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Ultimate load capacity of curved steel struts filled with higher strength concrete

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ULTIMATE LOAD CAPACITY of CURVED STEEL STRUTS FILLED WITH HIGHER STRENGTH CONCRETE

A thesis submitted in fulfillment of the requirements for the award of the degree of

Doctor of Philosophy

from

UNIVERSITY OF WOLLONGONG

by

MOHSEN GHASEMIAN, BE, MENGSC

DEPARTMENT OF CIVIL AND MINING ENGINEERING

March, 1997
DECLARATION

This is certify that the work presented in this thesis was carried out by the author in the Department of Civil and Mining Engineering, University of Wollongong, and has not been submitted to any other university or institute for a degree except when specifically indicated.

........................................
Mohsen Ghasemian
1997
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ABSTRACT

An experimental and theoretical study concerning the ultimate load behaviour of curved steel struts infilled with higher strength concrete has been carried out. As well, a nonlinear finite element model for investigating the elasto-plastic behaviour of such elements has been carried out. The analysis accounts simultaneously for both the geometrical and material nonlinearities. Different stress-strain relationships of the material are assumed to take into account strain hardening as well as residual stress. This study involves the structural characteristics of the composite sections under compressive axial loads.

A total number of 78 composite as well as 11 hollow section curved struts have been tested for the different structure forms. Material and composite curved strut tests have been performed on circular sections of 60.4 mm outside diameter and wall thickness 2.3 mm ERW curved struts, (with a low strain hardening ratio, and with radii of curvatures equal to 2000mm, 4000mm and 10000mm), 60.4 mm outside diameter and wall thickness 3.9 mm seamless composite curved struts (with a high strain hardening ratio and radii of curvatures equal to 2000mm and 4000mm). The struts were tested as pinned ended struts and loaded concentrically.

The composite curved steel struts subjected to compressive load were analysed both by a established theoretical method by assuming the initial deflected shape to be part of a sine wave and also by using a non-linear finite element method. The theoretical method greatly simplifies the analysis in comparison with other methods, and gives results which in most cases are close to the maximum loads. All computational procedures of the theoretical method are programmed on a computer and also a finite element package (Nastran) has been used to investigate the behaviour of strut over the elasto-plastic part of stress-strain diagram. In this case the results are close to the maximum loads obtained experimentally.
The ultimate load capacity of curved composite struts is extensively investigated by numerical experiments. The computed results show that the maximum load is influenced mainly by the slenderness and initial deflection at mid-height of tube, but the steel strength, concrete strength and diameter to thickness ratio are also found to be significant.

By comparing the theoretical results (intersection of elastic and plastic curves) with experimental results, it is shown that the theoretical method can predict with reasonable accuracy the experimental maximum loads. The error was -12% to +13% for ERW struts with 2000 mm initial radius of curvature, -2% to +3% for ERW struts with 4000 mm initial radius of curvature, -9% to +9% for ERW with 10000 mm initial radius of curvature, 0 to +11% for seamless struts with 2000 mm initial radius of curvature and -4% to 16% for seamless struts with 4000 mm initial radius of curvature.

The theoretical load-deflection behaviour of the as-received curved struts obtained from Nastran compared well with the experimental results. The residual stress effect due to initial curvature is taken into account by using different material properties (stress-strain) across the cross-section of the curved struts. In addition, the interaction of the concrete core and the steel tube have been modelled by the utilisation of gap elements to form an analytical model for the composite sections. The differences between the maximum loads obtained from the finite element method and experimental results is -5% to +6%.

Design methods and various ultimate load design formulae are investigated and it is found that no single formulae gives accurate results over all ranges of the significant parameters.
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NOTATION

\( A_e \) = Elementary area
\( D_\varepsilon \) = Ductility factor
\( d\varepsilon \) = Increment strain vector
\( d\sigma \) = Increment stress vector
\( d \) = External diameter of a circular cross-section tube
\( de \) = Deflection at a 0.75 % of the ultimate axial load
\( D_e \) = Elasticity matrix for plane stress
\( d_n \) = Depth of neutral axis
\( d_{sc}, d_{st} \) = Distance of compressive and tensile steel areas from the neutral axis
\( d_u \) = Deflection at ultimate axial load
\( e \) = Initial deflection at mid height
\( ERW \) = Electric Resistance Welded
\( E_s \) = Strain hardening modulus
\( E_t, E_T, E_x \) = Tangent modulus in inelastic range
\( E_{TC} \) = Tangent modulus of concrete
\( f'c \) = Unconfined compressive strength of concrete
\( F_0 \) = Equivalent nodal forces due to initial strains
\( F_s \) = Vector of all nodal
\( I_g \) = Second moment of the gross cross-section area
\( I_s \) = Second moment of steel cross-section area
\( K \) = An empirical factor
\( L \) = Straight length of a member
\( LVDT \) = Linear Variable Displacement Transduce
\( M_n \) = Ultimate bending moment at mid-height
\( P \) = Lateral pressure
\( P_c \) = Axial load carried by the concrete core
\( P_E \) = Euler load
\( P_H \) = Ultimate load corresponding to the hoop failure mode
\( P_L \) = Ultimate load corresponding to longitudinal failure mode
\( P_s \) = Axial load carried by the steel tube
\( P_s \) = Squash load

\( P_{TM} \) = Tangent modulus buckling load

\( P_u \) = Ultimate load capacity of short circular column

\( P_{u1} \) = Cross-section strength

\( P_{u2} \) = Elastic strength of composite curved steel strut

\( q \) = Nodal force vector

\( q_0 \) = Equivalent nodal force vector due to an initial strain \( \varepsilon_0 \)

\( R \) = Initial radius of curvature

\( u \) = A global displacement vector

\( U \) = Displacement vector

\( y \) = Distance of the neutral axis from tensile extreme fibre

\( y_i \) = Coordinate of the centroid of the elementary area

\( Z_{sc}, Z_{st}, Z_c \) = Lever arms of forces from the plastic centroid (Fig. 3.9)

\( \sigma \) = A plane stress vector

\( \lambda \) = Slenderness ratio

\([\Delta F]_R \) = Unbalanced force vector

\( \varepsilon_0 \) = Initial strain vector

\( \sigma_{11}, \sigma_{22} \) = Maximum and minimum principal stresses

\( \sigma_c \) = Concrete stress

\([C_1]\) = Transformation matrix

\( \sigma_{CL} \) = The longitudinal stress in the concrete

\( \sigma_{CR} \) = Radial stress in the concrete

\( \{f^c\} \) = Column matrix representing the internal elastic force components induced at grid points

\( \{F^e\} \) = Column vector representing the external forces

\([K^e]\) = Element stiffness matrix

\([K^s]\) = Structural stiffness matrix

\([K_T]\) = Tangent stiffness matrix of the member

\( \varepsilon_{SH} \) = Hoop strain

\( \sigma_{SH} \) = Hoop stress in steel

\( \varepsilon_{SL} \) = Longitudinal strain

\( \Delta_u \) = Additional deflection at mid-height

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\( \{u^i\} \) = Column matrix representing nodal displacement of the element

\( \Delta l_0, \Delta l_n \) = Incremental arc-lengths

\( \Delta \lambda_0 \) = Increment factor

\( I_0, I_d \) = The number of iterations

\( C_s \) = Scalar measure of the degree of nonlinearity.

\( \delta r \) = Variable iterative displacement vector

\( \beta \) = Line search tolerance

\( \delta F \) = Iterative force

\( \rho \) = Desired convergence factors

\( \epsilon \) = Strain

\( \epsilon_y \) = Yield strain

\( \epsilon_p \) = Proportional strain

\( \epsilon_x \) = Strain defined in the range between the proportional and yield strain

\( \{\Delta\}_R \) = Incremental displacement due to the residual force

\( [F] \) = Applied force vector

\( [B] \) = Strain / displacement

\( [\sigma_0] \) = Internal stress of the structure

\( \Delta \lambda_i \) = Load multiplier

\( D_p \) = Stress-dependent plastic component
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1. CHAPTER ONE

INTRODUCTION

1.1 INTRODUCTION

Tubular members have been widely used in roof structures, sports stadia, industrial buildings and space structures primarily for economy and for aesthetic architectural purposes. They have many advantages such as great torsional rigidity and offer local strength against impact loading. However, the study and research on the structural behaviour of curved structural members has been limited, and no design graphs have been provided in the codes of practice for such members. Curved steel tubular members are increasingly used in modern building; for example, curved hollow tubes in space trusses have been used at Sydney Airport as shown in Fig.1.1.

Tubular members are often cold formed during fabrication, and they may also be cold formed during erection. Typical manufactured tubes are classified according to the forming process and the heating conditions used in manufacture as follows:

1-Electric resistance welding (ERW): A strip is cold-formed by rolls into a circular shape, and the edges are heated to welding temperature by resistance to the flow of an electric current. This process is referred to as “cold-formed and electric resistance welded”.

2-Seamless: A heated bar is pierced one or more times, and rolled with an internal mandrel.
Fig. 1.1 Space Truss of Sydney International Airport
One way in which to manufacture the curved circular tubular members is by a process that passes tubes several times through a set of rollers. As a consequence of successive steps in the curving process, residual stress patterns are caused by plastic deformation in both the longitudinal and circumferential directions, in addition to the tube manufacturing and welding residual stresses. The process of cold curving of such members causes the steel to be strained well into the plastic range.

In the design of multi-story buildings, piles and other structures, high strength concrete has been used because of advantages of high strength, and superior durability. Filling a tube with high strength concrete is an attractive proposition because it considerably increases the load-carrying capacity without increasing the size of the steel tube, and also increases the ductility of the combination tube. Local buckling of the tube wall is delayed because it can only buckle outwards. The hollow section provides a convenient formwork for the concrete and an adequate cover against impact or abrasion of the concrete surface. Finally, the concrete, in its contained condition, is able to sustain much higher stresses and strains than when it is uncontained.

Since visual inspection of the concrete during filling the curved tube is not possible, some difficulties such as compaction may be arise. Therefore, other means must be used to check that the concrete has been properly placed and compacted.

Stability and general behaviour of straight struts and tubular composite columns have been investigated for many years and significant theoretical achievements have been made by many investigators such as Neogi, et, al. (1969), and Rangan and Joyce (1992). There is a lack of information, however, about the ultimate load capacity as well as the elasto-plastic behaviour of hollow and composite curved struts subjected to compressive load. Prior to this investigation, some work had been carried out on the effects of the influence of strain hardening, strain aging and the Bauschinger effect on hollow curved steel strut load capacity (Schmidt and Mortazavi, 1993). The concrete filled curved steel strut is a new structural element.
Curved steel struts infilled with concrete subjected to compressive load herein are treated analytically as one-dimensional stress problems. Such an analysis is suitable for a slender metal column, a slender ordinary reinforced concrete column or a slender encased straight composite column. In a short thin walled concrete filled steel tube, when the lateral deformation of the concrete core is restrained by the steel shell, the core is stressed triaxially and the tube biaxially. The stress condition in the concrete core is similar to that in the core of a spirally reinforced concrete column. Therefore, a uniaxial stress analysis is appropriate only in the range where triaxial stresses are non-existent or small.

The main purpose of this study is to analyse and study the influence of initial radius of curvature, initial deflection at mid-height, slenderness, high strength concrete, strain hardening and residual stresses due to the forming and curving process on the ultimate load capacity of composite curved steel struts.

1.2 SCOPE of RESEARCH

The objectives selected for this investigation are to:

(1) establish a method (numerical algorithm and its program) to calculate the ultimate load capacity of curved composite steel tubular struts.

(2) develop a non-linear finite element analysis of the composite curved steel struts using different material properties (stress-strain curves) across the cross section to take into account residual stress as well as the scalar elements (Gap element) to model the interaction of the concrete core and the steel tube.

(4) report the details of material and 78 strut tests used to study the behaviour of curved steel struts infilled with higher strength concrete.
(4) make a comparison between experimental and theoretical results and to give a general discussion.

(5) summarise the research work conducted and draw conclusions and recommendations for future work.
2. CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

At the present time there is little work reported on the behaviour of slender hollow and composite curved compression struts. Composite curved strut load capacity largely depends on variables such as initial radius of curvature, slenderness ratio, concrete strength, yield strength, and material type and the process of production. Composite action between steel sections and concrete cover or core in composite columns is a question which requires detailed analysis because of the interaction of the various variables. It is shown that a similarity exists between the axial thrust behaviour of a curved member subjected to compressive load and an equivalent straight column. It is necessary to review, in addition to the information available on curved members, the work has already been done on straight composite columns, and the properties of steel and high performance concrete under uniaxial and multiaxial states of stress. Since the literature in these fields, except for curved struts, is extensive, only work that is directly relevant to this investigation will be reviewed.

Research on stability of straight struts and tubular composite columns has been carried out for many years and significant achievements have been made in theoretical analyses and experimental investigations. Numerical and theoretical methods have been proposed by Neogi, Sen and Chapman (1969), Bridge and Roderick (1978), Knowles and Park (1969), Rangan and Joyce (1992). However, there is little reported work on the stability of curved struts, especially curved composite struts. Recently, tubular steel arches filled with concrete were used to act as the false work to form the reinforced concrete Joshi Bridge Arch in Japan (1993).
2.2 GENERAL COLUMN BEHAVIOUR

It is known that the classical theories of inelastic buckling have been widely used in the ultimate strength design of structures. Among these theories, the tangent-modulus theory is the most popular and preferred by engineers (Mansour (1986)). This is because the tangent-modulus theory is much easier to use and gives lower critical loads than other theories, which ensures rather safe designs.

The first correct explanation of the behaviour of a column subjected to concentrically loaded failing by buckling in the inelastic range was given by Shanley in 1947. He considered a simplified column model consisting of two rigid legs connected at the center by a two flange elasto-plastic hinge. Using the Shanly approach, Duberg and Wilder (1950) calculated the complete load-deflection curve of an idealized H-section column with flexibility over the full length. They concluded that the tangent modulus load is a lower bound estimate of the buckling load, and for practical purposes it may be considered as the critical load of a concentrically loaded column.

The failure of an eccentrically loaded steel column is due to lack of equilibrium between the internal and external bending moments. A general and exact theory for determining the maximum load of an eccentrically loaded column was proposed by von Karman in (1910). By assuming a different deformation behaviour regarding the variation of strain distribution following the increase of load, Karman proposed the reduced-modulus theory. The deflected shape of column was determined by numerical integration of angle changes along the column length. Westergaard and Osgood (1928) assumed the deflected shape to be part of a cosine wave and showed that this assumption did not significantly impair the accuracy of the result. It should be noted that a simple deflected shape with one arbitrary constant, like a part cosine wave, only satisfies equilibrium and compatibility at the center section and at the ends.

The reduced-modulus theory has been accepted as an upper bound theory of inelastic buckling of columns. Nevertheless, certain paradoxes do exist in both the tangent
modulus and reduced-modulus theories. Consequently it has been difficult to say that there is a uniquely exact theory of inelastic buckling.

The reliability of Shanley’s plastic buckling model based on the nonlinear finite element analysis of column was investigated by Chen (1996). He proposed an iteration method based on improving a modified Newton-Raphson scheme for obtaining converged solutions of discretized nonlinear algebraic systems. The continuum modeling of the finite element method has the advantage of directly discretizing structural domains without imposing mechanical constraints on the structural system.

2.3 MATERIAL PROPERTIES OF CONCRETE

In general, concrete is a heterogeneous, viscoelastic material which can carry considerable compression but very little tension. To simplify analysis, concrete is generally idealised as a homogenous, isotropic material having no tensile strength. In flexural problems it is assumed that the stress-strain curve is identical with that for uniform compression.

High performance concrete, is here defined as concrete grades above 50 MPa. The higher strength grades offer potential for use in the lower storeys of multi-storey buildings where the maximum axial loading occurs. High performance concretes also offer superior durability and stiffness. In the design of tall buildings in Australia and in other countries high performance concrete has been used not only for strength but also for its superior stiffness (Attard, 1992).

There are several state of the art reports on high performance concretes such as the FIP/CEB Bulletin 197 (1990), the Cement and Concrete Association publication, High Strength Concrete (1992), and some codes of practice, such as the Norwegian NS 3473 (1989). The current Australian Standard for Concrete Structures AS3600-1988 is only intended for concrete grades up to 50 MPa, and for normal strength concrete
providing ties in accordance with the provisions of AS3600 provides for some ductility (Hwee and Rangan (1989)). However, for high performance concrete, which dilates less and requires higher confining pressures to achieve the same order of ductility as in normal strength concrete, much larger lateral steel volumes are required.

It is well recognised that high performance concretes are much more brittle than normal strength concretes (Attard and Mendis, 1993) (Fig.2.1). In normal strength concrete the mortar and aggregates have higher strengths than the strength of the resulting concrete. Failure is initiated in the weak transition zone between the aggregate and the mortar. For high strength concrete, the mortar as well as, in most cases, the aggregates have similar strength as the resulting concrete (Setunge et al., 1992). Cracks at failure are reported to be smooth with little transfer of shear through mechanical interlock.

### 2.3.1 Uniaxial Stress-Strain Relationship

In ultimate strength design, the stress-strain relationship and the modulus of the concrete (\(E_c\)) are the key properties required. There are a few published papers regarding experimental stress-strain curves for high strength concrete tested in uniaxial compression. High strength concrete is extremely brittle and therefore requires a very stiff testing machine with servo-control in order that the descending branch of the stress-strain curve to be traced. The complete concrete stress-strain compressive curve is given in CEB/FIP. In accordance with CEB-FIP ("International " 1970), the compressive stress is represented as a function of compressive strain by

\[
\sigma = \frac{\sigma_0 \epsilon (a - 206,600 \epsilon)}{1 + b \epsilon}
\]  

where \(\sigma\) is in MPa, and where

\[
a = 39000(\sigma_0 + 7.0)^{0.953}
\]
Fig. 2.1 Typical Stress-Strain Relationship for Concrete of Various Grades
\[ b = 65600(\sigma_0 + 10.0)^{-1.085} - 85.0 \quad (2.3) \]

in which \( \sigma_0 = 0.85 \) represents the peak stress on each curve and \( f_c \) is compressive strength of concrete.

Based on limited results up to 90 MPa, the following result was proposed by Fafitis and Shah (1986). For the ascending branch

\[ f = f_{co} \left( 1 - \left( 1 - \frac{\varepsilon}{\varepsilon_{co}} \right)^A \right) \quad \varepsilon \leq \varepsilon_{co} \quad (2.4) \]

where \( f_{co} \) is the peak stress under uniaxial compression, and \( f \) is the stress corresponding to a strain of \( \varepsilon \).

and for the descending branch

\[ f = f_{co} e^{-k(\varepsilon-\varepsilon_{co})^{1.5}} \quad \varepsilon > \varepsilon_{co} \quad (2.5) \]

where the strain at peak stress, \( \varepsilon_{co} \), for gravel aggregates is

\[ \varepsilon_{co} = \frac{f'_c}{E_c} \frac{3.78}{\sqrt{f'c}} \quad (2.6) \]

and for crushed aggregates is

\[ \varepsilon_{co} = \frac{f'_c}{E_c} \frac{4.26}{\sqrt{f'c}}, \quad \text{and} \]

\[ A = \varepsilon_{co} / f_{co}, \quad K = 24.7 f_{co} \quad (2.7) \]
Experimental work to obtain the complete stress-strain behaviour of high-strength concrete (HSC) under compression was carried out by Hsu and Hsu in 1994. They concluded that, on average, the strain corresponding to the peak stress for the HSC is greater than that for the normal strength concrete. Therefore, the constant values of 0.002 and 0.003 for the strain corresponding to the peak stress and the ultimate strain, as specified by ACI Committee 318, are conservative. As well, the crack patterns for the HSC show that the broken surface of the concrete cylinders is smoother, and this fracture surface passes through the aggregates.

The ascending branch of the stress-strain curve is steeper and the strain at peak load is slightly higher for high performance concrete than for normal strength concrete. The proportional limit occurs at a higher stress, typically 80 to 90% of the peak stress (for normal strength concretes the proportional limit is between 40 to 60% of the peak stress). The descending branch or the softening curve is almost vertical. For high strength concrete the stress strain curve can be described as approximately linear elastic up to the peak, and then brittle with almost complete loss of load carrying capacity with little increase of strain (Kostovos (1983)). This implies that a triangular stress block might be more suitable for unconfined high strength concrete in flexure.

Most published empirical formulae for the static elastic modulus of normal concrete $E_c$, are related to the compressive strength and the surface dry unit weight of the concrete. The formula in AS3600 for normal strength concrete is based on the extensive work of Pauw (1960). The AS3600 formula is quoted as

$$E_c = 0.043 \rho^{1.5} \sqrt{f_{cm}} \pm 20\%$$  \hspace{1cm} (2.8)

where $\rho$ is the surface dry unit weight, and $f_{cm}$ is the mean concrete strength. In most codes, other than in Australia, the mean strength is replaced by the characteristic strength. The in-situ strength of concrete is approximately 85% of the standard cylinder strength, and if the mean strength is approximately 1.2 times the characteristic strength, then
\[ f_{cm} = 1.2 \left(0.85 f'_c \right) = f'_c \] (2.9)

From the above equation it seems there was little value in changing the original formula (Attard, 1992). For normal weight concrete with a density of 2400 kg/m³, Eqn (2.8) can be written as

\[ E_c = 5056 \sqrt{f'_c} \pm 20\% \] (2.10)

For high strength concrete, Eqn (2.8) provides a reasonable fit to the experimental data. A lower bound fit is predicted by the formula proposed by Carrasquillo et al. (1981),

\[ E_c = (3320 \sqrt{f'_c} + 6900) \left(\rho / 2320\right)^{1.5} \] (2.11)

2.3.2 Confined Stress-Strain Relationship

In composite tubular steel and concrete struts, there is the possibility of enhancement of the concrete strength due to the confinement of the concrete. The stress strain curve of confined concrete can be predicted for different types of confinement. Confinement of concrete by transverse reinforcement in reinforced concrete columns is approximately the same as the confinement provided in composite circular hollow sections. Variables which play important roles in determining the behaviour of the confined concrete are the amount of lateral reinforcement, and the steel strength, and Poission’s ratio of the concrete after cracking especially. Almost all the analytical models for confinement were based on experimental results obtained from small-scale tests on simple tie configurations. In most of the tests the ratio of the area of the core bounded by the center line of the perimeter tie to the gross area of the specimen was small compared with the values commonly used in practice (Kavoosi, 1993).
The effect of lateral restraint on the compressive strength of concrete can be calculated by the following simple relationship by Richart (1928), Richart (1929) and Balmer (1949):

\[ F = F_0 + kp \]  \hspace{1cm} (2.12)

where \( F \) is the ultimate strength of the member when restrained by a lateral pressure \( p \), \( F_0 \) is the ultimate strength when \( p \) is zero, and \( k \) is a constant depending on the characteristics of the concrete mix and the lateral pressure. Richart et al. (1929) found the average values of the coefficient for the tests he conducted to be \( k = 4.1 \). Also, Lohr (1934) proposed concrete encased in steel shells. Encased concrete was applied to a new type of reinforced concrete column consisting of a steel tube encasing a concrete cylinder which is longer than the steel tube. Lohr proposed a design formula based on a brief series of tests, but stated that further investigations should be carried out, especially with regard to the determination of the maximum safe direct compressive stress for the concrete encased in a thin tube of steel. In this case, Moreell (1935) pointed out that the Lohr column acted the same manner as a spirally reinforced concrete column and therefore the idea of a tube filled with concrete was not new.

Various models were proposed by Chan (1955), Roy and Sozen (1964), Sargin (1971), Kent and Park (1971), Sheikh and Uzumeri (1980), and Sheikh (1982). One of the most practical models in this field is the modified Kent and Park model (Park, et al. 1982) that is a function of transverse reinforcement and concrete specifications. The relationship for the stress enhancement factor, \( K \), can be presented as the following equation:

\[ K = 1 + \frac{\rho_s f_{yh}}{f_c} \]  \hspace{1cm} (2.13)

where \( \rho_s = \) ratio of the volume of transverse reinforcement to the volume of concrete core measured from the outside of the hoop, and \( f_{yh} = \) yield strength of the transverse reinforcement. The modified model assumes that for the unconfined concrete core, the
maximum stress reached is $Kf_c$, and the strain corresponding to the maximum stress is 0.002K. The detailed form of this model for the stress-strain behaviour of concrete, according to Fig.2.2, can be shown as follows:

Range AB ($\varepsilon_c < 0.002K$)

$$f^* = Kf_c \left[ \frac{2 \varepsilon_c}{0.002K} + \left( \frac{\varepsilon_c}{0.002} \right)^2 \right]$$ (2.14)

Range BC ($\varepsilon_c > 0.002K$)

$$f^* = Kf_c \left[ 1 - Z_m (\varepsilon_c - 0.002K) \right]$$ (2.15)

but not less than $0.2Kf'_c$, in which

$$Z_m = \frac{0.5}{3 + 0.29f_c} + \frac{3}{145f_c - 1000} \frac{\rho_s}{V_{sh}}^{h'_c - 0.002K}$$ (2.16)

and:

$$Z_m = \frac{\tan \theta_m}{Kf'_c}$$ (2.17)

and in which $f'_c$ is in MPa; $K$ is as given in Eq. (2.13); $h'_c$= width of the concrete core measured to the outside of the peripheral hoop; and $sh$ = centre spacing of hoop sets.

The falling branch of the curve is suggested to be a straight line whose slope, $\theta_m$, is a function of concrete cylinder strength, ratio of width of confined concrete to spacing of ties, and ratio of volume of tie steel to volume of concrete core (Park et al. (1982)).

Under a triaxial stress state provided by confining pressure and the applied load, the stress-strain behaviour of high strength concrete changes with increasing strength and
plastic deformation due to the confining pressure (Setunge et al., 1992). It is difficult to formulate a general theory for the deformation behaviour of confined concrete because the uniaxial stress-strain curve is nonlinear and Poisson’s ratio is a function of the stress. Based on the extensive triaxial work on high strength concrete cylinders by Setunge, Attard and Darvall (1992), the following equation was proposed for very high strength confined concrete:

\[
\frac{f}{f_0} = \frac{ax + bx^2}{1+cx + dx^2} \quad \text{where} \quad x = \frac{\varepsilon}{\varepsilon_0} \tag{2.18}
\]

with the peak confined stress \(f_0\) and corresponding strain \(\varepsilon_0\), and \(a, b, c\) and \(d\) are constants.

![Fig. 2.2 Modified Kent and Park Model for Stress-Strain Behaviour of Concrete Confined by Rectangular Steel Hoops.](image-url)

Fig.2.2 Modified Kent and Park Model for Stress-Strain Behaviour of Concrete Confined by Rectangular Steel Hoops.
2.4 STEEL MATERIAL AND PROCESS OF PRODUCTION

Steel tubular members, in practice, are often cold formed during fabrication, and may also be cold formed during erection. Cold forming or straining leads to a decrease in yield strength, referred to as the Bauschinger effect, if inelastic strain occurs in the opposite direction from the initial inelastic straining. Research has been carried out in this area by Pavlovic and Stevens (1981), Morgan and Schmidt (1985) and Schmidt, Lu and Morgan (1989).

Curved hollow steel tubular members subjected to compressive load often experience inelastic local buckling failure combined with a dramatic reduction in load carrying capacity and ductility thereafter in cases of relatively small wall thickness. Prior inelastic bending deformations might cause a reduction in the buckling stress of mild steel struts subjected to direct compression (Pavlovic and Stevens, 1981). The reduction is due to the Bauschinger Effect apart from residual stresses.

Steel tubes filled with concrete have many advantages when used as columns in structures. By using concrete infilled steel struts, the buckling load of the curved steel tube subjected to compressive load will be increased, and also the ductility of the concrete core will be improved due to confinement by the steel tube. In the design of multi-storey buildings, piles and other structures, high strength concrete has been used because of advantages of high strength, superior durability and a reduction in the require amount of steel area.

The ultimate load capacity of curved hollow steel struts not only relates to variables such as initial radius of curvature, slenderness ratio, and yield strength, but also to variables such as material type and the process of production (Schmidt and Mortazavi, 1993). Turning attention to the strut as a structural element, limited work has been reported which includes the influence on strut load capacity of strain hardening, strain aging, the Bauschinger effect and residual stresses. "Strain hardening" is the term used
to define the increase in strength with increasing strain as plastic deformation or flow occurs beyond the yield point (Morgan and Schmidt, 1985).

"Strain aging" is the term used to describe any increase in strength or reappearance of a discontinuous yield phenomenon occurring on reloading in the same direction of strain as applied by the initial inelastic load. Strain aging is considered (Baird, 1963) to be due to the migration of carbon and nitrogen atoms to dislocations causing locking. Other changes also follow from the strain aging phenomenon. The discontinuous yield phenomenon normally returns, the ultimate tensile strength may be increased, and the elongation to fracture may be reduced. Baird (1963) has given an explanation of the effects as a multistage process. The first stage is the formation of atmospheres of carbon and nitrogen around the dislocation caused by the prestraining. As a consequence, the yield stress increases and a reduction occurs in the elongation at the lower yield stress. The second stage occurs when precipitates form along the dislocations; the yield stress continues to rise, the elongation at lower yield remains constant, but the ultimate tensile stress increases, and the elongation to fracture is reduced.

Chajes et al. (1963) have discussed at length the effects of cold-stretching flat sheets of steel. They showed that the effects of the cold work were directional. The Bauschinger effect can be described in terms of three parameters: strain, stress or strain energy as discussed by Abel et al. (1972), who explained that the principal causes are believed to be associated with elastic stress and/or anisotropy in the resistance to dislocation motion. The Bauschinger effect was observed in the longitudinal direction, together with an inverse effect in a direction normal to the direction of straining. After an initial inelastic tensile strain, straining in tension longitudinally causes an increase in tensile yield strength, but causes a reduction in compressive yield strength. In the transverse direction the opposite occurs; the compressive yield strength increases, but the tensile yield strength decreases. Chajes et al. (1963) referred to this effect as an "Inverse Bauschenger Effect". Such effects have been discussed also by Pascoe (1971).
Bouwkamp (1975) carried out axial compressive load tests on seamless and electric-welded steel pipes with slenderness ratios between 40 and 120. The test results agreed reasonably well with predicted load values using the tangent-modulus expression. It was found that local plastic buckling caused a drastic reduction of the post buckling strength.

Chen (1977) investigated experimentally the magnitude and distribution of longitudinal and circumferential residual stresses in fabricated steel tubular columns. Stub column tests, and the strength and behaviour of 10 full-scale fabricated cylindrical columns of medium slenderness ratios of 48 and 70 were investigated. It was concluded that theoretical ultimate load analysis based on the tangent modulus theory of an initially straight column underestimated the strength of fabricated tubular members. Except for the shortest columns, these variations were from 8 % to 16 %. It appeared that the transition from general plastic yielding to a local buckling type of failure occurred at a diameter-to-thickness ratio (D/t) of about 60 for all slenderness ratios tested.

Chan and Kitipornchai (1986) investigated the inelastic post-buckling behaviour of beam-columns of circular hollow section. A finite element technique was employed to study the geometric and material nonlinearities by continuously updating the geometry of the element and by modifying the element stiffness for plasticity (the idealized elastic perfectly plastic stress-strain model was assumed), taking into account the influence of strain unloading. Incremental equilibrium equations were formulated in an updated Lagrangian framework. The iterative arc-length technique (Crisfield, 1981) was employed to trace the pre- and the post-buckling load-deflection paths. Moment-axial-force-curvature relationships were not needed in the analysis, as only the fundamental stress-strain relationship of the material was required. Kitipornchai et al. (1987) modified the method proposed (Chan and Kitipornchai, 1986) and studied the geometric and material nonlinear large deflection behaviour of structures comprising thin-walled rectangular hollow sections. The influence of various types of residual stress, initial geometrical imperfections, load eccentricity and yielding of material were incorporated in the analysis. The idealized elastic perfectly plastic stress strain
relationship was assumed, strain hardening was neglected, but the effects of strain unloading were included.

An extensive experimental and theoretical work was carried out to investigate the behaviour of tensile-prestrained straight hollow-steel struts subjected to compressive load by Lu and Schmidt (1990). They analyzed the practical influence on the hollow steel strut load capacity of strain hardening, strain aging and the Bauschinger effect. It was found that the influence of the Bauschinger effect was more significant on the tubes with small initial imperfections. In the range of initial imperfections considered, the Bauschinger effect was more dominant than strain hardening and strain aging. Considering the influence of strain aging, strain hardening and the Bauschinger effect, the reduction in load capacities of the prestrained struts was clearly seen in comparison with the load capacities of the corresponding as-received struts. This loss of load capacity was due to the Bauschinger effect on load reversal. Strain aging reduced this reduction. As well, to investigate the influence of the residual stresses set up during the tube making process on the steel tubular strut capacity, tests on stress-relief-annealed tubes were also performed. For the finite element modeling, three stress-strain relationships of the steel were assumed with respect to the as-received, prestrained in tension and fully-aged, and prestrained in tension and unaged material. In the analysis, the cross-section of the element was divided into a finite number of elementary areas. The structure tangent stiffness matrix was obtained by using a series of transformation matrices to update the element geometry. Theoretical curves were established including the effects of initial imperfections, slenderness ratios and initial geometrical imperfections on the strut load capacities and the post-buckling behaviour. Theoretical results were in good agreement with those obtained from experimental results.
2.5 COMPOSITE COLUMNS

Serious studies on the structural behaviour of concrete filled steel tubes began in the decade 1950’s. An extensive experimental investigation of the properties of concrete filled steel tubes was carried out in Germany by Kloppel and Goder in 1954-1955. They took into account the length of columns, whereas in previous investigations only short members (stub columns) were considered. It was recognised that the columns could fail by column buckling, by material failure or both. A method was prescribed for calculating the Euler buckling load and the collapse load of the columns based on an experimentally derived modular ratio.

Gardener and Jacobson (1967) predicted the ultimate load of short concrete filled steel tubes and also the buckling load of long concrete filled steel tubes from an experimentally determined load deflection curve of a stub column of similar dimensions. They used the tangent modulus method to predict the buckling loads which were 0 to 16.8 percent conservative. As with the long columns the composite steel stub columns which yielded first in the longitudinal direction were tested. The failure loads which were calculated from the sum of the failure loads of the steel and concrete acting alone were significantly lower than the measured failure loads. However, the combined steel stress states, and the circumferential stress in the steel, gave good agreement with the measured failure loads. The maximum value of the lateral restraint factor $K$ which was used to calculate the ultimate load of short columns appeared to be in the region of 4.1.

An experimental investigation into the structural behaviour of concrete-filled spiral welded steel tubes under axial load was carried out by Gardener (1968). Several spiral welded pipe columns were made and tested to check on the applicability of conventional design methods to this type of column. The allowable loads were calculated using the steel properties taken from compressive tests. By comparing the load-strain curves for some long columns it appeared that the plain concrete stiffness was equal to the long concrete-filled spiral welded steel tube column stiffness. This
was illogical, and led to the conclusion that the concrete cylinders were not representative of the concrete in the long column. This would be due to inadequate compaction of the long columns compared with that of the cylinders. The writer herein believes that estimating the tangent modulus for the steel from uniaxial test results is incorrect if the steel was biaxially loaded.

The elasto-plastic behaviour of straight pin-ended, concrete-filled tubular steel columns, loaded either concentrically or eccentrically about one axis, was studied numerically by Neogi, Sen and Chapman (1969). They used uniaxial stress-strain curves for steel and concrete in their analysis. In order to determine the load-deflection curve, the differential equation governing the bent equilibrium configuration of an eccentrically-loaded column was derived by equating internal and external forces and moments at a displaced section. When calculating external moments the deflection $\delta$ of the section due to the applied load added to the end eccentricity. The deflected shape was then calculated by integrating this equation along the length of the column. To determine the complete load-deflection curve of the column, lateral deflection and axial load values were calculated for a series of equilibrium shapes defined by increments of curvature at the central cross-section. The peak of this curve gave the maximum load. By assuming the deflected shape to be part of a cosine wave the calculation was greatly simplified. They claimed that for practical purposes the part cosine wave deflected shape assumptions gave sufficiently accurate results for pin-ended eccentrically loaded columns. In this case, the maximum load calculated according to this assumption was always conservative, but not more than 5 percent below the value given by the exact shape calculation. The concrete stress-strain curve was represented by a single non-dimensional equation (Desayi (1964)).

Experimental investigations by Bridge (1976) have revealed that the concrete filled steel tubes have the ability to continue to carry a substantial proportion of their maximum loads for further deformation beyond that at maximum load. The ductility, tenacity or toughness of the tube also prevented or delayed local buckling failure, which would have curtailed the ductile range of behaviour.
Ghosh (1977) carried out an experimental and theoretical study on strengthening of slender hollow steel columns by filling with concrete. Tests were performed on long columns under combined axial and transverse bending. The columns had a slenderness ratio as high as 129. Ghosh concluded that concrete increased the load- and moment-carrying capacity without increasing the size of the column. Accordingly, thinner-walled steel columns filled with concrete could be used at considerable savings and without loss of strength. Although the behaviour up to the ultimate load was not established, the average of the deflection curves, assuming cracked and uncracked concrete sections, gave sufficiently accurate results on which to predict the behaviour of slender concrete-filled columns up to fairly high loads. However, the tests were limited in nature, and further testing was needed to establish a revised design standard to allow an increase in the capacity of long, slender columns due to the contribution of the concrete fill. Pumping was found to be an effective and economical way of filling the steel pipes and once the crews on the job became familiar with the process, they were able to fill up to 35 pipes, 15 m high, in a single shift with one pump.

Bridge and Roderick (1978) performed tests on encased I-section including members made up from two or more steel components as shown in Fig.2.3. They examined the behaviour of such members up to collapse with and without the battens. All columns tested were of the same cross section consisting of two 3-in. (76-mm) ×1-1/2in. (38-mm) 4.60-lb/ft (6.81-kg/m) steel channels, encased in concrete to give 2 in. (51 mm) of cover all around. The results of all the tests are summarized in Table 2.1.

They developed a theoretical model that enabled them to take into account the full range of linear characteristics of the material. Theoretical data were obtained from an analysis developed as an extension of the original version by Roderick and Rogers (1969) derived for encased rolled steel joists bent about their minor axis. The theoretical method was based on determining the equilibrium deflected shape of a column for successively higher values of load; the maximum load was defined as the value at which the slope of the load deflection relationship was zero.
Fig. 2.3 Details of Cross-Section and Battened Columns Used by Bridge and Roderick (1978).
Table 2.1 Summary of Data Columns Tested by Bridge and Roderick (1978)

<table>
<thead>
<tr>
<th>Column number</th>
<th>Type</th>
<th>Axis of bending</th>
<th>Eccentricity (inches)</th>
<th>Maximum Load, (kips)</th>
<th>Observed Load / Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Observed</td>
<td>Theoretical</td>
</tr>
<tr>
<td>(a) 7-ft Composite Columns Bent About Major Axis</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CC1</td>
<td>No battens</td>
<td>Major</td>
<td>0</td>
<td>270</td>
<td>273</td>
</tr>
<tr>
<td>CC2</td>
<td>No battens</td>
<td>Major</td>
<td>0.8</td>
<td>196</td>
<td>201</td>
</tr>
<tr>
<td>CC3</td>
<td>No battens</td>
<td>Major</td>
<td>1.5</td>
<td>159</td>
<td>151</td>
</tr>
<tr>
<td>(b) 7-ft Composite Columns Bent About Other than Major Axis</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CC4</td>
<td>No battens</td>
<td>Minor</td>
<td>1.5</td>
<td>117</td>
<td>110</td>
</tr>
<tr>
<td>CC5</td>
<td>No battens</td>
<td>49° to Major</td>
<td>0.8</td>
<td>158</td>
<td>150</td>
</tr>
<tr>
<td>(c) 10-ft Composite Columns Bent About Major Axis</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CC6</td>
<td>End battens</td>
<td>Major</td>
<td>0.8</td>
<td>21</td>
<td>20</td>
</tr>
<tr>
<td>CC7</td>
<td>4Int.battens</td>
<td>Major</td>
<td>0.8</td>
<td>53</td>
<td>51</td>
</tr>
<tr>
<td>(d) 10-ft Composite Columns Bent A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CC8</td>
<td>End battens</td>
<td>Major</td>
<td>0.8</td>
<td>147</td>
<td>147</td>
</tr>
<tr>
<td>CC9</td>
<td>No battens</td>
<td>Major</td>
<td>1.5</td>
<td>110</td>
<td>105</td>
</tr>
<tr>
<td>(e) 10-ft Battened Composite Column Bent About Major Axis</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CC10</td>
<td>4 Int bat.</td>
<td>Major</td>
<td>0.8</td>
<td>150</td>
<td>143</td>
</tr>
<tr>
<td>(f) 7-ft Composite Column Bent in Double Curvature</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CC11</td>
<td>No battens</td>
<td>Major</td>
<td>2.0</td>
<td>159</td>
<td>154</td>
</tr>
</tbody>
</table>

They established the load-moment-curvature relationship for the column cross section. The residual stresses were assumed to be treated independently, and the other basic assumptions were as follows;
1- For the case of biaxial bending the plane of deformation was the same as the plane of the applied end moments.

2- The complete stress-strain function for concrete was expressed in polynomial form in which

\[ \sigma_c = F_c [\varepsilon] \]  
\[ F_c [\varepsilon] = a_1 \varepsilon + a_2 \varepsilon^2 + a_3 \varepsilon^3 + a_4 \varepsilon^4 \text{ for } \varepsilon > \varepsilon_{ct} \]  
\[ F_c [\varepsilon] = 0 \text{ for } \varepsilon < \varepsilon_{ct} \]

where \( \varepsilon_{ct} \) was the tensile strain at which the concrete was assumed to crack. The constants \( a_1, a_2, a_3, \text{ and } a_4 \) were determined from cylinder or column test results using a method of least squares developed by Smith and Oranguan (1969).

3- The stress-strain relationship for steel was calculated from

\[ \sigma_s = F_s [\varepsilon] \]  
\[ F_s [\varepsilon] = E_s \varepsilon \text{ for } -\varepsilon_{sy} < \varepsilon < \varepsilon_{sy} \]  
\[ F_s [\varepsilon] = f_{sy} \text{ for } \varepsilon \geq \varepsilon_{sy} \]  
\[ F_s [\varepsilon] = -f_{sy} \text{ for } -\varepsilon_{sy} \geq \varepsilon \]

in which \( \varepsilon_{sy} \) was the yield strain and \( f_{sy} \) was the steel yield strength.

4- The initial imperfection \( c_0 \) at the mid-height of the column in the required direction \( \beta \) to the principal axes was taken as

\[ c_0 = c_{ox} \cos^2 \beta + c_{oy} \sin^2 \beta \]
They determined the load-deformation relationship of the column right up to the collapse load, the collapse load and the relationship of the maximum moment developed at collapse to the full flexural strength of the member. A typical load-deflection of the columns is shown in Fig.2.4. They concluded that the differences in observed and theoretical collapse loads were in the range of -2% and +6%. In such members, the function of transmitting shear force between channels were performed by the concrete encasement in addition to the battens. There were a range of slender columns of the form considered, for which collapse occurred by instability before the full flexural strength of the member was developed.

According to Rangan and Joyce (1992) the strength of eccentrically loaded straight slender steel tubular columns filled with high-strength concrete can be calculated by performing an analysis for the cross-section at mid-height based on assuming a sine function for the deflection of the column. The study involved testing nine slender composite columns under eccentric compression. The compressive strength of concrete was 67 MPa. The eccentricity of the applied compressive load was equal at both ends, and the columns were subjected to single curvature bending. Test parameters were the effective length of the column and eccentricity of the applied load. Test results are summarized in Table2.2. They assumed that the ultimate axial load capacity \( P_u \) of a slender eccentrically loaded steel tubular column filled with concrete was reached when the maximum moment \( M_u \) was equal to the ultimate bending moment \( M_n \) at the mid-height of the column. The deflection of the column due to creep (or long-term effects) and imperfections in the steel sheath were treated as an additional deflection. The value of \( P_u \) was related to \( M_u \)

\[
M_u = P_u (\epsilon + \Delta_{cp} + \Delta_u) \quad (2.27)
\]
Fig. 2.4 Load-Deflection Relationships for Columns CC6, CC7, CC8 and CC10 (Bridge and Roderick (1978)).
where \( e \) is the eccentricity of axial load, \( \Delta_{cp} \) is the creep deflection and \( \Delta_u \) is the deflection at failure. The value of \( P_u \), the curvature \( \kappa_u \), and the deflection \( \Delta_u \) at mid-height were related by,

\[
\kappa_u = \left( \frac{\pi^2}{L^2} \right) \Delta_u
\]  

(2.28)

and \( \varepsilon_1 \) was the strain at the extreme compressive fiber (Fig.2.4) given by,

\[
\varepsilon_1 = 0.003 \frac{d_n}{(d_n - t)}
\]  

(2.29)

where \( t \) was the wall thickness of the steel tube. The curvature also was given by,

\[
\kappa_u = \varepsilon_1 / d_n
\]  

(2.30)

\[
\kappa_u = 0.003 / (d_n - t)
\]  

(2.31)

in which \( d_n \) is the depth of neutral axis. It was assumed that the tubular steel section was split into two strips about the neutral axis. The area of concrete in the compression zone was assumed to be lumped at its centroid. Similarly, the area of steel in the tension zone and in the compression zone were concentrated at their respective centroids.
Rangan and Joyce used the following equations to estimate the strength of slender composite columns,
\[ P_u = C_c + C_s - T \]  \hspace{1cm} (2.32)

\[ M_n = (C_s z_{sc} + T z_{st} + C_c z_c) \]  \hspace{1cm} (2.33)

in which \( C_s = A_{sc} \sigma_{sc} \); \( T = A_{st} \sigma_{st} \); \( C_c = A_c \sigma_c \); and \( z_{sc}, z_{st}, \) and \( z_c \) were the lever arms of forces from the plastic centroid (Fig. 2.5). The steel stresses were \( \sigma_{sc} = E_s \varepsilon_1 d_{sc}/d_n \) and \( \sigma_{st} = E_s \varepsilon_1 d_{st}/d_n \). When the steel stresses were larger than the yield stress, they were taken as \( f_{sy} \). \( E_s \) was the modulus of elasticity of the steel, and \( d_{sc} \) and \( d_{st} \) were the distances of the respective centroids of the steel areas from the neutral axis.

The calculation procedure of the strength of eccentrically loaded straight and slender steel tubular column filled with high-strength concrete is briefly explained below;

1- Select a suitable value for the depth of neutral axis \( d_n \). For this value of \( d_n \), calculate \( \varepsilon_1 \) by Eq. (2.29), and calculate all stresses.

2- For these values, calculate \( P_n \) and \( M_n \) by Eqs. (2.32), (2.33). Also, calculate \( \kappa_u \) and \( \Delta_u \) by Eqs. (2.31) and (2.28).

3- Calculate \( \Delta_{cp} \) and estimate deflections due to initial imperfections in the steel sheath when they are significant and add these to \( \Delta_{cp} \).

4. For these values of \( \Delta_u \) and \( \Delta_{cp} \), take \( P_u = P_n \) and calculate \( M_u \) by Eq. (2.27). The selected value of \( d_n \) is accepted when \( M_u = M_n \) and strength of the column is \( P_u \).

Rangan and Joyce found that there was good correlation between tested and calculated results as well as with those available in the literature. The mean value of test/calculated for 27 columns (including literature data) was 1.17, with a coefficient
of variation of 16 percent. The calculated strengths were very conservative for columns with small eccentricity (e/D=0.05), perhaps due to the assumption that ε_u=0.003 at failure for the concrete.

Shakir-Khalil (1993) reported tests on pushout strength of concrete-filled steel hollow sections. The tests showed clearly that the resistance of the specimens to the pushout load was a function of the shape and size of the steel hollow section used, and also the condition of the steel-concrete interface. He mentioned that for the tested specimens with an oiled steel-concrete interface, resulted in halving the bond resistance of the concrete-filled steel hollow sections. The test results clearly showed how sensitive the bond strength was to the interface conditions. He used steel-concrete interface lengths of approximately 200, 400 and 600mm, i.e. in the ratio of 1:2:3. The failure loads of each group of the specimens, dry or oiled, were not in a similar ratio. The average bond strength result for each group indicated that the bond strength of circular specimens was on average about 82 % and 64 % higher than that of the rectangular specimens for the ‘dry’ and ‘oiled’ conditions, respectively. The ‘dry’ specimens gave an average bond strength that were about twice those of the ‘oiled’ specimens for both the rectangular and circular specimens. In general, Shakir-Khalil concluded that the bond strength, as based on the pushout failure load, was rather sensitive to the local imperfections of the steel-concrete interface and also to the overall longitudinal variations in the dimensions of the steel hollow sections.

Kavoossi and Schmidt (1993) carried out an experimental and theoretical study on the behaviour of confined higher strength concrete (HSC) in steel circular hollow sections with emphasis on the dependence of the radial stress on the axial stress in the confined concrete. The results of the study showed that the brittle behaviour of higher strength concrete could be modified considerably by confining the concrete in a steel tube. They concluded that the apparent elastic parameters, such as the modulus of elasticity and Poisson's ratio in a hollow steel tube under axial compression depended on the dimensions of the specimen. In the tests of confined concrete, the concrete mass was loaded, and the steel tube yielded longitudinally. This effect showed that the axial load, in a very short length, transferred load to the steel tube. The increase of the
compressive strength of the confined concrete in a steel tube was in fair agreement with the proposed equation by Cai (1986). The ductility factor, the ratio of load at any specific strain over the ultimate axial load, showed that the concrete encased in thick walled tubes was more ductile than the concrete in thin walled tubes.

2.6 CURVED MEMBERS AND ARCHES

As mentioned in the previous section the ultimate load capacity of concrete infilled curved steel struts largely depends on variables such as initial curvature, slenderness ratio, concrete strength, yield strength of steel, and the effect of confining pressure on the compressive strength of the concrete. Imperfections of shape of the member are of lesser importance than for the case of straight struts due to the dominant initial curvature.

In general, arches support their loads by a combination of axial compression and bending actions, except in the case of uniform radial loading, which is resisted by uniform axial compression alone. Thus their in-plane behaviour varies from the buckling failure of members under end axial compression to the plastic collapse of flexural members. The relative importance of the compression and bending actions depends on the arch loading, and on its rise/span ratio.

Elastic buckling of arches under loadings which produce pure axial compression was investigated by Walter and Austin (1971). They computed the critical axial thrust in an arch conveniently by the following equation which is identical in form to that used for straight compression members,

\[ P_e = \frac{\alpha EI}{S^2} = \frac{\pi^2 EI}{(kS)^2} \]  

(2.34)

in which \( P_e \) = the critical axial compressive force; \( E \) = Young's modulus; \( I \) = second moment of area of the cross section; \( S \) = one-half the length of the arch axis; \( \alpha \) = a
coefficient, and \( k = \) the effective length factor. The values of \( \alpha \) and \( k \) were calculated for parabolic, catenary, and circular arches for rise-span ratios of 0.1, 0.2, 0.3, 0.4, and 0.5, a complete range of practical values.

Arches in uniform compression caused by uniform radial loading generally fail by buckling in two half waves, and at loads very close to those of axially loaded columns whose lengths are equal to the developed half-length of the arch (\( S/2 \)), (Pi and Trahair, 1995). Current studies of the failure of arches with significant horizontal thrusts suggest that an adequate design procedure is to use the non-linear interaction equation (Trahair 1995),

\[
\frac{1}{\lambda_c} + \frac{1}{\lambda_p(1 - 1/\lambda_e)} = 1
\]

in which \( \lambda_c \) is the load factor for failure under uniform compression, \( \lambda_e \) is the load factor for elastic buckling, and \( \lambda_p \) is the load factor for plastic collapse.

2.7 THEORETICAL SOLUTION PROCEDURE

To analyse theoretically the effects mentioned above on composite curved strut load capacity, the theoretical model must include both geometrical and material nonlinearity. In addition, the post-buckling behaviour needs to be tracked (analysed), as the characteristics of the post-limit-point behaviour of the structure can be related to its imperfection sensitiveness. In order to determine the pre- and post-buckling behaviour of the structure, the nonlinear load-deflection path has to be followed. The conventional Newton-Raphson procedure may have difficulty in surpassing the limit point on the load deflection curve where the tangent stiffness matrices approach singularity. The iterative process may then fail in the neighbouring region of the limit point. It is necessary to employ a more powerful iterative procedure to trace the load-deflection curve beyond the critical point.
The arc-length method technique has been independently introduced by Riks (1972 and 1979) and Wempner (1971). Both authors limit the load step by a constraint equation; that is, the generalized “arc length” of the tangent at a converged point of the load-displacement curve is fixed to a prescribed value. Then the iteration path follows a “plane” normal to the tangent path. The Newton-Raphson method is not often used with the finite element method, the modified Newton-Raphson method (m.N-R) being generally preferred (Zienkiewicz, 1977). When using the m.N-R procedure, the tangent stiffness matrix is neither re-formed nor re-factorized at each iteration but is, instead, held fixed. Even having substituted the m.N-R method for the N-R method for the N-R procedure, Riks’technique is still not suitable for use with the standard finite element method, because the direct simultaneous solution of the relevant equations destroys the symmetric banded nature of the equilibrium equations on their own (with a loading parameter $\lambda$ taken as a constant). The technique has been modified by Crisfield(1981) and Ramm(1981), so that it is suitable for use with the finite element method. In addition to the “constant-arc-length”, the step size may be scaled by relating the number of iterations used in the previous step to a desired value. If material nonlinearities are involved smaller load steps should be defined to avoid drifting. Whenever a negative element in the triangularized stiffness matrix is encountered unloading is initiated.

Bergan et al. (1978) use the “current stiffness parameter” , to predict the local maximum or minimum. They then suppress the equilibrium iterations in the neighbourhood of the extreme (limit) point and reverse the sign of the load following a change in the sign of the determinant of the tangent stiffness matrix. The suppression of the equilibrium iterations dictates the provision of very small load increments close to the limit point and also leads to a local drift from equilibrium.

Bathe et al. (1983) presented an algorithm for the automatic incremental solution of nonlinear element equations in static analysis. The procedure used two different constraints depending on the response and load level considered; i.e., the spherical constant arc-length and a constant increment of external work. It was designed to
calculate the pre- and post-buckling collapse response of general structures. Eigensolutions for calculating the linearized buckling response were also discussed.

The theoretical inelastic behaviour of tubular columns and beam-columns has been studied extensively by Hays and Santhanaman (1979), Supple and Collins (1982), and notably by Chen and his co-workers (Han and Chen, 1983; Sugimoto and Chen, 1985; Toma and Chen, 1983). Most of the earlier analytical methods presented on the inelastic biaxial bending of beam-columns ignored the effect of strain unloading (Saleeb and Chen, 1981) and so could not predict accurately the post-buckling behaviour.

Smith et al. (1979) investigated the buckling strength and post-collapse behaviour of tubular bracing members including damage effects by using incremental finite element methods. The influence of initial deflection and residual stresses were considered. By applying incremental end-shortening displacements instead of loads, it is possible in many cases to carry analysis well into the post-collapse range without numerical difficulty. However, the authors found that, for some columns, the incremental stiffness matrix $K$ was found to become non-positive definite just after the peak load was reached and the solution procedure broke down. In such cases, a static post-collapse load-shortening curve was computed by incrementing lateral displacements, say at mid-length on the column with end-shortening displacements unrestrained and with axial force as an undetermined parameter. Compression tests were also carried out on a series of 16 tubes representing off-shore steel bracing members at about $1/4$ scale so as to provide experimental confirmation of the theoretical results, referring particularly to post-collapse behaviour and the performance of damaged members.

2.8 Summary

Concrete filled steel tubular struts are an important component in modern structures, such as tall buildings, and bridges. It has been seen that little work has been carried
out on the stability and ultimate load capacity of such structures subjected to an eccentric load. Various methods such as using exact and a cosine wave for deflected shape (Neogi, 1967), determining the equilibrium deflected shape for successively higher values of load (Bridge, 1978) and using an iteration process (Rangan, 1992) have been used to calculate ultimate load capacity of slender columns filled with concrete subjected to eccentric load.
3. CHAPTER THREE

THEORETICAL DEVELOPMENT

3.1  GENERAL

The aim of this study is primarily to present a simple theoretical and computational method based on assumptions which will enable to be estimated the ultimate load capacity of curved composite struts. As well, in order to predict the load-deformation behaviour of curved composite steel strut elements over the entire elastic-plastic range the finite element method will be used. The effects of all parameters which significantly influence load-deformation will be taken into account.

Such theoretical procedures were required to make a full comparison with the experimental results which will be described in Chapter four. The calculation procedures were programmed in a computer whenever needed. Iterative methods were employed to determine the ultimate load capacity of the curved struts. A finite element program (Nastran) enabled material and geometric nonlinearity to be taken into account to predict the detailed load-deflection behaviour.

3.1.1  Basic Assumptions

The following basic assumptions have been made:

1. Plane sections before deformation remain plane after deformation.
2. Complete interaction takes place between the tube and the core, i.e. there is no longitudinal or circumferential slip.

3. Failure due to local buckling of the steel tube does not occur.

4. Concrete cannot carry tensile stress.

5. The concrete stress-strain curve is identical in pure compression and flexure.

6. Yielding is governed by direct stress only. Shear strains due to bending are not considered.

7. Strains are small but displacements and rotations can be arbitrarily large.

These simplifying assumptions are made by most workers in the field of reinforced concrete and composite columns.

### 3.1.2 Uniaxial Stress-Strain Curve for Steel

It is essential to obtain the stress-strain relationship for the modeling used in any structural analysis. Based on the observation of material behaviour, the stress-strain relationship of the electric resistance welded (ERW) steel stub column is approximated by a bilinear curve upon loading, as shown in Fig.3.1. To take into account the effect of curving and residual stress, the stress-strain relationship of the prestrained stub column is approximated by a fourth curve, as shown in Fig.3.2. Stress, $\sigma$, may be related to strain, $\varepsilon$, by following equations:

1. ERW tube material (see Fig.3.1)

   $\sigma_i = E_s \varepsilon_i$ for $|\varepsilon| < |\varepsilon_y|$  \hspace{1cm} (3.1)

   $= \sigma_y + (\varepsilon_i - \varepsilon_y)E_s$ (loading) for $|\varepsilon| \geq |\varepsilon_y|$  \hspace{1cm} (3.2)

   $E (\varepsilon_i - \varepsilon_{pi})$ (unloading)  \hspace{1cm} (3.3)
\[ \varepsilon_{\text{pi}} = \varepsilon_{\text{max}} - (\varepsilon_y + (\varepsilon_{\text{max}} - \varepsilon_y)) \frac{E_s}{E} \]  

(3.4)

2. Prestrained material (see Fig.3.2)

\[ \sigma_i = E \varepsilon_i \quad \text{for} \quad |\varepsilon_i| < |\varepsilon_y| \]  

(3.5)

\[ \sigma_s + (\varepsilon_i - \varepsilon_s)E_i \quad \text{for} \quad |\varepsilon_s| \leq |\varepsilon| > |\varepsilon_y| \]  

(3.6)

\[ \sigma_x + (\varepsilon_i - \varepsilon_x)E_x \quad \text{for} \quad |\varepsilon_x| \leq |\varepsilon_i| > |\varepsilon_y| \]  

(3.7)

\[ \sigma_y + (\varepsilon_i - \varepsilon_y)E_s \quad \text{(loading)} \quad \text{for} \quad |\varepsilon_s| \leq |\varepsilon| > |\varepsilon_y| \]  

(3.8)

\[ (\varepsilon_i - \varepsilon_{\text{pi}})E \quad \text{(unloading)} \]  

(3.9)

\[ \varepsilon_{\text{pi}} = \varepsilon_{\text{max}} - (\varepsilon_{\text{sn}} + (\varepsilon_{\text{max}} - \varepsilon_y)) \frac{E_s}{E} \]  

(3.10)

where \( \varepsilon_{\text{sn}} = \sigma_y / E \)  

(3.11)

The incremental stress-strain relationship can be found by a variation of the above equations. In the elastic range \(|\varepsilon_i| < |\varepsilon| < |\varepsilon_s|\) and on the unloading and reloading paths \(|\sigma_i| < |\sigma_y|\) and \(|\varepsilon_i| < |\varepsilon_s|\), and \(\delta \sigma_i = E \delta \varepsilon_i\).

In the inelastic range \(|\varepsilon_i| > |\varepsilon_s|\), \(\delta \sigma_i = E_i \delta \varepsilon_i\) or \(\delta \sigma_i = E_x \delta \varepsilon_i\) or \(\delta \sigma_i = E_s \delta \varepsilon_i\), depending on the inelastic model used.

3.1.3 Uniaxial Stress-Strain Curve for Concrete

The complete concrete stress-strain curve is obtained by using the CEB-FIP ("International" 1970) model as shown in Fig.3.3. In accordance with the "International " (1970) model, the compressive stress is represented as a function of compressive strain by
Fig. 3.1 Idealised Stress-Strain Relationship For ERW Steel Tubes
Fig. 3.2 Idealised Stress-Strain Relationship For Prestrained Material
Fig. 3.3 Stress-Strain Relationship of Concrete
\[ \sigma = \frac{\sigma_0 \varepsilon (a - 206,600 \varepsilon)}{1 + b \varepsilon} \]  

(3.12)

where \( \sigma \) is in MPa, and where

\[ a = 39000(\sigma_0 + 7.0)^{-0.953} \]  

(3.13)

\[ b = 65600(\sigma_0 + 10.0)^{-1.085} - 85.0 \]  

(3.14)

in which \( \sigma_0 = 0.85 f_C \) represents the peak stress on each curve.

### 3.1.4 Biaxial Yield Criterion for Steel

As the steel tube is stressed biaxially, it is necessary to assume a biaxial yield criterion for the steel. The von Mises criterion has been used in this study (Fig. 3.4); it should be noted that all this criterion refers to the first yield of a linear elastic material. In the inelastic range, the failure envelope can remain identical with the first yield envelope only if the material does not strain harden, i.e. if it has a linearly elastic-perfectly plastic uniaxial stress-strain curve.

The yield criterion of Von-Mises represents an ellipse in the biaxial plane. The principal stresses at first yield are therefore related by the equation

\[ \sigma_{11}^2 + \sigma_{22}^2 - \sigma_{11} \sigma_{22} = \sigma_y^2 \]  

(3.15)

where \( \sigma_{11} \) and \( \sigma_{22} \) are maximum and minimum principal stresses, respectively.
When considering the difference in compression and tension yield capacity of a material, a modified Von-Mises criterion is also suggested in the following equation (Raghava, 1973)

\[(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2+2(C-T)(\sigma_{11}+\sigma_{22}+\sigma_{33})=2CT\]  \hspace{1cm} (3.16)

where, T, C are yield strength in uniaxial tension and compression respectively, and \(\sigma_{11}, \sigma_{22}\) and \(\sigma_{33}\) are principal stresses in a three dimensional principal stress space. According to Raghava this criterion was originally proposed by Schleicher in 1926, and was suggested independently by Stassi-D’Alia in 1967. The von-Mises yield criterion was also modified for anisotropic behaviour of materials by Theocaries (1989).

In addition to the Von-Mises yield criterion, the Tresca yield criterion of maximum shear theory assumes that yield will occur when the maximum shear stress reaches the value of maximum shear stress occurring under simple tension. The maximum shear stress is equal to half the difference between the maximum and minimum principal stresses. Therefore, since \(\sigma_{11} = \sigma_{22} = 0\), under simple tension the maximum shear stress at yield is \(\sigma_{11}/2\). Accordingly, yield will occur when any one of six following conditions is reached:

\[
\begin{align*}
\sigma_{11}-\sigma_{22} &= \pm \sigma_y \\
\sigma_{22}-\sigma_{33} &= \pm \sigma_y \\
\sigma_{33}-\sigma_{11} &= \pm \sigma_y
\end{align*}
\]  \hspace{1cm} (3.17, 3.18, 3.19)

These equations represent a cylindrical shape with a hexagonal cross-section, along the normal axis to the \(\pi\)-plane \((\sigma_{11}+\sigma_{22}+\sigma_{33} = 0)\) from the center of coordinates in a \(\sigma_{11}, \sigma_{22}, \sigma_{33}\) coordinate system. The cross-section of this cylinder with the plane of \((\sigma_{11}-\sigma_{22})\) is shown in Fig. 3.5.
Fig. 3.4 Von Mises Criterion for a Linearly Elastic-Perfectly Plastic Material

Fig 3.5 Maximum Shear Stress Theory of Tresca Yield Criterion
3.2 TRIAXIAL ANALYSIS of CONCENTRICALLY LOADED STUB COLUMNS

3.2.1 General Behaviour

When the steel and the concrete of the composite column are subjected to the compressive load in the early stage of loading the Poisson’s ratio for concrete is smaller than that for steel. The steel expands more than the concrete and the steel tube has no restraining effect on the concrete core. These two materials would separate unless the cohesion between the steel and the concrete was sufficient to maintain contact. This cohesion exerts an inward radial pressure on the tube, therefore producing a small hoop compressive stress in the steel.

As the longitudinal strain increases, an outward radial pressure develops at the steel concrete interface due to expanding and cracking concrete inside the tube, thereby setting up a hoop tension in the tube as shown in Fig.3.6. As long as the steel remains elastic, the stresses can be determined from the strains. As the load is further increased the steel tube yields in longitudinal compression, and both longitudinal and lateral strains continue to increase. When the column fails the hoop strain in the tube is also in excess of the uniaxial yield strains.

Failure may be initiated by a local wrinkling of the tube as shown in Fig.3.7. It is difficult to establish criteria for the occurrence of this phenomenon. If this possible failure mode is excluded, simple relations can be established for the failure load of a stocky column regarding the behaviour of the steel tube and the concrete core.
3.2.2 Failure Load Corresponding to The Hoop and Longitudinal Failure Mode of Concentrically Loaded Stub Column

The following equations were suggested by Richart (1928), Lohr (1934), Moreel (1935) and Neogi (1967) to calculate the longitudinal stress and radial stress in the concrete core at the ultimate stage of loading:

$$\sigma_{CL} = f'_c + K \cdot \sigma_{CR}$$  \hspace{1cm} (3.20)

where $f'_c$ = unconfined compressive strength of concrete, $K = \text{an empirical factor}$, and $\sigma_{CR} = \text{radial stress on concrete}$ (Fig. 3.6). For a thin-walled tube the hoop stress $\sigma_{SH}$ may be assumed to be constant over the wall thickness (Fig. 3.6). Considering the equilibrium of a semi-circular half of the steel shell,
\[
\sigma_{CR} = \sigma_{SH} \cdot \frac{2t}{di} \tag{3.21}
\]

where \(t\) is the wall thickness of the tube and \(di\) is the internal diameter for circular tube.

Eq. 3.20 can be expressed as:

\[
\sigma_{CL} = f'c + K \cdot \sigma_{SH} \cdot \frac{2t}{di} \tag{3.22}
\]

The total load carried by the concrete filled tube is

\[
P = A_c \sigma_{CL} + A_s \sigma_y \tag{3.23}
\]

\[
A_c (f'c + K \cdot \sigma_{SH} \frac{2t}{di}) + A_s \sigma_{SL} \tag{3.24}
\]

Fig. 3.7 Failure Modelling of Confined Concrete Due to Local Buckling
The failure loads can be calculated from

\[ P_H = A_c (f'_c + K \cdot \sigma_y \frac{2t}{d_i}) \]  \hspace{1cm} (3.25) \\

\[ P_L = A_c f'_c + A_s \sigma_{SL} \]  \hspace{1cm} (3.26)

where \( P_H \) is the ultimate load corresponding to the hoop failure mode and \( P_L \) is the ultimate load corresponding to the longitudinal failure mode. The load \( P_L \) is the same as the squash load of an ordinary reinforced concrete or composite column. For a thin walled tube,

\[ \frac{2t}{d_i} \approx \frac{A_s}{2A_c} \]

Substituting this relation in Eq.3.25, and assuming \( K=4 \) (Neogi, 1967), this equation may be rewritten as

\[ P_H = A_c f'_c + 2 A_s \sigma_y \]  \hspace{1cm} (3.27)

which indicates that at failure, the steel is twice as effective in hoop tension as longitudinal compression. The variation of strength with wall thickness is shown in Fig.3.8 for a particular tube, assuming \( K=4 \). For a practical tube size i.e. \( 8 < d/t > 50 \), the hoop failure load is always greater than the longitudinal failure load (Neogi, 1967).
The ultimate strength of circular hollow sections filled with concrete has been conveniently described by different codes of practice. But the generality of the recommended equations to include the wide range of material properties for steel tubes and concrete furnish considerable differences between the computed strength of composite columns with the actual strengths.

The mechanical behaviour of higher strength concrete is entirely different with the ordinary strength concrete. The higher strength concrete is more brittle than ordinary concrete. Restrictions for use of higher strength concrete in the filled circular hollow sections has not been specified as accurately as recommended in specifications for reinforced concrete. In the case of thin walled tubes, the brittle behaviour of the higher strength concrete and the formation of major cracks in the concrete mass cannot be prevented by the steel tube. This behaviour of circular hollow sections filled with higher strength concrete leads to unpredictable behaviour, and in some cases it may cause brittle failure in the structural member.
An experimental research program was carried out on the prediction of ultimate load capacity of CHS filled with higher strength concrete and the structural behaviour of these elements in elastic and plastic stages of loading by Kavoossi and Schmidt (1993). To achieve the most realistic model of these columns, they used two methods of loading: load on both steel tube and the concrete core, and load on steel tube only. They suggested the following equation for computing the ultimate load capacity of short circular hollow sections filled with concrete:

\[ P_u = 0.9A_s f_y + A_c [1.065 f'_c + 2.148(t/2)f_y] \]  

(3.28)

where \( A_s \) is steel cross section area, \( A_c \) is concrete cross section area, \( t \) is wall thickness, \( f_y \) is steel yield strength, and \( f'_c \) is concrete cylinder strength.

### 3.3 UNIAXIAL TANGENT MODULUS ANALYSIS of CONCENTRICALLY LOADED PIN ENDED COMPOSITE COLUMNS

The first theory for the maximum load of an axially loaded elastic pin ended column was obtained by L. Euler in 1757. The failure load \( P_E \) is given by

\[ P_E = \frac{\pi^2 EI}{L^2} \]  

(3.29)

where \( P_E \) is known as the Euler load, \( EI \) flexural rigidity, and \( L \) is length of pin ended column. Euler recognised that this equation applied only to long slender columns in which the material remains linearly elastic. Euler's theory was verified by a series of experiments carried out by von Karman and others about 1900.
The tangent modulus buckling load $P_{TM}$ is that value of the load $P$ which simultaneously satisfies the equations

$$P_{TM} = p' = \frac{\pi^2}{l^2} (E_{TS} I_s + E_{TC} I_c)$$

and

$$P_{TM} = p'' = A_s \sigma_{SL} + A_c \sigma_{CL}$$

where $E_{TC}$ is the tangent modulus of concrete, $E_{TS}$ is the tangent modulus of steel, $l$ is effective length of the column, $A_c$ is the concrete cross-section area, $A_s$ is the steel cross-section area, $I_c$ and $I_s$ are second moments of areas of the concrete and the steel respectively. The above equations can be solved by a graphical method. These formulas can be adapted to allow for various conditions of fixity at the ends of the columns by replacing $l$ by the effective length $KL$ where $K$ is a constant depending on the degree of fixity.

### 3.3.1 Stress-Strain Relationship

When the tube has a thin wall it is assumed that the steel is subjected only to two normal stresses. The radial stress acting on the element is small and is neglected; the circumferential stress is assumed to be constant over the thickness. As long as the steel is in the linear elastic range, the steel and concrete stresses can be calculated from the steel strains, using the following equations

$$\sigma_{SL} = \frac{E_s}{1 - \nu_s^2} (\varepsilon_{SL} + \nu_s \varepsilon_{SH})$$
\[
\sigma_{SH} = \frac{E_s}{1 - \nu_s^2} (\varepsilon_{SH} + \nu_s \varepsilon_{SL}) \tag{3.33}
\]

\[
P_c = P - P_s \tag{3.34}
\]

\[
P_c = P - A_s \cdot \sigma_{SL} \tag{3.35}
\]

\[
\sigma_{CL} = \frac{P_c}{A_c} \tag{3.36}
\]

\[
\sigma_{CR} = \sigma_{SH} \cdot \frac{2t}{d_i} \tag{3.37}
\]

where \(P_c\) is the axial load carried by the concrete core, \(P_s\) is the axial load carried by the steel tube, \(\varepsilon_{SL}\) and \(\varepsilon_{SH}\) are longitudinal and hoop strains, \(\sigma_{SL}\) and \(\sigma_{SH}\) are longitudinal and hoop stresses in the steel, \(\sigma_{CL}\) and \(\sigma_{CR}\) are longitudinal and radial stresses in the concrete, respectively. As it is also assumed that there is no longitudinal or circumferential slip between the steel and concrete,

\[
\varepsilon_{SL} = \varepsilon_{CL} \text{ and } \varepsilon_{SH} = \varepsilon_{CR}
\]

If the concrete is assumed to behave like a linear elastic material in the range, the concrete strains are given by the following equations:

\[
\varepsilon_{CL} = \frac{1}{E_c} \cdot (\sigma_{CL} - \nu_c \sigma_{CR}) \tag{3.38}
\]

\[
\varepsilon_{CR} = \frac{1}{E_c} \cdot (\sigma_{CR} - \nu_c (\sigma_{CL} + \sigma_{CR}) \tag{3.39}
\]

where \(E_c\) and \(\nu_c\) are the initial concrete modulus and Poisson’s ratio respectively; both are assumed to remain constant in this range.
3.4 ULTIMATE STRENGTH of COMPOSITE CURVED STRUTS

The failure load of a curved strut is the item of prime interest to the designer, but unfortunately there is no easy means of obtaining it. At failure there is a complex interaction for the cross section behaviour such as the spread of plasticity, position of neutral axis as well as the deflected form of the strut. One of the methods, that may be employed, is an iteration process.

An iteration process was used to determine the strength of the curved composite tubes. The ultimate load capacity was calculated from the intersection point of inelastic and elastic behaviour, which will be described later. The calculation procedure was programmed in a computer by using Fortran 77. The flow diagram is shown in Fig.3.9.

3.4.1 Elastic Behaviour

To avoid more complex analysis it was assumed that there was an initial sinusoidal deflected shape. Maximum strength of the members subjected to compressive load largely depends on the slenderness of the members. Based on Bar-Spring model of a axially loaded column (Warner et al., 1989) the end thrusts ($P_{u2}$ is shown in Fig.3.10) in a curved strut can be related to the initial deflection ($e$) and additional deflection ($\Delta_u$) at mid-height (see Fig.3.10) as

$$e + \Delta_u = e \frac{1}{1 - P_{u2}/P_{crit}}$$  \hspace{1cm} (3.40)

$$P_{u2} = \Delta_u P_{crit}/(e + \Delta_u)$$  \hspace{1cm} (3.41)
where $P_{crit}$ is the elastic critical load for a pin-ended strut, that is, the Euler load $\pi^2 EI / L^2$. Here $L$ is assumed to be the straight length for convenience, and $EI$ the effective flexural rigidity, according to ACI (1977), is taken as,

$$EI = \frac{1}{5} E_c I_g + E_s I_s$$  \hspace{1cm} (3.42)

where $I_g$ is the second moment of the gross cross-section area and $I_s$ is the second moment of steel cross-section area.

### 3.4.2 Inelastic Behaviour

The cross section strength, at mid-height, $P_{ul}$, was calculated as shown in Fig.3.11. The tubular steel section was divided into two strips about the neutral axis. The area of steel in the compression zone is denoted by $A_{sc}$, and the area of tension by $A_{st}$. Similarly, the area of concrete in compression is $A_c$. It was assumed that the area of steel in tension and in compression, and the concrete in compression are lumped at their centroids (Rangan and Joyce, 1992). The Hognestad parabola (1951) was used to calculate the concrete stress.

$$\sigma_c = f' c [2(\varepsilon_c/\varepsilon_0) - (\varepsilon_c/\varepsilon_0)^2]$$  \hspace{1cm} (3.43)
Chapter Three Theoretical Development

Fig. 3.9 Flow Diagram For Load-Deflection Curve Program Assuming An Initial Sinusoidal Deflected Shape.
where $f''_c = 0.85f'_c$; $f'_c$ = the compressive cylinder strength of concrete; $\varepsilon_0$ = the strain in the concrete at a stress of $f''_c$, and is taken as 0.002; and $\varepsilon_c$ is determined from the strain diagram of the cross section. According to the strain diagram of the cross section the additional curvature can be calculated as
\[ \kappa_u = \frac{0.003}{(d_n-t)} \]  \hspace{1cm} (3.44)

The maximum compressive strain of the concrete before failing, \( \varepsilon_{cu} \), is taken as 0.003 (ACI 1989). It was assumed when the extreme fibre compressive strain of the concrete reaches 0.003 that failure occurs. Initial curvature (\( e \)) and additional deflection (\( \Delta u \)) are shown in Fig. 3.10.

The axial load, \( P_{u1} \), is related to the ultimate bending moment at mid-height, \( M_n \), initial deflection, and additional deflection by,

\[ P_{u1} = C_c + C_s - T \]  \hspace{1cm} (3.45)

\[ M_n = (C_s z_{sc} + T z_{st} + C_c z_c) \]  \hspace{1cm} (3.46)

\[ (e+\Delta u) = \frac{M_n}{P_{u1}} \]  \hspace{1cm} (3.47)

in which \( e \) = initial deflection at mid-height ; \( \Delta u \) = additional deflection at mid height ; \( C_s = A_{sc} \sigma_{sc} ; T = A_{st} \sigma_{st} ; C_c = A_c \sigma_c \); and \( z_{sc}, z_{st}, \) and \( z_c \) are the lever arms of forces from the plastic centroid (Fig.3.11). The steel stresses are \( \sigma_{sc} = E_s \varepsilon_1 d_{sc}/d_n \) and \( \sigma_{st} = E_s \varepsilon_1 d_{st}/d_n \). When the steel stresses are larger than the yield stress, they are taken as \( f_{sy} \). \( E_s \) is the modulus of elasticity of the steel, and \( d_{sc} \) and \( d_{st} \) are the distances of the respective steel areas from the neutral axis.

To obtain the inelastic load-deflection curve, the first step is to determine \( P_{u1} \) when \( \Delta u = 0 \) and Eqs 3.45, 3.46 and 3.47 are satisfied with respect to the calculated out of balance forces. If the calculated out of balance forces is less than tolerance the next step, after modifying \( d_n \), allowing \( P_{u1} \) and \( M_n \) to be determined from Eqs 3.45 and 3.46, \( \Delta u \) is calculated from Eq. 3.47. As mentioned before, the calculation of \( P_{u1} \) is based on an iteration process which is described more in detail as follows:
1. Select an initial value for the depth of the neutral axis, \( d_n \), i.e., 10% of \( d \). For this value of \( d_n \) calculate the curvature by Eq. 3.44 and hence calculate \( \sigma_{Sc}, \sigma_{St}, \sigma_c \).

2. From these values, calculate \( P_{ui} \) and \( M_n \) by Eqs. 3.46 and 3.47. Also calculate the ratio of \( M_n/P_{ui} \). If the value of this ratio is equal to \( e \) then after modifying \( d_n \) calculate \( P_{ui} \) and \( M_n \). The additional deflection at mid-height, \( \Delta_u \), can be calculated by using Eq. 3.44 for this particular load.

3. If the value of \( M_n/P_{ui} \) is not equal to \( e \), select an increment to \( d_n \) i.e. 0.1% of \( d \), then repeat steps 1 to 2.

The above procedure can be programmed on a computer. In this case, it is more convenient to convert equations (3.41) and (3.45) into the form

\[
P_{ui} - P_{u2} = 0 \quad (3.48)
\]

\[
(C_c + C_s - T) - (\Delta_u \frac{P_{crit}}{e + \Delta_u}) = 0 \quad (3.49)
\]

or

\[
F(\varepsilon) = 0 \quad (3.50)
\]

To solve this equation a value of \( \varepsilon \) is found, by iteration, which decreases the residual below a pre-assigned tolerance limit. In this method the flexural rigidity is assumed to be constant along the curved tube length. Consequently, all section along the curved tube length have the same flexural rigidity as the central sections.

It is noted that the sine wave assumption was adopted from the elastic behaviour theory (Eq. 3.41), and was then adopted for the rigid-plastic theory, where equilibrium was investigated at mid-height only (Eqs 3.45, 3.46 and 3.47). As well, for a particular load the value of the additional elastic deflection was added to the plastic deflection because
of the contribution of the elastic deflection in the plastic collapse stage. Consequently, the total deflection at mid-height was larger than the plastic deflection alone. Load-additional deflection relationships for the curved struts infilled with higher strength concrete with different initial radii of curvature as well as different slendernesses in the elastic and plastic stages are shown in Figs.3.12 to 3.39. The ultimate load capacity of the struts calculated from an intersection point of the elastic response alone and the elastic-plastic response is listed in Tables 3.1 and 3.2. The results will be compared with the experimental results, and also with the results obtained from the other theoretical method.

### 3.4.3 Comparison of Theoretical Results with Rangan and Joyce (1992)

The strength calculation method presented by Rangan and Joyce (1992) was used to calculate the ultimate load capacity of curved steel struts infilled with high strength concrete. In order to make a comparison between the theoretical method herein with that of Rangan and Joyce the strength results obtained from both methods are given in Tables 3.1 and 3.2. It can be seen that there is good agreement in some cases depending on the initial deflection at mid-height. The results of these methods will be compared with experimental results.

An iteration process similar to the Rangan method, which is described in Chapter Two, was used. The procedure is briefly described below: the value of $P_u$ is related to $M_u$ as,

$$M_u = P_u (e + \Delta u) \quad (3.51)$$

where $e$ is the initial deflection at mid height, and $\Delta u$ is the additional deflection at failure. The additional curvature $\kappa_u$ can be related to the deflection $\Delta u$ at mid-height by;

$$\kappa_u = (\pi^2 / L^2) \Delta u \quad (3.52)$$
The strain $\varepsilon_1$ at the extreme compressive fibre (see Fig. 3.11) can be calculated by

$$\varepsilon_1 = 0.003 \frac{d_n}{(d_n - t)}$$

(3.53)

where $t$ is the wall thickness of the steel tube. The curvature also is given by,

$$\kappa_u = \frac{\varepsilon_1}{d_n}$$

(3.54)

$$\kappa_u = 0.003 / (d_n - t)$$

(3.55)

in which $d_n$ is the depth of neutral axis.

It was assumed that the tubular steel section was split into two strips about the neutral axis. The area of concrete in the compression zone was assumed to be lumped at its centroid. Similarly, the area of steel in the tension zone and in the compression zone were concentrated at their respective centroids.

According to Rangan and Joyce the strength of the composite curved steel struts subjected to compressive axial load was calculated by using the following Eqs. (see Fig. 3.11)

$$P_u = C_C + C_S - T$$

(3.56)

$$M_n = (C_S z_{sc} + T z_{st} + C_C z_c)$$

(3.57)

in which $C_S = A_{SC} \sigma_{SC}$; $T = A_{ST} \sigma_{ST}$; $C_C = A_C \sigma_C$; and $z_{sc}$, $z_{st}$, and $z_c$ were the lever arms of forces from the plastic centroid. The steel stresses were $\sigma_{SC} = E_S \varepsilon_1 d_{SC}/d_n$, and $\sigma_{ST} = E_S \varepsilon_1 d_{ST}/d_n$. When the steel stresses were larger than the yield stress, they were taken as $f_{sy}$. $E_s$ was the modulus of elasticity of the steel, and $d_{SC}$ and $d_{ST}$ were the distances of the respective centroids of the steel areas from the neutral axis.
The calculation procedure of the strength is briefly explained below:

1- Select a suitable value for the depth of neutral axis $d_n$. For this value of $d_n$, calculate $\varepsilon_1$ by Eq. (3.53), and calculate all stresses.

2- For these values, calculate $P_n$ and $M_n$ by Eqs. (3.56), (3.57). Also, calculate $\kappa_u$ and $\Delta_u$ by Eqs. (3.55) and (3.52).

3. For these values of $\Delta_u$ and $e$, take $P_u = P_n$ and calculate $M_u$ by Eq. (3.51). The selected value of $d_n$ is accepted when $M_u = M_n$ and strength of the column is $P_u$. 
Table 3.1 Ultimate Load Capacity of ERW Curved Steel Struts infilled with Higher Strength Concrete Obtained From Theoretical Method

<table>
<thead>
<tr>
<th>No</th>
<th>R (mm)</th>
<th>L (mm)</th>
<th>e (mm)</th>
<th>$P_{ult}$ (kN) Theoretical method</th>
<th>$P_{ult}$ (kN) Rangan method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2000</td>
<td>775</td>
<td>30</td>
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Table 3.2 Ultimate Load Capacity of Seamless Curved Steel Struts infilled with Higher Strength Concrete Obtained From Theoretical Method

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<th>e (mm)</th>
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Fig. 3.12 Additional Deflection at Mid-Height vs Load For the Electric Resistance Welded (ERW) Tube with R=2000mm and L=775mm.

Fig. 3.13 Additional Deflection at Mid-Height vs Load For the Electric Resistance Welded (ERW) Tube with R=2000mm and L=1176mm.
Fig. 3.14 Additional Deflection vs Load For The Electric Resistance Welded Tube with \( R=2000 \text{mm} \) and \( L=1559 \text{mm} \).

Fig. 3.15 Additional Deflection vs Load For The Electric Resistance Welded Tube with \( R=2000 \text{mm} \) and \( L=1745 \text{mm} \).
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Fig. 3.19 Additional Deflection at Mid-Height vs Load For The Electric Resistance Welded (ERW) Tube with R=4000mm and L=1176 mm.
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Fig. 3.21 Additional Deflection at Mid-Height vs Load For The Electric Resistance Welded (ERW) Tube with $R=4000\text{mm}$ and $L=1755\text{ mm}$. 
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Fig. 3.23 Additional Deflection at Mid-Height vs Load For The Electric Resistance Welded (ERW) Tube with R=4000mm and L=3114 mm.
Fig. 3.24 Additional Deflection at Mid-Height vs Load For The Electric Resistance Welded (ERW) Tube with $R=10000\,\text{mm}$ and $L=765\,\text{mm}$.

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Fig. 3.28 Additional Deflection at Mid-Height vs Load For The Electric Resistance Welded (ERW) Tube with R=10000mm and L=2265 mm.

Fig. 3.29 Additional Deflection at Mid-Height vs Load For The Electric Resistance Welded (ERW) Tube with R=10000mm and L=3020 mm.
Fig. 3.30 Additional Deflection at Mid-Height vs Load For The Seamless Steel Tube with R=2000mm and L=743 mm.

Fig. 3.31 Additional Deflection at Mid-Height vs Load For The Seamless Steel Tube with R=2000mm and L=1125 mm.
Fig.3.32 Additional Deflection at Mid-Height vs Load For The Seamless Steel Tube with R=2000mm and L=1484 mm.

Fig.3.33 Additional Deflection at Mid-Height vs Load For The Seamless Steel Tube with R=2000mm and L=1685 mm.
Fig. 3.34 Additional Deflection at Mid-Height vs Load for the Seamless Steel Tube with R=2000mm and L=2220 mm

Fig. 3.35 Additional Deflection at Mid-Height vs Load for the Seamless Steel Tube with R=4000mm and L=745 mm
Fig. 3.36 Additional Deflection at Mid-Height vs Load For The Seamless Steel Tube with $R=4000\,\text{mm}$ and $L=1120\,\text{mm}$

Fig. 3.37 Additional Deflection at Mid-Height vs Load For The Seamless Steel Tube with $R=4000\,\text{mm}$ and $L=1480\,\text{mm}$
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Fig. 3.38 Additional Deflection at Mid-Height vs Load For The Seamless Steel Tube with $R=4000\text{ mm}$ and $L=1680\text{ mm}$

Fig. 3.39 Additional Deflection at Mid-Height vs Load For The Seamless Steel Tube with $R=4000\text{ mm}$ and $L=2225\text{ mm}$
3.5 FINITE ELEMENT MODELLING of CURVED STRUTS INFILLED WITH CONCRETE

3.5.1 Introduction

The output of any FE analysis is influenced by the parameters used in the various stages of the modeling procedure. Analysis usually produces some results which may or may not be correct/accurate. There is no direct message or index to indicate the appropriateness of the parameters used in the model. Therefore, it is important to calibrate and optimise these parameters by preparing simple models that have theoretical or obvious solutions. The results from the FE analysis are then compared with the results obtained from other theoretical solutions or from other experimental tests. Some of these parameters are modified until a reasonable correlation between the FE results and theoretical and/or experimental results is achieved.

For composite structures such as concrete infilled steel struts, it is not always an easy task to analyse behaviour before and beyond the peak load. The difficulties are mainly a result of the highly nonlinear behaviour when concrete structures undergo very large deformations. Although many researchers over a long period have considered this problem, as yet a systematic and reliable package has not been developed. Bergan’s method (1978, 1980) can pass the limiting points, though it is not the best solution. Warner’s deformation approach (1987 and 1992) is also able to tackle the same problem. However, the direct stiffness method hampers the efficiency and application as far as nonlinear analysis is concerned. Crisfield’s arc-length method (1981) is probably the most successful numerical technique for analysing reinforced concrete structures up to and somewhere beyond their peak loads. A description of the arc-length method for the nonlinear analysis of concrete structures will be presented.
A finite element method was applied here for the nonlinear analysis of curved composite struts. The curved composite strut was divided into segments and treated as a space structure after segmentation. The sections were divided into finite elements of steel and concrete in order to calculate their tangent stiffness properties at different levels of strain. Accounting for both material and geometrical nonlinearities, the algorithms can analyse concrete infilled struts up to the ultimate load and somewhere beyond the limit points.

### 3.5.2 Material and Geometric Nonlinearities

Nonlinearity in concrete columns is due to both the material behaviour and to changes in geometry. Material nonlinearity is caused by several phenomena, such as steel yielding, concrete cracking, crushing, etc. Another important type of nonlinearity is that due to changes in geometry. One of the most significant of these nonlinearities is the P-Δ effect in slender columns. Nonlinearities due to material plasticity and geometrical change are handled by an iterative procedure based on the modified tangent stiffness approach. By taking into account geometric nonlinearity, the computing time increases due to the increased number of iterations. However, the accuracy of predicting the load-deflection curve increases dramatically if the nonlinear analysis is able to consider geometric nonlinearity in addition to material nonlinearity.

### 3.5.3 Basic Theory of Finite Element Method

The FEM seeks to analyses a continuum problem in terms of sets of nodal forces and displacements for a discretised domain (Cook, 1981). The procedure involves a set of routines which generate the stiffness matrix $[K^e]$ and initial load vector $\{f^e\}$ for all elements. These data and applied external loads, together with boundary conditions are
used to determine the nodal displacements for the whole structure. In these computations three major equations are repeatedly used.

(a) The characteristic equation of the elements; it is defined by the material property and the nodal displacement of the grid points surrounding the element.

\[
\{f^e\} = [K^e] \{u^e_i\} 
\]  

(3.58)

where:

\(\{f^e\}\) = column matrix representing the internal elastic force components induced at grid points

\([K^e]\) = element stiffness matrix

\(\{u^e_i\}\) = column matrix representing nodal displacement of the element

(b) the structural stiffness matrix; it is defined by compiling the element stiffness matrices together.

\[
[K^s] = \Sigma [K^e] 
\]  

(3.59)

where:

\([K^s]\) = the structure stiffness matrix.

It should be noted that freedom conditions of individual grid points will affect the construction of the structure stiffness matrix.

(c) the characteristic equation of the structure; it is set up as follows:
\[ \{F^s\} = [K^s] \{u^s\} \]  \hspace{1cm} (3.60) 

where

\[ \{F^s\} = \text{column vector representing the external forces.} \]

To solve any finite element problem using the displacement method, the following procedure should be carried out precisely:

- construct stiffness matrix for every element

- construct structural stiffness matrix (using equation 3.59)

- calculate displacement of those grid points which have any displacement (using Equation 3.53)

- calculate forces induced in every element (using Equation 3.58).

It is evident that one of the major problems to be overcome is determination of the element stiffness matrices. This is because: (i) the element stiffness is dependent upon the geometry and material properties of the element, (ii) elements take many and varied forms depending on the shape of the structures, and (iii) the material properties are strain dependent. Time dependency is not considered herein.

3.5.4 Elastic and Elastic-Plastic Constitutive Relationship

As mentioned before, the element stiffness matrix is fundamentally representative for both the shape and material properties of the element. The stiffness matrix explains the relationship between applied stresses and induced strains within the element. In other
words, if the strains within the element are known then the stresses can be calculated, or vice versa.

For an elastic stress-strain relationship the generalised Hook's law for plane stress can be expressed as follows:

\[ \sigma = D_e (\varepsilon - \varepsilon_0) \]  

where

\[ \sigma = \text{a plane stress vector, and it is defined by Equation 3.62}, \]

\[ \sigma = \{\sigma_x, \sigma_y, \tau_{xy}\} \]

\[ D_e = \text{the elasticity matrix for plane stress; and it is given by Equation 3.63} \]

\[ D_e = \frac{E}{1-v} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & 1-v \end{bmatrix} \]

\[ \varepsilon_0 = \text{initial strain vector such as thermal strain, shrinkage etc.; and it is defined by Equation 3.57}; \]

\[ \varepsilon_0 = \{\varepsilon_{x0}, \varepsilon_{y0}, \gamma_{xy0}\} \]

\[ \varepsilon = \text{strain vector; this vector at any point within an element is defined in terms of displacements by the relationship expressed in Equation 3.65} \]
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$$\varepsilon = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} \end{bmatrix}$$  \hspace{1cm} (3.65)$$

The strain vector for a three dimensional system becomes,

$$\varepsilon = \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} \end{bmatrix} + \begin{bmatrix} \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial x} \end{bmatrix} + \begin{bmatrix} \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial x} + \frac{\partial u}{\partial x} \\ \frac{\partial w}{\partial x} \end{bmatrix}$$  \hspace{1cm} (3.66)$$

and for an elastic, isotropic and homogenous solid, the stress-strain state is defined as,

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \frac{E}{(1+v)(1-2v)} \begin{bmatrix} 1-v & v & v \\ v & 1-v & v \\ v & v & 1-v \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} \hspace{1cm} (3.67)$$

where, $$\zeta = \frac{1}{2} - v$$

If a shape function, which defines displacements within an element when the ith element
do.f. has unit value and other element d.o.f. are zero, is substituted into Eqn.3.65, then a
general relationship can be written in the matrix notation as given in Equation 3.68;
\[ \varepsilon = B \cdot U \tag{3.68} \]

where \( U \) is displacement vector, matrix \( B \) is independent of the position within the element and is derived from the shape function; \( u \) and \( v \) displacement fields are defined as thus \( u = \{u_i; u_j; u_k\} \) and \( v = \{v_i; v_j; v_k\} \) would be a global displacement vector shown in Fig. 3.40.

\[ q = [K^e] \cdot U - q_0 \tag{3.69} \]

Where:

\( q = \) nodal force vector

\( U = \) nodal displacement vector

Fig. 3.40 Nodal Displacement Vector Calculation.
\( q_0 \) = equivalent nodal force vector due to an initial strain \( \varepsilon_0 \)

\[ [K^e] \] = stiffness matrix of the element which is denoted by Equation 3.70;

\[ [K^e] = B^T \cdot D_e \cdot B \cdot t \cdot A \]  \hspace{1cm} (3.70)

Where \( t \) and \( a \) are thickness and area of the element, respectively.

Finally, with assemblage of Equation 3.69 including all elements, the equilibrium equation of the whole structure can be easily obtained as following:

\[ F^s = [K^s] \cdot U - F_0 \] \hspace{1cm} (3.71)

Where

\( F^s \) = vector of all nodal loads

\( K^s \) = global stiffness matrix of the assembled structure

\( U \) = vector of all nodal displacements

\( F_0 \) = equivalent nodal forces due to initial strains

For non-linear materials where a structure is in an elastic-plastic state, the stress-strain increments are generally related by Equation 3.72;

\[ d\sigma = D_{ep} \, d\varepsilon \] \hspace{1cm} (3.72)

Where:
$D_{ep}$ = elastic-plastic matrix; and it is given by Equation 3.75;

\[ d\sigma = \text{incremental stress vector} \]

$de = \text{incremental strain vector which is the sum of an incremental elastic-strain vector, } de_e, \text{ and an incremental plastic-strain vector, } dep, \text{ as follows:} \]

\[ de = de_e + dep \]

(3.73)

and according to Plastic Theory, due to Prandt-Reuss (quoted by Yamada et al., 1968 and 1969), $dep = \lambda \frac{\partial F}{\partial \sigma}$ where $F$ is the yield criterion, $\lambda$ is a non-negative scalar defining the magnitude of the plastic strain, $\frac{\partial F}{\partial \sigma}$ gives the direction of the plastic flow, and $de_e$ is an incremental elastic-strain vector which can be derived from Equation 3.61 in the following form:

\[ de_e = D_{\sigma}^{-1}d\sigma \]

(3.74)

\[ D_{ep} = De - Dp \]

(3.75)

Where $De$ is the elasticity matrix and $Dp$ is the stress-dependent plastic component, denoted as:

\[ D_p = \left[ D_e \left[ \frac{\partial F}{\partial \sigma} \right] \right] \cdot \left[ \frac{\partial F}{\partial \sigma} \right] \cdot D_e \]

(3.76)
where $H'$ is the slope of the stress-strain curve in the plastic region.

### 3.5.5 Tangent Stiffness Matrix

The incremental finite element method for a problem having geometric nonlinearity requires that equilibrium equations must be written with respect to the current deformed geometry. Therefore element-to-member transformations must account for geometric change. In the elastic-plastic range, due to the shift in the centroid of the remaining elastic core caused by the partial yielding of the member, the applied load which was concentric before yielding now becomes eccentric. Basic geometrical properties required for the evaluation of the element tangent matrix are the cross-sectional area, $A$, and the second moment of area $I_x$, which are calculated numerically by subdividing the cross-section into $n$ elementary areas, depending on the accuracy required.

\[
A_e = \sum_{i}^{n} \Delta A_i \tag{3.77}
\]

\[
I_{xe} = \sum_{i}^{n} Y \tag{3.78}
\]

in which $A_e$ is the area of the elastic core, and $y_i$ is the coordinate of the centroid of the elementary area $\Delta A_i$. The elementary area $\Delta A_i$ is calculated by $\Delta A_i = \frac{E_T}{E} \left( \frac{A}{n} \right)$ or $\Delta A_i = \frac{E_x}{E} \left( \frac{A}{n} \right)$, or $\Delta A_i = \frac{E_s}{E} \left( \frac{A}{n} \right)$ in which $E_T$, $E_x$, $E_s$ are tangent moduli in the different ranges (see Fig. 3.2).

For each applied load increment, a corresponding displacement increment may be calculated based on the tangent stiffness relationship. At the end of each load cycle, the
displacement increment is added to give accumulated total displacements, and the geometric properties of the elements, including the total eccentricity due to the shift in the position of the centroid, are updated.

The tangent stiffness matrix $[K_T]$ of the member is obtained by assembling the updated stiffnesses of all the elements, i.e.,

$$[K_T] = \sum_{i=1}^{m} [C_1][C_2][K_{L}+K_{G}][C_2^T]C_1^T$$  \hspace{1cm} (3.79)

where $m$ is the number of elements, $[C_1]$ is the transformation matrix relating the local coordinate axes $x, y$ to the global coordinate axes $x, y$, and $[C_2]$ is the transformation matrix accounting for the load eccentricity (Kitipornchai and Chan, 1987). Matrix $[C_1]$ may be found in any standard text (see for example, Gere et al., 1965).

The incremental equilibrium equations may be expressed by the linearized stiffness expression,

$$[\Delta F] = [K_T]\{\Delta r\}$$  \hspace{1cm} (3.80)

where $[\Delta F]$ is the incremental nodal force vector and $\{\Delta r\}$ is the incremental displacement vector.
3.5.6 Method of Solution

3.5.6.1 Arc-length method

When the Newton-Raphson method (NR), or modified Newton-Raphson method (mNR), is used as a standard process in nonlinear analysis, constant loads are applied in each increment. The nature of these kinds of algorithm precludes the passing of limiting points (Sun, Bradford and Gilbert, 1993). Although many other techniques have been introduced to attempt to overcome this dilemma, such as those of Bergan (1980), Ricks (1979) and Wempner (1971) etc., they are not efficient for solving highly nonlinear problems such as the behaviour of ductile reinforced concrete structures. Recently, Crisfield (1983) suggested using the arc-length method with a combination of a line-search technique to analyse concrete structures.

In the arc-length method, the load increment is not going to be chosen arbitrarily, rather it is determined under an arc-length constraint equation. For an arbitrary referenced load increment vector \([\Delta F_a]\) the tangent displacement vector may be calculated as,

\[
\{\Delta r_a\} = [K_T]^{-1} [\Delta F_a]
\]  

(3.81)

At the \(i\)th iteration within the \(j\)th load cycle, the incremental displacement due to residual forces is given by,

\[
\{\Delta\} R = [K_T]^{-1} [\Delta F] R
\]  

(3.82)

in which \(\{\Delta\} R\) is the incremental displacement due to the residual force \([\Delta F] R\)

\[
[\Delta F] R = \int [B]^T [\sigma_o] dv - [F]
\]  

(3.83)
\[ [\Delta F]_R = \text{Unbalanced force vector} \]

\[ [F] = \text{Applied force vector} \]

\[ [B] = \text{Strain / displacement matrix} \]

\[ [\sigma_0] = \text{Internal stress of the structure} \]

The accumulated displacement increment, \( \{\Delta r\}^i_j \), after the \( i \)th iteration in the \( j \)th cycle is given by

\[
\{\Delta r\}^i_i = \{\Delta r\}^i_{i-1} + \Delta \lambda_i \{\Delta r_i\} + \{\Delta r\}_R \quad (3.84)
\]

in which \( \Delta \lambda_i \) is the load multiplier to be determined. Imposing the arc-length constraint condition, the arc-length \( \Delta L \) is specified.

\[
\{\Delta r\}^T_i \{\Delta r\}_i = \Delta L^2 \quad (3.85)
\]

For the first iteration of \( j \)th load cycle,

\[
\{\Delta r\}^0_i = \{\Delta r\}_R = 0 \quad (3.86)
\]

Consequently,

\[
\Delta L^2 = \{\Delta r\}_i^T \{\Delta r\}_i \quad (3.87)
\]

Substituting Equation 3.87 into Equation 3.84, we obtain

\[
\Delta \lambda_i = \frac{\Delta L}{\sqrt{\{\Delta r_i\}_i^T \{\Delta r_i\}_i}} \quad (3.88)
\]
For the second and subsequent iteration, Equation 3.88 becomes,

\[
\{[\Delta r]^j_{i+1} + \Delta \lambda_i \{\Delta r_a\} + [\Delta r]_R \}^T \{[\Delta r]^j_{i+1} + \Delta \lambda_i \{\Delta r_a\} + [\Delta r]_R \} = \Delta L^2
\]  

(3.89)

This equation can be written as

\[
A \Delta \lambda_i^2 + B \Delta \lambda_i + C = 0
\]  

(3.90)

in which

\[
A = \{\Delta r_a\}^T \{\Delta r_a\} 
\]  

(3.91)

\[
B = 2\{[\Delta r]^j_{i+1} + [\Delta r]_R \}^T \{\Delta r\}_a \]  

(3.92)

\[
C = \{[\Delta r]^j_{i+1} + [\Delta r]_R \}^T \{[\Delta r]^j_{i+1} + [\Delta r]_R \} - \Delta L^2
\]  

(3.93)

The two roots of this scalar quadratic equation will be designated \( \Delta \lambda_{i1} \) and \( \Delta \lambda_{i2} \). To avoid "doubling back" on the original load/deflection path, the "angle" between the incremental displacement vector, \( \Delta r_{i,1} \) before the present iteration, and the incremental vector, \( \Delta r_{i,1} \) after the current iteration within jth load cycle should be positive. The "angles" \( \psi_1 \) and \( \psi_2 \) are given by

\[
\psi_1 = \Delta r^T \_1 \Delta r_{i,1} \quad \psi_2 = \Delta r^T \_2 \Delta r_{i,1}
\]  

(3.94)

The appropriate root for \( \Delta \lambda_{i1} \) or \( \Delta \lambda_{i2} \) is that one which gives a positive angle. However, if both angles are positive, then that one closer to the linear solution should be chosen.

\[
\Delta \lambda_i = \frac{C}{B}
\]  

(3.95)
Once $\Delta \lambda_i$ is determined, the displacement increments from the current iteration and the total applied force can be determined:

$$\{\Delta r\}^i = \{\Delta r\}^i_{i-1} + \Delta \lambda_i \{\Delta r\}_s + \{\Delta r\}_R \quad (3.96)$$

The total displacement $\{r\}_i$ and the total applied force, $[F]_i$, are

$$\{r\}_i = \{r\}_{i-1} + \{r\}_i \quad (3.97)$$

$$[F]_i = [F]_{i-1} + \Delta \lambda_i [F]_s \quad (3.98)$$

If an incremental solution strategy based on iterative methods is to be effective, realistic criteria should be used for the termination of the iterative process. At the end of each iteration, the solution obtained should be checked to see whether it has converged within a preset tolerance or whether the iteration is diverging. Different types of convergence criteria will be discussed later.

### 3.5.6.2 Automatic incremental procedures

A number of procedures have been advocated for calculating a changing increment size. Den Heijer and Rheinboldt (1981) have related the increment size to the curvature of the non-linear path. This method requires both the tangential predictor and the difference between the displacement vectors at the current and the previous load levels. Bergan et al. (1978, 1980) suggested an approach based on the current stiffness parameter. Crisfield (1981) advocated a procedure where the new increment factor $\Delta \lambda_n$ and the old increment factor $\Delta \lambda_0$ are correlated with the help of an iteration ratio ($I_n/I_0$) as given below,

$$\Delta \lambda_n = \Delta \lambda_0 \left(\frac{I_n}{I_0}\right)^r \quad (3.99)$$
where $I_0$ is the number of iterations which were required for the old increment factor $\Delta \lambda_0$ and $I_d$ is the number of iterations desired for new increment factor $\Delta \lambda_n$. The parameter $r$ was set to unity in his work. Ramm (1981, 1982), however, suggested that $r$ should be set to $1/2$. Crisfield (1991), then, extended the above relationship with $r = 1/2$ to the arc-length method as below.

$$\Delta l_n = \Delta l_0 \left( \frac{I_d}{I_0} \right)^{1/2}$$

where, $\Delta l_n$ and $\Delta l_0$ are the old and new incremental arc-lengths respectively. This technique leads to the provision of small increments when the response is mostly non-linear and large increments when the response is mostly linear.

Relationship (3.99) can be applied both for a load increment and a displacement increment. As the arc-length increment controls both the load and the displacement at the same time, equation (3.100) controls both simultaneously.

It is, however, recommended that a maximum ($N_{\text{max}}$) and also a minimum increment size ($N_{\text{min}}$) should also be specified as described, along with the desired number of increments while using equation (3.100).

Bergan et al. (1978, 1980) introduced a new and very useful index called the current stiffness parameter and denoted by $C_s$ which gives some scalar measure of the degree of non-linearity. This parameter is defined as

$$C_s = \frac{K}{K_0}$$

where, $K$ and $K_0$ are the initial and the current stiffness matrices respectively.
The measurement of the current stiffness parameter is very helpful when the material behaviour is highly non-linear, as in the case of concrete.

Bergan et al. have also advocated a new technique of automatic increments based on the concept of a stiffness parameter. They used the same equation (3.99), but replaced the right hand side of the equation with,

\[
\Delta \lambda_n = \Delta \lambda_0 \left( \frac{\Delta C_{sd}}{\Delta C_{so}} \right) \tag{3.102}
\]

where, \( \Delta C_{sd} \) is the desired change in the current stiffness and \( \Delta C_{so} \) is the previous achieved change in stiffness.

Crisfield (1991), later, used the above concept of the automatic increment in the arc-length method. He also recommended to switch from load or displacement control to arc-length control as the limit point is reached for the case when material, like concrete, exhibits softening behaviour.

### 3.5.6.3 Line search technique

This is one of the acceleration techniques used with different iterative techniques to achieve higher rate of convergence by obtaining the direction from an iterative procedure like the modified Newton-Raphson method. In the line search technique, the variable iterative displacement vector \( \delta r \) is written as \( \eta \delta \bar{r} \) where \( \delta \bar{r} \) is kept constant while the newly introduced scalar \( \eta \) is kept as the only variable. Therefore, the following equation can be written as,

\[
r^{i+1} = r^i + \eta^i \delta \bar{r} \tag{3.103}
\]
The scalar $\eta$ is called the step length. For a simple iterative procedure, when the line search is not included, $\eta$ is set to unity. The $\delta r$ is the fixed direction and $r^i$ is the fixed displacement vector at the end of the $i$th iteration. The iterative displacement $\delta r$ or $\eta \delta r$ may be calculated by many different ways. However, the line search concept seeks the scalar which is the step-length for $i$th iteration $\eta^i$ such that the total potential energy $\phi$ at $r^{i+1}$ is stationary in the direction of $\eta^i$, i.e.

$$\frac{\partial \phi}{\partial \eta^i} |_{\eta^i} = \frac{\partial \phi}{\partial \eta^i} |_{\eta^i} \frac{\partial r}{\partial \eta^i} |_{\eta^i} = g^{i+1}(r^{i+1}(\eta^i))T \delta r^i = s_j(\eta^i) = 0$$  \hspace{1cm} (3.104)

where $g$ is the out-of-balance force or gradient of the total potential energy. For clarity, this can be written as

$$s(\eta) = g(\eta)^T \delta r = 0$$  \hspace{1cm} (3.105)

or,

$$s(\eta) = \delta r^T g(\eta) = 0$$  \hspace{1cm} (3.106)

The relationship (3.104) indicates that $\tan \alpha$ in Fig.(3.41) is zero when the search is to be exact. However, in practice, it is inefficient to apply an exact line search and instead a slack line search is adopted with the aim of making the new modulus of the $j$th iteration, $s(\eta^i_j)$, small in comparison with the basic modulus of $s(\eta^i_0)$, i.e.,

$$\frac{|s(\eta^i_j)|}{|s(\eta^i_0)|} < \beta$$

where $\beta$ is the line search tolerance or slack tolerance. The optimum value of $\beta$ has been found to be 0.8 by Crisfield (1983) and Foster and Gilbert (1990).

The line search concept was first applied to non-linear finite element analysis by Irons and Elswaf (1970). Since then many researchers have used this technique as a method of
acceleration in the iterative procedures such as N-R and mN-R methods for quick convergence. It has been found by Foster and Gilbert (1990) that when a highly nonlinear material relationship is used, such as in concrete, convergence may not be obtained unless an acceleration technique like a line search is used.

\[
\tan \alpha_0 = -s_0 = -\delta r^T g_0
\]

\[
\tan \alpha = -s(\eta) = -\delta r^T g(\eta)
\]

\[
\text{Range of 'slack' soln}
\]

\[
|\delta r| \leq \rho |r|
\]  \hspace{1cm} (3.108)

**Fig.3.41 Line search technique (Crisfield, 1983)**

3.5.6.4 Convergence Criteria

Different methods of monitoring convergence are available. Convergence criteria can be adapted, based on force or displacement, as illustrated below.
where $\delta r$ and $r$ are the iterative displacement changes and the total displacement respectively. Similarly, $\delta F$ and $F$ are the iterative force and the total force respectively, and $\rho_1$ and $\rho_2$ are the desired convergence factors in the convergence criteria based on out-of-balance force and displacement error respectively. Equation (3.108) defines the convergence criteria based on displacement control, while equation (3.109) defines the convergence based on force control.

Alternatively, convergence criteria can also be defined based on energy as below:

$$|\delta r^T g| \leq \rho_3 |r^T F|$$

(3.110)

where $\rho_3$ is the desired convergence factor in the convergence criterion based on energy.

### 3.6 MSC/NASTRAN

MSC/NASTRAN is a general purpose 3-D finite element program which can be used for static and dynamic stress and displacement analysis of structures, solid and fluid systems (MSC/NASTRAN Theoretical Manual 1981 and MSC/NASTRAN User’s Manual 1991). This program is capable of analysing structural systems using a combination of different finite elements. In addition, a GAP element is also included in Nastran for modelling structural separation and frictional effects. This element was used to model the effect of elements sliding over each other. Nastran can be used to perform linear and non-linear analysis, which provide both geometric and material non-linear solutions.
The broad categories of material non-linearity are distinguished as non-linear elastic and plastic material. The stress strain relationship may be non-linear for both cases however, in the case of nonlinear elasticity, the unloading curve follows the loading curve, and there is no permanent deformation in the specimen. In the plastic case, the unloading curve again is different from the loading path, so that there is a permanent deformation in the specimen.

In Nastran, functions can be created to define X versus Y tables of information. In nonlinear analysis, these tables can be used to define varying materials or loads. In order to input the stress-strain curves of the concrete and steel materials different functions in the form of tables were made.

The primary Nastran input is the Bulk Data card. These cards are used to define the structural model and the various pools of data which may be selected by Case Control at execution time.

The solution sequence Sol.66 in Nastran provides static solutions for both large displacement and material nonlinearities. The three primary operations are (1) applied load increments, (2) internal force equilibrium error limits, and (3) element stiffness matrix updates. The primary solution operations in Solution 66 are controlled with NLPARM Bulk Data Card. In the present analysis the Arc-Length incremental solution strategies with the line search technique, a particular subcase of NLPARM in nonlinear static analysis (SOL 66) were used.

The goal in the Nastran nonlinear static analysis capability of Sol. 66 is to simulate a specific physical structure undergoing a specific load history. Nonlinear structural solutions are typically obtained from a trial and error search procedure for a particular loading or displacement increment. The search procedure starts from a particular stress and position state and terminates when the basic equations are satisfied within a known tolerance.
As mentioned before, a number of criteria have been proposed for the yielding of solids. MSC/NASTRAN provides for four of these criteria viz., Von Mises, Tresca, Mohr-Coulomb and Drucker-Prager. The former two are most commonly used in plastic analysis of ductile materials like steel, while the latter two are suitable for analysis of frictional or brittle materials such as concrete, soil and to some extent rock.

3.6.1 Mesh Pattern and Gap Element

The most important matter in the finite element modelling of the curved composite struts is the number of the elements. If the number of the elements increases to infinity then the displacement and stresses converge to their true values. The convergence is generally related to the size of the element and the order of the polynomial approximation inside the element. On the other hand, the computer time and required memory space increase when the number of elements increases. Thus, a compromise between the accuracy of the results, computer time and cost should be made.

A large variety of element types (mesh patterns) have been used to model 3-D structures, but the selection of a particular pattern depends on the geometrical and physical characteristics of the structure. In this research, a SOLID element was used to model the curved composite strut. The choice of the element geometry in the present analysis was based on a careful review of the finite element types available in the library of the Nastran program. Finally, the solid elements proved to be the most suitable for discretization of the concrete and steel. Three different material types (concrete, unprestrained and prestrained steel) were used.

In order to create the segments along composite curved struts the following steps were carried out. In the first step, the surfaces were generated so that the steel and concrete elements were built apart (0.4 mm distance between concrete and steel elements) and connected by GAP elements. In this case the plane elements were used for steel and concrete as shown in Fig.3.42 (20 strips along the diameter). In the second step, the
volumes, including the number of segments, were generated by revolving the original surfaces (omitting the surfaces simultaneously) around a vector (the axis of revolution). In addition to specifying the axis of revolution, the angle through which the surfaces were rotated was defined.

As the loading may cause parts of the structures to come in contact or to separate in composite steel tubes, in this model the steel and concrete are connected to each other using special transitional elements called GAP elements. The mechanical properties of the gap elements such as the axial stiffness before and after closure, and the frictional properties in the case of sliding have to be defined in Nastran.

A fixed model, with no distance between steel and concrete elements, and another that included Gap elements between the steel and concrete elements, were created to examine the accuracy of the different models. The use of Gap elements greatly increased the accuracy of the modelling of the curved composite struts. Mechanical properties of gap elements used in the Gap Models are given in Table 3.3. Figs 3.43 and 3.44 illustrate the structural characteristics and the load-displacement curves of the Gap element employed for modelling the material.
Since a Gap element is considered to have non-linear behaviour, a non-linear method of solution was used to analysis the models.

\[ \mu_y, K_t \]

\[ \mu_z, K_t \]

\[ K_{xa} \] stiffness in X direction after closure
\[ K_{xb} \] stiffness in X direction before closure
\[ K_t \] shear stiffness in Y-Z Plane
\[ \mu_y \] friction coefficient in Y direction
\[ \mu_z \] friction coefficient in Z direction

U, V and W are displacements in the X, Y and Z directions, respectively

Fig 3.43 Structural Characteristics of Transitional Elements Called GAP Elements

\[ \text{slope} = K_t U_0 \]

\[ \text{slope} = K_a \]

F₀ initial load on the gap element
U₀ initial opening of the gap
\[ K_a = \text{axial stiffness after closure (N/mm)} = \frac{EA}{L} \times 10^5 \]

\[ K_b = \text{axial stiffness before closure (N/mm)} \text{ was taken as } K_a \times 10^{-7} \]

\[ F_f = \begin{cases} F_x & \text{if } F_x > 0 \\ 0 & \text{if } F_x < 0 \end{cases} \]

\[ F_f: \text{frictional resistance of the gap element} \]

\[ F_x: \text{normal load over the gap element} \]

\[ \mu: \text{frictional coefficient of the gap} \]

Fig. 3.44 Load-Displacement Characteristic of Gap Elements.

Table 3.3 Mechanical properties of gap elements used in Gap Models.

<table>
<thead>
<tr>
<th>( u_0 )</th>
<th>( F_0 )</th>
<th>( K_a )</th>
<th>( K_b )</th>
<th>( K_t )</th>
<th>( \mu ) = \mu_e</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0</td>
<td>1.1x10^9</td>
<td>110</td>
<td>7.0x10^8</td>
<td>0.6</td>
</tr>
</tbody>
</table>

\( u_0 = \text{initial thickness of the gap element (mm)} \)

\( F_0 = \text{initial load on the gap element (N)} \)

where \( L \) is the radius element size, \( E \) is the elastic modulus of concrete, and \( A \) is the contact area of the element.

\[ K_t = \text{shear stiffness when gap element is closed, can be } \mu_y \times K_a, \text{ (N/mm)} \]
\[ \mu_y, \mu_z = \text{coefficient of friction in the Y and Z directions.} \]

### 3.6.2 Residual Stress

In achieving the desired initial radius of curvature, a set of residual stresses across the cross section of the tube are produced by the rolling process. The reduction of stiffness due to the residual stresses (and Bauschinger effect) can cause a decrease in the carrying capacity of the curved strut.

To take into account residual stresses there are two general methods to obtain the column strength. Analytically, making use of the residual stress distribution (either measured or assumed) along with the stress-strain diagram for the material, the strength may be expressed as a function of the second moment of area of the unyielded part of the cross section and the slenderness ratio. An alternate method is to determine experimentally an average stress-strain relationship from a short section of rolled shape containing residual stress. Column strength can be determined using the tangent modulus of this average stress-strain curve.

To account for the effects of early yielding due to the curving process, the 1.57 % prestrained stub columns in tension, which will be presented in Chapter 4, were tested to obtained a stress-strain relationship as shown in Fig.3.45. Referring to Fig.3.2, the idealised stress-strain relationship for prestrained material, the slopes of stress-strain which were used in the finite element modelling are \( E = 200 \text{ GPa}, E_t = 50 \text{ GPa}, E_x = 17 \text{ GPa}, E_s = 6 \text{ GPa} \) (see Fig. 3.2).

The cross-section of the composite curved struts was broken into elementary areas (Fig.3.46). The steel cross-section contains twenty four discrete areas. The effects of early yielding and residual stress were taken into account by using two different stress-strain diagrams for the steel material (normal and 1.57% prestrained in tension stub column test results). It was assumed that after loading the twelve elements of the steel
cross-section which are located near to the symmetry line of the cross-section follow the stress-strain curve of the unprestrained stub column (electric resistance welded tube), the remainder of the elements follow the stress-strain curve of the prestrained stub column (ERW tube). Referring to Fig.3.1, the idealised stress-strain relationship for the ERW steel tubes, the slopes of stress-strain relationship are taken as $E=200 \text{ GPa}$, $E_s=30 \text{ GPa}$. As mentioned before, the stress-strain curve for the high strength concrete is taken from the CEB-FIP model, and will be discussed in Chapters 4 and 5.

![Stress-Strain Curve of the Unprestrained Stub Column and of 1.57% Prestrained Stub Column (Electric Resistance Welded Tube).](image_url)

Fig. 3.45. Stress-Strain Curve of the Unprestrained Stub Column and of 1.57% Prestrained Stub Column (Electric Resistance Welded Tube).
3.6.3 Loading Conditions

Because of symmetry conditions, only half of the curved composite strut was modelled and analysed. The half length was divided into 8 segments and compressive load was applied at the support as shown in Figs. 3.47a and 3.47b (4 segments are shown in the Figure). It was assumed that there was a roller constraint at the support and a fixed condition at the mid-span, so that it can only move up or down.

The magnitude of the load or displacement increment is important, especially when small values of load or displacement cause a large change in response. The response characteristics of the pre- and post-yield regions are very different. Composite curved struts may deflect close to a linear response in the pre-yield region. The pre-yield phase refers to the portion of the F versus δ curve prior to the tension/compression steel yielding, and the post-yield phase begins after the tension steel has yielded. The post-yield phase experiences large deflections for small incremental loads. Consequently, convergence difficulties are encountered when using Nastran in the region neighbouring
the limit point. To overcome these difficulties the convergence parameters may change through the load increments. To control accurately the loading process, compressive loads were applied incrementally in 40 steps and to identify the peak load, small increments of 1 kN per step were used near the critical points.

The load increment, in conjunction with displacement, can be used to solve the problems such as unstable behaviour (negative stiffness) in Nastran. It needs to be noted that to trace the unloading path, the enforced displacements can be used instead of the applied loads for achieving a first estimate of the solution. The enforced displacement could be chosen at any location and could be assumed in any direction. The problem may be restarted with a boundary constraint change.

Deformed and undeformed shapes of the curved strut with 4000 mm initial radius of curvature and 1515 mm straight length are shown in Fig.3.48. Colour contour plots of stresses due to compression and bending are also shown in Fig.3.48. All of the composite curved steel struts failed under combined flexure-compression conditions. The load-lateral deflection curves of the curved pin-ended steel tubular struts of different initial deflections at mid-height found from the Nastran results using 8 segments are shown in Figs.3.49-3.57. For comparative purposes, the specific dimensions of the tubes were chosen for the finite element method so that the results could be compared directly with the experimental results. For comparison, the results using the intersection method are also plotted in Figs.3.49-3.57. It will be shown that there is good agreement between the finite element and experimental results in Chapter Six. The finite element analysis has the advantage over the elastic and plastic intersection curve method is that it determines the complete strut behaviour including deflection, stress and strain anywhere within the strut.
3.6.4 Convergence Criteria in Nastran

Nastran has the power and flexibility to easily include the various parameters and solution methods required for nonlinear analysis. Three different convergence criteria are available in Nastran. They are based on the tolerance limit of (a) displacement, (b) out-of-balance force, and/or (c) work. Any combination of these can be set as the required convergence criterion. The tolerance limits can be set depending on the problem and the degree of accuracy required. Two different convergence criteria based on the tolerance limit of both displacement and out-of-balance force were used. The convergence criterion was based on the out-of-balance force and used in the initial load increments, and was based on displacement as the critical point was approached because the deflection was very high and the use of a convergence criterion based on the tolerance limit of out-of-balance force caused numerical computational problems. The convergence tolerance for out-of-balance force was taken as 0.5% to 1%. 
Fig. 3.47a Constraint Conditions at Support and at Mid-span

Fig. 3.47b Finite Element Modelling of Half of Composite Curved Strut (half of strut is divided into 4 segments in figure)
Fig. 3.48 Colour Contour plot of Bending Stresses and Deformed and Undefomed Shape of Half Composite Curved Strut Subjected to Compressive Load and divided into 8 segments (R=4000 mm and L=1540 mm)
Chapter Three  Theoretical Development

Fig. 3.49 Finite Element and Theoretical Results (elastic and plastic behaviour) of ERW Composite Curved Strut (R=2000 mm and L=775 mm)

Fig. 3.50 Finite Element and Theoretical Results (elastic and plastic behaviour) of ERW Composite Curved Strut (R=2000 mm and L=1176 mm)
Fig. 3.51 Finite Element and Theoretical Results (elastic and plastic behaviour) of ERW Composite Curved Strut (R=2000 mm and L=1559 mm)

Fig. 3.52 Finite Element and Theoretical Results (elastic and plastic behaviour) of ERW Composite Curved Strut (R=4000 mm and L=765 mm)
Fig. 3.53 Finite Element and Theoretical Results (elastic and plastic behaviour) of ERW Composite Curved Strut (R=4000 mm and L=1176 mm)

Fig. 3.54 Finite Element and Theoretical Results (elastic and plastic behaviour) of ERW Composite Curved Strut (R=4000 mm and L=1540 mm)
Fig. 3.55 Finite Element and Theoretical Results (elastic and plastic behaviour) of ERW Composite Curved Strut (R=10000 mm and L=765 mm)

Fig. 3.56 Finite Element and Theoretical Results (elastic and plastic behaviour) of Composite Curved Strut (R=2000 mm and L=1141 mm)
3.7 COMPARISON of CALCULATED MAXIMUM LOAD using the INTERSECTION METHOD with those calculated from NASTRAN

A comparison between the ultimate load capacity obtained from the finite element analysis (having different numbers of segments for determining the differences of the results) and calculated from the intersection point of the elastic and plastic curves for the ERW strut with 4000 mm initial radius of curvature was made. Consider a pin-ended composite curved steel tubular strut of length 1515 mm, outside diameter 60.3mm, wall thickness 2.3mm, initial radius of curvature 4000mm, the ultimate load calculated from the intersection point of elastic and plastic curves is,

\[ P_{ul} = 37.2 \text{ kN} \]
The limit points calculated by using Nastran, and using different numbers of elements along the length, are determined as follows. The elements along the tube (segments) are of equal length for any one subdivision.

\[ P_{ult2} = 41.4 \text{ kN} \text{ (2 elements, whole length of the strut)} \]

\[ P_{ult4} = 37.9 \text{ kN} \text{ (4 elements used)} \]

\[ P_{ult8} = 37.51 \text{ kN} \text{ (8 elements used)} \]

\[ P_{ult12} = 37.46 \text{ kN} \text{ (12 elements used)} \]

\[ P_{ult16} = 37.4 \text{ kN} \text{ (16 elements used)} \]

\[ P_{ult20} = 37.4 \text{ kN} \text{ (20 elements used)} \]

Compared with the intersection method, the percentage errors with respect to the number of elements adopted are shown in Table 3.4. The curve of error percentage versus number of elements is plotted in Fig.3.58. From Table 3.4 it can be seen that the result of the finite element analysis having sixteen elements is close to the result of the intersection of the elastic and plastic curves.

<table>
<thead>
<tr>
<th>No. of Elements</th>
<th>Percentage Errors %</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>11.2</td>
</tr>
<tr>
<td>4</td>
<td>1.9</td>
</tr>
<tr>
<td>8</td>
<td>0.8</td>
</tr>
<tr>
<td>12</td>
<td>0.7</td>
</tr>
<tr>
<td>16</td>
<td>0.5</td>
</tr>
<tr>
<td>20</td>
<td>0.5</td>
</tr>
</tbody>
</table>
3.8 SUMMARY

This Chapter presents results obtained from simple theoretical analysis (elastic and plastic behaviour) and from nonlinear finite element analysis by using the Nastran package for composite curved steel struts. The ultimate load capacity and load-deflection behaviour of the curved struts were calculated from intersection point of elastic and plastic behaviour, and will be compared in Chapter Six with experimental results and also with finite element analyses.

The half of the curved struts were divided into eight segments. Solid elements were used in the finite element modelling of the steel and the concrete. Steel and concrete were connected by using a nonlinear Gap element. The arc-length method (Crisfield) combined with the line search technique was used for the solution method. Different convergence criteria (based on out-of-balance of load or displacement) were selected for the solution.
4. CHAPTER FOUR

EXPERIMENTAL WORK

4.1 INTRODUCTION

To determine experimentally the range of the ultimate load capacities of the concrete infilled curved tubes, 78 pin-ended composite tubes, as well as 11 hollow sections, have been tested under different conditions.

It is also necessary to determine the properties of the concrete and the steel tubes which are used. Therefore, tensile and compressive tests have been conducted to provide basic data in the elastic and inelastic ranges for the steel tubes used herein.

4.2 GENERAL FEATURES of EXPERIMENTS

4.2.1 Number, Scale and Purpose of the Tests

Three series of curved composite steel and concrete tests are described in this chapter; the material properties and specimen details are given in Tables 4.1-4.8. As well, load versus deflection, load versus strain and load versus curvature results are shown in Figs.4.27-4.63. A comparison will be made between Stress Relief Annealed and normal steel struts infilled with concrete. As well, the increase of load capacity of the composite curved struts in compare to hollow curved struts will be discussed.
All test specimens are designated by a three symbol code. The first letter specifies the initial radius of curvature of the test specimens such as “T” denotes 2000 mm initial radius of curvature, “F” 4000 mm initial radius of curvature and “N” 10000 mm initial radius of curvature. The second letter specifies some characteristic of the test series; i.e. “E” stands for Electric Resistance Welded tubes and “M” for Mild steel seamless tubes. The third symbol, a number, denotes the position of the test within its own series. For example, TE7 denotes an ERW tube with 2000 mm initial radius of curvature, the test being the seventh in this series.

The first series, TE1-NE20 consisted of 44 cold-formed curved ERW tubes with 60.4 mm OD and 2.3 mm wall thickness. The nominal straight length varied from 775 mm to 3115 mm. It was measured between two knife edges. Initial deflection at mid-height (e) was also measured between center lines (Fig.4.1).

![Fig 4.1 The Length and Initial Deflection at Mid-height Defined for Curved Struts](image)

Three different radii of initial curvature, 2000 mm, 4000 mm, and 10000 mm were selected to examine the effects of small and large initial radii of curvature. Four stress-
relief-annealed composite curved struts (R=10000) were also tested in this series. The purpose of these tests was to study the overall behaviour of composite curved tubes over a significant range of curvatures.

The second series, TM1-FM17 consisted of 34 hot-finished mild steel seamless tubes with 60.4mm OD and 4 mm wall thickness. The nominal straight length varied from 743 mm to 2225 mm. Two different initial radii of curvature, 2000 mm and 4000 mm were selected. These series had seven stress relief annealed (SRA) curved tubes. This type of tube may have the greatest application in practice, especially for columns in multi-storey building construction. This diameter was chosen because of the capacity limitation of the testing machine.

Stress relief annealing (SRA) treatment was performed in a furnace for 30 minutes at 620 °C, followed by air cooling. This process was adopted so as to relieve the residual stresses set up during the tube curving process.

The third series, TH1-NH3 consisted of 11 ERW hollow section tubes with 60.4 mm OD and 2.3 mm wall thickness. The initial radii of curvature were 2000 mm, 4000 mm and 10000 mm. The purpose of these tests was to make a comparison between hollow and composite curved struts. The nominal straight length varied from 770 mm to 1755 mm.

### 4.2.2 Curving Procedure of Tubular Steel Strut

The rolling procedure used to produce the appropriate radius of curvature for the tubular steel struts is shown in Figs.4.2-4.4. Three single rollers were used to curve the tubular struts. The rollers were chosen to fit the tubular struts before rolling. Two rollers were fixed, while the third roller was free to move. The tubular struts passed through the rollers. The position of the third roller was changed after each pass to get as close as possible to the specified radius of curvature.
The number of passes needed for the curving process depends on the type and the shape of the section being curved and the specified radius of curvature. For the tubular struts used in this thesis, seven passes were needed to obtain the 2000mm radius of curvature, and four passes were needed to obtain the 4000mm radius of curvature.

A set was used to control the existing radius of curvature of the struts after each pass.

The dimensions of the set are shown below

As soon as points A, B and C touched the rolled specimen at the same time, the desired radius of curvature was obtained.

4.2.3 Instrumentation

All tubes were tested as a pin-ended column. All loading was of short duration; the effects of sustained or repeated loading were not investigated. Struts and material tests up to 2000 mm long were performed in a 50 tonne capacity closed loop hydraulic testing system (Instron machine) with controlled rates of ram travel. This machine consists of three major units:

a) a reaction loading frame
b) an electronic control console

c) a hydraulic power pack

The role of the load frame is to provide a rigid, stable mounting frame to apply force to a test specimen. The control console provides full digital facilities for control and monitoring of the overall testing system. The rates of deformation applied to a specimen and axial deformation measurements can be precisely controlled. With hydraulic testing system, energy can be transmitted to the specimen using high pressure hydraulic fluid acting on a double sided piston (actuator). The role of the actuator is to apply controlled forces and deflections to a specimen constrained within a load frame. The servo-hydraulic loading frame is shown in Fig.4.5.

Because of a length limitation for testing the longer specimens in the servo hydraulic machine an additional loading frame was made. The longer specimens were placed in a horizontal position on rollers, and load was applied horizontally. A 50 kN hydraulic jack was used for load application and was attached to a load cell for recording the force level. Photographs of the Instron machine and the horizontal loading frame are shown in Figs.4.6 and 4.7, respectively.
Fig. 4.2 The arrangement of the Rollers

Fig. 4.3 Checking curvature by using scale and string
Fig. 4.4 The specimen after curving
Fig. 4.5 Reaction Loading Frame showing Actuator and Transducers
Fig. 4.6 Instron Machine
Fig. 4.7 Loading Frame for Long Specimens
4.3 PREPARATORY WORK

4.3.1 Dimensions of the Steel Tubes

The ratio of the area of steel to the area of concrete in the tubes affects the proportions of the load carried by the steel and by the concrete. As well, the ratio of the length to the diameter is related to slenderness ratio it will affect the behaviour of the tube infilled with concrete. In this investigation the ratio of the steel and concrete areas, and the length to the diameter ratio, were varied. Dimensions and properties of the steel used in the experimental tests are shown in Table 4.1.

4.3.2 Concrete Mix Design

The concrete strength and the stress-strain behaviour can be varied by varying the water/cement ratio, the aggregate/cement ratio, the aggregate grading, the curing time, the curing conditions and cement type. In addition to such ratios and conditions, high performance concrete can be affected by changing the amount of silica fume and superplactisizer.

Different mix proportions based on investigations by ACI Committee 363, Carrasquillo and Carrasquillo (1988), and Hwee and Rangan (1990), were designed with 10 mm maximum aggregate size to obtain the target strength of 70 MPa. The minimum aggregate size adopted herein is largely determined by the inside dimensions of the tube. The concrete should be sufficiently workable to ensure complete compaction, but at the same time the concrete must not segregate. Superplasticizer was added to the mixture to bring the concrete to a flowing consistency with a slump of 150 mm. The mix proportions adopted are given in Table 4.2.
### Table 4.1 Dimensions of the Steel Tubes Used in Tests

<table>
<thead>
<tr>
<th>Outside Dimensions</th>
<th>60.39 mm OD (ERW)</th>
<th>60.39 mm OD (Seamless)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Wall Thickness t</td>
<td>2.5 mm</td>
<td>4.00 mm</td>
</tr>
<tr>
<td>Average Wall Thickness t (R=2000 mm)</td>
<td>2.33 mm</td>
<td>3.8</td>
</tr>
<tr>
<td>Average Wall Thickness t (R=4000 mm)</td>
<td>2.36 mm</td>
<td>3.8</td>
</tr>
<tr>
<td>Average Wall Thickness t (R=10000 mm)</td>
<td>2.36 mm</td>
<td>—</td>
</tr>
<tr>
<td>Area of Steel (sq. mm) As</td>
<td>455</td>
<td>709</td>
</tr>
<tr>
<td>Second moment of area (mm⁴) Iₕ</td>
<td>190800</td>
<td>283000</td>
</tr>
<tr>
<td>rs = √ Iₕ/As</td>
<td>20.5</td>
<td>20</td>
</tr>
<tr>
<td>Area of the Concrete Core of the Tube (sq. mm) Ac</td>
<td>2410</td>
<td>2156</td>
</tr>
<tr>
<td>Average O.D (R=2000 mm)</td>
<td>60.72 mm</td>
<td>60.45 mm</td>
</tr>
<tr>
<td>Average O.D (R=4000 mm)</td>
<td>60.5 mm</td>
<td>60.57 mm</td>
</tr>
<tr>
<td>Average O.D (R=10000)</td>
<td>60.3 mm</td>
<td>—</td>
</tr>
</tbody>
</table>

### Table 4.2. Concrete Mix Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement content (Type A and ordinary)</td>
<td>550 kg/m³</td>
</tr>
<tr>
<td>Water/cement ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Aggregate/cement ratio</td>
<td>2.4</td>
</tr>
<tr>
<td>10 mm gravel</td>
<td>880 kg/m³</td>
</tr>
<tr>
<td>Sand</td>
<td>440 kg/m³</td>
</tr>
<tr>
<td>Silica fume</td>
<td>14.1 kg/m³</td>
</tr>
<tr>
<td>Superplastisizer</td>
<td>17 ml/kg of cement</td>
</tr>
<tr>
<td>Slump</td>
<td>150 mm</td>
</tr>
<tr>
<td>Compressive strength at 28 days</td>
<td>70 MPa</td>
</tr>
</tbody>
</table>
The combined aggregate grading for the mix is given as follows

Percent passing (metric sieve size, mm)

<table>
<thead>
<tr>
<th>Sieve sizes, mm</th>
<th>19</th>
<th>9.5</th>
<th>4.75</th>
<th>2.36</th>
<th>1.18</th>
<th>0.6</th>
<th>0.3</th>
<th>0.15</th>
<th>0.075</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent passing</td>
<td>100</td>
<td>84</td>
<td>30</td>
<td>24</td>
<td>21</td>
<td>15</td>
<td>6</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

4.3.3 Casting and Curing Procedure

A base plate was welded to each tube to retain the concrete; each tube was located in a vertical position in order to cast the concrete. Concrete was supplied in three different batches for the three differently curved pipes. The concrete was dropped into the tube from the top. Four specimens, with a radius of initial curvature of 2 m, were located in a horizontal position, and the concrete was cast into the tube from both ends due to the large curvature and long length. Concrete was cast in layers, and the tubes were vibrated on a vibrating table. Various types of supporting frames and holding down devices were used for keeping the tube in the vertical position, depending upon the length of the specimen, while the concrete was being vibrated.

Considerable care had to be taken while vibrating the concrete to ensure proper compaction. The specimens were cured in a humidity room or by keeping the top end covered with wet sacking for 7 days (see Figs.4.8 and 4.9). The free water in the concrete inside the tube could not escape while the concrete was being cured. It was not possible to test all specimens 28 days after the concrete was cast. The time of casting and the time of testing was governed by the availability of a suitable testing frame.
Fig. 4.8 Specimens After Casting Concrete
Fig 4.9 Curing Specimens in the Humidity Room
4.4 MATERIAL PROPERTIES

4.4.1 Steel Properties

Tensile and compressive tests were performed to determine the uniaxial strength and the stress-strain curve of the steel. The detail of the tensile and compressive specimens is shown in Figs. 4.10 and 4.11. The tests consisted of tensile tests and stub compression tests for the steel tubes. Tensile tests for the Electric Resistance Welding (ERW) and hot-rolled seamless steel tubes were performed by using the Instron machine as shown in Figs 4.12 and 4.13. The complete stress-strain curves were provided. Based on the tensile test data and the shape of the stress-strain curve, a uniform value of $E_s = 200,000$ MPa was assumed for all cases.

The chemical composition of the ERW tube was as follows:

<table>
<thead>
<tr>
<th>%C</th>
<th>%P</th>
<th>%Mn</th>
<th>%Si</th>
<th>%S</th>
<th>%Cr</th>
<th>%Ni</th>
<th>%Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.060</td>
<td>0.025</td>
<td>0.20</td>
<td>0.013</td>
<td>0.015</td>
<td>0.014</td>
<td>0.026</td>
<td>0.028</td>
</tr>
</tbody>
</table>
and for the chemical composition of the seamless tube was as follows:

<table>
<thead>
<tr>
<th>%C</th>
<th>%P</th>
<th>%Mn</th>
<th>%Si</th>
<th>%S</th>
<th>%Cr</th>
<th>%Ni</th>
<th>%Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.19</td>
<td>0.017</td>
<td>0.72</td>
<td>0.25</td>
<td>0.009</td>
<td>0.024</td>
<td>0.025</td>
<td>0.036</td>
</tr>
</tbody>
</table>

Based on recommendations of the Column Research Council (1956), the stub column specimens were 200 mm long and their ends were machined parallel to close limits (200 mm is approximately 3 times the least lateral dimension of the tubes). The specimens were obtained from at least 300 mm away from the ends of a length of tube to allow for any adverse cooling effects which may have arisen during the manufacture of the tubes.

All stub columns were tested in compression. The axial shortening vs load of the stub columns was plotted by using an extensometer of gauge length 90 mm. The specimens were tested in the Instron machine using the same bearing apparatus as was used for the testing curved struts. A small load was applied to center the stub columns to prevent rotation and therefore ensure an axial deflection.

The results of the tensile and squash tests on ERW and seamless tubes are listed in Table 4.3 and 4.4 and shown in Figs.4.14 and 4.15. The average result of the ERW squash tests is 371 MPa, of the Seamless squash tests is 350 MPa. The average value of elasticity modulus E of the ERW squash tests is 209.5 MPa, of the Seamless tubes is 211 MPa. The stress-strain diagrams of the stub columns are shown in Fig.4.16 and Fig.4.17. The initial modulus of elasticity and the bilinear and trilinear curve parameters (used for prestrained stub columns which will be described) were deduced by fitting two and three straight lines to these curves for theoretical calculations. It should be noted that considerable variations in the material properties may occur over the cross-section
and along the length of the tube, which are not taken into account in the theoretical calculations.

In addition to these tensile and compressive tests on ERW tube, tests were carried out to investigate the effect of initial prestraining as well as the residual stresses. For this purpose, the tubes were prestrained in tension to a defined percentage elongation. The prestraining was carried out in the Instron machine, using a Demec gauge to check the extensions.

The prestrained stub column test specimens were performed on 201mm length (length 3.2 times the diameter) cut from the prestrained tube in tension to 0.75% and 1.5%, respectively. The selected prestrain values were related to the maximum strains in the extreme fibres of the tubular section after curving. The stress-strain relationships are shown in Figs. 4.18 and 4.19. It is noted that a value of 0.75% tensile strain will be between yield and strain hardening. By considering the dynamic nature of yielding in mild steel (Lay, 1982) the material will either be elastic or will be at the strain hardening. It is not possible to obtain a 0.75% yield prestrain that is uniform along the length of the specimens, therefore, an average between gauge points was used for calculations.

Curvature can be related to strain by following Eqs.

$$\frac{1}{R} = \frac{\varepsilon}{y}$$

If $R = 2000 \text{ mm}$

$$\varepsilon_1 = \frac{y}{R} \frac{30.2}{2000} = 1.5\%$$

If $R = 4000 \text{ mm}$

$$\varepsilon_2 = \frac{30.2}{4000} = 0.75\%$$

where $y$ is the distance of the neutral axis from tensile extreme fibre.
Chapter Four Experimental Work

Fig. 4.10 Tensile Specimen

Fig. 4.11 Compressive Specimen
Fig 4.12 ERW Stub Column Specimen
Fig. 4.13 Seamless Steel Stub column
Fig. 4.14 Stress-Strain Curve for Seamless Tube

Fig. 4.15 Stress-Strain Curve for ERW Tube
Table 4.3 Squash and Tensile Stub Column Results of ERW Tubes

<table>
<thead>
<tr>
<th>Test No</th>
<th>$\sigma_{0.2}$ MPa</th>
<th>$\sigma_u$ MPa</th>
<th>$\sigma_u / \sigma_{0.2}$</th>
<th>$E$ GPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE1</td>
<td>387</td>
<td>410</td>
<td>1.05</td>
<td></td>
</tr>
<tr>
<td>TE2</td>
<td>384</td>
<td>408</td>
<td>1.06</td>
<td></td>
</tr>
<tr>
<td>CE1</td>
<td>369</td>
<td></td>
<td></td>
<td>205</td>
</tr>
<tr>
<td>CE2</td>
<td>373</td>
<td></td>
<td></td>
<td>214</td>
</tr>
</tbody>
</table>

TE1, TE2 = tensile tests  
CE1, CE2 = compressive tests  
Strain Hardening Ratio (SHR)=$\sigma_u / \sigma_{0.2}$

Table 4.4 Squash and Tensile Stub Column Results of Seamless Tubes

<table>
<thead>
<tr>
<th>Test No</th>
<th>$\sigma_{0.2}$ MPa</th>
<th>$\sigma_u$ MPa</th>
<th>$\sigma_u / \sigma_{0.2}$</th>
<th>$E$ GPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>TS1</td>
<td>362</td>
<td>483</td>
<td>1.33</td>
<td></td>
</tr>
<tr>
<td>TS2</td>
<td>363</td>
<td>485</td>
<td>1.33</td>
<td></td>
</tr>
<tr>
<td>CS1</td>
<td>353</td>
<td></td>
<td></td>
<td>217</td>
</tr>
<tr>
<td>CS2</td>
<td>348</td>
<td></td>
<td></td>
<td>205</td>
</tr>
</tbody>
</table>

TS1 and TS2 = tensile tests  
CS1 and CS2 = compressive tests  
Strain Hardening Ratio (SHR)=$\sigma_u / \sigma_{0.2}$
Fig. 4.16 Compressive Stress-Strain Relationship of ERW Steel Stub Column

Fig. 4.17 Compressive Stress-Strain Relationship of Seamless Steel Column
Fig. 4.18 ERW Stub Column Test Prestrained 0.75 % in Tension

Fig. 4.19 ERW Steel Stub Column Test Prestrained 1.5 % in Tension
4.4.2 Tests on Concrete Specimens

In order to determine the strength of the concrete core of the infilled steel struts compression tests were carried out on 34 unconfined concrete cylinders. The cylinders were cast at the same time as the tubes were filled with concrete, and were tested at approximately the same time as the infilled columns. The cylinders with a nominal 100 mm diameter and 200 mm long were used as the main control specimens. The concrete cylinders were capped at one end with a restrained natural rubber pad. The rubber with a nominal Shore A Durometer hardness of 50 as described in AS 1523 was used. In spite of the differences in placing, compacting and curing, it was assumed that the strength $f'_c$ of the concrete inside the tube was similar to the average cylinder strength.

The 100 mm $\times$ 200 mm cylinders were tested in a 1800 kN capacity Avery Compression Testing Machine. The longitudinal strain was measured between two planes along the concrete by using a dial gauge and a steel cylinder frame. It consisted of a fixture to the cylinder at three points in each of the two planes between which strain was being measured. A typical set up of such a frame is shown in Fig.4.20. The change in distance between the planes is measured on a 0.01 mm dial gauge. The distance between the two planes was initially 90 mm (set by using a standard bar to fix the stainless steel buttons 90 mm apart in the longitudinal direction). It was not possible to take strain readings after the peak load as the load was decreasing because of crushing of the concrete cylinders due to high strain energy stored in the test machine. The concrete cylinder strength results and the age of the tests are given in Table 4.5.

Companion 50 mm $\times$ 100 mm cylinders were made and tested in some cases in the Instron machine with a controlled rate of ram travel. These allowed the plotting of load vs. axial deflection by using an extensometer of gauge length 90 mm as shown in Fig.4.21 similar to that used for the steel stub column tests. In spite of the high maximum load and the strain energy stored in the testing machine which would be relatively considerable some readings on the falling branch were possible because of the
deformation control of the testing machine. Curves obtained from the experimental results are shown in Figs. 4.22 and 4.23.

Table 4.5 Concrete Cylinder Strength and Age at Test

<table>
<thead>
<tr>
<th>Batch No.</th>
<th>Age at test days</th>
<th>Strength MPa</th>
<th>Batch No.</th>
<th>Age at test days</th>
<th>Strength MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>49</td>
<td>68</td>
<td>4</td>
<td>94</td>
<td>71.8</td>
</tr>
<tr>
<td>1</td>
<td>49</td>
<td>71</td>
<td>5</td>
<td>94</td>
<td>75.1</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
<td>72</td>
<td>5</td>
<td>100</td>
<td>77</td>
</tr>
<tr>
<td>1</td>
<td>81</td>
<td>72.3</td>
<td>2</td>
<td>94</td>
<td>76</td>
</tr>
<tr>
<td>3</td>
<td>68</td>
<td>67.5</td>
<td>4</td>
<td>94</td>
<td>73.2</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
<td>80</td>
<td>3</td>
<td>100</td>
<td>82.2</td>
</tr>
<tr>
<td>2</td>
<td>98</td>
<td>75.1</td>
<td>1</td>
<td>55</td>
<td>71.9</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
<td>78</td>
<td>1</td>
<td>43</td>
<td>68</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
<td>77.5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.5 STRUT TESTS AND TEST PROCEDURE

In order to determine the influence of the initial radius of curvature, initial deflection, slenderness ratio, diameter/thickness ratio, and concrete strength and steel strength, 89 struts were tested. The struts were curved in the three different radii of initial curvature, 2 m, 4 m, and 10 m. The number and results of the tests are listed in Tables 4.6-4.8 and are described as follows:

1) Twelve ERW curved composite struts with 2000 mm initial radius of curvature (Numbers 1-12)

2) Twelve ERW curved composite struts with 4000 mm initial radius of curvature (Numbers 13-24)

3) Twenty ERW curved composite struts with 10000 mm initial radius of curvature (Numbers 25-44)

4) Seventeen seamless curved composite struts with 2000 mm initial radius of curvature (Numbers 45-61)

5) Seventeen ERW curved composite struts with 4000 mm initial radius of curvature (Numbers 62-78)

6) Eleven ERW curved hollow struts (Numbers 79-89)
Fig. 4.20 Concrete Cylinder Test By Using Avery Compression Testing Machine
Fig. 4.21 Concrete Cylinder Test By Using Instron Machine
Fig. 4.22 Experimental Result for Stress-Strain Relationship of Concrete Tested by using Instron Machine

Fig. 4.23 Experimental Result for Stress-Strain Relationship of Concrete Tested by using Avery Machine
The range of the initial radius of curvature was selected between 2000 mm and 10000 mm in order to examine the effects of large and small initial curvatures. Axial load versus lateral deflection plots for the ERW and seamless struts are shown in Figs 4.27-4.56. Also, axial load versus strain and axial load versus curvature plots are given in Figs 4.57-4.58 and 4.59-4.63, respectively. Curved steel struts infilled with higher strength concrete will be compared with hollow struts, and Stress Relief Annealed (SRA) struts with as-received struts under the same conditions of geometry and slenderness.

As mentioned before all tests up to 2000 mm long were performed in a 50 tonne capacity servo-hydraulic testing machine. All tests were carried out at controlled rates of ram travel of 0.005 mm per second up to the peak load, then 0.05 mm per second while following the descending branch of the curve. In order to read the strain, in some tests which included strain gauges, the load was held in a sequence i.e. 0, 5 kN, 10 kN etc so that the strain readings were possible. This procedure was followed in the rising branch and a few readings were taken in the falling branch of the curve.

In this study all tests were performed as a pin-ended column. Knife-edges have been used extensively and it appears that successful results can be obtained by using them (Knowels, 1967). Consequently, parallel high-strength-steel knife-edges were used and attached to the top and bottom parallel plattens of the test machine as shown in Fig.4.24. In order to simulate the pin ended condition a V-notch support was used. The V-notch plates were bolted to the base plates which were welded to the tubes. The straight length of each column was measured between two knife-edges seatings as shown in Fig.4.1. The tubes were fixed against rotation parallel to the knife-edges and therefore column buckling did not occur in this direction. Little deformation of the knife-edges occurred during the testing program other than a slight radiusing of the knife edge, which caused minimal rotation restraint or moment of resistance at the support.

Deflections relevant to the overall behaviour of the strut were measured along two perpendicular diametral planes. Shortening and lateral deflection were measured by using Linear Voltage Displacement Transducer (LVDT) and dial gauges reading to 0.01
mm. The LVDT's were used for the short specimens up to 2000 mm length, and dial gauges were used for the long specimens. The LVDT's for the shortening measurement were mounted on the Instron machine.

Strains were measured by electrical resistance strain gauges and a demountable mechanical (Demec) gauge. Electrical resistance gauges were type LE-5 with resistance $120 \pm 0.3 \Omega$ with a 1 mil (0.03) tough, flexible film backing. The gauge factor varied from 2.05 to 2.11, and each gauge was virtually free from transverse sensitivity. The gauges were stuck to the cleaned steel surface with M-Bond 200. The gauges were connected to a strain indicator through a twelve channel switchbox. The sensitivity of the measuring equipment was about 2 $\mu$e.

4.5.1 Mechanism of Collapse

4.5.1.1 ERW Struts

In general, the final failure mechanism of the ERW struts can be classified in three different stages.

In stage one, the specimen underwent lateral deflections beyond the peak loads. In stage two ripple marks formed at mid-span on the concave side of the strut. In stage three, necking marks formed at mid-span on the convex side of the strut which lead to tearing and fracture of the steel and overall failure of the specimen.

Struts having slenderness ratio less than 100 for three different initial radii of curvature used in these series of tests followed through the three stages and finally failed as a result of ripples on the concave side and fracture on the convex side after the peak load had been reached. Necking and fracture are the consequence of the small strain hardening ratio (SHR for ERW struts = 1.05) of which did not allow a significant spread of plasticity. Final fracture within the neck initiates from concentrations of slip bands.
This kind of failure is named as C in Table 4.6. The load-lateral deflection of these struts normally dropped sharply after the peak load.

Some longer struts underwent large lateral deflection beyond the peak load and failed due to fracture on the tension side due to large tensile stresses; no ripples occurred on the compression side. This kind of failure is named as F in such Table 4.6. No ripples nor fractures were observed on the longest struts with 2000 and 4000 mm initial radii of curvatures and 3000 mm straight length. This type of failure is named as D.

4.5.1.2 Seamless struts

As the manufacture of these struts is completely different from ERW struts, furnishing a high strain hardening ratio (SHR=1.33), therefore it can be expected that the final failure mechanism should be different. Necking and fracture did not occur on the struts beyond the peak load. Leuder’s bands were observed on the concave side after the peak load had been reached for struts with L/r < 80. This kind of failure is named as Y in Table 4.7. No sign of Leuder’s band was observed on the remainder of the longer struts.

4.5.1.3 SRA struts

Stress relief annealed ERW curved composite specimens deformed laterally with large mid-span deflections, no necking or fracture occurred, and only slightly visible ripple marks on the compression side of the struts (SHR=1.05). Struts having visible ripple marks at failure are named as R in Table 4.6. The stress relief annealed seamless struts also deformed laterally with large mid-span deflection, however, no sign of Leuder’s bands was observed on the struts (SHR=1.33).
4.6 SUMMARY

The primary objective of the experimental work was to report tests on ERW and seamless composite curved steel struts with 2000 mm, 4000 mm and 10000 mm initial radii of curvatures. The work furnishes a benchmark for the theoretical model developed in Chapter 3, and therefore enables consideration to be given to the various factors which influence composite curved strut load capacity. Two different steel materials with significantly different strain hardening ratios (\( \text{SHR}_{\text{ERW}} = 1.05 \) and \( \text{SHR}_{\text{seamless}} = 1.33 \)) were used. The influence of factors such as the concrete infill, yield strength, initial radius of curvature and initial deflection at mid-height is reported. The combined influence of the factors exercise a significant effect on ultimate load capacity. The discussion of the present experimental results in detailed form will be given in Chapter 5.
Fig. 4.24 Knife-edges Used In The Strut Tests
Fig. 4.25 Deflected Shape of ERW Strut After Buckling

Fig. 4.26 Deflected Shape of Seamless strut After Buckling
## Table 4.6 Ultimate Load Capacity of Higher Strength Concrete Infilled ERW Curved Struts

| No. | Specimen No. | R (mm) | Length(L) (mm) | e (mm) | L/r | e/L | \(P_{\text{ult test}}\) (kN) |_type of failure_
<table>
<thead>
<tr>
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\(R = \) Initial Radius of Curvature (mm)
\(e = \) Initial deflection at mid-height (mm) measured between center lines as shown in Fig.4.1.
\(r = \) Radius of Gyration of Hollow Section

*Type of failure: F = Failure due to only fracture  \(R = \) Failure due to only ripple  
\(C = \) Failure due to both ripple and fracture  
\(D = \) Ductile (no ripple and facture has been observed)

\(Y = \) Leuder’s band development
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SRA = Stress Relief Annealed
Table 4.7 Ultimate Load Capacity of Higher Concrete Infilled Seamless Curved Struts

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Types of failure:
- Y: Yielding
Table 4.8 Ultimate Load Capacity of ERW Curved Tubular Hollow Sections

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<th>No.</th>
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<th>Length (L) (mm)</th>
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<th>L/r</th>
<th>e/L</th>
<th>$P_{ult.}$ (kN)</th>
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Fig. 4.27 ERW Tubes with 2000 mm Initial Radius of Curvature
Fig. 4.28 ERW Tubes with 4000 mm Initial Radius of Curvature

Fig. 4.29 ERW Tubes with 10000 mm Initial Radius of Curvature
Fig. 4.30 Seamless Tubes with 2000 mm Initial Radius of Curvature

Fig. 4.31 Seamless Tubes with 4000 mm Initial Radius of Curvature
Chapter Four Experimental Work

Fig. 4.32 ERW Tubes with 2000 mm Initial Radius of Curvature

Fig. 4.33 ERW Tubes with 4000 mm Initial Radius of Curvature
Fig. 4.34 ERW Tubes with 10000 mm Initial Radius of Curvature

Fig. 4.35 Seamless Tubes with 2000 mm Initial Radius of Curvature
Fig. 4.36 Seamless Tubes with 4000mm Initial Radius of Curvature

Fig. 4.37 Load versus end-shortening for struts TM3 & TM4
Fig. 4.38 Load versus end-shortening for Struts TM6 and TM7

Fig. 4.39 Load versus end-shortening for Struts TM11 and TM10
Fig. 4.40 Load versus end-shortening for Struts FM3 & FM4

Fig. 4.41 Load versus end-shortening for Struts FM5 & FM6
Fig. 4.42 Load versus end-shortening for Struts FM10 & FM11

Fig. 4.43 Load versus end-shortening for Struts NE3 & NE4
Fig. 4.44 Load versus end-shortening for Struts NE7 & NE8

Fig. 4.45 Load versus end-shortening for Struts NE9 & NE12
Fig. 4.46 Load versus Lateral Deflection at Mid-height for Struts TE1 & TH1

Fig. 4.47 Load versus Lateral Deflection at Mid-height for Struts TE3 & TH2
Fig. 4.48 Load versus Lateral Deflection at Mid-height for Struts TE5 & TH3

Fig. 4.49 Load versus Lateral Deflection at Mid-height for Struts TE7 & TH4
Fig. 4.50 Load versus Lateral Deflection at Mid-height for Struts FE1 & FH1

Fig. 4.51 Load versus Lateral Deflection at Mid-height for Struts FE4 & FH2
Fig. 4.52 Load versus Lateral Deflection at Mid-height for Struts FE5 & FH3

Fig. 4.53 Load versus Lateral Deflection at Mid-height for Struts FE7 & FH4
Fig. 4.54 Load versus Lateral Deflection at Mid-height for Struts NE1 & NH1

Fig. 4.55 Load versus Lateral Deflection at Mid-height for Struts NE5 & NH2
Fig. 4.56 Load versus Lateral Deflection at Mid-height for Struts NE13 & NH3

Fig. 4.57 Load versus Extreme Fiber Strains at Mid-Height of ERW Tubes
Fig. 4.58 Load versus Extreme Fiber strains at Mid-Height of Seamless Tubes

Fig. 4.59 Load versus Curvature for Struts FE3 & FE5
Fig. 4.60 Load versus Curvature for Struts TE3 & TE7

Fig. 4.61 Load versus Curvature for Strut NE13
Fig. 4.62 Load versus Curvature for Struts FM7 & FM13

Fig. 4.63 Load versus Curvature for Struts TM9 & TM14
5. CHAPTER FIVE

DISCUSSION OF EXPERIMENTAL RESULTS

5.1 INTRODUCTION

The primary objective of this study is to obtain the ultimate load capacities of pin-ended curved tubular struts infilled with higher strength concrete subjected to axial load. Considering the experimental results described in Chapter 4, the ultimate load capacity of such members under different geometric and material conditions will be discussed. Also, a comparison will be made between hollow and concrete infilled curved steel struts.

5.2 FAILURE TYPES of CONCRETE and STEEL STUB COLUMN CYLINDERS

Considering the experimental results, three types of failure occurred after unconfined testing 100mm x 200mm concrete cylinders (Figs 5.1 and 5.2).

i) Failure due to tensile splitting action causing a longitudinal crack

ii) Shear failure in which slippage occurred along a plane of weakness

iii) Crushing of the concrete at the top or bottom of the cylinder due to a local bearing failure. This often occurred if the end of the concrete cylinder was not planar. When failure occurred at the top or bottom of the cylinder the measured strains at failure do not give the true stress-strain relationship of concrete at maximum stress because there
is a release of strain in the failure zone (Barnard 1964). Therefore, the stress-strain relationships obtained experimentally will be affected by the position of the failure zone.

The complete concrete stress-strain relationships calculated from the CEB-FIB ("International " 1970) method by using Eqs 2.1-2.3 and the Fafitis and Shah (1986) model by using Eqs 2.4-2.7 (see Chapter 2) are shown in Fig.5.3. The stress-strain relationship of the higher strength concrete obtained from experimental results (using an Instron Machine) is also shown in Fig.5.3. As can be seen the ascending branch of the stress-strain curve obtained by the experimental work is close to both, the FIP and Fafitis models. However, the descending branch of the curve is not as sharp as the curve calculated from the FIP method. The descending branch of the stress-strain curve of the concrete could be influenced by the stiffness of the testing machine and the release of strain energy. Higher strength concrete is more brittle than the normal concrete, and therefore it is difficult to control strain rate in the Instron machine after the peak load when the concrete fails suddenly. As mentioned in Chapter 3 the FIP model was used in the theoretical calculations.

An ERW steel stub column specimen after testing is shown in Fig.5.4. All ERW stub column specimens failed by plastic local buckling close to the top bearing surface. The stress-strain curve of such a tube is shown in Fig.5.6. A seamless stub column specimen after failure is shown in Fig.5.5. The flaking of the mill scale of the stub column due to shear failure can be clearly seen in the figure. The stress-strain relationship of this specimen is shown in Fig.5.7.
Fig. 5.1 Shear Failure of Concrete Cylinder

Fig. 5.2 Longitudinal Splitting and Local Bearing Failure
Fig. 5.3 Stress-Strain Relationship of Concrete Using different Models
Fig. 5.4 ERW Stub Column after Testing

Fig. 5.5 Seamless Stub Column after Testing
Fig. 5.6 Compressive Stress-Strain Relationship of ERW Steel Stub Column

Fig. 5.7 Compressive Stress-Strain Relationship of Seamless Steel Column
5.3 OVERALL STRUT RESULTS

5.3.1 Curved Steel Struts Infilled With Higher Strength Concrete

5.3.1.1 ERW Tubes

In general, by a simple inspection of the load-lateral deflection for the ERW curves shown in Figs 4.27-4.29, the peak load is found to decrease as the length of the specimens increases. The peak load decrease can be attributed to the increasing initial deflection with increasing slenderness of the struts.

From the load-lateral deflection curve for TE1 as shown in Fig. 4.27, the load up to 120 kN is approximately linear. Above this load, the rate of development of deflection increased as the section of tube became progressively inelastic. Immediately after the maximum load of 123 kN, the deflections increased rapidly. At this stage the load began to decrease with further increase in deflection. When the load had decreased to 55 kN ripples of local buckling were observed on the concave side at near mid-height. The tube suddenly fractured on the tensile face when the load was 50 kN.

The load-deflection curves for the longer struts i.e. TE7, TE9 and TE11 given in Fig. 4.27, show that there is a significant plateau after the peak load, which follows from the large initial radius of curvature as well as the large spread of plasticity in comparison with the shorter specimens along the length of tube in the tension and compressive region at mid-span. The curves are approximately linear up to 50% of the peak load. In the case of struts more than 3000 mm long, the falling branch of the load deflection curve was not followed due to the limitation of rotation of the knife-edges. Local buckling was not observed for the long specimens (L>3000 mm).

From the load-deflection relationships shown in Figs 4.27, 4.28 and 4.29, considering struts with the same straight length and different initial radii of curvature, i.e., TE1, FE1
and NE1, as a result of increasing the initial radius of curvature the peak load increased due to decreasing the initial deflection at mid-height. The load-deflection curves are nearly similar except in some cases, i.e., in NE13(L=1515mm), the load dropped sharply after the peak load because of the small region of plasticity on the tension face in comparison with specimen TE5(L=1559mm) and fractured when the load was 20 kN.

The ERW tubes had a localised plastic hinge owing to the restricted spread of plasticity, and consequently they tended to fracture on the convex face when subjected to the consequential high tensile load strains. The spread of the plastic area before failure for the long specimens with large initial curvature was more significant. Specimen TE6 after testing is shown in Fig.4.25. The deflected shape indicates that the curved strut bent in symmetrical single curvature, with a "plastic hinge" approximately at the center during the falling branch of the P-δ curve. The tube eventually fractured on the tension side with associated local buckling on the compression side.

5.3.1.2 Seamless tubes

Load-deflection curves for seamless struts are given in Figs 4.30-4.31 and Figs 4.35-4.36. Considering the load-lateral deflection relationship of specimen TM1 shown in Fig.4.30, the curve up to 120 kN is almost linear. With increasing load the rate of development of deflection increased slightly up to 146 kN. After the load fell away to 120 kN flaking of the mill scale was observed on the concave face in the compression region of tube, and at 45 kN the test was stopped. Due to the large deflection the falling branch of the load-deflection curve was followed until the maximum possible of central deflection was obtained, which was limited by the rotation capacity of the knife-edge.

The load-deflection curve up to 50% loading before peak load for the longer specimens, TM14 and TM16, is almost linear. They also had flat plateau and large deflection after the peak load. As mentioned in the previous section the deflection was recorded as far as possible by the limits allowed by rotation of the knife edges. Specimen TM14 after failure is shown in Fig.4.26.
As can be seen from previous sections there is a significant difference between the type of failure between both ERW and seamless composite curved struts even though the geometric condition is identical. This can be related to the manufacturing process and material structure of the struts.

5.3.2 SRA and As-Received Curved Struts

Load-end shortening relationships for seamless Stress Relief Annealed (SRA) and as-received composite curved struts with 2000 mm, 4000 mm and 10000 mm initial radius of curvature are shown in Figs 4.37-4.45. In general, there is a reduction in the ultimate load capacity of SRA struts in comparison with the as-received struts. From the load-end shortening relationships for the seamless struts, i.e. TM3 and TM4(SRA), given in Fig.4.37, it can be seen that the ultimate load capacity of the SRA curved struts decreases by up to 10%. However, the post peak load curve, does not change significantly.

Load-end shortening results for ERW struts with 10000mm initial radius of curvature, are given in Figs.4.43-4.45. Considering Fig.4.44 the decrease of the ultimate load capacity of the specimen NE8 due to SRA was about 15% in contrast with the as-received specimen NE7. From the load-deflection curve in Fig.4.43 the greatest difference between specimens NE3 and NE4 (SRA) is the value of the deflection at failure (fractured). The ratio of deflection at failure for the NE4 (SRA) over to the deflection at failure for the NE3 is about 3.3, which is significant.

The reduction in the ultimate load capacities of stress-relief-annealed curved struts was due to the drop in the yield stress after the annealing process. It could be related to the process of heating and the rate of cooling of struts. Stress relief annealing, the process of heating and then slowly cooling in a furnace, decreased strength levels, removed residual stresses, and increased the ductility.
5.3.3 Hollow and Concrete Infilled Struts

So far, all the discussion has focused on the behaviour of concrete filled specimens and how their varying steel condition, initial mid-span deflection and slenderness affects the load carrying capacity and the failure mechanism. For a more practical evaluation of the advantages and disadvantages of filling hollow circular steel struts with higher strength concrete mix a comparison is required between the behaviour of unfilled struts and concrete filled struts.

A straightforward comparison can be made on the difference in behaviour and load capacity of concrete filled and infilled specimens by the inspection of load-deflection plots, peak load and failure type.

Load-lateral deflection relationships for hollow and composite ERW curved steel struts with identical initial radius of curvature and the slenderness are shown in Figs 4.46-4.56. The ultimate load capacity of short hollow curved struts increased as a result of filling with higher strength concrete. The ratio of the maximum load of the composite curved struts over to the maximum load of the hollow struts versus slenderness ratio of the curved hollow struts for different initial radii of curvatures is shown in Fig. 5.8. The figure shows that the plots are linear in nature and the slope of the line obtained from struts with 4000 mm initial radius of curvature is almost the same as the line obtained from struts with 10000 mm initial radius of curvature.

The percentage increase in strength from the smallest slenderness ratio ($\lambda=39$) for 2000 mm to 10000 mm initial radii of curvatures is about 80%. This ratio for struts with 4000 mm initial radius of curvature is about 50%. Therefore, the increase of maximum load due to filling concrete for the struts having this slenderness ratio and 4000 mm initial radius of curvature is not as significant as struts with 2000 mm and 10000 mm initial radii of curvatures.
The percentage increase in strength for struts with 2000 mm and 4000 mm initial radius of curvature and slenderness ratio of 87 is about %10. This ratio for struts with 10000 mm initial radius of curvature is about %37. As the strength of higher strength concrete in compression is significant, the strength of hollow curved struts with small initial curvature increases considerably.

![Graph showing percentage decrease in strength vs slenderness ratio.](image)

**Fig. 5.8** Percentage Decrease in strength vs Slenderness ratio

ERW specimens TE1, TH1, NE1 and NH1 after testing are shown in Figs. 5.18 and 5.19. The overall shape of the two specimens after failure was different. The unfilled specimens had finally buckled locally in an inward direction (specimens TH1 and NH1). The mechanism of failure for the unfilled struts occurred as local buckling of the inside region or compression side and a sharp angle (kink) in the outside region or the tension side.
Concrete filled specimens (TE1, NE1) had a slightly bent form after failure so that outward local buckling occurred on the concave face for both specimens before tearing on the convex face, where the steel underwent local necking before it fractured.

From the load-deflection curves in Fig.4.46 the load is linear up to 65 kN for the ERW hollow specimen TH1. As can be seen the initial slope (initial stiffness) of the curves for both hollow and composite curved struts is nearly the same. After the peak load for the hollow strut, 67 kN, the fall off branch of the curve was smooth to 55 kN. Immediately after this load, the slope of curve changed sharply and local buckling on the compression side as well as a kink on the tension side was observed. On the other hand, the descending branch of the curve of the composite specimen, TE1, dropped smoothly even though local buckling occurred at nearly 65 kN. In this case, a significant change in the slope of falling part of curve did not appear.

In general, the initial slopes (initial stiffnesses) of the load-deflection curves of the ERW longer hollow struts are smaller than of the composite struts with identical length and initial radius of curvature as shown in Figs 4.47-4.49. The general view of the curves (hollow and composite struts) shown in Figs 4.50-4.56 follows in a similar manner to the strut curves which are already explained.

5.3.4 Post-Peak load behaviour and Ductility

The post-peak load capacity of tubular curved struts may be defined as the capacity of the members to sustain the additional bending stresses as the members pass the ultimate load. These stresses are the result of the bending moment due to the increased eccentricity of the axial load with respect to the deflected post-peak load shape. It can be assumed that the post peak load capacity of curved tubular struts will be significant in the case of long curved struts where the applied stresses at or near the peak load are small in comparison with shorter struts.
Ductility has an important role in the stability of structural frames, especially for the elements that are under compressive force systems. Therefore, a realistic criterion for evaluation of the ductile behaviour of the elements such as composite hollow section columns is required for the necessary computation of the stability of structures. In this study, by using experimental results, the ductility factor is calculated as

\[ D_e = \frac{\mu_e}{\mu_u} \]  

(5.1)

where \( D_e \) = ductility factor, \( \mu_e \) = end-shortening (axial) deflection at an arbitrarily defined 0.75% of the ultimate load, and \( \mu_u \) = end-shortening deflection at ultimate load.

From Table 5.1 it can be seen that for the ERW specimens with 2000 mm initial radius of curvature, the ratio of the deflection at 75% of ultimate load, to the deflection at ultimate load, \( \frac{\mu_e}{\mu_u} \), varied between 3.5 and 6.5. As mentioned before, ductility is significant for the longer specimens (ERW and seamless struts) because of the flat plateau of the load deflection curve after the peak load. Also, in comparison with the hollow struts with the same slenderness ratio, the ratio increases significantly, i.e. for the ERW TH4 (hollow) and ERW TE7 (composite), from 3 to 6.5, respectively.

It is seen that the change of the ductility factor for ERW composite curved struts with 4000 mm and 10000 mm initial radii of curvatures is not large for the range of straight lengths i.e. for the specimen FE1, the ratio is 4 and for the specimen FE7 the ratio is 4.5. However, in comparison with hollow curved struts it increases significantly. The factor is 1.6 for the NE1 (R=10000 mm, L=780 mm and composite) and 2.8 for the NH1 (R=10000 mm, L=770 mm and hollow). The increase of the ductility factor can be related to the flat plateau of the peak load and also the post peak load of the specimen NH1 in comparison with the specimen NE1. The peak load of the NE1 specimen is the highest peak load in these series of the tests due to it having the smallest initial deflection at mid-height and smallest straight length.
Referring to Table 5.2 the post peak behaviour for the as-received curved seamless steel struts filled with concrete was improved by the stress relief anneal processing, for instance the ratio for the seamless specimens TM6 and TM7 (SRA) as shown in Fig. 5.9a and normalised load-end shortening in Fig. 5.9b increase from 4.8 to 6.5. However, in the other cases it did not change significantly for the stress relief annealed struts compared with the as-received struts, i.e. FM3 and FM4 (SRA) as shown in Fig. 5.10a and normalised curves in Fig. 5.10b. The figure shows that the load-deflection curves of the specimens FM3 and FM4 (SRA) are close in the pre-peak load range and also after the peak load for small end shortening. However, after the peak load when the load was nearly 0.75P_{ult}, differences between the curves started to increase.

From above discussion, and Tables 5.1 and 5.2, it can be concluded that the ductility factor depends on the different factors such as material properties (for instance hollow against composite and as-received against stress-relief annealed) and also on the distribution of the critical stresses and plasticity after the peak load, which can be influenced by changing initial radius of curvature and straight length. For example, it is clearly seen for struts with 2000 mm and 4000 mm initial radii of curvature and different slenderness ratio; the ductility factor increased i.e. from 3 to 6.5 (L=1745 mm and R=2000 mm), from 2.3 to 4.5 (L=1755 mm and R=4000 mm) due to the infill concrete.
Fig. 5.9a Load-end shortening Curves of Seamless Specimens TM6 and TM7

Fig. 5.9b Normalised Load-end shortening Curves of Seamless Specimens TM6 and TM7
Fig 5.10a. Load-end shortening Curves of Seamless Specimens FM3 and FM4

Fig 5.10b. Normalised Load-end shortening Curves of Seamless Specimens FM3 and FM4
Table 5.1 Ductility of Hollow and Composite Curved Electric Resistance Welded Steel Struts

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<th>No</th>
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<th>L (mm)</th>
<th>Condition</th>
<th>$\mu_0$ (mm)</th>
<th>$\mu_e$ (mm)</th>
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Table 5.2 Ductility of Stress Relief Annealed and As-Received Composite Curved Seamless Steel Struts.

<table>
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<th>L/mm</th>
<th>Condition</th>
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<th>$\mu_e$ mm</th>
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$\mu_u$ = Axial deflection (end shortening) at $P_{ult}$

$\mu_e$ = Axial deflection (end shortening) at $0.75P_{ult}$

5.3.5 Load-Strain Curves

5.3.5.1 ERW Composite steel struts

Considering Chapter 4, typical load-strain curves for the ERW composite curved tubes are given in Figure 4.57. Strains in the tension and compression sides of the specimen TE7 (ERW, R=2000 mm and L=1745 mm) were 4200 and 3250 microstrain at the peak load respectively. The strain in the convex face was observed in excess of 19000 microstrain. This large value can be related to the significant bending stresses which were carried by the strut. In specimen NE13 (ERW) the critical buckling strain for such strut was 2704 microstrain. The maximum recorded strain after the peak load was 12,900 microstrain for the tension side and 8750 microstrain for the compression side.
Specimens with a straight length larger than 1500 mm \((L/r>70)\) yielded first in the longitudinal direction in the tension face for the three sets of the initial radii of the curvatures used in this series of tests. At low loads, the ratio of circumferential strain over to longitudinal strain, \(\varepsilon_{sc} / \varepsilon_{sl}\), for the steel at all points of measurement was less than the Poisson’s ratio for steel. At the first yield load, the ratio was approximately equal to the Poisson’s ratio.

5.3.5.2 Seamless composite steel struts

Typical load-strain curves for the seamless composite curved tubes are given in Figure 4.58. The strains for some seamless struts i.e. FM7 (\(R=4000\) mm and \(L=1120\) mm) were not possible to be recorded after the peak load due to flaking of the mill scale and peeling off of the strain gauges. The flaking of the mill scale in the specimen was observed only in the compression region. The steel tube of the specimen FM7 yielded first in the longitudinal direction on the compression side due to the small slenderness ratio and the initial deflection at mid-height. At the peak load the strain in the compression side was about 4000 microstrain and in the tension side was about 2100 microstrain. Also, for the specimen TM14 (seamless, \(R=2000\) mm and \(L=1685\) mm) the compression and tensile strains were about 3200 and 3000 microstrain, respectively.

5.3.6 Load-Curvature Curves

Typical load-curvature relationships obtained from the recorded strains of the ERW composite curved steel struts are shown in Figs 5.11-5.14 (also shown in Figs.4.59-4.61). By increasing the axial load, the changing curvature was significant at mid-height of the tube due to maximum eccentricity (initial deflection at mid-height). The influence of the straight length on the load-curvature curves with identical initial radius of curvature can be seen i.e. in Figs 5.11 (TE3 and TE7) and 5.12 (FE3 and FE5). In general, the fall-off branch of such curves after the peak load for the short ERW specimens \((L<1500\text{mm and }L/r<70)\) was more significant as a result of the small region on the tension side reaching a plastic condition. The longer specimens such as TE7
(R=2000 mm and L=1745 mm) had a nearly flat plateau after the peak load. Referring to Table 5.3 the ductility factor for the TE3 (R=2000 mm and L=1176 mm) is 2.1 and for the TE7 is 2.4.

The struts having different initial radius of curvature with nearly the same straight length are shown in Figs 5.13-5.14. The differences in the descending branch of the curves of the FE3 (R=4000 mm and L=1181 mm) and TE3 in Fig.5.13 and of the NE13 (R=10000 mm and L=1515 mm) and FE5 (R=4000 mm and L=1540 mm) in Fig.5.14 are significant. From Table 5.3 the ductility factor is 2.2 for the FE3, 1.7 for the NE13 and 2.08 for the FE5. Due to decreasing the initial radius of curvature the ductility factor increased. It can be attributed to the spread of plasticity along the length of the strut at mid-length after the maximum load was reached. During the falling branch of the load-curvature relationship of the struts with a large initial curvature and a long straight length, the increase in curvature was not concentrated at the central section. Therefore, the area which reached the yield condition for longer specimens was larger than for the short specimens.

As can be seen in Table 5.3 the ductility factor for seamless composite curved steel struts also increased with increasing length and decreasing initial radius of curvature. For example, the ductility factor is 1.87 for TM9 (R=2000 mm and L=1125 mm) and 2.9 for TM14 (R=2000 mm and L=1685 mm). The load-curvature curves of the specimens TM9 and TM14 are shown in Fig.5.15.

As mentioned before, the deformed shape of the seamless struts at failure (at the end of test) were completely different from the ERW struts having identical straight length and initial deflection at mid-height due to different material characteristics. In order to make a comparison between ERW and seamless struts the normalised load-curvature curves were calculated with respect to the corresponding squash load ($N_{uo}$), initial radius of curvature and nearly identical straight length (load / squash load versus curvature / initial curvature). The specimens TM14 and TE7 are shown in Fig.5.16, and FM13 and FE5 in Fig.5.17, respectively. It is noted that the cross-section area of ERW and seamless strut is different, so a comparison in a normalised format is more reasonable.
In general, the ascending and descending branch of curves i.e. TM14 and TE7 are different. The initial slope (initial stiffnesses) of the normalised load-curvature curves and the value of the ratio of load/squash load of the seamless struts are larger than ERW struts.

It is noted that according to ACI (1971) the effective flexural rigidity of a composite tube \( (E_d)_{com} \) can be calculated from Eq. 3.42. The effective rigidity of the cross section of the ERW struts over the seamless struts using Eq. 3.42 is nearly 66%. However, by referring to Figure 5.16, the ratio of the initial stiffness of the load-curvature curve of the ERW specimen TE7 in the normalised condition over the seamless specimen TM14 is nearly 0.75. From Figure 5.17 such a ratio for the ERW FE5 and the seamless FM13 is nearly 0.69. It can be clearly seen that the seamless struts have a larger rigidity in comparison with the ERW struts for those struts having an identical straight length and an initial deflection at mid-height when considered in the normalised condition.

From Fig. 5.16 the ratio of the maximum load capacity / squash load of the ERW strut TE7 (0.057) is larger than the seamless strut TM14 (0.063). Such ratio at maximum load is 0.12 for FE5 and 0.16 for FM13. It can be concluded that seamless struts can carry larger strength in comparison with ERW struts. The ductility factor of the TM14 \( (\mu=2.9) \) is also larger than the TE7 \( (\mu=2.4) \) and also of the FM13 \( (\mu=2.27) \) is larger than FE5 \( (\mu=2.08) \).
Table 5.3 Ductility Factor of Load-curvature Curves of Composite Curved Steel Struts

<table>
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<th>R (mm)</th>
<th>( c_u ) (1/mm)</th>
<th>( c_e ) (1/mm)</th>
<th>( \mu = c_e / c_u )</th>
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\( c_u \) = Curvature at ultimate load (\( P_{ult} \))

\( c_e \) = Curvature at 0.95%\( P_{ult} \)

Fig. 5.11 Load versus Curvature for ERW Struts TE3 and TE7
Chapter Five Discussion of Experimental work

Fig. 5.12 Load versus Curvature for ERW Struts FE3 and FE5

Fig. 5.13 Load versus Curvature for ERW Struts FE3 and TE3
Fig. 5.14 Load versus Curvature for ERW Struts NE13 and FE5

Fig. 5.15 Load versus Curvature for Seamless Struts TM9 and TM4
Fig. 5.16 Normalised Load versus Curvature for Struts TM14 and TE7

Fig. 5.17 Normalised Load versus Curvature for Struts FM13 and FE5
Fig. 5.18 Specimens TE1 & TH1 after Testing
Fig. 5.19 Specimens NE1 & NH1 after Testing
6. CHAPTER SIX

COMPARISON AND DISCUSSION OF

THEORETICAL AND EXPERIMENTAL RESULTS

6.1 GENERAL

The details of the theoretical development and experimental program have been
described in Chapters Three and Four, respectively. As mentioned in Chapter Four a
total number of 78 composite curved steel struts as well as 11 hollow curved steel struts
were tested. The following sections of this Chapter generally compare the theoretical
and experimental results and give a relevant discussion. The parameters which affect the
ultimate load capacity of composite curved steel struts will be described.

6.2 COMPARISON OF THE THEORETICAL AND
EXPERIMENTAL LOAD-DEFORMATION CURVES

Load-deflection curves obtained from the experimental and elastic and plastic analyses
for the Electric Resistance Welded struts are shown in Figs 6.1-6.9. As the test results of
the two sets of struts were reasonably close to each other, the specific dimensions of
only one set of specimens for strut testing were used to compute the theoretical curves.

From Fig.6.1, for the strut with 2000 mm initial radius of curvature (L=775 mm), it can
be seen that the elastic curve lies on the linear branch of the experimental curve,
however, there is a difference between the plastic curve and the fall off portion of the experimental curve. Considering Figs 6.1-6.4 for the struts with the same initial radius of curvature, with increasing straight length and the initial deflection at mid-height, the intersections of the elastic and the plastic curves and also plastic curves, are close to the maximum loads and the post buckling curves of the experimental results, respectively. This can be related to the straight length as well as the initial deflection at mid height, which could influence the Equations 3.41 and 3.47, and consequently the initial slope of the elastic and the plastic curves.

By comparing the elastic and plastic curves of the ERW strut (R=-4000 mm and L=775 mm) with the ascending and descending branch of the experimental curve, respectively, in Fig.6.5, there is a considerable difference for both curves. With increasing slenderness the difference of such curves is significantly reduced as shown in Figs 6.5-6.9. This reduction occurs in the same manner for the seamless struts with 2000 mm and 4000 mm initial radii of curvatures as shown in Figs 6.10-6.19.

Load-deflection curves obtained from the finite element analysis, the elastic and plastic responses and experimental results are shown in 6.20-6.28. It can be seen that there is good agreement between the finite element and experimental results in comparison with the elastic and plastic results. All load deflection curves obtained from finite element analysis displayed the same characteristics as the experimental load-deflection curves.

One of the important reasons for the difference of the results of these two theoretical methods are that in the method of intersection of the elastic and the plastic curves it is assumed that all sections along the curved strut have the same stiffness as the central section. As the reduction in stiffness due to bending, is greatest at the center, the overall stiffness of the strut is consequently underestimated. On the other hand, in the finite element modelling the curved composite strut was divided into segments so that they have different stiffnesses in comparison with the central segment. The sections throughout the segment were divided into finite elements of steel and concrete in order to calculate their tangent stiffness properties at different levels of strain.
6.3 COMPARISON AND DISCUSSION OF THE THEORETICAL AND EXPERIMENTAL MAXIMUM STRUT LOADS

The ultimate load capacity of composite curved steel struts obtained from the experimental results are compared with the results calculated from the intersection of the elastic and the plastic curves, and from Rangan and Joyce (1992), as given in Tables 6.1-6.2. Rangan and Joyce investigated eccentrically loaded struts, their work is included here as a possible model for the ultimate load calculation.

Referring to Table 6.1 the range of the ratio of test/predicted results, \( \eta_1 = \frac{P_{\text{test}}}{P_{\text{th}}} \) (\( P_{\text{th}} \) = ultimate load obtained from the elastic and plastic curves) of the ERW struts with 2000mm initial radius of curvature is between 0.88 and 1.19, the arithmetic mean is 1.03, and the standard deviation is 0.115, respectively, for the intersection method. The ratio \( \eta_2 = \frac{P_{\text{test}}}{P_{\text{rang}}} \) (\( P_{\text{rang}} \) = obtained from the Rangan method) is between 0.66 and 1.22; the arithmetic mean is 0.93, and the standard deviation is 0.21.

The ratio \( \eta_1 \) for ERW struts with 4000 mm initial radius of curvature is between 0.92 and 1.06; the arithmetic mean is 1.0 and the standard deviation is 0.02. The ratio \( \eta_2 \) is between 0.68 and 1.12; the arithmetic mean is 0.89 and the standard deviation is 0.15.

The ratio \( \eta_1 \) for the ERW struts with 10000 mm initial radius of curvature is between 0.88 and 1.08; the arithmetic mean is 1.0 and the standard deviation is 0.075. The ratio \( \eta_2 \) is between 0.58 and 1.08; the arithmetic mean is 0.76 and the standard deviation is 0.186.

The \( \eta \) ratios are conservative for the shortest struts (L=775mm). It can be due to the effects of triaxial stresses that can influence the maximum load at this length. The straight length between supports and the initial deflection at mid-height affect the initial slope, and therefore the intersection of the elastic and plastic curves.
As can be seen for the struts with a straight length between 1000 mm and 2000 mm the ratio $\eta_1 = \frac{P_{\text{Test}}}{P_{\text{The}}}$ is close to unity for the intersection method, and there is very good agreement between the experimental and theoretical results for most cases in comparison with the Rangan method. As mentioned in Chapter Three, with the intersection method the additional deflection at mid-height for the plastic curve was calculated based on an iteration process (using Eq.3.47). However, in the Rangan method the maximum additional deflection at mid-height was calculated by assuming a sine function for the maximum deflection at mid-height (using Eq.3.52). This assumption can influence the maximum load obtained with the Rangan method.

Considering Table 6.2 the ratio $\eta_1$ for seamless struts with 2000 mm initial radius of curvature is between 1.0 and 1.11; the arithmetic mean is 1.05 and the standard deviation is 0.043. The ratio $\eta_2$ is between 0.77 and 1.19; the arithmetic mean is 0.902 and the standard deviation is 0.17. The ratio $\eta_1$ for the seamless struts with 4000 mm initial radius of curvature is between 0.96 and 1.16; the arithmetic mean is 1.06 and the standard deviation is 0.084. The ratio $\eta_2$ is between 0.7 and 1.01; the arithmetic mean is 0.87 and the standard deviation is 0.131.

It can be seen that the standard deviation of the ratio $\eta_1$ for the seamless struts is less than the standard deviation of the ratio $\eta_1$ for ERW struts with 2000 mm initial radius of curvature. However, it is larger than the standard deviation of the $\eta_1$ ratio for the ERW struts with 4000 mm initial radius of curvature. The difference is attributed to the different material characteristics and the process of the fabrication of the seamless struts in comparison with the ERW struts, which could influence the intersection point of the elastic and plastic curves.

In general, good agreement can be noticed between the experimental and finite element results (given in Table 6.3) especially in the load range close to the failure load. The arithmetic mean of the ratio $\eta_3 = \frac{P_{\text{Test}}}{P_{\text{Fin}}}$ for the ERW struts is 1.06 and the standard deviation is 0.041.
Despite taking into account the residual stress, several other factors, such as initial deflection at mid-height (eccentricity) and slenderness ratio, can influence the maximum load calculated by the intersection method which could be different from that given by the finite element method. The finite element analysis predicts the strut failure load more accurately than the intersection point of the elastic and plastic curves. One of the important reasons is that in calculating the rigidity of the composite struts from an approximate method (i.e. ACI) which is explained in Chapter Three, this rigidity is taken as constant along the length the strut.

6.3.1 Main Parameters Which Could Influence Ultimate Load Capacity

The ultimate load capacity, $P_{ult}$, of a curved steel strut infilled with concrete can be a function of six parameters: external diameter ($d$), wall thickness ($t$), straight length between supports ($L$), initial deflection at mid-height ($e$), $f_y$ and $f'_c$. The first four, which are geometrical properties, can be specified by the non-dimensional parameters $d/t$, $L/d$ and $e/d$, together with diameter $d$. It is noted that $e$ and $L$ can be influenced by the initial radius of curvature ($R$) in curved structures. The material properties cannot, however, be represented by the non-dimensional parameter $f_y/f'_c$, because the concrete and steel stress-strain curves are not similar. Both $f_y$, $f'_c$ must be specified independently. To non-dimensionalise the ultimate load $P_{ult}$, it is divided by the squash load $P_{sq}$ which is a function of the parameters $d$, $t$, $f_y$ and $f'_c$. The ratio $P_{ult}/P_{sq}$ also gives the value of the reduction coefficient $\alpha$.

The ratios of $L/d$, $e/d$ and $P_{ult}/P_{sq}$ ($P_{ult}$ calculated from intersection point of elastic and plastic curves) are given in Tables 6.4 and 6.5. The variation of $\alpha (=P_{ult}/P_{sq}$ a reduction coefficient $)$ versus $e/d$ and $e/L$ for constant values of initial radius of curvature is plotted in Figs 6.29 and 6.30, respectively.
The maximum value of $\alpha$ is 1.0 as the ultimate load $P_{\text{ult}}$ cannot be greater than the squash load $P_{\text{sq}}$; this value must correspond to $L/d = 0$, $e/d=0$. Fig.6.29 shows that the $\alpha$-$e/d$ curves are quasi-hyperbolic in nature, having a steep slope in the range $0<e/d<2$ and approaching zero as $e/d$ tends to infinity. It can be seen that all graphs nearly lie over each other except for struts with 10000 mm initial radius of curvature, where there is a small difference. It will be shown that the ultimate load capacity of curved struts largely depends on the ratio $e/d$.

The reduction coefficient $\alpha$ versus $L/d$ ratio is shown in Fig.6.30. The slope of curves is not as sharp as $\alpha$-$e/d$ curves for the small range of $L/d$ ratio. With increasing $L/d$ ratio all graphs approach one another and at infinity they are getting close to zero. Also, the reduction coefficient $\alpha$ vs $e/L$ is plotted in Fig.6.31. Referring to Tables 6.4 & 6.5 the range of variation of the geometrical parameters is 10-50 for $L/d$, and 0.5-11 for $e/d$; these ranges cover most cases of practical importance in curved struts.
Table 6.1 Ultimate Load Capacity of ERW Composite Curved Steel Struts Obtained From Experimental and Theoretical Results

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<th>e (mm)</th>
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<th>$P_{\text{ult}}$ (kN) Th. Res.</th>
<th>$P_{\text{ult}}$ (kN) Ran. Res.</th>
<th>$\eta_1$ ($P_{\text{Test}} / P_{\text{Th}}$)</th>
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<td>16.9</td>
<td>24.4</td>
<td>0.91</td>
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$P_{\text{Th}}$ = obtained from intersection of elastic and plastic curves

$P_{\text{Rang}}$ = obtained from Rangan method
### Table 6.2 Ultimate Load Capacity of Seamless Composite Curved Steel Struts Obtained From Experimental and Theoretical Results

<table>
<thead>
<tr>
<th>No</th>
<th>R (mm)</th>
<th>L (mm)</th>
<th>e (mm)</th>
<th>$P_{ult}$ (kN) Test Res.</th>
<th>$P_{ult}$ (kN) Th. Res.</th>
<th>$P_{ult}$ (kN) Ran. Res.</th>
<th>$\eta_1$ ($P_{Test}/P_{Th.}$)</th>
<th>$\eta_2$ ($P_{Test}/P_{Ran.}$)</th>
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<td>29.35</td>
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### Table 6.3 Ultimate Load Capacity of ERW Curved Steel Struts Obtained From Experimental, Finite Element and Intersection Results

<table>
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<th>No</th>
<th>R (mm)</th>
<th>L (mm)</th>
<th>e (mm)</th>
<th>$P_{ult}$ (kN) Test Res.</th>
<th>$P_{ult}$ (kN) Th. Res.</th>
<th>$P_{ult}$ (kN) Fin. Res.</th>
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<td>0.96</td>
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<tr>
<td>4</td>
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<td>157.1</td>
<td>1.01</td>
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$P_{Fin}$ = obtained from finite element method
Table 6.4 Reduction Coefficient for Electric Resistance Welded Composite Curved Struts

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<th>R (mm)</th>
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<th>L/d</th>
<th>e/d</th>
<th>$\frac{P_{ult}}{P_{sq}}$</th>
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<td>12.8</td>
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<td>0.38</td>
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<tr>
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<td>0.103</td>
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<td>0.157</td>
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<td>0.03</td>
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Table 6.5 Reduction Coefficient for Seamless Composite Curved Struts

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<tr>
<th>R (mm)</th>
<th>e/l</th>
<th>L/d</th>
<th>e/d</th>
<th>$\frac{P_{ult}}{P_{sq}}$</th>
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</thead>
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<tr>
<td>2000</td>
<td>0.041</td>
<td>12.3</td>
<td>0.56</td>
<td>0.37</td>
</tr>
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<td>2000</td>
<td>0.058</td>
<td>18.6</td>
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<td>0.017</td>
<td>12.3</td>
<td>0.22</td>
<td>0.51</td>
</tr>
<tr>
<td>4000</td>
<td>0.035</td>
<td>18.5</td>
<td>0.65</td>
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<tr>
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<td>0.07</td>
<td>36.8</td>
<td>2.52</td>
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</table>
Fig. 6.1 Load-Deflection Curves of ERW Struts (R=2000mm and L=775mm) Obtained From Experimental and Elastic and Plastic Results

Fig. 6.2 Load-Deflection Curves of ERW Struts (R=2000mm and L=1559mm) Obtained From Experimental and Elastic and Plastic Results
Chapter Six  Comparison and Discussion of Theoretical and Experimental Results

Fig.6.3 Load-Deflection Curves of ERW Struts (R=2000mm and L=1745mm) Obtained From Experimental and Elastic and Plastic Results

Fig.6.4 Load-Deflection Curves of ERW Struts (R=2000mm and L=2290mm) Obtained From Experimental and Elastic and Plastic Results
Chapter Six  Comparison and Discussion of Theoretical and Experimental Results

Fig. 6.5 Load-Deflection Curves of ERW Struts (R=4000mm and L=775mm) Obtained From Experimental and Elastic and Plastic Results.

Fig. 6.6 Load-Deflection Curves of ERW Struts (R=4000mm and L=1176mm) Obtained From Experimental and Elastic and Plastic Results.
Fig. 6.7 Load-Deflection Curves of ERW Struts (R=2000mm and L=1540mm) Obtained From Experimental and Elastic and Plastic Results

Fig. 6.8 Load-Deflection Curves of ERW Struts (R=2000mm and L=1775mm) Obtained From Experimental and Elastic and Plastic Results
Fig. 6.9 Load-Deflection Curves of ERW Struts (R=4000 mm and L=2255 mm) Obtained From Experimental and Elastic and Plastic Results

Fig. 6.10 Load-Deflection Curves of Seamless Struts (R=2000 mm and L=743 mm) Obtained From Experimental and Elastic and Plastic Results
Fig. 6.11 Load-Deflection Curves of Seamless Struts (R=2000mm and L=1125mm) Obtained From Experimental and Elastic and Plastic Results

Fig. 6.12 Load-Deflection Curves of Seamless Struts (R=2000mm and L=1484mm) Obtained From Experimental and Elastic and Plastic Results
Fig. 6.13 Load-Deflection Curves of Seamless Struts (R=2000mm and L=1685mm) Obtained From Experimental and Elastic and Plastic Results

Fig. 6.14 Load-Deflection Curves of Seamless Struts (R=2000mm and L=2220mm) Obtained From Experimental and Elastic and Plastic Results
Fig. 6.15 Load-Deflection Curves of Seamless Struts (R=4000mm and L=745mm) Obtained From Experimental and Elastic and Plastic Results

Fig. 6.16 Load-Deflection Curves of Seamless Struts (R=4000mm and L=1120mm) Obtained From Experimental and Elastic and Plastic Results
Fig. 6.17 Load-Deflection Curves of Seamless Struts (R=4000mm and L=1480mm) Obtained From Experimental and Elastic and Plastic Results

Fig. 6.18 Load-Deflection Curves of Seamless Struts (R=4000mm and L=1680mm) Obtained From Experimental and Elastic and Plastic Results
Fig. 6.19 Load-Deflection Curves of Seamless Struts (R=4000mm and L=2225mm)
Obtained From Experimental and Elastic and Plastic Results

Fig. 6.20 Load-Deflection Curves of ERW Struts (R=2000mm and L=775mm)
Obtained From Experimental and Finite Element and Elastic and Plastic Results
Fig. 6.21 Load-Deflection Curves of ERW Struts (R=2000 mm and L=1176 mm)
Obtained From Experimental and Finite Element and Elastic and Plastic Results

Fig. 6.22 Load-Deflection Curves of ERW Struts (R=2000 mm and L=1559 mm)
Obtained From Experimental and Finite Element and Elastic and Plastic Results
Fig. 6.23 Load-Deflection Curves of ERW Struts (R=4000 mm and L=765 mm) Obtained From Experimental and Finite Element and Elastic and Plastic Results

Fig. 6.24 Load-Deflection Curves of ERW Struts (R=4000 mm and L=1176 mm) Obtained From Experimental and Finite Element and Elastic and Plastic Results
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Fig. 6.25 Load-Deflection Curves of ERW Struts (R=4000 mm and L=1540 mm)
Obtained From Experimental and Finite Element and Elastic and Plastic Results

Fig. 6.26 Load-Deflection Curves of ERW Struts (R=10000 mm and L=765 mm)
Obtained From Experimental and Finite Element and Elastic and Plastic Results
Fig. 6.27 Load-Deflection Curves of ERW Struts (R=10000 mm and L=1141 mm)
Obtained From Experimental and Finite Element and Elastic and Plastic Results

Fig. 6.28 Load-Deflection Curves of ERW Struts (R=10000 mm and L=1515 mm)
Obtained From Experimental and Finite Element and Elastic and Plastic Results
Fig. 6.29 Reduction Coefficient vs Initial Deflection at Mid-height / External Diameter

Fig. 6.30 Reduction Coefficient vs Straight Length / External Diameter
6.4 DESIGN FORMULAE

There is no guidance in a code of practice such as AS 3600, ACI 318, Eurocode 4 to design hollow/composite curved steel struts, although there is some guidance for the design of the concrete filled tubes in such codes. Composite curved steel struts subjected to compressive load having a large initial deflection at mid-height cannot be entirely designed as slender columns with small imperfections and/or end eccentricities by using a code of practice. However, there is a large similarity in designing such elements.

O’Shea and Bridge (1994) investigated the design of circular concrete filled steel tubes. They found that the strength model in Eurocode 4 accurately predicted the strengths of concrete filled tubes for a range of end eccentricities and column slendernesses in comparison with ACI-318.
For designing composite straight tubes the Eurocode 4 approach is similar to that for steel members where a column curve is used to determine the column strength under axial load. The ACI-318 method uses the traditional reinforced concrete approach (similar to AS 3600-1988) in that a minimum load eccentricity is used in the determination of the column strength under axial load.

Eurocode 4 uses the strength limit state whereby the action effect resulting from the design (factored) loads must be less than the corresponding member strength (resistance) determined from the member geometry and material properties. The use of this code is limited to the concrete strengths not greater than 50 MPa and diameter to wall thickness ratios less than 90 \((235/f_y)\) (O'Shea and Bridge), the latter value ensures that local buckling of the walls does not affect the strength. However, O'Shea and Bridge argued that the Eurocode 4 method may be suitable for use with high strength concrete, and for tubes with diameter to wall thickness ratios \(d/t\) greater than 90 \((235/f_y)\) where the strength of the tube wall can be reduced by local buckling.

For short columns where the strength is not affected by slenderness the interaction diagram defining the strength of a cross-section may be used directly. Rules by which reinforced concrete columns are estimated to be short have been derived by Bridge and Seevarantnam (1987) and incorporated in AS 3600. Similar rules can be used for composite columns.

In general, rational empirical formulae for calculating the maximum load of an eccentrically loaded column can be grouped into two broad classes: reduction coefficient formulae, and interaction formulae, which will be explained below.

### 6.4.1 Eurocode 4

The slender column strength \(N_c\) in axial compression can be calculated by the following equation
Chapter Six  Comparison and Discussion of Theoretical and Experimental Results

\[ N_c = \chi N_{u0} \]  \hspace{1cm} (6.1)

where \( N_{u0} = A_t f_y + A_c f_c \), \( \chi \) is the reduction factor due to slenderness \( \lambda \) obtained from the appropriate column curve A in Eurocode 2 for the bare steel tubes. This is identical to the procedure for determining \( \alpha_c \) in AS 4100-1990 for steel columns from an appropriate column curve defined by \( \alpha_b = -1.0 \) or -0.5 for bare steel tubes. At the level of the axial capacity \( \chi N_{u0} \), there is a corresponding value of moment \( \mu_k M_{u0} \) (see Fig. 6.32) which can be considered as the moment arising from the imperfection of the column.

The strength of a member in combined bending and compression can be calculated by using an interaction diagram such as shown in Fig.6.32. In this case the reduction factor due to imperfections can be decreased linearly with axial force to (O'Shea and Bridge, 1994)

\[ \chi_n = \chi (\beta + 1)/4 \]  \hspace{1cm} (6.2)

where \( \beta \) is the ratio of the smaller to the larger end moments, and is positive when member is bent in double curvature.

Fig.6.32 Interaction Curve for Cross-section Strength (O'Shea and Bridge, 1994)
6.4.2 ACI-318

The design procedure is based on the following equation (use of a moment magnifier $\delta$ for second-order effects)

$$M^* = \delta M_0^*$$

(6.3)

where

$$\delta = C_m / (1 - N^*/N_{cr})$$

(6.4)

and

$$N_{cr} = \pi^2 EI/l^2$$

(6.5)

$$C_m = 0.6 - 0.4\beta \geq 0.4$$

(6.6)

where $M_0^*$ is the maximum moment from the application of the design loads (first-order elastic analysis) and $M^*$ is the maximum moment including second order effects and $l$ is the effective length of the column and $EI$ is the effective rigidity. The cross-section strength in combined bending and axial force can be determined using a rectangular stress block of $0.85f_c$ for the concrete acting over a depth less than the neutral axis depth by a factor $\beta_1 (= \nu$ in AS 3600-1988). The strength interaction is the locus of $N_m$, $M_m$ ($m$ referring to the maximum load of an eccentrically loaded column) of values as shown in Fig. 6.33.

6.4.3 Reduction Coefficient Formula

A strength reduction formulae approach is based on the assumption that the maximum load of an eccentrically loaded column, $N_m$, can be calculated from the stub column strength by using reduction coefficients, which attempt to allow for the decrease in strength due to the slenderness of the column and the eccentricity of loading.
Such a formulae can be written in the form

\[ N_m = \alpha_c \alpha_d N_{\text{uo}} \]  \hspace{1cm} (6.7)

where

- \( \alpha_c \) = the slenderness reduction coefficient
- \( \alpha_d \) = the eccentricity reduction coefficient
- \( N_{\text{uo}} \) = the stub column strength

For a uniaxial analysis \( N_{\text{uo}} \) is assumed to be the squash load for the section. \( \alpha_c \), the reduction factor due to slenderness, can be obtained from the appropriate column curve in a code of practice, e.g. AS 4100-1990.

Based on the relationship between the yield stress and the average stress causing first yield in a homogeneous section (Robertson, 1925 and Neogi, 1967) \( \alpha_d \), a function of eccentricity, may be calculated as follows,

\[ \alpha_d = \frac{1}{1 + \gamma} \]  \hspace{1cm} (6.8)

where \( \gamma = c \frac{e}{d} \), \( e \) is the eccentricity, \( d \) is the external diameter, and \( c \) is a coefficient which is suggested by Neogi (1967) to be taken as 5 for the composite tubular columns.

Equation 6.7 can be rewritten as:

\[ N_m = \frac{1}{1 + c \frac{e}{d}} \alpha_c N_{\text{uo}} \]  \hspace{1cm} (6.9)
\( \alpha_c \) can be calculated from Australian Standard 4100 as

\[
\alpha_c = \xi \left\{ 1 - \left[ 1 - \left( \frac{90}{\xi \lambda} \right)^2 \right]^{1/2} \right\}
\]  

(6.10)

where \( \lambda \) (modified compression member slenderness) is equal to \( \lambda_n + \alpha_\theta \alpha_a \), \( \alpha_a \) is a compression member slenderness modifier which can be taken as -0.5 for bare steel tubes, \( \alpha_\theta \) is a compression member cross-section parameter, \( \lambda_n \) is the nominal slenderness and \( \xi \) is a coefficient which depends on the imperfection parameter (\( \eta \)) and \( \lambda \). They can be determined from,

\[
\alpha_a = \frac{2100(\lambda_n - 13.5)}{\lambda_n^2 - 15.3\lambda_a + 2050}
\]  

(6.11)

\[
\xi = \frac{(\lambda / 90)^2 + 1 + \eta}{2(\lambda / 90)^2}
\]  

(6.12)

and the imperfection parameter \( \eta \) is

\[
\eta = 0.00326(\lambda - 13.5) \geq 0
\]  

(6.13)

### 6.4.4 Interaction formula

The basis of the design procedure is the determination of the cross-section strength which is usually expressed in terms of an interaction diagram (Fig.6.33). There are various possible shapes of the interaction diagram. Formulae for steel columns can be represented by curves of the type A, B or C (Trahair and Bradford, 1984). For a short concrete column the shape should be similar to curve D because the maximum moment-
carrying capacity of the section is developed when there is an axial load, which reduces cracking of the concrete.

The most commonly used interaction formulae is the straight line formulae given by

$$\frac{N_m}{N_{uo}} + \frac{M_m}{M_{uo}} = 1$$  \hspace{1cm} (6.14)

where $M_{uo}$ is bending moment with no axial load and $M_m$ is bending moment which exists with axial load ($N_m$).

![Diagram showing interaction curve for cross-section strength with labeled regions A, B, C, and D, and the typical curve for a short concrete column.]

Fig. 6.33 Interaction Curve for Cross-section Strength
6.5 PROPOSED ULTIMATE LOAD FORMULAE

6.5.1 Reduction Coefficient Formula

The reduction coefficient is proposed to be used to the calculate ultimate load capacity of composite curved steel struts infilled with high strength concrete. It is noted that the ultimate load of such struts largely depends on the initial deflection at mid-height which can be related to the straight length and the initial radius of curvature. The formulae have been based on following equation (Neogi, 1967)

\[
\alpha_d = \frac{1}{1 + 5 \frac{e}{d}}
\]

(6.15)

The results of \( \alpha_d N_{uo} \) (ignoring the slenderness effect), \( \alpha_d \alpha_c N_{uo} \), and experimental results, which were presented in Chapter 4, are given in tables 6.6 and 6.7. It can be seen that there is good agreement between the results of \( \alpha_d N_{uo} \) and the experimental results for struts with 2000 mm initial radius of curvature and a straight length between 1157 mm and 3080mm. However, such agreement for struts with 4000 mm initial radius of curvature and straight length between 1176 and 1755, and for struts with 10000 mm initial radius of curvature and straight length 1141 mm is good, but not for struts with a straight length of 2255-3114. This method takes into account the initial deflection at mid-height for calculation of the ultimate load capacity, therefore the method gives more accurate result for a particular straight length (say, 1500 mm) for a strut with 2000 mm initial radius of curvature in comparison with struts with 4000 mm and 10000 mm initial radii of curvatures but with identical straight lengths. As well, there is good agreement nearly in most cases for seamless struts (R=2000 mm and 4000 mm).
6.5.2 Interaction Formula

A straight line interaction formula, based on equation 6.14, has been used. In the formula \( N_{uo} \) is multiplied by the slenderness reduction coefficient \( \alpha_c \). In this case \( M_{uo} \) is calculated by assuming a rectangular stress block for the concrete, and the values of the steel and concrete stresses are taken as \( f_y \) and \( f_c \), respectively. The results are compared with results obtained from other methods in Tables 6.6 and 6.7.

6.6 DISCUSSION on the PROPOSED FORMULAE

None of the two formulae (Eqs 6.14 and 6.15) is entirely satisfactory. However, by referring to Tables 6.6 and 6.7 the interaction formulae probably gives more accurate results in most cases. The proposed methods are easy to use, but suffer from some disadvantages such as the slenderness problem in the calculation of \( N_{m1} \), or \( N_{m2} \) which is very conservative in comparison with the test results. The value of \( C=5.0 \) probably is suitable for composite curved steel struts, and \( C=2.5 \) for hollow curved steel struts.

Considering Table 6.6 the range of the ratio of \( \eta_4 \left( \frac{P_{test}}{\alpha_d N_{uo}} \right) \) is between 0.56 and 1.47 for ERW steel struts, the arithmetic mean is 0.98 and the standard deviation is 0.26. The range of the ratio of \( \eta_5 \left( \frac{P_{test}}{\alpha_d \alpha_c N_{uo}} \right) \) is between 1.16 and 4.27 for ERW steel struts, the arithmetic mean is 1.84 and the standard deviation is 0.84. The range of the ratio of \( \eta_6 \) is between 0.76 and 1.56 for ERW steel struts, the arithmetic mean is 1.09 and the standard deviation is 0.22.

In general, by referring to Table 6.6, it can be seen that the interaction formula gives more accurate results in comparison with other formulae. The arithmetic mean of the ratio \( \eta_6 \left( \frac{P_{test}}{N_m} = \text{obtained from Eq.6.14} \right) \) is 1.09, and also the standard deviation of \( \eta_6 \), is the smallest value for this range of the calculations. The arithmetic mean of the \( \eta_5 \)
(using the reduction coefficient which is obtained from both eccentricity and slenderness problems) is very conservative (1.89 for ERW struts) in comparison with other formulae.

Considering Table 6.7 the range of the ratio of $\eta_5$ is between 0.97 and 1.56 for seamless steel struts, the arithmetic mean is 1.16 and the standard deviation is 0.18. The range of the ratio of $\eta_5$ is between 1.34 and 2.64 for seamless steel struts, the arithmetic mean is 1.79 and the standard deviation is 0.44. The range of the ratio of $\eta_6$ is between 0.9 and 1.42 for seamless steel struts, the arithmetic mean is 1.05 and the standard deviation is 0.17.

Table 6.7 shows that the reduction formula gives results $\eta_4$ (obtained from only the eccentricity problem) which are, in general, more conservative and more accurate in comparison with other formulae for seamless struts. In this case it seems that the influence of the initial deflection at mid-height in the calculation of the ultimate load for the seamless struts is more significant than is the case with the ERW struts. The reason for this can be due to the differences of the fabrication process and material characteristics of the seamless struts in comparison with the ERW struts.
Table 6.6 Comparison of Maximum Loads Calculated from Design Formulae with Loads from Experiments (ERW struts)

<table>
<thead>
<tr>
<th>L (mm)</th>
<th>R (mm)</th>
<th>( \alpha_d )</th>
<th>( \alpha_c )</th>
<th>( N_{m1} ) (kN)</th>
<th>( N_{m2} ) (kN)</th>
<th>( N_{m3} ) (kN)</th>
<th>( \eta_4 )</th>
<th>( \eta_5 )</th>
<th>( \eta_6 )</th>
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<td>0.92</td>
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\[ N_{m1} = \alpha_d N_{uo} \quad N_{m2} = \alpha_d \alpha_c N_{uo} \quad N_{m3} = \text{Interaction formula (straight line)} \]

\[ \eta_4 = \frac{P_{Test}}{N_{m1}} \quad \eta_5 = \frac{P_{Test}}{N_{m2}} \quad \eta_6 = \frac{P_{Test}}{N_{m3}} \]
Table 6.7 Comparison of Maximum Loads Calculated from Design Formulae with Loads from Experiments (seamless struts)

<table>
<thead>
<tr>
<th>L (mm)</th>
<th>R (mm)</th>
<th>α_d</th>
<th>α_c</th>
<th>N_{m1} (kN)</th>
<th>N_{m2} (kN)</th>
<th>N_{m3} (kN)</th>
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7. CHAPTER SEVEN

CONCLUSIONS AND RECOMMENDATIONS

7.1 CONCLUSIONS

Based on the experimental and theoretical results outlined in previous Chapters, the following conclusions are drawn.

(a) A relatively simple method to calculate ultimate load capacity of curved steel struts infilled with higher strength concrete is presented. The method uses the intersection point of elastic and plastic curves. An effective flexural rigidity was assumed to obtain the elastic curve. The plastic curve was calculated based on an iteration process which is described in Chapter 3. The steel cross section in each iteration was divided into two strips about the neutral axis. In order to determine the inelastic strength it was assumed that the area of steel in tension and in compression are lumped at their centroids.

Elastic and inelastic curves have been obtained for different initial radii of curvature, initial deflection and slenderness. In the analysis and in order to allow for the contribution of the elastic deflection to the plastic deflection for a particular load the elastic deflection was added to the plastic collapse curve. There is good agreement between such curves and the experimental curves.

In general, the intersection of the elastic and plastic curves gives sufficiently accurate results for the determination of the ultimate load capacity of pin-ended composite curved steel struts. The arithmetic mean of test/predicted results for Electric Resistance Welded (ERW) struts is 1.01 and the standard deviation is 0.076. The arithmetic mean for seamless struts is 1.05 and the standard deviation is 0.063.
The theoretical ultimate load capacity of composite curved struts subjected to compressive load having an L/d ratio greater than 12 is in very good agreement with the results of experimental results. For shorter struts the test failure loads are higher than the calculated loads. It can be due to the effects of triaxial stresses which are more significant in this case. The main parameters which could influence the intersection point may be the initial deflection at mid-height as well as slenderness, but the effects of d/t (diameter/thickness), $f'_c$ and $f_y$ are also considerable.

(b) A nonlinear finite element model using the Nastran package for tracing the elastic and inelastic load-deformation path of curved steel struts infilled with higher strength has been developed. The accuracy of the results of the FE analysis of the composite curved steel struts largely depends on proper and careful FE modelling, and accuracy in simulating the properties of the steel-concrete interface.

The analysis accounts simultaneously for both the geometrical and material nonlinearities. In particular, the model takes into account a generalised stress-strain relationship and the influence of strain hardening. To take into account the effects of residual stresses and the Bauschinger effect different stress-strain relationships of the material are assumed; a bilinear curve for the as-received stub columns and a quadrilinear curve for the prestrained in tension stub columns. The concrete and steel tubes are modelled using solid elements, and the interface between the steel and concrete is modelled using gap elements.

A solution method, the so-called arc length method (Crisfield), has been used in analysing the struts. The most difficult aspect of solving the composite curved steel struts was selecting the convergence criteria. It was concluded that using different convergence criteria based on out-of-balance forces, and displacement before and after the limit points, were the most practical solution.

The model has been used to trace out the inelastic load-deflection response of pin-ended ERW struts with varying initial radius of curvature. All load-deflection curves obtained from finite element analysis displayed the same characteristic as the experimental
curves. In general, there is good agreement between the finite element, elastic and plastic curves and experimental curves. The arithmetic mean of test/predicted maximum load is 1.01 and the standard deviation is 0.041.

(c) In order to determine the influence of high strength concrete, the initial radius of curvature, residual stresses including the Bauschinger effect, and the initial deflection at mid-height on the load capacity, 78 composite curved steel struts were tested. Two different types of manufactured steel struts, ERW struts and seamless, were used. Load-lateral deflection curves as well as load-end shortening curves were recorded. Longitudinal and circumferential strains at mid-height were measured during loading for some struts.

A total of 40 as-received ERW composite curved steel struts with an average strain hardening ratio (SHR) of 1.05 were tested. Three different initial radii of curvatures, R=2000 mm, R= 4000mm and 10000mm were adopted to curve the steel struts. The straight length (L) varied between 700 mm and 3200 mm.

A total of 27 as-received seamless composite curved steel struts with a higher strain hardening ratio (SHR=1.33) were tested in order to determine the influence of strain hardening on the ultimate load capacity, and the general failure of composite curved struts. Two different initial radii of curvatures, 2000 mm and 4000 mm, were used for curving the struts. The straight length varied between 700 mm and 2250 mm.

The influence of the initial radius of the curvature and initial deflection at mid-height is significant on the ultimate load capacity of both ERW and seamless composite curved struts. With decreasing initial deflection at mid-height (increasing initial radius of curvature) the ultimate load greatly increases for an identical straight length, i.e. the ratio of the peak load of the ERW strut with 2000 mm initial radius of curvature (straight length (L) = 775 mm and initial deflection at mid-height (e) = 30 mm) over that of the ERW strut with 4000 mm initial radius of curvature (L=765 mm and e= 20 mm) is about 79 %. This ratio for seamless struts with an identical geometry is about 80%.
The ultimate load capacity behaviour of the struts with the same initial radius of curvature largely depends on the straight length (slenderness ratio) and initial deflection at mid-height. The ultimate load decreases with increasing slenderness ratio, i.e. the ratio, of the peak load of the ERW strut with 2000 mm initial radius of curvature and \( L = 775 \) mm over to the strut having \( L = 2290 \) mm is about 7.1%. This ratio for the seamless strut with identical geometry is about 8.6%.

To investigate the influence of the residual stresses and Bauschinger effect developed during the curving process on the curved steel struts, tests on stress-relief annealed tubes (SRA) were performed on 4 ERW struts with 10000 mm initial radius of curvature, and 7 seamless struts with 2000 mm and 4000 mm initial radii of curvature.

Almost in all cases, as a result of stress-relief annealing process, the ultimate load capacity decreased and the post buckling behaviour improved. The average of reduction was about 8% compared with the as-received ERW struts. The reduction in the ultimate load capacities of the stress-relief-annealed curved struts was due to the drop in the yield stress after the annealing process, although the curved struts were then free of residual stresses induced during the curved tube manufacturing process. As the absolute value of the ultimate load capacities of the as-received composite curved strut results (i.e. FM3 and FM4 (SRA) in Fig.5.10a) were slightly above the corresponding stress-relief-annealed results (free of residual stresses), the Bauschinger effect is not significant for the curved struts with 2000, 4000 and 10000 mm initial radii of curvatures. A yardstick by which to measure the influence of the Bauschinger effect was to compare the ultimate normalised load capacity (load / squash load) of the composite curved struts in both the stress-relief-annealed and as-received conditions. The normalised as-received composite curved strut results also lie above the corresponding stress-relief-annealed struts (i.e. FM3 and FM4 (SRA) in Fig.5.10b).

Experimental work was carried out on 11 ERW hollow curved struts in order to determine the influence of higher strength concrete on the ultimate load capacity of curved struts. As concrete has significant strength in compression the influence of initial deflection at mid-height (eccentricity) on the percentage of the increase of the strength is
clearly seen. The percentage increase in strength for the composite strut with 10000 mm initial radius of curvature and a slenderness ratio of 39 is about 80% with respect to the hollow strut having an identical geometry. The percentage increase in strength for struts with 2000mm and 4000mm initial radii of curvature and slenderness ratio of 87 is 10%. In general, the percentage increase in strength varied between 10% and 80% for the this range of the ERW strut tests. The initial slopes (initial stiffnesses) of the load-deflection curves of the hollow curved struts having slenderness ratio larger than 60 increased due to the infill concrete.

The post-peak load capacity of composite curved struts may be defined as the capacity of the members to sustain the additional bending stresses as the members pass the ultimate load. In this study for the load-end shortening curves, the ductility factor was calculated as the ratio of the deflection at 0.75% of the ultimate load over to the deflection at the ultimate load (Eq. 5.1).

The change of the ductility factor is significant for the ERW specimens with 2000 mm initial radius of curvature with increasing slenderness ratio (varied between 3.5 and 6.5). However, it is not significant for the struts with 4000 mm and 10000 mm initial radii of curvature. In comparison with hollow curved struts the ductility factor increases significantly. The factor is 1.6 for the NE1 (R=10000 mm, L=780 mm and composite) and 2.8 for NH1 (R=10000 mm, L=770 mm and hollow). The increase of the ductility factor can be attributed to the flat plateau of the peak load and also the post peak load of the specimen.

The post peak behaviour for the as-received curved seamless steel struts filled with concrete was improved by the stress relief anneal processing. For example, the ratio for the seamless specimens TM6 and TM7 (SRA) increase from 4.8 to 6.5. The ductility factor depends on the different factors such as material properties (hollow against composite, and as-received against stress-relief annealed) and also on the spread of the critical stresses and strains which can be affected by the changing initial deflection in the curved struts.
Strains on the tension and compression sides of some of the ERW and seamless composite curved struts were recorded. The strain in the convex face of the ERW long specimens (L>1500mm) was observed in excess of 19000 microstrain (i.e. TE7, R=2000 mm and L=1745 mm). This large value can be related to the significant bending stresses which were carried by the strut. The steel tube of such specimens yielded first in the longitudinal direction in the tension side. The strains of some of the seamless composite curved struts were also recorded. In short specimens i.e. FM7 (R=4000 mm and L=1120 mm) the steel tube yielded first in the longitudinal direction on the compression side due to the small slenderness and the initial deflection at mid-height. At the peak load the strain on the compression side was about 4000 microstrain and on the tension side was about 2100 microstrain.

The load-curvature relationships were obtained from the recorded strains for both ERW and seamless composite curved struts. The ductility factor for such curves was calculated as the curvature at 0.95% of the ultimate load over to the curvature at the ultimate load because of an almost flat plateau of the post peak load curves. The influence of initial radius of the curvature and slenderness ratio on the ductility factor is clearly seen. With decreasing initial radius of curvature and increasing slenderness ratio the ductility factor usually improved. For example, the factor for the ERW specimen FE5 (R=4000 mm and L=1540 mm) is 2.08 and for ERW specimen NE 13 (R=10000 mm and L=1515 mm) is 1.7. The ductility factor for seamless specimen TM9 (R=2000 mm and L=1125 mm) is 1.87 and for seamless specimen TM14 (R=2000 mm and L=1685 mm) is 2.9.

During the falling branch of the load-curvature relationship of the struts with a large initial curvature and a long straight length (i.e. R=2000 mm and L>1500 mm), the increase in curvature was not concentrated at the central section. Therefore, the area which reached the yield condition for longer specimens was larger than for the short specimens.

As the cross-section areas of ERW and seamless struts are different, so a comparison in a normalised format is more reasonable. In order to make a comparison between ERW
and seamless struts the normalised load-curvature curves were used with respect to the corresponding squash load ($N_{uo}$), initial radius of curvature and nearly identical straight length (load / squash load versus curvature / initial curvature). In general, the normalised load deflection curves of the seamless struts lie above the ERW strut curves. It can be concluded that seamless struts can carry a larger load in comparison with the ERW struts. The ductility factor of the TM14 ($\mu=2.9$) is also larger than the TE7 ($\mu=2.4$).

The overall shape of the ERW hollow and composite curved steel struts after failure was different. The unfilled specimens had finally buckled locally in an inward direction. The mechanism of failure for the unfilled struts occurred as local buckling of the inside region or compression side and a sharp angle (kink) in the outside region or the tension side. The concrete filled specimens had a slightly bent form after failure.

In general, the ERW composite curved specimens having a slenderness ratio less than 100 ($L<2000$ mm) failed as a result of local buckling on the concave face and fracture on the convex face after the peak load reached. Necking and fracture are the consequence of the small strain hardening ratio ($SHR_{ERW}=1.05$) which did not allow a significant spread of plasticity. The fracture within the neck initiates from localised plasticity. Some longer struts underwent large lateral deflection beyond the peak load and failed due to fracture on the tension side due to large tensile stresses; no local buckling occurred on the compression face. No local buckling nor fracture were observed on the longest struts with 2000 mm and 4000 mm initial radii of curvatures and 3000 mm straight length. Stress relief annealed ERW composite curved specimens deformed laterally with significant mid-span deflections, but with no necking and fracture, and only slightly visible local buckling at failure.

Necking and fracture did not occur on the seamless composite curved struts (furnishing a high strain hardening ratio $SHR_{seamless}=1.33$) beyond the peak load due to different material characteristics. Leudcr's bands were observed after the peak load had been reached for struts with $L/r < 80$ on the concave side. No sign of Leudcr's band was observed on the remainder of the longer struts, and also the stress relief annealed seamless curved composite struts.
(d) Different approximate design methods, based on codes of practice such as AS 3600, Eurocode 4 and literature, are proposed to calculate ultimate load capacity of the composite curved steel struts. The interaction formulae developed herein, which are explained in Chapter 6, give better results than the reduction coefficient formulae in some cases. The value of the reduction coefficient \( \alpha_d \) (Eq. 6.7) for curved struts depends significantly on the values of the \( e/d \) and \( L/d \). It is clearly seen that the influence of the initial deflection at mid-height and slenderness ratio on the maximum loads calculated from approximate design methods is highly significant.

### 7.2 Further Work

The theoretical and experimental work completed has been confined to pin-ended composite curved steel struts with 2000mm, 4000mm and 10000 mm initial radii of curvatures.

As modern structures use arches in framed systems, the theoretical analysis should be extended to include end restraints and eccentricity. It should be also extended to curved tubular struts subjected to lateral loads as the biaxial bending loads can be also important in curved tubular frames. The design of structural joints in composite curved struts need study as the differences in strut behaviour when the steel or concrete alone is loaded directly, or the two are loaded together, should also be investigated. In tube framing structures the connections provide end restraints and the buckling of the curved strut should be delayed by such end restraints. The local strength of the connection and the transfer of load from the strut to the strut should be studied.

As ductility is an important problem in high strength concrete a limitation on the compressive capacity of the concrete exists in the codes of practice for the design of composite steel struts. The study of such problem can be useful where high strength concrete is used in the core of the composite struts considered herein.
More tests are needed on curved struts with a L/d ratio between 5 and 12. Tests are needed on the composite curved steel struts with rectangular cross-sections. Tests can be carried out on the composite curved steel struts with initial radii of curvatures between 2000mm and 4000mm, and between 4000mm and 10000mm in order to investigate the influence of initial deflection in these ranges of the initial radii of curvatures.
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APPENDICES

APPENDIX I COMPUTER PROGRAM

The following calculation is part of a Fortran program that was used to determine the ultimate load capacity of a curved composite strut based on the assumption that the steel and concrete areas are lumped at their centroids. The program here calculates the steel and concrete areas.

14 k=l
   Do 12 l=1,k,l
   Rc=(d-2.0*t)/2.0
   R=d/2.0
   cu=0.003/(dn-t)
   def=cu*l**2/(9.8696)
   q=dn-t-Rc
   if (q-0)5,10,10
5 vl=sqrt (Rc**2.0-q**2.0)
   zl=Asin(q/Rc)
   Ac=(3.14*Rc**2)/2.0+zl*Rc**2+vl*q
   dc1=(3.14*Rc**2)/2.0*((4*Rc)/(3*3.14)+q)
   dc2=(zl*Rc**2*q-((0.6666667)*Rc*(((sin(zl/2))**2)/(zl/2))))
   dc3=(q**2*vl/3)
   dcs=dc1+dc2+dc3
   dc=dcs/Ac
   v2=sqrt (R**2.0-q**2.0)
   z2=Asin(q/R)
   dsc1=(3.14*R**2)/2.0*((4*R)/(3*3.14)+q)
   dsc2=(z2*R**2*(q-((0.6666667)*R*(((sin(z2/2))**2)/(z2/2))))
   dsc3=(q**2*v2/3.0)
\[\text{dsc} = (dsc_1 + dsc_2 + dsc_3 - dcs)/(Asc - Ac)\]
\[\text{Asc} = Asc - Ac\]
\[\text{dst} = 2.0*(R^3)/3.0 - dsc_3 - 2.0\]
\[\text{dst} = (z_2*R^3*(0.666667)*(\sin(z_2/2.0)^2)/(z_2/2.0))\]
\[\text{dst} = \text{dst}_1 - \text{dst}_2\]
\[\text{Ast} = (3.14*R^2)/2.0 - Z_2*R^2 - q*v_2\]
\[\text{dct} = 2.0*(Rc^3)/3.0 - dc_3 - 2.0\]
\[\text{dct} = (z_1*Rc^3*(0.666667)*(\sin(z_1/2.0)^2)/(z_1/2.0))\]
\[\text{dct} = \text{dct}_1 - \text{dct}_2\]
\[\text{Asct} = ((3.14*Rc^2)/2.0 - z_1*Rc^2 - q*v_1)\]
\[\text{Ast} = \text{Ast}_1 - \text{Asct}\]
\[\text{.dst} = \text{dst}_3 - \text{dct}_3\]
\[\text{eps} = 0.003 * \text{dn}/(\text{dn} - t)\]
\[\text{eps} = \text{eps}_1 * dc/dn\]
\[\text{zc} = dc + (d/2.0 - dn)\]
\[\text{zsc} = dsc + (d/2.0 - dn)\]
\[\text{zst} = dst - (d/2.0 - dn)\]

Go to 15

5 \[q = Rc + t - dn\]
\[v_1 = \sqrt{(Rc^2 - q^2)}\]
\[z_1 = \sin(q/Rc)\]
\[dcdc_1 = 2.0*(Rc^3)/3.0\]
\[dcdc_2 = (z_1*Rc^3*(0.666667)*(\sin(z_1/2.0)^2)/(z_1/2.0))\]
\[dcdc_3 = 2.0*q^2*v_1/3\]
\[\text{Ac} = (3.14*Rc^2)/2.0 - z_1*Rc^2 - v_1*q\]
\[v_2 = \sqrt{(R^2 - q^2)}\]
\[z_2 = \sin(q/R)\]
\[\text{Asc} = (3.14*R^2)/2.0 - z_2*R^2 - q*v_2\]
\[dsc_1 = 2.0*(R^3)/3.0\]
\[dsc_2 = (z_2*R^3*(0.6666667)*(\sin(z_2/2.0)^2)/(z_2/2.0))\]
Appendices

\[ \text{ddsc}_3 = (q^{**2}v_2/3.0) \]
\[ \text{ddsc} = \text{ddsc}_1 - \text{ddsc}_2 - \text{ddsc}_3 \]
\[ \text{ddcc}_1 = 2*(Rc^{**3})/3.0 \]
\[ \text{ddcc}_2 = (z_1*Rc^{**3}(0.666667)*(\sin(z_1/2.0)^{**2})/(z_1/2.0)) \]
\[ \text{ddcc}_3 = (q^{**2}v_1/3.0) \]
\[ \text{ddcc} = \text{ddcc}_1 - \text{ddcc}_2 - \text{ddcc}_3 \]
\[ \text{Asc}_2 = \text{Asc} - \text{Ac}_2 \]
\[ \text{dsc}_2 = (\text{ddsc} - \text{ddcc})/\text{Asc}_2 - q \]
\[ \text{ddst}_1 = (3.14*R^{**2})/2.0 + (4*R)/(3*3.14) + q \]
\[ \text{ddst}_2 = (z_2*R^{**2} - q - (0.66666667)*R*(((\sin(z_2/2))^{**2})/(z_2/2))) \]
\[ \text{ddst}_3 = (q^{**2}v_2/3.0) \]
\[ \text{Asst} = (3.14*R^{**2})/2.0 + z_2*R^{**2} + q*v_2 \]
\[ \text{ddct}_1 = (3.14*Rc^{**2})/2.0 + (4*Rc)/(3*3.14) + q \]
\[ \text{ddct}_2 = (z_1*Rc^{**2} - q - (0.66666667)*R*(((\sin(z_1/2))^{**2})/(z_1/2))) \]
\[ \text{ddct}_3 = (q^{**2}v_1/3.0) \]
\[ \text{Asct} = (3.14*Rc^{**2})/2.0 + z_1*Rc^{**2} + q*v_1 \]
\[ \text{Ast}_2 = \text{Asst} - \text{Asct} \]
\[ \text{ddst} = ((\text{ddst}_1 + \text{ddst}_2 + \text{ddst}_3) - (\text{ddct}_1 + \text{ddct}_2 + \text{ddct}_3))/\text{Ast}_2 \]

12 continue
15 dn = dn + 0.1

go to 14

Dimension a(40),b(40),c(40),aa(40),bb(40),cc(40),aaa(40),bbb(40)
1 ,ccc(40),an1(40),an2(40),eps1(40),sigc(40),sigst(40),z(40),
1 fm1(40),fm2(40),fm3(40),p1(40),p2(40),p3(40),dv(40),
1 f(40),y(40),x(40)

do 1 I = 1,40
a(I) = 0
b(I) = 0
c(I) = 0
aa(I)=0
bb(I)=0
cc(I)=0
aaa(I)=0
bbb(I)=0
ccc(I)=0
an1(I)=0
an2(I)=0
eps1(I)=0
sigc(I)=0
sigst(I)=0
z(I)=0
p1(I)=0
p2(I)=0
p3(I)=0
fm1(I)=0
fm2(I)=0
fm3(I)=0
x(I)=0
f(I)=0
y(I)=0
continue

call neul(a,b,c,aa,bb,cc,aaa,bbb,ccc,an1,an2,eps,eps1,sigc,sigst,z,p1,p2,p3,fm1,fm2,fm3,fmtot,ptot)

1 h=x/9.0
2 dr=eps/dn
3 do 2 I=1,9
4 write (*,3)dr
5 format (2x,'dr=',f20.9)
6 if (I-1)6,6,7
   dv(I)=1/2*h**2*r0+w0
7 x(I)=x(I)+h
f(I)=sqrt(r1**2-x(I)**2)-sqrt(r1**2-(xl**2))
y(I)=yO-((f0-f(I))+w0-dv(I))
go to 8

7  II=I+1
III=I-1
III=I-2
x(I)=x(III)+h
if (I-2) 10,10,11

dv(I)=1/2*h**2*dr+2*dv(III)-w0
go to 12

dv(I)=h**2*dr+2*dv(III)-dv(III)

f(I)=sqrt(r1**2-x(I)**2)-sqrt(r1**2-(xl**2))
y(I)=y(I)-(f(I)-f(II)+dv(I))
write (*,5) y(I),f(I)

5  format (2x,'y(I)',f20.9,2x,,f(I)=',f20.9)

8  call neu(a,b,c,aa,bb,cc,aaa,bbb,ccc,dn,Rc,t,R,d,an1,an2,eps,
1   eps1,sigc,sigst,z,p1,p2,p3,fm1,fm2,fm3,fmtot,ptot,dr)
2  continue
if (y(I)-5)13,14,14

13  dn=dn+0.1
go to 15

14  stop
end

subroutine neu(a,b,c,aa,bb,cc,aaa,bbb,ccc,dn,Rc,t,R,d,an1,an2
1   ,eps,eps1,sigc,sigst,z,p1,p2,p3,fm1,fm2,fm3,fmtot,ptot,dr)

dimension a(40),b(40),c(40),aa(40),bb(40),cc(40),aaa(40),bbb(40)
3   ,ccc(40),an1(40),an2(40),eps1(40),sigc(40),sigst(40),z(40)
3   ,p1(40),p2(40),p3(40),fm1(40),fm2(40),fm3(40)

hh=0
hhh=0
h1=0
totp1=0
totp2 = 0
totp3 = 0
totfml = 0
totfm2 = 0
totfm3 = 0
lll = 26
eps = 0.001
do 110 kll = 1, 10
dn = 10

do 111 kjj = 1, 60
if (dn - 30.2) 41, 42, 42

41 do 2 j = 1, lll
   hj = j
   a(j) = ((d)/26.0)*hj
   q = Rc+t-a(j)
   if (a(j) - dn) 6, 7, 7
   vl = sqrt(Rc**2-q**2)
   zl = Asin(q/Rc)
   Ac2 = (3.14*Rc**2)/2.0-zl*Rc**2-vl*q
   b(j) = Ac2-c(j)
nn = j + 1
   c(nn) = Ac2
   hh = hh + b(j)
   v2 = sqrt(R**2-q**2)
   z2 = Asin(q/R)
   Asc = (3.14*R**2)/2.0-z2*R**2-q*v2
   Asc2 = Asc-Ac2
dsc2 = (ddsc-ddcc)/Asc2-q
   bb(j) = Asc2-cc(j)
   ii = j + 1
   cc(ii) = Asc2
   hhh = bb(j) + hhh
an1(nn)=a(j)
an2(j)=((a(j)-an1(j))/2.0+an1(j))
eps1(j)=eps*(dn-an2(j))/dn
if (eps1(j)-0.002)<87,87,88
87 sigc(j)=0.85*71*(2*(eps1(j)/0.002)-(eps1(j)/0.002)**2)
go to 89
89 if (eps1(j)-0.0019)<25,26,26
25 sigst(j)=200000*eps1(j)
go to 27
26 sigst(j)=370
27 z(j)=30.2-((a(j)-an1(j))/2.0+an1(j))
p1(j)=sigc(j)*b(j)+sigst(j)*bb(j)
totp1=p1(j)+totp1
fm1(j)=sigc(j)*b(j)*z(j)+sigst(j)*bb(j)*z(j)
totfm1=fm1(j)+totfm1
go to 2
7 if (a(j)-30.2)<46,35,35
46 v2=sqrt (R**2.0-q**2.0)
z2=Asin(q/R)
v1=sqrt (Rc**2.0-q**2.0)
z1=Asin(q/Rc)
Astt=(3.14*R**2.0)/2.0+z2*R**2+q*v2
Asct=(3.14*Rc**2.0)/2.0+z1*Rc**2+q*v1
Ast2=Astt-Asct
bbb(j)=ccc(j)-Ast2
kl=j+1
ccc(kl)=Ast2
if (bbb(j) - 0)<52,53,53
53 h1=h1+bbb(j)
nn=j+1
an1(nn)=a(j)
an2(j)=((a(j)-an1(j))/2.0+an1(j))
eps1(j)=eps*(an2(j)-dn)/dn
if (eps1(j)-0.0019)>31,32,32
31 sigst(j)=200000*eps1(j)
go to 33
32 sigst(j)=370
33 z(j)=30.2-((a(j)-an1(j))/2.0+an1(j))
p2(j)=sigst(j)*bbb(j)
totp2=p2(j)+totp2
fm2(j)=sigst(j)*bbb(j)*z(j)
totfm2=fm2(j)+totfm2
go to 2
52 nn=j+1
an1(nn)=a(j)
go to 2
35 q=a(j)-t-Rc
v2=sqrt (R**2.0-q**2.0)
z2=Asin(q/R)
v1=sqrt (Rc**2.0-q**2.0)
z1=Asin(q/Rc)
Ast1=(3.14*R**2)/2.0-Z2*R**2-q*v2
Asct=((3.14*Rc**2)/2.0-zl*Rc**2-q*v1)
Ast=Ast1-Asct
bbb(j)=ccc(j)-Ast
kl=j+1
ccc(kl)=Ast
if (bbb(j) .gt. 0) h1=h1+bbb(j)
if (j .eq. ll1-1) then
bbb(j)=Ast
else
ENDIF
if (bbb(j) - 0.0) 2,2,36
36 nn=j+1
an1(nn)=a(j)
an2(j)=((a(j)-an1(j))/2.0+an1(j))
eps1(j)=eps*(an2(j)-dn)/dn
if (eps1(j)-0.0019)37,38,38
37 sigst(j)=200000*eps1(j)
go to 39
38 sigst(j)=370
39 z(j)=((a(j)-an1(j))/2.0+an1(j))-30.2
p2(j)=sigst(j)*bbb(j)
totp3=p2(j)+totp3
fm2(j)=sigst(j)*bbb(j)*z(j)
totfm3=fm2(j)+totfm3
if (j .eq. 25) go to 45
2 CONTINUE
45 fmtot=totfml-totfm2+totfm3
ptot=totp1-totp2-totp3
difm=fmtot-fmn
write (*,117)difm,fmtot,fmn
117 format (2x,'difm=',f20.10,2x,'fmtot=',f20.10,2x,'fmn=',f20.10)
if(difm-1000)116,116,114
116 dr=eps/dn
go to 43
114 dn=dn+5
if (dn-57)111,103,103
111 continue
100 write (*,86)fmtot,dn,ptot
86 format (2x,'fmtot=',f20.9,2x,'dn=',f10.5,2x,'ptot=',f20.9)
103 eps=eps+0.001
if (eps-0.007)110,110,43
110 continue
42 do 60 j=1,l11
   hj=j
a(j)=((d)/26.0)*hj
if(a(j)-30.2)8,9,9
q=Rc+t-a(j)
v1=sqrt (Rc**2-q**2)
z1=Asin(q/Rc)
Ac2= (3.14*Rc**2)/2.0-z1*Rc**2-v1*q
b(j)=Ac2-c(j)
nn=j+1
c(nn)=Ac2
hh=hh+b(j)
v2=sqrt (R**2-q**2)
z2=Asin(q/R)
Asc=(3.14*R**2)/2.0-z2*R**2-q*v2
Asc2=Asc-Ac2
dsc2=(ddsc-ddcc)/Asc2-q
bb(j)=Asc2-c(j)
i=j+1
cc(ii)=Asc2
hhh=bb(j)+hhh
nn=j+1
an1(nn)=a(j)
an2(j)=((a(j)-an1(j))/2.0+an1(j))
eps1(j)=eps*(dn-an2(j))/dn
if (eps1(j)-0.002)93,93,94
93 sigc(j)=0.85*71*(2*(eps1(j)/0.002)-(eps1(j)/0.002)**2)
go to 95
95 if (eps1(j)-0.0019)63,64,64
63 sigst(j)=200000*eps1(j)
go to 65
64 sigst(j)=370
65 z(j)=30.2-((a(j)-an1(j))/2.0+an1(j))
p1(j)=sigc(j)*b(j)+sigst(j)*bb(j)
totpl = p1(j) + totpl
fm1(j) = sigc(j) * b(j) * z(j) + sigst(j) * bb(j) * z(j)
totfm1 = fm1(j) + totfm1

go to 60

9 if (a(j) - dn) < 67, 68, 68

67 q = a(j) - t - Rc
v1 = sqrt (Rc^2.0 - q^2.0)
z1 = A * sin(q/Rc)

Ac = ((3.14 * Rc^2) / 2.0 + z1 * Rc^2 + v1 * q)
b(j) = Ac - c(j)

mm = j + 1
c(mm) = Ac
hh = hh + b(j)
v2 = sqrt (R^2.0 - q^2.0)
z2 = A * sin(q/R)

Asc = ((3.14 * R^2) / 2.0 + z2 * R^2 + q * v2)

Ascc = Asc - Ac

bb(j) = AScc - CC(J)

K = J + 1

CC(K) = AScc

hhh = bb(j) + hhh

nn = nn + 1

an1(nn) = a(j)
an2(j) = ((a(j) - an1(j)) / 2.0 + an1(j))
eps1(j) = eps * (dn - an2(j)) / dn

if (eps1(j) < 0.002) go to 90, 90, 91

90 sigc(j) = 0.85 * 71 * (2 * (eps1(j) / 0.002) - (eps1(j) / 0.002))

go to 92

92 if (eps1(j) < 0.0019) go to 76, 77, 77

76 sigst(j) = 200000 * eps1(j)

go to 78

77 sigst(j) = 370
78 \[ z(j) = \frac{(a(j) - an1(j))}{2.0} + an1(j) - 30.2 \]

79 \[ p1(j) = \text{sigc}(j) \times b(j) + \text{sigst}(j) \times bb(j) \]

80 \[ \text{totp2} = p1(j) + \text{totp2} \]

81 \[ fml(j) = \text{sigc}(j) \times b(j) \times z(j) + \text{sigst}(j) \times bb(j) \times z(j) \]

82 \[ \text{totfm2} = fml(j) + \text{totfm2} \]

68 \[ q = a(j) - t - R_c \]

69 \[ v2 = \sqrt{R**2.0 - q**2.0} \]

70 \[ z2 = \text{Asin}(q/R) \]

71 \[ v1 = \sqrt{R_c**2.0 - q**2.0} \]

72 \[ z1 = \text{Asin}(q/R_c) \]

73 \[ \text{Astl} = \frac{3.14 \times R**2}{2.0} - Z2 \times R**2 - q \times v2 \]

74 \[ \text{Asct} = \frac{(3.14 \times R_c**2)}{2.0} - z1 \times R_c**2 - q \times v1 \]

75 \[ \text{Ast} = \text{Astl} - \text{Asct} \]

76 \[ bbb(j) = ccc(j) - \text{Ast} \]

77 \[ kl = j + 1 \]

78 \[ ccc(kl) = \text{Ast} \]

79 \[ \text{if} \ (j \ .eq. \ III - 1) \ \text{then} \]

80 \[ bbb(j) = \text{Ast} \]

81 \[ \text{else} \]

82 \[ \text{ENDIF} \]

83 \[ \text{if} \ (\text{bbb}(j) - 0.0) \ \text{60,60,70} \]

70 \[ h1 = h1 + bbb(j) \]

80 \[ nn = j + 1 \]

81 \[ an1(nn) = a(j) \]

82 \[ an2(j) = (a(j) - an1(j))/2.0 + an1(j)) \]

83 \[ \text{eps1}(j) = \text{eps} \times (an2(j) - dn)/dn \]

84 \[ \text{if} \ (\text{eps1}(j) - 0.0019) \ \text{72,73,73} \]

72 \[ \text{sigst}(j) = 200000 \times \text{eps1}(j) \]

73 \[ \text{go to} \ 74 \]

73 \[ \text{sigst}(j) = 370 \]

74 \[ z(j) = (a(j) - an1(j))/2.0 + an1(j)) - 30.2 \]
Appendices

\[ p2(j) = \text{sigst}(j) \times \text{bbb}(j) \]
\[ \text{totp3} = p2(j) + \text{totp3} \]
\[ \text{fm2}(j) = \text{sigst}(j) \times \text{bbb}(j) \times z(j) \]
\[ \text{totfm3} = \text{fm2}(j) + \text{totfm3} \]

if (j .eq. lll - 1) go to 81

60  continue

81  \text{fmtot} = \text{totfm1} - \text{totfm2} + \text{totfm3}
\[ \text{ptot} = \text{totp1} + \text{totp2} - \text{totp3} \]
\[ \text{write} (*) , 85 \text{fmtot}, dn, \text{ptot}, \text{eps} \]

85  \text{format} (2x,'fmtot=',f20.9,'dn1=',f10.5,2x,'ptot=',f20.9,1,'eps=',f20.9)

43  return

end

subroutine neu1(a,b,c,aa,bb,cc,aaa,bbb,ccc,dn,Rc,t,R,d,an1,an2,1,eps,eps1,sigc,sigst,z,p1,p2,p3,fm1,fm2,fm3,fmtot,ptot)

dimension a(40),b(40),c(40),aa(40),bb(40),cc(40),aaa(40),bbb(40),ccc(40),an1(40),an2(40)
\[ \text{hh} = 0 \]
\[ \text{hhh} = 0 \]
\[ \text{hl} = 0 \]
\[ \text{totp1} = 0 \]
\[ \text{totp2} = 0 \]
\[ \text{totp3} = 0 \]
\[ \text{totfm1} = 0 \]
\[ \text{totfm2} = 0 \]
\[ \text{totfm3} = 0 \]
\[ \text{l11} = 26 \]
\[ \text{if}(dn-30.2) \text{41}, \text{42}, \text{42} \]

41  \text{do} 2 \text{j}=1,l11
\[ \text{hj} = \text{j} \]
\[ \text{a}(j) = ((d)/26.0) \times \text{hj} \]


\[ q = Rc + t - a(j) \]

\[
\text{if } (a(j) - dn) > 6,7,7
\]

\[ v1 = \sqrt{(Rc^2 - q^2)} \]

\[ z1 = \text{Asin}(q/Rc) \]

\[ Ac2 = (3.14 * Rc^2) / 2.0 - z1 * Rc^2 - v1 * q \]

\[ b(j) = Ac2 - c(j) \]

\[ nn = j + 1 \]

\[ c(nn) = Ac2 \]

\[ hh = hh + b(j) \]

\[ v2 = \sqrt{(R^2 - q^2)} \]

\[ z2 = \text{Asin}(q/R) \]

\[ Asc = (3.14 * R^2) / 2.0 - z2 * R^2 - q * v2 \]

\[ Asc2 = Asc - Ac2 \]

\[ dsc2 = (ddsc - ddcc) / Asc2 - q \]

\[ bb(j) = Asc2 - cc(j) \]

\[ ii = j + 1 \]

\[ cc(ii) = Asc2 \]

\[ hhh = bb(j) + hhh \]

\[ an1(nn) = a(j) \]

\[
\text{an2(j) = ((a(j) - an1(j))/2.0 + an1(j))} \]

\[ eps1(j) = \text{eps} * (dn - an2(j))/dn \]

\[
\text{if } (eps1(j) - 0.002) > 87,87,88
\]

\[ \text{sigc(j) = } 0.85 * 71 * (2 * (eps1(j)/0.002) - (eps1(j)/0.002)^2) \]

\[
\text{go to 89}
\]

\[ \text{if } (eps1(j) - 0.0019) > 25,26,26
\]

\[ \text{sigst(j) = } 200000 * \text{eps1(j)} \]

\[
\text{go to 27}
\]

\[ \text{sigst(j) = 370} \]

\[ z(j) = 30.2 - ((a(j) - an1(j))/2.0 + an1(j)) \]

\[ p1(j) = \text{sigc(j)*b(j)+sigst(j)*bb(j)} \]

\[ \text{totp1 = p1(j) + totp1} \]

\[ \text{fm1(j) = sigc(j)*b(j)*z(j)+sigst(j)*bb(j)*z(j)} \]

totfml = fm1(j) + totfml

go to 2

7 if (a(j) - 30.2) > 46, 35, 35

46 \[ v2 = \sqrt{(R**2.0 - q**2.0)} \]
\[ z2 = \text{Asin}(q/R) \]
\[ v1 = \sqrt{(Rc**2 - q**2)} \]
\[ z1 = \text{Asin}(q/Rc) \]
\[ \text{Astt} = (3.14*R**2)/2.0 + z2*R**2 + q*v2 \]
\[ \text{Asct} = (3.14*Rc**2)/2.0 + z1*Rc**2 + q*v1 \]
\[ \text{Ast2} = \text{Astt} - \text{Asct} \]
\[ bbb(j) = ccc(j) - \text{Ast2} \]
\[ kl = j + 1 \]
\[ ccc(kl) = \text{Ast2} \]
if (bbb(j) - 0) > 52, 53, 53

53 \[ h1 = h1 + bbb(j) \]
\[ nn = j + 1 \]
\[ an1(nn) = a(j) \]
\[ an2(j) = ((a(j) - an1(j))/2.0 + an1(j)) \]
\[ \text{eps1}(j) = \text{eps}*(an2(j) - dn)/dn \]
if (eps1(j) - 0.0019) > 31, 32, 32

31 \[ \text{sigst}(j) = 200000*\text{eps1}(j) \]

go to 33

32 \[ \text{sigst}(j) = 370 \]

33 \[ z(j) = 30.2 - ((a(j) - an1(j))/2.0 + an1(j)) \]
\[ p2(j) = \text{sigst}(j) * bbb(j) \]
\[ \text{totp2} = p2(j) + \text{totp2} \]
\[ \text{fm2}(j) = \text{sigst}(j) * bbb(j) * z(j) \]
\[ \text{totfm2} = \text{fm2}(j) + \text{totfm2} \]

go to 2

52 \[ nn = j + 1 \]
\[ an1(nn) = a(j) \]

go to
35  \( q = a(j) - t - Rc \)
\[ v_2 = \sqrt{R^2 - q^2} \]
\[ z_2 = \text{Asin}(q/R) \]
\[ v_1 = \sqrt{Rc^2 - q^2} \]
\[ z_1 = \text{Asin}(q/Rc) \]
\[ A_{st1} = \frac{3.14 * R^2}{2.0} - Z_2 * R^2 - q * v_2 \]
\[ A_{stct} = \frac{3.14 * Rc^2}{2.0} - z_1 * Rc^2 - q * v_1 \]
\[ A_{st} = A_{st1} - A_{stct} \]
\[ bbb(j) = ccc(j) - A_{st} \]
\[ kl = j + 1 \]
\[ ccc(kl) = A_{st} \]
if (bbb(j) > 0) then
\[ h_1 = h_1 + bbb(j) \]
if (j == 111 - 1) then
\[ bbb(j) = A_{st} \]
else
ENDIF
if (bbb(j) <= 0.0) then
\[ nn = j + 1 \]
\[ an1(nn) = a(j) \]
\[ an2(j) = (a(j) - an1(j))/2.0 + an1(j) \]
\[ eps1(j) = eps*(an2(j) - dn)/dn \]
if (eps1(j) > 0.0019) then
\[ sigst(j) = 200000 * eps1(j) \]
go to 39
37  sigst(j) = 370
38  sigst(j) = 370
39  \( z(j) = ((a(j) - an1(j))/2.0 + an1(j)) - 30.2 \)
\[ p2(j) = sigst(j) * bbb(j) \]
\[ \text{totp3} = p2(j) + \text{totp3} \]
\[ fm2(j) = sigst(j) * bbb(j) * z(j) \]
\[ \text{totfm3} = fm2(j) + \text{totfm3} \]
if (j == 25) then go to 45
2 CONTINUE
45    fmtot=totfm1-totfm2+totfm3
     ptot=totp1-totp2-totp3
     write (*,86)fmtot,dn,ptot
86    format (2x,'fmtot=',f20.9,'dn=',f10.5,2x,'ptot=',f20.9)
     GO TO 43
42    do 60 j=1,ill
       hj=j
       a(j)=((d)/26.0)*hj
       if(a(j)-30.2)8,9,9
8    q=Rc+t-a(j)
    v1=sqrt(Rc**2-q**2)
    z1=Asin(q/Rc)
    Ac2= (3.14*Rc**2)/2.0-z1*Rc**2-v1*q
    b(j)=Ac2-c(j)
    nn=j+1
    c(nn)=Ac2
    hh=hh+b(j)
    v2=sqrt(R**2-q**2)
    z2=Asin(q/R)
    Asc=(3.14*R**2)/2.0-z2*R**2-q*v2
    Asc2=Asc-Ac2
    dsc2=(ddsc-ddcc)/Asc2-q
    bb(j)=Asc2-cc(j)
    ii=j+1
    cc(ii)=Asc2
    hhh=bb(j)+hhh
    nn=j+1
    an1(nn)=a(j)
    an2(j)=((a(j)-an1(j))/2.0+an1(j))
    eps1(j)=eps*(dn-an2(j))/dn
    if (eps1(j)<0.002)93,93,94
93    sigc(j)=0.85*71*(2*(eps1(j)/0.002)-(eps1(j)/0.002)**2)
go to 95

95 if (eps1(j)-0.0019)63,64,64

63 sigst(j)=200000*eps1(j)
go to 65

64 sigst(j)=370

65 z(j)=30.2-((a(j)-an1(j))/2.0+an1(j))
p1(j)=sigc(j)*b(j)+sigst(j)*bb(j)
totp1=p1(j)+totp1

fm1(j)=sigc(j)*b(j)*z(j)+sigst(j)*bb(j)*z(j)
totfm1=fm1(j)+totfm1

go to 60

9 if (a(j)-dn)67,68,68

67 q=a(j)-t-Rc
v1=sqrt (Rc**2.0-q**2.0)
z1=Asin(q/Rc)

Ac= (3.14*Rc**2)/2.0+z1*Rc**2+v1*q
b(j)=Ac-c(j)

mm=mm+1

cc(mm)=Ac
hh=hh+b(j)

v2=sqrt (R**2.0-q**2.0)
z2=Asin(q/R)

Asc=(3.14*R**2)/2.0+z2*R**2+q*v2

Ascc=Asc-Ac

bb(j)=ASCC-CC(J)

K=J+1

CC(K)=ASCC

hhh=bb(j)+hhh

nn=nn+1

an1(nn)=a(j)

an2(j)=((a(j)-an1(j))/2.0+an1(j))

eps1(j)=eps*(dn-an2(j))/dn
if(eps1(j)-0.002)90,90,91
90   sigc(j)=0.85*71*(2*(eps1(j)/0.002)-(eps1(j)/0.002))
go to 92
92   if (eps1(j)-0.0019)76,77,77
76   sigst(j)=200000*eps1(j)
go to 78
77   sigst(j)=370
78   z(j)=((a(j)-an1(j))/2.0+an1(j))-30.2
   pl(j)=sigc(j)*b(j)+sigst(j)*bb(j)
totp2=pl(j)+totp2
   fm1(j)=sigc(j)*b(j)*z(j)+sigst(j)*bb(j)*z(j)
totfm2=fm1(j)+totfm2
68   q=a(j)-t-Rc
   v2=sqrt (R**2.0-q**2.0)
z2=Asin(q/R)
v1=sqrt (Rc**2.0-q**2.0)
z1=Asin(q/Rc)
   Ast1=((3.14*R**2)/2.0-Z2*R**2-q*v2)
   Asct=((3.14*Rc**2)/2.0-z1*Rc**2-q*v1)
   Ast=Ast1-Asct
   bbb(j)=ccc(j)-Ast
   kl=j+1
   ccc(kl)=Ast
   if (j .eq. lll-1) then
      bbb(j)=Ast
   else
      ENDIF
if (bbb(j) - 0.0) 60,60,70
70   h1=h1+bbb(j)
nn=j+1
   an1(nn)=a(j)
an2(j)=((a(j)-an1(j))/2.0+an1(j))
eps1(j) = eps*(an2(j)-dn)/dn
if (eps1(j)-0.0019) 72, 73, 73
72  sigst(j) = 200000*eps1(j)
go to 74
73  sigst(j) = 370
74  z(j) = ((a(j)-an1(j))/2.0+an1(j))-30.2
    p2(j) = sigst(j)*bbb(j)
totp3 = p2(j) + totp3
    fm2(j) = sigst(j)*bbb(j)*z(j)
totfm3 = fm2(j) + totfm3
if (j .eq. lll-1) go to 81
60  continue
81  fmtot = totfm1 - totfm2 + totfm3
    ptot = totp1 + totp2 - totp3
    write (*,85) fmtot, dn, ptot, eps
85  format (2x,'fmtot=', f20.9, ',dn=', f10.5, ',ptot=', f20.9, ',eps=', f20.9)
43  return
end
APPENDIX II EXECUTIVE CONTROL DECK, CASE CONTROL DECK AND BULK DATA DECK IN NASTRAN PACKAGE

$ Executive Control Deck Follows:

ID MSC/N
SOL NLSTATIC
TIME 10000
CEND

$ Case Control Deck Follows:

DISPLACEMENT = ALL
SPC_FORCE = ALL
OLOAD = ALL
MPCFORCE = ALL
FORCE = ALL
STRESS = ALL
STRAIN = ALL
SPC = 1
SUBCASE 1
LOAD = 1
NLPARM = 1
SUBCASE 2
LOAD = 2
NLPARM = 2
SUBCASE 3
LOAD = 3
NLPARM = 3
SUBCASE 4
LOAD = 4
NLPARM = 4
SUBCASE 5
LOAD = 5
NLPARM = 5
SUBCASE 6
LOAD = 6
NLPARM = 6
SUBCASE 7
LOAD = 7
NLPARM = 7
SUBCASE 8
LOAD = 8
NLPARM = 8
SUBCASE 9
LOAD = 9
NLPARM = 9
SUBCASE 10
LOAD = 10
NLPARM = 10
SUBCASE 11
LOAD = 11
NLPARM = 11
SUBCASE 12
LOAD = 12
NLPARM = 12

BEGIN BULK (the following data is based on material properties which was used in finite element)

$ Property 1 : plate
PSHELL  1 1 0.25 1 1 0.+PR  1
+PR  1 -0.125 0.125

$ Property 2 : concrete
PSOLID 2 1 0 0
$ Property 3: steel1
PSOLID 3 2 0 0
$ Property 4: steel2
PSOLID 4 3 0 0
$ Property 5: gap
PGAP 5 0.4 0. 1.1E+9 110. 7.E+8 0.6 0.6
$ Material 1: concrete
MATS1 1 1 NLELAST
$ Function 1: concrete
TABLES1 1 +
+ -3.4E-3 -6.93 -2.2E-3 -58.84 -1.8E-3 -56.07 -1.6E-3 -52.7+
+ -1.4E-3 -48.4 -1.2E-3 -43.26 -1.E-3 -37.38 -8.E-4 -30.87+
+ -6.E-4 -23.8 -4.E-4 -16.24 -2.E-4 -8.32 0. 0.+
MAT1 1 42000. 0.23 0. 0. 0. +MT 1
+MT 1 58.84
$ Material 2: steel1
MATS1 2 PLASTIC 30000. 1 1 370.
MAT1 2 200000. 0.3 0. 0. 0. +MT 2
+MT 2 370. 370.
$ Material 3: steel2
MATS1 3 2 NLELAST
$ Function 2: steel
TABLES1 2 +
+ -0.014 -400. -6.E-3 -350. -3.2E-3 -300. -1.2E-3 -200.+
+ 0. 0. 0.0012 200. 0.0032 300. 0.006 350.+
+ 0.014 400.ENDT
MAT1 3 200000. 0.3 0. 0. 0. +MT 3
+MT 3 370. 370.
86603 0.497660.048314