
<td>77.27°</td>
</tr>
<tr>
<td>C.E.M.A. 6</td>
<td>44.94°</td>
<td>70.30°</td>
<td>77.25°</td>
</tr>
<tr>
<td>M.H.E.A. 1986</td>
<td>45.30°</td>
<td>70.37°</td>
<td>77.28°</td>
</tr>
<tr>
<td>KORZEN</td>
<td>44.94°</td>
<td>70.37°</td>
<td>77.28°</td>
</tr>
<tr>
<td>BOOST</td>
<td>30.30°</td>
<td>42.65°</td>
<td>47.50°</td>
</tr>
<tr>
<td>GOLKA</td>
<td>LOWER: 52.41°</td>
<td>LOWER: 71.84°</td>
<td>LOWER: 77.92°</td>
</tr>
<tr>
<td>DUNLOP</td>
<td>37.59°</td>
<td>49.07°</td>
<td>52.87°</td>
</tr>
<tr>
<td>GOODYEAR</td>
<td>59.36°</td>
<td>73.55°</td>
<td>78.70°</td>
</tr>
</tbody>
</table>
A comparison graph for the slow-speed trajectory predictions is presented in Figure 5. Where trajectory curves for different models vary by no more than 2% for both the lower and upper trajectories, at a vertical drop height of 4m, they have been grouped together.

It is evident from Table 3 that as the pulley diameter increases, an increasing number of methods converge to a similar discharge angle, resulting in only four curves being produced for a pulley diameter of 1.5m.

![Figure 5](image)

**Figure 5** Slow speed condition, horizontal conveyor, lower and upper trajectory path

Pulley diameter, \( D_p = 1.0 \) m, belt velocity, \( V_b = 1.25 \) m/s

### 3.2 FAST-SPEED TRAJECTORY COMPARISONS

In order to investigate the possible changes in trajectory profile due to belt speed, two fast-speed belt conditions were selected, \( V_b = 3.0 \) m/s and \( V_b = 6.0 \) m/s. A comparison was made between the discharge velocities and pulley diameters and is presented in Table 4. As can be clearly seen from all the values for the lower trajectory limit, the discharge velocity is equal to the belt speed, the variations lie with the upper trajectory discharge velocities. The slight variations with the C.E.M.A. (1966; 1979; 1994; 1997; 2005) and M.H.E.A. (1986) methods lie with the variation in material height as the upper discharge velocity is calculated based on a ratio of velocities and radii. The upper discharge velocity for the Booth (1934) method is identical to the C.E.M.A. (2005) due to the material height from C.E.M.A. (2005) being assumed. For both belt speeds selected for this comparison, the Golka (1992; 1993) method uses Case 2, as explained in section 2.6 and as such the discharge velocity for both the lower and upper trajectories is equivalent to the belt speed. Korzen by default uses the belt speed as the discharge velocity for the lower, central and upper trajectories as previously stated.

In Figure 6 and Figure 7 the inclusion of air drag by Korzen (1989) results in a trajectory prediction clearly lower than all other methods. If air drag is neglected in the Korzen (1989) method, the resulting trajectory prediction is located amongst the other methods.
Table 4 Discharge velocities versus pulley diameter for a belt speed of \( V_b = 3.0 \text{ m/s} \)

<table>
<thead>
<tr>
<th>TRAJECTORY METHOD</th>
<th>( D_p = 0.5 \text{ m} )</th>
<th>( D_p = 1.0 \text{ m} )</th>
<th>( D_p = 1.5 \text{ m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C.E.M.A. 1, 2, 4, 5</td>
<td>L: 3.000 U: 4.109</td>
<td>L: 3.000 U: 3.566</td>
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In section 2.6 it was explained that Golka (1992; 1993) uses divergent coefficients to obtain a better approximation of the trajectory paths. Without explanation of how these divergent coefficients have been determined, it is hard to justify using this method with any level of accuracy. If on the other hand the divergent coefficients are neglected in the calculations, the resulting trajectories are identical to the Korzen (1989) method when air drag is neglected as the equations are identical.

In both cases, the early C.E.M.A. (1966; 1979; 1994; 1997) and M.H.E.A. (1986) methods generate the highest trajectory curve and as the discharge velocity increases, the variation from the other curves becomes more defined.

Figure 6 High speed condition, horizontal conveyor, lower and upper trajectory path
Pulley diameter, \( D_p = 1.0 \text{ m} \), belt velocity, \( V_b = 3.0 \text{ m/s} \).
5 CONCLUSION

This paper has presented a number of the more widely used and/or readily available trajectory prediction methods published in the literature. They range in complexity from the basic, Dunlop (1982) and Goodyear (1975) to the complex, Booth (1934), Golkha (1992; 1993) and Korzen (1989). Some methods include a multitude of parameters such as C.E.M.A. (1966; 1979; 1994; 1997; 2005) and M.H.E.A. (1986) while others incorporate parameters which no others address, divergent coefficients (Golkha 1992; 1993) and air drag (Korzen 1989).

For slow-speed belt conditions there appears to be two distinct groupings of trajectory predictions, see Figure 5, resulting from the differing determination of the discharge angle. It is obvious that both groupings cannot be correct but without trajectory data from the field to compare these groups, no definitive conclusion can be presented at this stage. For fast-speed belt conditions an observation made was that some of the basic methods approximated a trajectory which was also predicted by the more complex methods, see Figure 7. Incorporating air drag into the trajectory predictions (Korzen 1989) causes a dramatic variation from the other trajectory methods. When considering air drag, it makes sense that material will drop away much quicker than the other methods but whether it is truly representative of an actual trajectory needs to be explored further.

6 FUTURE INVESTIGATIONS

The next stage of the comparison process is to construct an experimental test facility to experimentally validate trajectories generated for a range of belt speeds and inclination angles. This validation process is planned and with the aid of a high-speed digital video camera facility capable of up to 3000 frames per second, precision measurement of material trajectories will be achieved. These will be directly compared to the methods presented above for the corresponding conveying parameters. From this experimental comparison, a series of recommendations will be produced as to which trajectory method(s) are the most accurate and reliable for both slow-speed and fast-speed conveying conditions. It may also be found that components from several of these methods need be combined to produce a more accurate method.
7 ACKNOWLEDGEMENTS
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8 NOMENCLATURE

ARABIC
\( a_1 \) height to material centroid \( m \)
\( a_w \) proportionality factor 
\( b \) belt thickness \( m \)
\( C \) constant of integration 
\( d_m \) elementary mass of material stream \( kg \)
\( D_p \) pulley diameter \( m \)
\( g \) gravity \( m \ s^{-2} \)
\( h \) material depth \( m \)
\( h_d \) material depth at discharge \( m \)
\( R_b \) radius to outer belt surface \( m \)
\( R_c \) radius of material centroid/centre \( m \)
\( R_h \) radius to outer depth of material surface \( m \)
\( R_p \) pulley radius \( m \)
\( t \) increment time for trajectory path \( s \)
\( v(x) \) resultant velocity of inclined freefall \( m \ s^{-1} \)
\( V_1 \) discharge velocity of lower boundary \( m \ s^{-1} \)
\( V_2 \) discharge velocity of upper boundary \( m \ s^{-1} \)
\( V_b \) belt velocity \( m \ s^{-1} \)
\( V_{cr} \) critical velocity \( m \ s^{-1} \)
\( V_d \) velocity of material at discharge point \( m \ s^{-1} \)
\( V_s \) tangential velocity of material at discharge point \( m \ s^{-1} \)
\( w_b \) belt width \( m \)
\( X \) distance travelled along tangent line of belt and pulley \( mm \)
\( x \) horizontal distance at which \( y(x), \xi(x) \) and \( v(x) \) are calculated \( m \)
\( Y \) distance material falls below line of discharge \( mm \)
\( y(x) \) y component of trajectory of particle freefall \( m \)

GREEK
\( \alpha \) initial material discharge angle measured from the vertical \( \circ \)
\( \alpha_b \) belt inclination angle \( \circ \)
\( \alpha_d \) material discharge angle measured from the vertical trajectory \( \circ \)
\( \alpha_{d2} \) material discharge angle measured from the vertical for upper trajectory \( \circ \)
\( \alpha_r \) angle at which slip begins to occur \( \circ \)
\( \varepsilon_1 \) divergent coefficient \( - \)
\( \varepsilon_2 \) divergent coefficient \( - \)
\( \eta_f \) air viscosity \( Ns \ m^{-2} \)
\( \mu \) coefficient of friction \( - \)
\( \mu_k \) coefficient of kinematic friction \( - \)
\( \mu_s \) coefficient of static friction \( - \)
\( \sigma_a \) adhesion stress \( kPa \)
\( \xi(x) \) trajectory direction angle \( \circ \)
\( \rho_b \) bulk density of material \( kg \ m^{-3} \)
\( \psi \) wrap angle around discharge pulley \( \circ \)
9 REFERENCES


