Adaptive control of power system stabilising devices

Feroze Coowar
University of Wollongong

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ADAPTIVE CONTROL OF POWER SYSTEM STABILISING DEVICES

A thesis submitted in fulfilment of the requirements for the award of the degree of

DOCTOR OF PHILOSOPHY

from

THE UNIVERSITY OF WOLLONGONG

BY

FEROZE COOWAR, B.Sc. (Hons)

Department of Electrical and Computer Engineering

1991
To Father Jacob
Abstract

A frequency domain approach is used to analyse low frequency oscillations in a power system where the oscillations are viewed as the result of disturbances originating from a section of the network and reflected on the remainder of the network. Available control theory for counteracting exogenous disturbances are thus readily applicable (e.g. the internal model principle). Further, this approach is shown to be capable of revealing risky situations which otherwise could not be detected. The gain characteristics of closed loop transfer functions, which reflect the effect of disturbances on power angle oscillations, are shown to be crucial in identifying frequency ranges where amplified oscillatory disturbances can occur.

The adopted analytical approach also provides additional insight in the development of power system stabiliser tuning rules, which are usually based on eigenvalue analysis. This leads to a proposal for restructuring the transfer function blocks in the excitation control system such that the negative damping effects are decoupled without compromising the benevolent effects of other control loops. Simulation results confirm the validity of this proposal. A special feature of the new structure is the location of a notch filter, acting as an on/off switch, in the forward path of the negative damping signal. This provides the necessary dynamic gain reduction for "switching off" the detrimental signals. It is shown that, while a considerable reduction in the tie-line mode oscillations can be achieved, the phenomenon of amplified oscillations can still occur, albeit at a reduced magnitude.
A self-tuning control scheme is therefore proposed which is able to provide both positive damping characteristics and disturbance rejection. A control strategy based on the internal model principle is used to directly cancel tie-line oscillatory disturbances. The thesis describes the internal model principle and its implementation in a power system for disturbance rejection purposes. Since the self-tuner requires on-line estimation of the system parameters, an efficient recursive algorithm for the estimation routine is developed. The algorithm makes use of some properties of the Cholesky matrix decomposition techniques. The numerical stability of the estimation algorithm is improved through the updating of the triangular factors of an augmented information matrix and a fixed width data window, rather than a forgetting factor, is used to cater for parameter drifts.

Simulation studies performed on a typical excitation control system model are presented, demonstrating clearly the benefits of incorporating the internal model principle in an adaptive power stabilising scheme. Results show effective cancellation of tie-line time-varying oscillatory disturbances and enhanced damping behaviour of the power system.
Declaration

This is to certify that the work reported in this thesis was done by the author, unless specified otherwise, and that no part of it has been submitted in a thesis to any other university or similar institution

Feroze Coowar
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I also thank Dr. Philip O. Ogunbona for his constant companionship and enlightening philosophical discussions.

The work involved in the development of this thesis would not have been complete without the assistance of other members of staff of the Department of Electrical and Computer Engineering, particularly Dr. Joe F. Chicharo, Professor Chris D. Cook, Mr. Peter Costigan, Ms Maree J. Fryer, Dr. Sarrath Perrera and Dr. Don Platt.

Thanks are also due to the Australian Electricity Supply Industry Research Board for part funding of the research.

I thank my wife Marianne for her patience, understanding and never-failing support throughout the course of this work.

To my Mum and Dad, sister Rosie and nephew Hachem: Thank you for your support and trust.
# List of symbols and abbreviations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A(s)</td>
<td>denominator of $G_{DRF}(s)$</td>
</tr>
<tr>
<td>A/D</td>
<td>analogue to digital converter</td>
</tr>
<tr>
<td>$A_0 \ldots A_4$</td>
<td>coefficients of denominator of $G_{DRF}(s)$</td>
</tr>
<tr>
<td>$A_k \ldots A_{k6}$</td>
<td>coefficients of denominator of $G_{TGR}(s)$</td>
</tr>
<tr>
<td>$A_{V0} \ldots A_{V5}$</td>
<td>coefficients of denominator of $G_{AVR}(s)$</td>
</tr>
<tr>
<td>B(s)</td>
<td>numerator of $G_{servo}(s)$</td>
</tr>
<tr>
<td>$B_0, B_1$</td>
<td>coefficients of numerator of $G_{servo}(s)$</td>
</tr>
<tr>
<td>$B_{k0} \ldots B_{k4}$</td>
<td>coefficients of numerator of $G_{TGR}(s)$</td>
</tr>
<tr>
<td>$B_{V0} \ldots B_{V3}$</td>
<td>coefficients of numerator of $G_{AVR}(s)$</td>
</tr>
<tr>
<td>C(s)</td>
<td>numerator of $G_{DRF}(s)$</td>
</tr>
<tr>
<td>$C_0 \ldots C_2$</td>
<td>coefficients of numerator of $G_{DRF}(s)$</td>
</tr>
<tr>
<td>D</td>
<td>diagonal matrix</td>
</tr>
<tr>
<td>d(s)</td>
<td>disturbance signal</td>
</tr>
<tr>
<td>D/A</td>
<td>digital to analogue converter</td>
</tr>
<tr>
<td>$D=\frac{d}{dt}$</td>
<td>differential operator</td>
</tr>
<tr>
<td>e</td>
<td>error signal</td>
</tr>
<tr>
<td>$E()$</td>
<td>statistical expectation</td>
</tr>
<tr>
<td>E</td>
<td>generator internal voltage</td>
</tr>
<tr>
<td>$E_{ld}$</td>
<td>generator field voltage</td>
</tr>
<tr>
<td>$E_q$</td>
<td>voltage proportional to the direct axis flux</td>
</tr>
<tr>
<td>$G_{AVR}(s)$</td>
<td>transfer function relating $\Delta \delta$ to $\Delta P_d$ when DGR block is cascaded with AVR</td>
</tr>
<tr>
<td>$G_{DRF}(s)$</td>
<td>transfer function relating $\Delta \delta$ to $\Delta P_d$</td>
</tr>
<tr>
<td>$G_{servo}(s)$</td>
<td>transfer function relating $\Delta \delta$ to $\Delta V_r$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>$G_{TGR}(s)$</td>
<td>transfer function relating $\Delta \delta$ to $\Delta P_d$ when DGR block is cascaded with $K_5$</td>
</tr>
<tr>
<td>$H_g(s)$</td>
<td>transfer function of lag compensator</td>
</tr>
<tr>
<td>$i_{qo}$</td>
<td>armature current, quadrature axis component</td>
</tr>
<tr>
<td>$i_{do}$</td>
<td>armature current, direct axis component</td>
</tr>
<tr>
<td>$I_m$</td>
<td>imaginary part of a complex quantity</td>
</tr>
<tr>
<td>$k$</td>
<td>delay index</td>
</tr>
<tr>
<td>$K$</td>
<td>amplitude of a disturbance signal</td>
</tr>
<tr>
<td>$K_c$</td>
<td>general gain constant</td>
</tr>
<tr>
<td>$K_d$</td>
<td>damping factor of synchronous machine</td>
</tr>
<tr>
<td>$K_e$</td>
<td>gain constant of AVR</td>
</tr>
<tr>
<td>$K_f$</td>
<td>feedback gain constant</td>
</tr>
<tr>
<td>$K_g$</td>
<td>gain constant of lag compensator</td>
</tr>
<tr>
<td>$K_{ge}$</td>
<td>damping factor of a generator</td>
</tr>
<tr>
<td>$K_s$</td>
<td>gain constant of PSS</td>
</tr>
<tr>
<td>$K(t)$</td>
<td>gain vector</td>
</tr>
<tr>
<td>$K_1$</td>
<td>parameter representing a change in electrical torque for a change in rotor angle with constant flux linkages in the d-axis</td>
</tr>
<tr>
<td>$K_2$</td>
<td>parameter representing a change in electrical torque for a change in d-axis flux linkages with constant rotor angle</td>
</tr>
<tr>
<td>$K_3$</td>
<td>parameter representing impedance factor</td>
</tr>
<tr>
<td>$K_4$</td>
<td>parameter representing demagnetising effect of a change in rotor angle</td>
</tr>
<tr>
<td>$K_5$</td>
<td>parameter representing a change in terminal voltage with change in rotor angle for constant $E_q$</td>
</tr>
</tbody>
</table>
$K_6$ parameter representing a change in terminal voltage with change in $E_q$ for constant rotor angle

$L$ lower triangular matrix

$M$ information matrix

$M$ generator constant of inertia

$p$ root of a characteristic equation

$p^*$ complex conjugate of $p$

$\hat{P}$ amplitude of a sinusoidal disturbance

$P_m$ mechanical input power to generator

$P(t)$ covariance matrix

$q^{-1}$ backward shift operator defined as $q^{-1} f(t) = f(t-1)$

t time variable

$T$ superscript, stands for transpose

$T$ a general time constant

$T_a$ time constant of PSS

$T_{do}$ field open circuit time constant

$T_s$ sampling interval

$U$ upper triangular matrix

$u(t), u(z)$ input variable

$v$ set point reference

$V$ busbar voltage

$V_0$ infinite bus voltage

$V_{do}$ armature voltage, direct axis component

$V_{qo}$ armature voltage, quadrature axis component

$V_{t,v_{to}}$ terminal voltage

$w(t)$ stochastic disturbance
\( W \)  
data window width  
\( X \)  
equivalent reactance between \( E \) and \( V \)  
\( X_d' \)  
 transient reactance, direct axis component  
\( X_d \)  
 reactance, direct axis component  
\( X_e \)  
equivalent system reactance  
\( X_L \)  
 line reactance  
\( X_q \)  
 reactance, quadrature axis component  
\( \mathbf{x}(t) \)  
 measurement vector  
\( y(t), y(z) \)  
 output variable  
\( z \)  
 frequency domain operator  
\( \alpha \)  
 factor that characterises a frequency  
\( \hat{\alpha} \)  
 estimate of \( \alpha \)  
\( \delta \)  
 angle of oscillation  
\( \Delta \delta \)  
 change in angle of oscillation  
\( \Delta E_q' \)  
 change in d-axis flux linkages  
\( \Delta P_e \)  
 change in electrical power  
\( \Delta P_d \)  
 oscillatory disturbance power  
\( \Delta T \)  
 change in torque (synchronising and damping)  
\( \Delta T_d \)  
 change in damping torque  
\( \Delta V_r \)  
 change in reference voltage  
\( \Delta V_s \)  
 change in voltage output of PSS  
\( \Delta V_t \)  
 change in terminal voltage  
\( \Delta \omega \)  
 change in generator output angular frequency  
\( e(t) \)  
 prediction error  
\( \phi \)  
 state vector  
\( \lambda \)  
 forgetting factor  
\( \Pi \)  
 product of factors  
\( \phi \)  
 angle of displacement
\( \theta \)
parameter vector

\( \hat{\theta} \)
estimate of \( \theta \)

\( \omega_d \)
angular frequency of a disturbance

\( \omega_f \)
prewarping angular frequency

\( \omega_g \)
natural angular frequency of a generator

\( \omega_n \)
natural angular frequency of oscillation

\( \xi(t) \)
Gaussian noise function

**Abbreviations**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AVR</td>
<td>Automatic Voltage Regulator</td>
</tr>
<tr>
<td>BCSSP</td>
<td>BBC System Simulation Package</td>
</tr>
<tr>
<td>DGR</td>
<td>Dynamic Gain Reduction</td>
</tr>
<tr>
<td>ECS</td>
<td>Excitation Control System</td>
</tr>
<tr>
<td>FLOP</td>
<td>Floating Point Operation</td>
</tr>
<tr>
<td>GMV</td>
<td>Generalised Minimum Variance</td>
</tr>
<tr>
<td>IMP</td>
<td>Internal Model Principle</td>
</tr>
<tr>
<td>MRAC</td>
<td>Model Reference Adaptive Control</td>
</tr>
<tr>
<td>MV</td>
<td>Minimum Variance</td>
</tr>
<tr>
<td>PA</td>
<td>Pole Assignment</td>
</tr>
<tr>
<td>PSS</td>
<td>Power System Stabiliser</td>
</tr>
<tr>
<td>SG</td>
<td>Synchronous Generator</td>
</tr>
<tr>
<td>SM</td>
<td>Synchronous Motor</td>
</tr>
<tr>
<td>STC</td>
<td>Self-Tuning Control</td>
</tr>
<tr>
<td>STR</td>
<td>Self-Tuning Regulator</td>
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A self-tuning scheme for power system disturbance rejection

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Chapter 1

Introduction
Introduction

1.1 Thesis objective

This thesis sets out to provide relevant analysis to explain the phenomenon of amplified low frequency oscillations which appear on interconnected power system tie-lines, despite the presence of 'correctly' tuned Power System Stabilisers (PSS) in the Excitation Control Systems (ECS).

A self-tuning scheme, based on the Internal Model Principle (IMP), is proposed for counteracting these low frequency oscillations and for resolving the conflict between the requirements for transient stability and those for dynamic stability. The proposed scheme involves the development of an efficient estimation model which makes use of some properties of the Cholesky decomposition techniques. Existing Dynamic Gain Reduction (DGR) measures are examined and a new approach to providing such measures is also proposed.

1.2 Background to low frequency oscillations in a power system

In an interconnected power system, individual or groups of generating plants are connected together via tie-lines, all sharing in the supply of electrical energy to various load centres. To maximise the efficiency of operation of an interconnected power system while providing the ability to respond to contingencies, it is usual to schedule operation of all generators such that the total variable costs (mostly fuel and operation and maintenance costs) are
demand balance only on higher cost generators. A limitation on the
ability to operate a given low-cost generator will therefore cause the
overall generation costs to rise, with poor utilisation of that plant.

Such limitations may be caused by relatively weak transmission tie-
lines to the system, which may require that an operational margin
of transmission capability be retained to ensure that synchronous
stability is maintained following large disturbances (i.e. transient
stability) or to prevent the occurrence of poorly damped or
sustained low frequency oscillations which may be excited by small
changes such as fluctuating load or system switching (steady-state
stability). Weak ties can occur either through delays in construction
of needed transmission augmentations; temporarily as a result of
loss of parallel lines due to maintenance or fault; or as a deliberate
planning alternative, such that the kinds of control measures that
are addressed in this thesis are relied upon to effect a substantial
saving in the cost of transmission works.

The alternative to adequate control of damping, with attention also
to transient response, may therefore be costly restrictions on the
operating power range and flexibility of generators. In an effort to
reduce the effects of these low frequency oscillations and extend
the dynamic stability range of the power system, additional control
equipment, in the form of PSS, are commonly installed.

Low frequency oscillations that are of concern to the dynamic
stability of a power system occur in the frequency range of
approximately 0.2 to 2.5 Hz [1]. Within this range, there are two
dominant troublesome modes. These are referred to as the local
dominant troublesome modes. These are referred to as the local mode and the inter-tie mode. The local mode results from the oscillation of synchronous generators at one location against the rest of the system producing an oscillatory signal which is reflected on the power output of the generator. Typically, this mode of oscillation occurs at the upper end of the frequency range mentioned above.

In an interconnected network, various groups of synchronous generators may be connected over long tie-lines. Oscillations of groups of generators against other groups are reflected in power angle oscillations and hence in the power flow over the tie-lines. This gives rise to the inter-tie mode of oscillation [2]. Typically, these oscillations occupy the lower end of the frequency range of interest due to the effective larger inertia value.

Under steady state conditions, the low frequency oscillations described above do not constitute a hazard to the dynamic stability of the power system. However, when a disturbance such as a sudden change in system configuration or power demand occurs due to, perhaps, the switching on or off of a power line or load, these oscillations may grow to unacceptable levels if the damping in the power system is inadequate. In these circumstances, additional damping is provided by a PSS through a supplementary control loop in the ECS. A positive damping torque signal, derived from either the frequency of the generator output or the speed of the rotor or the instantaneous generator power or a combination of any 2 of these 3 signals, with adequate gain and phase advance is injected at the summing junction where the AVR is connected.
While all generators in the system may be affected by tie-line oscillations, not all generators may be located in situations where damping of the oscillations can be influenced by control measures. This applies particularly to generators that are relatively weakly connected to the bulk of the system. In this case the aim of control cannot be to eliminate the forced tie-line oscillations but to desensitise the remote generator to it.

Use of any of the signals described above involves some disadvantages. The signal derived from the instantaneous generator power is used as a proxy for accelerating power, but does not take account of mechanical power variations, while the speed-derived one is subject to interference from the torsional modes of oscillation. The frequency-derived signal is associated with a fair level of noise. The choice of the feedback signal or signals, to some extent, depends on the torsional modes and is a matter of experience gained with the use of the particular signal or signals. In this thesis, the feedback signal employed is frequency-derived and is therefore based on the time derivative of the change in the angle of oscillation $\Delta \delta$.

1.3 A brief perspective of the low frequency oscillation problem

In a multi-machine environment, numerous low frequency modes are present and special care is taken to adjust PSS parameters so as to damp out these oscillations over a wide frequency range. Correct PSS parameter settings ensure a flat gain characteristic of the transfer function that reflects the effect of disturbances on
power angle oscillations \((G_{\text{DRF}}(s))\), around the frequency corresponding to the local mode. While flattening the \(|G_{\text{DRF}}(j\omega)|\) peak due to the local mode, the derivative supplementary excitation feedback loop, involving the PSS, can create another peak at the lower frequency range which may match oscillatory tie-line disturbances, thereby causing amplified oscillations.

For weakly connected systems, there is the additional detrimental effects of negative damping which is introduced through the voltage reset signal at the AVR summing junction. The AVR usually has a relatively high gain for field-forcing action and a high speed of response for coping with rapid voltage changes. The negative damping signals are therefore amplified by the AVR dynamics and may contribute to the destabilisation of the dynamic performance of the ECS.

1.4 Thesis review and organization

The thesis first seeks an explanation to the phenomenon of amplified tie-line low frequency oscillations in a power system despite the presence of 'correctly' tuned PSS in the ECS. In chapter 2, a frequency domain approach is used to analyse these oscillations. The gain characteristics of a closed loop transfer function \(G_{\text{DRF}}(s) = \Delta\delta/\Delta P_d\), which reflects the effect of disturbances on power angle oscillations, is shown to be crucial in identifying frequency ranges where amplified oscillatory disturbances can occur. It is shown that tuning power system stabilisers (PSS) based on eigenvalue analysis lacks the insight gained from the explicit evaluation of \(G_{\text{DRF}}(s)\) gain response. The adopted approach
identifies a risky situation which could not be detected from an eigenvalue pattern.

Chapter 3 presents a new method of decoupling negative damping oscillations without compromising the benevolent effects of other control loops in the ECS. A notch filter, acting as an on/off switch, located in the forward path of the negative damping signal, provides the necessary Dynamic Gain Reduction (DGR) for "switching off" the detrimental signals. A restructuring of the transfer blocks in the ECS that allows the decoupling of the negative damping signals is proposed. It is shown that, while a considerable reduction in the tie-line mode oscillations can be achieved, the phenomenon of amplified oscillations analysed in chapter 2 can still occur, though at a reduced magnitude.

A different approach for coordinating the actions of the PSS and the AVR while decoupling negative signals is next explored.

Disturbing frequencies can drift with operating conditions. A self-tuning regulator (STR) is therefore proposed in chapter 6. But, since the STR requires on-line estimation of the system parameters, an efficient recursive algorithm for the estimation routine is developed in chapter 4. The algorithm makes use of some properties of the Cholesky matrix decomposition techniques. The numerical stability of the estimation algorithm is improved through the updating of the triangular factors of an augmented information matrix. A fixed width data window, rather than a forgetting factor, is used to cater for parameter drifts.
One of the main contributions of the present work is the development of an adaptive scheme for coordinating power system stabilising devices in an ECS. In chapter 5, the mathematical background necessary for such development is presented. First, a description of adaptive control systems from a historical perspective is given. A specific approach which leads to a concrete rule for the development of various controller algorithms is then described. This necessitates the development of a predictive model. The minimum variance algorithm which is the basis for many other self-tuning schemes is then presented and is followed by the generalised minimum variance algorithm. An alternative and equivalent scheme, i.e. the pole assignment control, is briefly summarised for the sake of completeness. Finally, a general and flexible control structure is developed.

A Self-tuning Control (STC) scheme is proposed in chapter 6 which is able to provide both positive damping characteristics and disturbance rejection. A control strategy based on the Internal Model Principle (IMP) is used to directly cancel tie-line oscillatory disturbances. Chapter 6 describes the IMP and its implementation in a power system for disturbance rejection purposes. Simulation studies performed on a typical ECS model, demonstrating clearly the benefits of incorporating the IMP in an adaptive power stabilising scheme, are presented. Results show effective cancellation of tie-line time-varying oscillatory disturbances and enhanced damping behaviour of the power system.

Conclusions are presented in chapter 7.
1.5 Original contributions of this thesis

The original contributions of the present work in evolving this thesis are summarised below, with relevant author's publications given in square brackets and fully referenced in Appendix V.

* A frequency domain analysis is used to explain the phenomenon of amplified low frequency oscillations which appear on power system tie-lines despite the presence of 'correctly' tuned PSS in the ECS. This is achieved through the frequency domain analysis of forced low frequency oscillations in a power system [R1-R2].

* A novel approach for decoupling negative damping signals in a power system is proposed and is shown to have distinct advantages over current methods of implementation. The proposed scheme uses a DGR block cascaded with K₅ [R3].

* An efficient recursive algorithm for the estimation of system parameters is developed. The numerical stability of the estimation algorithm is improved through the updating of the triangular factors of an augmented information matrix. A fixed width data window is used to cater for parameter drifts [R4].

* The IMP is incorporated in a STR scheme which is able to provide positive damping characteristics and disturbance rejection. The conflict between the requirements for transient stability and those for dynamic stability in a power system is resolved [R5].
Chapter 2

Frequency domain analysis of low frequency oscillations in power systems
Abstract

A frequency domain approach is used to analyse power system forced low frequency oscillations. The gain characteristics of a closed loop transfer function $G_{DRF}(s) = \Delta \delta / \Delta P_d$, which reflects the effect of disturbances on power angle oscillations, is shown to be crucial in identifying frequency ranges where amplified oscillatory disturbances can occur. It is shown that tuning Power System Stabilisers (PSS) based on eigenvalue analysis lacks the insight gained from the explicit evaluation of $G_{DRF}(s)$ gain response. The adopted approach identifies a risky situation which could not be detected on an eigenvalue pattern. For weakly connected systems, the inherent negative damping effects along with the PSS derivative feedback action result in a situation where a PSS tuned to locate the eigenvalues corresponding to the natural mode $\omega_n$ at well damped locations, can still cause a hazardous situation. While flattening the $|G_{DRF}(j\omega)|$ peak at $\omega_n$, the derivative supplementary excitation feedback loops can create another peak which may match oscillatory tie-line disturbances, thereby causing amplified oscillations.

2.1 Introduction

Low frequency oscillations occurring in a power system may be short-lived or may persist for long periods, depending on the level of damping in the system. Sustained or increasing oscillations may impose severe limitations on the power transfer capability of the system. It is important that damping signals be introduced to counteract these oscillations. PSS are thus used to modulate the
generator excitation output to produce a component of electrical torque on the synchronous machine rotor, which is in phase with the rotor speed variations [1].

The damping requirements for satisfactory system performance are based on analytical studies. These studies fall under the umbrella of power system dynamic stability. They are undertaken, for instance, to identify troublesome modes of oscillations; develop rules for determining which generators in a system should be equipped with stabilisers; determine the general characteristics of those stabilisers and decide on tentative stabiliser and regulator parameter settings [3]. Different methods of analysis have been used by various investigators in these studies.

The Routh-Hurwitz criterion tests stability by examining the sign of the real part of the roots of the denominator polynomial of a transfer function. The system is deemed unstable if any root has a positive real part. This method has been used by investigators such as Messerle [4], Grove [5] and Booth et al [6]. Whilst it gives a simple means of determining asymptotic stability, its main disadvantage is that it gives no indication of the degree of stability of the system, since no information is available on how the poles of the transfer function move with variations in gain constants.

A method which is capable of indicating how the stability of a system relates to the variations in gain constants is the root-locus method. This method is quick to implement and involves plotting the poles and zeros of a transfer function on an Argand diagram. By varying the value of the gain constants, the migratory paths of the
poles and zeros are charted. Poles tending towards the right-hand plane of the diagram lead to an unstable system. This method has been employed successfully by Bollinger et al [7] and Oradat et al [8] in evaluating parameter settings for PSS. Stapleton [9] used this method to investigate the effect of voltage parameter settings on the stability and dynamic response of an Excitation Control System (ECS). One drawback with this method is the presence of an infinite number of poles and zeros, which makes the establishment of any rules somewhat clumsy.

The Nyquist criterion gives a test for the right-half plane poles of a closed-loop system. Its advantage is that stability considerations can be performed on a closed-loop system based on frequency response data. Given a closed-loop system with forward path transfer function $AG(s)$ and unity negative feedback, a necessary and sufficient condition for asymptotic stability is that the map of the Nyquist contour on the $G(s)$-plane, corresponding to a clockwise traverse, should make a number of anticlockwise encirclements of the point $(-1/A, 0)$ equal to the number of poles possessed by $G(s)$ in the right-half plane. The Nyquist contour is a path in the s-plane consisting of a segment of the imaginary axis, from $-jR$ to $jR$, together with a semicircle of radius $R$ in the right-half plane to close the contour. Where there are poles on the imaginary axis, the contour is indented so as to avoid them [10].

This method has been used by Ewart et al [11] and Aldred et al [12] for determining the stability of synchronous machines. It is capable of assessing the degree of stability in the system and is useful for
practical compensation studies [13]. However, it involves extensive computation.

Messerle [4] and Heffron et al [14] used the analogue computation approach to determine stability. This approach is based on the realisation of an electric circuit using adders, multipliers and integrators or differentiators to represent the dynamical system under investigation. This hardware set-up essentially solves a differential equation which reflects the characteristics of the dynamical system. The coefficients of the equation can be adjusted for optimum output using potentiometers. With this approach, close to real system performance can be observed. However, it is a lengthy process and is time-consuming.

A method which is well suited for the selection of the parameter values of stabilising devices is the domain separation one, which was developed by Venikov [15]. The method rests on the establishment of the characteristic equation of a system and its separation into real and imaginary parts, explicitly in terms of the parameters under investigation, as functions of frequency and damping. Two parameters are studied at a time. These are plotted as a function of frequency for particular values of damping in a coordinate system with the two parameters as axes. It is then possible to establish the optimum values for these parameters based on curves which allow a separation of improved damping domain from a domain with worse damping. Peneder et al [16] have applied this method for selecting optimum values for regulator parameters. Among other users of this method have been Yu et al [17] and Surana et al [18].
Eigenvalue analysis is, by far, the most common method used for large interconnected power systems and lends itself readily to digital computer applications. Eigenvalues are roots of the characteristic polynomial of a system and as such characterise its performance. They may be real numbers or, more often, complex ones for dynamical systems. The real part conveys information about the amount of damping, while the imaginary part is related to the natural frequency of oscillation of the corresponding mode [19]. Eigenvalues are functions of the design and control parameters of the system so eigenvalue sensitivity analysis, whereby eigenpatterns are mapped out in the s-domain for small changes in parameter values, give a good indication of the degree of stability of the system. Obata et al [20], Martins [21] and Ajjarapu [22] are among many investigators who have used this approach in studies on power system dynamic stability.

2.2 Frequency domain analysis based on transfer function evaluation

The methods of analysis described above aim at establishing relative stability measures for control purposes. Whilst classical control methods rely on phase and gain margins, modern control design aim at locating the eigenvalues at well damped positions. Forced low frequency oscillations in power system analysis however, necessitate further criteria. External oscillatory disturbances have frequencies which may be far from the crossover frequency of the control loop. Relative stability considerations do not provide sufficient information as to whether these external disturbances would be properly attenuated. The frequency of the tie-line mode is
usually lower than that of the local mode, and so it is important to explicitly evaluate the gain of the transfer function \( G_{\text{DRF}}(s) = \Delta \delta / \Delta P_d \), which reflects the effect of disturbances on power angle oscillations, in the frequency range of interest. The gain characteristics are dependent on the zeros as well as the poles, and so eigenvalues (the poles) do not provide sufficient insight.

The eigenvalues that correspond to the natural local modes are depicted on \( |G_{\text{DRF}}(j\omega)| \) versus frequency graphs as peaks with a magnitude which is dependent on the relative damping ratio \( \xi \). Typically, for a unit not equipped with a PSS, the graph has only one peak corresponding to the natural mode. The main motivation for using PSS is to reduce this peak by improving the damping factor. The supplementary PSS control loop modifies \( G_{\text{DRF}}(s) \) by introducing additional poles and zeros which reshape the magnitude of the frequency response. Due to typical design of PSS, the additional inserted dynamics, while reducing the magnitude of the natural mode peak, may produce another peak at a lower frequency range.

A control loop properly designed in terms of relative stability measures (e.g. gain and phase margins), can still amplify oscillatory disturbances with a frequency far from the crossover point. In section 2.4, it is shown that the derivative feedback incorporated in the PSS control loop introduces zeros in the transfer function \( G_{\text{DRF}}(s) \) which amplifies the gain characteristics in the range of frequencies below the natural mode. Incautious design of PSS can thus contribute to amplifying external forced oscillations. There may be potentially risky consequences in the following situations:
(i) when the source of oscillation is a large disturbance, such as a mine hoist, with a frequency lower than or close to the natural frequency of a nearby generator [23].

(ii) when the frequency of the inter-tie mode is usually lower than the natural (local) mode in multi-area interconnected power systems [24-25].

The effects of low frequency oscillations on a model power system are analysed in the following sections. The unregulated machine is considered first, in section 2.3, in order to demonstrate the amplification effect without the complexity of the excitation system. The effects of the excitation control loops are taken into account in section 2.4. Simulation results are presented in section 2.5, followed by conclusions in section 2.6. Generalisation to multimachine networks is straightforward. In Appendix I, a systematic derivation of a polynomial matrix is described which evaluates disturbance rejection/attenuation functions in a general multimachine environment.

2.3 Forced oscillation model for an unregulated machine

A linearised model is used in the analysis of low frequency oscillations to describe the machine dynamics and the associated excitation control loops. Fig. 2.1 shows the standard excitation control model used for the analysis of low frequency oscillations. Modelling details of the transfer blocks and a description of the differential constants are given in Appendix I.
In this section, an unregulated machine model ($\Delta P_e = 0$) is first considered. The results are generalised in section 2.4 to account for the effects of the AVR/PSS control loops.

Consider an unregulated synchronous generator supplying power to infinite bus bars, B, through a reactance $X_L$, as shown by Fig. 2.2. The generator dynamics is a transfer function description of the non-linear swing equation given by:
\[ M \frac{d^2 \delta}{dt^2} + K_d \frac{d \delta}{dt} = P_m - \frac{EV}{X} \sin \delta \]  

(2.1)

where \( M \) is the generator constant of inertia
- \( K_d \) is the damping factor
- \( P_m \) is the mechanical input to the generator
- \( E \angle \delta_g \) is the generator internal voltage
- \( V \angle \delta_1 \) is the busbar voltage phasor
- \( X \) is the equivalent reactance between \( E \) and \( V \)
- \( \delta = \delta_g - \delta_1 \) is the load angle between the internal voltage of the generator and the busbar voltage

The power/load angle characteristics of the machine are given in Fig. 2.3.

![Fig. 2.3 Power/Load angle characteristics of synchronous generator.](image)

Fig. 2.3 Power/Load angle characteristics of synchronous generator.
Considering an operating load angle $\delta_0$ and small oscillations producing changes $\Delta P_e$ in electrical power and $\Delta \delta$ in load angle, equation (2.1) can be linearised to give

$$\frac{d^2 (\Delta \delta)}{dt^2} + K_{ge} \frac{d(\Delta \delta)}{dt} = -\left(\frac{EV}{XM \cos \delta_0}\right) \Delta \delta \tag{2.2}$$

where

$$K_{ge} = \frac{K_d}{M} = 2 \xi \omega_n$$

Suppose the synchronous generator were subjected to an oscillatory disturbance $\Delta P_d$, as depicted by the block diagram of Fig. 2.4.

![Fig. 2.4 Synchronous generator subjected to a disturbance](image)

Let $\Delta P_d (t) = K \sin (\omega_d t + \phi) \tag{2.3}$

Equation (2.2) then becomes
\[
\frac{d^2 (\Delta \delta)}{dt^2} + K_{ge} \frac{d(\Delta \delta)}{dt} + \left( \frac{EV}{XM} \cos \delta_0 \right) \Delta \delta = K \sin (\omega_d t + \phi) \tag{2.4}
\]

or

\[
(D^2 + K_{ge} D + \omega_g^2) \Delta \delta = K \sin (\omega_d t + \phi) \tag{2.5}
\]

where

the differential operator \( D = \frac{d}{dt} \)

and

\[
\omega_g^2 = \frac{EV}{XM} \cos \delta_0
\]

Equation (2.5) will be used to demonstrate the effect of the ratio \( \omega_d / \omega_g \) on the magnitude of the generator oscillation. A useful interpretation of this equation is to consider \( \Delta \delta \) as the output of a dynamic system \( G_{DRF}(s) \) injected with a sinusoidal signal of magnitude \( K \) and frequency \( \omega_d \). \( G_{DRF}(s) \) is, therefore a second order all-pole model given by:

\[
G_{DRF}(s) = \frac{1}{(s+p)(s+p^*)} \tag{2.6}
\]

where

\[
p = -K_{ge} \pm \frac{\sqrt{K_{ge}^2 - 4 \omega_g^2}}{2}
\]
Fig. 2.5 shows the location of the poles in the s-domain. The gain of the function is given by:

\[
\text{Gain} = \frac{1}{|A| |B|}
\]

where A and B are vectors shown in Fig. 2.5

![Diagram showing s-domain poles of GDRF(s)]

(a) \( \omega_d > \omega_g \)  
(b) \( \omega_d < \omega_g \)

Fig. 2.5 s-domain poles of \( G_{DRF}(s) \)

Fig. 2.5a depicts the case where \( \omega_d > \omega_g \). For negligible generator damping:

\[ \text{Im} \{p\} \approx \omega_g \]
where \( \text{Im} \{ \} \) stands for the imaginary part of a complex quantity. Thus \( \omega_d < \frac{\sqrt{4 \omega_g^2 - K_{ge}^2}}{2} \) represents the case where the frequency of the oscillatory disturbance is less than that of the generator. Comparing Fig. 2.5a with 2.5b, it can be seen that the gain of the function decreases as the ratio \( \omega_d / \omega_g \) increases beyond unity. A critical condition arises when \( \omega_d \) approaches the value of \( \omega_g \). This produces a small value of \( |A| \cdot |B|\) thus causing amplification of the generator oscillation and endangering the system stability.

An alternative way of describing the effect of the disturbance is to consider the frequency response of the generator unit.

From equation (2.5), it follows that:

\[
\frac{\Delta \delta(s)}{\Delta P_d(s)} = \frac{1}{s^2 + K_{ge} s + \omega_g^2} \tag{2.7}
\]

For a sinusoidal disturbance of frequency \( \omega_d \), equation (2.7) becomes

\[
\frac{\Delta \delta(j\omega)}{\Delta P_d(j\omega)} = \frac{1}{\omega_g^2 - \omega_d^2 + jK_{ge}\omega_d} \tag{2.8}
\]

It can be seen, from equation (2.8), that the magnitude of the response, \( \frac{\Delta \delta(j\omega)}{\Delta P_d(j\omega)} \), will increase for the case when the disturbing frequency approaches that of the generator unit. The situation is aggravated for frequencies in the lower range and for low values of \( K_{ge} \).
2.3.1 Numerical example of frequency amplification in a power system

A numerical example is used to demonstrate the effect of the frequency characteristics on the generator angle of swing. In this example, an unregulated synchronous generator SG, with natural frequency $\omega_g$ is connected to a larger system via transmission lines. This is shown in Fig. 2.6 where B1 is a robust busbar supplying a large synchronous motor SM.

![Fig. 2.6 A sample power system](image)

The special, but often encountered, power system configuration shown in Fig. 2.6 leads to an identifiable problem related to low frequency oscillations experienced by generator SG. Sudden impact loading on the motor initiates low frequency oscillations which are reflected on busbar B1 producing oscillations in the synchronous generator [23].
A low frequency oscillation model for the time domain simulation of
the power system in Fig. 2.6, may be developed from equation (2.4).
The corresponding block diagram representation for simulation
using the BBC simulation package BCSSP [26] is shown in Fig. 2.7
(integrators are shown as blocks with integral signs).

![Block Diagram](image)

**Fig. 2.7** Block diagram representation of equation (2.4)
for amplified oscillation studies

Time domain simulations were obtained for three different
disturbances with frequencies of $\omega_m = 3.7$, $2\pi$ and 12 rad/sec
respectively. The disturbance level $K$ and the generator natural
frequency $\omega_g$ were fixed for the three cases (at $K = 3.95$ and
$\omega_g=2\pi$). The generator damping factor $\xi = K_{ge} / 2\omega_g$ was assumed to
be 0.15 to demonstrate the effect of poorly damped generator
oscillations.
Fig. 2.8 shows the simulated generator power angle oscillations for the three cases under consideration. These results illustrate the effect of a forced disturbance on the magnitude of the power angle oscillations. Three different magnitudes of oscillation were obtained for the same disturbance level. In one case this magnitude was almost three times as high as the other, which emphasises the role of the ratio $\omega_m/\omega_g$. At a frequency $\omega = \omega_g$ the function $|G_{DRF}(j\omega)|$ exhibits a peak value, so a forced excitation with a frequency $\omega_m = \omega_g$ would naturally be amplified.
The generator oscillation $\Delta \delta$ can be modelled as the output of a filter with a transfer function given by equation (2.6) and injected with a sinusoidal disturbance. For negligible generator damping, using Laplace transform analysis, a forced oscillation with a frequency $\omega_m = \omega_g = \omega_n$ results in:

$$\Delta \delta (s) = \frac{K}{(s^2 + \omega_n^2)^2}$$

producing an unstable time response given by

$$\Delta \delta (t) = a \sin \omega_n t + b t \cos \omega_n t$$

where $a$ and $b$ are time independent constants.

2.4. Effects of excitation control loops

The analysis in section 2.3.1 emphasises the effect of the frequency response of $G_{DRF}(s)$ on the amplitude of the forced low frequency oscillations. The all-pole transfer function given by equation (2.4) ignores the excitation control loops. When the effects of the automatic voltage regulator (AVR) and the PSS are included, the result is a higher order transfer function with a number of additional poles and zeros depending on modelling details.

The design of PSS aims to flatten the peak at the natural frequency $\omega_n$ and thereby reduce the risk of resonance-like amplification.
However, the additional zeros are typically located far from the origin of the $s$ domain, giving an amplified frequency response in a lower frequency range. In this section, it is shown that another peak may be created, depending on the type of control apparatus used and their associated parameter settings. A serious situation may arise in interconnected systems if the tie-line power oscillation frequency lies close to the frequency of the newly created peak. To illustrate this phenomenon, one particular PSS configuration is considered. However other excitation control configurations are likely to exhibit similar characteristics for specific parameter settings.

In Fig. 2.1, the supplementary excitation signal generated by the PSS is given by:

$$\Delta V_s = \frac{K_s s^2}{T_a s + 1} \Delta \delta$$  \hspace{1cm} (2.9)

For low frequency oscillations produced by a disturbance $\Delta P_d$, the change in the swing angle $\Delta \delta$ may be modelled as follows:

$$\Delta \delta (s) = G_{DRF}(s) \Delta P_d (s)$$  \hspace{1cm} (2.10)

where

$$G_{DRF}(s) = \frac{C(s)}{A(s)} = \frac{C_0 s^2 + C_1 s + C_2}{A_0 s^4 + A_1 s^3 + A_2 s^2 + A_3 s + A_4}$$  \hspace{1cm} (2.11)

$$A_0 = K_3 T_d' \frac{T_o}{T_a} M$$
\[ A_1 = K_3 \, T_{do} \cdot M + (1 + K_e K_3 K_6) \, T_a \cdot M + K_d K_3 \, T_a \cdot T_{do} \]

\[ A_2 = (1 + K_e K_3 K_6) \, M + K_d K_3 \, T_{do} + (1 + K_e K_3 K_6) \, K_d \, T_a + K_1 K_3 \, T_a \cdot T_{do} + K_e K_2 K_3 \]

\[ A_3 = (1 + K_e K_3 K_6) \, K_d + K_1 K_3 \, T_{do} + (1 + K_e K_3 K_6) \, K_1 \, T_a - K_2 K_3 \, T_a \cdot (K_4 + K_e K_5) \]

\[ A_4 = (1 + K_e K_3 K_6) \, K_1 - K_2 K_3 \, (K_4 + K_e K_5) \]

\[ C_0 = K_3 \, T_{do} \cdot T_a \]

\[ C_1 = (1 + K_e K_3 K_6) \, T_a + K_3 \, T_{do} \]

\[ C_2 = 1 + K_e K_3 K_6 \]

Block diagram manipulation, as shown in Appendix I, results in equation (2.11) for the case when the flux decay transfer function is represented by a first order delay having a gain of \( K_3 \) and time constant \( K_3 \, T_{do} \). The generating unit dynamics are described by the following characteristic equation:

\[ M s^2 + K_d s + K_1 = 0 \quad (2.12) \]

\( K_1 \) in the above equation is used instead of \( M \omega_n^2 \) to maintain consistency with the literature. At this stage the AVR block in
Fig. 2.1 is modelled by a dynamic gain constant $K_e$. The variation of $K_e$ with frequency will be accounted for later. The shape of the frequency characteristics is determined by the pole/zero locations. Eigenvalue analysis is used to determine the eigenvalue (pole) pattern for a specific control parameter setting. A common practice is to check the pole locations for relative stability considerations. Control apparatus are then tuned to confine the critical poles at well damped locations. The location of the poles is an essential criterion for assessing the relative stability of the system but it is the complete pole/zero pattern which determines the positions of the peaks and troughs of the frequency response amplitude.

In interconnected systems the low frequency oscillations experienced by a unit may be initiated by other modes from neighbouring units, through a tie-line power disturbance $\Delta P_d$ (see Fig. 2.1). In practice $\Delta P_d$ consists of sinusoidal components (for example $a_i \sin \omega_i t$, say) with a known frequency range ($0.4 \pi \leq \omega_i t \leq 5\pi$) [1]. A power disturbance $\Delta P_d = a_i \sin \omega_i t$ would result in a swing oscillation amplitude $|\Delta \delta| = a_i |G_{DRF}(j\omega_i)|$. To avoid the possibility of amplified oscillations, it is necessary to reduce the gain $|G_{DRF}(j\omega_i)|$.

It is important to note that the eigenvalue analysis based on the dynamics of a local unit cannot detect the amplified frequency range. The amplification effect is dependent on the zeros of the system, while eigenvalue analysis is solely dependent on the poles. Even a multi-machine eigenvalue pattern can only detect amplification due to resonance arising from modes located close to each other. To identify the amplified frequency range, explicit
evaluation of the amplitude response $|G_{\text{DRF}}(j\omega)|$ for all the involved units based on both pole and zero pattern is necessary. An incident was reported in reference [27] where a system became unstable at a frequency lower than the natural mode. Eigenvalue analysis based on the worst case system conditions failed to detect or explain it.

Comparing the poles/zeros of the transfer function given by equation (2.11) to those of the unregulated machine model of equation (2.4), it can be seen that the excitation control loop introduces additional poles and zeros in $G_{\text{DRF}}(s)$. A further insight can be gained through the analysis of the polynomial $C(s)$ which describes the zeros. As indicated by equation (2.11),

$$C(s) = K_3 T_d' T_a s^2 + [ (1 + K_e K_3 K_6 ) T_a + K_3 T_d' ] s + 1 + K_e K_3 K_6$$

(2.13)

A time constant $T$ and a gain constant $K_c$ are defined as follows:

$$T = \frac{K_3 T_d'}{1 + K_e K_3 K_6}$$

(2.14)

$$K_c = 1 + K_e K_3 K_6$$

(2.15)

Using these definitions along with equation (2.13) gives

$$C(s) = K_c (1 + T s) (1 + T_a s)$$
The gain characteristics of $G_{DRF}(s)$ can be estimated from the pole/zero pattern in a similar way to that shown in Fig. 2.5. Because of the additional poles and zeros introduced, the gain characteristics satisfy the condition:

$$|G_{DRF}(j\omega)| \propto |j\omega - \frac{1}{T}| |j\omega - \frac{1}{T_a}|$$

where $\propto$ stands for proportionality.

Typical values of PSS parameters indicate that the contribution of the vectors to the magnitude of $|G_{DRF}(j\omega)|$ due to the pole locations, does not significantly change at the lower frequency range. For the range of frequency, where $\omega < \text{Min}\left[\frac{1}{T}, \frac{1}{T_a}\right]$ the following approximation can be made:

$$|j\omega - \frac{1}{T}| |j\omega - \frac{1}{T_a}| \approx \frac{1}{TT_a}$$

This represents a considerable amplification for lower values of time constants $T$ and $T_a$. The PSS time constant $T_a$ is usually given a small value to produce enough phase advance to counteract the delay associated with the excitation system. The amplification introduced in the lower frequency range is also attributed to higher AVR gain which results in a smaller time constant $T$ as indicated by equation (2.14).
2.5 Simulation results

In this section, the effects of the PSS gain and time constant $T_a$, as well as the AVR gain constant $K_e$, on the lower frequency range of $|G_{DRF}(s)|$ are demonstrated. Simulation experiments were performed on the ECS model of Fig. 2.1. The parameters of the machine and its associated controls are given in Appendix I. Parameter sensitivity experiments were based on the system represented by equation (2.11). Both gain and phase characteristics of the frequency response were obtained for the set of coefficients given in Table 2.1 below.

Table 2.1 Coefficients of transfer function of equation (2.11)

<table>
<thead>
<tr>
<th>Parameter Value</th>
<th>$C_0$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$A_0$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
</tr>
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<tbody>
<tr>
<td>$T_a = 0.15$</td>
<td>0.27</td>
<td>2.5</td>
<td>4.8</td>
<td>0.0022</td>
<td>0.023</td>
<td>0.61</td>
<td>2.6</td>
<td>5.2</td>
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<td>0.017</td>
<td>0.42</td>
<td>2.1</td>
<td>5.2</td>
</tr>
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<td>0.017</td>
<td>0.7</td>
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<td>0.017</td>
<td>0.42</td>
<td>2.1</td>
<td>5.2</td>
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<tr>
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<td>0.016</td>
<td>0.24</td>
<td>2.0</td>
<td>2.3</td>
</tr>
<tr>
<td>$K_e = 5$</td>
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<td>1.9</td>
<td>1.8</td>
<td>0.00072</td>
<td>0.016</td>
<td>0.18</td>
<td>1.9</td>
<td>1.4</td>
</tr>
</tbody>
</table>
Only one parameter at a time was changed. The other parameters as given in Appendix I, remained unchanged.

(a) $G_{DRF}(s)$ Gain characteristics

(b) $G_{DRF}(s)$- Phase characteristics

Fig. 2.9. Implication of PSS time constant settings on forced disturbances
Fig. 2.9a shows the $G_{DRF}(s)$ frequency response for the given set of parameters and for different values of the PSS time constant $T_a$. An eigenvalue (pole) based analysis would indicate that $T_a = 0.05$ second is a satisfactory value because a substantial improvement is obtained for the natural mode of oscillation. The explicit evaluation of the $G_{DRF}(s)$ frequency response clearly identifies a serious problem which the eigenvalue analysis fails to detect. The gain at the lower frequency range is considerably amplified. The magnitude of the oscillations due to the natural mode is reduced, but external oscillatory disturbances with frequencies around 4 rad/sec, can create a serious problem. A further improvement of the natural mode using an eigenvalue pattern (a further shift to the left of the s domain) would create a higher peak at the lower frequency range.

Fig. 2.9b indicates that the phase angle of $G_{DRF}(s)$ at the natural frequency of oscillation is about $90^0$ for a correctly tuned stabiliser. This ensures the phase advance angle, introduced by the PSS counteracts the inherent system delay so that the supplementary signal $\Delta V_s$ produces a power signal $\Delta P_e$ (see Fig.2.1) in phase with $d(\Delta \delta)/dt$. This stabilising effect however, ceases at the lower frequency range since the angle $\angle G_{DRF}(j\omega)$ is considerably reduced.
Fig. 2.10 $G_{DRF}(s)$ as affected by stabiliser gain
A satisfactory PSS design should aim to reduce the gain \(|G_{\text{DRF}}(s)\)| for the whole range of expected low frequency oscillations. Further simulations have been conducted to investigate the role of the stabiliser gain \(K_s\).

The results of these are shown in Fig. 2.10 It can be seen that adjusting \(K_s\) to improve the natural mode of oscillation would have a similar detrimental effect on the forced oscillations in the lower frequency range. Varying \(T_a\) and/or \(K_s\) would seem only to provide a compromise. Extra flexibility is obtained by coordinating the tuning of both AVR and PSS.

In equation (2.11), \(K_s\), \(T_a\) and \(K_e\) are the only parameters available to the designer to shape the \(G_{\text{DRF}}(s)\) frequency response. The other parameters are either machine, network or load dependent. \(K_s\) and \(T_a\) are frequency independent constants but the dynamic gain constant \(K_e\) varies with frequency. The AVR includes compensating circuits which determine its frequency characteristics. This extra degree of freedom can be utilised in the design. A set of graphs showing the role of \(K_e\) is included in Fig. 2.11.
Fig. 2.11 G_{DRF}(s) as affected by AVR dynamic gain
The results depicted in Fig. 2.11 are based on different fixed values of $K_e$. $K_e = 50$ is a suitable value for the higher frequency range but it has undesirable effects at the lower frequency range. A lower value would however produce the opposite effect. It is possible to design the AVR compensating circuits to provide the necessary dynamic gain adjustment so it would appear that a promising solution can be achieved. The value of $K_e$ in the lower frequency region should not be set below the limit that would result in deterioration in voltage regulation accuracy [28].

Similar effects can be obtained by cascading the PSS with a compensator to provide dynamic gain adjustment to the PSS loop. As indicated in Fig. 2.10, lower values of $K_s$ are more suitable for the lower frequency range. The design of the reset block (washout) should not have any effect on the PSS dynamic gain in the lower frequency oscillation range but if necessary the design can be modified to achieve some dynamic gain reduction. In the next chapter, the effect of various dynamic gain reduction measures for the AVR loop on dynamic and transient behaviour of the ECS is presented.

2.6 Conclusions

Various methods of analysis in the study of power system dynamic stability have been reviewed. A phenomenon pertaining to the amplification of low frequency oscillations in a power system has been identified. First, this phenomenon has been explained by studying the interaction of the natural frequency of a relatively
small synchronous generator with that of a large synchronous motor in a system, where the two machines are connected to a large network via transmission lines. The excitation control loops of the generator were then included in a frequency domain analysis, based on the evaluation of the gain of the transfer function $G_{DF}(s) = \Delta \delta / \Delta P_d$, which reflects the effect of disturbances on power angle oscillations.

A special insight in the study of forced oscillations in electric power systems has been gained through the analysis of the frequency characteristics of the transfer function $G_{DF}(s)$ which relates the swing angle change $\Delta \delta$ to the disturbing power signal $\Delta P_d$. It follows that the magnitude of oscillation can be modelled as the output of a filter injected with a sinusoidal input. This interpretation can clearly identify a serious problem associated with peaks in $|G_{DF}(j\omega)|$ which result in amplification of forced oscillatory disturbances. Eigenvalue analysis cannot detect this phenomenon so it is imperative to explicitly evaluate the $G_{DF}(s)$ gain response.

It has been shown that, what seems to be a properly tuned PSS, while improving the damping of the local mode, can amplify forced disturbances which may occur at a lower frequency range that matches the inter-tie mode. The reason behind this amplification has been explained through the analysis of the polynomial that describes the zeros introduced by the PSS loop. The effects of the controlled parameters $K_s$, $T_a$ and $K_e$ on the $G_{DF}(s)$ gain response have also been discussed together with the implication on PSS design.
Chapter 3

Dynamic gain reduction measures
A method of decoupling negative damping oscillations without compromising the benevolent effects of other control loops in the ECS is presented. A notch filter, acting as an on/off switch, located in the forward path of the negative damping signal, provides the necessary Dynamic Gain Reduction (DGR) for "switching off" the detrimental signals. A restructuring of the transfer blocks in the ECS is proposed, allowing the decoupling of the negative damping signals. Additional insight into the effect of negative damping signals and their subsequent decoupling is provided through the examination of loop and transfer block signals.

3.1 Introduction

In the preceding chapter, frequency domain analysis of the transfer function \( G_{DRF}(s) \) which relates the swing angle change \( \Delta \delta \) to a disturbing power signal \( \Delta P_d \), revealed that a PSS, which is tuned to locate the eigenvalues corresponding to the natural mode of oscillation at well damped locations, can still cause a hazardous situation.

It was also established that, what seemed to be a proper choice for the value of the dynamic constant, \( K_e \), of the AVR in one frequency range, was clearly detrimental for the power system performance in another frequency range. Adjusting \( K_e \) over the range of frequency of interest, hence providing Dynamic Gain Reduction (DGR) of the AVR, is a possible solution.
DGR is also desirable for attenuating negative damping signals which are introduced at the summing junction of the AVR where the PSS is also connected. The feedback path of these signals is through the dynamic constant $K_5$, which represents the change in terminal voltage ($\Delta V_t$) for a small change in rotor angle ($\Delta \delta$) at constant d-axis flux linkages [29]. Expressed in mathematical form:

$$
K_5 = \frac{\Delta V_t}{\Delta \delta} = \left. \frac{X_q}{X_e + X_q} \frac{V_{do}}{V_{to}} V_o \cos \delta_o \right|_{E'_{eq}} - \frac{X_d}{X_e + X_d} \frac{V_{go}}{V_{to}} V_o \sin \delta_o
$$

(3.1)

The explicit formulation of $K_5$ is useful in explaining the role of the involved factors. These factors are estimated from field tests and it is difficult to obtain an accurate value for $K_5$. Some insight may be gained by examining the two terms on the right hand side of equation (3.1). For weakly connected systems, the load angle $\delta$ is relatively large, as is the equivalent reactance $X_e$. Under these conditions, the second term on the right hand side predominates and $K_5$ is negative. The effect of $K_5$ on the component of damping torque, produced by voltage regulator action response due to voltage deviations which are initiated by rotor angle deviations can be quantified.
Referring to Fig. 2.1, the pertinent transfer blocks that describe this effect are shown below, with the AVR and field flux decay blocks represented by their respective dynamic constant and transfer function.

\[ \Delta \delta \rightarrow K_5 \rightarrow \Sigma \rightarrow K_e \rightarrow \frac{K_3}{1 + s K_3 T_{do}} \rightarrow K_2 \rightarrow \Delta T \]

Fig. 3.1 Effect of $K_5$ on damping torque

At low frequencies, the demagnetising effect of $K_4$ on the component of damping torque is negligible. The loop involving this constant is therefore omitted from Fig. 3.1. From this figure, the component of torque (synchronising and damping, $\Delta T$) due to a change in rotor angle and its effects on voltage is given by:

\[
\frac{\Delta T}{\Delta \delta} = \frac{-K_2 K_3 K_5 K_e}{s (K_3 T_{do}) + (1 + K_3 K_6 K_e)}
\] (3.2)

The damping torque component for any given oscillation frequency is developed in phase with the synchronous machine rotor speed. Its magnitude is obtained from equation (3.2) as
\[ |\Delta T_d| = \frac{K_2 K_5 K_e T_{d_0} \omega}{(\omega T_{d_0'})^2 + (1/K_3 + K_3 K_6 K_e)^2} \Delta \delta \]  

(3.3)

As can be seen from equation (3.3), when \( K_5 \) is negative, a destabilising torque is introduced in the excitation control. The situation is aggravated by a high value of \( K_e \), which is necessary for field forcing action.

Dynamic Gain Reduction (DGR) of the AVR gain has been proposed as a cautious measure to weaken the feedback loop that causes negative damping [29]. The gain characteristics are lowered at a particular frequency range, by inserting a series compensator with the AVR block. This attenuates the unwanted destabilising signals. Unfortunately, the resulting reduction in bandwidth also attenuates the benevolent PSS signals. Reducing the AVR forward path gain reduces the ability of the excitation control to enhance transient stability and field forcing functions.

In section 3.2, various methods of implementing DGR measures are investigated. A method for restructuring the DGR blocks is obtained which results in the following distinct advantages:

(a) The loop which is the source of negative damping undergoes dynamic gain reshaping. This attenuates the effects of external disturbances without sacrificing the fast response of the excitation block which is beneficial for transient stability.
(b) The supplementary excitation control loop, involving the PSS, is not subjected to gain reduction. Therefore its function is not impeded over any frequency range.

(c) The risky consequence of the phenomenon described earlier, namely the amplification of $G_{D RF}(s)$ gain characteristics at a lower frequency, is reduced.

Section 3.3 presents the results of simulation experiments followed by conclusions in section 3.4.

3.2 Investigations of various DGR measures

DGR implementation usually relies on a form of series compensator cascaded with the AVR block and aims to reduce the gain of the forward path of the excitation loop in a particular frequency range. This section describes the effect of DGR on forced low frequency oscillations and proposes a restructuring of the commonly adopted implementation.

In power system low frequency analysis, increasing power angle oscillations can be attributed to incorrect control parameter settings because the unregulated machine has inherent positive damping. As established earlier, the feedback loop involving $K_5$ (see Fig.2.1) can also be a source of negative damping. Examination of Fig.2.1 shows that the path $K_5$, $\Sigma_1$, $\Sigma_2$, $\Sigma_3$ and $\Sigma_4$ represents a positive feedback loop for negative values of $K_5$. PSS are used to counteract this effect by the addition of a supplementary excitation
signal adjusted to produce a component of positive damping. The design and tuning of PSS have been widely investigated [30-32]. In a multi-machine environment, which can produce a number of low frequency disturbances, special care is taken to adjust the PSS parameters so that the disturbances are damped over a wider frequency range.

Modern excitation controls, by virtue of their fast response have improved the transient stability of interconnected systems. However, the greater bandwidth of the excitation control, resulted in amplification of the negative damping signal that is generated by the previously mentioned feedback path between Δδ and the measured voltage ΔVₜ. It has been suggested that the DGR of the AVR weakens this path and thus reduces the negative damping effect [33].

For further insight into the role of DGR in excitation control systems, the block diagram of Fig. 2.1 is simplified as shown in Fig.3.1. The frequency characteristics of the machine dynamics are those of a lightly damped system and appear above the machine dynamics block. Typical frequency characteristics of the other transfer blocks are shown by the graph adjacent to the respective block. In these graphs, |G| represents the magnitude of the response while ω is the frequency.
Fig. 3.1 depicts the major components of an excitation control system involving a commonly adopted DGR scheme. The DGR block, comprising typically of a band-stop filter with corner frequencies in the range 0.1 to 10 Hz is located in the forward path of the AVR. In this configuration, both the benevolent stabilising signals of the PSS and the detrimental destabilising signals of the inherent negative damping are attenuated.
An alternative approach, whereby the DGR block is inserted as a preshaping filter for $\Delta V_t$ is shown in Fig. 3.3. With this approach, the main objective of the scheme, which is to decouple the negative damping effects while maintaining the stabilising effect of the PSS, is achieved. This is illustrated by considering two distinct control loops in the excitation control system. One is formed by the PSS which, when correctly adjusted, has a positive damping effect on the system. In general, once the PSS is finely tuned, the signals produced from this loop should not be restrained. The other control loop, involving feedback from the network through the
dynamic constant $K_5$, can be a destabilising one by virtue of the negative damping signals finding a path through that loop.

Comparing the two system configurations depicted in Fig. 3.2 and 3.3, the following observations can be made:

(a) The loop $K_5$ which is the source of negative damping undergoes the same transient gain reduction in each case.

(b) In the configuration of Fig. 3.3, the supplementary stabilising control loop of the PSS is not subjected to transient gain reduction and therefore its function is not impeded in any frequency range.

3.3 Simulation results

Simulation experiments were performed on the excitation control system shown in Fig. 2.1. Modelling details of the transfer function and parameter settings are given in Appendix I. Frequency response and time domain simulations were first obtained for the case when the excitation control system was not equipped with DGR. Time domain simulations for this and other cases are based on the injection of a unit step function. The corresponding transfer function is given by the following equation:

$$G_{DRF}(s) = \frac{\Delta \delta}{\Delta P_d} = \frac{C_0 s^2 + C_1 s + C_2}{A_0 s^4 + A_1 s^3 + A_2 s^2 + A_3 s + A_4}$$

(3.4)
The coefficients $C_t$ and $A_t$ which were used in the simulations are given in Table 3.1.

**Table 3.1** Coefficients of equation (3.4)

<table>
<thead>
<tr>
<th>Parameter value</th>
<th>$C_0$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$A_0$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_a = 0.12$</td>
<td>0.22</td>
<td>2.4</td>
<td>4.8</td>
<td>0.0017</td>
<td>0.021</td>
<td>0.55</td>
<td>2.5</td>
<td>5.2</td>
</tr>
<tr>
<td>$T_a = 0.08$</td>
<td>0.14</td>
<td>2.2</td>
<td>4.8</td>
<td>0.0012</td>
<td>0.019</td>
<td>0.48</td>
<td>2.3</td>
<td>5.2</td>
</tr>
<tr>
<td>$T_a = 0.05$</td>
<td>0.09</td>
<td>2.0</td>
<td>4.8</td>
<td>0.00072</td>
<td>0.017</td>
<td>0.42</td>
<td>2.1</td>
<td>5.2</td>
</tr>
<tr>
<td>$T_a = 0.03$</td>
<td>0.054</td>
<td>1.9</td>
<td>4.8</td>
<td>0.00043</td>
<td>0.016</td>
<td>0.39</td>
<td>2.0</td>
<td>5.2</td>
</tr>
</tbody>
</table>

*— $T_a=0.12$  >•— $T_a=0.08$  — $T_a=0.05$  — $T_a=0.03$

**Fig.3.4** Frequency characteristics of $G_{DRF}(s)$

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Fig. 3.4 shows the gain characteristics of $G_{\text{DRF}}(s)$ for various values of PSS time constant $T_a$. As commented upon earlier, a value of $T_a=0.05$ second seem to be quite satisfactory in damping out the local mode of oscillation. Tie-line oscillations around a frequency of 4 rad/sec are, however, undamped and may present a hazardous situation for the system. The transient performance of the power system with a PSS time constant of 0.05 second, as depicted in Fig. 3.5, is satisfactory.

Two different methods of DGR implementation are now simulated. In each case, a first order lag compensator given by

$$H_g(s) = \frac{1}{1 + 40s}$$
was used. One method involved locating the compensator in the forward path of the AVR of the system shown in Fig.2.1 as shown in Fig. 3.1. The corresponding transfer function for this implementation is derived in Appendix II, and is given by the equation:

\[
G_{AVR}(s) = \frac{\Delta \delta}{\Delta P_d} = \frac{BV_0 s^3 + BV_1 s^2 + BV_2 s + BV_3}{AV_0 s^5 + AV_1 s^4 + AV_2 s^3 + AV_3 s^2 + AV_4 s + AV_5} \quad (35)
\]

The second method refers to the location of the DGR in cascade with K5. This is depicted in Fig.3.3. The transfer function for this implementation is derived in Appendix II and is given by

\[
G_{TGR}(s) = \frac{\Delta \delta}{\Delta P_d} = \frac{BK_0 s^4 + BK_1 s^3 + BK_2 s^2 + BK_3 s + BK_4}{AK_0 s^5 + AK_1 s^4 + AK_2 s^3 + AK_3 s^2 + AK_4 s + AK_5 s + AK_6} \quad (36)
\]

The coefficients of equations (3.5) and (3.6) are given in Table 3.2 below:

**Table 3.2 Coefficients of G\(_{AVR}(s)\) and G\(_{TGR}(s)\)**

<table>
<thead>
<tr>
<th>G(_{AVR}(s))</th>
<th>BV(_0)</th>
<th>BV(_1)</th>
<th>BV(_2)</th>
<th>BV(_3)</th>
<th>AV(_0)</th>
<th>AV(_1)</th>
<th>AV(_2)</th>
<th>AV(_3)</th>
<th>AV(_4)</th>
<th>AV(_5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>3.6</td>
<td>76</td>
<td>44</td>
<td>4.8</td>
<td>0.029</td>
<td>0.64</td>
<td>4.8</td>
<td>76</td>
<td>18</td>
<td>5.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>G(_{TGR}(s))</th>
<th>BK(_0)</th>
<th>BK(_1)</th>
<th>BK(_2)</th>
<th>BK(_3)</th>
<th>BK(_4)</th>
<th>AK(_0)</th>
<th>AK(_1)</th>
<th>AK(_2)</th>
<th>AK(_3)</th>
<th>AK(_4)</th>
<th>AK(_5)</th>
<th>AK(_6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>144</td>
<td>2960</td>
<td>1760</td>
<td>232</td>
<td>4.8</td>
<td>1.16</td>
<td>25.2</td>
<td>640</td>
<td>3000</td>
<td>800</td>
<td>228</td>
<td>5.2</td>
</tr>
</tbody>
</table>
Frequency response and time domain simulations were obtained for the two methods of DGR implementation. Results are shown in Figures 3.6, 3.7 and 3.8.

Fig.3.6 Gain characteristics - DGR cascaded with AVR

Fig.3.7 Gain characteristics - DGR cascaded with K5
These results lead to the following observations:

(a) when the DGR block is cascaded with the AVR, the stabilising effect of the PSS is impeded, since the magnitude of $G_{DGR}(s)$ at the natural mode $\omega_n \approx 11$ rad/sec is higher than when DGR is not used.

(b) when cascaded with $K_5$, DGR implementation retains the stabilising effect of the PSS since the gain at the natural mode is not amplified.

(c) In both cases the lower frequency peak has been shifted to a location which is unlikely to be of potential risk, since it does not match the expected tie-line mode.
(d) the transient response of the power system to a unit step function, when the AVR is equipped with DGR, implemented as a first order lag compensator and cascaded either with the AVR or with K₅, has become more sluggish. The power angle oscillations take over 25 seconds to settle to the steady state value, compared to about 1.5 seconds when the power system is not equipped with DGR. (refer to Figs. 3.5 and 3.8).

The above observations indicate, in terms of dynamic stability considerations, that, implementing dynamic gain compensation as a preshaping filter for the measured terminal voltage has distinct advantages over the commonly adopted approach of cascading the DGR block with the AVR. However, transient stability in both cases deteriorates.

For voltage regulation action, the preshaping filter however, should satisfy two requirements:

(a) high d.c. gain, which reduces the steady state regulation error

(b) a bandwidth of the excitation control loop wide enough to produce the required response for transient stability considerations, including responsive field forcing action.

To meet these requirements, a bandstop preshaping filter with a stop range matching the expected frequencies associated with tie-line oscillatory disturbances can be used. A narrow bandstop filter (notch filter) would seem to be an appropriate choice. This notch filter may be viewed as an on/off switch which, when located in
series with $K_5$, will "switch off" the negative damping signals when $K_5$ is negative.

Further simulations, based on this concept were performed. Frequency response and time domain simulations were obtained, using the power system model of Fig. 2.1, for cases where $K_5$ was assigned the following values: -0.097, 0 and +0.097. In table 3.3 and the graphs that follow, these values are referred to as $K_5$ negative, $K_5 = 0$ and $K_5$ positive. The corresponding transfer function is that of equation (3.4) and the relevant coefficients for $A_1$ and $C_1$ are shown in Table 3.3 below.

**Table 3.3** Coefficients of $G_{DRF}(s)$ for different values of $K_5$.

<table>
<thead>
<tr>
<th>Parameter value</th>
<th>$A_0$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$C_0$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_5$ Negative; $K_e=50$</td>
<td>0.09</td>
<td>2.2</td>
<td>8.5</td>
<td>0.00072</td>
<td>0.019</td>
<td>0.73</td>
<td>2.4</td>
<td>1.0</td>
</tr>
<tr>
<td>$K_5$ Negative; $K_e=25$</td>
<td>0.09</td>
<td>2.0</td>
<td>4.8</td>
<td>0.00072</td>
<td>0.017</td>
<td>0.42</td>
<td>2.1</td>
<td>5.2</td>
</tr>
<tr>
<td>$K_5$ Negative; $K_e=10$</td>
<td>0.09</td>
<td>1.9</td>
<td>2.5</td>
<td>0.00072</td>
<td>0.016</td>
<td>0.24</td>
<td>2.0</td>
<td>2.3</td>
</tr>
<tr>
<td>$K_5$ Negative; $K_e=5$</td>
<td>0.09</td>
<td>1.9</td>
<td>1.8</td>
<td>0.00072</td>
<td>0.016</td>
<td>0.18</td>
<td>1.9</td>
<td>1.4</td>
</tr>
<tr>
<td>$K_5 = 0$; $K_e=50$</td>
<td>0.09</td>
<td>2.2</td>
<td>8.5</td>
<td>0.00072</td>
<td>0.019</td>
<td>0.73</td>
<td>2.3</td>
<td>8.0</td>
</tr>
<tr>
<td>$K_5 = 0$; $K_e=25$</td>
<td>0.09</td>
<td>2.0</td>
<td>4.8</td>
<td>0.00072</td>
<td>0.017</td>
<td>0.42</td>
<td>2.1</td>
<td>4.2</td>
</tr>
<tr>
<td>$K_5 = 0$; $K_e=10$</td>
<td>0.09</td>
<td>1.9</td>
<td>2.5</td>
<td>0.00072</td>
<td>0.016</td>
<td>0.24</td>
<td>1.9</td>
<td>1.9</td>
</tr>
<tr>
<td>$K_5 = 0$; $K_e=5$</td>
<td>0.09</td>
<td>1.9</td>
<td>1.8</td>
<td>0.00072</td>
<td>0.016</td>
<td>0.18</td>
<td>1.9</td>
<td>1.2</td>
</tr>
<tr>
<td>$K_5$ Positive; $K_e=50$</td>
<td>0.09</td>
<td>2.2</td>
<td>8.5</td>
<td>0.00072</td>
<td>0.019</td>
<td>0.73</td>
<td>2.2</td>
<td>6.0</td>
</tr>
<tr>
<td>$K_5$ Positive; $K_e=25$</td>
<td>0.09</td>
<td>2.0</td>
<td>4.8</td>
<td>0.00072</td>
<td>0.017</td>
<td>0.42</td>
<td>2.0</td>
<td>3.2</td>
</tr>
<tr>
<td>$K_5$ Positive; $K_e=10$</td>
<td>0.09</td>
<td>1.9</td>
<td>2.5</td>
<td>0.00072</td>
<td>0.016</td>
<td>0.24</td>
<td>1.9</td>
<td>1.5</td>
</tr>
<tr>
<td>$K_5$ Positive; $K_e=5$</td>
<td>0.09</td>
<td>1.9</td>
<td>1.8</td>
<td>0.00072</td>
<td>0.016</td>
<td>0.18</td>
<td>1.9</td>
<td>0.96</td>
</tr>
</tbody>
</table>
Fig. 3.9a Effects of $K_e$ on the frequency characteristics of $G_{DRF}(s)$ for $K_5$ negative

Fig. 3.9b Effects of $K_e$ on the frequency characteristics of $G_{DRF}(s)$ for $K_5 = 0$
Fig. 3.9c Effects of $K_e$ on the frequency characteristics of $G_{DRF}(s)$ for $K_5$ positive.

Fig. 3.9d Effects of $K_5$ on the frequency characteristics of $G_{DRF}(s)$ for $K_e = 50$. 

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Fig. 3.9e Effects of $K_5$ on the frequency characteristics of $G_{DRF}(s)$ for $K_e = 25$

Fig. 3.9f Effects of $K_5$ on the frequency characteristics of $G_{DRF}(s)$ for $K_e = 10$
The results shown in Fig. 3.9 (a-g) confirm the expectations arising from equation (3.3). With $K_5$ negative, positive feedback is introduced in the excitation control, causing amplification of a mode of oscillation around a frequency of 4 rad/sec. This oscillation, however, ceases when $K_5$ is positive, indicating that a positive component of damping torque is now developed and this counteracts the oscillations in the lower frequency range of interest.

When the path for negative damping signals is opened ($K_5 = 0$), the amplification effect is reduced, but not eliminated. The implication of this observation is that, although negative damping signals have been decoupled, low frequency oscillations from other sources still persist, albeit at a reduced magnitude. The persistence of this
oscillatory mode is attributed to the introduction of additional zeros by the PSS, which adversely reshape the system dynamics. This phenomenon was explained in chapter 2.

Time domain simulations of $G_{\text{DRF}}(s)$ as depicted in Fig 3.10 (a-g) show that transient performance is better than in the case where DGR is implemented as a lag compensator. These graphs confirm that a relatively high AVR gain is desirable for field forcing action. The transient performance of the excitation control system is enhanced with positive values of $K_5$.

Fig.3.10a Effects of $K_e$ on the transient performance of power system represented by $G_{\text{DRF}}(s)$ for $K_5$ negative
Fig. 3.10b Effects of $K_e$ on the transient performance of power system represented by $G_{D_{RF}}(s)$ for $K_5 = 0$

Fig. 3.10c Effects of $K_e$ on the transient performance of power system represented by $G_{D_{RF}}(s)$ for $K_5$ positive
Fig. 3.10d Effects of $K_5$ on the transient performance of power system represented by $G_{DRF}(s)$ for $K_e = 50$

Fig. 3.10e Effects of $K_5$ on the transient performance of power system represented by $G_{DRF}(s)$ for $K_e = 25$
Fig. 3.10f Effects of $K_5$ on the transient performance of power system represented by $G_{DRF}(s)$ for $K_e = 10$

Fig. 3.10g Effects of $K_5$ on the transient performance of power system represented by $G_{DRF}(s)$ for $K_e = 5$
A summary of the various methods of implementing DGR in an excitation control system and their effects on the dynamic and transient stability of the system is set out below.

<table>
<thead>
<tr>
<th>Condition of excitation control system</th>
<th>Effect on dynamic stability</th>
<th>Effect on Transient stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>No DGR</td>
<td>amplification of oscillatory mode at 4 rad/sec</td>
<td>satisfactory</td>
</tr>
<tr>
<td>DGR cascaded with AVR as lag compensator</td>
<td>amplification at 11 rad/sec (PSS impeded)</td>
<td>sluggish</td>
</tr>
<tr>
<td>DGR cascaded with K5 as lag compensator</td>
<td>no amplification at either the 4 or 11 rad/sec mode</td>
<td>sluggish</td>
</tr>
<tr>
<td>DGR cascaded with K5 as a notch filter</td>
<td>reduced amplification at the 4 rad/sec mode</td>
<td>satisfactory</td>
</tr>
</tbody>
</table>

The above summary shows that, under certain conditions, DGR measures are successful in improving the dynamic stability of a power system by decoupling negative damping signals. Transient performance under those same conditions is satisfactory. It would seem that the conflict between the field forcing action of the AVR (servo action) and the injection of positive damping signals of the PSS (regulation action) is resolved. However, the phenomenon of the amplified mode of oscillation described in chapter 2 can still occur, though at a reduced magnitude. A different approach to coordinating the actions of the PSS and AVR for field forcing action, decoupling of negative signals and for disturbance rejection,
is required. Since disturbing frequencies can drift with operating conditions, any approach should be adaptive. A self-tuning regulator (STR) is proposed in chapter 6. The STR requires on-line estimation of the system parameters. In chapter 4, an efficient recursive algorithm for the estimation routine is presented.

3.4 Conclusions

The source of negative damping signals in a power system has been reviewed. Various methods of decoupling these signals through dynamic gain reduction measures have been investigated. It has been shown that providing dynamic gain reduction through a notch filter, acting as an on/off switch, cascaded with the dynamic constant $K_5$ effectively "switches off" the detrimental signals.
Chapter 4

Estimation routines
Abstract

An efficient recursive algorithm, based on some properties of the Cholesky matrix decomposition techniques, for the estimation of system parameters is developed. It is shown that numerical stability is improved through the updating of the triangular factors of an augmented information matrix. A fixed width data window is used to cater for parameter drifts.

4.1 Introduction

Self-tuning control is based on the certainty equivalence principle of stochastic control theory. This principle establishes that a control strategy can be implemented in two steps. The first step consists of a parameter identification scheme, such as the Recursive Least Squares (RLS), to obtain the best parameter estimates from input/output data. The second step relates to the control policy which is based on the estimated parameters.

The particular control policy employed in the present work will be discussed in chapter 6. In this chapter, the accent is placed on the identification scheme. An estimation model is derived together with an efficient recursive estimation algorithm. The pertinent features of the estimation algorithm are:

(a) numerical stability is improved through the updating of the triangular factors of an augmented information matrix,
(b) a fixed width data window, rather than a forgetting factor, is used to cater for parameter drifts, and

(c) a special rearrangement of the parameters of the estimation model can be performed in order to minimize the need for excessive parameter projection during the initial phase of adaptation. This feature will be best described after the self-tuning control scheme had been analysed in chapter 6.

Section 4.2 describes the discrete-time model of the Excitation Control System (ECS) while section 4.3 discusses system identification. In section 4.4 the proposed estimation model is developed. Section 4.5 presents the estimation algorithm followed by conclusions in section 4.6.

4.2 The discrete-time excitation control system model

In self-tuning control, both the control algorithm and the identification scheme are based on a linear discrete-time model. A linear model representation of the ECS in continuous time was adopted in chapter 2. The equivalent discrete time model is derived in this section prior to discussing the identification scheme. The Laplace domain transfer function G_{DRF}(s) of the ECS given is Fig. 2.1, in generalised form, is

\[
G_{DRF}(s) = \frac{C_m s^m + C_{m-1} s^{m-1} + \ldots + C_0}{A_n s^n + A_{n-1} s^{n-1} + \ldots + A_0} : \quad m \leq n
\]  

(4.1)
The discrete time transfer function $G_{DRF}(s)$ may be obtained by one of many mapping methods from the $s$-plane to the $z$-plane. Some of the methods available are the forward difference, the backward difference, the bilinear transformation and the bilinear transformation with frequency prewarping [34]. In practice the forward difference method is avoided since, often the left-half of the $s$-plane is mapped into the region of instability in the $z$-plane. Of the remaining methods, Ogata [34] advises to try a few alternate forms of the equivalent discrete time functions before the final discrete time function is arrived at. This is usually achieved after satisfactory results have been obtained from a digital computer simulation.

The commonly used mapping methods and their corresponding mapping equations are given in table 5.1.

**Table 5.1 Mapping methods and equations**

<table>
<thead>
<tr>
<th>Mapping method</th>
<th>Mapping equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward difference</td>
<td>$s = \frac{1 - z^{-1}}{T_s z^{-1}}$</td>
</tr>
<tr>
<td>Backward difference</td>
<td>$s = \frac{1 - z^{-1}}{T_s}$</td>
</tr>
<tr>
<td>Bilinear transformation</td>
<td>$s = \frac{2}{T_s} \frac{1 - z^{-1}}{1 + z^{-1}}$</td>
</tr>
<tr>
<td>Bilinear transformation with frequency prewarping</td>
<td>$s = \frac{\omega_f}{\tan (\omega_f T_s /2)} \frac{1 - z^{-1}}{1 + z^{-1}}$</td>
</tr>
</tbody>
</table>
In table 5.1, $T_s$ is the sampling interval and $\omega_f$ is the frequency at which it is required to eliminate scale distortion which occurs when the bilinear transformation method is used.

The discrete-time transfer function of equation (4.1) is

$$G_{DRF}(z) = z^{-1} \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_{n-1} z^{-n+1}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}}$$

Equation (4.2) is expressed in a form which is suitable for analysis in the frequency domain, and for implementation in the time domain it can be conveniently expressed as follows:

$$y(t) + a_1 y(t-1) + a_2 y(t-2) + \ldots + a_n y(t-n) = b_0 u(t-1) + b_1 u(t-2) + \ldots + b_{n-1} u(t-n) \quad (4.3)$$

where $t$ is the discrete time index.

The frequency domain operator $z$ can be replaced by the backward shift operator $q^{-1}$. The latter is defined such that
\[ q^{-1} f(t) = f(t-1) \]

where \( f(\cdot) \) stands for any discrete-time signal.

Equation (4.3) may be rearranged to give

\[
y(t) = q^{-1} \frac{b_0 + b_1 q^{-1} + b_2 q^{-2} + \ldots + b_{n-1} q^{-(n+1)}}{1 + a_1 q^{-1} + a_2 q^{-2} + \ldots + a_n q^{-n}} \cdot u(t) \quad (4.4)
\]

In compact form

\[
y(t) = q^{-1} \frac{B(q^{-1})}{A(q^{-1})} \cdot u(t) \quad (4.5)
\]

where

\[
B(q^{-1}) = b_0 + b_1 q^{-1} + b_2 q^{-2} + \ldots + b_{n-1} q^{-(n+1)}
\]

\[
A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} + \ldots + a_n q^{-n}
\]

For a system with time delay represented by \( q^{-k} \), equation (4.5) becomes

\[
y(t) = q^{-k} \frac{B(q^{-1})}{A(q^{-1})} \cdot u(t) \quad (4.6)
\]
Equation (4.6) is the general discrete time transfer function representation of a rational continuous time transfer function.

4.3 System identification

In self-tuning control it is necessary to determine the unknown parameters of the discrete time system model on-line. The process of experimentally obtaining a system model which best represents the essential dynamic characteristics of the controlled plant is known as system identification [35].

System identification comprises both model structure and parameter identification. The model structure of a power system is usually known \textit{a priori} and only parameter identification need be performed. Various algorithms are available for this purpose. These can be implemented on-line or off-line. The off-line algorithms make use of accumulated data which are processed several times. These algorithms are therefore generally more accurate than the on-line ones but are invariably more computationally burdensome. On-line algorithms are necessary for self-tuning schemes because the data must be processed in real time.

The RLS identification method is the most commonly used in self-tuning control. With this method, the computation is relatively simple and parameter convergence is fast [36]. Its main features are summarised below. These will serve as a basis for the development of the proposed estimation algorithm.
The predictive model of equation (4.6) for the case where Gaussian noise is included is given by:

\[ y(t) = -a_1 y(t-1) - a_2 y(t-2) - \ldots + b_0 u(t-k) + b_1 u(t-k-1) + \ldots + b_{n-1} u(t-k-n) + \xi(t) \]  

(4.7)

In compact form, equation (4.7) can be expressed as

\[ y(t) = \theta^T \phi + \xi(t) \]  

(4.8)

where \( \theta \) is the parameter vector consisting of unknown model parameters and \( \phi \) is the state vector consisting of past input and output data as defined below:

\[ \theta^T = [-a_1 -a_2 \ldots b_0 \ldots b_{n-1}] \]

\[ \phi^T = [y(t-1) y(t-2) \ldots u(t-k) \ldots u(t-k-n)] \]

where T stands for transposition.

Equation (4.8) is the estimation model. Estimates of the elements of the unknown parameter vector \( \hat{\theta} \) can be updated during each sampling interval, using the RLS algorithm.
\[ \hat{\theta}(t) = \hat{\theta}(t-1) + K(t) \varepsilon(t) \]  \hspace{1cm} (4.9)

where \( \hat{\theta}(t) \) is the new estimate of the parameter vector, and \( \hat{\theta}(t-1) \) is the old estimate of the parameter vector.

\[
K(t) = \frac{P(t-1) \phi(t)}{\left[ I + \phi^T(t) P(t-1) \phi(t) \right]} \text{ is the gain vector.}
\]

\[
P(t) = \left[ I - K^T(t) \phi(t) \right] P(t-1) \text{ is the covariance matrix, and}
\]

\[
\varepsilon(t) = \left[ y(t) - \hat{\theta}^T(t-1) \phi(t) \right] \text{ is the prediction error.}
\]

This algorithm is well suited for time-invariant systems where \( \hat{\theta}(t) \) converges to its 'true' value with time. The gain vector \( K(t) \), the prediction error \( \varepsilon(t) \) and the covariance matrix also tend to zero.

For systems where the parameters drift with time, as is typical in self-tuning applications, parameter convergence cannot be guaranteed since the prediction error \( \varepsilon(t) \) becomes less significant with time. The estimation algorithm then does not respond to any parameter changes. It is then necessary to periodically reset the covariance matrix to its initial value or to a value which is dependent on the latest covariance matrix value. In doing so, the stored information in the \( K(t) \) matrix is lost.

A way of overcoming this problem is to gradually forget past data, in particular past errors. This is achieved by incorporating a forgetting factor which exponentially discounts old data as follows:
\[ P(t) = \frac{[I - K^T(t) \phi(t)] P(t-1)}{\lambda} \]  

(4.10)

where \( \lambda \) is the forgetting factor.

This algorithm gives satisfactory parameter tracking properties for a time-varying system provided the latter is properly excited. When applied to a power system, where the excitation test signal is kept relatively small in order not to affect normal operating conditions, the system becomes poorly excited during long periods of sustained steady state operation. The prediction error then tends to zero and causes the matrix \( P(t-1) \) in equation (4.10) to increase exponentially. This sometimes leads to the so called "covariance matrix blow-up", which results in an undesirable burst of control activity which may endanger system stability [37].

To counteract the problem of covariance blow-up, various methods based on variable forgetting factor algorithms have been proposed [38-39]. In these algorithms, the forgetting factor is made a function of the data. The forgetting factor may vary from close to unity for the condition when the prediction error is small in order to retain much of the old data, or it may assume a much lower value when the error is large in order to increase the estimation sensitivity.

An alternative to the variable forgetting factor method is the fixed width data window [40]. With this method, a data window of a fixed
number of samples is used to collect relevant measured data such that the remote past train of data is disregarded. This method does not suffer from the drawback of covariance matrix blow up and is able to track drifting parameters. The proposed estimation routine makes use of this method and is discussed further in the next sections.

4.4 A proposed estimation routine

The estimation model given by equation (4.8) is the basis for the proposed estimation routine. Stated in terms of the discrete time operator, the model is given by the following equation:

\[ P_k = \theta^T \phi_k \]  \hspace{1cm} (4.11)

The off-line least squares estimate of the parameter vector \( \theta \) [36], based on a fixed data window of width \( W \) samples is

\[ \theta = \left[ \Phi^T(k) \Phi(k) \right]^{-1} \Phi^T(k) P(k) \]  \hspace{1cm} (4.12)

where

\[ \Phi^T(k) = \left[ \phi(k-W+1) \phi(k-W) \ldots \phi(k) \right] \]

\[ P^T(k) = \left[ p(k-W+1) p(k-W) \ldots p(k) \right] \]
Ideally, the data window should include a large number of samples. However, to limit the computational burden, the width should only include the relevant information. For further development, an augmented information matrix $M$ and a measurement vector $X$ are defined as follows:

$$M(k) = \begin{bmatrix} P^T(k) & P(k) & P^T(k)\Phi(k) \\ \Phi^T(k)P(k) & \Phi^T(k) & \Phi(k) \end{bmatrix}$$

$$X^T(k) = \begin{bmatrix} P(k) & \Phi^T(k) \end{bmatrix}$$

Using the above definitions, the information matrix can be updated as follows:

$$M(k+1) = M(k) + X(k)X^T(k) - X(k-W+1)X^T(k-W+1) \quad (4.13)$$

The information matrix is updated by including the information measured by $X(k)$ and dropping that of $X(k-W+1)$ which occurred $W$ samples earlier. The recursive algorithm is derived below. It is based on Cholesky matrix decomposition techniques [41] from which

$$M(k) = L(k) \quad (4.14)$$
\[ M^{-1}(k) = U(k) \, U^T(k) \quad (4.15) \]

where \( L(k) \) is a lower triangular matrix and \( U(k) \) is an upper triangular matrix. The proposed algorithm is based on the following two lemmas:

Lemma 1: The off-line least squares solution of equation (4.12) is given by

\[ \theta_1 = - U_{n,n-1}^T / U_{n,n} \quad ; \quad i = 1, 2, \ldots, n-1 \quad (4.16) \]

where \( U_{i,j} \) is the \((i,j)\) element of the matrix \( U \) and \( \theta_1 \) is the \( i \)th element of the vector \( \theta \). The proof of this lemma is given in Appendix III. It is based on the results of reference [42].

Lemma 2: The following relationship holds

\[ L^{-1}(k) = U^T(k) \]

The proof of this lemma follows directly from the uniqueness of the Cholesky decomposition of positive definite matrices [41].

Applying the above lemmas, the recursive least squares estimation routine can be achieved via the following steps:

Step 1: Use equation (4.13) to update the information matrix \( M \)
Step 2: Perform the Cholesky factorisation of equation (4.14)

Step 3: Invert the matrix L to obtain $U^T$ which includes the solution as the last row (see equation (4.16))

A substantial reduction in the necessary computations can be achieved by adopting the following procedure:

(a) Based on rank two modification of Cholesky factors, steps 1 and 2 can be combined as one. The factor $L(k+1)$ can thus be directly computed from $L(k)$, $X(k-W+1)$ and $X(k)$. This involves $2n^2$ floating point operations (FLOPS) and $n$ square roots, where $n$ is the dimension of the matrix $M$.

(b) The triangular matrix inversion need not be completed. It is sufficient to compute the last row of the inverted matrix (see equation (4.16)). This involves $n^2 / 2$ FLOPS.

The algorithm therefore reduces to the following steps:

Step 1: Perform the following rank two modification

$$L(k+1) \quad L^T(k+1) = L(k) \quad L^T(k) + X(k) \quad X^T(k) - X(k-W+1) \quad X^T(k-W+1)$$

Step 2: Compute the last row of $L^{-1}$.

The above algorithm is used to estimate the parameter vector $\theta$. 
4.5 The estimation algorithm

The proposed recursive algorithm is presented below in a modular form and equation (4.13) is rewritten appropriately:

\[ M(k + 1) = M(k) + X(k) \cdot C \cdot X^T(k) \]

where \( C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \)

and \( X = \begin{bmatrix} X(k) & X(k - W + 1) \end{bmatrix} \)

The algorithm processes a \( n \times n \) triangular array \( \tilde{L} \), a \( n \times 2 \) rectangular array \( X \) and a \( 2 \times 2 \) square array \( C \). Upon entry into the routine, \( X \) and \( C \) are as defined above and \( \tilde{L} = L^{-1}(k) \). Upon exit from the routine, \( \tilde{L} = L^{-1}(k + 1) \) and both \( X \) and \( C \) are destroyed. The updated triangular matrix overwrites the old one and the time index \( k \) is not required in the algorithm. The subscripts in the algorithm refer to the row and column locations of the matrix elements respectively. For programming convenience, step 2 of the algorithm uses the notation \( U^T \) for the inverted matrix \( \tilde{L}^{-1}(k+1) \).

To initiate the algorithm during the first \( W \) samples, the measurement vector \( X(k - W + 1) \), which is not available is replaced by a zero vector. The algorithm fails if the system is not persistently excited. In that case, the variable \( t \) in the algorithm takes a very small or negative value.
The elements of the matrix C are not indexed and the following notation is used instead:

\[ \alpha = C_{1,1} , \quad \beta = C_{2,2} , \quad \gamma = C_{1,2} = C_{2,1} \]

The estimation algorithm comprises two steps:

Step 1: Rank 2 modification

loop 1 : For \( i = 1 \) to \( n \)
\[
\begin{align*}
    d &= \tilde{L}_{4,1} \\
    \omega &= x_{i,1} \alpha + x_{i,2} \gamma \\
    q &= x_{i,1} \gamma + x_{i,2} \beta \\
    t &= d^2 + x_{i,1} \omega + x_{i,2} q \\
    \text{if } t < \varepsilon \text{ stop} \\
    \tilde{L}_{4,1} &= \sqrt{t} \\
    \text{if } i = n \text{ Exit} \\
    \omega &= \omega / \tilde{L}_{4,1} \quad ; \quad q = q / \tilde{L}_{4,1} \\
\end{align*}
\]

loop 2: For \( j = i + 1 \) to \( n \)
\[
\begin{align*}
    \tilde{L}_{j,1} &= \tilde{L}_{j,1} / d \\
    x_{j,1} &= x_{j,1} - x_{i,1} \tilde{L}_{j,1} \\
    x_{j,2} &= x_{j,2} - x_{i,2} \tilde{L}_{j,1} \\
    \tilde{L}_{j,1} &= \tilde{L}_{j,1} \tilde{L}_{4,1} + x_{j,1} \omega + x_{j,2} q \\
    \alpha &= \alpha - \omega^2 \\
    \beta &= \beta - q^2 \\
    \gamma &= \gamma - \omega q \\
\end{align*}
\]
End loop 2

End loop 1
Step 2: Computation of the last row of $U^T$

(i.e. the last row of $\bar{L}^{-1} (t + 1)$)

$$U_{n,n} = 1 / \bar{L}_{n,n}$$

$$U_{n,j} = \left[ - \sum_{k=1}^{n-j} U_{n,j+k} L_{j+k,j} \right] / L_{j,j}, j = n-1, \ldots, 1$$

$$\theta_i = -U_{n,n-i} / U_{n,n}, i = 1, \ldots, n-1$$

Examination of the above steps reveal that the estimation algorithm requires

$$\frac{5}{2} n^2 + O(n)$$

FLOPs per sample

This modest computational burden admits application on a relatively simple computing facility. The estimation algorithm is included in the self-tuning control scheme which is proposed in chapter 6. Results of simulations are therefore deferred until then.

4.6 Conclusions

A perspective on the subject of identification in a dynamical system has been presented with particular reference to the recursive least squares method. Based on some properties of the Cholesky matrix decomposition techniques, an efficient recursive estimation algorithm has been developed. The algorithm makes use of a fixed width data window, rather than a forgetting factor, to cater for parameter drifts. The numerical stability of the estimation
algorithm is improved through the updating of the triangular factors of an augmented information matrix. The involved computational burden is modest and involves $2.5 n^2$ floating point operations + $n$ square roots for a $n \times n$ matrix during each sampling period.
Chapter 5

Adaptive techniques
Abstract

One of the main contributions of the present work is the development of an adaptive scheme for coordinating power system stabilising devices in an Excitation Control System (ECS). In this chapter, the mathematical background necessary for such development is presented. First, a description of adaptive control systems from a historical perspective is given. A specific approach which leads to a concrete rule for the development of various controller algorithms is then described. This necessitates the development of a predictive model. The Minimum Variance (MV) algorithm which is the basis for many other self-tuning schemes is then presented and is followed by the Generalised Minimum Variance (GMV) algorithm of Clarke and Gawthrop [43]. An alternative and equivalent scheme, i.e. the pole assignment control is briefly summarised for the sake of completeness. Finally, a general and flexible control structure following Egardt [44] is developed.

5.1 Introduction

The ability of an ECS to respond to fluctuations in system characteristics depends principally on the nature of the fluctuation and the type of excitation control the system is equipped with. When a large disturbance occurs on the tie-line, it is essential that the excitation control respond quickly and effectively in order to maintain the generator in synchronism.
The development of fast-acting AVR's has contributed to the improvement of the transient performance of power systems. However, the wider bandwidth that has been imposed on the frequency characteristics of the excitation control loop has resulted in a deterioration of the dynamic stability of the system. The application of DGR measures, which attempt to satisfy the requirements of both transient and dynamic stability, has been investigated in chapter 3.

Spontaneous low frequency oscillations are another source of fluctuations which the ECS has to contend with. These oscillations are due to lack of damping of the electromechanical mode of the interconnected system [45]. To counteract these oscillations, additional positive damping torque signals are provided by a Power System Stabiliser (PSS) through a supplementary excitation control loop. The aim is to extend the dynamic stability range of the power system by stabilising the oscillations.

The PSS circuit has appropriate phase-lead characteristics which compensate for the phase-lag between the exciter output and the electrical torque signals. The tuning of the PSS is based on one or more of the analytical methods discussed in chapter 2. Because of the changes in the phase characteristics of the system, the final PSS parameter settings are usually aimed at maintaining an acceptable dynamic performance within the range of frequency of interest (0.2 to 2.5 Hz) rather than targeting an optimal performance [33].
The conventional PSS performs satisfactorily over a certain range of operating conditions. However, large changes in the operating conditions of the synchronous generator as well as changes in the load and/or network configurations can produce adverse damping effects which may render the PSS less effective. Adaptive PSS schemes have therefore been proposed to overcome these difficulties. Different methods are available for the design of an adaptive PSS [46-49]. It is apt that a review of different adaptive techniques be given next. This will also serve as a preface to the proposed self-tuning scheme which is developed in chapter 6.

Sections 5.2-5.5 give a description of adaptive control systems from a historical perspective. In section 5.6 a predictive model is developed. The MV algorithm is presented in section 5.7 and the GMV algorithm in section 5.8. The pole assignment control is briefly summarised in section 5.9. A general and flexible control structure is then developed in section 5.10 followed by conclusions in section 5.11.

**5.2 Adaptive control**

In this section, a historical perspective on the development of adaptive control systems is described. Adaptive control is an area of control theory which deals with systems that have the ability to continuously adapt to changing operating conditions. The word 'adaptive' in adaptive control calls for caution when defining possible control strategies. Such controls as open-loop control, closed-loop feedback control with fixed parameters and closed-loop feedback control based on changing parameters have different
degrees of ability to alter their behaviour in sympathy with the changing process conditions. Åström [50] points out that "a meaningful definition of adaptive control, which would make it possible to look at a regulator hardware and software and decide if it is adaptive or not is still lacking". He therefore adopts the definition that "adaptive control is a special type of nonlinear feedback". The implication here is that feedback control and variable feedback gains instead of constant gains are necessary conditions for the control to be adaptive.

The concept of adaptive control developed in the 1950's with research into the design of control systems for high performance aircraft [51]. The performance characteristics of these aircraft, based on a fixed parameter, linear feedback controller, was satisfactory for one operating condition. The wide range of operating conditions of the aircraft required that the controller automatically change its control parameters and feedback gains to match the changing operating conditions. At the time, suitable hardware for the controller implementation and theory on adaptive control were not available and this first attempt at implementing an adaptive controller failed [52].

Research on adaptive control in the 1960's focussed on control systems for the guidance and tracking of space vehicles [53]. During that period, advances in the theory of stochastic control, system identification and estimation theory provided fertile ground for further development in adaptive control theory. With the advent of the microcomputer in the 1970's, it became easier and cheaper to implement adaptive controllers. Industrial applications of
adaptive control, particularly in the area of process control, were successfully undertaken. This trend continued with added vigour in the 1980's which saw the emergence of commercially available adaptive controllers. These controllers were not restricted to process control applications only, but have also found their way in power system excitation control [54].

Now, in the 1990's, it seems that the power engineering world is looking increasingly towards adaptive control strategies for enhancing dynamic stability measures [55-57]. Various adaptive schemes have been developed. Among these, three main classes may be recognized, namely gain scheduling control, model reference adaptive control and self-tuning control. These schemes lead to different ways of adjusting the controller parameters.

5.3 Gain scheduling

In this kind of control, the measurable process variables that best represent the characteristics of the process dynamics are obtained for a range of operating conditions. A schedule of controller gains, based on some design technique, corresponding to selected points in the operating range of the process is then established. This constitutes a look-up table of controller terms which is used to adjust the controller gain setting to its best value for each range of measured values [58].
Gain scheduling is simple to operate and provides a quick means of controller parameter adjustment for changing operating conditions. One disadvantage is that the look-up table must be established in advance of plant operation and where fine tuning is required, a large number of controller parameters corresponding to the different operating conditions must be obtained. Furthermore, the data obtained may be corrupted by noise or offset problems.

For a complex process, there may be more than one set of parameters to adjust based on one or more performance objectives [58]. It then becomes important to specify a mathematical model which would yield a desired response with which the process' response could be compared. The type of algorithm required for such a situation forms the basis of model reference control.
5.4 Model adaptive reference control

In Model Reference Adaptive Control (MRAC), a mathematical model specifies the desired ideal output which the plant is required to approximate for a given command input [59]. The problem is then to determine the control input to the plant. The output of the controlled plant \( y(t) \) is continually compared to that of the model \( y_m(t) \). The error signal \( e = y_m(t) - y(t) \) is used to drive an adjustment mechanism which changes the controller parameters such that the error tends to zero.

![Block diagram of model reference adaptive control](image)

Fig. 5.2 Model reference adaptive control

The block diagram of Fig. 5.2 illustrates the model reference adaptive control method where two control loops may be distinguished. There is the inner loop consisting of the plant and the controller and an adaptive outer loop. The advantage of this
loop configuration is that, in the event of the adaptive loop failing, the controller will still function, though at a sub-optimal level.

The original controller adjustment rule, known as the 'MIT rule' [59] is given by the following equation:

\[
\frac{d\theta}{dt} = -k \ e \ \text{grad}_\theta \ e
\]  

(5.1)

where \( e \) is the error between the model output and the actual plant output. The components of the vector \( \theta \) are the adjustable controller parameters and the components of the vector \( \text{grad}_\theta \ e \) are the sensitivity derivatives of the error with respect to the adjustable parameters. The constant \( k \) is a design parameter which determines the adaptation rate.

The original adjustment rule was motivated by the assumption that the controller parameters \( \theta \) change much more slowly than the other system variables. To direct the square of the error \( e \) to zero, the controller parameters are then changed in the direction of the negative gradient of \( e^2 \).

Equation (5.1) can be rewritten in the form

\[
\theta(t) = -k \int_0^t e(s) \ \text{grad}_\theta \ e(s) \ ds
\]  

(5.2)
The adjustment mechanism described by equation 5.2 can be thought of as consisting of three parts: a linear filter for computing sensitivity derivatives from plant inputs and outputs, a multiplier and an integrator.

The MRAC method discussed above is referred to as a direct one, since the adjustment rule provides for direct updating of the controller parameters. Extensions to this method have led to self-tuning techniques. These are discussed next.

5.5 Self-tuning control

Self-Tuning Control (STC), originally developed as a discrete-time method, relates to a controller which is able to self-adjust its parameters according to the changing system conditions. The controller may be regarded as consisting of two stages. The first stage is the identification one. A reduced order, linear discrete-time model is assumed to represent the essential dynamics of the plant and is regularly updated so as to make it identical to the plant. In the second stage, the updated parameters are treated as the 'true' parameters of the plant and are used to compute the controller parameters to give the new control signal. The two stages are performed during each sampling interval.
Fig. 5.3 General self-tuning control structure

Fig. 5.3 shows the general structure of a self-tuning control scheme. Two control loops can be depicted:

(i) the normal feedback loop which consists of the plant and feedback controller, and

(ii) the information loop which continually adjusts the controller parameters, based on a parameter estimation technique.

In Fig. 5.3, the plant model parameters are explicitly estimated from sampled input/output data. This method is known as explicit STC. In some other self-tuning algorithms, it is possible to re-parameterise the plant model so that it is expressed in terms of the controller parameters. The controller design step is thus eliminated and the controller parameters, rather than the plant
model ones, are estimated. This method is known as the implicit STC.

Self-tuning control was first proposed in the late 1950's by Kalman [60] at a time when computing facilities and suitable hardware were lacking. Practical applications, initially in the area of process control, became feasible as microprocessors became available. In the early 1970's, Åström and Wittenmark [61] introduced a self-tuning regulator which made use of the recursive least squares parameter estimation and was based on a control strategy known as MV. The objective of this regulator is to minimise the variance of the output, without putting any constraints on the magnitude of the control signal. The next development in self-tuning algorithms came with the introduction of the GMV control by Clarke and Gawthrop [43,62]. This type of control uses a general form of performance index which includes control effort and set point variations. Wellstead et al [63] next developed the pole assignment controller where the objective is to determine the controller parameters such that the closed-loop poles are placed at desired locations.

5.6 Predictive model

A predictive model which will be used in the various algorithms of self-tuning control is now derived.
Consider a plant given by the discrete-time transfer function
\[ q^{-k} \frac{B_1(q^{-1})}{A_1(q^{-1})} \] and subjected to stochastic disturbances as shown in Fig. 5.4 below.

The disturbances may arise, for instance, from the introduction of measuring devices or from unpredictable noisy variations. The situation depicted in Fig. 5.4 can be described by the following equation:

\[ y(t) = q^{-k} \frac{B_1(q^{-1})}{A_1(q^{-1})} u(t) + \frac{C_2(q^{-1})}{A_2(q^{-1})} \xi(t) \]  

(5.3)

where \( \xi(t) \) is Gaussian noise.

By algebraically manipulating the two terms on the right-hand side of equation (5.3), the following equation is obtained.
\[
y(t) = q^{-k} \frac{B(q^{-1})}{A(q^{-1})} u(t) + \frac{C(q^{-1})}{A(q^{-1})} \varepsilon(t) \quad (5.4)
\]

where \( A = A_1 A_2 \), \( B = B_1 A_2 \) and \( C = C_2 A_1 \), assuming \( A_1 \) and \( A_2 \) have no common factors.

The model described by equation (5.4) can be represented by the block diagram of Fig. 5.5.

At the instant \( t+k \), the output of the system represented by Fig. 5.5, is given by

\[
y(t+k) = \frac{B(q^{-1})}{A(q^{-1})} u(t) + \frac{C(q^{-1})}{A(q^{-1})} \varepsilon(t+k) \quad (5.5)
\]

Fig. 5.5 Block diagram of predictive model

At the instant \( t+k \), the output of the system represented by Fig. 5.5, is given by

\[
y(t+k) = \frac{B(q^{-1})}{A(q^{-1})} u(t) + \frac{C(q^{-1})}{A(q^{-1})} \varepsilon(t+k) \quad (5.5)
\]
The second term on the right-hand side of equation (5.5) represents the contribution of noise. It can be broken into two components as follows:

\[
\frac{C(q^{-1})}{A(q^{-1})} \xi(t+k) = F(q^{-1}) \xi(t+k) + q^{-k} \frac{G(q^{-1})}{A(q^{-1})} \xi(t+k) \quad (5.6a)
\]

and

\[
A(q^{-1}) = \frac{C(q^{-1}) - q^{-k} G(q^{-1})}{F(q^{-1})} \quad (5.6b)
\]

At this stage, the argument \( q^{-1} \) is dropped for convenience. This will also be the case elsewhere provided there is no ambiguity.

Combining equations (5.5) and (5.6b) results in

\[
y(t+k) = \frac{G}{C} y(t) + \frac{F}{C} B u(t) + F \xi(t+k) \quad (5.7)
\]

The last term on the right-hand side of equation (5.7) represents the noise components at instant \( t+k \). These are unpredictable [64]. The optimum prediction of \( y(t+k) \) at time \( t \), therefore is

\[
y(t+k | t) = \frac{G}{C} y(t) + \frac{F}{C} B u(t) \quad (5.8)
\]
and the prediction error is

$$\tilde{y}(t+k|t) = F\zeta(t+k) \quad (5.9)$$

### 5.7 Minimum variance control

MV control is a simple form of optimal control, where the control signal is determined such that the predicted value of the output is equal to a desired value. The idea is to minimise the variance of the output. The predictive model described by equations (5.8) and (5.9) is used. The objective function $I$ may be expressed as follows;

$$I = \mathbb{E}\left\{\left[\hat{y}(t+k|t) + \tilde{y}(t+k|t)\right]^2\right\} \quad (5.10)$$

where $\mathbb{E}$ is the statistical expectation operator.

The output will approach its optimum value as the prediction error $\tilde{y}(t+k|t)$ tends toward zero so that in equation (5.10) the product of $\hat{y}(t+k|t)$ and $\tilde{y}(t+k|t)$ is negligible. The objective function then reduces to

$$I = \mathbb{E}\left\{\left[\hat{y}(t+k|t)\right]^2\right\} + \mathbb{E}\left\{\left[\tilde{y}(t+k|t)\right]^2\right\} \quad (5.11)$$
The prediction error $\bar{y}(t+k \mid t)$ is unaffected by the control $u(t)$ as the former is composed of the contribution of noise at instants $t+1$ to $t+k$. So, the objective function $I$ is minimum when $\bar{y}(t+k \mid t)$ is set to zero by the choice of control $u(t)$. From equation (5.8)

$$Gy(t) + FBu(t) = 0 \quad (5.12)$$

$$u(t) = - \frac{G}{FB} y(t) \quad (5.13)$$

Equation (5.13) is the feedback law for MV control. The main features of this type of control are summarised below:

(a) The zeros of the system, given by polynomial $B$, are explicitly cancelled by the controller poles. For non-minimum phase situations, any error in the mathematical model will result in an unstable closed-loop system [37].

(b) For an implicit self-tuner, where the feedback parameters are estimated directly, the parameter estimates $\hat{G}$ and $\hat{E}$ ($E=FB$) in equation (5.12) are not unique since this equation may be multiplied by an arbitrary constant without affecting the calculation of $u(t)$. This may lead to very large or small values with possible numerical problems developing [37].

(c) When a large control signal is required, saturation of the control output due to physical system limits will occur. This may lead to deterioration of the system response and sometimes result in instability [37].
(d) The MV control algorithm is simple and thus easy to implement.

5.8 Generalised minimum variance control

As pointed out in the previous section, MV control makes no attempt to penalise the control effort, nor is it suitable for non-minimum phase situations; nor does it ensure the optimal following of set point variations. These difficulties are overcome by the GMV controller proposed by Clarke and Gawthrop [43 and 62]. Their method involves the prediction of an auxiliary output $\phi(t)$ defined in terms of a user-specified transfer function $P$ such that

$$\phi(t) \triangleq Py(t)$$

The identity given by equation (5.6a) then becomes

$$\frac{PC}{A} = F + q^{-k} \frac{G}{A}$$

The control objective is to minimise a general cost function given by

$$I = E \left\{ \phi^2(t+k) \right\}$$

where

$$\phi(t+k) = Py(t+k) + Q u(t) - R v(t)$$
\( v \) is the set point and \( P, Q \) and \( R \) are polynomials in the backward shift operator. In Appendix IV, it is shown that the control law required to minimise the variance of \( \phi \) is

\[
    u(t) = - \frac{G}{H} y(t) - \frac{E}{H} v(t)
\]  

(5.17)

where

\[
    H = BF + QC 
\]  

(5.18)

\[
    E = - RC 
\]  

(5.19)

The block diagram of Fig. 5.6 represents a structure of the GMV control scheme, based on equation (5.17).

Fig. 5.6 GMV control scheme

At the instant \( t \), the output of the system is
\[ y(t) = - q^{-k} \frac{B G}{A H} y(t) - q^{-k} \frac{B E}{A H} v(t) + \frac{C}{A} \xi(t) \] (5.20a)

or

\[ A H y(t) + q^{-k} B G y(t) = - q^{-k} B E v(t) + C H \xi(t) \] (5.20a)

So,

\[ y(t) = - q^{-k} \frac{B E}{A H + q^{-k} B G} v(t) + \frac{C H}{A H + q^{-k} B G} \xi(t) \] (5.21)

Using equations (5.15), (5.18) and (5.19), the output \( y(t) \) can be obtained in terms of \( P, Q \) and \( R \) as

\[ y(t) = q^{-k} \frac{B R}{P B + Q A} v(t) + \frac{B F + Q C}{P B + Q A} \xi(t) \] (5.22)

It can be shown (see Appendix IV) that the control input is given by

\[ u(t) = \frac{A R}{P B + Q A} v(t) - \frac{G}{P B + Q A} \xi(t) \] (5.23)

The characteristic equation of the closed-loop (\( PB + QA = 0 \)) involves polynomials \( P \) and \( Q \) and the roots no longer depend solely on \( B \).
By the suitable choice of $P$ and $Q$, closed-loop stability is achieved.

The main features of GMV control are summarised below:

(a) The difficulties associated with non-minimum phase situations are overcome by the introduction of additional polynomials in the characteristic equation which can be selected to enhance stability.

(b) It is more complex than MV control and requires more computation.

(c) It is sensitive to parameter variation.

(d) A weighting factor polynomial ($Q$) helps in limiting the control output.

It is not always easy to choose suitable values for polynomials $P$ and $Q$ to ensure well damped locations for the closed-loop poles. An alternative self-tuning method which looks at the strategy of locating closed-loop poles to desired locations is the pole assignment control.

5.9 Pole assignment control

In Pole Assignment (PA) control, the controller parameters are chosen such that the closed-loop poles are placed at well damped
locations. There is no attempt to cancel the zeros of the plant with the poles of the controller. This method was first proposed by Edmunds [65] and was developed further by Wellstead et al [63].

The predictive model given by equation is again considered, i.e.

\[ y(t) = q^{-k} \frac{B(q^{-1})}{A(q^{-1})} u(t) + \frac{C(q^{-1})}{A(q^{-1})} \xi(t) \]  (5.24)

A standard feedback control law is assumed:

\[ u(t) = -\frac{G}{H} y(t) \]  (5.25)

Substituting equation (5.25) in (5.24) and rearranging gives

\[ y(t) = \frac{CH}{AH + q^{-k}BG} \xi(t) \]  (5.26)

The next step is to choose the coefficients of \( H \) and \( G \) such that

\[ AH + q^{-k}BG = CT \]  (5.27)
where $T$ is a polynomial in the form $\prod_{i} \left(1 - \alpha_{i} q^{-1}\right)$ and the $\alpha_{i}$ correspond to stable poles.

Using the identity given by equation (5.27), the closed-loop characteristics are then described by

$$y(t) = \frac{H}{T} \xi(t)$$

and

$$u(t) = -\frac{G}{T} \xi(t)$$

With suitable choice for the roots of polynomial $T$, closed-loop stability can be guaranteed, although good system response does not always follow. This constitutes the major feature of PA control.

5.10 A general STC structure

In the preceding sections, MRAC and STC have been treated as two separate and different members of the same family of adaptive control. Egardt [44] and later Landau [66] showed that there are more similarities than differences between the two approaches. MRAC assumes a predefined reference model whose output is the desired one for a given command signal. It is viewed more as a method for solving adaptive servo problems and the design is mostly performed in a deterministic framework. STC was originally proposed to solve the regulator problem in a stochastic framework [61].
In the proposed self-tuning scheme which is described in chapter 6, both the servo and regulator problems are addressed in the context of an ECS. A general self-tuning control structure is used for that purpose.

![Fig. 5.7 A general STC structure](image)

Fig. 5.7 shows a block diagram of a general STC scheme. $u_c$ is a reference signal and $R$, $S$ and $T$ are polynomials in the backward shift operator. $w(t)$ is a stochastic disturbance. The design method consists of a combination of zero cancellation with pole placement algorithms.

The servo problem arises as a result of changes in the reference signal while the disturbance $w(t)$ is assumed negligible. The output of the closed-loop system in Fig. 5.7 for these conditions is

$$y(t) = q^{-k} \frac{T}{AR + q^{-k}S} u_c$$

(5.30)
The objective of the controller design is to have the plant output \( y(t) \) equal to a reference model output \( y_m(t) \). Let the mathematical model be given by

\[
y_m(t) = \frac{q^{-k}}{A_m} u_c
\]  

(5.31)

where \( A_m \) is a polynomial in the backward shift operator whose roots are selected to be at desired locations.

Equating (5.30) to (5.31) results in

\[
T A_m = A R + q^{-k} S
\]  

(5.32)

This is the Diophantine equation, which when solved, yields the controller parameters. The choice of polynomial \( T \) is somewhat affected by initial values, but may, in general, be determined arbitrarily. This leads to many solutions to equation (5.32) for \( R \) and \( S \). For a unique solution, the degree of \( R \) must be less or equal to the time delay \( k-1 \). To avoid a non-causal control law, the leading parameter in polynomial \( R \), i.e. \( R(0) \) must not be zero. This condition sets the leading parameter in polynomial \( T \), i.e. \( T(0) \) to
be non-zero. The $R$ and $T$ polynomials are usually scaled so that $T(0)=R(0)=1$ [44].

When the effects of the disturbance are also considered, both servo action and disturbance rejection must be provided. The output of the closed-loop system of Fig. 5.7 then is

$$y(t) = q^{-k} \frac{T}{AR + q^{-k}S} \ u_c + q^{-k} \frac{CR}{AR + q^{-k}S} \ w(t)$$  \hspace{1cm} (5.33)

Since equation (5.31) satisfies the servo condition, equation (5.33) becomes:

$$y(t) = \frac{q^{-k}}{A_m} \ u_c + q^{-k} \frac{CR}{AR + q^{-k}S} \ w(t)$$ \hspace{1cm} (5.34)

The solution to equation (5.34) is dependent on the nature of the disturbance $w(t)$. When the latter is a moving average given by

$$w(t) = C \varsigma(t)$$ \hspace{1cm} (5.35)

where $\varsigma(t)$ consists of independent, zero-mean variables, then the optimal choice of polynomial $T$ is [44]
When the disturbance is sinusoidal, as is the case in power systems, an effective method is based on the application of the internal model principle. This requires manipulation of the polynomial $R$ and solving a modified Diophantine equation. These issues are discussed in the next chapter.

5.11 Conclusions

An insight into adaptive control has been given. The main strands of this important area of control engineering have been discussed. Self-tuning control strategies based on minimum variance, generalised minimum variance and pole assignment have been analysed. The corresponding control laws have been derived and their main features summarised. A general self-tuning control structure which allows both the servo and regulator problem to be addressed has been analysed. This serves as a preview to the self-tuning scheme which is proposed in the next chapter.
Chapter 6

A self-tuning scheme for power system disturbance rejection
Abstract

The current application of Self-Tuning Control (STC) in power systems is extended to provide both positive damping characteristics and disturbance rejection. A control strategy based on the Internal Model Principle (IMP) is used to directly cancel tie-line oscillatory disturbances. A Self-tuning Regulator (STR) is used which complements the action of the Power System Stabiliser (PSS) by injecting an amplitude-limited additional signal to the summing point of the supplementary excitation control loop. For reliability and efficiency, the self-tuner is made responsible for regulation (disturbance rejection) action, while the conventional Excitation Control System (ECS) performs the necessary servo action. A distinct feature of the proposed STR is the safeguard against lack of excitation of the information loop under normal operating conditions, in which case the STR is simply deactivated. Simulation studies performed on a typical power system excitation control model demonstrate clearly the benefits of incorporating the IMP in adaptive power stabilizing schemes. Results show effective cancellation of tie-line time-varying oscillatory disturbances and enhanced damping behaviour of the power system.

6.1 Introduction

Self-tuning PSS are used to provide positive damping signals over a wide range of operating conditions. In these stabilisers, the gain
settings are automatically adjusted on-line to follow closely the changing operating conditions. Self-tuning techniques based on minimum variance, pole-shifting and PID strategies have produced good results [67-69]. These techniques involve the identification of the parameters of the plant model through continual sampling of the input and output signals, thus keeping track of the operating conditions, and on the calculation of the control action during each sampling period, based on the particularly preselected control strategy. In all these applications, the emphasis has been to maintain good damping characteristics as the operating conditions change. One important feature of STC that has not been exploited in power systems is the direct decoupling of tie-line disturbances in addition to providing positive damping signals. This extra feature can be accommodated using a control strategy based on the IMP. In this strategy, the STR is equipped with a model of the disturbance dynamics which is used to cancel the disturbance itself. Clarification of this principle is provided in section 6.2.

Commonly adopted STC schemes [70,71] involve closing the supplementary excitation control loop by a computer. Lack of enough probing of that loop causes the estimator to exhibit a sluggish convergence behaviour. Since the probing ability is dependent on the disturbance level, the self-tuner stops tracking slow parameter drifts during steady state operating conditions. This may lead to a highly undesirable situation when the negative damping introduced via the Automatic Voltage Regulator (AVR) is not properly counteracted. Reliability considerations indicate that such a situation can be avoided by incorporating the STR in a loop
which is independent of the PSS and AVR and which becomes active only when sustained oscillations persist on the tie-lines.

The proposed STR scheme explores further the application of STC to power systems without compromising the benefits of the PSS. This is depicted in Fig. 6.1 where the power system and its associated control loops are represented by blocks 1, 2 and 3. The STR is represented by the block marked 'STR software'. $\Delta P_d$ is a tie-line oscillatory disturbance, $\Delta \delta$ is the change in the angle of oscillation $\delta$ while $u_R$ and $u$ are the reference and control signals respectively. For clarity, some of the feedback blocks have been omitted.

The scheme in Fig. 6.1 retains the benevolent effects of the PSS while an additional control loop comprising a STR provides the
necessary control action for directly cancelling the tie-line disturbances. The STR does not impede other control actions and is tuned to intervene only when the disturbance level exceeds a certain pre-set value, allowing the PSS to perform its useful but constrained role.

The general features of the regulator are summarized below:

* It allows direct cancellation of the disturbance mode without interfering with the effects of the PSS,

* It is completely fault-failure proof as it forms an independent control loop in the system and nullification of the probing ability of the regulator is of no consequence to the remaining control blocks,

* It is effective against time-varying sinusoidal disturbances since the estimation algorithm is based on a fixed width information window, and

* It is readily implementable on systems equipped with conventional PSS, with little modification involved.

This chapter is organized as follows: section 6.2 introduces the concept of the IMP with an application in a typical feedback control system. Section 6.3 describes and analyses the proposed self-tuning scheme based on the IMP. Simulation results are included in sections 6.4 and 6.8. Validation of a discrete-time reduced order model is given in section 6.5. The control signal necessary for the
adaptive control of the power system is derived in section 6.6 and a summary of the STR algorithm is given in section 6.7. Conclusions are drawn in section 6.9.

6.2 The Internal Model Principle

The Internal Model Principle (IMP) [72,73] is employed in control schemes to provide closed loop stability and output regulation for small variations in certain system parameters. It can be stated as follows:[74]

"A regulator is structurally stable if the controller utilizes feedback of the regulated variable, and incorporates in the feedback loop a suitably reduplicated model of the dynamic structure of the exogenous signals which the regulator is required to process".

Using the IMP, the output of a plant can be made to asymptotically track a reference signal while asymptotically rejecting a disturbance [75]. Application of the IMP in power systems offers flexibility in improving the damping behaviour of the system while simultaneously cancelling the effect of external disturbances. In this section, a simple example is used to demonstrate the merits of the IMP. Since PSS are usually designed as derivative feedback controllers which are tuned to improve the damping factors associated with the system modes, the following example makes use of such feedback to illustrate the additional feature that can be extended to a power stabilizing system for disturbance rejection purposes.
Consider the feedback control system represented by the block diagram of Fig.6.2 where $G(s)$ is a plant subjected to exogenous signals, namely reference signal $u(s)$ and disturbance signal $d(s)$. $H(s)$ is the derivative feedback controller and $K_f$ is a positive gain constant.

\[ y(s) = u(s) + d(s) \quad (6.1) \]

The plant output is given by the following equation:

\[ y(s) = \frac{1}{s^2 + (K_f + 2\zeta \omega_n)s + \omega_n^2} u(s) + \frac{1}{s^2 + (K_f + 2\zeta \omega_n)s + \omega_n^2} d(s) \quad (6.1) \]

In equation (6.1), the first term on the right-hand side represents servo action while the second the disturbance rejection function. It is clear that the damping factor has been improved from $\zeta$ to $1$. 

Fig.6.2 A feedback control system
However, the disturbance $d(s)$ has not been decoupled. Disturbance rejection may be achieved by implementing the IMP as a transfer block, consisting of the reduplicated model of the dynamic structure of the disturbance signal, inserted between the two summing points $\Sigma 1$ and $\Sigma 2$, as shown in Fig.6.3.

\[
\zeta + \frac{K_f}{2\omega_n}
\]

The plant output equation for the system in Fig. 6.3 is given by:

\[
y(s) = \frac{G(s)}{R(s) + G(s)H(s)} u(s) + \frac{G(s)R(s)}{R(s) + G(s)H(s)} d(s)
\]  

(6.2)

The second term on the right-hand side of equation (6.2) determines the response of the plant, as far as the effect of the disturbance is concerned. To effectively cancel the disturbance,
d(s) must be made equal to \( \frac{1}{R(s)} \). This means that when \( d(s) \) is a sinusoidal disturbance of frequency \( \omega \), this transfer block will take the form of \( \frac{1}{R(s)} = \frac{1}{s^2 + \omega^2} \) and when \( d(s) \) is a step function \( \frac{1}{R(s)} = \frac{1}{s} \). What remains of that term after the cancellation of \( d(s) \) is a decaying transient since \([ R(s) + G(s) H(s)]\) is a stable function. The plant steady state performance will then be dictated only by the first term on the right-hand side of equation (6.2).

The analysis may be extended to self-tuning schemes, as is demonstrated in the next section. However, the transfer function representing the power system has to be put in a form that can be represented by an equation similar to equation (6.1). The rest of this section is devoted to this alternative representation. The ECS described in chapter 1 is reproduced below for further analysis.

In Fig.6.4, \( \Delta P_d \) represents a tie-line power disturbance producing \( \Delta \delta \), a change in the power angle \( \delta \). Successive manipulations of the
transfer blocks of Fig.6.4 lead to the simplified diagram shown in Fig.6.5, where the disturbance rejection function is given by:

\[
G_{\text{DRF}}(s) = \frac{\Delta \delta}{\Delta P_d} = \frac{C_0 s^2 + C_1 s + C_2}{A_0 s^4 + A_1 s^3 + A_2 s^2 + A_3 s + A_4} \quad (6.3)
\]

and the transfer function representing servo action by:

\[
G_{\text{servo}}(s) = \frac{\Delta \delta}{\Delta V_r} = \frac{B_0 s + B_1}{A_0 s^4 + A_1 s^3 + A_2 s^2 + A_3 s + A_4} \quad (6.4)
\]

The coefficients of the above equations are based on modelling details of a typical ECS as given in Appendix I.

The alternative block diagram representation of Fig.6.5 allows the designer to analyse the effects of the controller on both the servo

![Fig.6.5 Alternative representation of power system](image-url)

The alternative block diagram representation of Fig.6.5 allows the designer to analyse the effects of the controller on both the servo
(damping) and regulation (disturbance rejection) performance of the power system. Section 6.3 provides this analysis for an adaptive scheme.

6.3 A self-tuning scheme for decoupling power system tie-line disturbances

A typical control scheme is responsible for two types of action: first, the servo action which aims at forcing the output to follow a command signal with appropriate speed and limited overshoot. The other function is to counteract the effect of disturbances such that the output remains constant at the level pre-selected by the set point. In the majority of control applications, these two functions are conflicting [76]. Excitation control in power systems emphasizes this conflict since servo action is required for both field-forcing and voltage level control, while regulation action aims at counteracting tie-line disturbances. A distinct additional feature results from the negative damping effects due to the inherent feedback through the dynamic variable $K_5$. The common practice is to use high pass filters (wash-out circuits) to restrict the PSS to the regulation action and a band stop filter (lag-lead compensator) in the forward path of the AVR as a cautious measure against negative damping effects [33]. Efficient schemes insert the field-forcing signal at a summing point that bypasses the dynamic gain reduction block.

STC schemes have been proposed to achieve both servo and regulation through a general feedback/feedforward controller
structure [77]. The proposed STR scheme provides a means of satisfying both the servo and regulation criteria. The key to the design is to equip the regulator with a duplicate model of the disturbance environment. According to the IMP, by placing the poles of the regulator at locations that match the poles of the disturbance model, the disturbance mode is thereby decoupled. Since the regulator is to be implemented digitally, it is necessary to use discrete-time analysis for the whole system.

Applying Z-transform to the transfer functions given by equations (6.3) and (6.4) results in the discrete-time representation of Fig.6.6 where

\[
\frac{q^{-k} C(q^{-1})}{A(q^{-1})} = Z [G_{DRF}(s)]
\]  

(6.5)

and

\[
\frac{q^{-k} B(q^{-1})}{A(q^{-1})} = Z [G_{servo}(s)]
\]  

(6.6)

Fig.6.6 Discrete time representation of power system model
In Fig.6.6 Δu(t), ΔPd (t) and Δδ(t) are small changes in the input control signal, the disturbance and the output angle of oscillation at the instant t respectively. k ≥ 1 is the system delay. A(q^{-1}), B(q^{-1}) and C(q^{-1}) are polynomials representing the system transfer functions as seen by ΔPd(t) and Δu(t).

For further analysis, a sampled sinusoidal tie-line power disturbance is described. It is known that oscillatory tie-line power disturbances are the most common type of disturbances affecting the dynamic stability of power systems. Considering a sampling period T_s second, a sinusoidal disturbance of frequency ω_0 and amplitude \( \hat{P} \) is represented by the following sequence [78]:

\[
ΔP_d (t) = \frac{q^{-1} K_p}{1 + \alpha q^{-1} + q^{-2}} \tag{6.7}
\]

where \( K_p = \hat{P} \sin \omega_0 T_s \)

\( \alpha = - 2 \cos \omega_0 T_s \) is a factor which characterizes the frequency of the disturbance.
Proper selection of the sampling period restricts the angle $\theta$ to a value between $5^\circ$ and $60^\circ$ (see Fig. 6.7).

The proposed self-tuning strategy is twofold:

(i) to decouple $\Delta P_d(t)$ so that it is not reflected on the output $\Delta \delta(t)$ and

(ii) to obtain an output which tracks a reference input $\Delta u_R(t)$.

Both requirements can be achieved using the proposed scheme shown in Fig. 6.8.
The STR is represented by the blocks within the frame. $R_1(q^{-1})$, $S(q^{-1})$ and $T(q^{-1})$ are polynomials which are to be determined to satisfy the control strategy requirements. A duplicate of the disturbance model poles, represented by the transfer block $\frac{1}{1+\alpha q^{-1} + q^{-2}}$, has been inserted in the forward path of the regulator as shown in Fig.6.8. Apart from that block, the remaining ones within the frame represent a typical structure of a self-tuning controller, as discussed in chapter 5.

The following analysis is provided to demonstrate that the proposed regulator can achieve the above mentioned design requirements. Combining the regulator of Fig.6.8 with the discrete time model of Fig.6.6 results in the system shown in Fig. 6.9.
Referring to Fig. 6.9 and using the linear superposition theorem, the following equation is obtained:

$$\Delta \delta(t) = \frac{q^{-k} T}{AR_1(1 + \alpha(t) q^{-1} + q^{-2}) + q^{-k} S} u_R(t)$$

$$+ \frac{q^{-k} C R_1 (1 + \alpha(t) q^{-1} + q^{-2})}{A R_1 (1 + \alpha(t) q^{-1} + q^{-2}) + q^{-k} S} \Delta P_d(t) \quad (6.8)$$

For clarity, the polynomial argument $q^{-1}$ has been dropped from equation (6.8). This will also be the case elsewhere provided there is no ambiguity. The first term on the right hand side of equation (6.8) depicts the servo action while the second term quantifies the effect of the disturbance on the power angle oscillation. The design requirements regarding servo action can be achieved by selecting...
polynomials \( R_1(q^{-1}), S(q^{-1}) \) and \( T(q^{-1}) \) to satisfy the following conditions:

\[
\frac{q^{-k} T}{A R_1(1 + a(t) q^{-1} + q^{-2}) + q^{-k} S} = \frac{q^{-k}}{A^m}
\]  \(6.9\)

which results in the Diophantine equation [6.15]:

\[
T A^m = A R_1(1 + \alpha(t) q^{-1} + q^{-2}) + q^{-k} S
\]  \(6.10\)

\(A^m\) is a polynomial selected to provide well damped factors to satisfy the servo requirements. Typically, applications of self-tuning control rely on a choice of \(A^m\) as a second order polynomial with preselected relative damping and natural frequency. In the proposed scheme, however, it is selected to be quite close to the actual polynomial \(A(q^{-1})\) since the existing PSS/AVR loops are assumed to be properly tuned to provide eigen patterns with well damped modes. To satisfy the regulation requirements, the second term on the right hand side of equation (6.8) should be nullified.

Combining equations (6.8) and (6.9) results in:

\[
\Delta \delta(t) = \frac{q^{-k}}{A^m} u_R(t) + \frac{q^{-k} CR_1 (1 + \alpha(t) q^{-1} + q^{-2})}{TA^m} \Delta P_d(t)
\]  \(6.11\)
For a general disturbance $\Delta P_d$, the following frequency domain relationship holds:

$$\Delta \delta(j\omega) \bigg|_{u_R=0} = \frac{C(j\omega) R(j\omega) \Delta P_d(j\omega)}{T(j\omega) A^m(j\omega)}$$

(6.12)

where $R = R_1(j\omega) [1 + \alpha(t)e^{-j\omega T_s} + e^{-2j\omega T_s}]$

Choosing $T(j\omega) = C(j\omega)$ results in:

$$\Delta \delta(j\omega) \bigg|_{u_R=0} = \frac{R(j\omega) \Delta P_d(j\omega)}{A^m(j\omega)}$$

(6.13)

which minimizes the effect of the disturbance on the output.

For a sinusoidal disturbance, using equation (6.7), equation (6.11) becomes to:

$$\Delta \delta(t) = \frac{q^{-k}}{A^m} u_R(t) + \frac{q^{-k} R_1(1 + \alpha(t) q^{-1} + q^{-2})}{A^m} \frac{q^{-1} K_p}{(1 + \alpha(t) q^{-1} + q^{-2})}$$

(6.14)
The second term on the right hand side of this equation represents a decaying sequence which decouples the disturbance from the output after a transient period. This term will decay faster by selecting the regulator polynomial $T(q^{-1})$ to be equal to the system polynomial $C(q^{-1})$. Clearly, the poles on the unit circle that represent the sustained sinusoidal disturbance have been cancelled due to the special structure of the selected regulator.

The above analysis uses algebraic manipulations of the transfer functions to justify the need of the term \( \frac{1}{1 + \alpha(t)q^{-1} + q^{-2}} \) to be cascaded with the regulator to decouple oscillatory disturbances. It is interesting to observe the similarity of this proposal with the dynamic gain reduction concept. With reference to equation (6.13), the term $R(j\omega)$ can be interpreted as a notch filter with the following characteristics:

\[
|R(j\omega)| = \begin{cases} 
  |R_1(j\omega)| & \omega_o - \Delta\omega < \omega < \omega_o + \Delta\omega \\
  0 & \omega = \omega_o
\end{cases}
\]  

(6.15)

where $\omega_o$ is the notch frequency and $\Delta\omega$ is a small deviation of the angular frequency.

For unity time delay where $k$ is equal to 1, the degree of polynomial $R_1$ is zero, i.e. $R_1$ is a constant (say $r$), and the filter characteristics will be as shown in Fig. 6.10.
Polynomials $T(q^{-1})$, $A_m(q^{-1})$ and $A(q^{-1})$ may be chosen as monic without loss of generality, which renders $r = 1$ (see equation (6.10)) and produces the typical characteristics of a digital notch filter.

It is obvious that other choices, e.g. band stop filters can also be used and implemented to replace the filter $\frac{1}{1 + \alpha(t) q^{-1} + q^{-2}}$. A wide band filter, however, has detrimental effects on the transient stability since it attenuates a wider range of frequency unnecessarily. The proposed adaptive scheme justifies the use of a narrow band stop filter, i.e. a notch filter implementation. In section 6.4, the proposed STR is tested under various simulation conditions.

6.4 Regulator parameters

The parameters of the regulator are obtained by solving the Diophantine equation (6.10). The following assumptions are made:
\[ \partial A = 2 \]
\[ \partial B = \partial C = 1 \]
\[ k = 1 \]  \hspace{1cm} (6.16)

where \( \partial \) stands for the degree of the polynomial.

It should be noted that the original model has higher dimensions as indicated in Appendix I. Simulation results, however, indicate good performance based on the lower order assumptions. The following solutions are obtained by combining equation (6.16) with (6.10):

\[ \partial A^m = 2 \]  \hspace{1cm} (a)
\[ \partial R_1 = k - 1 = 0 \]  \hspace{1cm} (b)
\[ \partial S = \max[ \partial A^m - k, n_a + 2 + \partial R_1 - k] \]  \hspace{1cm} (c)
\[ s_0 = c_0 a_1^m + c_1 - \alpha - a_1 \]  \hspace{1cm} (d)
\[ s_1 = c_1 a_1^m + c_0 a_2^m - a_2 - 1 - a_1 \alpha \]  \hspace{1cm} (e)
\[ s_2 = c_1 a_2^m - a_1 - a_2 \alpha \]  \hspace{1cm} (f)
\[ s_3 = -a_2 \]  \hspace{1cm} (g) \hspace{1cm} (6.17)
\[ r_0 = b_0 \]  \hspace{1cm} (h)
\[ r_{10} = b_0 \alpha + b_1 \]  \hspace{1cm} (k)
\[ r_{20} = b_1 \alpha + b_0 \]  \hspace{1cm} (l)
\[ r_{30} = b_1 \]  \hspace{1cm} (m)
\[ r_1 = 1 \]  \hspace{1cm} (n)
\[ t_0 = c_0 \]  \hspace{1cm} (o)
\[ t_1 = c_1 \]  \hspace{1cm} (p)

The STR may be made solely responsible for regulation action by selecting the model polynomial \( A^m(q^{-1}) \) to be very close to the
system polynomial $A(q^{-1})$ such that the STR does not change the servo performance of the loop.

6.5 Validation of model

A model order reduction technique was used to obtain a second order polynomial $A(q^{-1})$ which produces the best fit [79] for the frequency characteristics of the original system within the frequency range of interest such that:

\[
\min_{A,B} \left\| \frac{B(e^{j\omega T_s})}{A(e^{j\omega T_s})} - \frac{B_s(j\omega)}{A_s(j\omega)} \right\|_2 \quad : \quad \omega_L \leq \omega \leq \omega_H
\] (6.18)

\[
\min_{A,C} \left\| \frac{C(e^{j\omega T_s})}{A(e^{j\omega T_s})} - \frac{C_s(j\omega)}{A_s(j\omega)} \right\|_2 \quad : \quad \omega_L \leq \omega \leq \omega_H
\] (6.19)

where $\omega_L$ and $\omega_H$ are two frequency points selected to cover the low frequency oscillation range. The subscript $s$ is used for the original higher order polynomials and the symbol $\| \|_2$ stands for the Euclidian norm.
Fig. 6.11 Comparison of original and reduced second order system characteristics
The frequency characteristics of the original and reduced order models are shown in Fig. 6.11 and the numerical results are given in table 6.1 below.

### Table 6.1 Model order reduction details

<table>
<thead>
<tr>
<th>Continuous time - Original system</th>
<th>Discrete time - lower order system</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_s(s) = 0.0072 s^4 + 0.017 s^3 + 0.42 s^2 + 2.1 s + 5.2$</td>
<td>$A(q^{-1}) = 1 - 1.42 q^{-1} + 0.485 q^{-2}$</td>
</tr>
<tr>
<td>$B_s(s) = 0.518 s + 10.38$</td>
<td>$B(q^{-1}) = 0.423 q^{-1} - 0.228 q^{-2}$</td>
</tr>
<tr>
<td>$C_s(s) = 0.09 s^2 + 2 s + 4.8$</td>
<td>$C(q^{-1}) = 0.597 q^{-1} - 0.531 q^{-2}$</td>
</tr>
</tbody>
</table>

### 6.6 Control signal synthesis

The first order polynomial $T(q^{-1})$ was chosen to be equal to $C(q^{-1})$ where polynomial $C(q^{-1})$ satisfies:

$$R(q^{-1}) u(t) = C(q^{-1}) u_R(t) - S(q^{-1}) \Delta \delta(t)$$  \hspace{1cm} (6.20)

from which the control signal is obtained as:

$$u(t) = \frac{1}{r_0} [c_0 u_R(t) + c_1 u_R(t-1) - s_0 \Delta \delta(t) - s_1 \Delta \delta(t-1)$$

$$- s_2 \Delta \delta(t-2) - s_3 \Delta \delta(t-3) - r_{10} u(t-1) - r_{20} u(t-2)$$

$$- r_{30} u(t-3)]$$  \hspace{1cm} (6.21)
The steps involved in the self-tuning algorithm are summarized below:

6.7 Summary of STR algorithm

Step 1

Using recursive least squares, update parameters of polynomials $A(q^{-1})$ and $B(q^{-1})$.

Step 2

Update $\alpha$ estimate according to reference[44]

Step 3

Substitute estimated values in equation (6.17) (d) to (m)

Step 4

Compute control signal using equation (6.21)

6.8 Simulation results

Simulation studies were performed on an ECS, connected to a large network via a transmission line carrying an oscillatory disturbance. The parameters of the machine and its associated controls are
given in Appendix I. The system is represented by the polynomials
given in table 6.1. The discrete-time ECS model was equipped with
the STR accommodating the IMP and tests were performed under
the following conditions:

(a) A sustained tie-line oscillatory disturbance given by \( \Delta P_d(t) = 0.1 \sin (\omega_o t + \phi) \) p.u. where \( \omega_o \) is a fixed frequency, and a step change in the reference voltage were injected in the system. Three different disturbing frequencies, namely 3.14, 6.28 and 15.7 rad/sec were considered.

(b) A sustained tie-line time-varying oscillatory disturbance given by \( \Delta P_d(t) = 0.1 \sin (\omega_v t + \phi) \) p.u. where \( \omega_v \) is a time-varying frequency ranging from 15.7 to 3.14 rad/sec, and a step change in the reference voltage were then injected in the system.

Three different reference models were used in the tests. These are listed in the relevant graphs shown below.

The conditions described in (a) and (b) above were then applied to the continuous time ECS model which was equipped with a PSS with fixed parameters properly tuned to dampen the local modes of oscillation. The simulation results which follow show the time response of the ECS under the conditions given above.
Fig. 6.12 Power system subjected to a step reference input and a tie-line oscillatory disturbance with a frequency fixed at 3.14 rad/sec
(i) $A^m = 1$

(ii) $A^m = 1 - 0.5 q^{-1}$

(iii) $A^m = 1 - 1.412 q^{-1} + 0.485 q^{-2}$

(a) Time response

(b) Parameter convergence

Fig.6.13 Power system subjected to a step reference input and a tie-line oscillatory disturbance with a frequency fixed at 6.28 rad/sec
Fig. 6.14 Power system subjected to a step reference input and a tie-line oscillatory disturbance with a frequency fixed at 15.7 rad/sec
Fig. 6.15 Power system subjected to a step reference input and a tie-line oscillatory disturbance with varying frequency ranging from 15.7 to 3.14 rad/sec
In order to avoid a non-causal control law, the parameter $b_0$ was fixed to its estimated value of 0.423. In practice, this is not a hindrance [44]. This value or a reasonable approximation of it would have been established by a preliminary identification process.

Figs. 6.12-6.14 show the system response to a time invariant oscillatory disturbance. With the STR applied to the ECS, the disturbance is decoupled in all cases after a very short transient period. The system is more responsive when the reference model is $A^m = 1$, but the overshoot is the largest of the three cases considered. Overall, the STR performs very well.

Without the STR, the oscillations persist throughout the 30 second period. The magnitude of these oscillations are substantially reduced, but not eliminated, when the disturbing frequency is in the vicinity of 15.7 rad/sec, confirming the positive effect of the PSS at that frequency.

When an oscillatory disturbance with varying frequency ranging from 15.7 to 3.14 rad/sec appears on the tie-line, the STR is quite effective in decoupling it, doing so in around 18 seconds. There is no overshoot with the second order reference model. Without the STR, some attenuation occurs in the magnitude of the oscillation around the local mode. This magnitude, however increases to a steady value at the lower frequency range. Parameter convergence is satisfactory in all cases.

The above results confirm the effectiveness of the STR when equipped with the IMP. In these studies, the regulator performed
the two actions of servo and regulation such that the output followed the reference signal while decoupling the tie-line disturbance. A special feature of the proposed scheme is the flexibility offered to concentrate on either action. Further simulation experiments have been conducted to investigate the effect of these actions on the control signal magnitude.

Fig.6.16 shows the magnitude of the control effort and Fig.6.17 shows the control energy for two cases. In the first case, the regulator was made responsible for regulation only while in the other, the regulator provided for both regulation and reshaping the dynamics of the closed loop system such that \( A^m = 1 \). Clearly, the offered flexibility can be utilized to prevent the saturation that would occur if the control signal should exceed the band limit constraints. For instance, as shown in Fig.6.16, a 10% control signal limit would not be violated when the regulator is restricted to counteract the disturbance.

\[
\begin{align*}
K &> 3 \\
E &> E \\
U &> U
\end{align*}
\]

**Fig.6.16** Comparison of control effort required for models depicted in Fig.6.15 (i) and (ii)
The simulation studies presented in this section are based on the decoupling of a single sinusoidal disturbance. In a multi-machine environment, numerous modes of oscillation are present. As pointed out in the discussions to reference [33], it would be impractical to provide on-line optimal damping characteristics for all these modes. This scheme, whilst shown to be effective against a single sinusoidal disturbance, however may be adapted to counteract many modes. In such a case, the transfer block of Fig. 6.3 implementing the IMP would be modified to accommodate a higher order polynomial \(R(q^{-1})\) as follows:

\[
R(q^{-1}) = R_1 \left[ \prod_{i=1}^{i=m} (1 + \alpha_i q^{-1} + q^{-2}) \right]
\]  

(6.22)

where \(m\) is the number of modes.
Implementing equation (6.22) will result in a higher order regulator and increased computational burden.

6.9 Conclusions

A method of decoupling tie-line sinusoidal disturbances in a power system, based on the internal model principle, has been presented. The self-tuning regulator forms a loop which is independent of the AVR/PSS control loops and therefore can be implemented with little modification to the existing system. It can be used to complement the action of the PSS when sustained oscillatory disturbances persist on the tie-line. Unlike other self-tuning schemes where lack of probing ability prevents the estimator from tracking parameter variations, in this proposed scheme the complementary signal can be stopped under normal operating conditions, leaving the PSS/AVR loops to perform their normal excitation control functions. The STR has also been shown to be effective against time-varying sinusoidal disturbances with good convergence qualities.
Chapter 7

Conclusions
Conclusions

The work described in this thesis aimed at extending the dynamic stability range of power systems through an appropriate adaptive control scheme that would ensure good servo and disturbance rejection properties. The conflict between the requirements for servo action due to the AVR and for disturbance rejection as provided by the DGR/PSS has been resolved by basing the control strategy on the IMP.

The goal throughout this thesis has been to search for an efficient solution for control management of the power system stabilising devices. In the pursuit of this goal, it has been necessary to first understand and to explain the phenomenon of amplified low frequency oscillations which occur on power system tie-lines despite the presence of 'correctly' tuned PSS.

A typical ECS was considered. Frequency-domain analysis, based on the evaluation of the gain characteristics of the transfer function of the ECS which reflects the effect of tie-line disturbances on power angle oscillations, was carried out. A hazardous situation was identified, whereby a PSS tuned to dampen the local mode of oscillation caused amplification of forced disturbances from the tie-line at a lower frequency range. This amplification was explained by analysing the effects of the additional dynamics introduced by the poles and zeros of the PSS. It was shown that adjustment of the parameter settings of the AVR and PSS produced damping effects which were inadequate for counteracting tie-line disturbances as
well as decoupling negative damping signals which may be introduced through the dynamic variable $K_5(s)$.

A method of decoupling negative damping signals through DGR measures was proposed. This involved the location of a notch filter acting as an on/off switch cascaded with the dynamic variable $K_5(s)$ and hence in the forward path of the negative damping signals. The function of this notch is to switch off the damping signal when it has negative polarity but allow it to pass through when it is positive. This method was compared to current practice which insert the DGR block as a lag/lead compensator in series with the AVR block. The proposed method was shown to have distinct advantages over current methods of implementation. The proposed method provided DGR measures while enhancing the transient performance of the ECS. There was also marked improvement in the dynamic performance of the ECS. However, the effect of amplified forced oscillations still prevailed, albeit at a reduced magnitude.

From the understanding gained thus far regarding the phenomenon of amplified oscillations due to tie-line disturbances and the conflict between the functions of the AVR/DGR/PSS, it became evident that a global solution was desirable. This solution had to take into account the constraints placed on the various stabilising devices, for instance the limit on the magnitude of the PSS signal and the nature of the tie-line disturbances. These are known to be oscillatory with frequencies that drift within a certain range. Additionally, the parameters of the ECS themselves are not

\[ 149 \]
constant, being affected by such factors as load and/or network configurations.

An adaptive scheme based on self-tuning control techniques was therefore proposed. As self-tuning control involves the two steps of identification and control, it is important that, during the adaptation process, numerical stability be maintained in order to ensure the calculation of suitable control signals based on the estimated parameter values. Some attention was therefore paid to the development of an efficient estimation model. It is based on a fixed width data window rather than a forgetting factor to cater for parameter drifts. The algorithm makes use of some properties of the Cholesky matrix decomposition techniques. Numerical stability is improved through the updating of the triangular factors of an augmented matrix. It was also shown that the computational burden involved with the proposed algorithm required $2.5 n^2$ floating point operations plus $n$ square roots for a $n \times n$ matrix.

The idea of a global solution involving a self-tuning control strategy capable of satisfying both servo and disturbance rejection requirements led to the inclusion of the IMP. The structure of the STR incorporated a model of the disturbance environment which is used to cancel the disturbance itself. The advantages of the proposed self-tuning scheme are characterised by a control loop which is independent of other control loops in the ECS. As such, it can be implemented with little modification to the existing system. Unlike other self-tuning schemes where lack of probing ability prevents the estimator from tracking parameter variations, in this proposed scheme the complementary signal can be stopped under
normal operating conditions, leaving the PSS/AVR loops to perform their normal excitation control functions. The STR has been shown to be effective against time-varying sinusoidal disturbances with good convergence qualities. Above all, it has been shown that the proposed adaptive scheme is able to provide adequate damping signals for counteracting time-varying disturbances while simultaneously allowing effective field-forcing action from the AVR.
**Appendix I.1 - Modelling Details of the ECS**

**AVR gain constant** \( K_e \)

**Machine Dynamics transfer function**
\[
\frac{1}{M s^2 + K_d s + K_i}
\]

**Field flux decay transfer function**
\[
\frac{K_3}{1+K_3 T_{do} s}
\]

**PSS transfer function**
\[
\frac{K_9 s}{1 + T_a s}
\]

**Appendix I.2 - Parameters of the ECS**

The parameters of the ECS are defined in reference [29] and are reproduced below:

\[
K_1 = \frac{\Delta T_e}{\Delta \delta} \bigg|_{E_q'} = \frac{X_q - X_d'}{X_e + X_d'} i_qo \ V_o \sin \delta_o + \frac{V_{qo} V_o \cos \delta_o}{X_e + X_q}
\]

\[
K_2 = \frac{\Delta T_e}{\Delta E_q'} \bigg|_{\delta} = \frac{V_o \sin \delta_o}{X_e + X_d'}
\]

\[
K_3 = \frac{X_d' + X_e}{X_d + X_e}
\]

\[
K_4 = \frac{1}{K_3} \frac{\Delta E_q'}{\Delta \delta} = \frac{X_d - X_d'}{X_e + X_d'} \ V_o \sin \delta_o
\]
\[ K_5 = \frac{\Delta V_t}{\Delta \delta} = \frac{X_q V_{do}}{X_e + X_q V_{to}} V_o \cos \delta_o - \frac{X_d V_{ao}}{X_e + X_d V_{to}} V_o \sin \delta_o \]

\[ K_6 = \frac{\Delta V_t}{\Delta E'_{eq}} = \frac{X_e V_{ao}}{X_e + X_d V_{to}} \]

**Appendix I.3 - Parameter Values of the ECS**

\[ K_1 = 1.01 \quad K_2 = 1.15 \]
\[ K_3 = 0.36 \quad K_4 = 1.47 \]
\[ K_5 = -0.097 \quad K_6 = 0.417 \]
\[ K_d = 0.01 \quad K_e = 25 \]
\[ K_s = 0.0265 \quad M = 0.008 \]
\[ T_a = 0.05 \quad T_{do} = 5 \]

**Appendix I.4: Derivation of the disturbance rejection function**

\[ G_{DRF(s)} - \text{system without DGR} \]

Fig. A.1 (a) - (c) demonstrate successive simplification of the block diagram of the ECS. Based on the superposition principle, the effect of \( \Delta P_d \) is considered while \( \Delta V_T \) is taken as zero.
Fig. A.1 Transfer block reduction for evaluation of $G_{DRF}(s)$

$$H_1(s) = \frac{K_e K_2 K_3}{(1 + K_3 T'_d s) + K_e K_3 K_6}$$

$$H_2(s) = \frac{K_e K_5 s^2 - K_e K_5 T_a s - K_e K_5 - K_4 T_a s - K_4}{K_e (1 + T_a s)}$$

Therefore, the transfer function

$$G_{DRF}(s) = \frac{\Delta \delta}{\Delta P_d} = \frac{G(s)}{1 + G(s) H_1(s) H_2(s)}$$

$$= \frac{1}{M s^2 + K_d s + K_1} \frac{K_e K_5 K_2 K_3 s^2 - K_2 K_3 T_a (K_4 + K_e K_5) s - K_2 K_3 (K_4 + K_e K_5)}{[K_3 T'_d T_a s^2 + (T_a + K_3 T'_d) K_e K_3 K_6 T_a] s + 1 + K_e K_3 K_6 (M s^2 + K_d s + K_1)}$$

$$= \frac{C_0 s^2 + C_1 s + C_2}{A_0 s^4 + A_1 s^3 + A_2 s^2 + A_3 s + A_4}$$
where

\[ A_0 = K_3 \quad T_{do} \quad T_a \quad M \]

\[ A_1 = K_3 \quad T_{do} \quad M + (1 + K_e \quad K_3 \quad K_6) \quad T_a \quad M + K_d \quad K_3 \quad T_a \quad T_{do} \]

\[ A_2 = (1 + K_e \quad K_3 \quad K_6) \quad M + K_d \quad K_3 \quad T_{do} \quad + (1 + K_e \quad K_3 \quad K_6) \quad K_d \quad T_a \]
\[ + K_1 \quad K_3 \quad T_a \quad T_{do} \quad + K_e \quad K_s \quad K_2 \quad K_3 \]

\[ A_3 = (1 + K_e \quad K_3 \quad K_6) \quad K_d \quad + K_1 \quad K_3 \quad T_{do} \quad + (1 + K_e \quad K_3 \quad K_6) \quad K_1 \quad T_a \]
\[ - K_2 \quad K_3 \quad T_a \quad (K_4 \quad + K_e \quad K_5) \]

\[ A_4 = (1 + K_e \quad K_3 \quad K_6) \quad K_1 \quad - K_2 \quad K_3 \quad (K_4 \quad + K_e \quad K_5) \]

\[ C_0 = K_3 \quad T_{do} \quad T_a \]

\[ C_1 = (1 + K_e \quad K_3 \quad K_6) \quad T_a \quad + K_3 \quad T_{do} \]

\[ C_2 = 1 + K_e \quad K_3 \quad K_6 \]

The transfer function relating \( \Delta \delta \) to \( \Delta V_r \) may be derived in a similar manner to that of \( G_{\text{DRF}}(s) \) and is given by the following equation:

\[ G_{\text{servo}}(s) = \frac{B_0 \quad s + B_1}{A_0 \quad s^4 + A_1 \quad s^3 + A_2 \quad s^2 + A_3 \quad s + A_4} \]

where
Bo = Ke K2 K3 Ta
B1 = Ke K2 K3

The coefficients A0 to A4 are the same as those of GDRF(s).

Appendix I.5- Derivation of matrix-valued GDRF(s) in multi-machine networks

The previously discussed advantages of the explicit evaluation of GDRF(s) in the analysis of low frequency oscillations can also be applied to the multi-machine environment. In this Appendix, it is shown that programs available for eigenvalue analysis in power systems can be extended to derive a polynomial matrix involving a generalised form of disturbance rejection functions. The gain frequency characteristics of the matrix entries convey information regarding the reflection of vector-based disturbances ΔPd on the vector Δδ, where bold faced letters represent vector and matrix quantities.

State-space models are convenient for representing the system dynamic equations and are frequently used for eigenvalue analysis. Equations describing the generator swings along with the associated control apparatus are arranged in the following vector-matrix form:

\[
\frac{dx}{dt} = Ax + Bu, \quad y = c x
\]  \hspace{1cm} (I.1)
For frequency analysis Laplace transform is applied to the above equation such that

\[ s \mathbf{I} \mathbf{x}(s) = \mathbf{A} \mathbf{x}(s) + \mathbf{B} \mathbf{u}(s) \quad (1.2) \]

where \( \mathbf{I} \) is a unit matrix with the same dimension as that of the state vector \( \mathbf{x} \) and matrices \( \mathbf{A} \) and \( \mathbf{B} \) are assumed to be time invariant. Rearranging equation (1.2) gives

\[ \mathbf{x}(s) = \frac{\text{adj}[(s\mathbf{I} - \mathbf{A})][\mathbf{B}]\mathbf{u}(s)}{D(s)} \]

where \( \text{Adj} \) stands for adjugate of a matrix and the eigenvalues are the factors of the characteristic polynomial \( D(s) \).

Since the vector \( \Delta \delta \) is included in the state \( \mathbf{x} \), and \( \Delta \mathbf{P}_d \) is included in the control vector \( \mathbf{u} \), the reflection of disturbances on power angle oscillations is given by

\[
\Delta \delta_i = \frac{\mathbf{B}_{ii}(s)}{D(s)} \Delta \mathbf{P}_{di} + \sum_{j=1; j \neq i}^{n} \frac{\mathbf{B}_{ij}(s)}{D(s)} \Delta \mathbf{P}_{dj} \quad (1.3)
\]

where \( \mathbf{B}_{ij}(s) \) is the \( ij \)th element of matrix \([\text{adj}(s\mathbf{I} - \mathbf{A})][\mathbf{B}]\) and \( \mathbf{x} \in \mathbb{R}^n \).

Using rational functions, equation (1.3) can be rewritten as

\[
\Delta \delta_i(s) = \mathbf{G}_{diu}(s) \Delta \mathbf{P}_{di}(s) + \sum_{j=1; j \neq i}^{n} \mathbf{G}_{dj}(s) \Delta \mathbf{P}_{dj}(s) \quad (1.4)
\]
where $G_{d_{ii}}$ evaluates the reflection of the local tie-line disturbance while $G_{d_{ij}}$ reflects the disturbance $\Delta P_{d_{ij}}$ on $\Delta \delta_{i}$.

The gain characteristics of these transfer functions provide information regarding the amplification/attenuation of forced low frequency oscillations in a multi-machine environment.
Appendix II.1 ECS with DGR cascaded with AVR

The transfer function for the case when the DGR block is cascaded with the AVR is derived below. Similar transfer function reduction techniques to Appendix I are used.

Fig. A.2.1 Transfer block reduction for evaluation of $G_{AVR}(s)$
From Fig.A.2.1, it can be shown that

\[
G_{AVR}(s) = \frac{BV_0 s^3 + BV_1 s^2 + BV_2 s + BV_3}{AV_0 s^5 + AV_1 s^4 + AV_2 s^3 + AV_3 s^2 + AV_4 s + AV_5}
\]

where

\[
AV_0 = K_e K_g K_3 T_a T_{do} T_g M
\]

\[
AV_1 = M (K_e K_g K_3 T_{do} T_g + [(K_3 T_{do} + T_g) (K_e K_g T_a)])
\]

\[
+ K_d K_e K_g K_3 T_a T_{do} T_g
\]

\[
AV_2 = M [(K_3 T_{do} + T_g) (K_e K_g)] + [(1 + K_e K_g K_3 K_6) (K_e K_g T_a)]
\]

\[
+ K_d [K_e K_g K_3 T_{do} T_g + [(K_3 T_{do} + T_g) (K_e K_g T_a)]
\]

\[
+ K_e K_g K_1 K_3 T_a T_{do} T_g
\]

\[
AV_3 = M [(1 + K_e K_g K_3 K_6) (K_e K_g)]
\]

\[
+ K_d [(K_3 T_{do} + T_g) (K_e K_g)] + [(1 + K_e K_g K_3 K_6) (K_e K_g T_a)]
\]

\[
+ K_1 [K_e K_g K_3 T_{do} T_g + [(K_3 T_{do} + T_g) (K_e K_g T_a)]
\]

\[
+ K_e K_g K_2 K_3 (K_e K_g K_s - K_4 T_a T_g)
\]

\[
AV_4 = K_d [(1 + K_e K_g K_3 K_6) (K_e K_g)]
\]

\[
+ K_1 [(K_3 T_{do} + T_g) (K_e K_g)] + [(1 + K_e K_g K_3 K_6) (K_e K_g T_a)]
\]

\[
- K_e K_g K_2 K_3 (K_4 T_a + K_4 T_g + K_e K_g K_5 T_a)
\]

\[
AV_5 = K_1 [(1 + K_e K_g K_3 K_6) (K_e K_g)]
\]

\[
- K_e K_g K_2 K_3 (K_4 + K_e K_g K_5)
\]
\[ BV_0 = K_e K_g K_3 T_a T_d' T_g \]

\[ BV_1 = K_e K_g K_3 T_d' T_g + [(K_3 T_d' + T_g) (K_e K_g T_a)] \]

\[ BV_2 = [(K_3 T_d' + T_g) (K_g K_g)] + [(1 + K_e K_g K_3 K_6) (K_e K_g T_a)] \]

\[ BV_3 = (1 + K_e K_g K_3 K_6) (K_e K_g) \]

**Appendix II .2 ECS with DGR cascaded with K5**

The transfer function for the case when the DGR block is cascaded with \( K_5 \) is derived below.
From Fig. A.2.2, it can be shown that

$$G_{TGR}(s) = \frac{BK_0 s^4 + BK_1 s^3 + BK_2 s^2 + BK_3 s + BK_4}{AK_0 s^6 + AK_1 s^5 + AK_2 s^4 + AK_3 s^3 + AK_4 s^2 + AK_5 s + AK_6}$$

where

$$AK_0 = MK_e K_3 T_a T_{do} T_g T_g$$

$$AK_1 = M \left[(K_3 T_{do} T_g) (K_e T_a + K_e T_g)\right] + \left[(K_3 T_{do} + T_g) (K_e T_a T_g)\right] + K_d K_e K_3 T_a T_{do} T_g T_g$$

$$AK_2 = M \left[(K_e K_3 T_{do} T_g) + [(K_3 T_{do} + T_g) (K_e T_a + K_e T_g)\right] + [(1 + K_e K_g K_3 K_e) (K_e T_a T_g)]$$
\[ + K_d \left( [K_3 T_{do} T_g] (K_e T_a + K_e T_g) + \left[ \frac{K_3}{T_{do} + T_g} \right] (K_e T_a T_g) \right) \]

\[ + K_e \left( K_1 K_3 T_a T_{do} T_g + K_e K_e K_s K_2 K_3 T_g T_g \right) \]

\[ A_{K_3} = M \left( \left[ \frac{K_3}{T_{do} + T_g} \right] K_e + \left[ (1 + K_e K_g K_3 K_6) (K_e T_a + K_e T_g) \right] \right) \]

\[ + K_d \left( \left[ K_e K_3 T_{do} T_g \right] + \left[ \frac{K_3}{T_{do} + T_g} \right] (K_e T_a + K_e T_g) \right) \]

\[ + \left[ (1 + K_e K_g K_3 K_6) (K_e T_a T_g) \right] \]

\[ + K_1 \left( \left[ [K_3 T_{do} T_g] (K_e T_a + K_e T_g) + \left[ \frac{K_3}{T_{do} + T_g} \right] (K_e T_a + K_e T_g) \right] \right) \]

\[ + \left[ (1 + K_e K_g K_3 K_6) (K_e T_a T_g) \right] \]

\[ - \left[ (K_e K_2 K_3 T_g) (K_4 T_a + K_4 T_g + K_e K_g K_5 T_a) \right] \]

\[ + \left[ (K_e K_2 K_3) (K_e K_s - K_4 T_a T_g) \right] \]

\[ A_{K_4} = M K_e \left( 1 + K_e K_g K_3 K_6 \right) \]

\[ + K_d \left( \left[ K_3 T_{do} + T_g \right] K_e + \left[ (1 + K_e K_g K_3 K_6) (K_e T_a + K_e T_g) \right] \right) \]

\[ + K_1 \left( \left[ K_e K_3 T_{do} T_g \right] + \left[ \frac{K_3}{T_{do} + T_g} \right] (K_e T_a + K_e T_g) \right) \]

\[ + \left[ (1 + K_e K_g K_3 K_6) (K_e T_a T_g) \right] \]

\[ - \left[ (K_e K_2 K_3 T_g) (K_4 + K_e K_g K_3) \right] \left[ (K_4 T_a + K_4 T_g + K_e K_g K_5 T_a) \right] \]

\[ A_{K_5} = K_d K_e \left( 1 + K_e K_g K_3 K_6 \right) \]

\[ + K_1 \left( \left[ K_3 T_{do} + T_g \right] K_e + \left[ (1 + K_e K_g K_3 K_6) (K_e T_a + K_e T_g) \right] \right) \]

\[ - \left[ (K_e K_2 K_3 T_g) (K_4 + K_e K_g K_3) \right] \left[ (K_4 T_a + K_4 T_g + K_e K_g K_5 T_a) \right] \]

\[ A_{K_6} = K_1 K_e \left( 1 + K_e K_g K_3 K_6 \right) \]

\[ - \left[ (K_e K_2 K_3) (K_4 + K_e K_g K_3) \right] \]

\[ B_{K_0} = K_e K_3 T_a T_{do} T_g T_g \]
\[ \text{BK}_1 = [(K_3 T'_{do} T_g ) (K_e T_a + K_e T_g)] + [(K_3 T'_{do} + T_g ) (K_e T_a T_g)] \]

\[ \text{BK}_2 = (K_e K_3 T'_{do} T_g) + [(K_3 T'_{do} + T_g ) (K_e T_a + K_e T_g)] \]

\[ + [(1 + K_e K_g K_3 K_6) (K_e T_a T_g)] \]

\[ \text{BK}_3 = (K_3 T'_{do} + T_g ) K_e + [(1 + K_e K_g K_3 K_6) (K_e T_a + K_e T_g)] \]

\[ \text{BK}_4 = K_e (1 + K_e K_g K_3 K_6) \]
Appendix III - Proof of lemma 1 [cf equation (4.16)]

Consider the following matrix

\[
M(t) = \begin{bmatrix}
\sum_{t=t-k}^{t} V^2(i) & B^T \\
B & R
\end{bmatrix}
\]

Equation (III.1) can be factorised as

\[
M(t) = U(t) D(t) U^T(t)
\]

where \( U(t), D(t) \) and \( U^T(t) \) are upper triangular, diagonal and lower triangular matrices respectively.

The following relationship holds

\[
[U \ D \ U^T]^{-1} M(t) = I
\]

Manipulating equation (III.3) results in

\[
\begin{bmatrix}
1 & 0 \\
X & L
\end{bmatrix}
\begin{bmatrix}
d_0 \\
d_1
\end{bmatrix}
\begin{bmatrix}
1 & X^T \\
L^T & B\ R
\end{bmatrix}
\begin{bmatrix}
\sum_{t=t-k}^{t} V^2(i) & B^T \\
B & R
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & I
\end{bmatrix}
\]

where \( d_1 \) is a diagonal matrix. Expanding equation (III.4) gives
\[
\begin{bmatrix}
1 & 0 \\
X & L
\end{bmatrix}
\begin{bmatrix}
d_0 \sum_{t=t-k}^{t} V^2 (i) + d_0 X^T B & d_0 B^T + d_0 X^T R \\
d_1 L^T R & d_1 L^T R
\end{bmatrix}
= \begin{bmatrix} 1 & 0 \\ 0 & I \end{bmatrix}
\] (III.5)

from which

\[d_0 B^T + d_0 X^T R = 0\]

\(M(t)\) is a positive definite matrix and \(d_0 \neq 0\)

Therefore

\[X^T R = -B^T\] or \(X = -R^{-1} B\), which proves the lemma.
Appendix IV.1 - Derivation of the GMV Control Law

The model given by equation (5.4) is considered, i.e.

\[ y(t) = q^{-k} \frac{B}{A} u(t) + \frac{C}{A} \xi(t) \]  

(IV.1)

and an auxiliary output given by:

\[ \phi(t+k) = P y(t + k) + Q u(t) - R w(t) \]  

(IV.2)

where P, Q and R are weighting polynomials in \( q^{-1} \) and \( w \) is the set point.

The control objective is to minimise the cost function

\[ I = E \left\{ [ P y(t + k) + Q u(t) - R w(t)]^2 \right\} \]  

(IV.3)

Substituting equation (IV.1) in (IV.3) and rearranging gives

\[ I = E \left\{ [ \left( \frac{PB}{A} + Q \right) u(t) - R w(t) + \frac{PC}{A} \xi(t+k)]^2 \right\} \]  

(IV.4)

From equation (5.15)

\[ \frac{PC}{A} = F + q^{-k} \frac{G}{A} \]  

(IV.5)

Therefore

\[ \frac{PC}{A} \xi(t + k) = F \xi(t + k) + \frac{G}{A} \xi(t) \]  

(IV.6)
Combining equations (IV.6) and (IV.4)

\[ I = \mathbb{E} \left[ \left( \frac{PB}{A} + Q \right) u(t) - R w(t) + F \xi(t + k) + \frac{G}{A} \xi(t) \right]^2 \] (IV.7)

From equation (IV.1)

\[ \xi(t) = \frac{A}{C} y(t) - q^k \frac{B}{C} u(t) \] (IV.8)

Equation (IV.7) then becomes

\[ I = \mathbb{E} \left[ \left( \frac{PB}{A} + Q \right) u(t) - R w(t) + F \xi(t + k) + \frac{G}{A} \left( \frac{A}{C} y(t) - q^k \frac{B}{C} u(t) \right) \right]^2 \]

\[ = \mathbb{E} \left[ \left( \frac{B}{C} \left( \frac{PC}{A} - q^k \frac{G}{A} \right) + Q \right) u(t) - R w(t) + F \xi(t + k) + \frac{G}{C} y(t) \right]^2 \]

\[ = \mathbb{E} \left[ \frac{1}{C} \left( H u(t) + G y(t) + E w(t) + F \xi(t + k) \right)^2 \right] \] (IV.9)

where \( H = BF + QC \)

and \( E = -RC \)

Equation (IV.9), in expanded form, gives

\[ I = \mathbb{E} \left[ \left( \frac{1}{C} \left( H u(t) + G y(t) + E w(t) \right) \right)^2 \right. \]

\[ + \frac{2F}{C} \left( H u(t) + G y(t) + E w(t) \right) \xi(t + k) \]

\[ + [F \xi(t + k)]^2 \] (IV.10)
The disturbance $\xi(t)$ is a random uncorrelated zero-mean sequence and the expected value of the term $\left( \frac{2F}{C} (H u(t) + G y(t) + E w(t)) \right) \xi(t + k)$ will be zero.

Equation (IV.10) therefore becomes

$$I = E \left[ \left( \frac{1}{C} (H u(t) + G y(t) + E w(t)) \right)^2 + [F \xi(t + k)]^2 \right]$$  \hspace{1cm} (IV.11)

For minimum $I$

$$\frac{\partial I}{\partial u(t)} \left[ E \left[ \left( \frac{1}{C} (H u(t) + G y(t) + E w(t)) \right)^2 + [F \xi(t + k)]^2 \right] \right] = 0$$

from which the control law that minimises the variance of $\phi(t+k)$ is obtained as

$$u(t) = - \frac{G}{H} y(t) - \frac{E}{H} w(t)$$  \hspace{1cm} (IV.12)

**Appendix IV.2 - Derivation of the control signal for the GMV**

Given $PC = FA + q^{-k}G$ \hspace{1cm} (IV.13)

$$H = BF + QC$$ \hspace{1cm} (IV.14)

$$E = -RC$$ \hspace{1cm} (IV.15)

From equation (5.22)

$$y(t) = - q^{-k} \frac{BR}{PB + QA} w(t) + \frac{BF + QC}{PB + QA} \xi(t)$$ \hspace{1cm} (IV.16)

and from equation (IV.12)
\[ u(t) = -\frac{G}{H} y(t) - \frac{E}{H} w(t) \]  \hspace{1cm} (IV.17)

Combining equations (IV.16) and (IV.17) gives

\[ u(t) = q^{-k} \frac{G B R}{H(PB + QA)} w(t) - \frac{G(BF + QC)}{H(PB + QA)} \xi(t) - \frac{E}{H} w(t) \]  \hspace{1cm} (IV.18)

Substituting for \( q^{-k} G \) from equation (IV.13) into (IV.18) and rearranging, results in

\[ u(t) = -\frac{PCBR + FABR - PBE - EQA}{H(PB + QA)} w(t) - \frac{G(BF + QC)}{H(PB + QA)} \xi(t) \]  \hspace{1cm} (IV.19)

from which the control signal is obtained as

\[ u(t) = \frac{AR}{PB + QA} w(t) - \frac{G}{PB + QA} \xi(t) \]  \hspace{1cm} (IV.20)
Appendix V-Author's publications resulting from this thesis


References


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